

SCALE SPACE

HYUN SOO PARK

Mercury

Venus

Earth

Mars

Jupiter

Saturn

Uranus

Neptune





Template



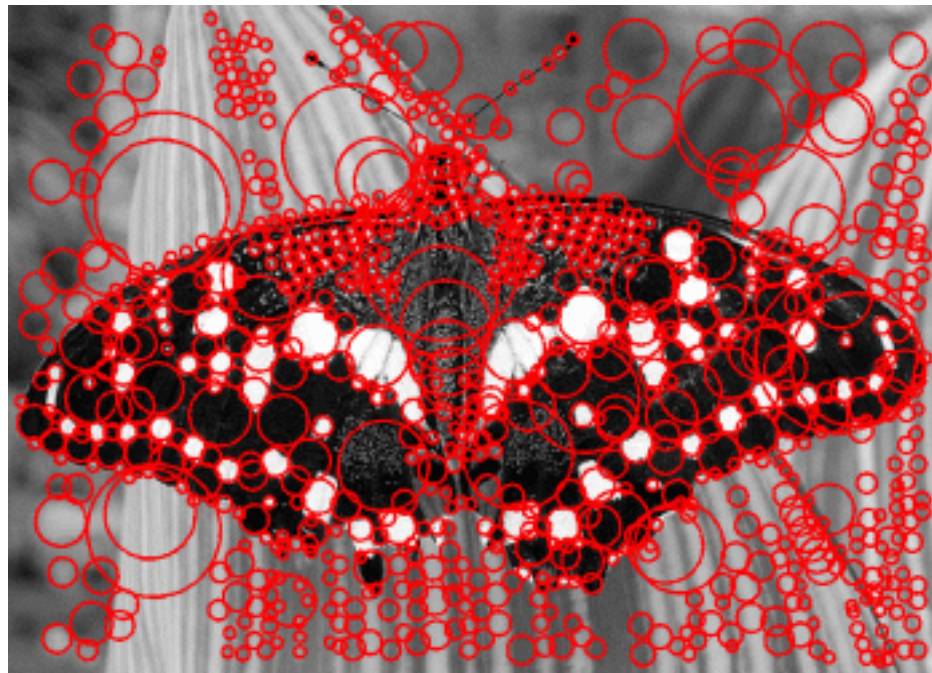
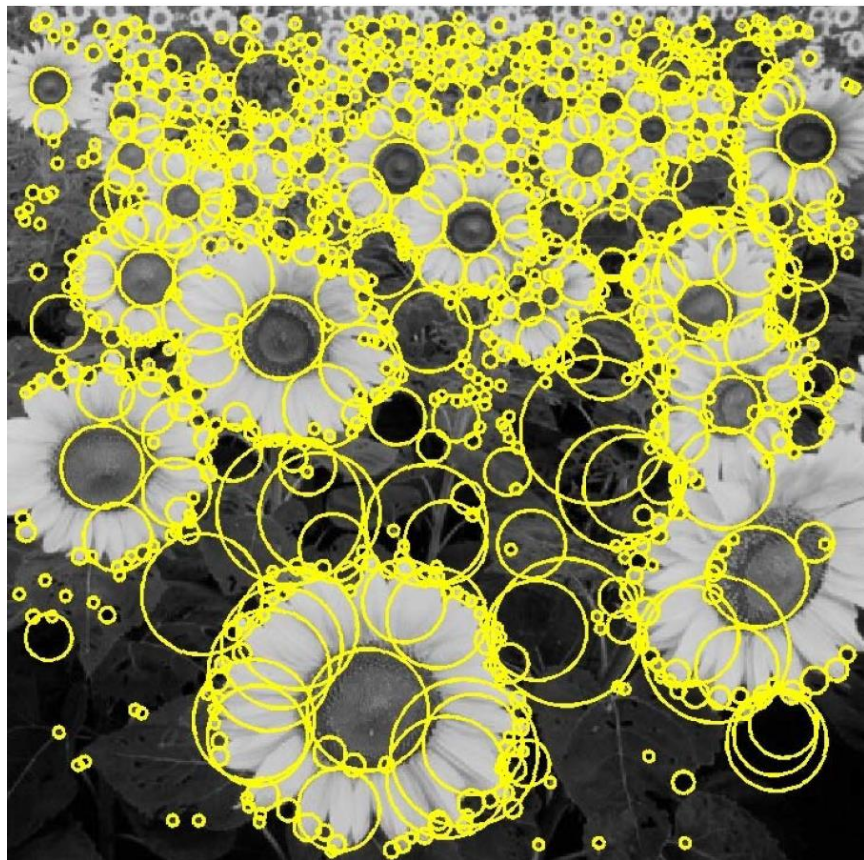
Template



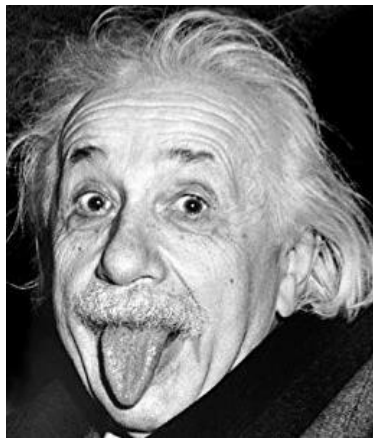
SCALE INVARIANT IMAGE REPRESENTATION



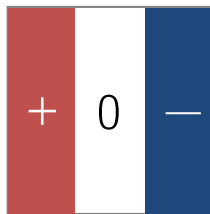
BLOB DETECTION ~ SCALE SELECTION



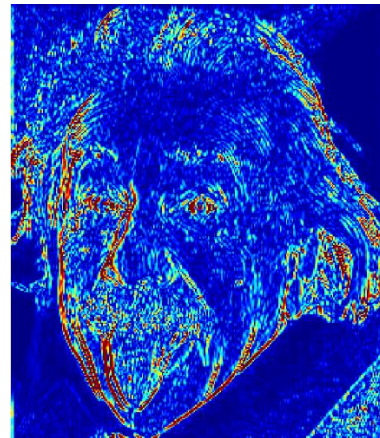
RECALL: IMAGE DIFFERENTIATION



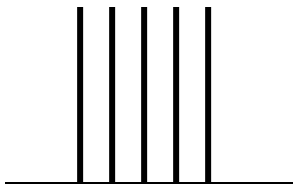
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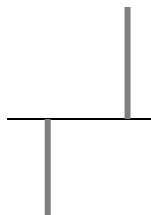
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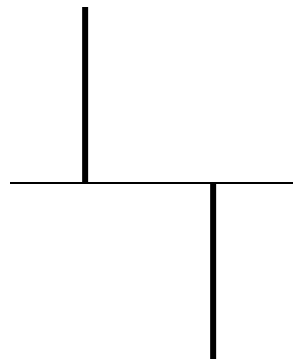
$$I \otimes z = \frac{\partial I}{\partial u}$$



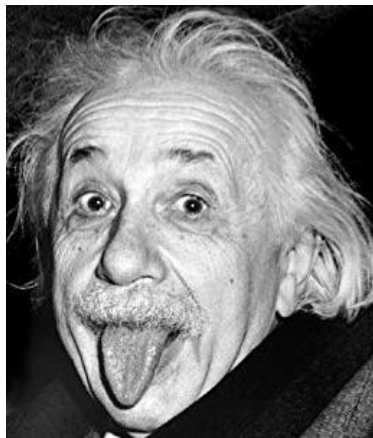
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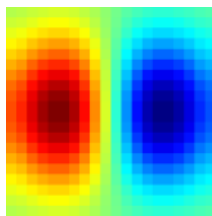
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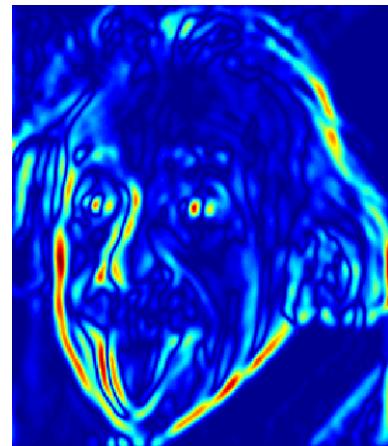
RECALL: SOBEL FILTER



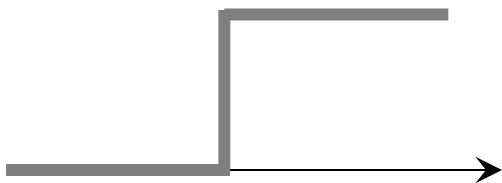
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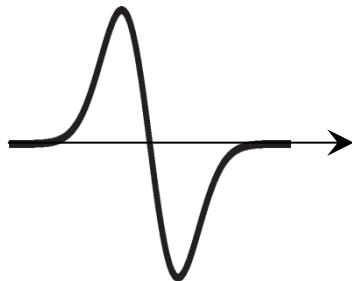


$$\frac{\partial G}{\partial u} = \frac{G(u+h) - G(u-h)}{2h}$$

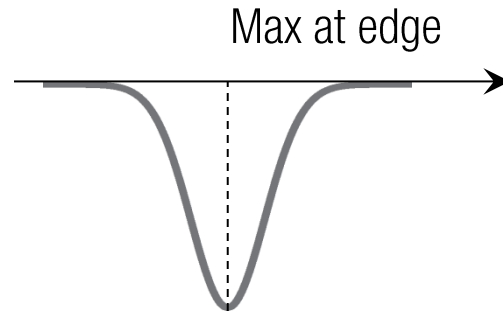


Edge

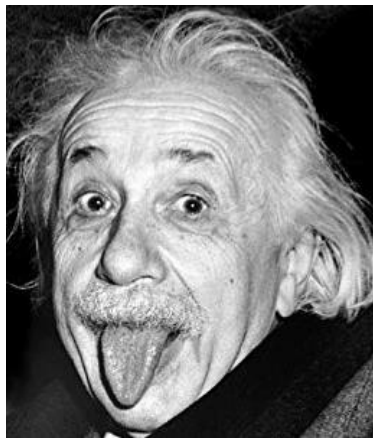
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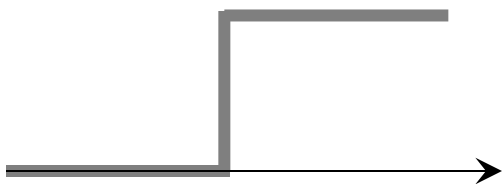
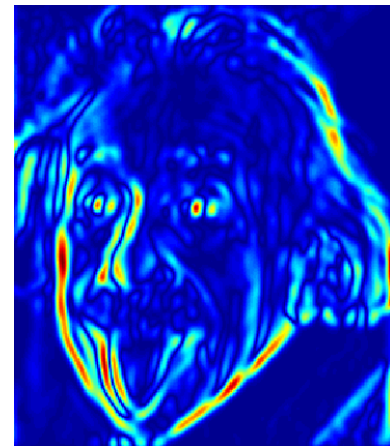


RECALL: SOBEL FILTER

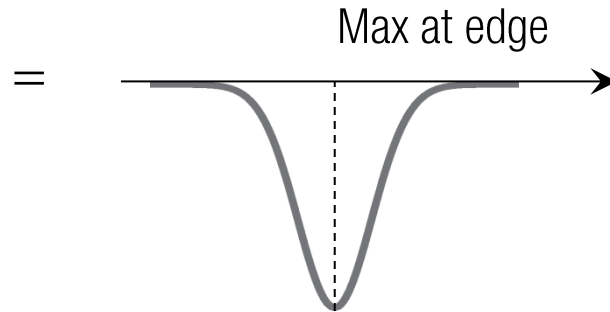
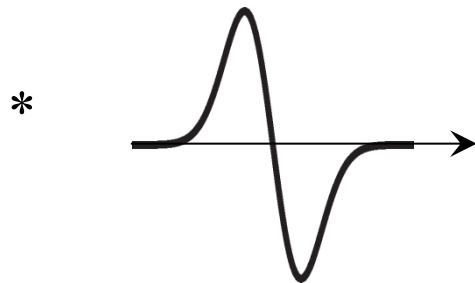


$$* \begin{array}{|c|c|c|} \hline + & 0 & - \\ \hline \end{array} * \begin{array}{|c|} \hline \text{Gaussian Kernel} \\ \hline \end{array} =$$

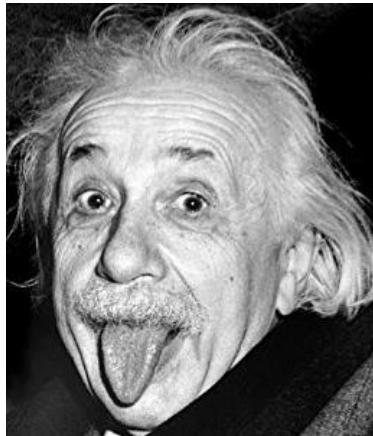
$$\frac{\partial G}{\partial u} = \frac{G(u+h) - G(u-h)}{2h}$$



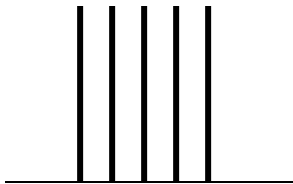
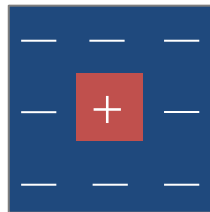
Edge



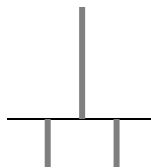
RECALL: IMAGE SHARPENING



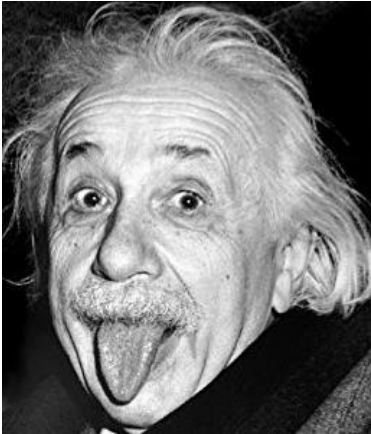
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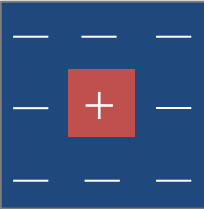
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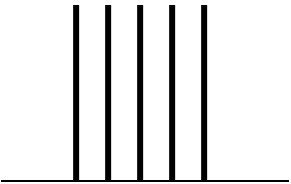
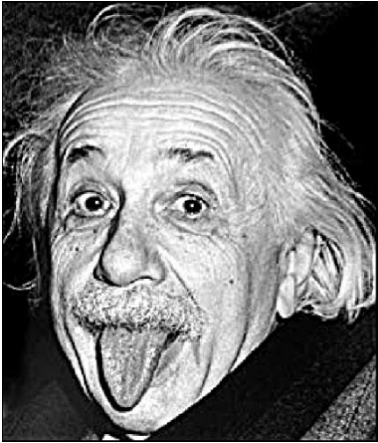
RECALL: IMAGE SHARPENING



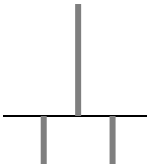
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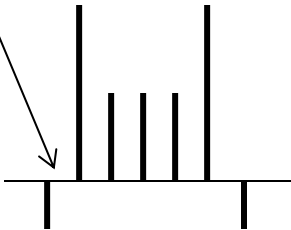
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*

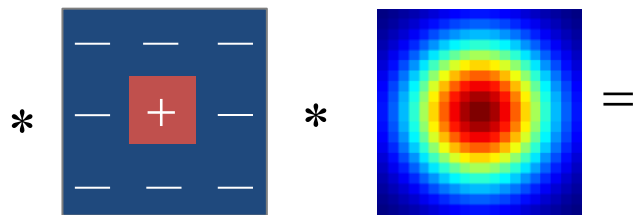
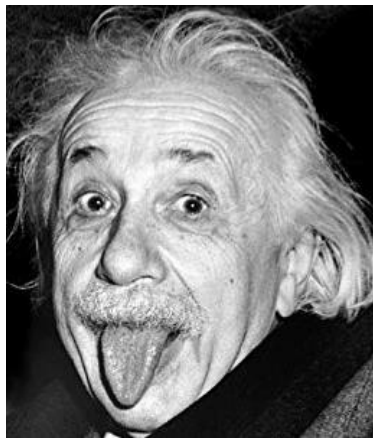


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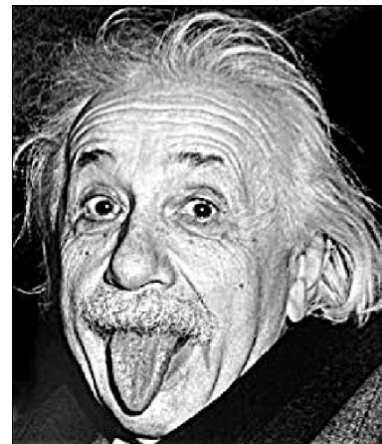


Zero crossing at the edge

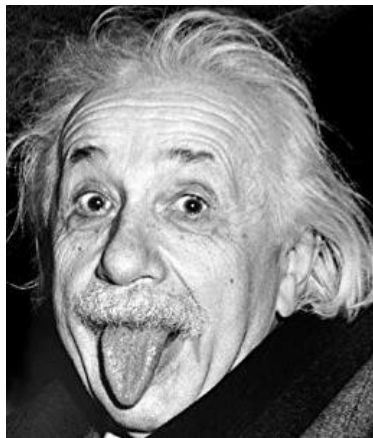
LAPLACIAN



$$\frac{G(u+h) + G(u-h) - 2G(u)}{h}$$

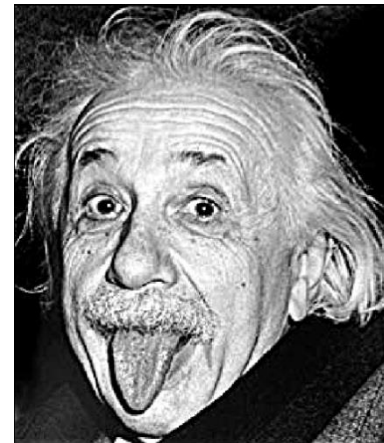


LAPLACIAN

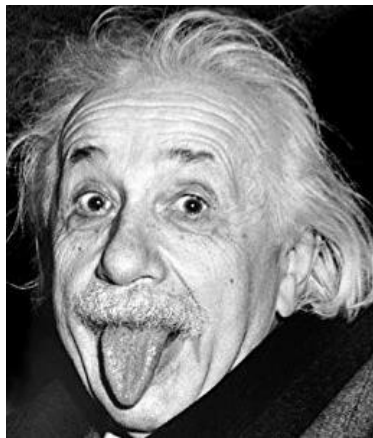


$$\begin{array}{c} * \quad \begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array} * \quad \begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array} = \\ \frac{G(u+h) - G(u)}{h} - \frac{G(u) - G(u-h)}{h} \end{array}$$

The equation illustrates the discrete Laplacian filter. It shows the original image being convolved with a kernel (a blue square with a red center and white crosses) and then with a second kernel (a heatmap showing a central peak). The result is the Laplacian image.



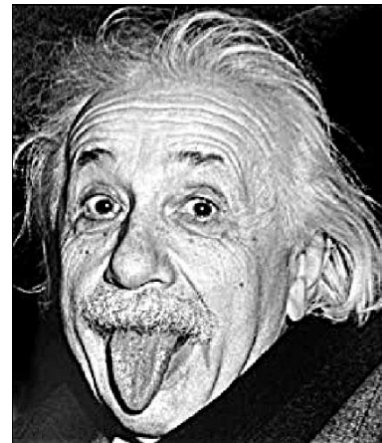
LAPLACIAN



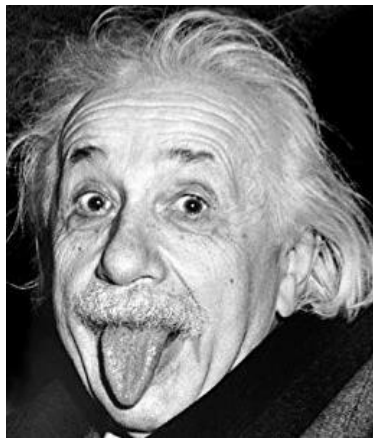
$$\lim_{h \rightarrow 0} \left(\frac{G(u+h) - G(u)}{h} - \frac{G(u) - G(u-h)}{h} \right) / h$$

The equation is visually represented by the following components:

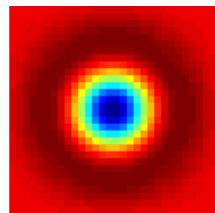
- A blue square with a red square in the center containing a white plus sign (+), representing the discrete Laplacian kernel.
- A heat map showing a central red circle fading to blue, representing the continuous Laplacian kernel.
- Two asterisks (*) indicating convolution.
- An equals sign (=) indicating the result of the operation.



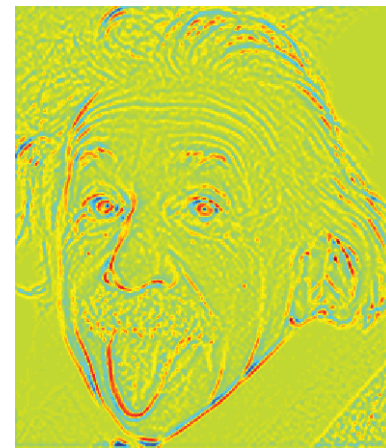
LAPLACIAN



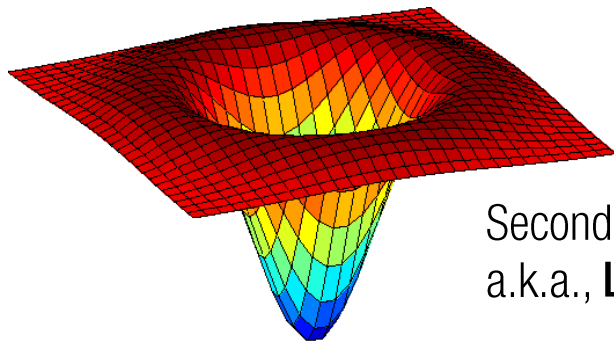
*



=

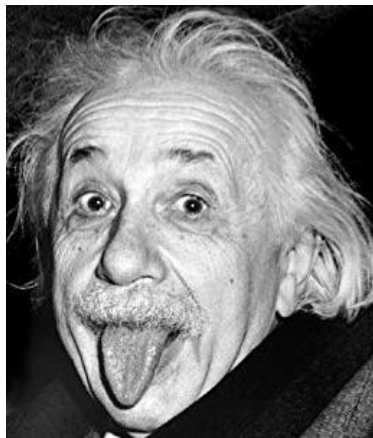


$$\nabla \cdot \nabla G = \nabla \left(\frac{\partial G}{\partial u} \mathbf{i} + \frac{\partial G}{\partial v} \mathbf{j} \right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2}$$



Second order derivative of Gaussian,
a.k.a., **Laplacian of Gaussian**

LAPLACIAN

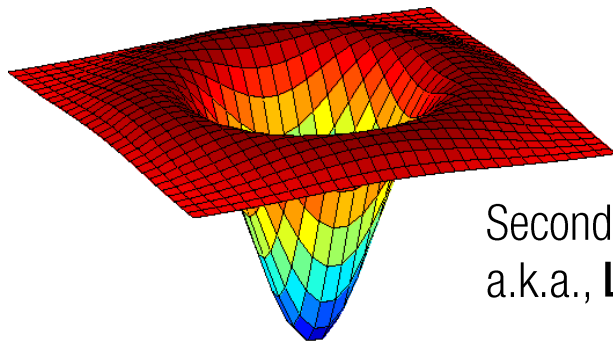
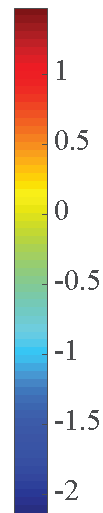
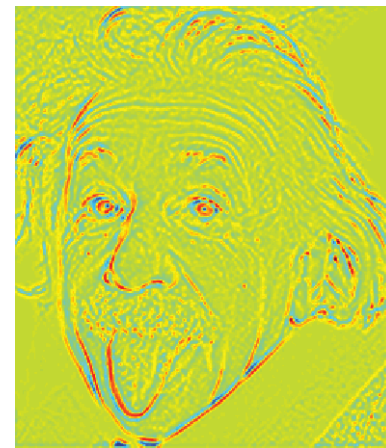


*

0	1	0
1	-4	1
0	1	0

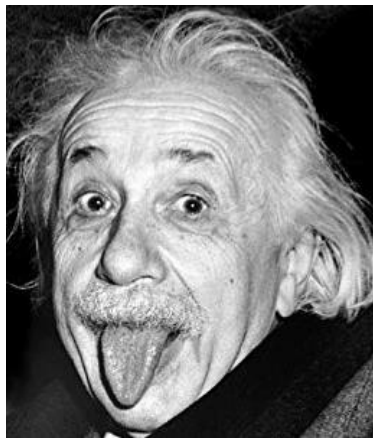
=

$$\nabla \cdot \nabla G = \nabla \left(\frac{\partial G}{\partial u} \mathbf{i} + \frac{\partial G}{\partial v} \mathbf{j} \right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2}$$



Second order derivative of Gaussian,
a.k.a., **Laplacian of Gaussian**

LAPLACIAN

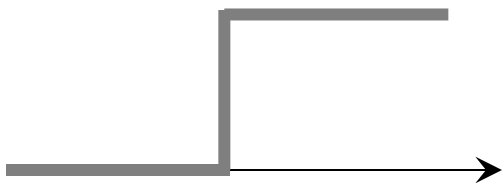
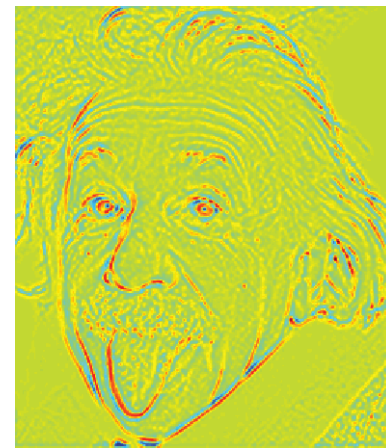


*

0	1	0
1	-4	1
0	1	0

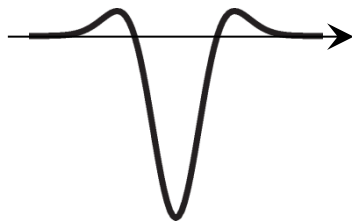
=

$$\nabla \cdot \nabla G = \nabla \left(\frac{\partial G}{\partial u} \mathbf{i} + \frac{\partial G}{\partial v} \mathbf{j} \right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2}$$

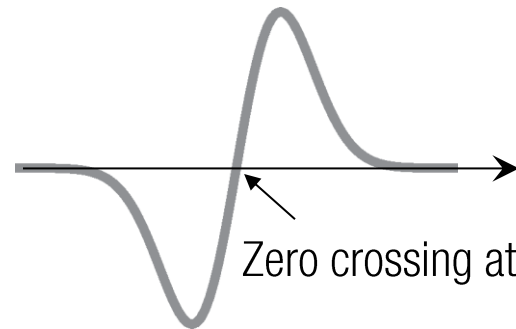


Edge

*

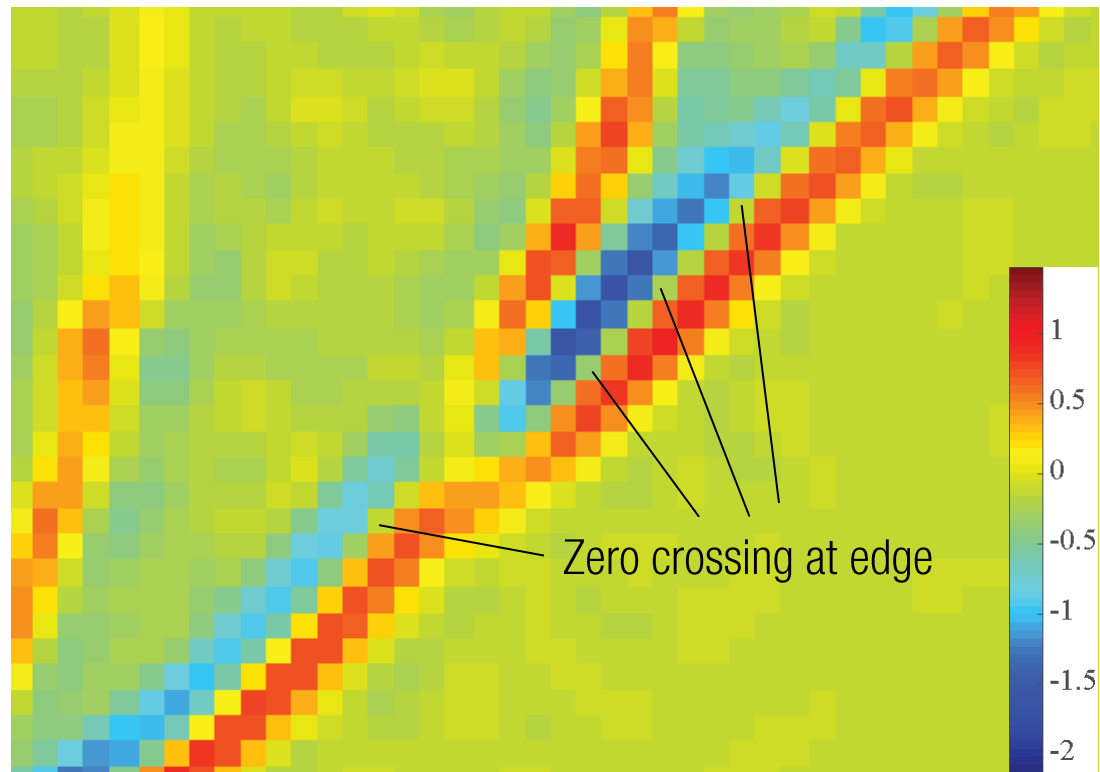
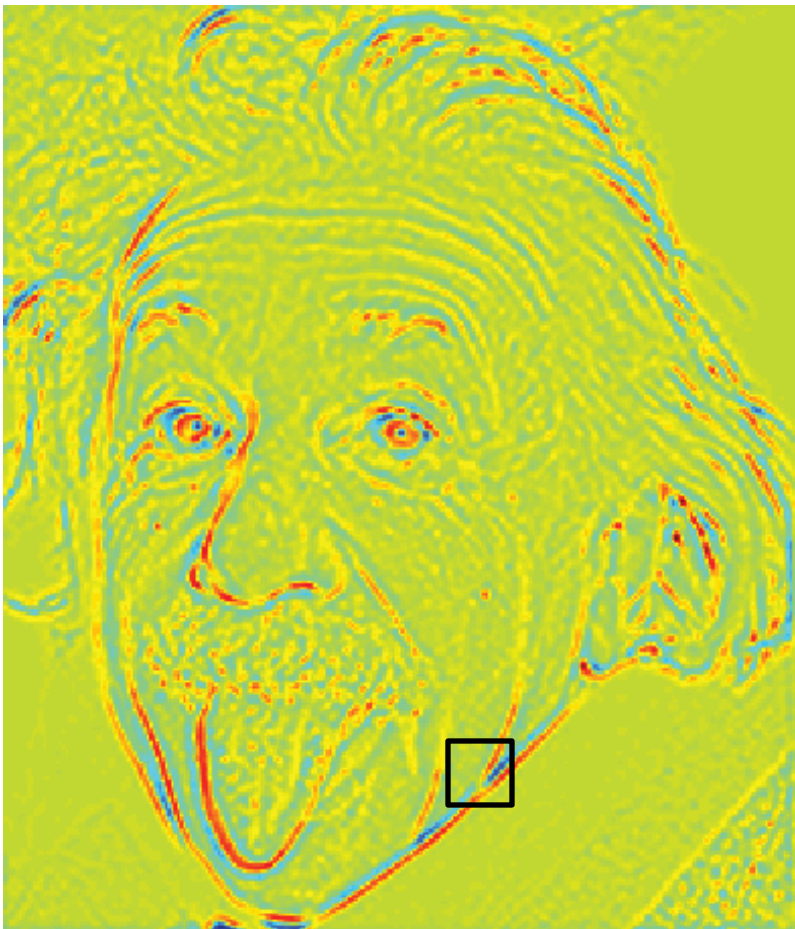


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Zero crossing at edge

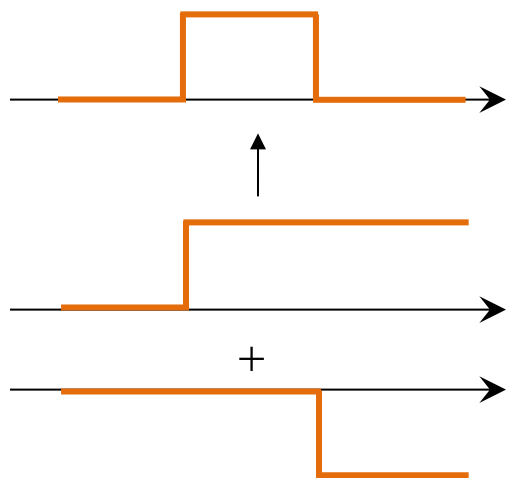
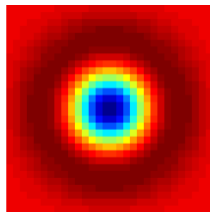
EDGE LOCALIZATION



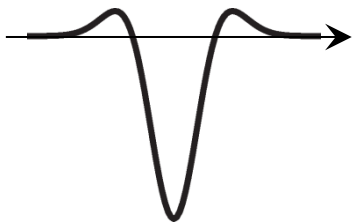
STRUCTURED EDGES (E.G., BLOB)



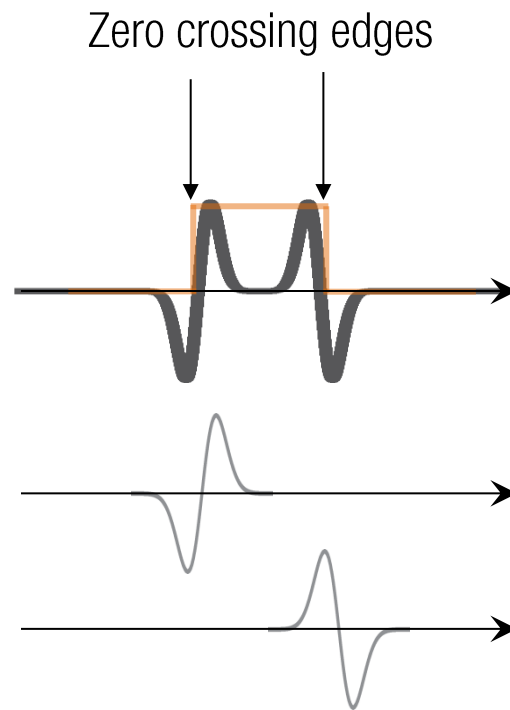
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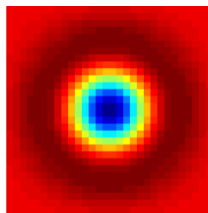
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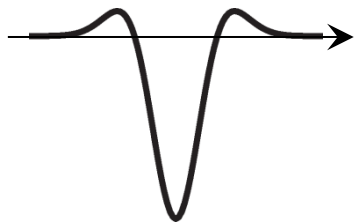
STRUCTURED EDGES (E.G., BLOB)



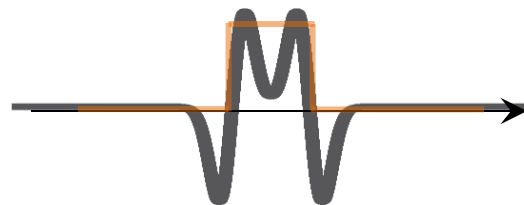
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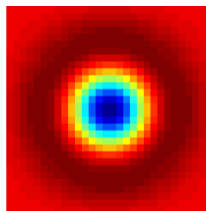
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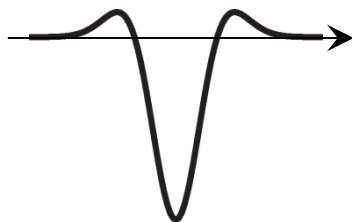
STRUCTURED EDGES (E.G., BLOB)



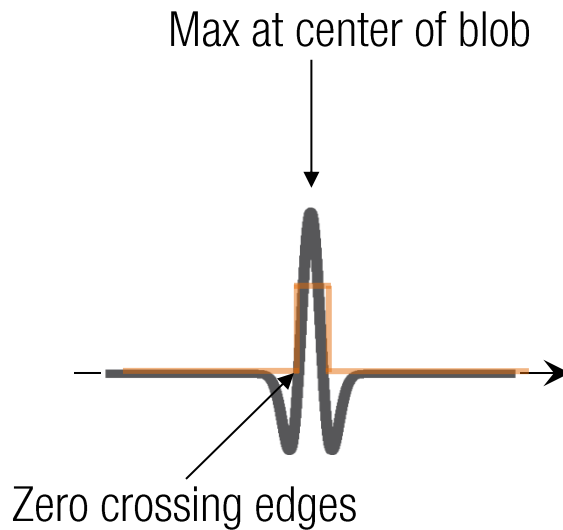
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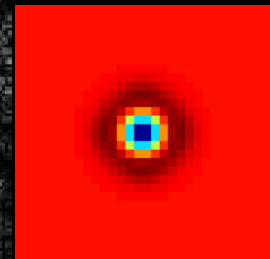
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STRUCTURED EDGES (E.G., BLOB)

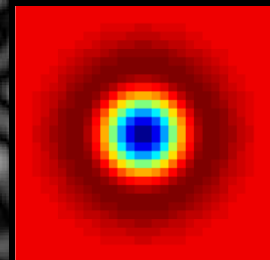


STRUCTURED EDGES (E.G., BLOB)



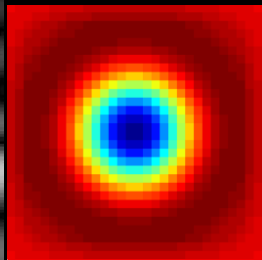
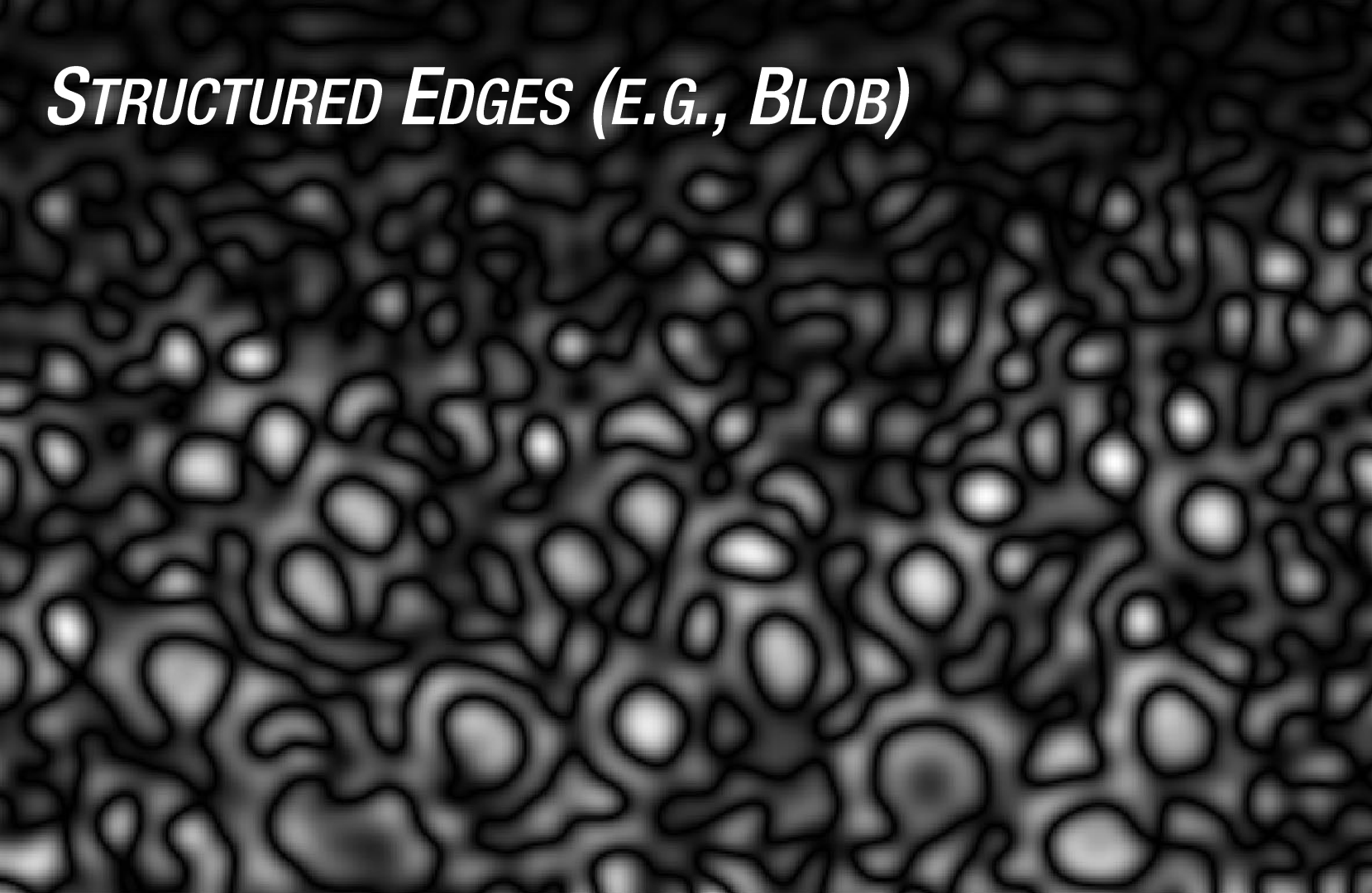
$$\sigma = 1$$

STRUCTURED EDGES (E.G., BLOB)



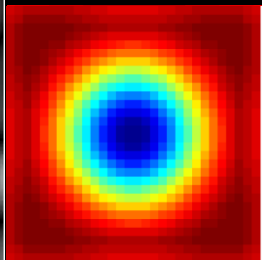
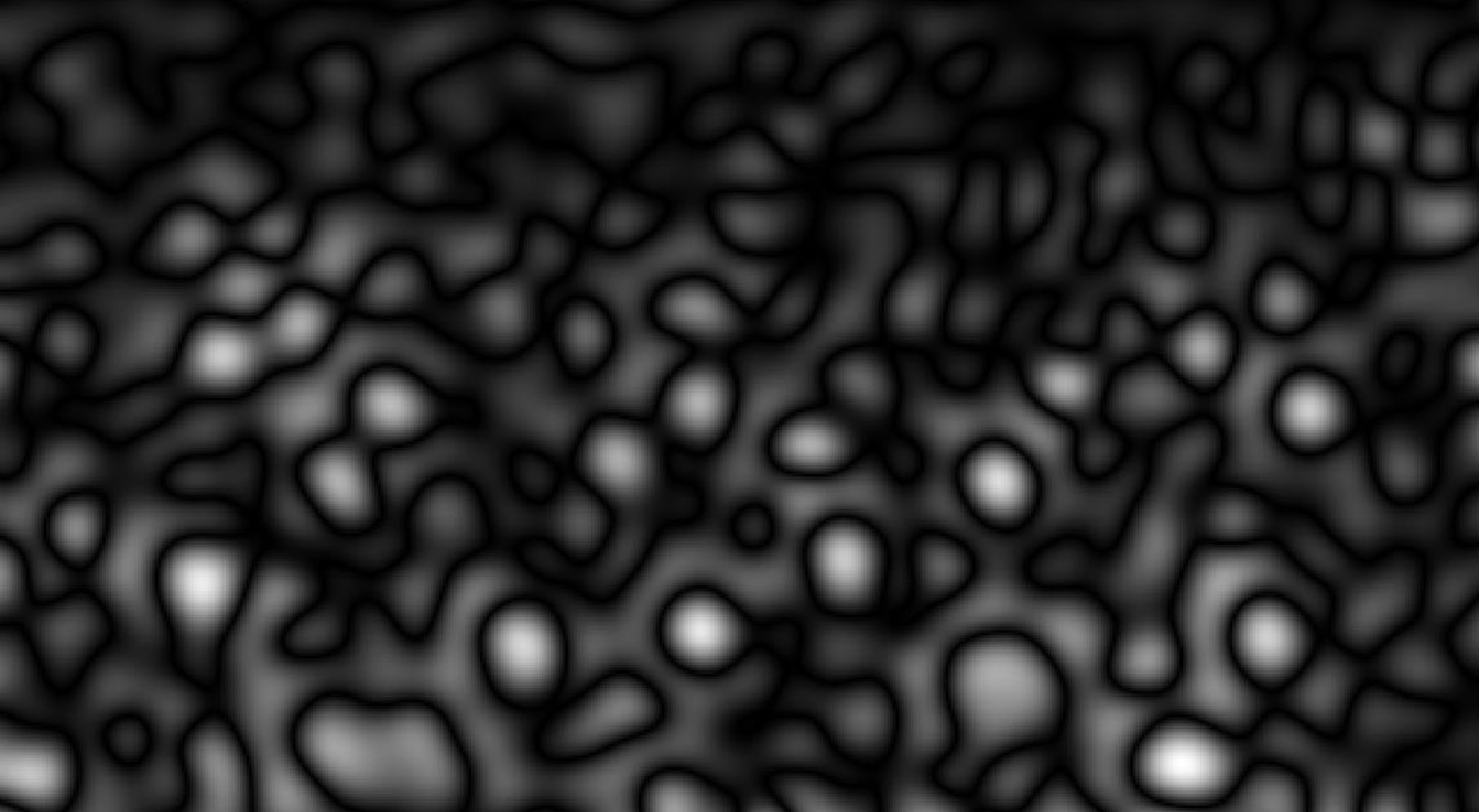
$$\sigma = 4$$

STRUCTURED EDGES (E.G., BLOB)



$$\sigma = 7$$

STRUCTURED EDGES (E.G., BLOB)



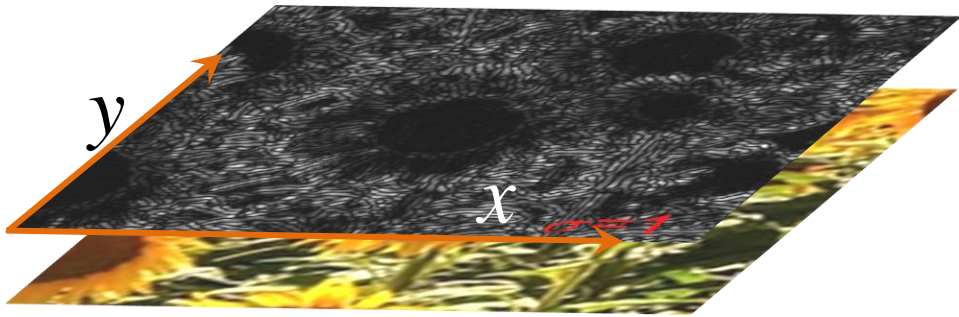
$$\sigma = 10$$

SCALE SPACE



$I(x, y)$

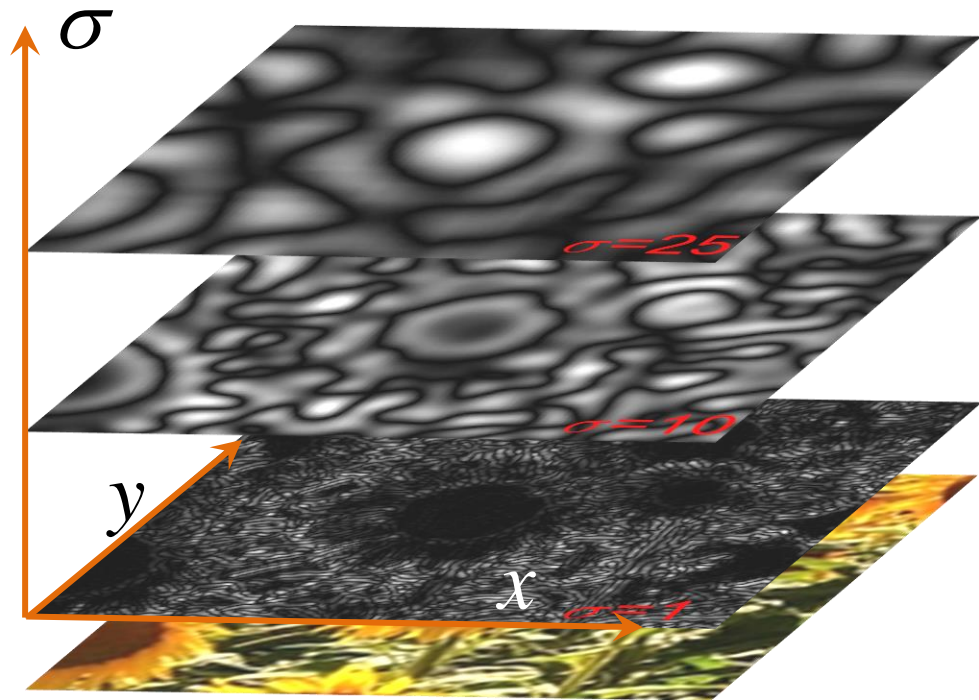
SCALE SPACE



$$I(x, y) * \underline{L(\sigma)}$$

Laplacian operator

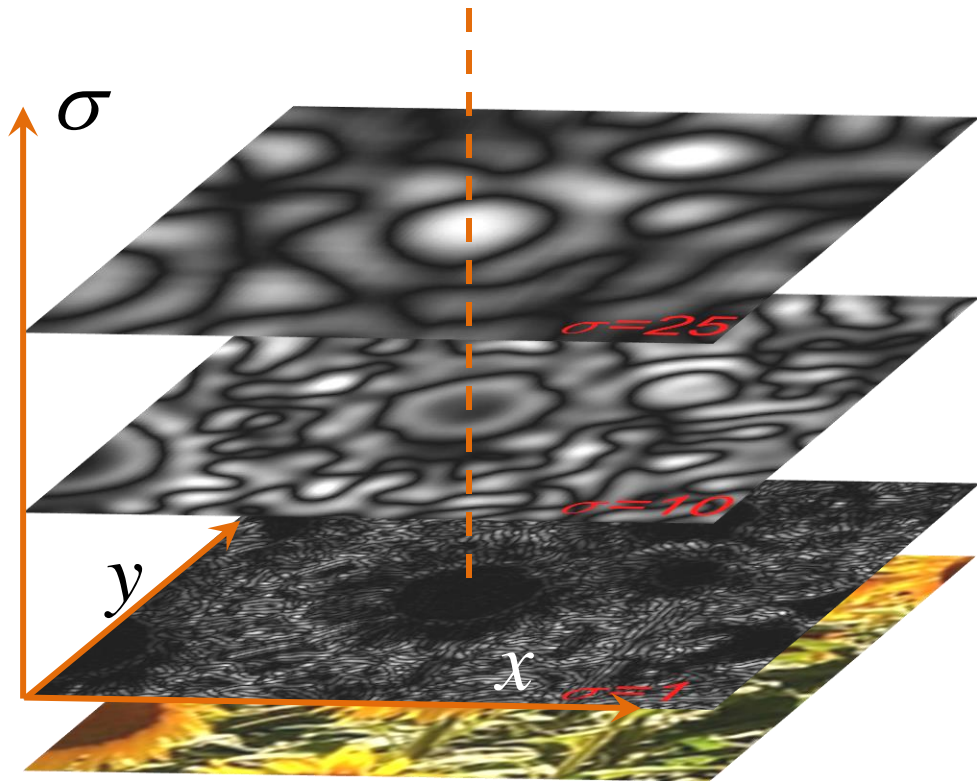
SCALE SPACE



$$\underline{D(x, y, \sigma) = I(x, y) * L(\sigma)}$$

Scale space response

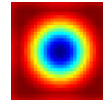
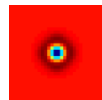
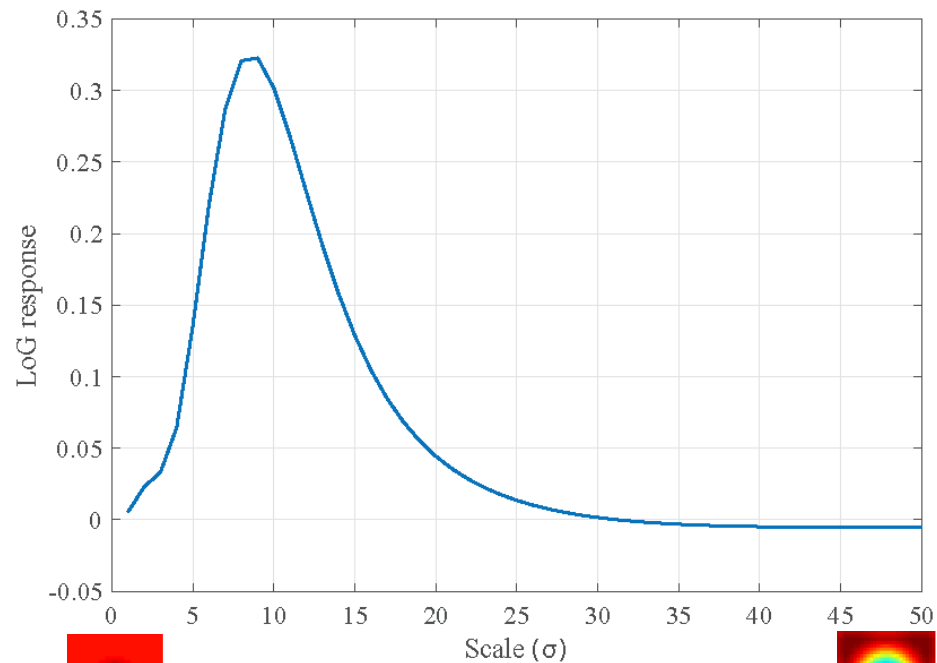
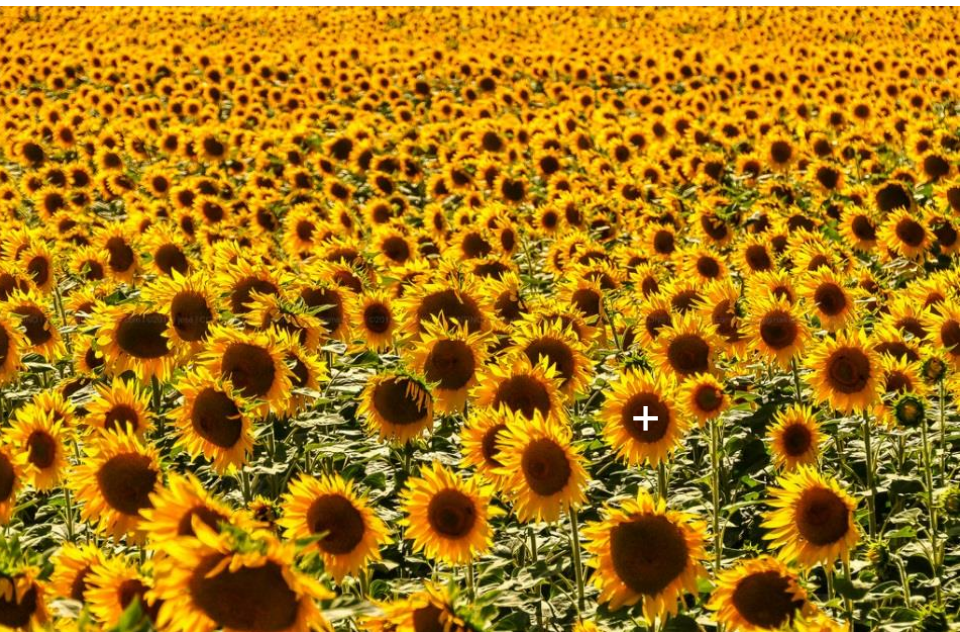
SCALE SPACE



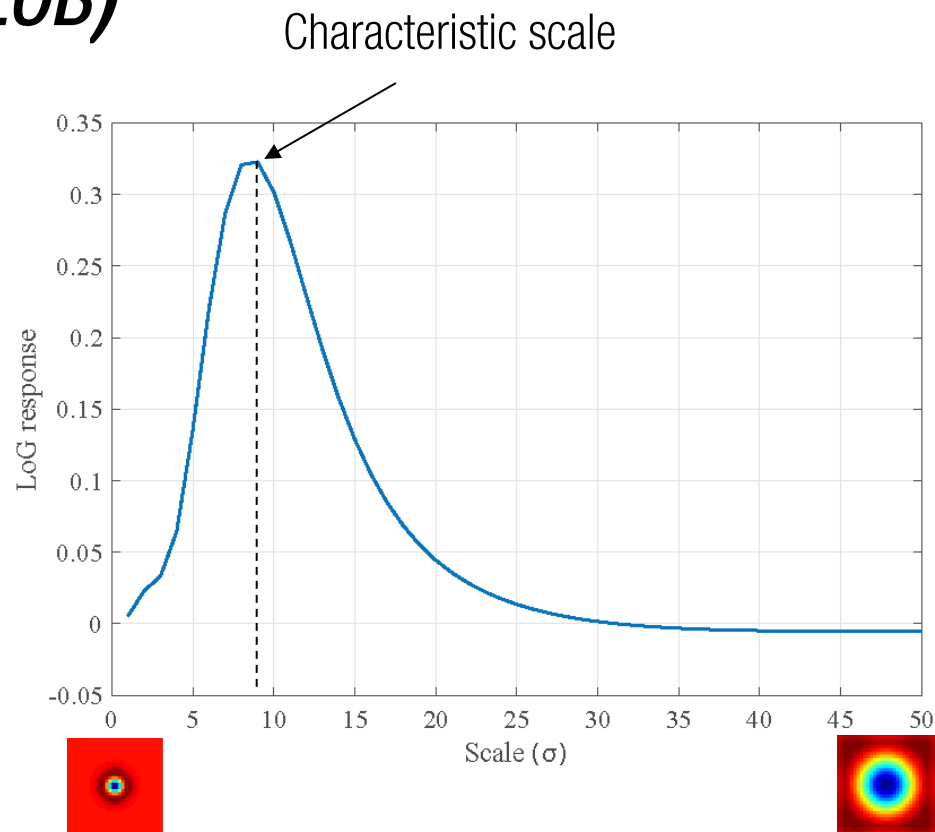
$$\underline{D(x, y, \sigma) = I(x, y) * L(\sigma)}$$

Scale space response

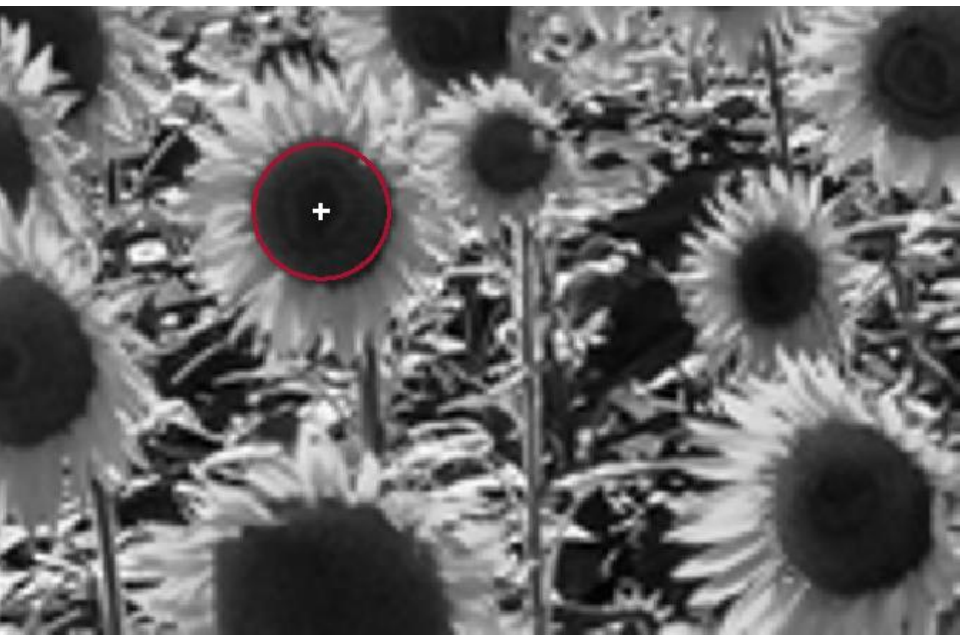
STRUCTURED EDGES (E.G., BLOB)



STRUCTURED EDGES (E.G., BLOB)

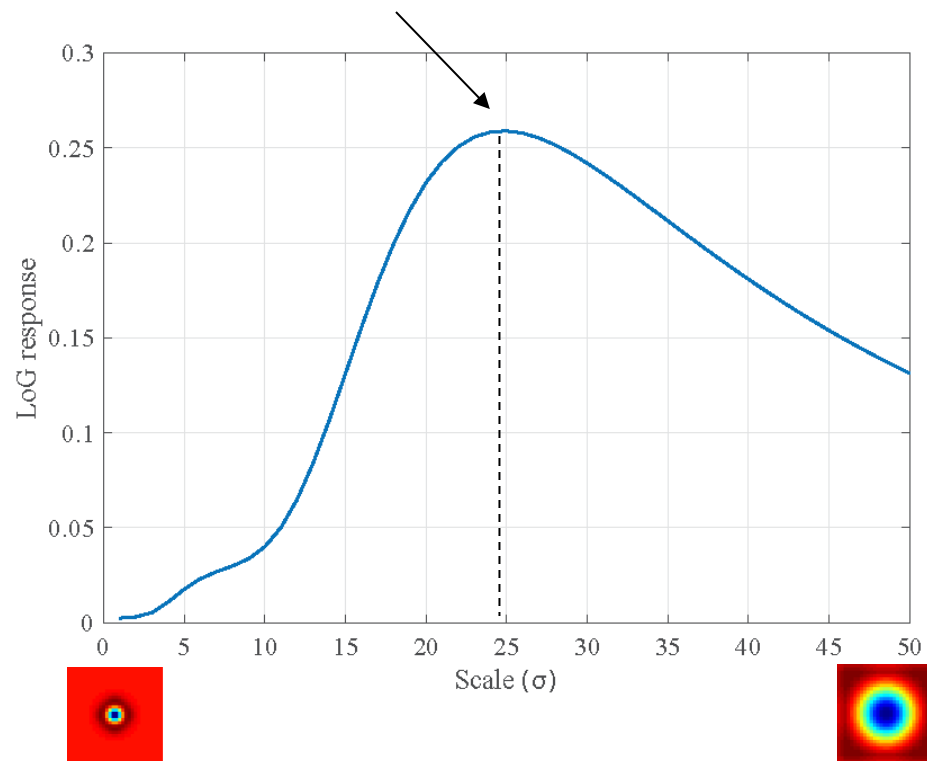


STRUCTURED EDGES (E.G., BLOB)



x3 bigger sunflower

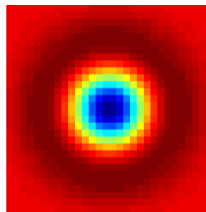
Characteristic scale



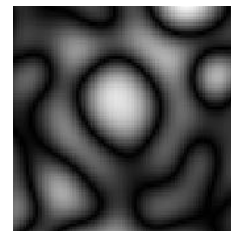
SCALE SELECTION



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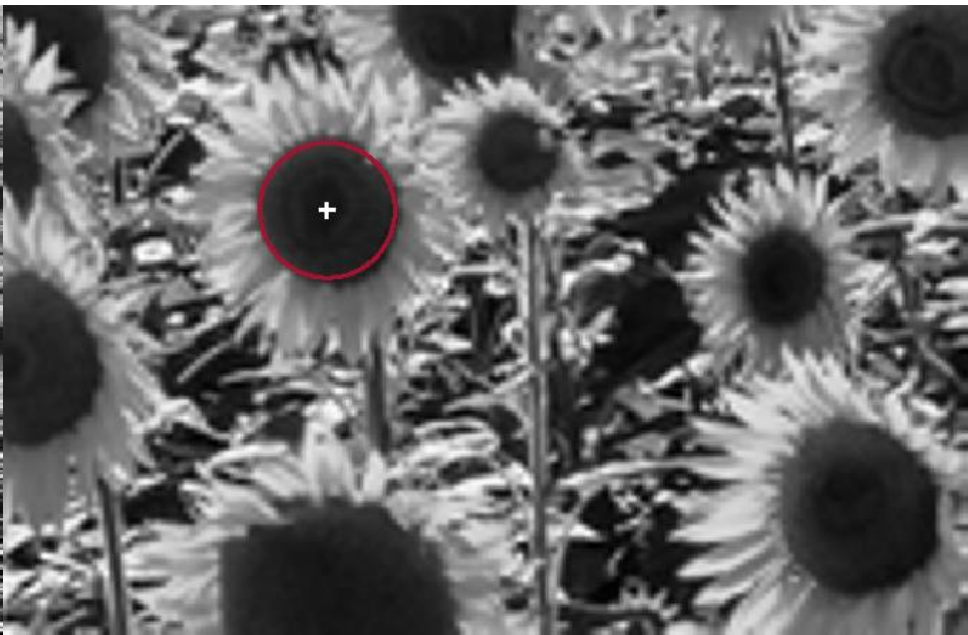


$$\sigma^2 \left(\frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2} \right)$$

Normalization factor

- The scale response (laplacian) is maximized when $\sigma = \frac{r}{\sqrt{2}}$
- The characteristic scale can be used to normalize the image, resulting in *scale invariant* image description.

SCALE NORMALIZATION



SCALE NORMALIZATION

