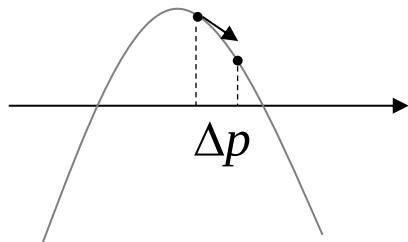
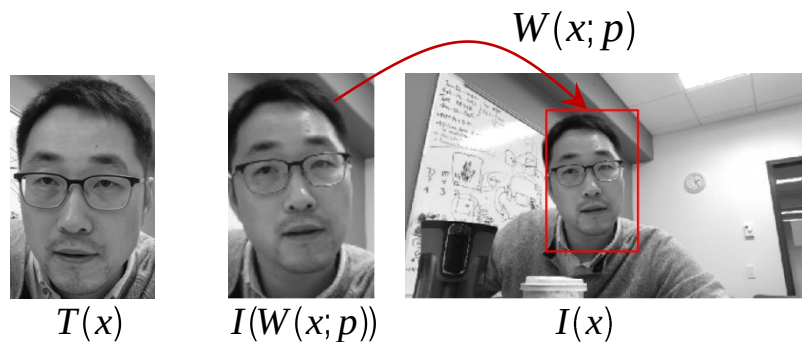


COMPOSITIONAL WARPING

HYUN SOO PARK



RECALL: IMAGE ALIGNMENT OBJECTIVE



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$



2. Find Δp that minimizes the obj. function at p

$$I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x) = 0 \longrightarrow \nabla I \frac{\partial W}{\partial p} \Delta p = T(x) - I(W(x; p))$$

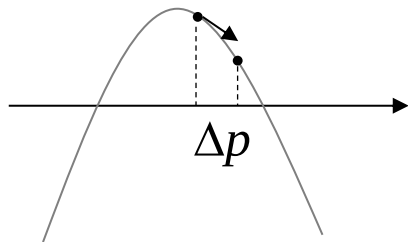
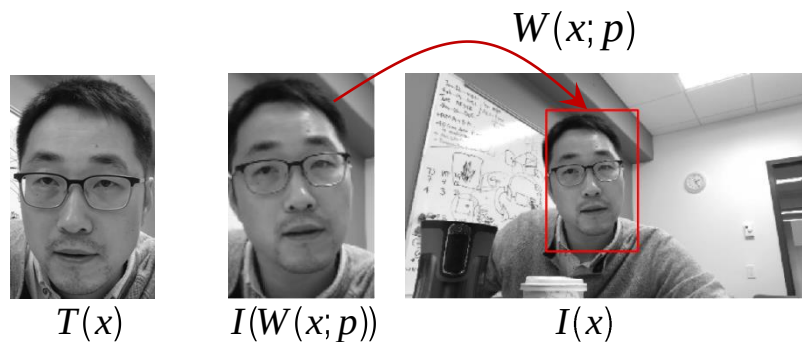
$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$$

$$\text{where } H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$$



3. Update $p \leftarrow p + \Delta p$

COMPUTATIONAL ASPECT



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$

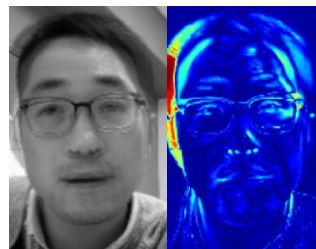


2. Find Δp that minimizes the obj. function at p

$$I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x) = 0 \longrightarrow \nabla I \frac{\partial W}{\partial p} \Delta p = T(x) - I(W(x; p))$$

$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$$

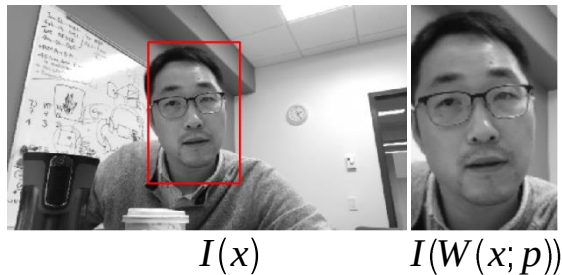
$$\text{where } H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$$



3. Update $p \leftarrow p + \Delta p$

LUCAS-KANADE ALGORITHM

1. Warp the target image $I(W(x; p))$



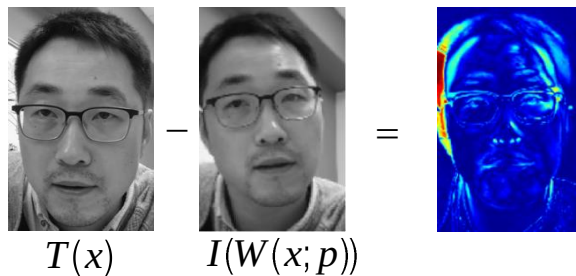
$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (I(x) - I(W(x; p)))$$

$$\text{where } H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$$

$$p \leftarrow p + \Delta p$$

LUCAS-KANADE ALGORITHM

1. Warp the target image $I(W(x;p))$
2. Compute the error image $T(x) - I(W(x;p))$



$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x;p)))$$

$$\text{where } H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$$

$$p \leftarrow p + \Delta p$$

LUCAS-KANADE ALGORITHM

1. Warp the target image $I(W(x; p))$
2. Compute the error image $T(x) - I(W(x; p))$
3. Warp the gradient image

$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$$

$$\text{where } H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$$

$$p \leftarrow p + \Delta p$$



$\nabla I(x)$



$\nabla I(W(x; p))$

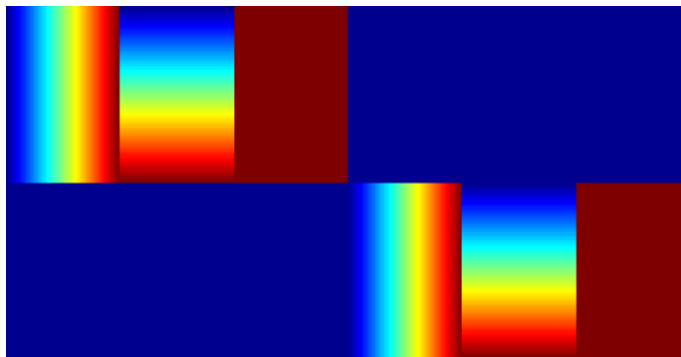
LUCAS-KANADE ALGORITHM

1. Warp the target image $I(W(x; p))$
2. Compute the error image $T(x) - I(W(x; p))$
3. Warp the gradient image $\nabla I(W(x; p))$
4. Compute Jacobian $\frac{\partial W}{\partial p}$

$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$$

$$\text{where } H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$$

$$p \leftarrow p + \Delta p$$



LUCAS-KANADE ALGORITHM

1. Warp the target image $I(W(x; p))$
2. Compute the error image $T(x) - I(W(x; p))$
3. Warp the gradient image $\nabla I(W(x; p))$
4. Compute Jacobian $\frac{\partial W}{\partial p}$
5. Compute steepest descent images $\nabla I \frac{\partial W}{\partial p}$

$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$$

$$\text{where } H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$$

$$p \leftarrow p + \Delta p$$



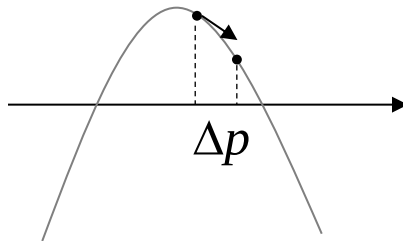
LUCAS-KANADE ALGORITHM

1. Warp the target image $I(W(x; p))$
2. Compute the error image $T(x) - I(W(x; p))$
3. Warp the gradient image $\nabla I(W(x; p))$
4. Compute Jacobian $\frac{\partial W}{\partial p}$
5. Compute steepest descent images $\nabla I \frac{\partial W}{\partial p}$
6. Compute Hessian $H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$
7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$

$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$$

$$\text{where } H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$$

$$p \leftarrow p + \Delta p$$



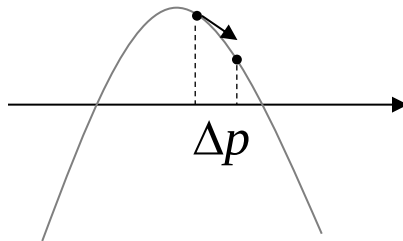
LUCAS-KANADE ALGORITHM

1. Warp the target image $I(W(x; p))$
2. Compute the error image $T(x) - I(W(x; p))$
3. Warp the gradient image $\nabla I(W(x; p))$
4. Compute Jacobian $\frac{\partial W}{\partial p}$
5. Compute steepest descent images $\nabla I \frac{\partial W}{\partial p}$
6. Compute Hessian $H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$
7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$
8. Update $p \leftarrow p + \Delta p$
9. Goto 1 unless $\|\Delta p\| < \varepsilon$

$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$$

where $H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$

$$p \leftarrow p + \Delta p$$



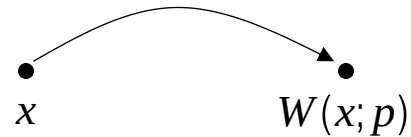
COMPUTATION BOTTLENECK

1. Warp the target image $I(W(x; p))$
2. Compute the error image $T(x) - I(W(x; p))$
3. Warp the gradient image $\nabla I(W(x; p))$
4. Compute Jacobian $\frac{\partial W}{\partial p}$
5. Compute steepest descent images $\nabla I \frac{\partial W}{\partial p}$
6. Compute Hessian $H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$
7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$
8. Update $p \leftarrow p + \Delta p$
9. Goto 1 unless $\|\Delta p\| < \varepsilon$

ADDITIVE VS. COMPOSITIONAL

Additive mapping

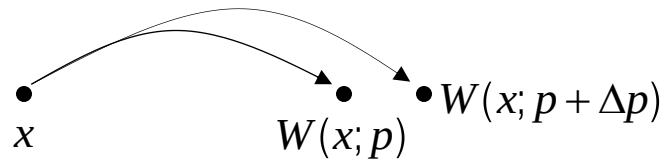
$$I(W(x; p + \Delta p)) \approx I(W(x; p))$$



ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \Big|_p \frac{\partial W}{\partial p} \Big|_p \Delta p$$



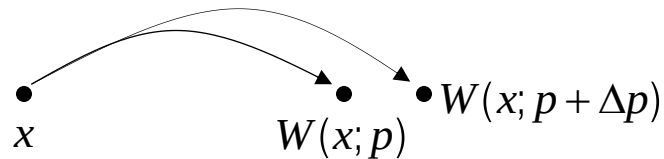
ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \Big|_p \frac{\partial W}{\partial p} \Big|_p \Delta p$$

Compositional mapping I

$$I(W(W(x; p); \Delta p))$$



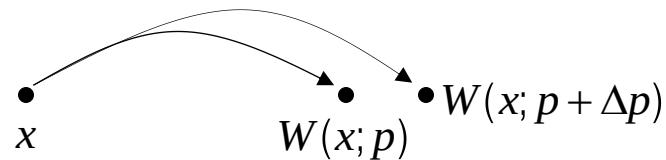
ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \Big|_p \frac{\partial W}{\partial p} \Big|_p \Delta p$$

Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(W(x; p); 0)) + \nabla I(W) \frac{\partial W}{\partial p} \Big|_p \Delta p$$



Note) $(f \circ g)' = (f' \circ g)g'$

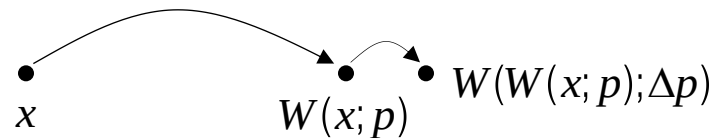
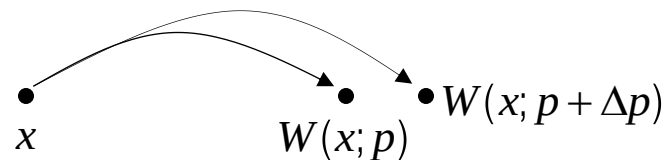
ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \Big|_p \frac{\partial W}{\partial p} \Big|_p \Delta p$$

Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(W(x; p); 0)) + \nabla I(W) \frac{\partial W}{\partial p} \Big|_p \Delta p$$



Note) $(f \circ g)' = (f' \circ g)g'$

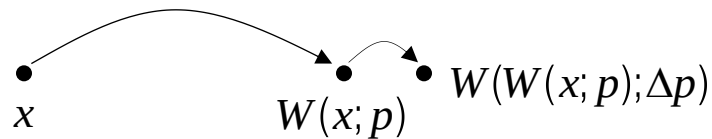
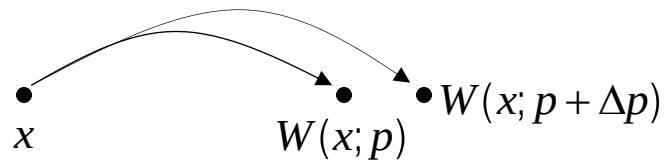
ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

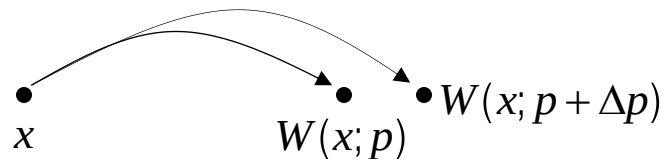


Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

ADDITIVE VS. COMPOSITIONAL

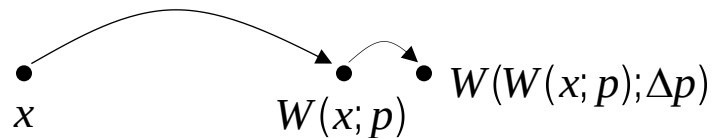
Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$



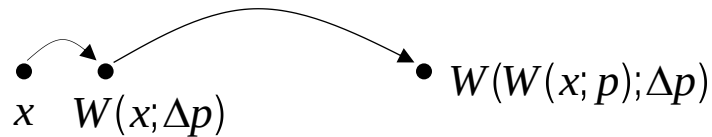
Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$



Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(W(x; 0); p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

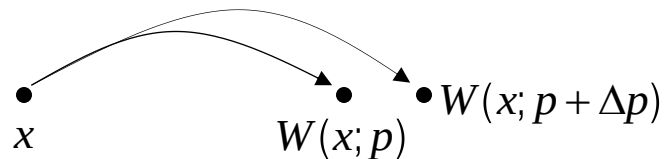


Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

ADDITIVE VS. COMPOSITIONAL

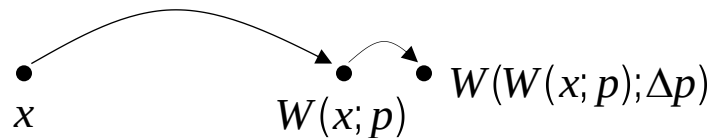
Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$



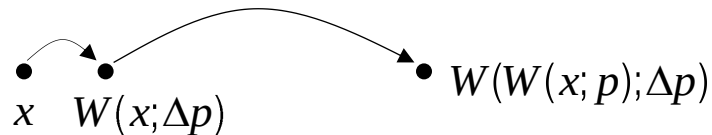
Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$



Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$



Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Big|_p \Delta p$$

Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Big|_p \Delta p$$

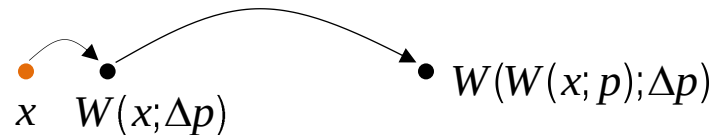
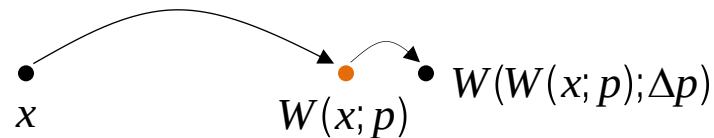
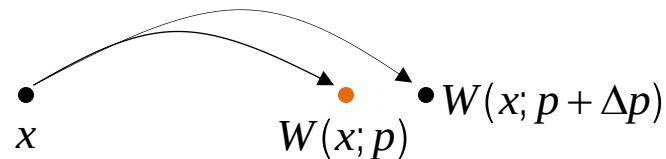
Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \boxed{\frac{\partial W}{\partial p} \Big|_0} \Delta p$$

constant

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

● Location of linearization



$\frac{\partial W}{\partial p} \Big|_0$ is constant

ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

constant

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

1. Warp the target image $I(W(x; p))$
2. Compute the error image $T(x) - I(W(x; p))$
3. Warp the gradient image $\nabla I(W(x; p))$
4. Compute Jacobian $\frac{\partial W}{\partial p}$
5. Compute steepest descent images $\nabla I \frac{\partial W}{\partial p}$
6. Compute Hessian $H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$
7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$
8. Update $p \leftarrow p + \Delta p$
9. Goto 1 unless $\|\Delta p\| < \varepsilon$

ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

constant

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

1. Warp the target image $I(W(x; p))$
2. Compute the error image $T(x) - I(W(x; p))$
3. Warp the gradient image $\nabla I(W(x; p))$
4. Compute Jacobian $\frac{\partial W}{\partial p}$
5. Compute steepest descent images $\nabla I \frac{\partial W}{\partial p}$
6. Compute Hessian $H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$
7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$
8. Update $p \leftarrow p + \Delta p$
9. Goto 1 unless $\|\Delta p\| < \varepsilon$

ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

$$W(x; p + \Delta p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 + \Delta p_1 & p_2 + \Delta p_2 & p_3 + \Delta p_3 \\ p_4 + \Delta p_4 & p_5 + \Delta p_5 & p_6 + \Delta p_6 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

constant

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

$$W(x; p + \Delta p) = \begin{bmatrix} 1 + p_1 + \Delta p_1 & p_2 + \Delta p_2 & p_3 + \Delta p_3 \\ p_4 + \Delta p_4 & 1 + p_5 + \Delta p_5 & p_6 + \Delta p_6 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

constant

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

constant

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

Abuse of notation

$$W(x; p + \Delta p) = \begin{bmatrix} 1 + p_1 + \Delta p_1 & p_2 + \Delta p_2 & p_3 + \Delta p_3 \\ p_4 + \Delta p_4 & 1 + p_5 + \Delta p_5 & p_6 + \Delta p_6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

ADDITIVE VS. COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

Compositional mapping I

$$I(W(W(x; p); \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_p \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

constant

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

Abuse of notation

$$W(x; p + \Delta p) = A(p + \Delta p) \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

$$W(W(x; \Delta p); p) = A(p)A(\Delta p) \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

LUCAS-KANADE

1. Warp the target image $I(W(x; p))$
2. Compute the error image $T(x) - I(W(x; p))$
3. Warp the gradient image $\nabla I(W(x; p))$
4. Compute Jacobian $\frac{\partial W}{\partial p}$
5. Compute steepest descent images $\nabla I \frac{\partial W}{\partial p}$
6. Compute Hessian $H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$
7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$
8. Update $p \leftarrow p + \Delta p$
9. Goto 1 unless $\|\Delta p\| < \varepsilon$

COMPOSITIONAL ALIGNMENT

1. Compute Jacobian $\frac{\partial W}{\partial p}$
-
2. Warp the target image $I(W(x; p))$
 3. Compute the error image $T(x) - I(W(x; p))$
 4. Warp the gradient image $\nabla I(W(x; p))$
 5. Compute steepest descent images $\nabla I \frac{\partial W}{\partial p}$
 6. Compute Hessian $H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$
 7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$
 8. Update $W(x; p) \leftarrow W(x; p) \circ W(x; \Delta p)$
 9. Goto 2 unless $\|\Delta p\| < \varepsilon$

DUALITY

$$p^* = \underset{p}{\text{minimize}} \sum_x \left(I(W(x; p)) - T(x) \right)^2$$



$T(x)$



$I(W(x; p))$

$$p^* = \underset{p}{\text{minimize}} \sum_x \left(I(x) - T(W^{-1}(x; p)) \right)^2$$



$T(W^{-1}(x; p))$



$I(x)$

INVERSE COMPOSITIONAL

Additive mapping

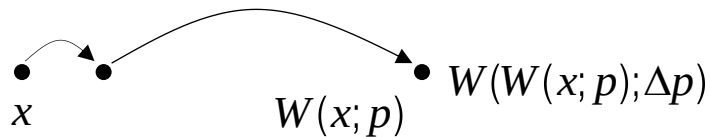
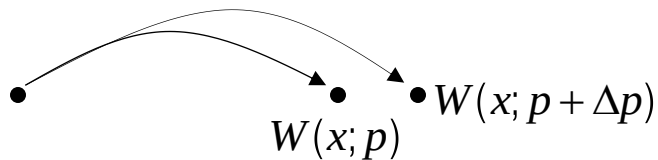
$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Big|_0 \Delta p$$

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

- Location of linearization



INVERSE COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

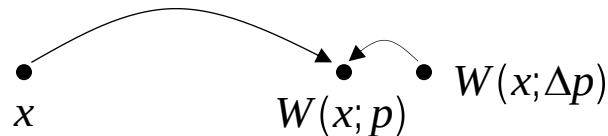
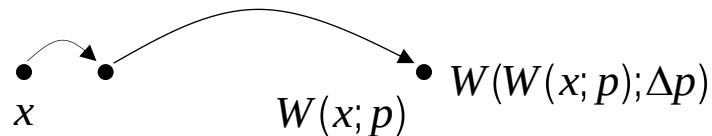
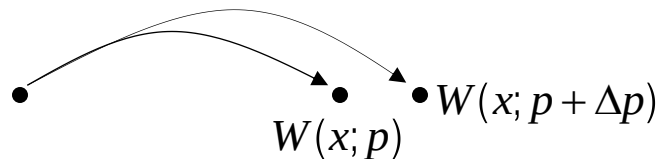
Inverse compositional mapping

$$I(W(x; p))$$

$$T(W(x; \Delta p)) \approx T(W(x; 0)) + \nabla T(x) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

● Location of linearization



INVERSE COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Big|_0 \Delta p$$

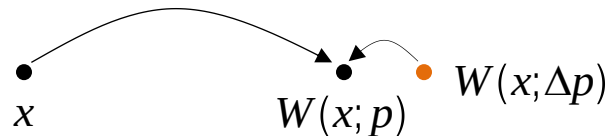
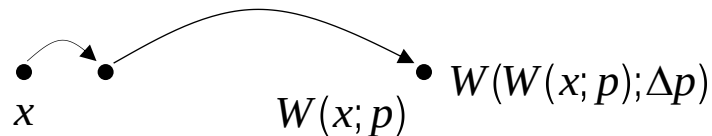
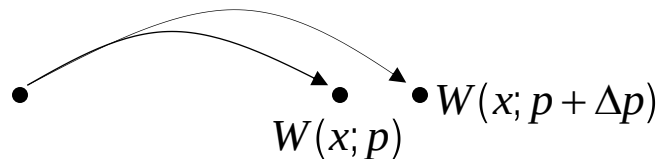
Inverse compositional mapping

$$I(W(x; p))$$

$$T(W(x; \Delta p)) \approx T(x) + \nabla T(x) \frac{\partial W}{\partial p} \Big|_0 \Delta p$$

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

● Location of linearization



DUALITY

$$p^* = \underset{p}{\text{minimize}} \sum_x \left(I(W(x; p)) - T(x) \right)^2$$



$T(x)$



$I(W(x; p))$

$$p^* = \underset{p}{\text{minimize}} \sum_x \left(I(W(x; p)) - T(W(x; \Delta p)) \right)^2$$



$T(W(x; \Delta p))$



$I(W(x; p))$

INVERSE COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

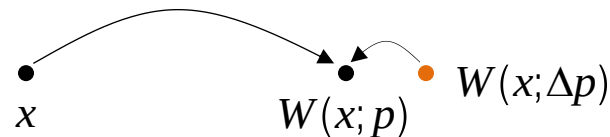
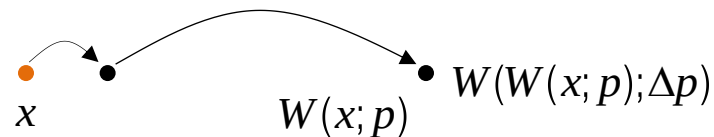
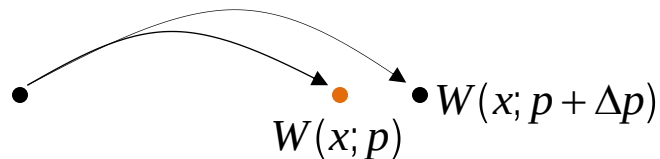
Inverse compositional mapping

$$I(W(x; p)) \quad \text{constant}$$

$$T(W(x; \Delta p)) \approx T(x) + \nabla T(x) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

● Location of linearization



INVERSE COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Big|_0 \Delta p$$

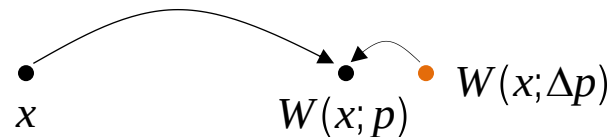
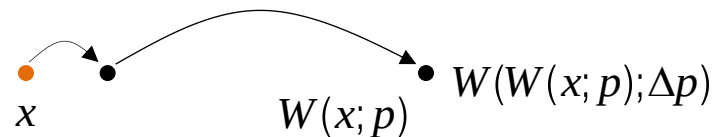
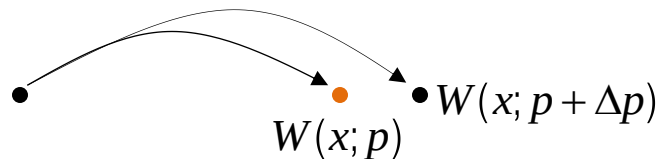
Inverse compositional mapping

$$I(W(x; p)) \quad \text{constant}$$

$$T(W(x; \Delta p)) \approx \boxed{T(x) + \nabla T(x) \frac{\partial W}{\partial p} \Big|_0 \Delta p}$$

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

● Location of linearization



INVERSE COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Delta p$$

$$W(x; p + \Delta p) = A(p + \Delta p) \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

$$W(x; p) \circ W(x; \Delta p) = A(p) A(\Delta p) \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Inverse compositional mapping

$$I(W(x; p)) \quad \text{constant}$$

$$T(W(x; \Delta p)) \approx \boxed{T(x) + \nabla T(x) \frac{\partial W}{\partial p} \bigg|_0} \Delta p$$

$$\text{Note) } (f \circ g)' = (f' \circ g)g' \quad \text{and} \quad W(x; 0) = x$$

INVERSE COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Delta p$$

$$W(x; p + \Delta p) = A(p + \Delta p) \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

$$W(x; p) \circ W(x; \Delta p) = A(p)A(\Delta p) \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Inverse compositional mapping

$$I(W(x; p)) \quad \text{constant}$$

$$T(W(x; \Delta p)) \approx \boxed{T(x) + \nabla T(x) \frac{\partial W}{\partial p} \bigg|_0} \Delta p$$

$$W(x; p) \circ W^{-1}(x; \Delta p) = A(p)A^{-1}(\Delta p) \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

INVERSE COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

Inverse compositional mapping

$$I(W(x; p)) \quad \text{constant}$$

$$T(W(x; \Delta p)) \approx T(x) + \nabla T(x) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

1. Compute Jacobian $\frac{\partial W}{\partial p}$

2. Warp the target image $I(W(x; p))$

3. Compute the error image $T(x) - I(W(x; p))$

4. Warp the gradient image $\nabla I(W(x; p))$

5. Compute steepest descent images $\nabla I \frac{\partial W}{\partial p}$

6. Compute Hessian $H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$

7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$

8. Update $W(x; p) \leftarrow W(x; p) \circ W(x; \Delta p)$

9. Goto 2 unless $\|\Delta p\| < \varepsilon$

INVERSE COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

Inverse compositional mapping

$$I(W(x; p)) \quad \text{constant}$$

$$T(W(x; \Delta p)) \approx T(x) + \nabla T(x) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

1. Compute Jacobian $\frac{\partial W}{\partial p}$
-
2. Warp the target image $I(W(x; p))$
3. Compute the error image $T(x) - I(W(x; p))$
4. The gradient image $\nabla T(x)$
5. Compute steepest descent images $\nabla T \frac{\partial W}{\partial p}$
6. Compute Hessian $H = \sum_x \left(\nabla T \frac{\partial W}{\partial p} \right)^T \left(\nabla T \frac{\partial W}{\partial p} \right)$
7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla T \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$
8. Update $W(x; p) \leftarrow W(x; p) \circ W^{-1}(x; \Delta p)$
9. Goto 2 unless $\|\Delta p\| < \varepsilon$

INVERSE COMPOSITIONAL

Additive mapping

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \Delta p$$

Compositional mapping II

$$I(W(W(x; \Delta p); p)) \approx I(W(x; p)) + \nabla I(W) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

Inverse compositional mapping

$$I(W(x; p)) \quad \text{constant}$$

$$T(W(x; \Delta p)) \approx T(x) + \nabla T(x) \frac{\partial W}{\partial p} \bigg|_0 \Delta p$$

Note) $(f \circ g)' = (f' \circ g)g'$ and $W(x; 0) = x$

1. Compute Jacobian $\frac{\partial W}{\partial p}$

2. Warp the target image $I(W(x; p))$

3. Compute the error image $T(x) - I(W(x; p))$

4. The gradient image $\nabla T(x)$

5. Compute steepest descent images $\nabla T \frac{\partial W}{\partial p}$

6. Compute Hessian $H = \sum_x \left(\nabla T \frac{\partial W}{\partial p} \right)^T \left(\nabla T \frac{\partial W}{\partial p} \right)$

7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla T \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$

8. Update $W(x; p) \leftarrow W(x; p) \circ W^{-1}(x; \Delta p)$

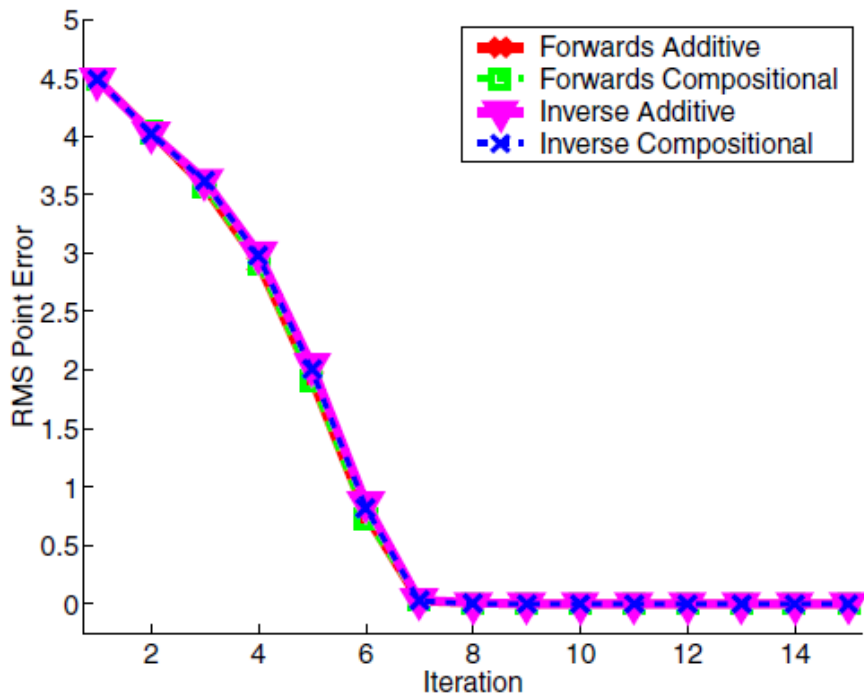
9. Goto 2 unless $\|\Delta p\| < \varepsilon$

COMPOSITIONAL ALIGNMENT

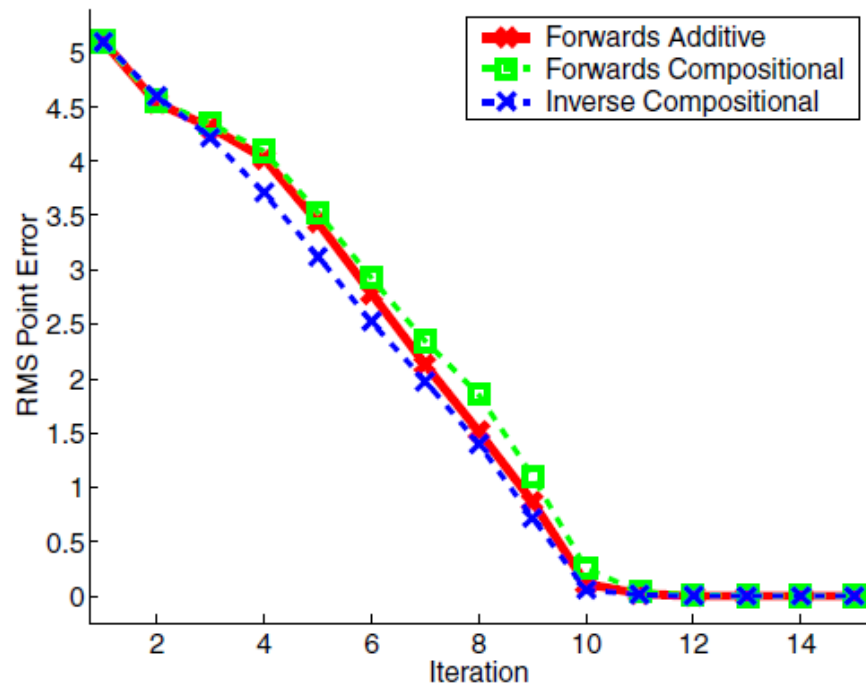
1. Compute Jacobian $\frac{\partial W}{\partial p}$
2. Warp the target image $I(W(x; p))$
3. Compute the error image $T(x) - I(W(x; p))$
4. Warp the gradient image $\nabla I(W(x; p))$
5. Compute steepest descent images $\nabla I \frac{\partial W}{\partial p}$
6. Compute Hessian $H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$
7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$
8. Update $W(x; p) \leftarrow W(x; p) \circ W(x; \Delta p)$
9. Goto 2 unless $\|\Delta p\| < \varepsilon$

INV. COMPOSITIONAL ALIGNMENT

1. Compute Jacobian $\frac{\partial W}{\partial p}$
4. Warp the gradient image $\nabla T(x)$
5. Compute steepest descent images $\nabla T \frac{\partial W}{\partial p}$
6. Compute Hessian $H = \sum_x \left(\nabla T \frac{\partial W}{\partial p} \right)^T \left(\nabla T \frac{\partial W}{\partial p} \right)$
2. Warp the target image $I(W(x; p))$
3. Compute the error image $T(x) - I(W(x; p))$
7. Compute $\Delta p = H^{-1} \sum_x \left(\nabla T \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$
8. Update $W(x; p) \leftarrow W(x; p) \circ W^{-1}(x; \Delta p)$
9. Goto 2 unless $\|\Delta p\| < \varepsilon$



(e) Example Convergence for an Affine Warp



(f) Example Convergence for a Homography

Table 5. Timing results for our Matlab implementation of the four algorithms in milliseconds. These results are for the 6-parameter affine warp using a 100×100 pixel template on a 933 MHz Pentium-IV.

				Step 3	Step 4	Step 5	Step 6	Total		
Pre-computation:										
Forwards Additive (FA)				–	–	–	–	0.0		
Forwards Compositional (FC)				–	17.4	–	–	17.4		
Inverse Additive (IA)				8.30	17.1	27.5	37.0	89.9		
Inverse Compositional (IC)				8.31	17.1	27.5	37.0	90.0		
	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7	Step 8	Step 9	Total
Per iteration:										
FA	1.88	0.740	36.1	17.4	27.7	37.2	6.32	0.111	0.108	127
FC	1.88	0.736	8.17	–	27.6	37.0	6.03	0.106	0.253	81.7
IA	1.79	0.688	–	–	–	–	6.22	0.106	0.624	9.43
IC	1.79	0.687	–	–	–	–	6.22	0.106	0.409	9.21



$I(x)$



$T(x)$



$I(W(x; p))$



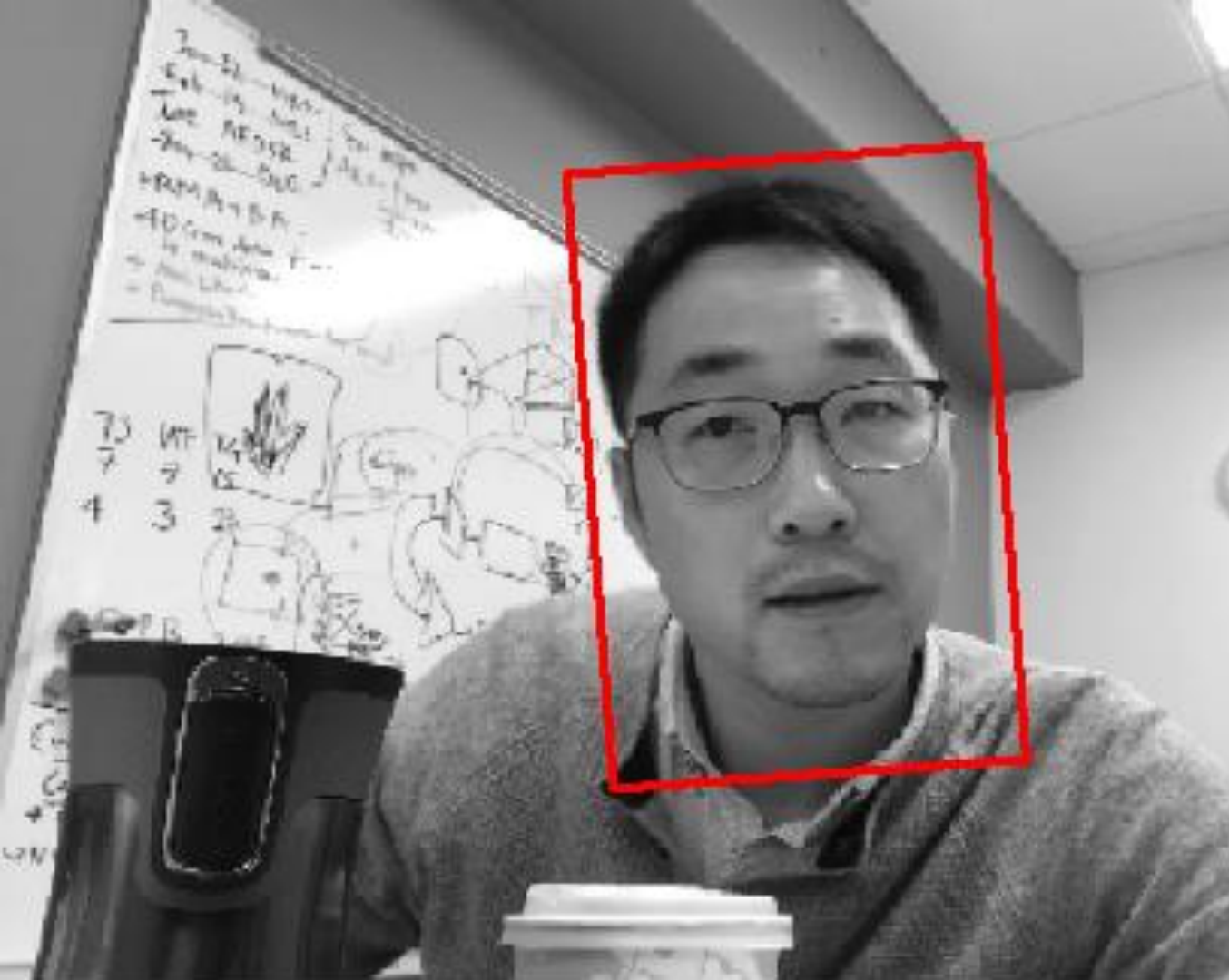
$0.5T(x) + 0.5I(W(x; p))$



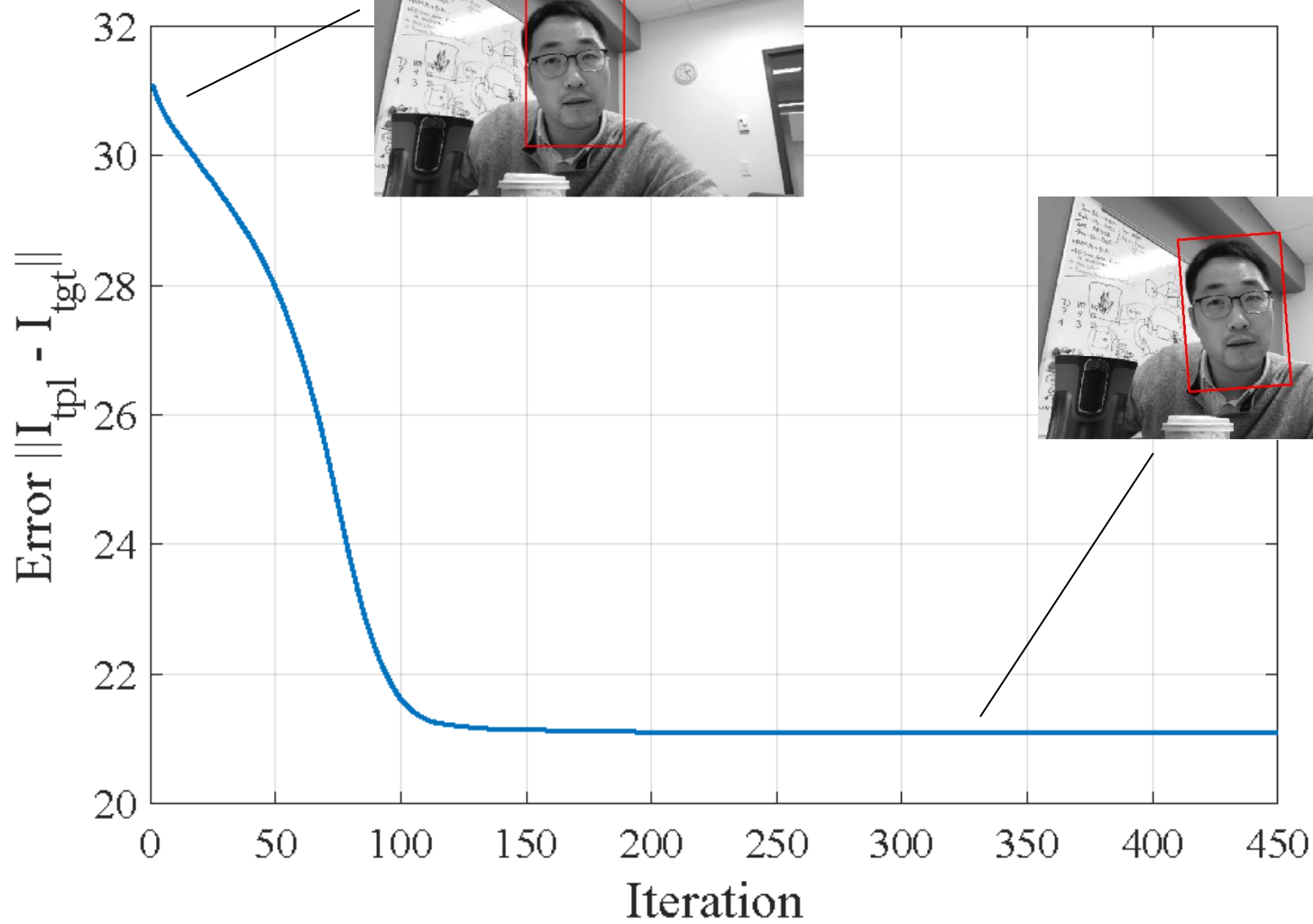
$|T(x) - I(W(x; p))|$

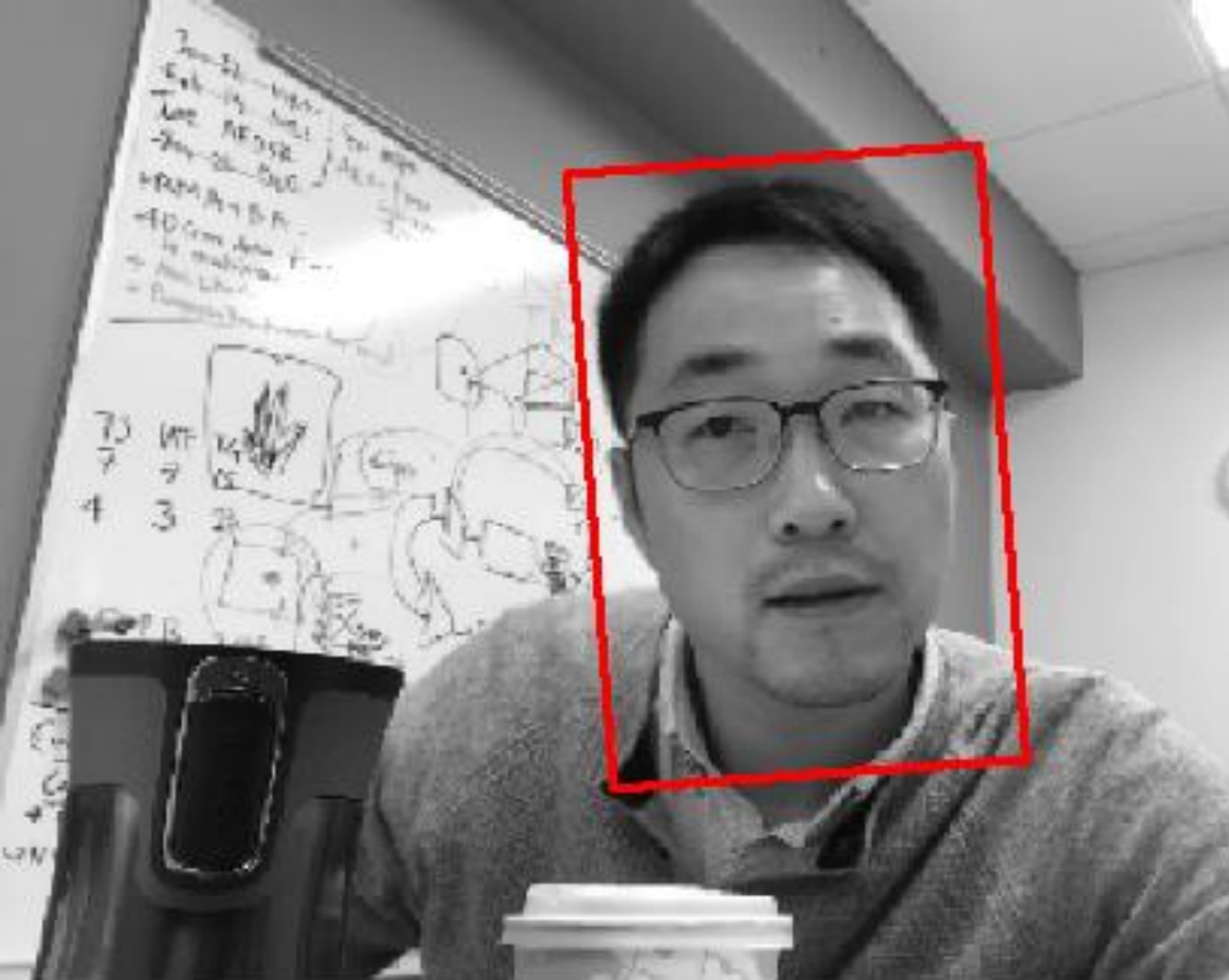
$$p \leftarrow p + \Delta p$$



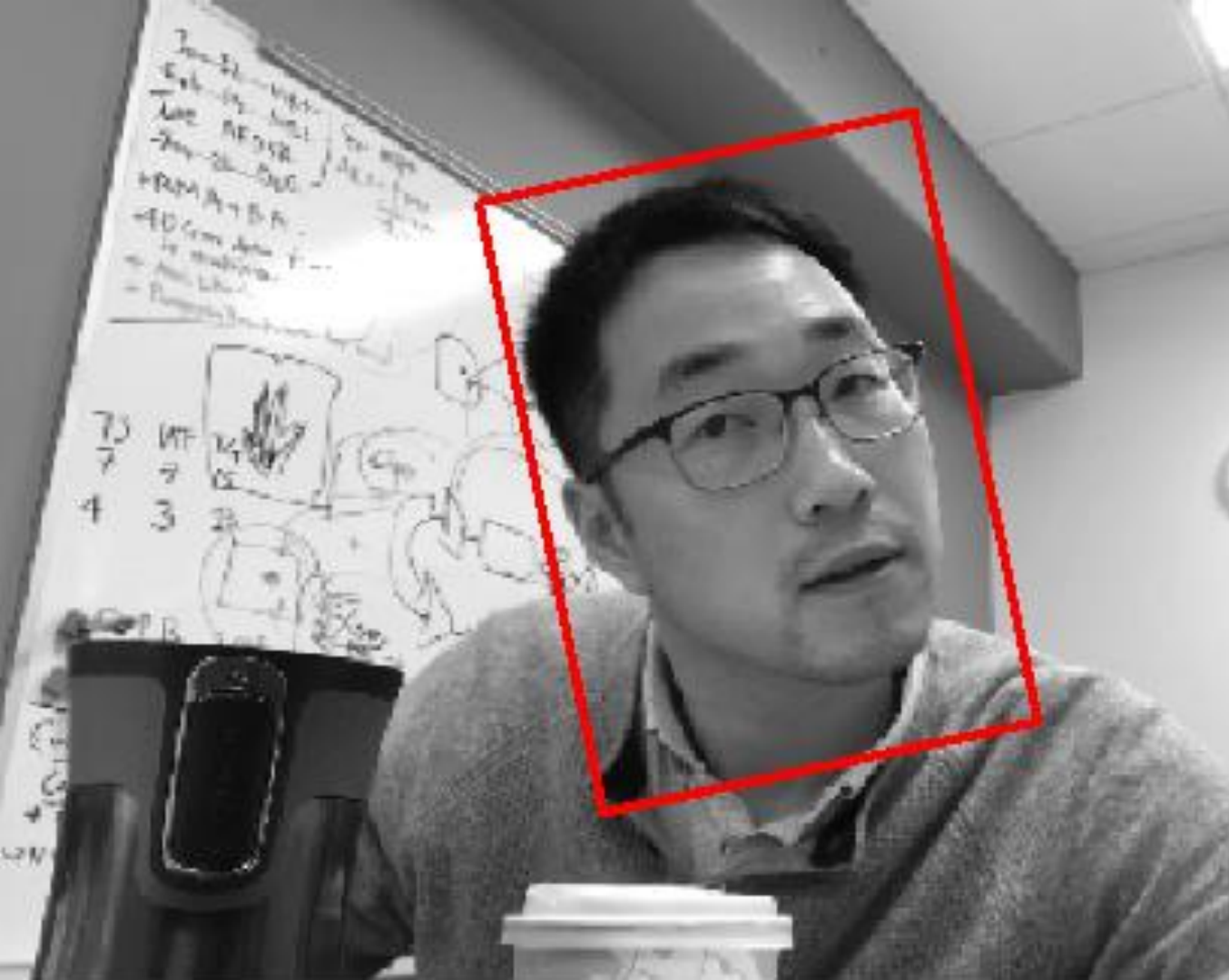


Error: 8.18









<https://www.youtube.com/watch?v=qeZ7H40BQv4>