Scale Space

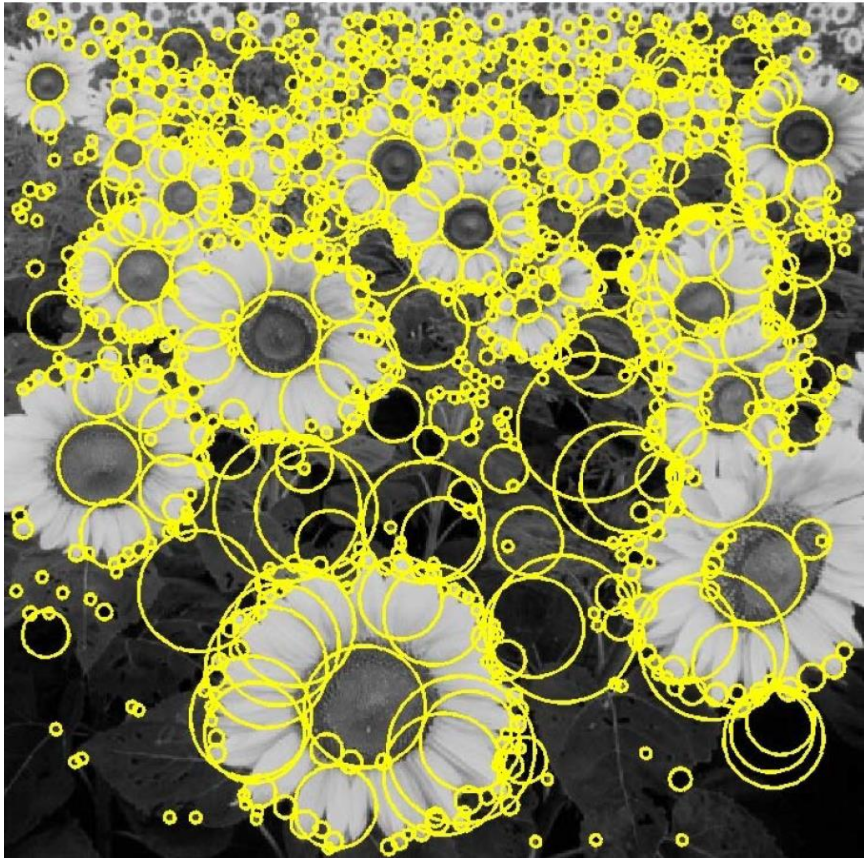
Hyun Soo Park

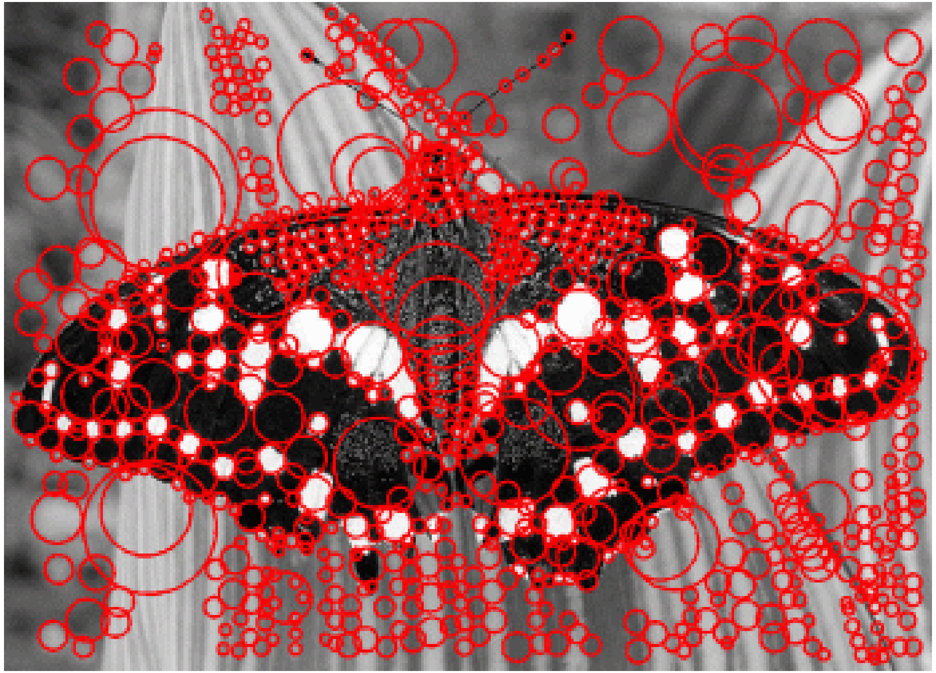
Template

Template

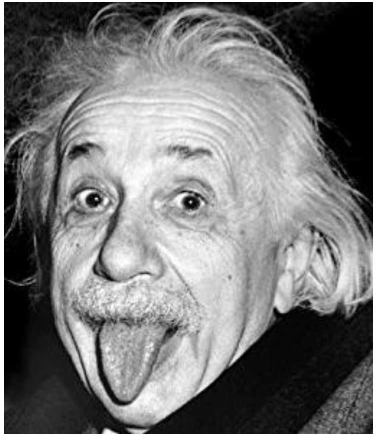
Scale Invariant Image Representation

Blob Detection ~ Scale Selection





Recall: Image Differentiation



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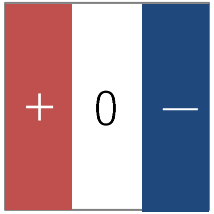

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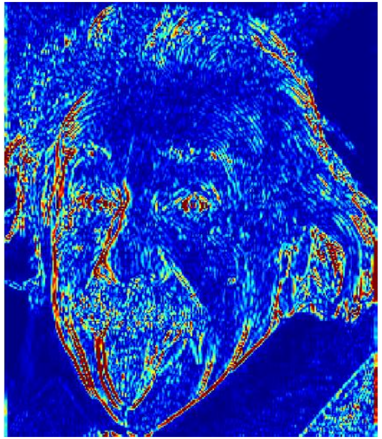
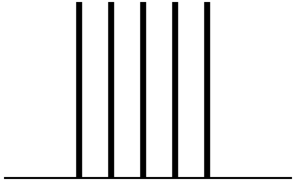


$$I \otimes z = \frac{\partial I}{\partial u}$$



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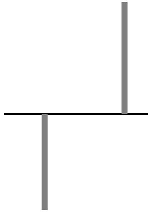

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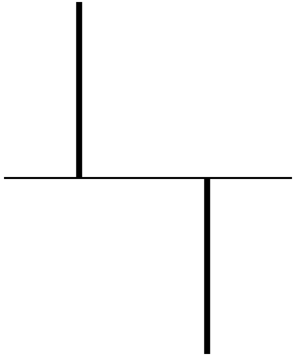
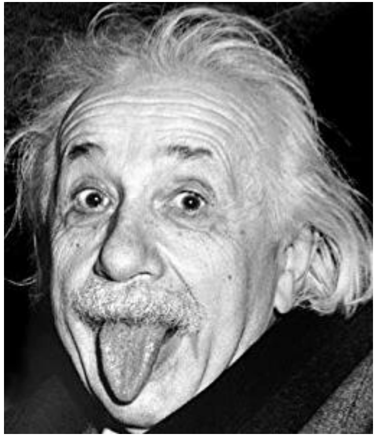


Recall: Sobel Filter



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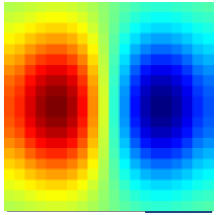
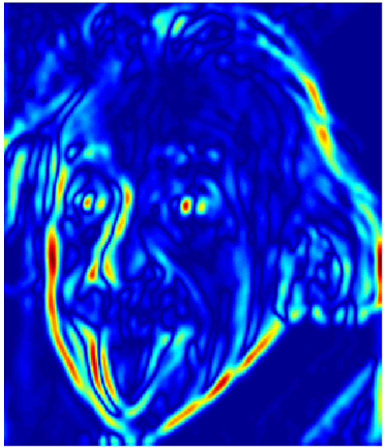
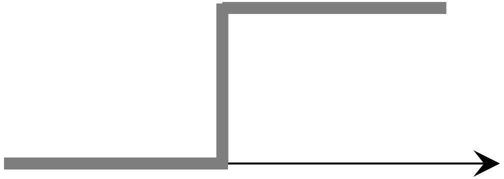


$$\frac{\partial G}{\partial u} = \frac{G(u+h) - G(u-h)}{2h}$$

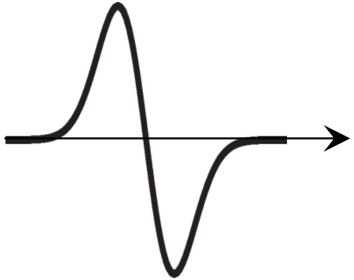
=





Edge

*



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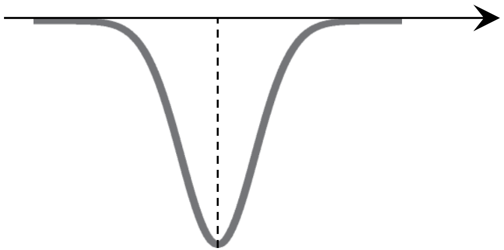
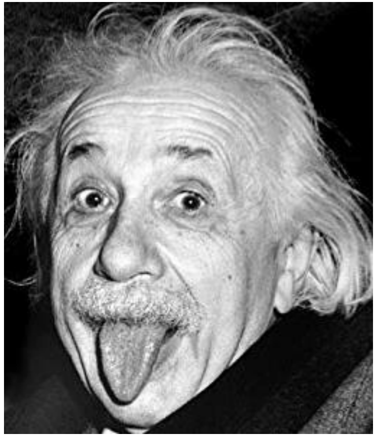


Recall: Sobel Filter



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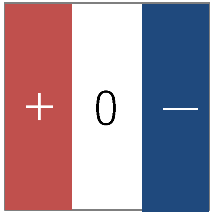

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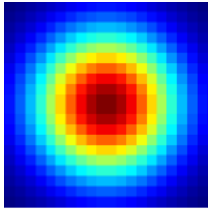

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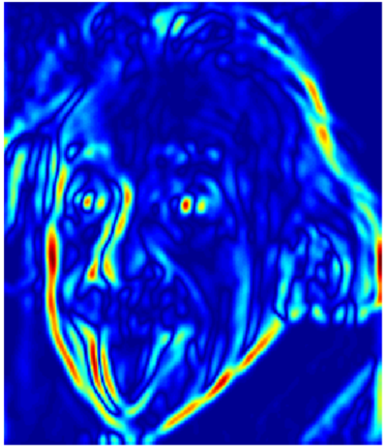
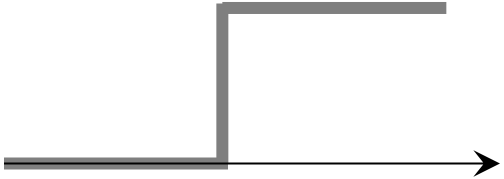
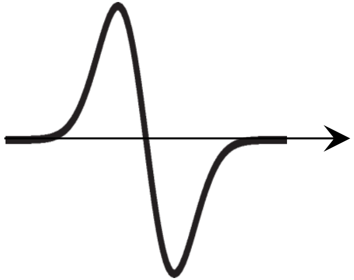


$$\frac{\partial G}{\partial u} = \frac{G(u+h) - G(u-h)}{2h}$$



Edge

$*$



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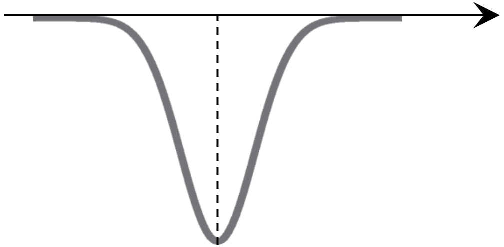
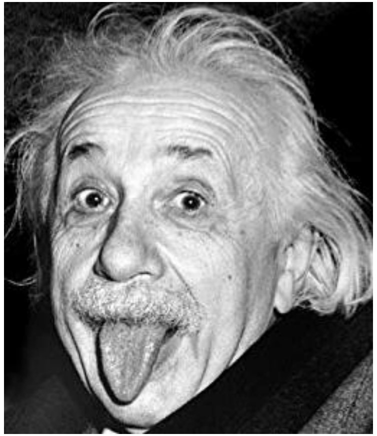
Max at edge



Recall: Image Sharpening



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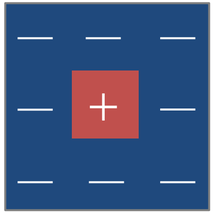

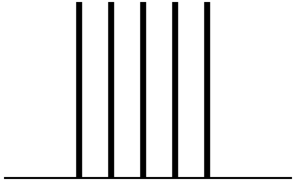

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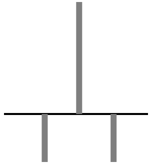
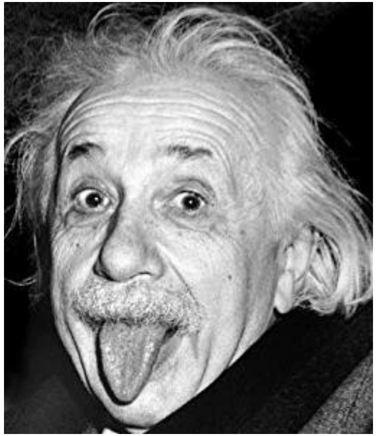


Recall: Image Sharpening



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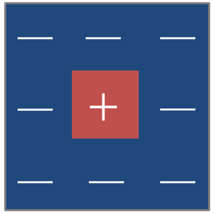

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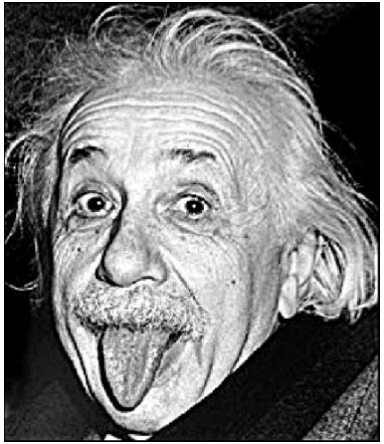
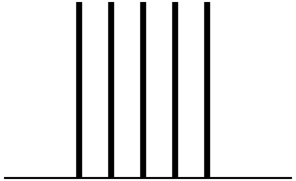


Zero crossing at the edge



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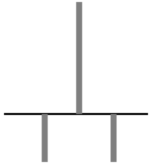

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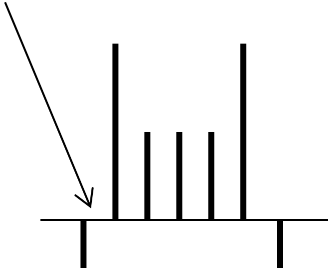
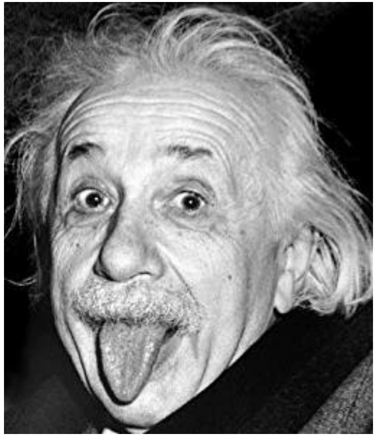


LAPLACIAN



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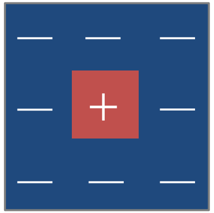

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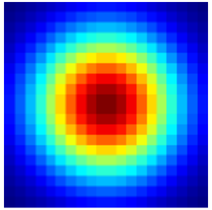

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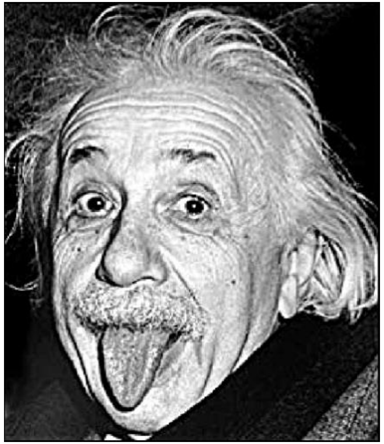
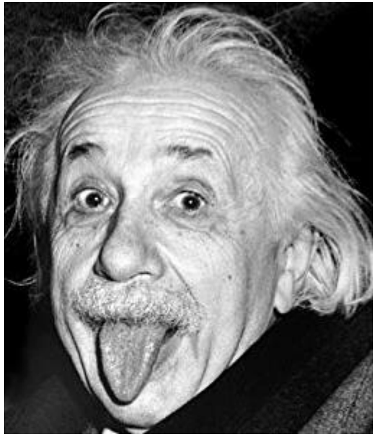


$$\frac{G(u+h)+G(u-h)-2G(u)}{h}$$

Laplacian



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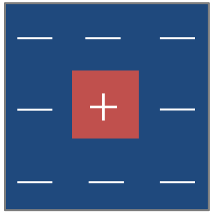

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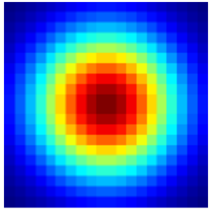

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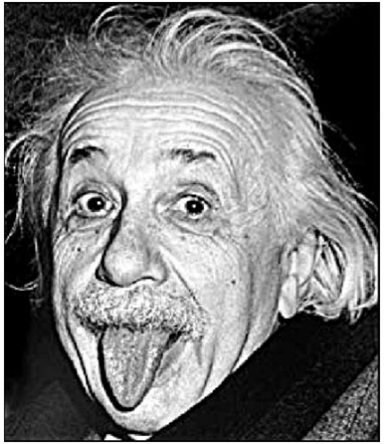
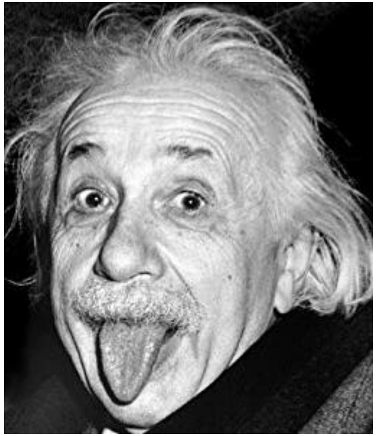


$$\frac{G(u+h)-G(u)}{h}-\frac{G(u)-G(u-h)}{h}$$

LAPLACIAN



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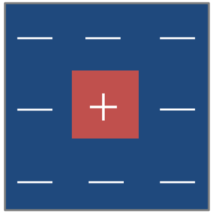

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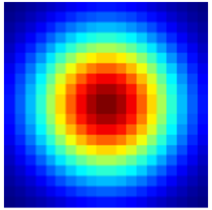

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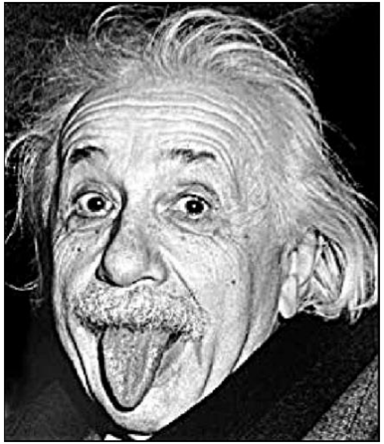
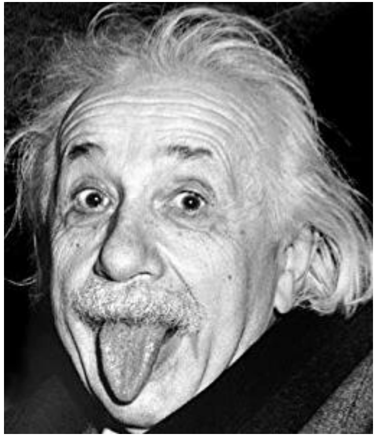


$$\lim_{h \to 0}\left(\frac{G(u+h)-G(u)}{h}-\frac{G(u)-G(u-h)}{h}\right)/\,h$$

LAPLACIAN



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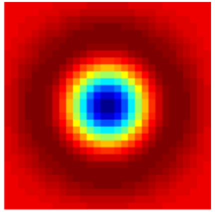

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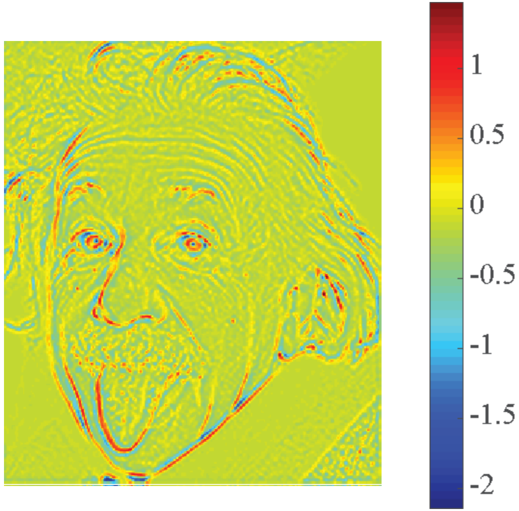
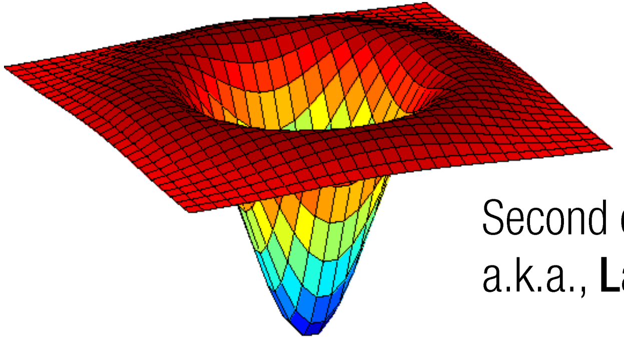
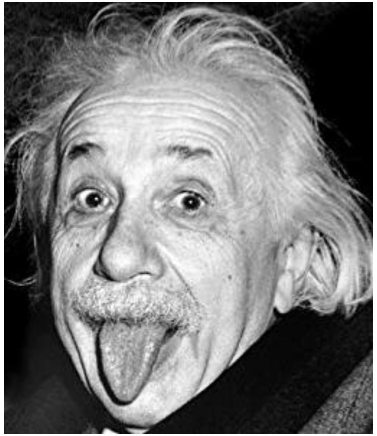


$$\nabla \cdot \nabla G = \nabla\left(\frac{\partial G}{\partial u}\mathbf{i} + \frac{\partial G}{\partial v}\mathbf{j}\right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2}$$



Second order derivative of Gaussian,
a.k.a., **Laplacian of Gaussian**

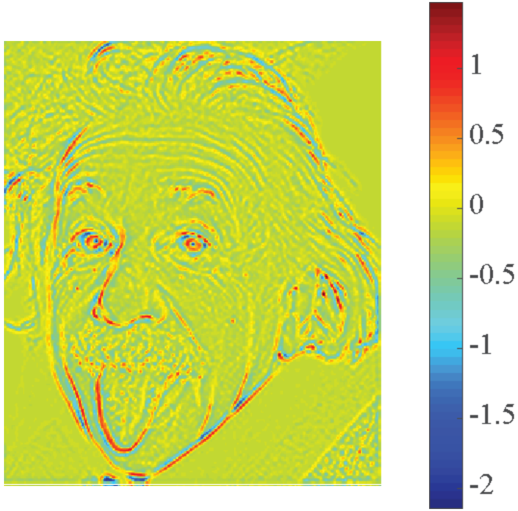
LAPLACIAN



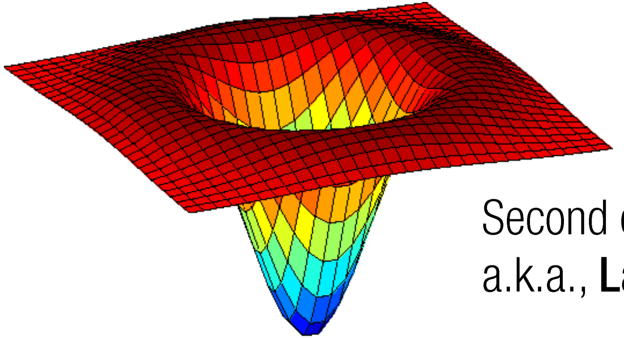
$*$

0	1	0
1	-4	1
0	1	0

$=$

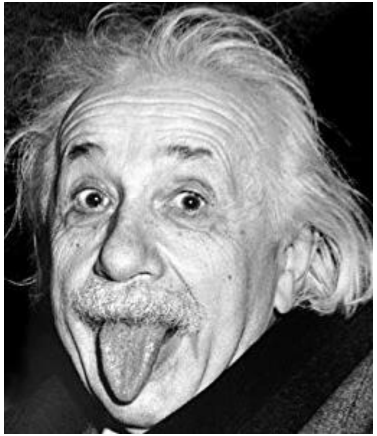


$$\nabla \cdot \nabla G = \nabla \left(\frac{\partial G}{\partial u}\mathbf{i} + \frac{\partial G}{\partial v}\mathbf{j} \right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2}$$



Second order derivative of Gaussian,
a.k.a., **Laplacian of Gaussian**

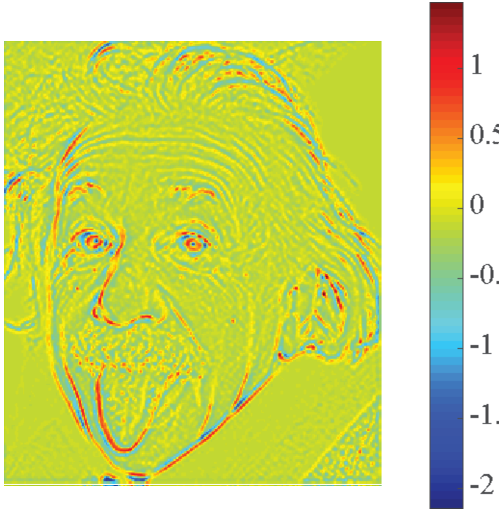
LAPLACIAN



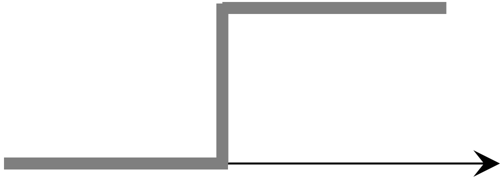
$*$

0	1	0
1	-4	1
0	1	0

$=$

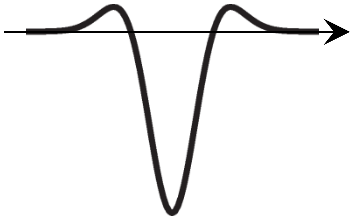


$$\nabla \cdot \nabla G = \nabla\left(\frac{\partial G}{\partial u}\mathbf{i} + \frac{\partial G}{\partial v}\mathbf{j}\right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2}$$



Edge

$*$



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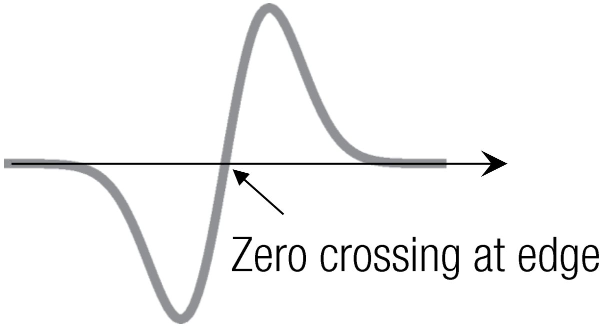
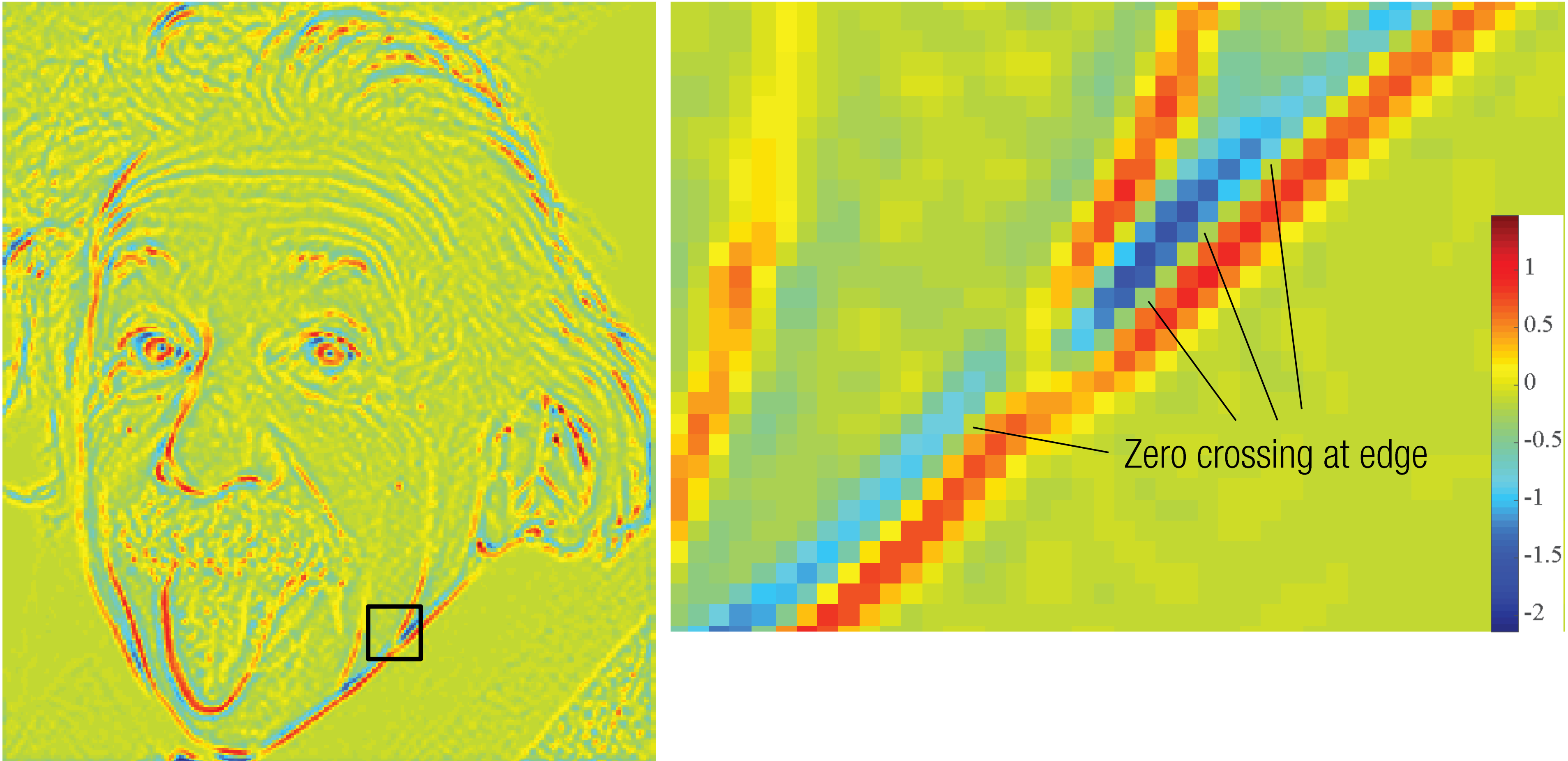
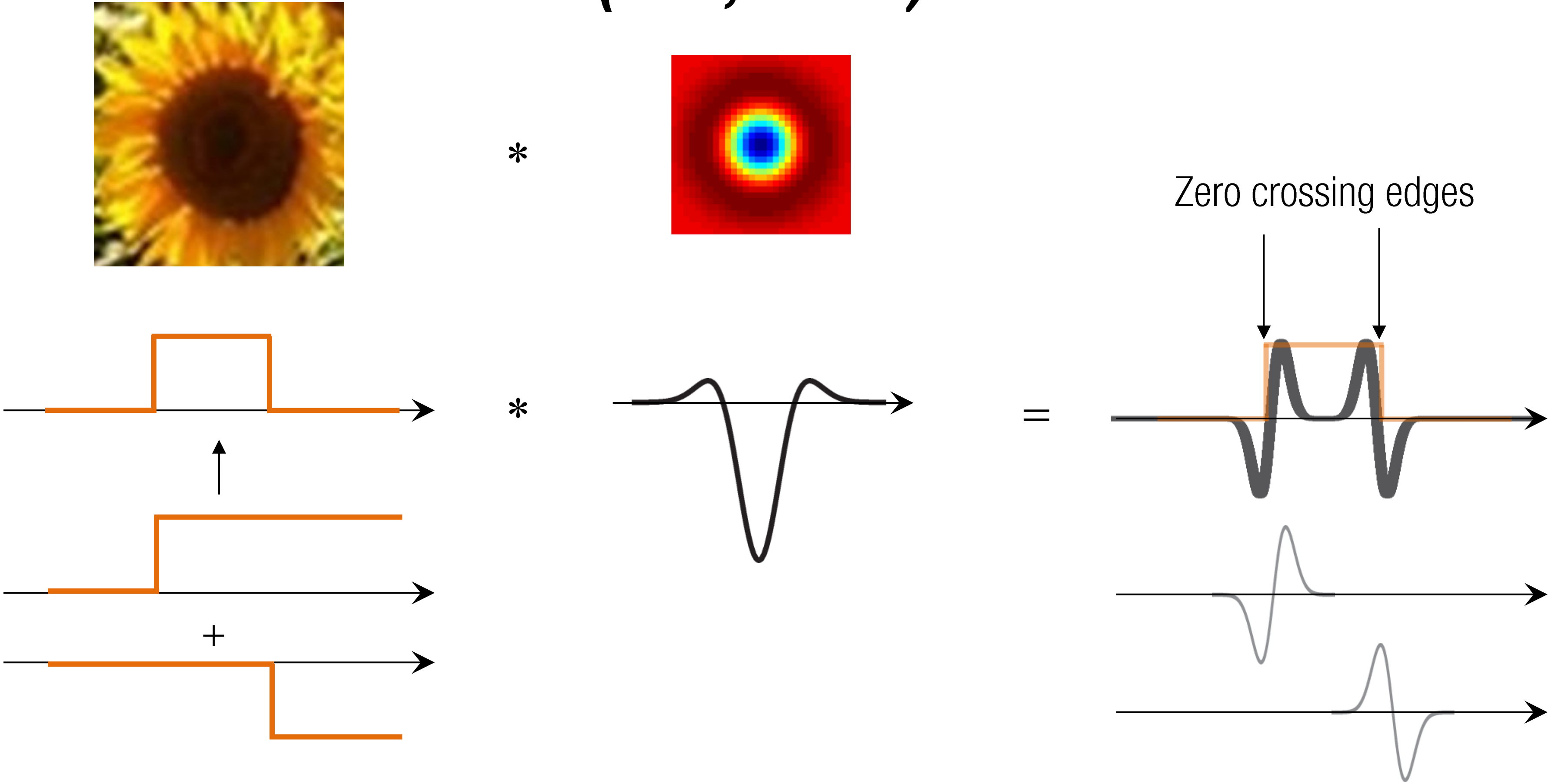


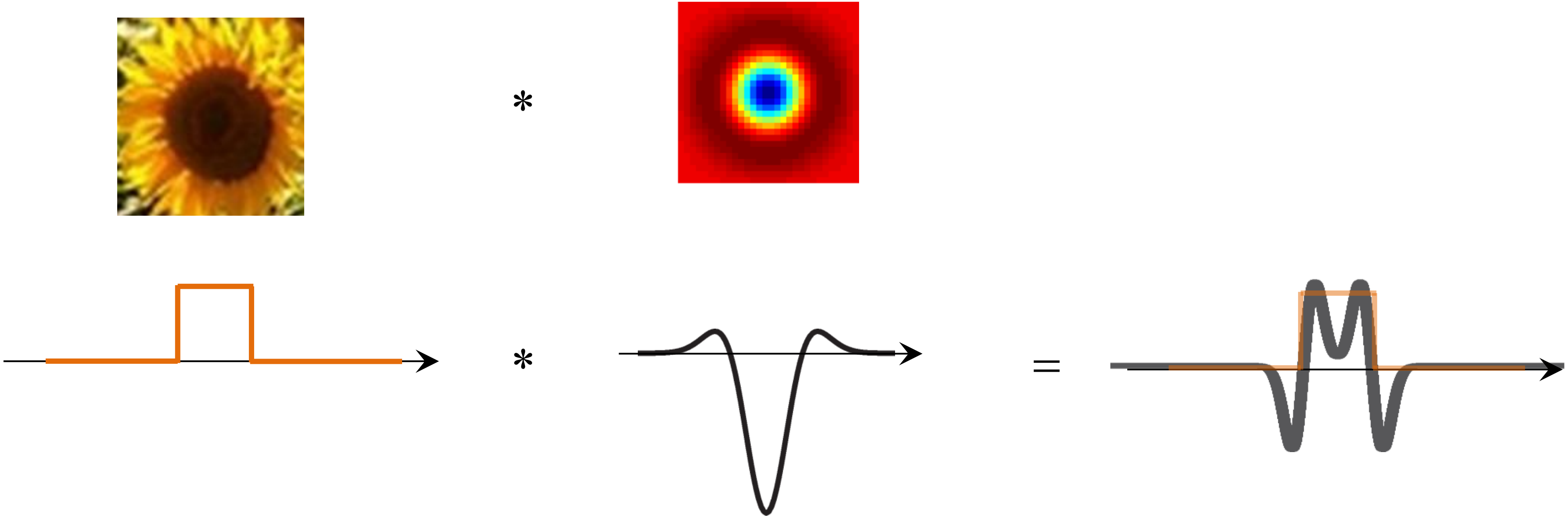
EDGE LOCALIZATION



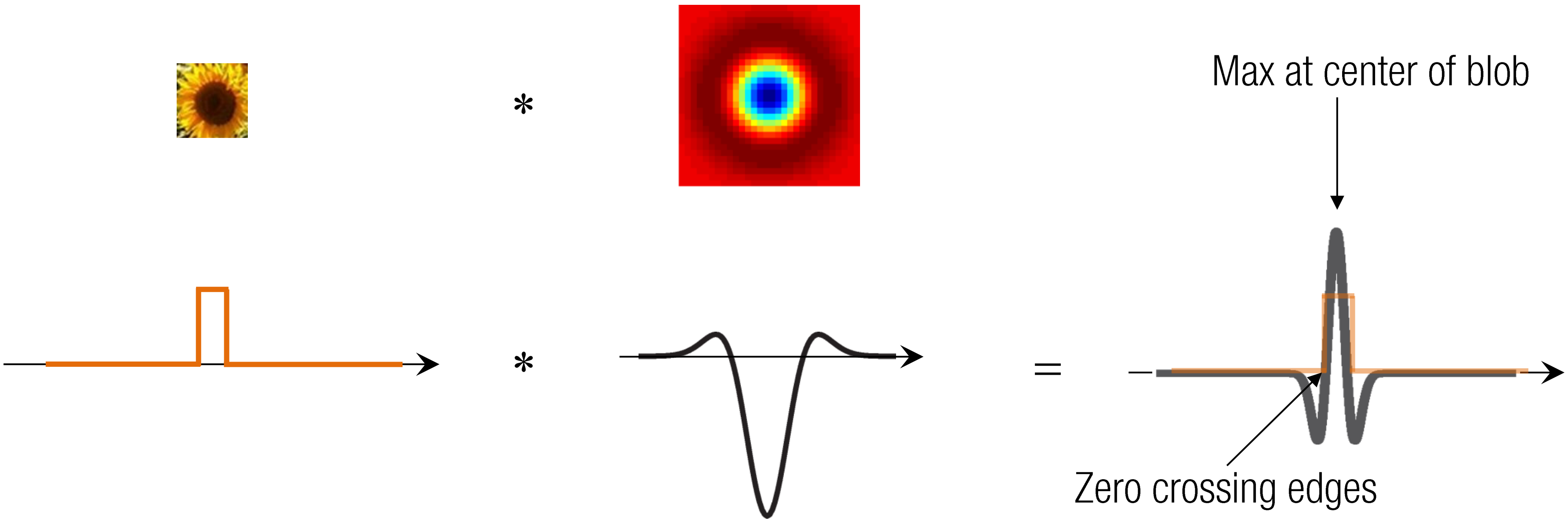
Structured Edges (e.g., Blob)



Structured Edges (e.g., Blob)

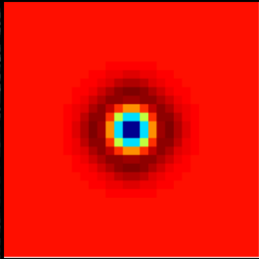


Structured Edges (e.g., Blob)



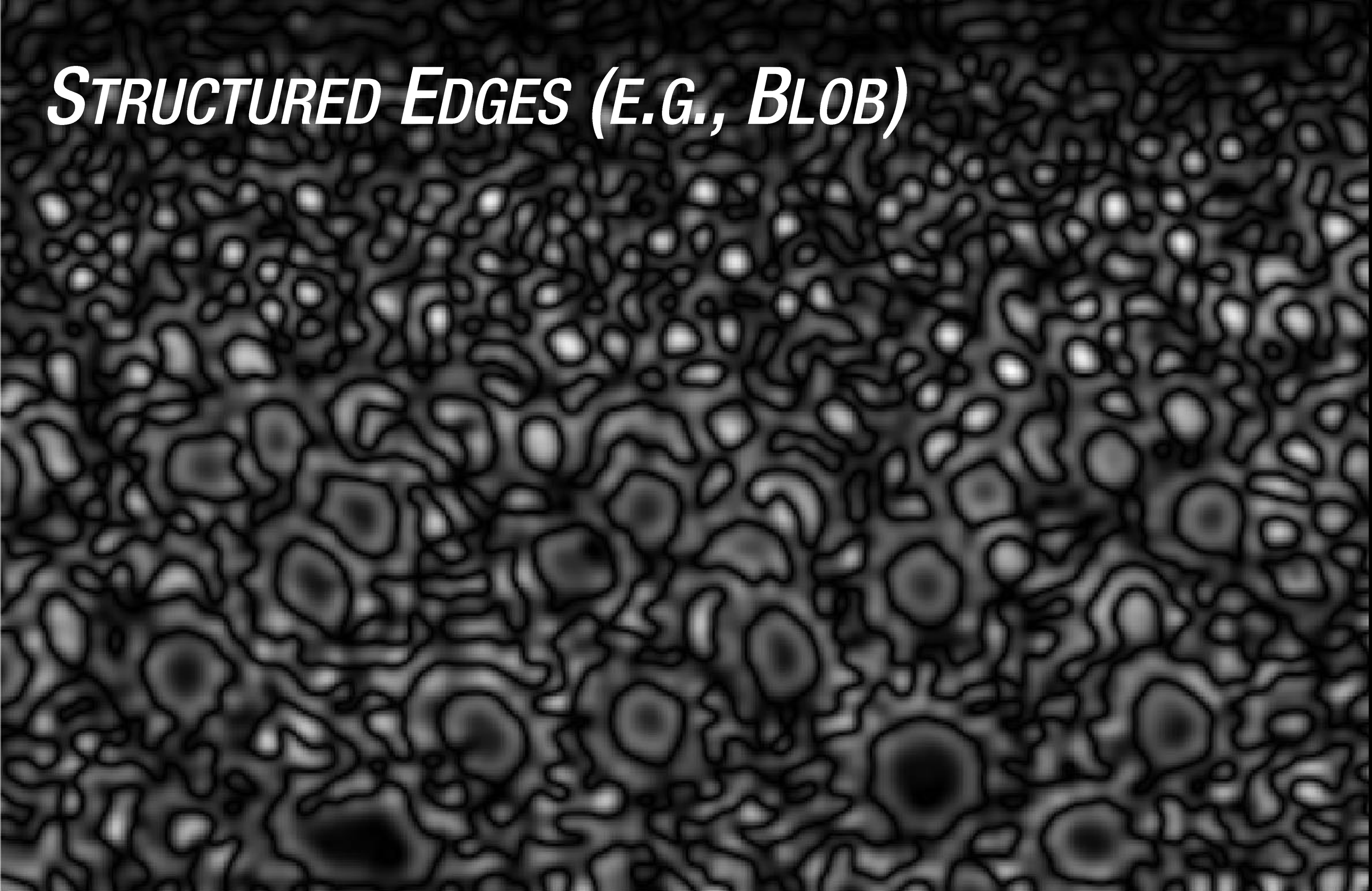
Structured Edges (e.g., Blob)

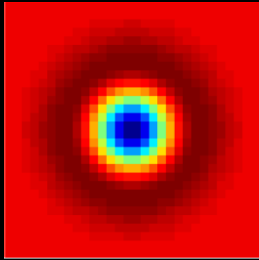
Structured Edges (e.g., Blob)



$\sigma = 1$

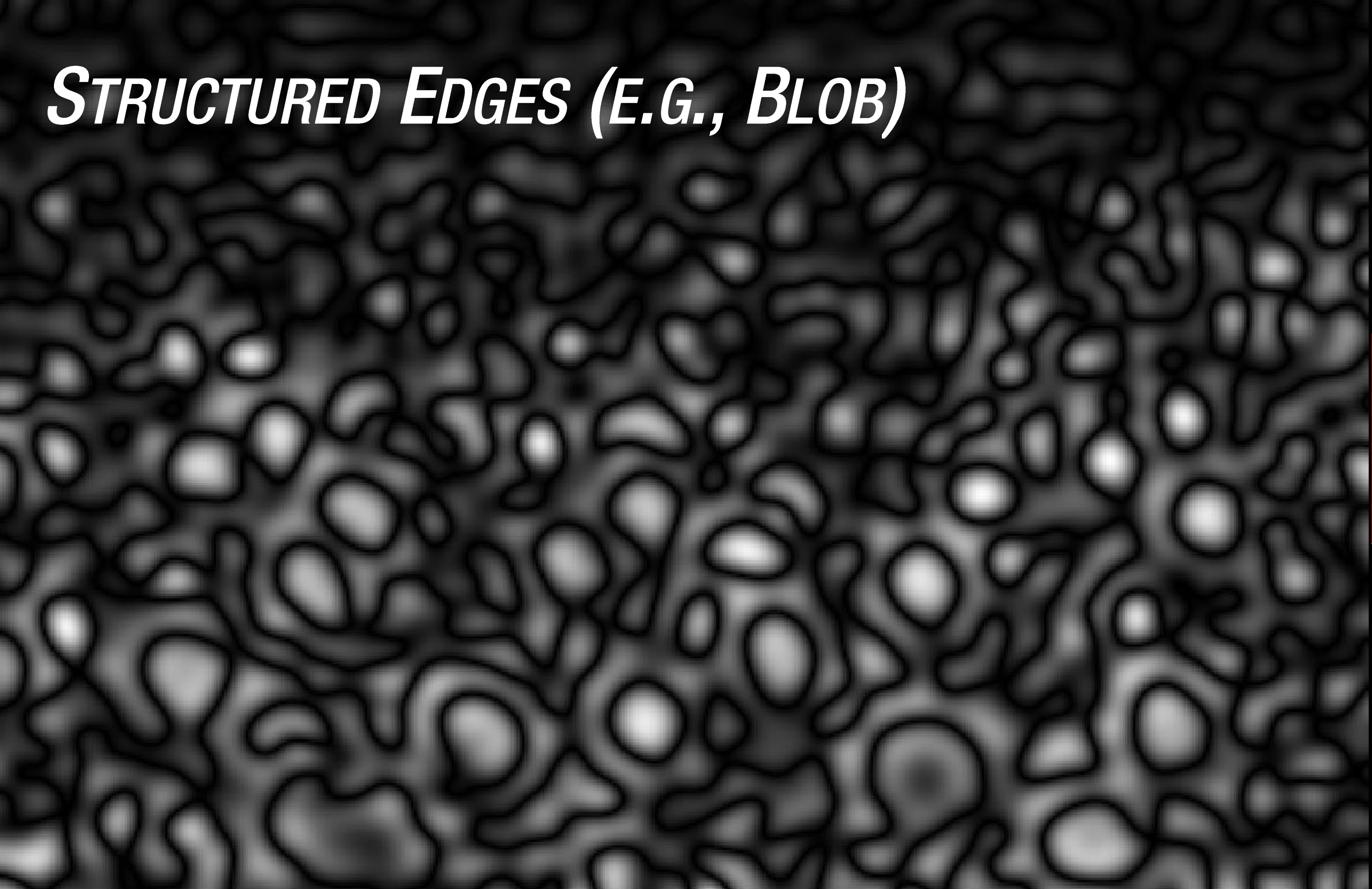
STRUCTURED EDGES (E.G., BLOB)

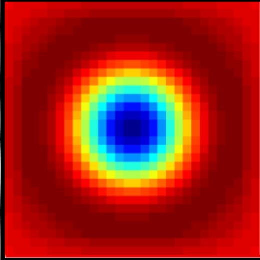




$\sigma = 4$

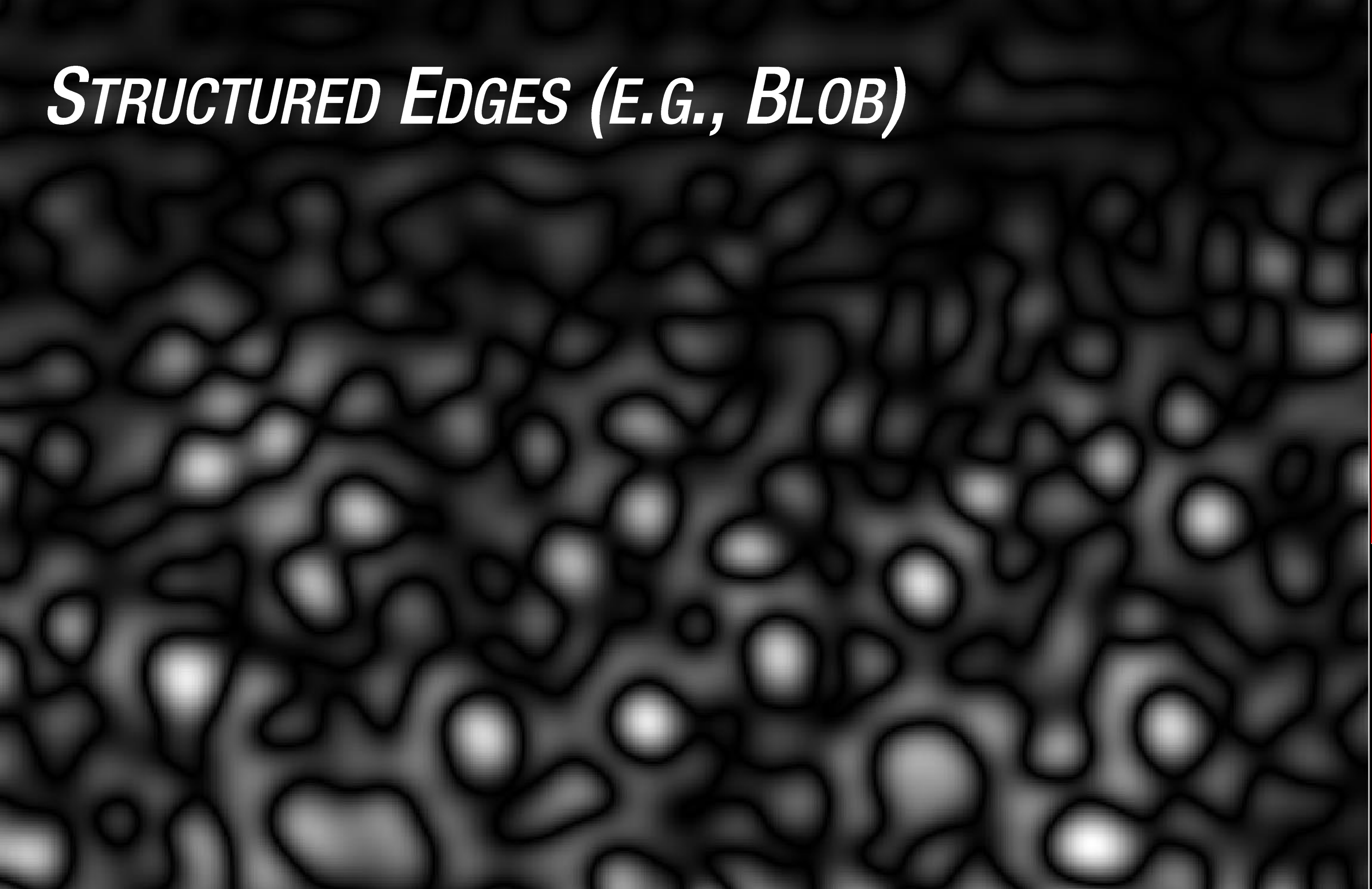
STRUCTURED EDGES (E.G., BLOB)

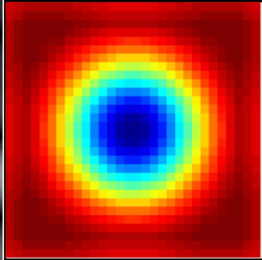




$\sigma = 7$

Structured Edges (e.g., Blob)



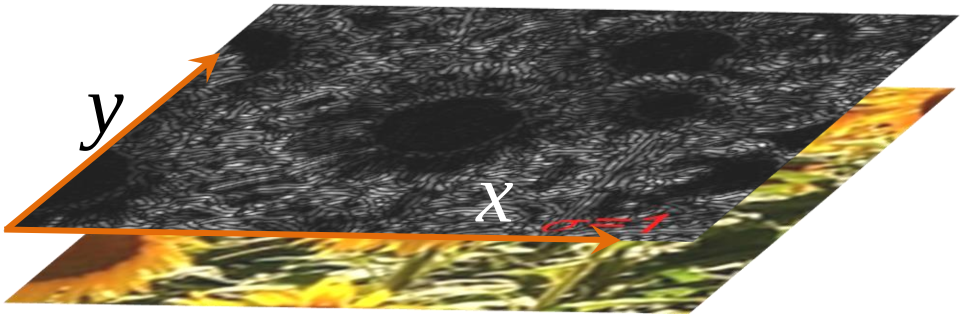


$\sigma = 10$

SCALE SPACE

$I(x, y)$

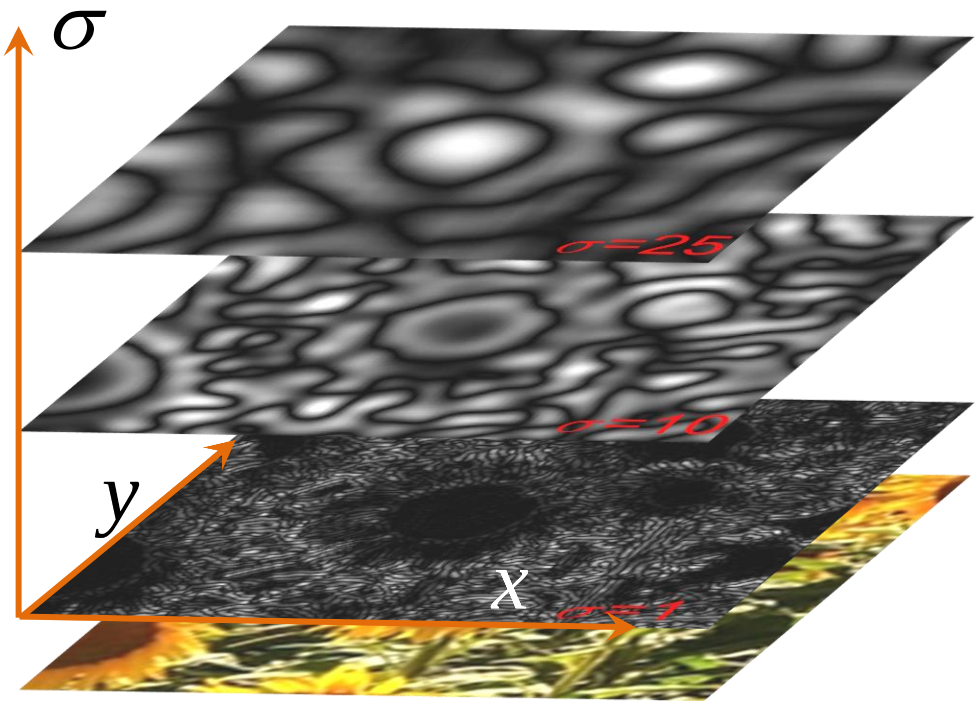
Scale Space



$$I(x,y) * \underline{L(\sigma)}$$

Laplacian operator

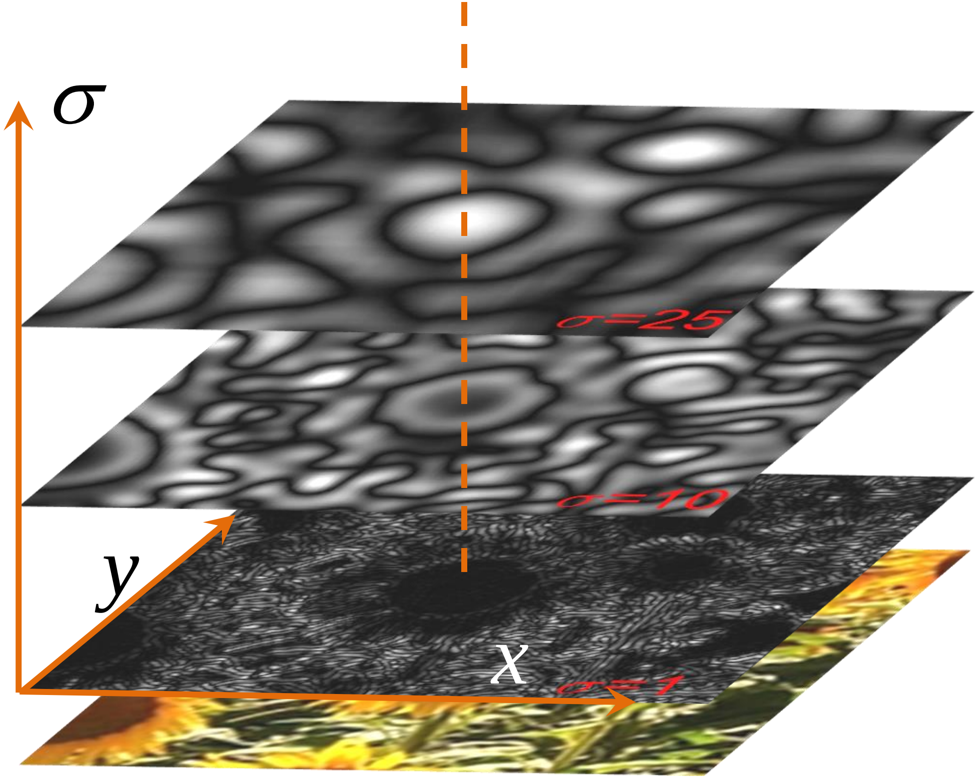
Scale Space



$$D(x,y,\sigma) = I(x,y) * L(\sigma)$$

Scale space response

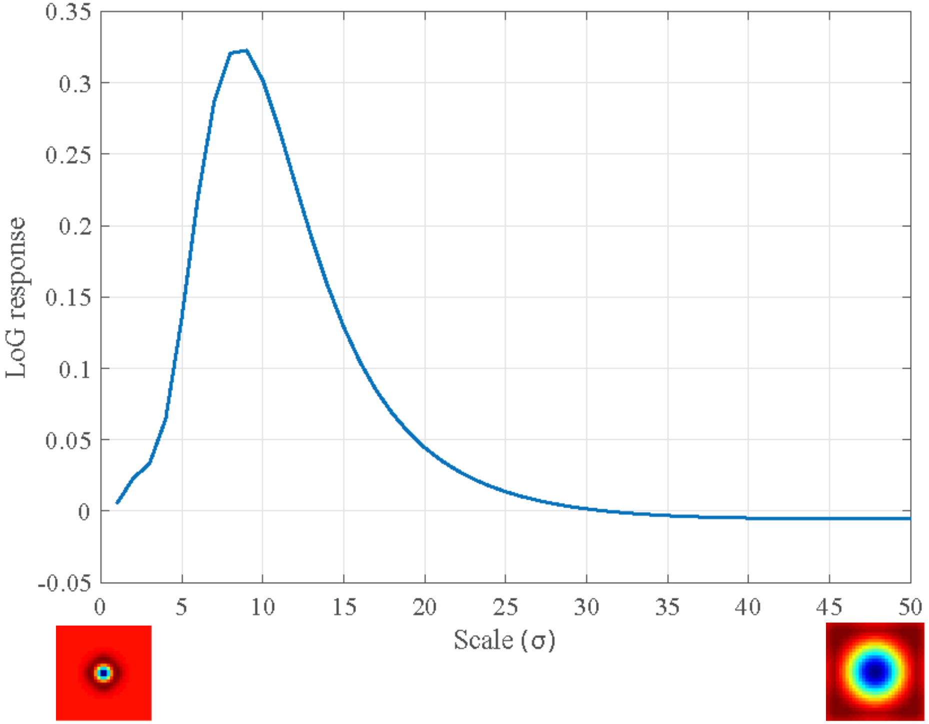
Scale Space



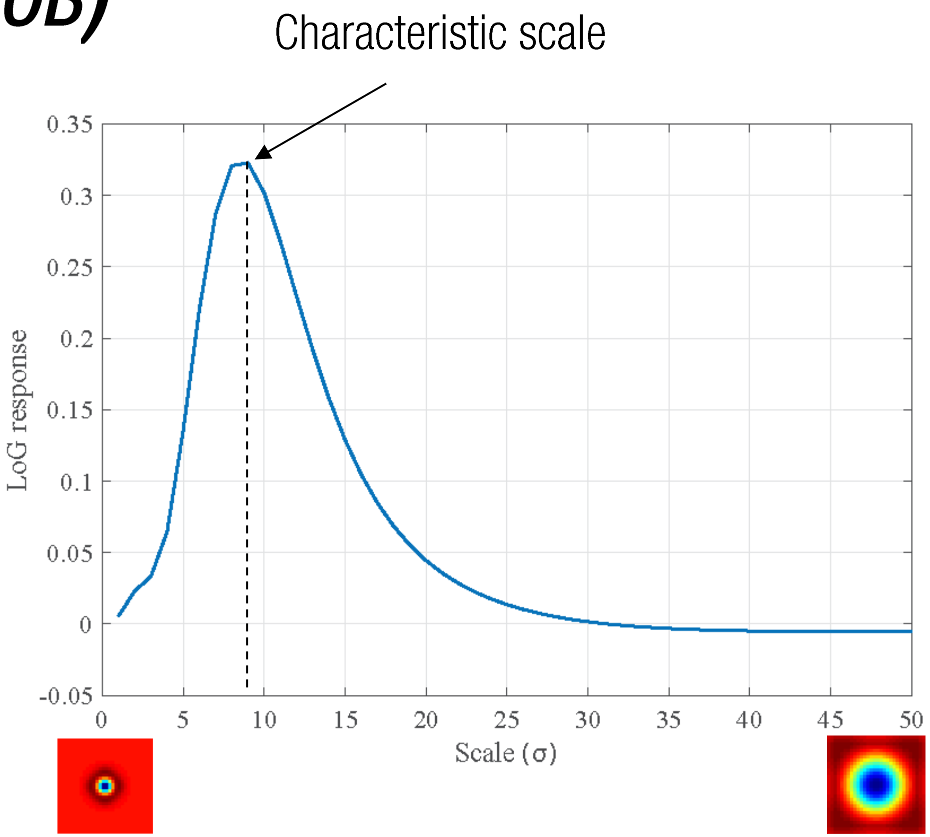
$$D(x,y,\sigma) = I(x,y) * L(\sigma)$$

Scale space response

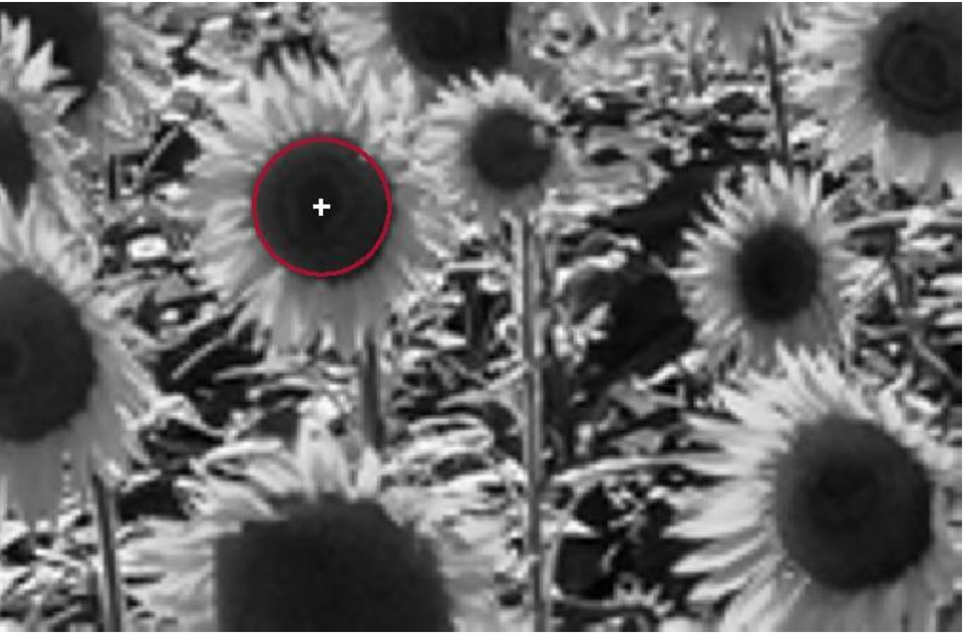
Structured Edges (e.g., Blob)



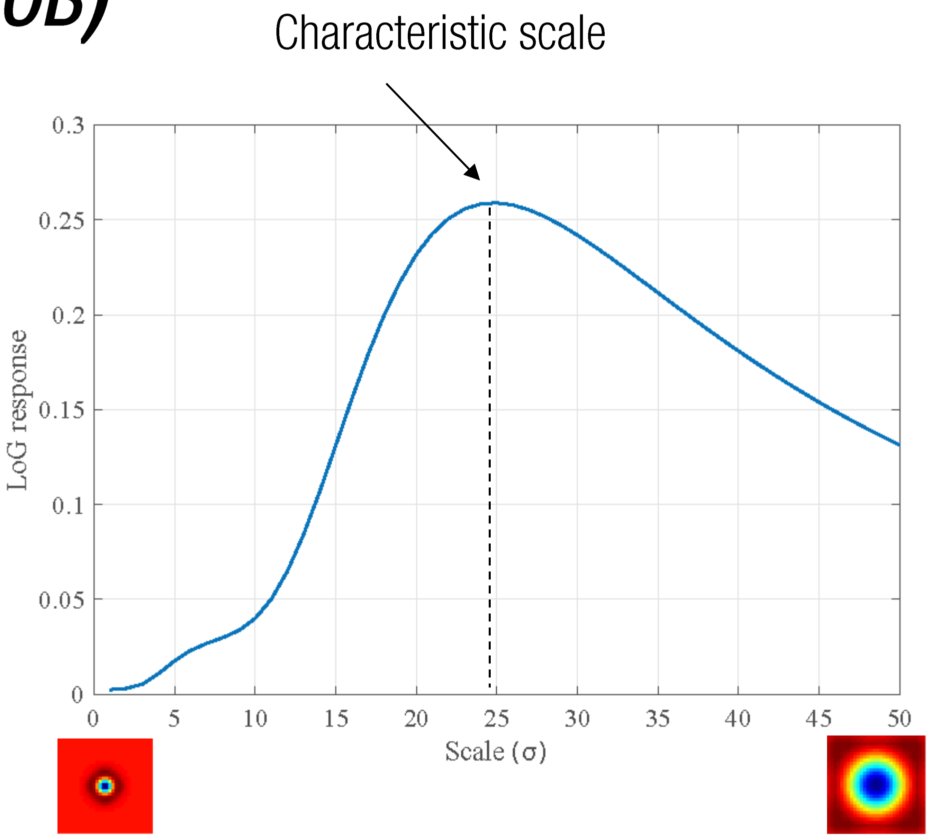
STRUCTURED EDGES (E.G., BLOB)



Structured Edges (e.g., Blob)

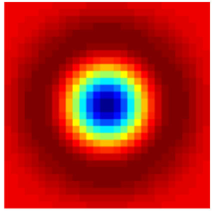


x3 bigger sunflower

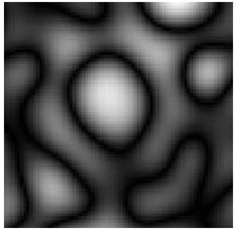


SCALE SELECTION

$*$



$=$

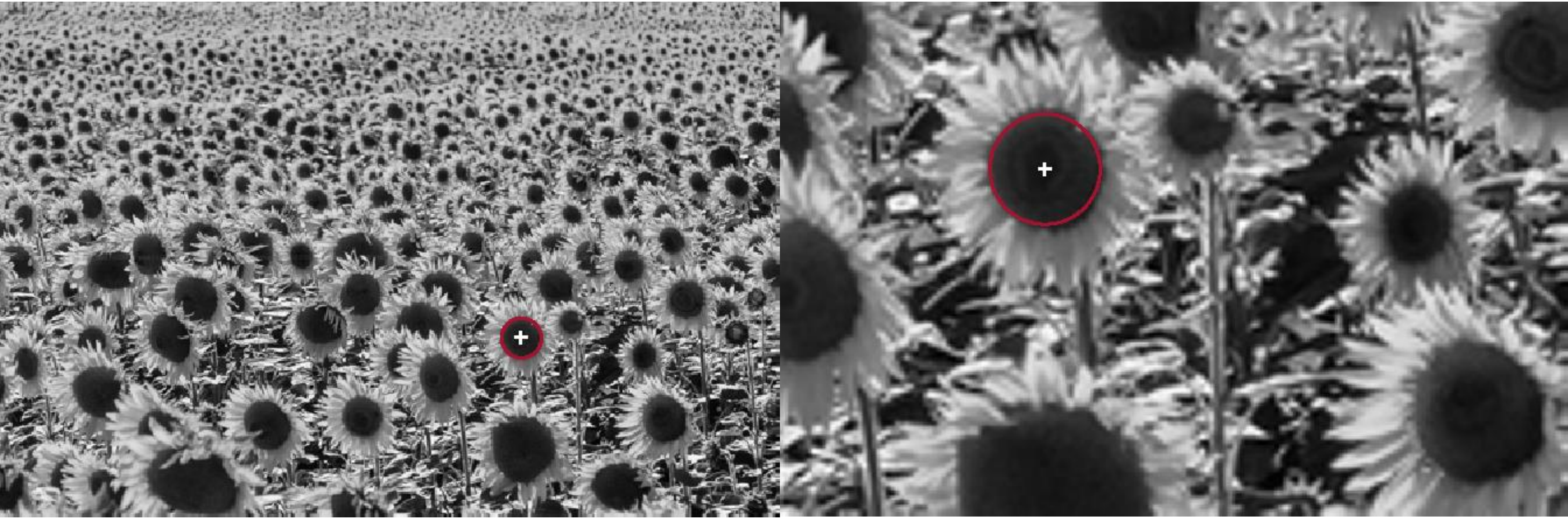


$$\sigma^2\left(\frac{\partial^2 G}{\partial u^2}+\frac{\partial^2 G}{\partial v^2}\right)$$

Normalization factor

- The scale response (laplacian) is maximized when $\sigma = \frac{r}{\sqrt{2}}$
- The characteristic scale can be used to normalize the image, resulting in *scale invariant* image description.

Scale Normalization



Scale Normalization