

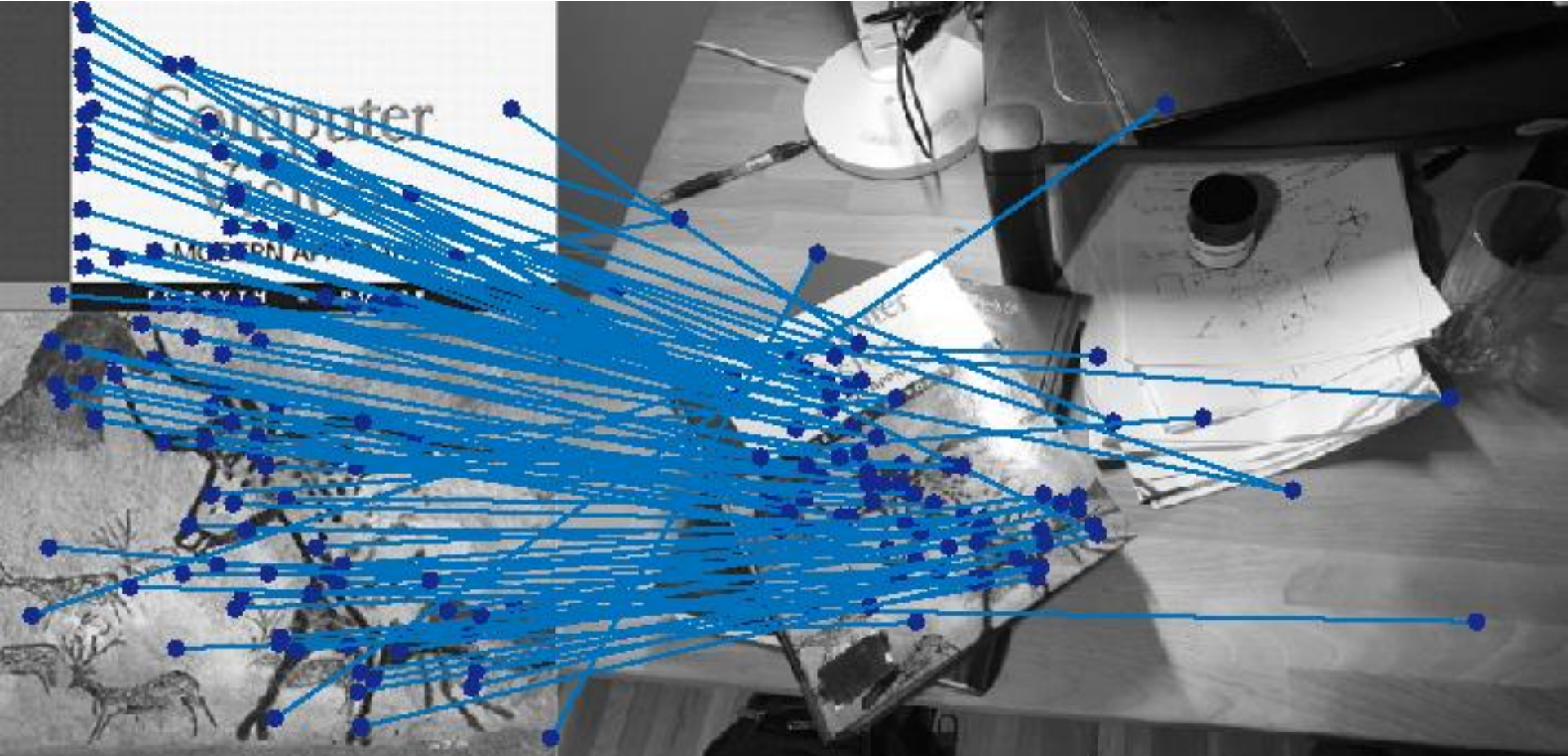
A photograph of a park scene with a large tree and a brick building in the background. The image is overlaid with a dense network of colorful lines (blue, green, yellow, orange, red, purple) that all converge at a single point in the center of the frame. These lines represent the RANSAC algorithm's process of finding a consensus set of points that fit a line model. The lines are of varying lengths and colors, indicating different iterations or confidence levels. The background is slightly blurred, emphasizing the geometric overlay.

# *RANSAC*

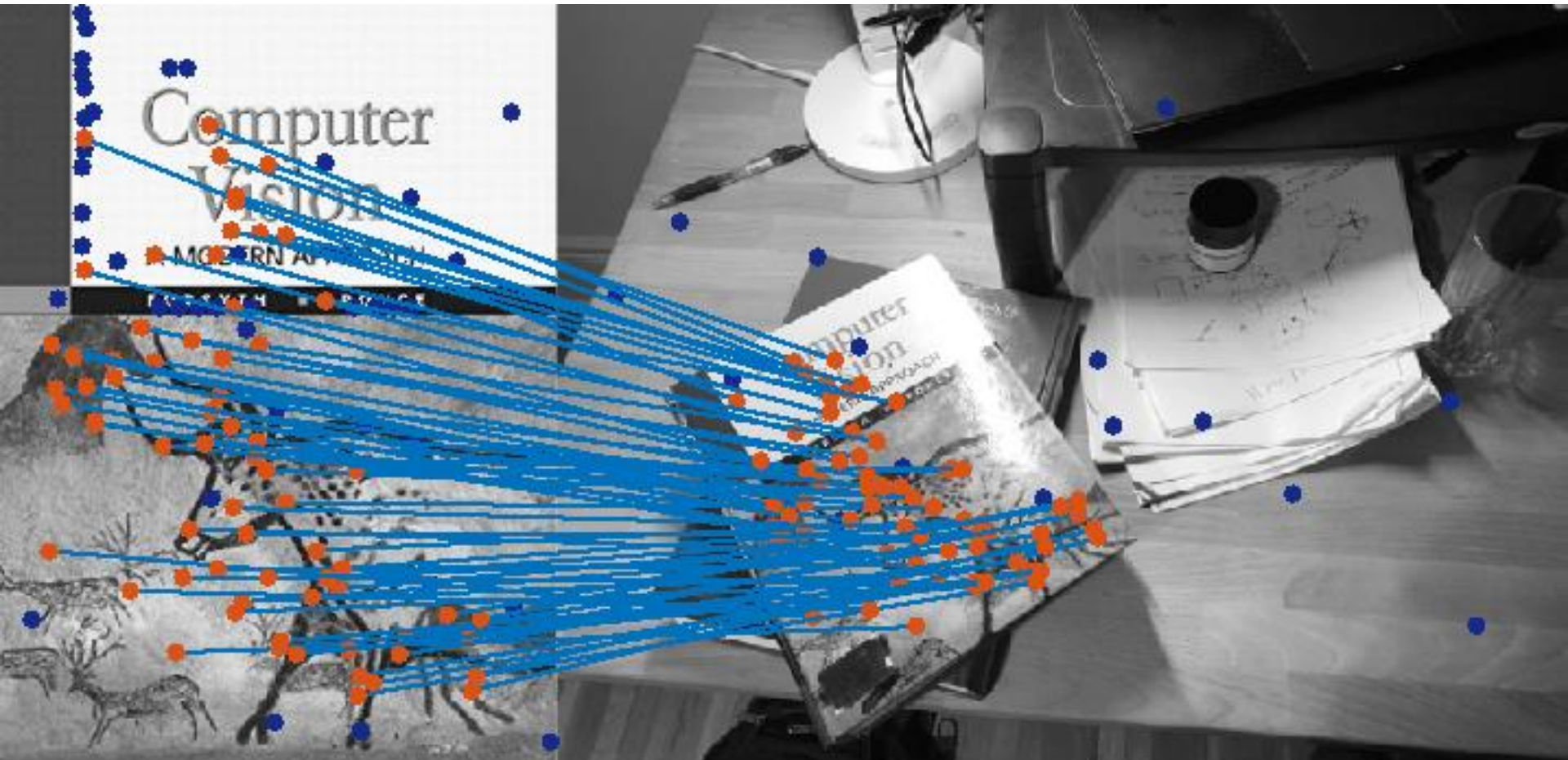
HYUN SOO PARK



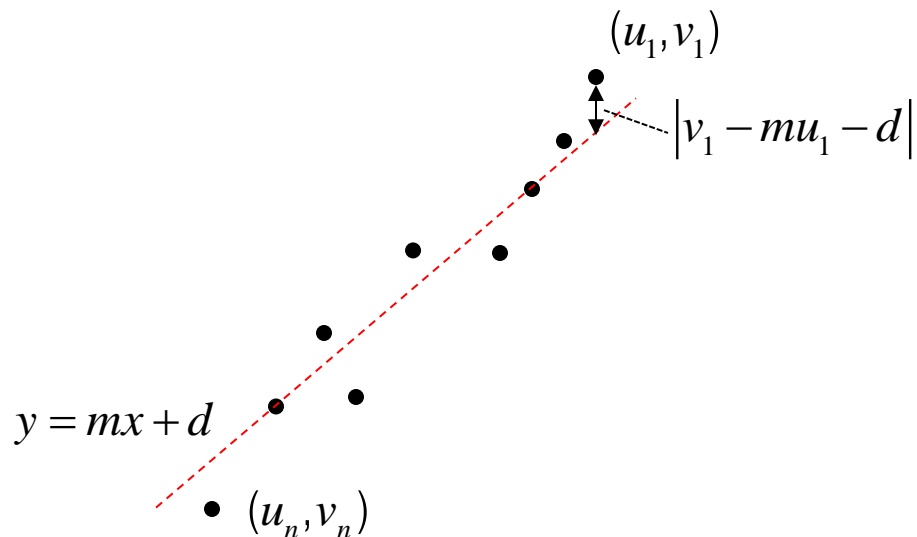
# *RECALL: LOCAL FEATURE MATCHING*



# *RECALL: ROBUST FILTERING*



# LINE FITTING



Given points:  $(u_1, v_1), \dots, (u_n, v_n)$

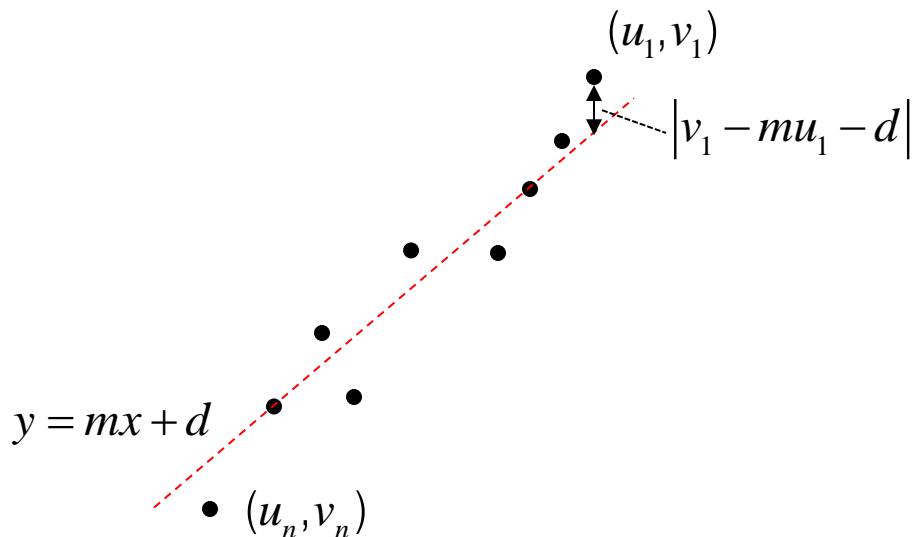
Find the best line:  $v_1 \approx mu_1 + d$

$\vdots$

$v_n \approx mu_n + d$

$$\longrightarrow \begin{bmatrix} u_1 & 1 \\ \vdots & \vdots \\ u_n & 1 \end{bmatrix} \begin{bmatrix} m \\ d \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

# LINE FITTING



Given points:  $(u_1, v_1), \dots, (u_n, v_n)$

Find the best line:  $v_1 \approx mu_1 + d$

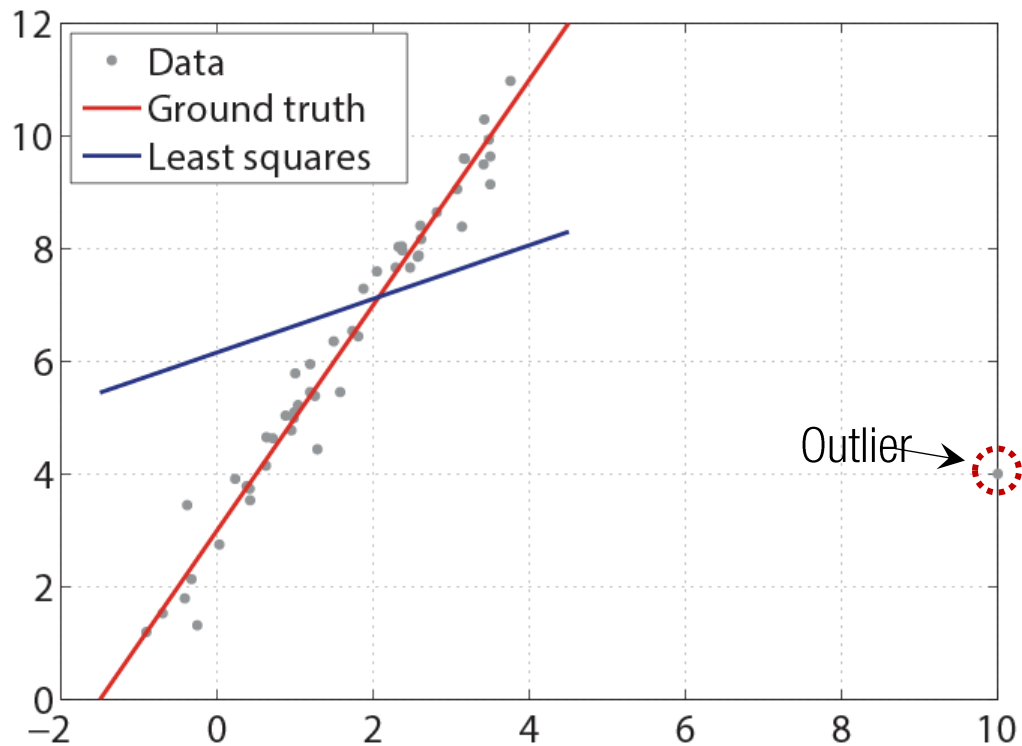
$\vdots$

$v_n \approx mu_n + d$

$$\begin{bmatrix} u_1 & 1 \\ \vdots & \vdots \\ u_n & 1 \end{bmatrix} \begin{bmatrix} m \\ d \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

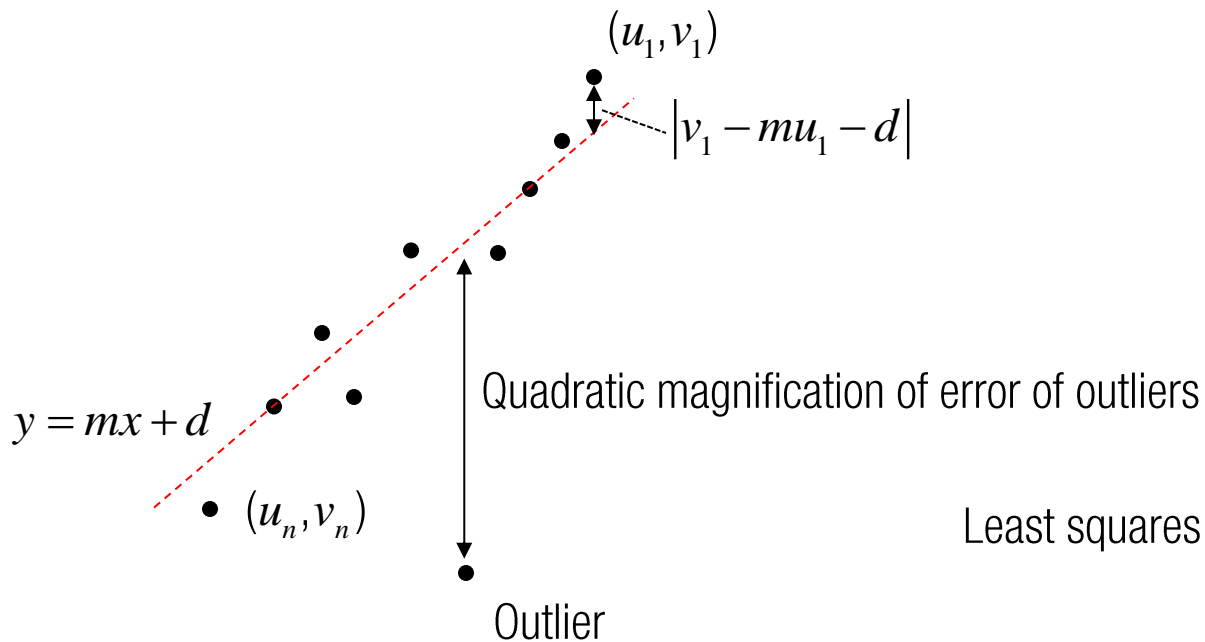
Least squares solution:  $x = (A^T A)^{-1} A^T b$

# OUTLIER



$$\begin{bmatrix} u_1 & 1 \\ \vdots & \vdots \\ A & \vdots \\ u_n & 1 \end{bmatrix} \begin{bmatrix} m \\ x \\ d \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ b \\ v_n \end{bmatrix}$$

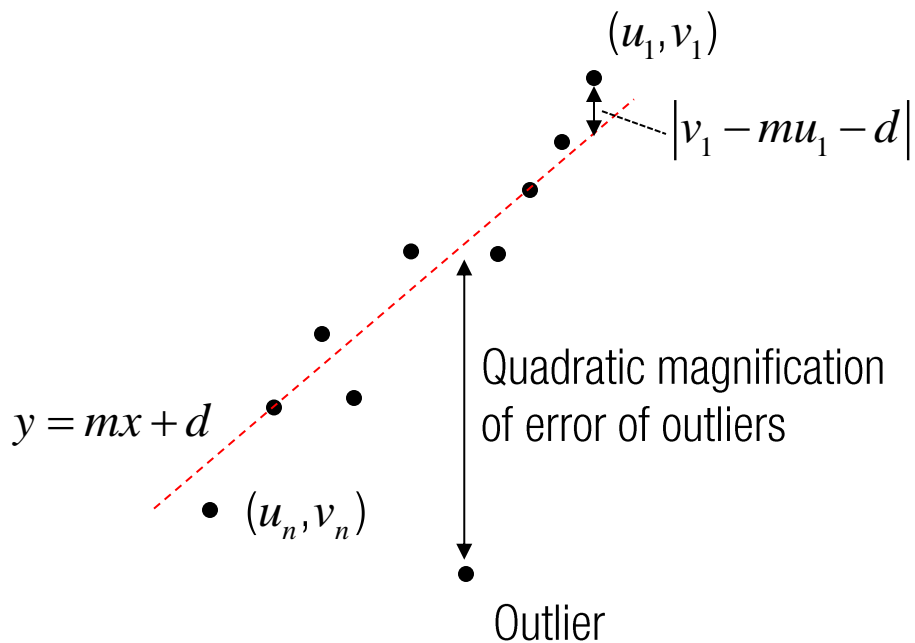
# OUTLIER SENSITIVITY



$$\begin{bmatrix} u_1 & 1 \\ \vdots & \vdots \\ A & \vdots \\ u_n & 1 \end{bmatrix} \begin{bmatrix} m \\ x \\ d \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ b \\ v_n \end{bmatrix}$$

Least squares solution:  $x = (A^T A)^{-1} A^T b$

# OUTLIER REJECTION STRATEGY



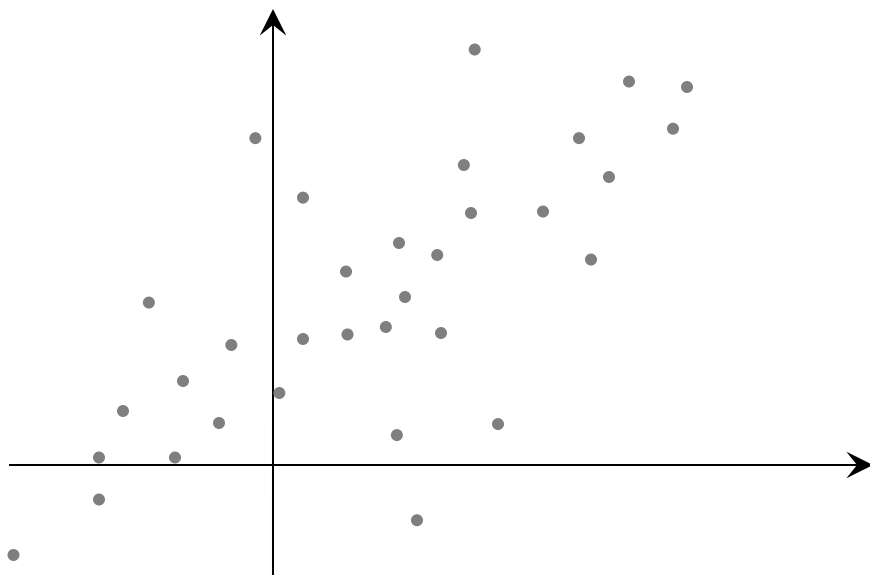
## Outlier rejection strategy:

To find the best line that explains the maximum number of points.

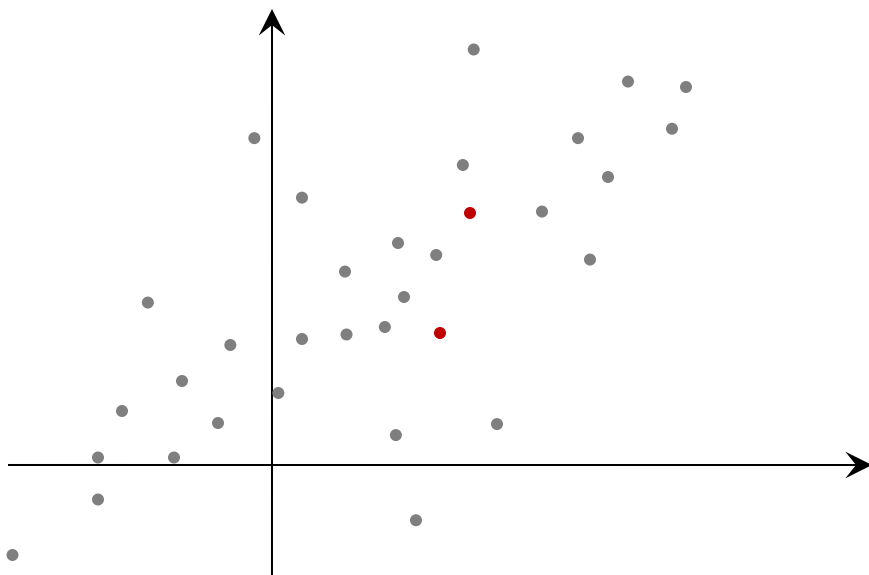
## Assumptions:

1. Majority of good samples agree with the underlying model (good apples are same and simple.).
2. Bad samples does not consistently agree with a single model (all bad apples are different and complicated.).



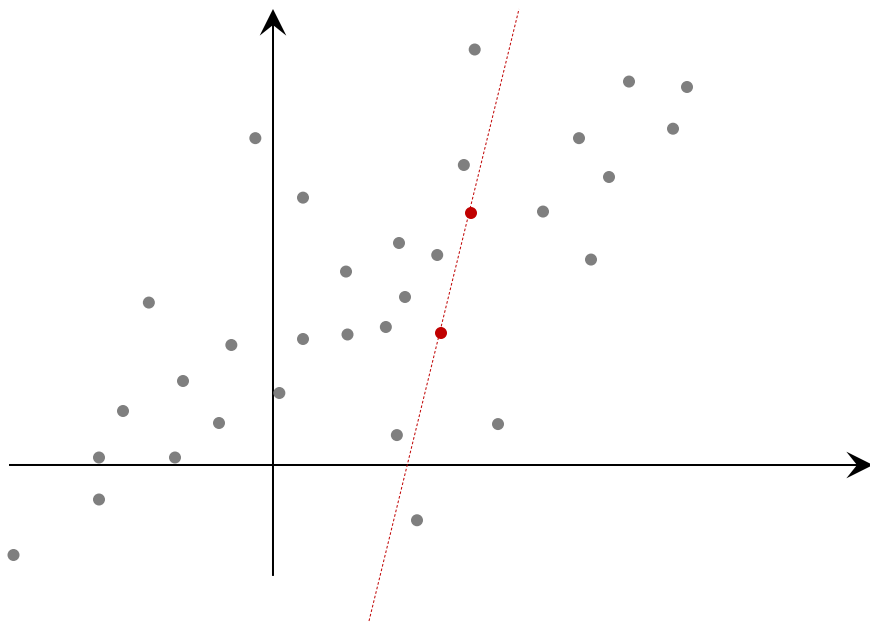


**RANSAC: RAndom SAmple Consensus**



1. Random sampling

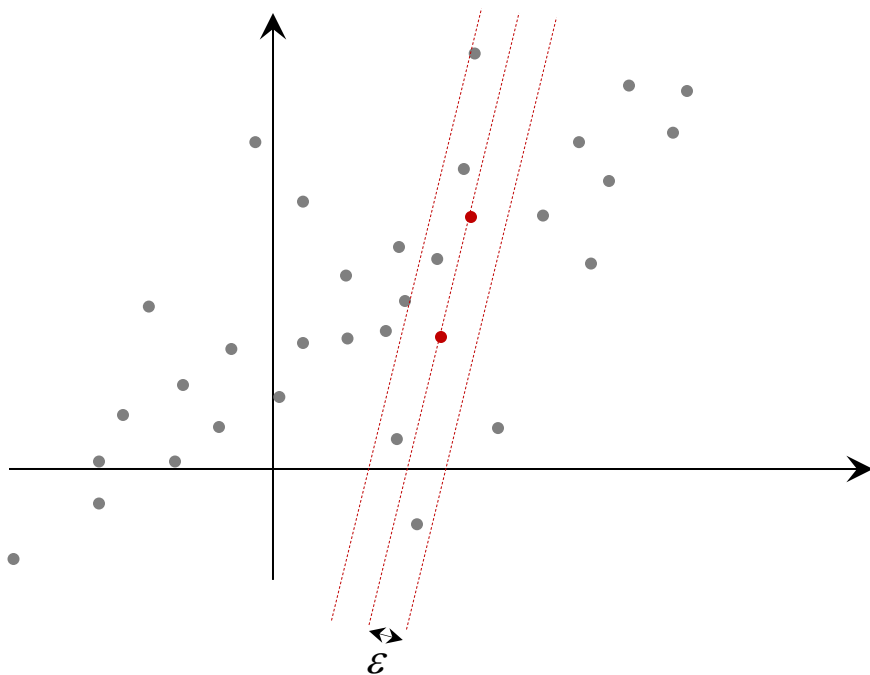
**RANSAC: RAndom SAmple Consensus**



1. Random sampling

2. Model building

**RANSAC: RANdom SAmple Consensus**

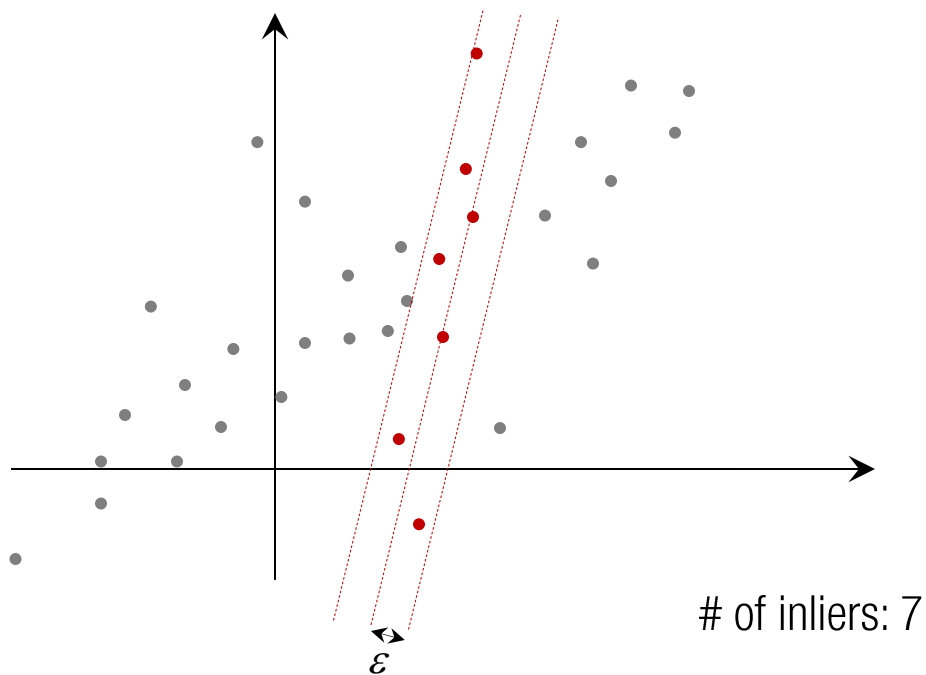


1. Random sampling

2. Model building

3. Thresholding

**RANSAC: RANdom SAmple Consensus**



1. Random sampling

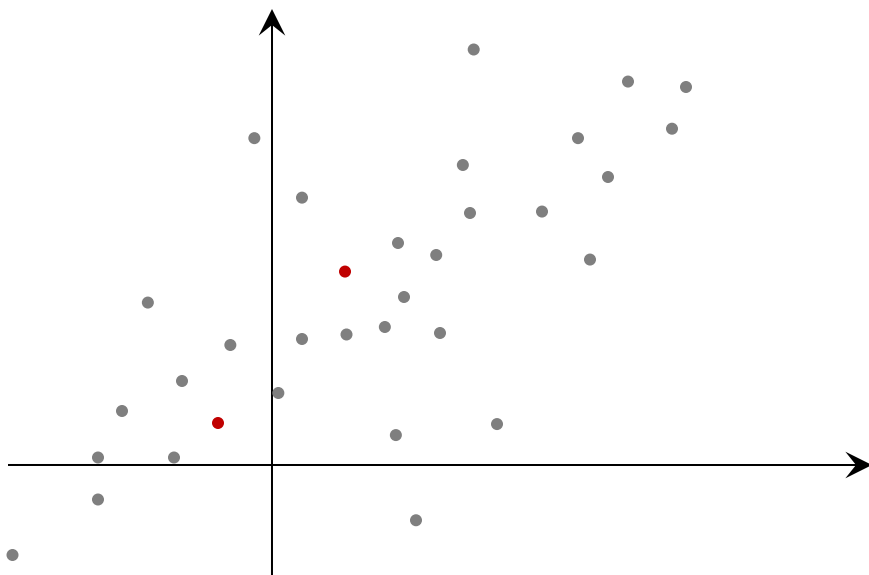
2. Model building

3. Thresholding

4. Inlier counting

**RANSAC: RANdom SAmple Consensus**





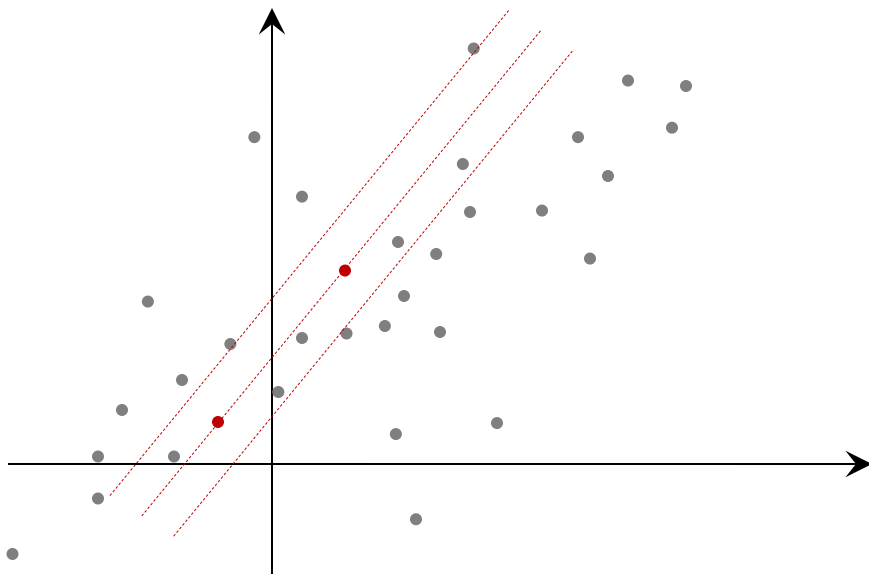
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

**RANSAC: RANdom SAmple Consensus**



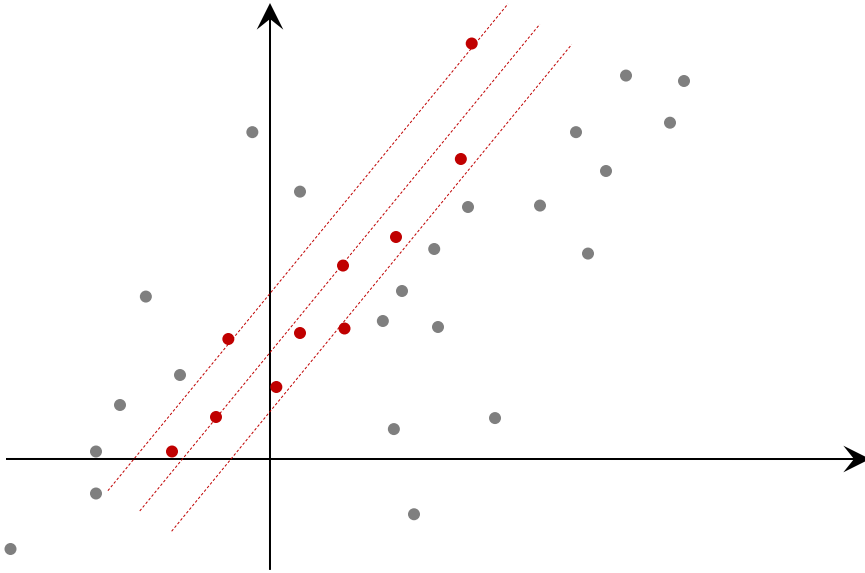
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

# RANSAC: RANdom SAmple Consensus



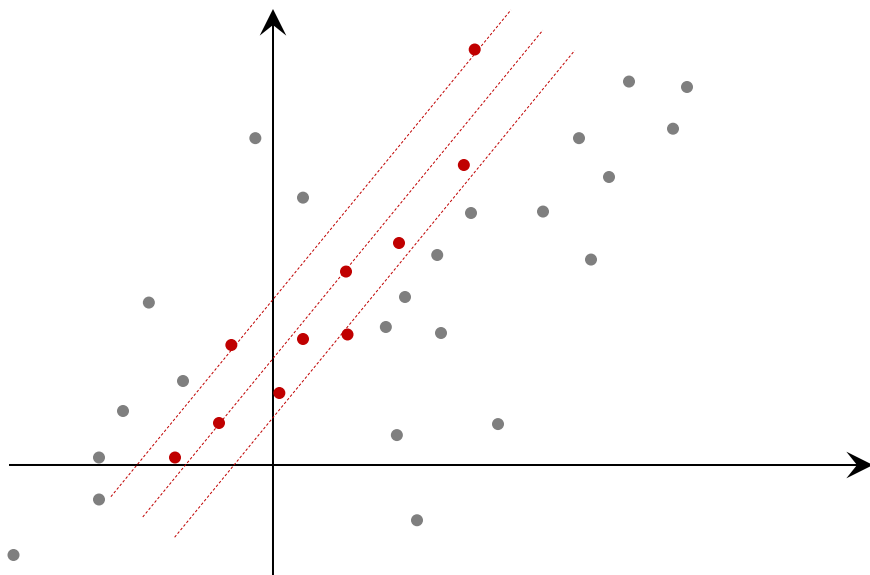
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

## RANSAC: RANdom SAmple Consensus



# of inliers: 10

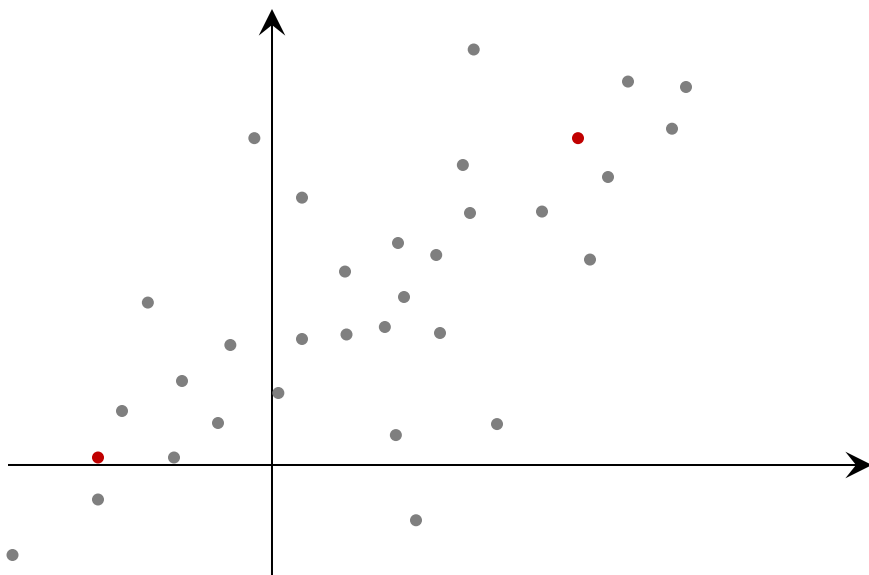
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

# RANSAC: RANdom SAMple Consensus



1. Random sampling

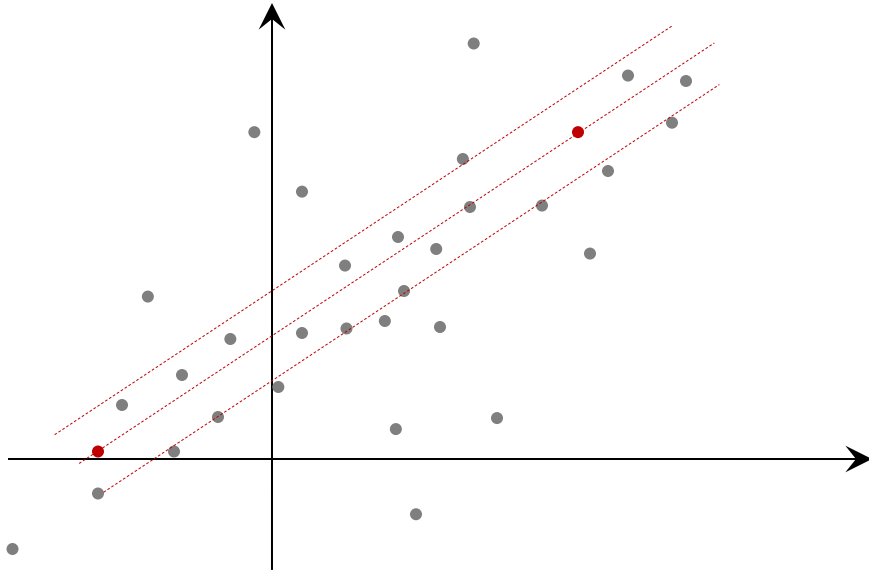
2. Model building

3. Thresholding

4. Inlier counting

# RANSAC: RANdom SAmple Consensus





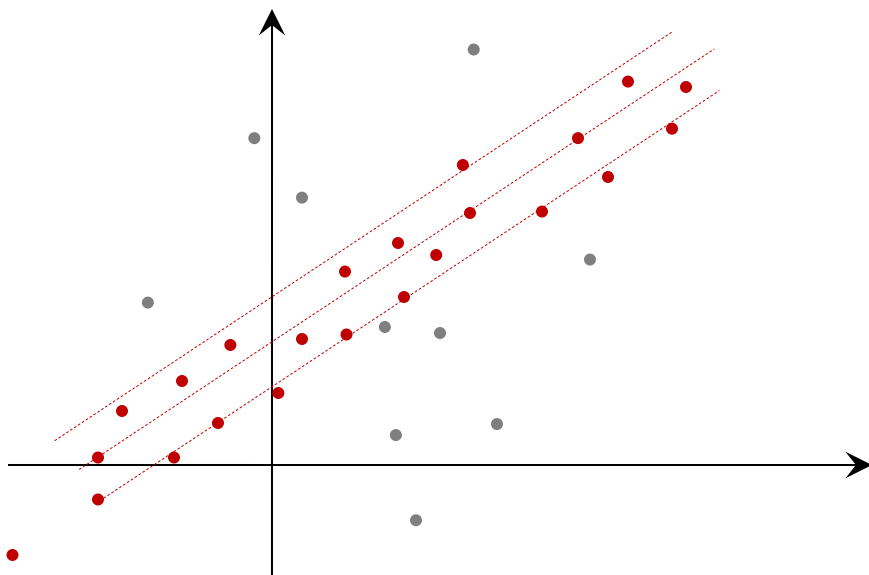
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

# RANSAC: RANdom SAmple Consensus



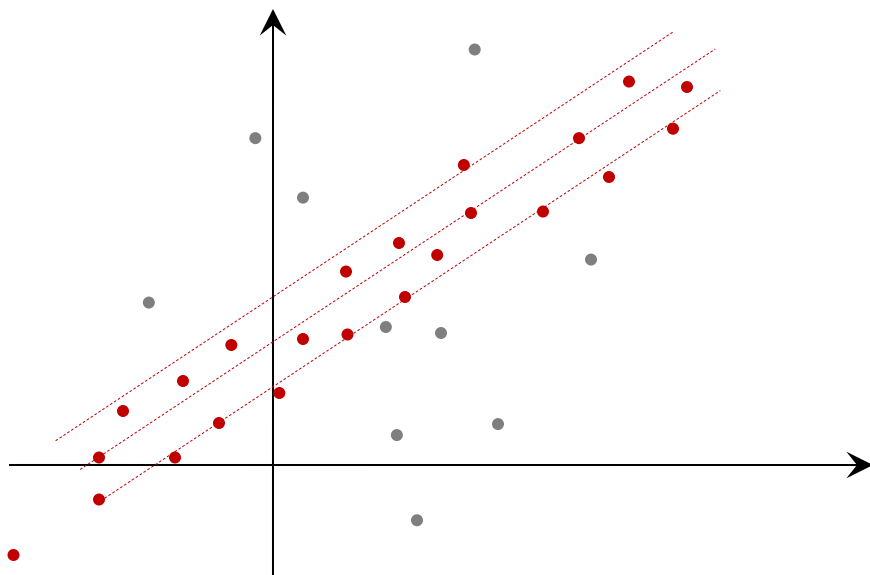
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

# RANSAC: RANdom SAmple Consensus



1. Random sampling

2. Model building

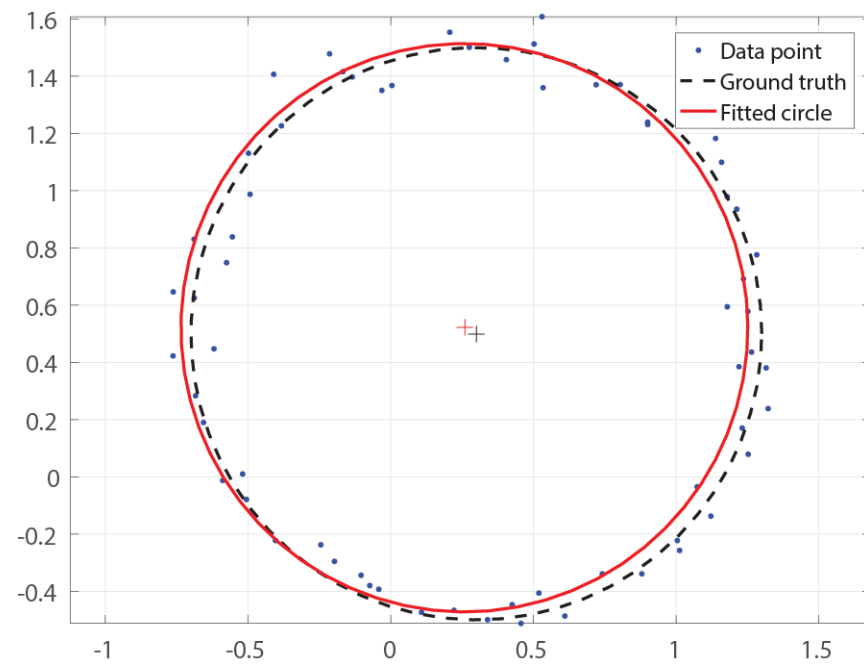
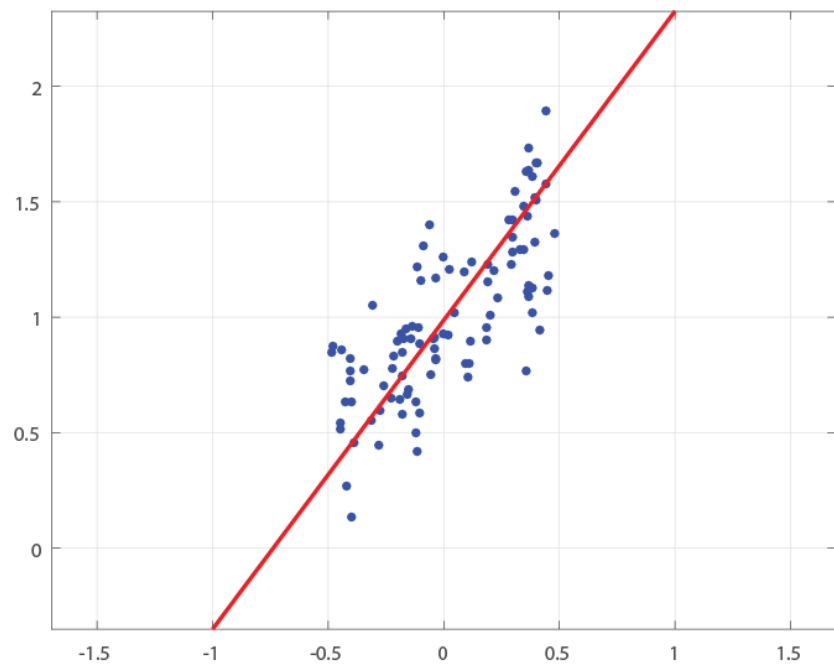
3. Thresholding

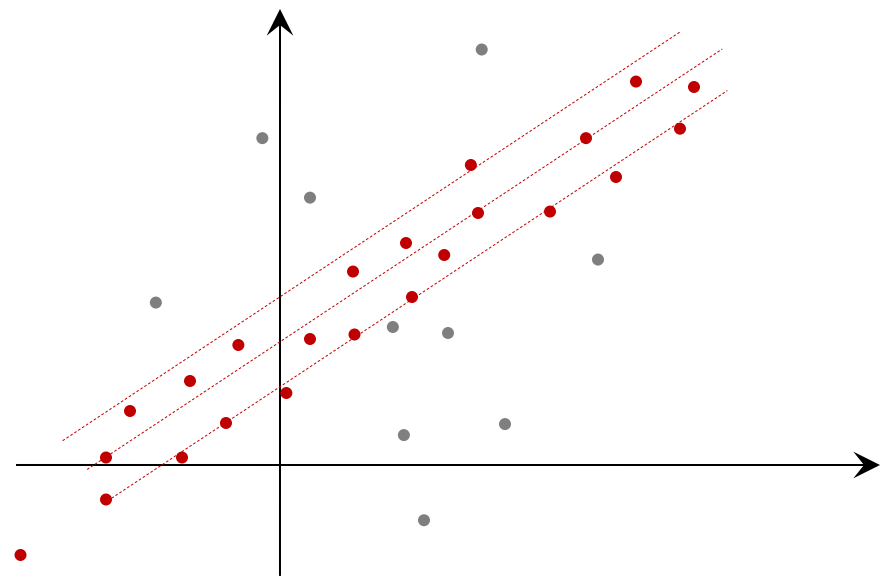
4. Inlier counting

# of inliers: 23

Maximum number of inliers

## RANSAC: RANdom SAmple Consensus

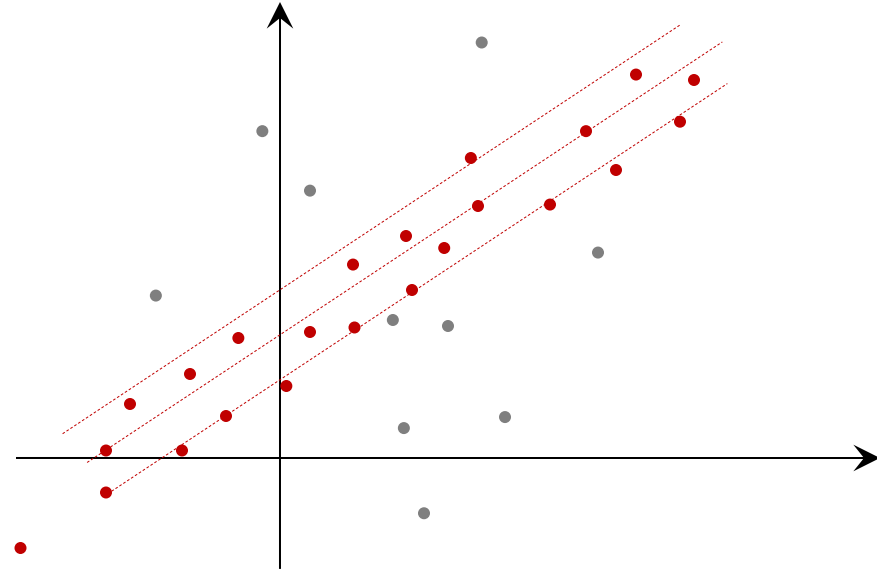




**Required number of iterations with  $p$  success rate:**

Prob. of success  $>$  Prob. of desired success  $p$

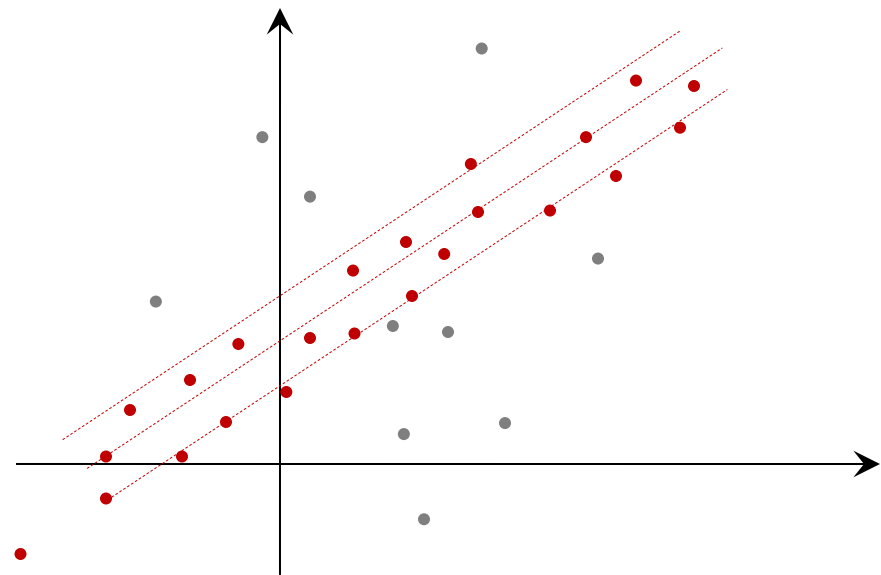




**Required number of iterations with  $p$  success rate:**

Prob. of success  $>$  Prob. of desired success  $p$

Prob. of success:  $1 - (1 - \text{prob. of success per trial})^k$



**Required number of iterations with  $p$  success rate:**

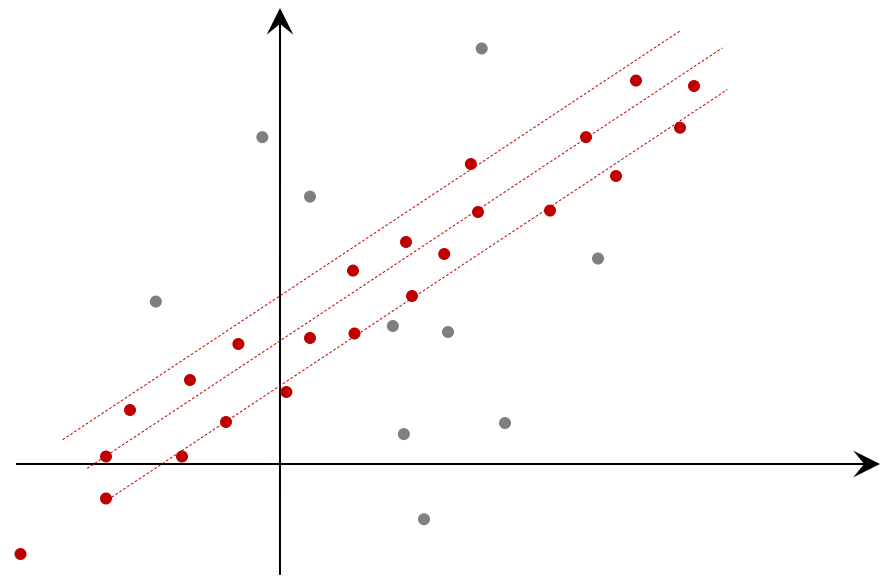
Prob. of success  $>$  Prob. of desired success  $p$

Prob. of success:  $1 - (1 - \text{prob. of success per trial})^k$

Prob. of success per trial:  $w^n$

where  $w = \frac{\text{\# of inliers}}{\text{\# of samples}}$

and  $n$  is the number of samples to build a model.



**Required number of iterations with  $p$  success rate:**

Prob. of success  $>$  Prob. of desired success  $p$

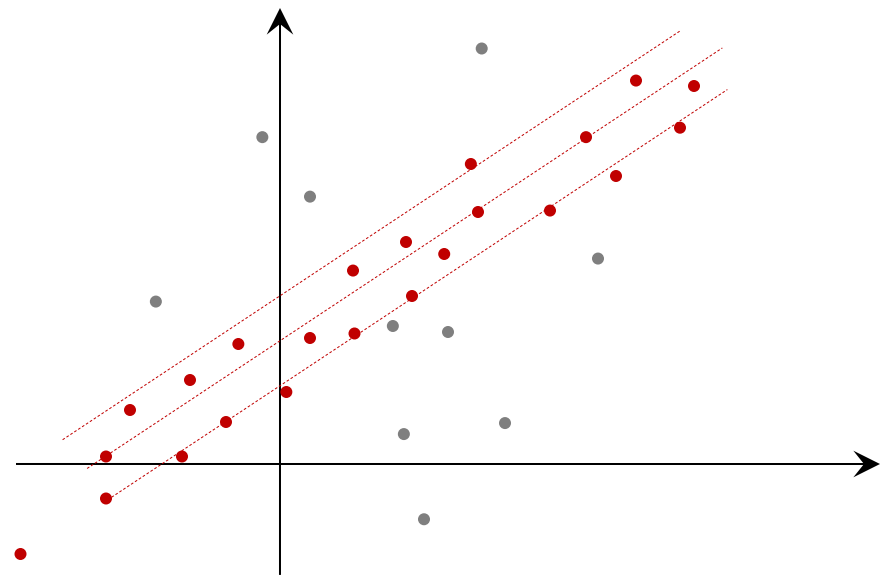
Prob. of success:  $1 - (1 - \text{prob. of success per trial})^k$

Prob. of success per trial:  $w^n$

where  $w = \frac{\text{\# of inliers}}{\text{\# of samples}}$

and  $n$  is the number of samples to build a model.

Prob. of success:  $1 - (1 - w^n)^k$



**Required number of iterations with  $p$  success rate:**

Prob. of success  $>$  Prob. of desired success  $p$

Prob. of success:  $1 - (1 - \text{prob. of success per trial})^k$

Prob. of success per trial:  $w^n$

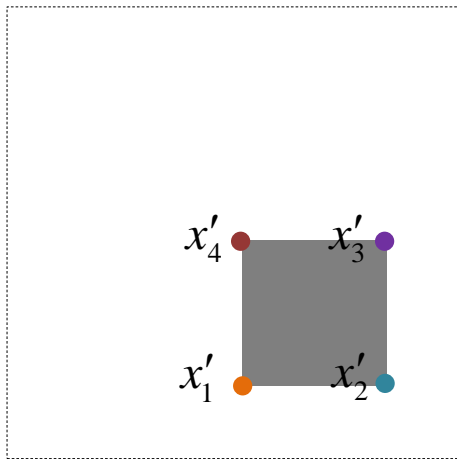
where  $w = \frac{\text{\# of inliers}}{\text{\# of samples}}$

and  $n$  is the number of samples to build a model.

Prob. of success:  $1 - (1 - w^n)^k$

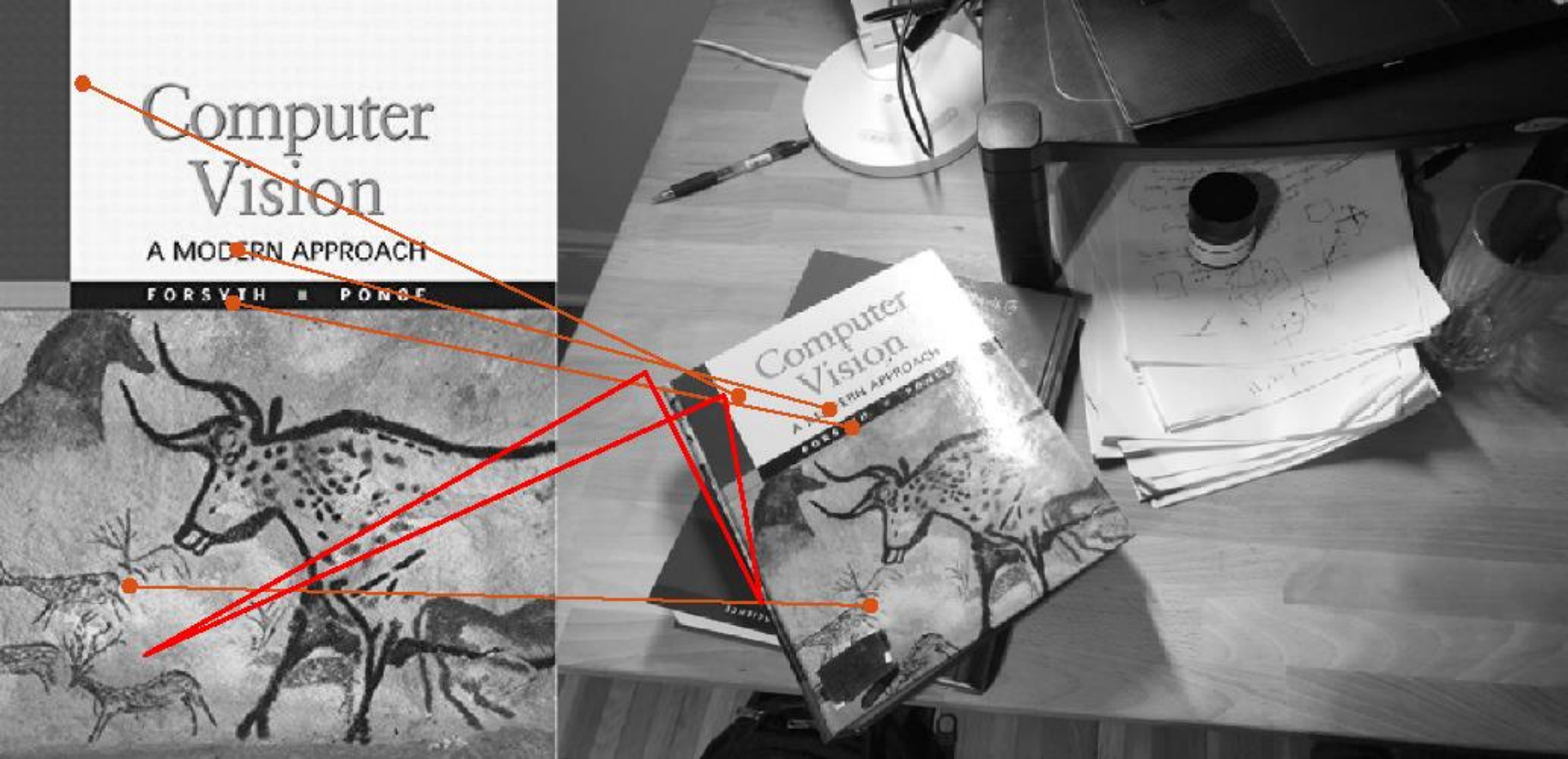
$$k = \frac{\log(1-p)}{\log(1-w^n)}$$

# RECALL: HOMOGRAPHY COMPUTATION



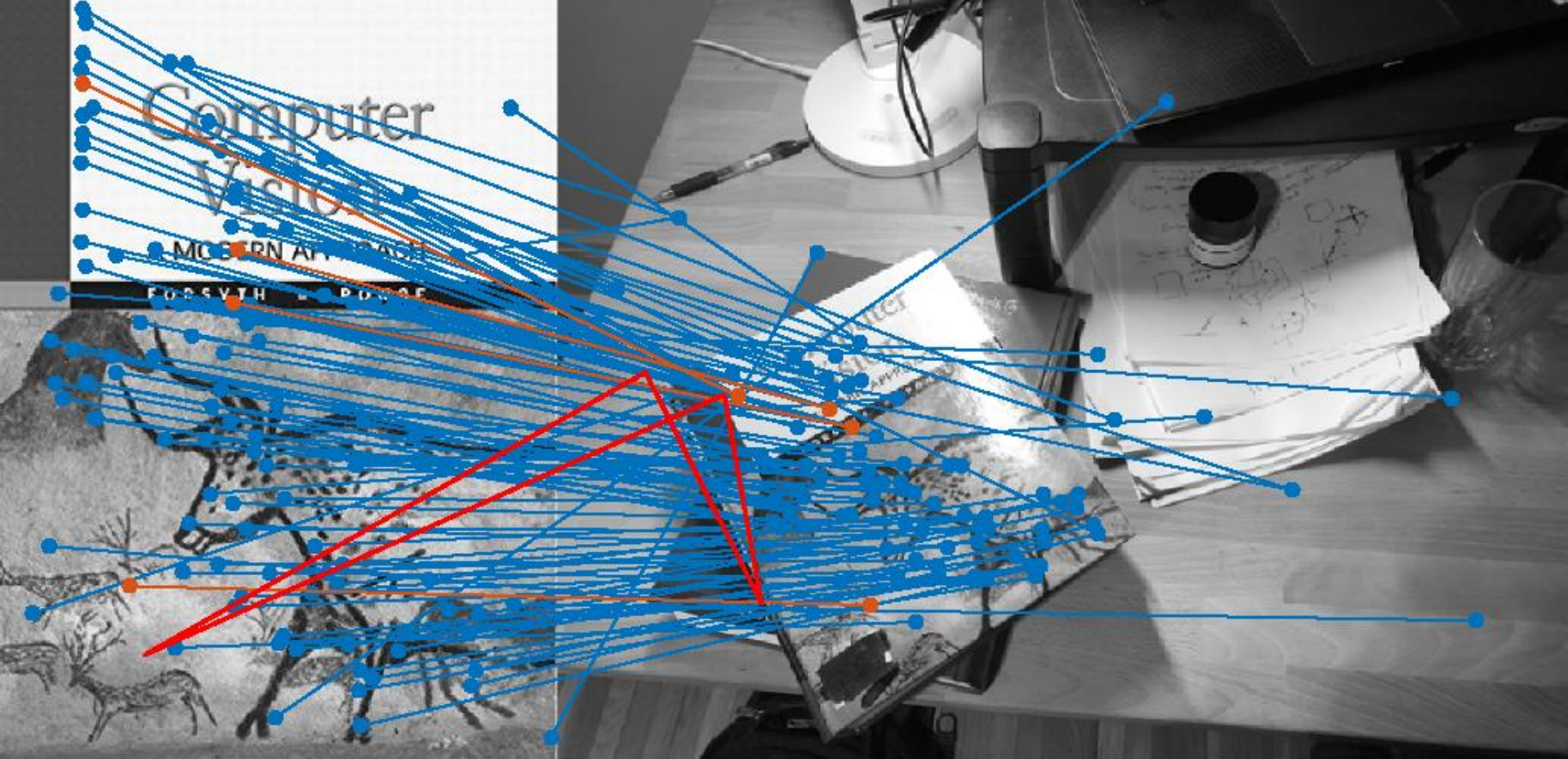
$$\begin{bmatrix}
 u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1 u'_1 & -v_1 u'_1 \\
 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1 v'_1 & -v_1 v'_1 \\
 u_4 & v_4 & 1 & 0 & 0 & 0 & -u_4 u'_4 & -v_4 u'_4 \\
 0 & 0 & 0 & u_4 & v_4 & 1 & -u_4 v'_4 & -v_4 v'_4
 \end{bmatrix}
 \begin{bmatrix}
 h_{11} \\
 h_{12} \\
 h_{13} \\
 h_{21} \\
 h_{22} \\
 h_{23} \\
 h_{31} \\
 h_{32}
 \end{bmatrix}
 x = \begin{bmatrix}
 u'_1 \\
 v'_1 \\
 u'_4 \\
 v'_4
 \end{bmatrix}$$

$$Ax = b \longrightarrow x = (A^T A)^{-1} A^T b$$



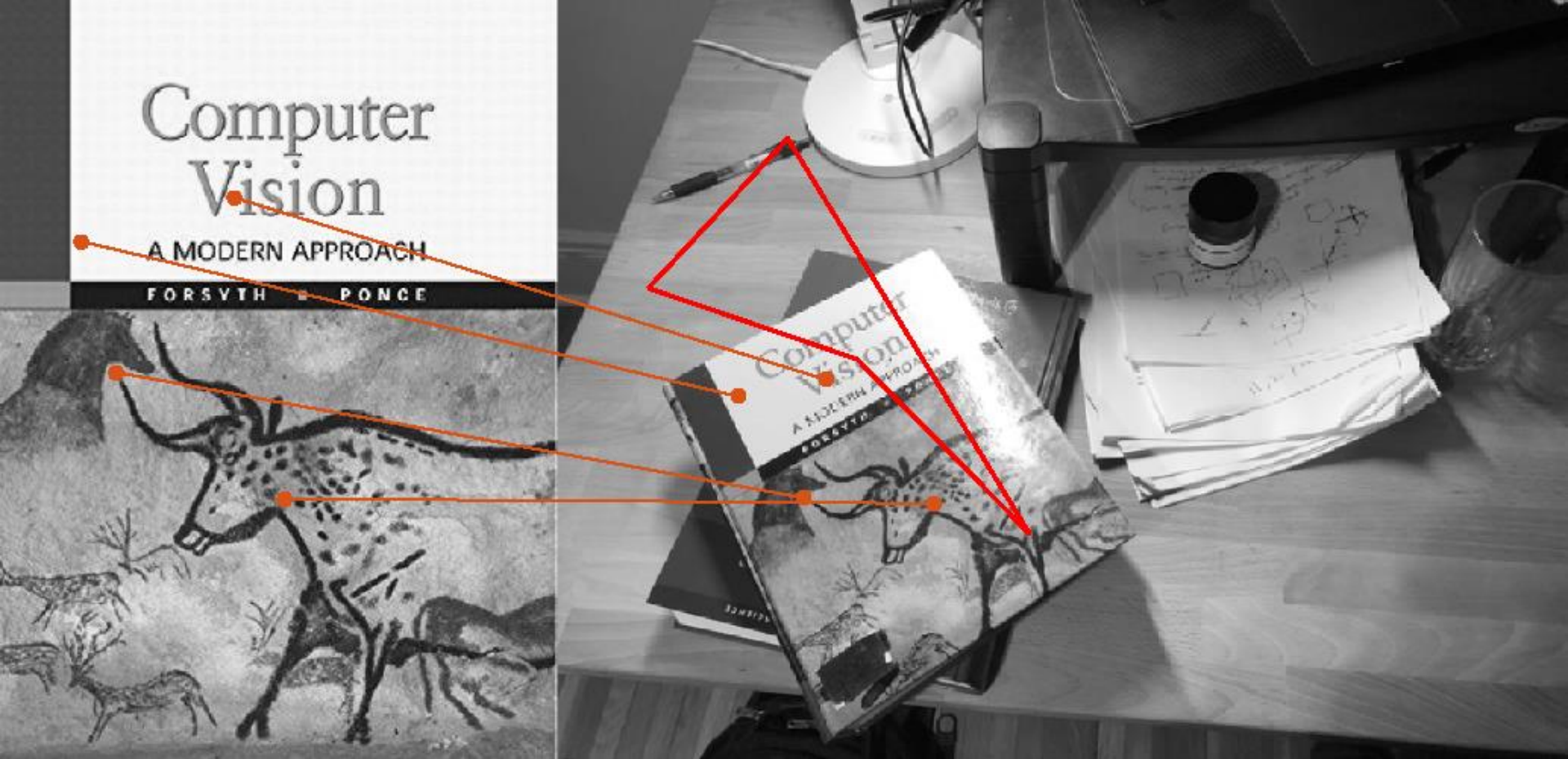
Homography from 4 random correspondences





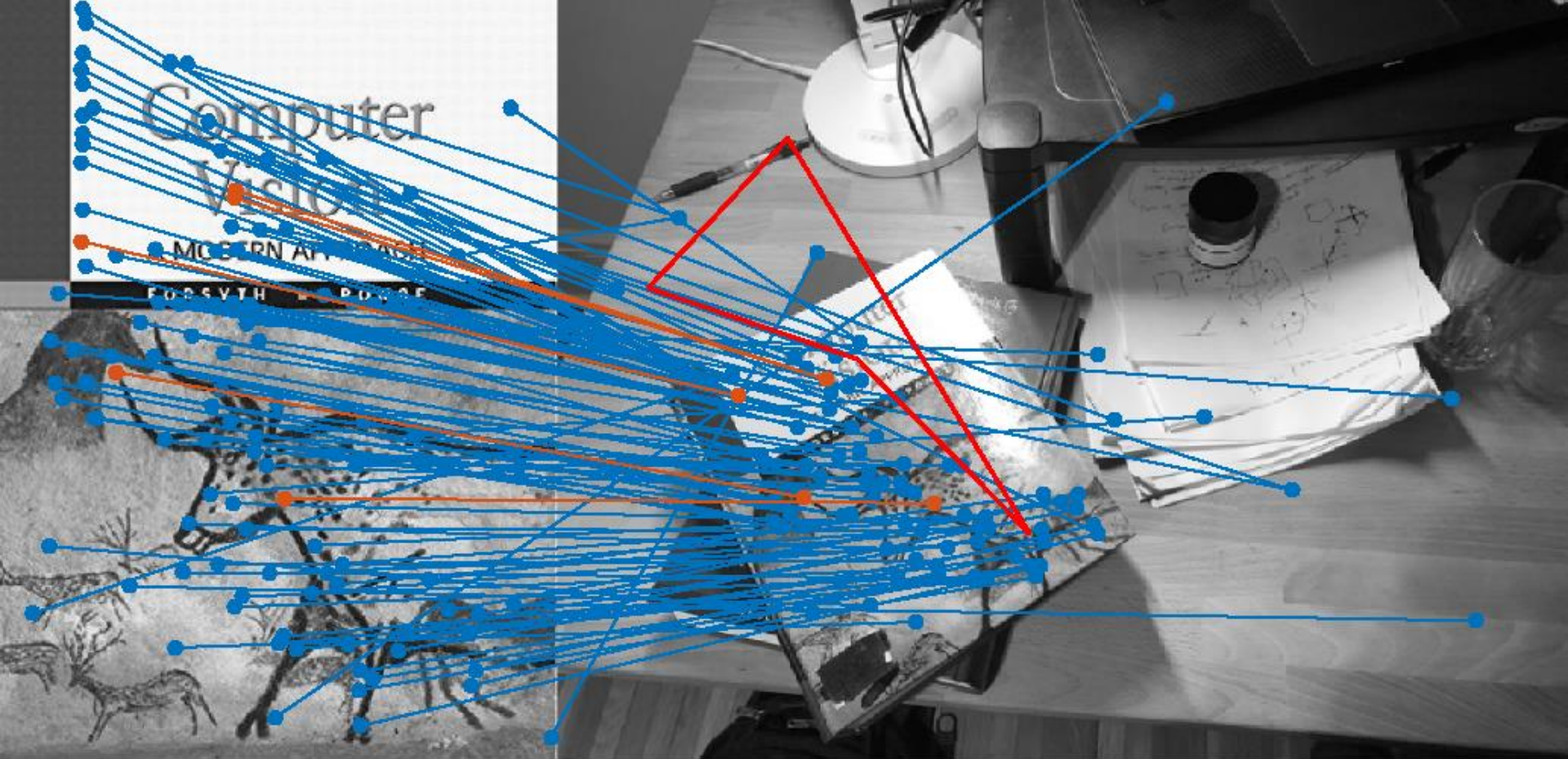
Inlier counting

Number of inliers: 5



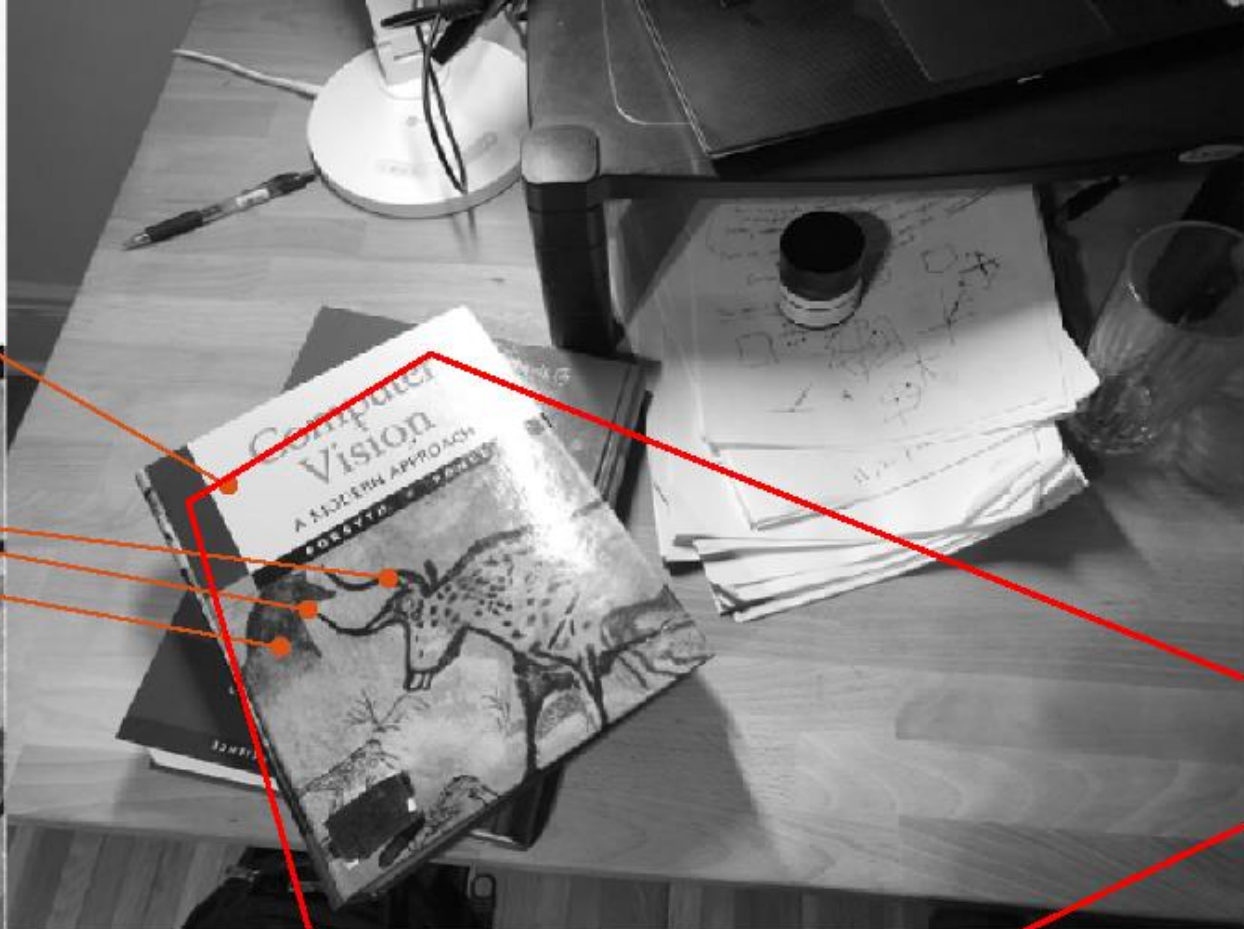
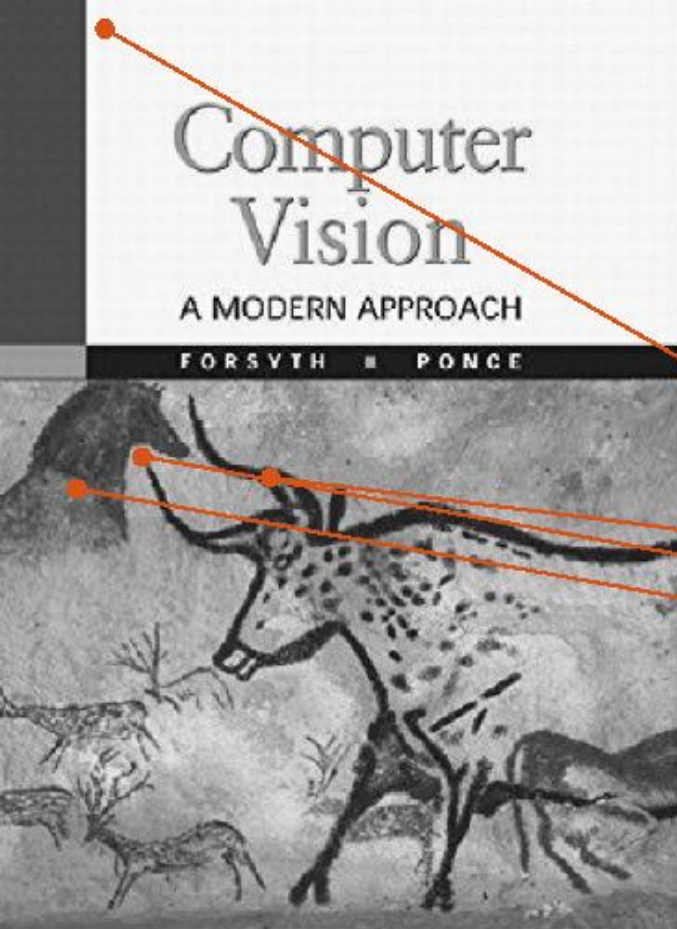
Homography from 4 random correspondences





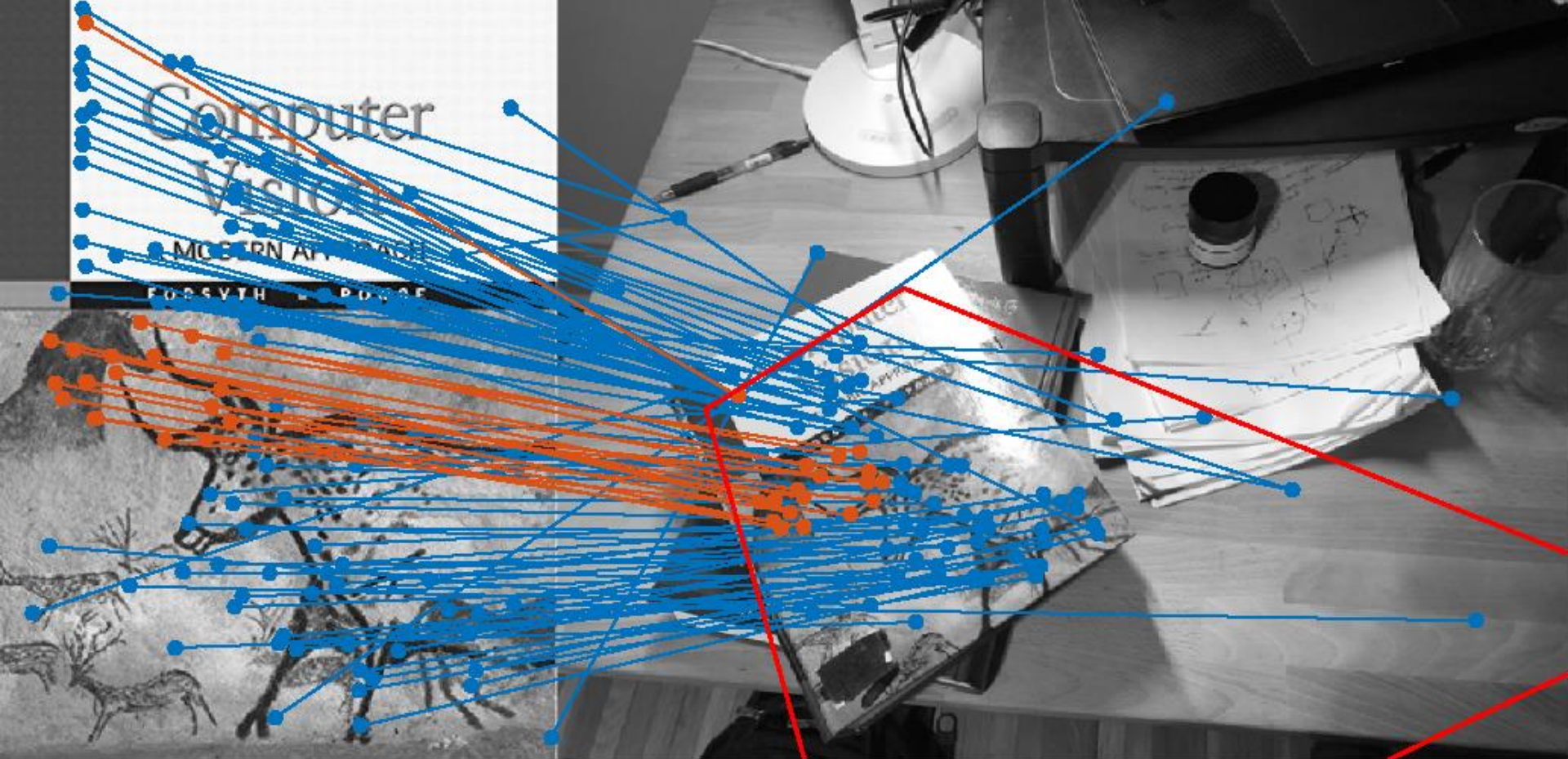
Inlier counting

Number of inliers: 8



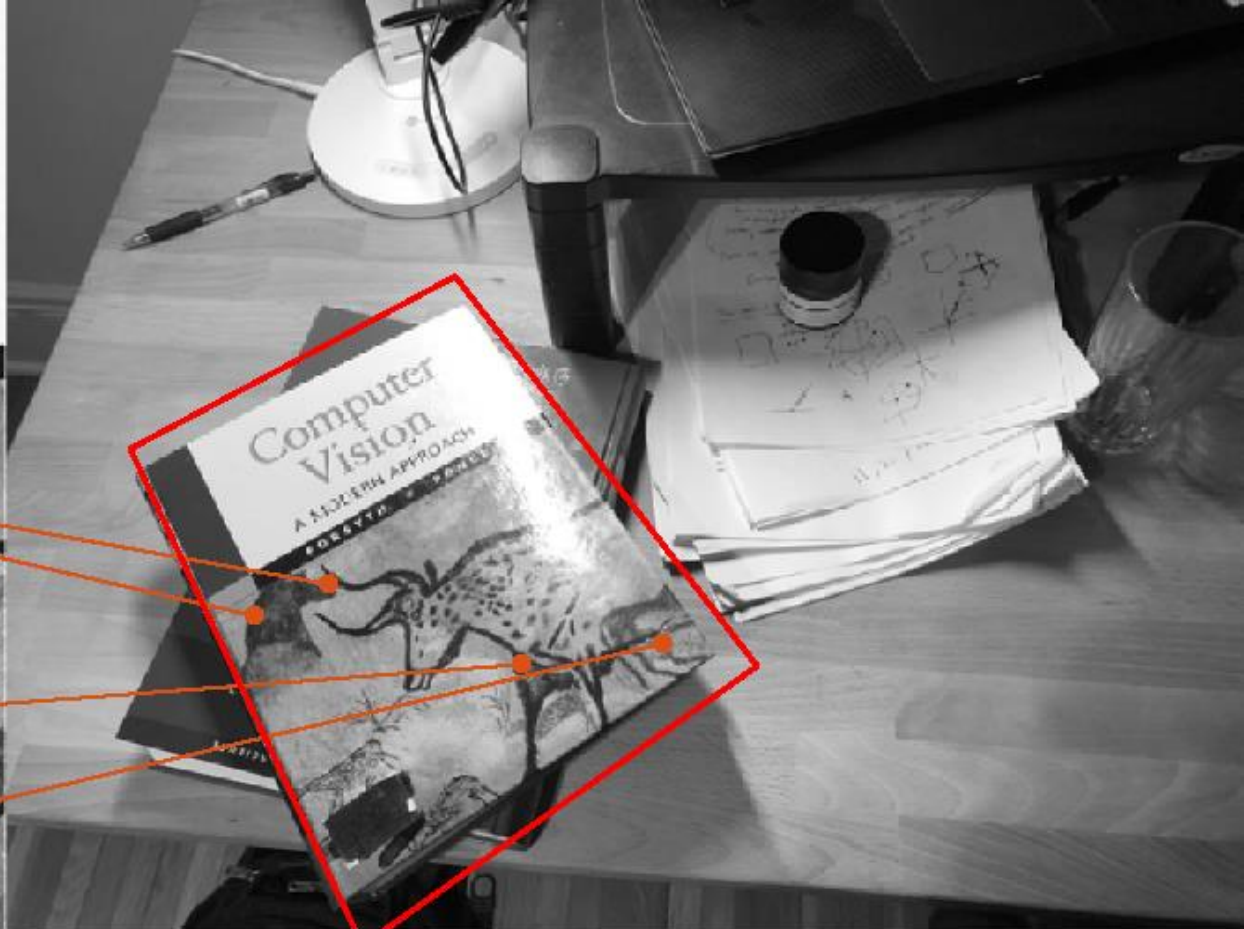
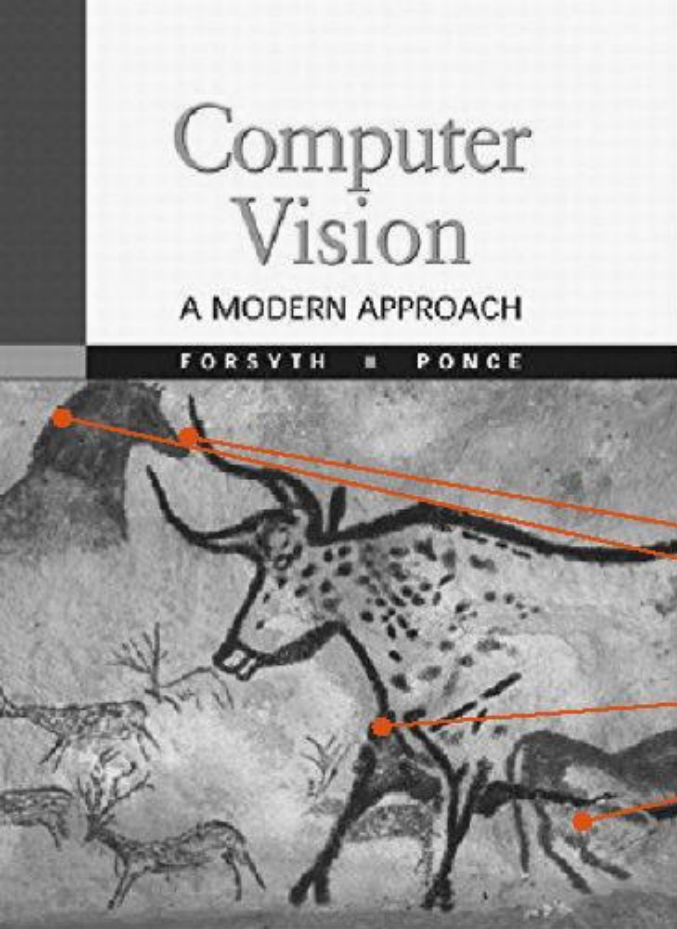
Homography from 4 random correspondences





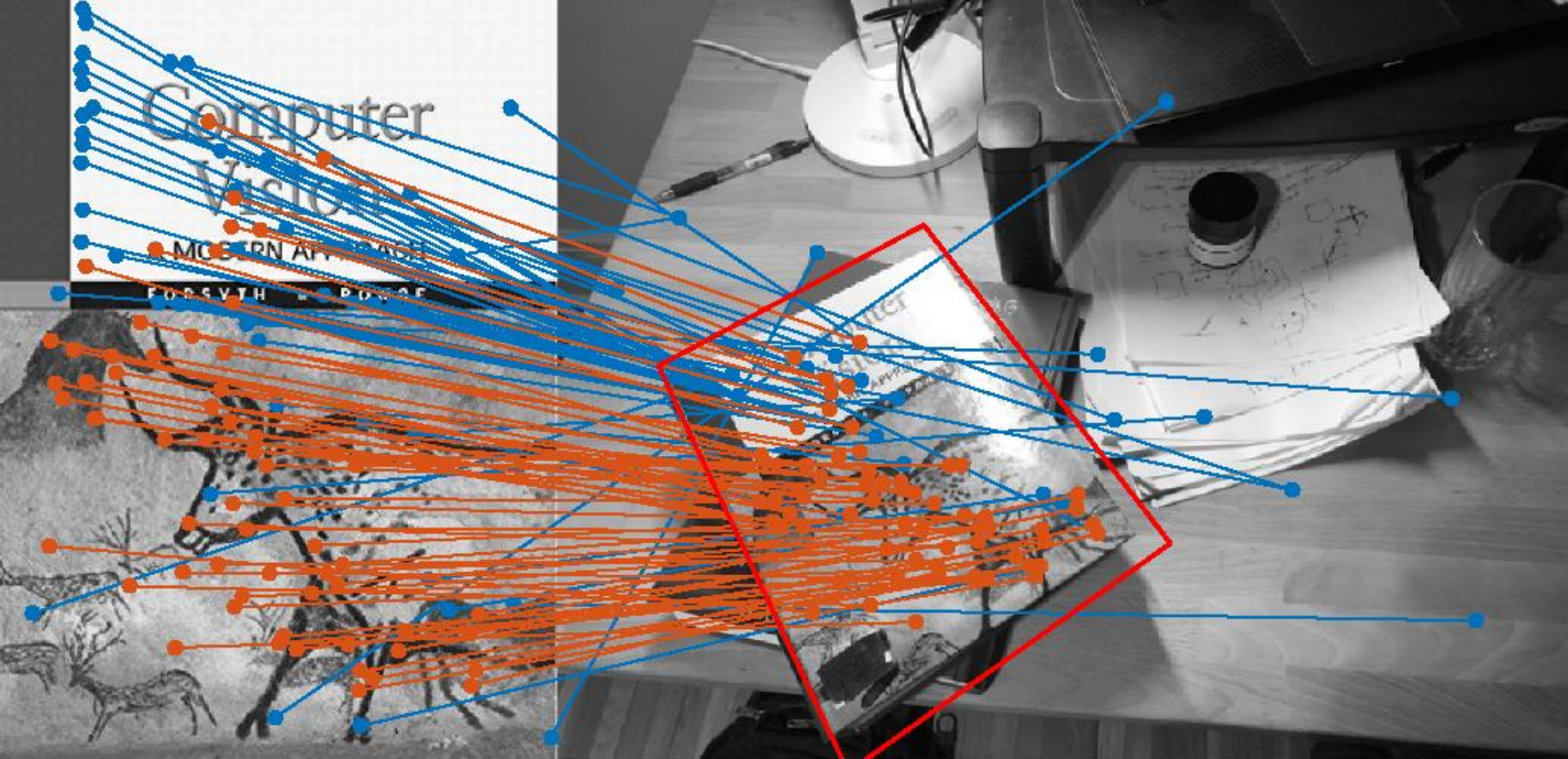
Inlier counting

Number of inliers: 25



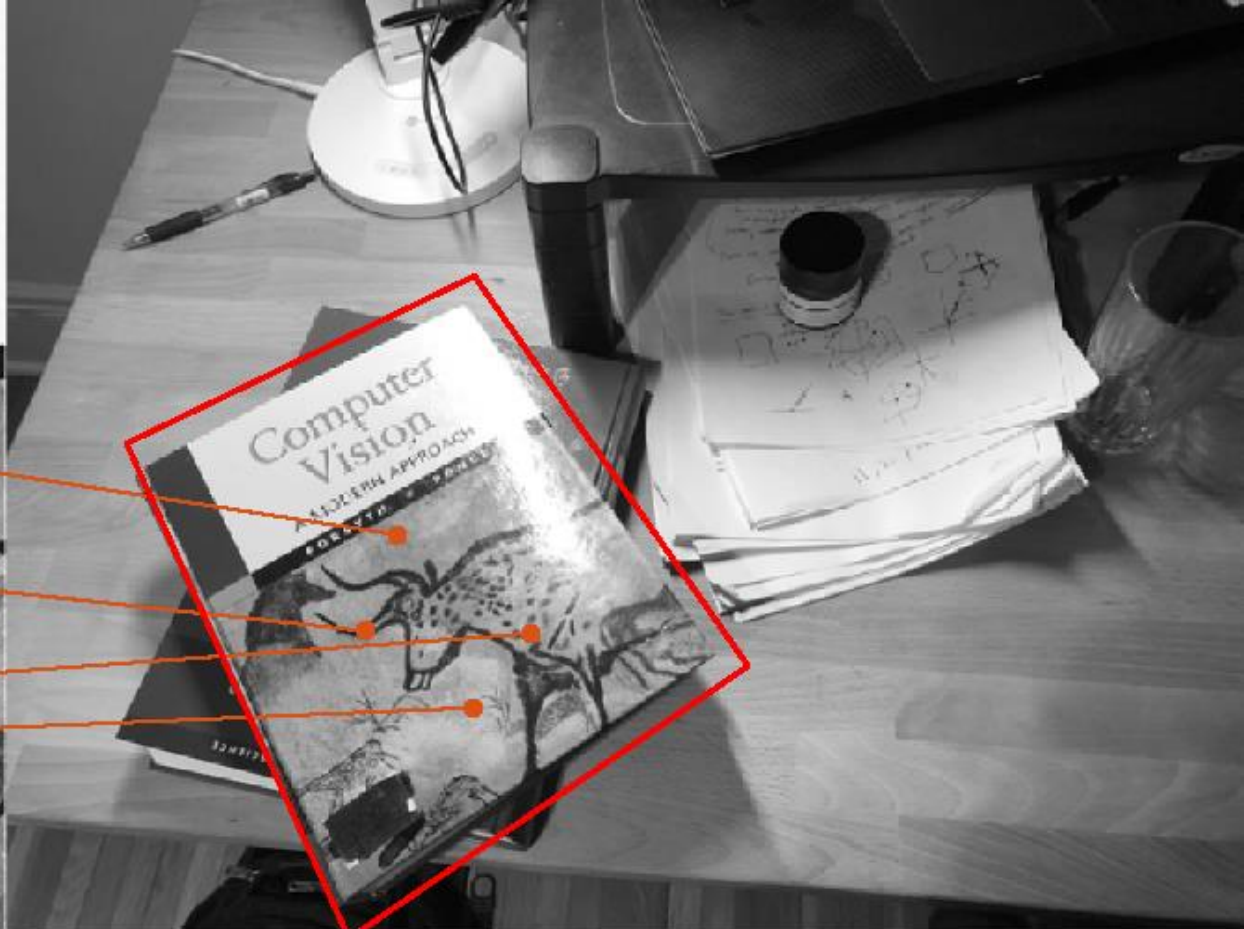
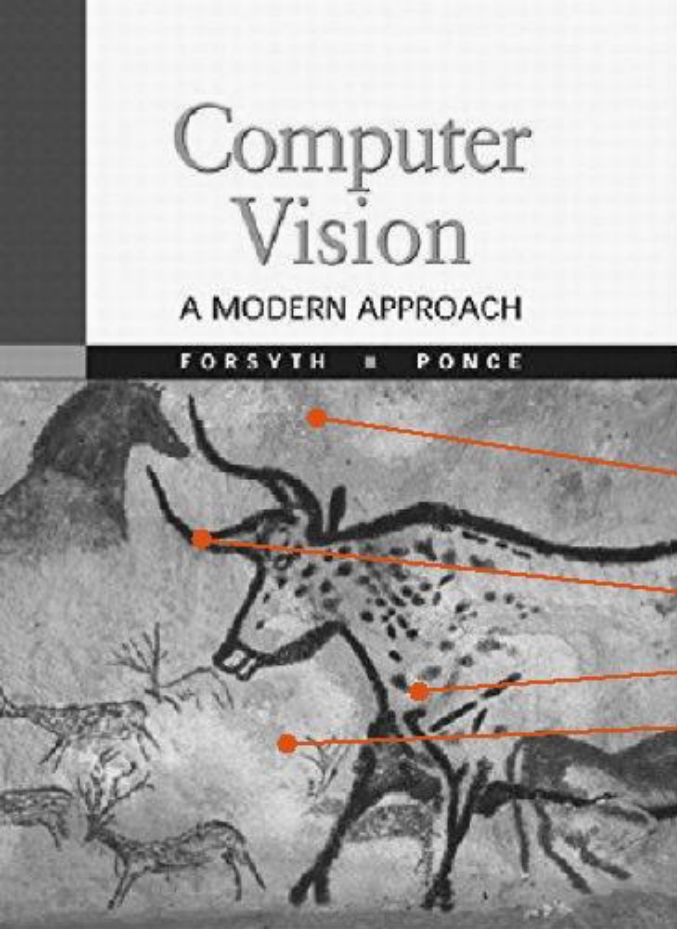
Homography from 4 random correspondences





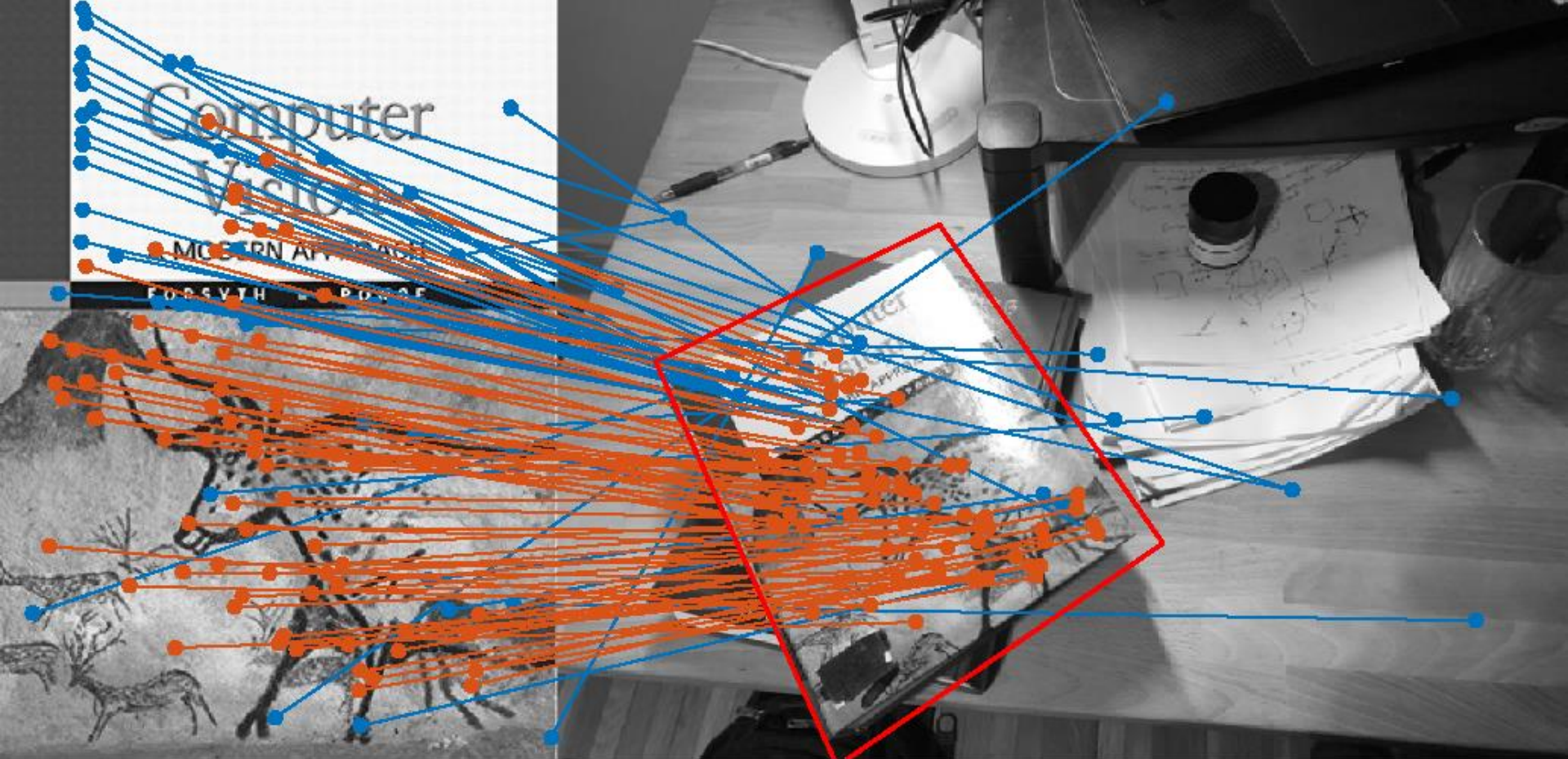
Inlier counting

Number of inliers: 76



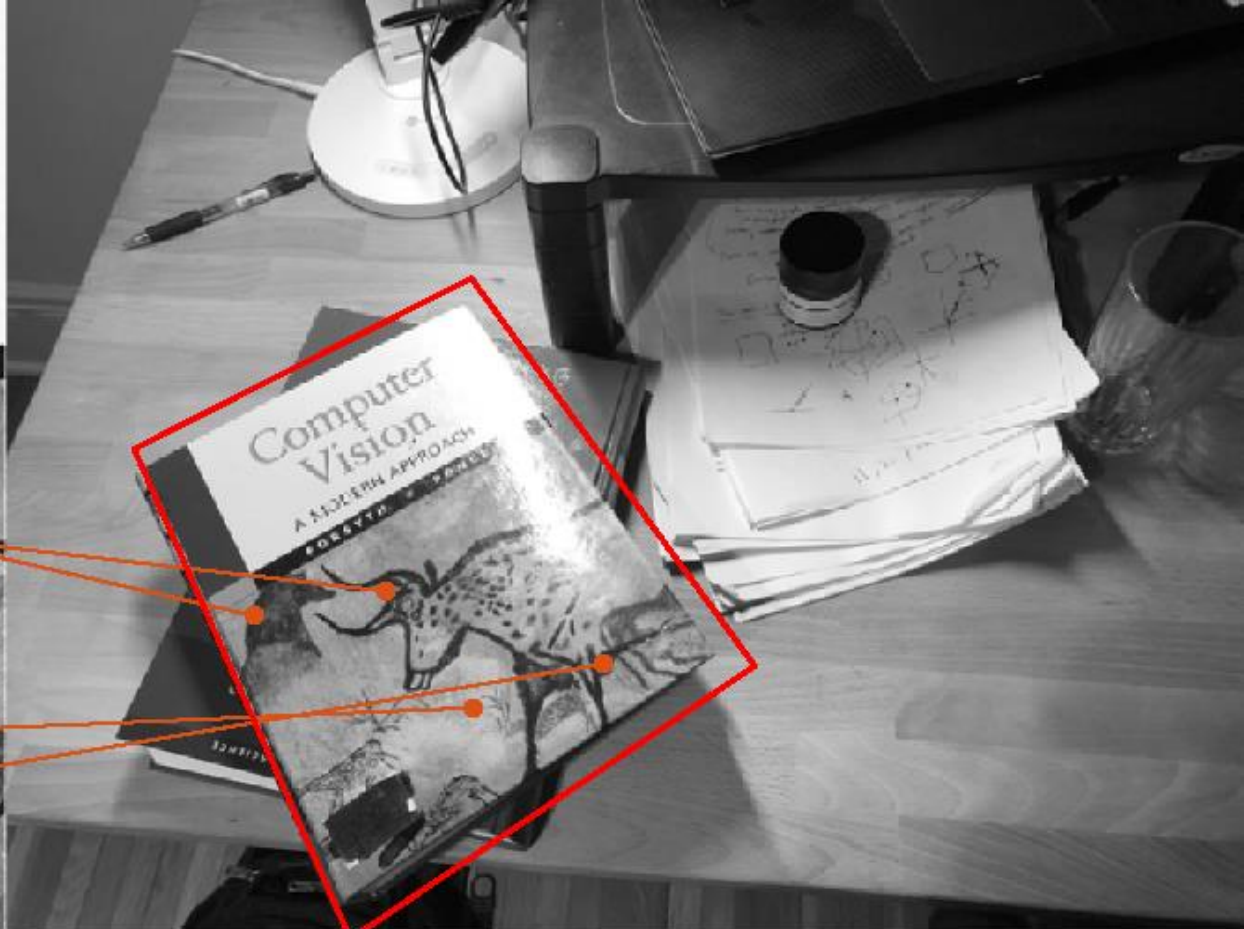
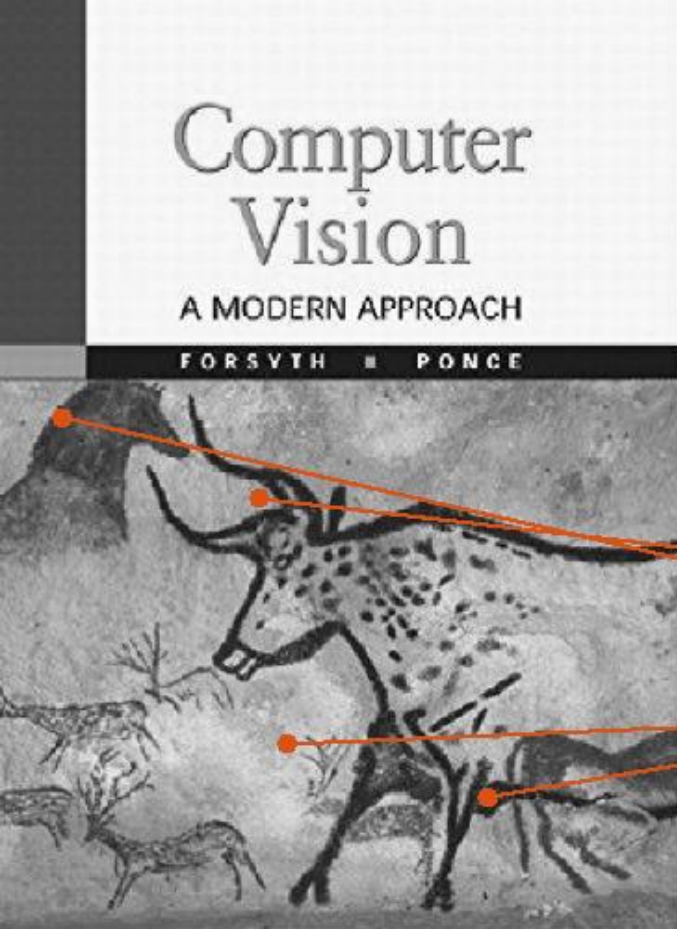
Homography from 4 random correspondences





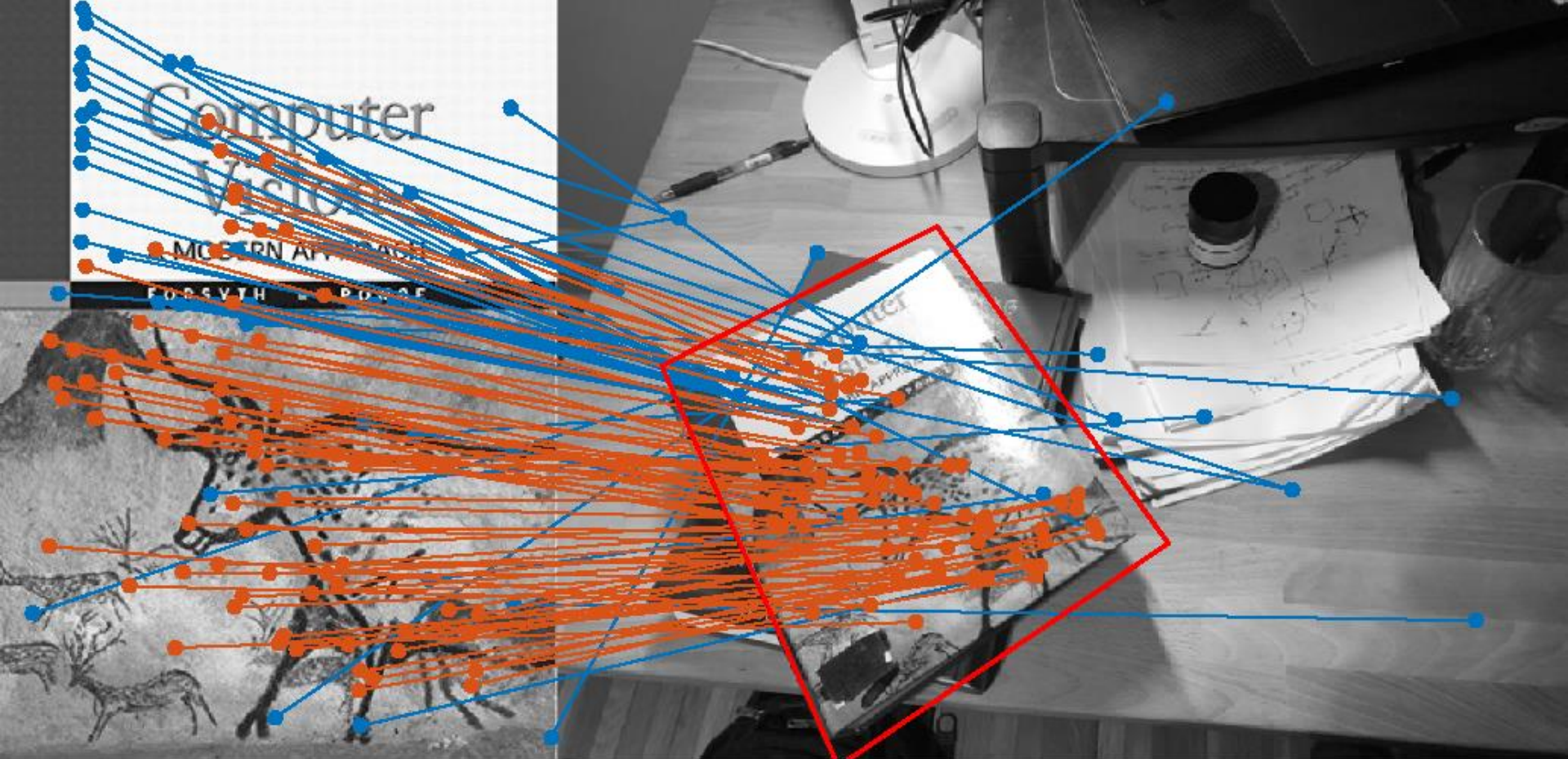
Inlier counting

Number of inliers: 83



Homography from 4 random correspondences

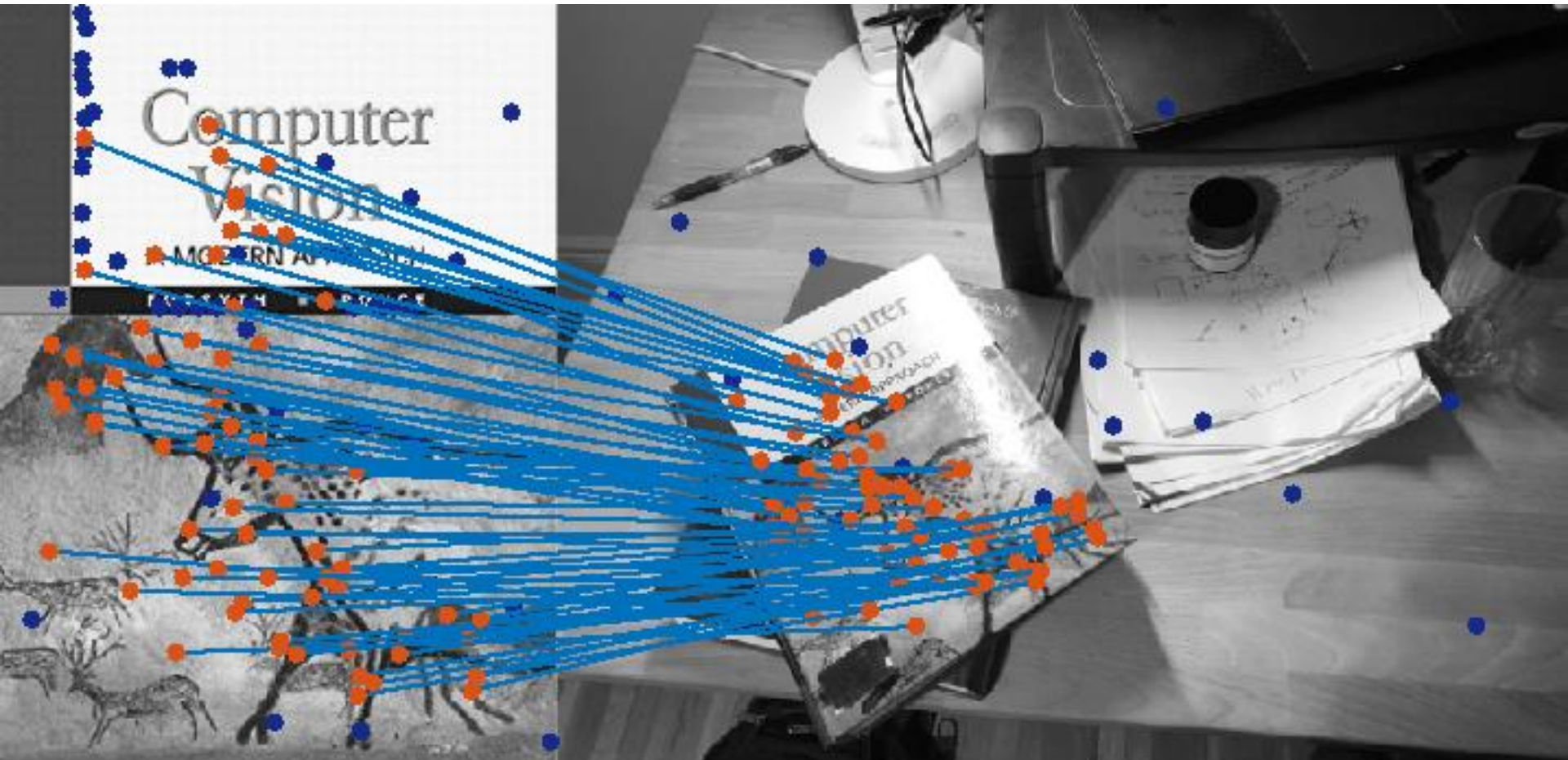




Inlier counting

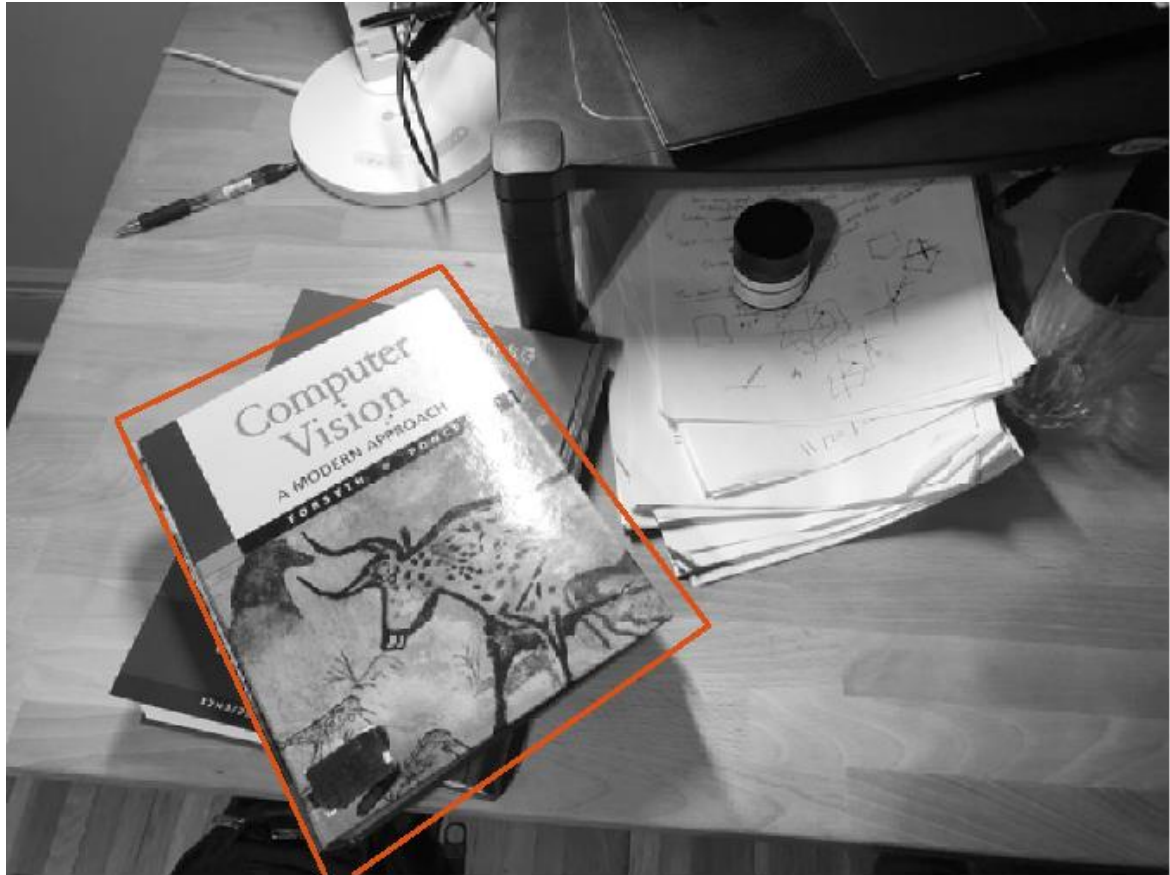
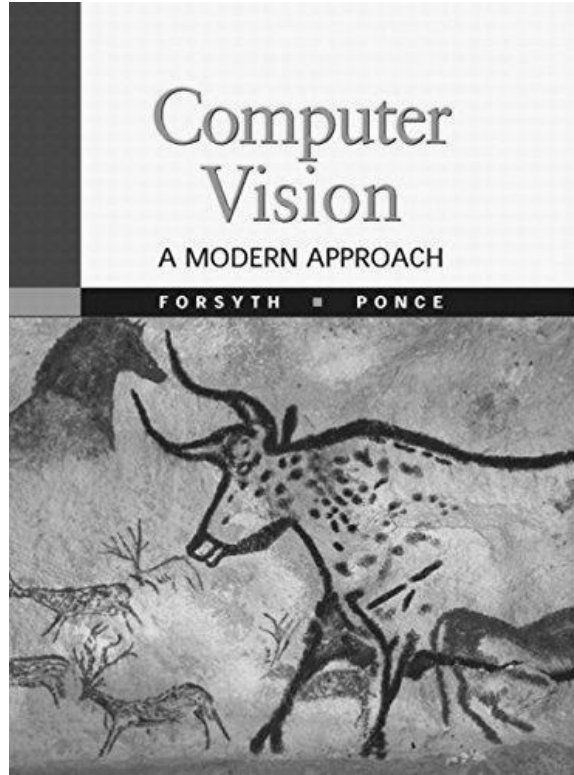
Number of inliers: 85

# *RECALL: ROBUST FILTERING*





# *RECALL: PARAMETRIC MODEL*



# *RECALL: IMAGE WARPING*

