

SCALE SPACE

HYUN Soo PARK

Mercury

Venus

Earth

Mars

Jupiter

Saturn

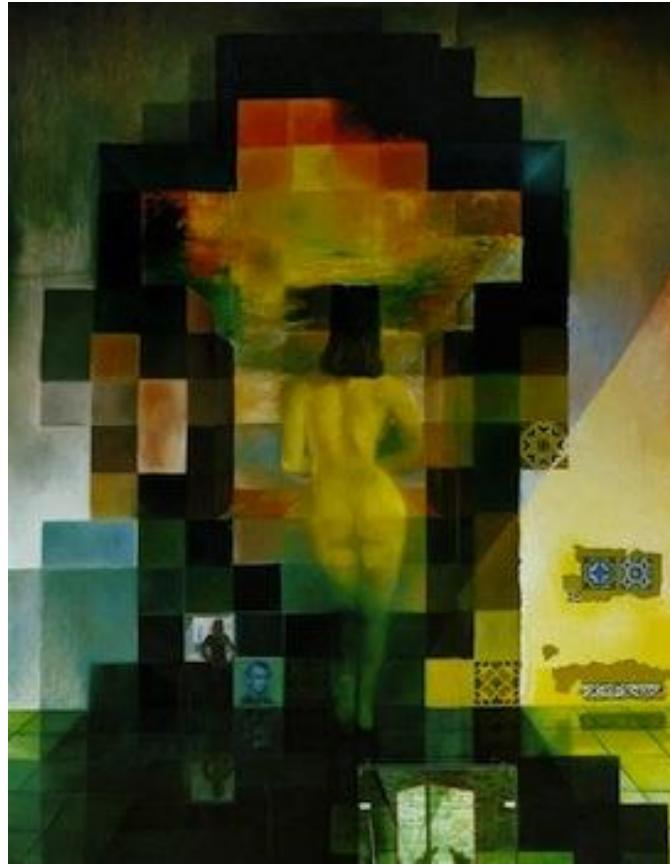
Uranus

Neptune





Salvador Dalí, Abraham Lincoln



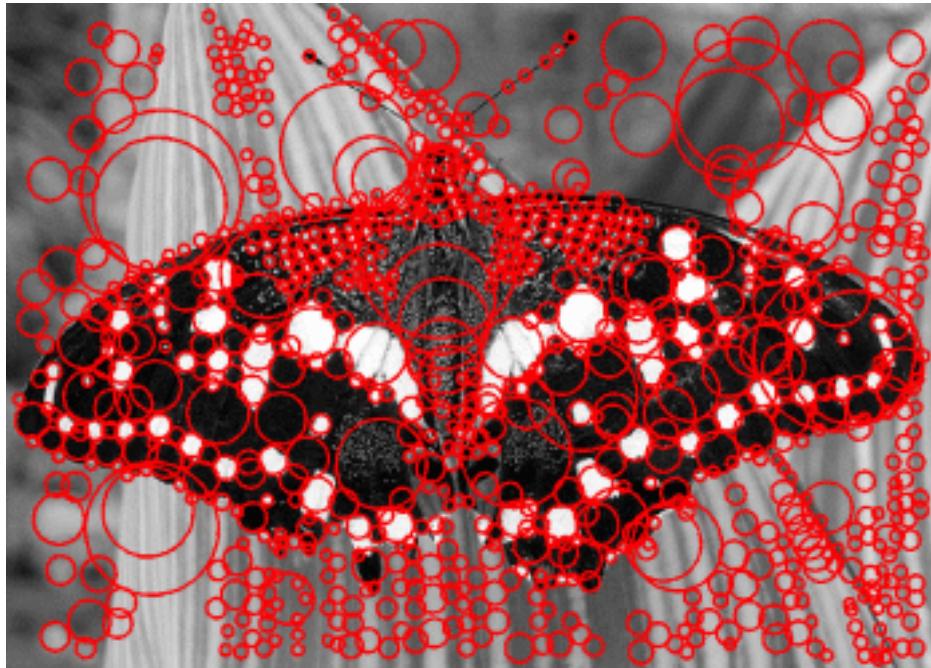
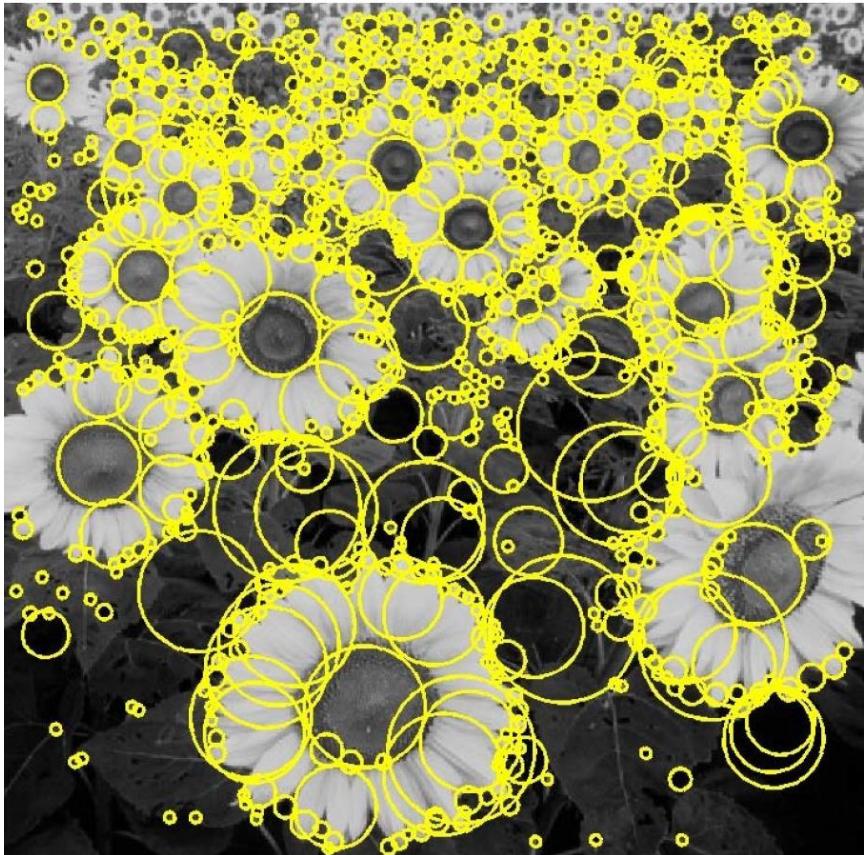
Salvador Dali, Abraham Lincoln



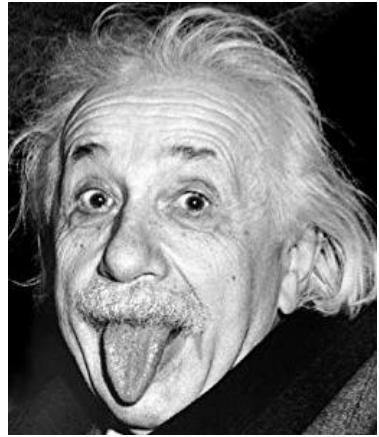
SCALE INVARIANT IMAGE REPRESENTATION



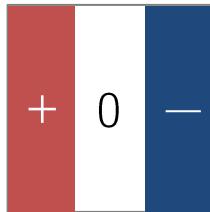
BLOB DETECTION ~ SCALE SELECTION



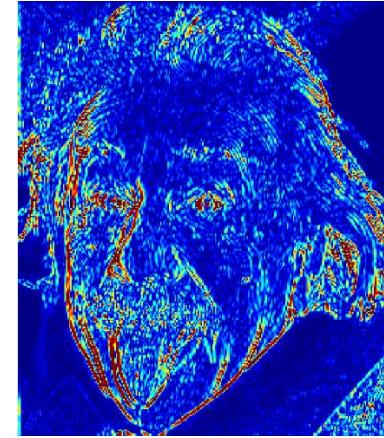
RECALL: IMAGE DIFFERENTIATION



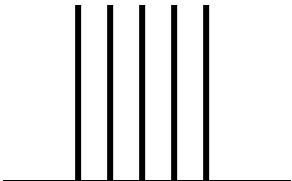
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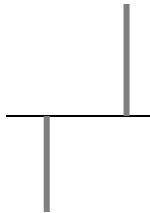
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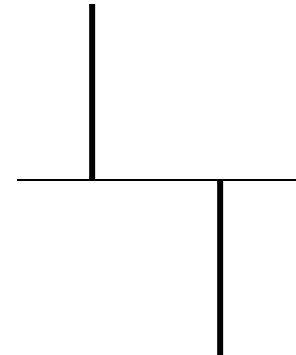
$$I \otimes z = \frac{\partial I}{\partial u}$$



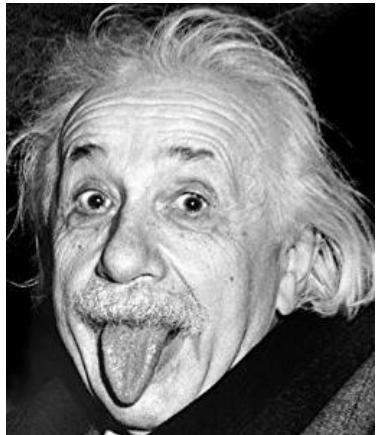
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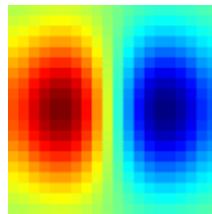
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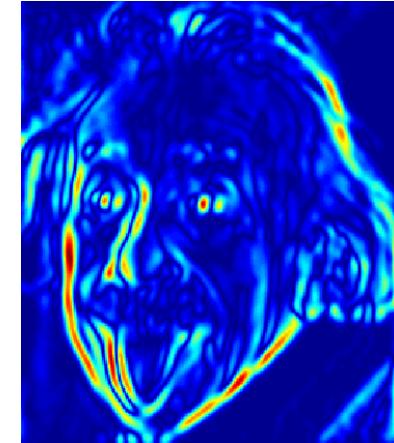
RECALL: SOBEL FILTER



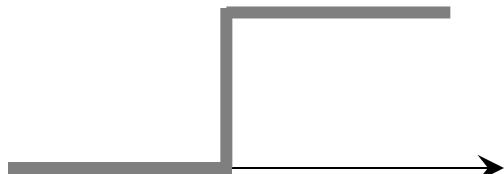
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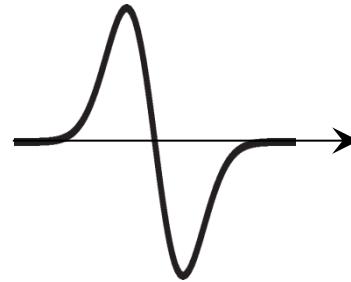


$$\frac{\partial G}{\partial u} = \frac{G(u+h) - G(u-h)}{2h}$$

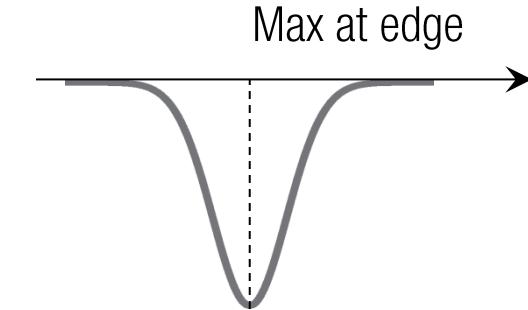


Edge

*

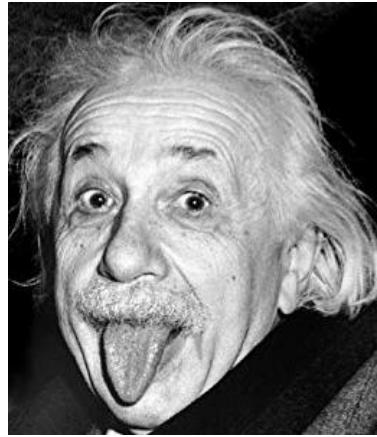


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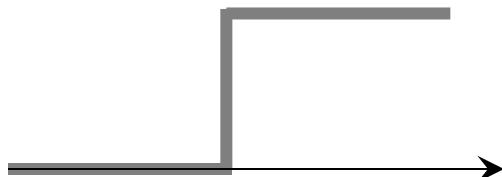
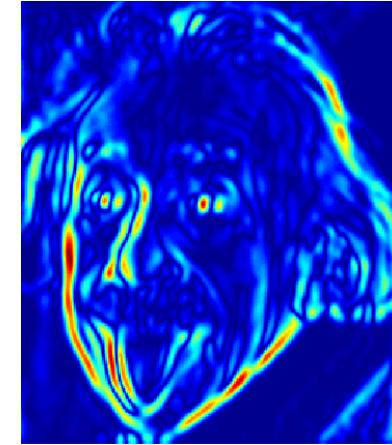
Max at edge

RECALL: SOBEL FILTER

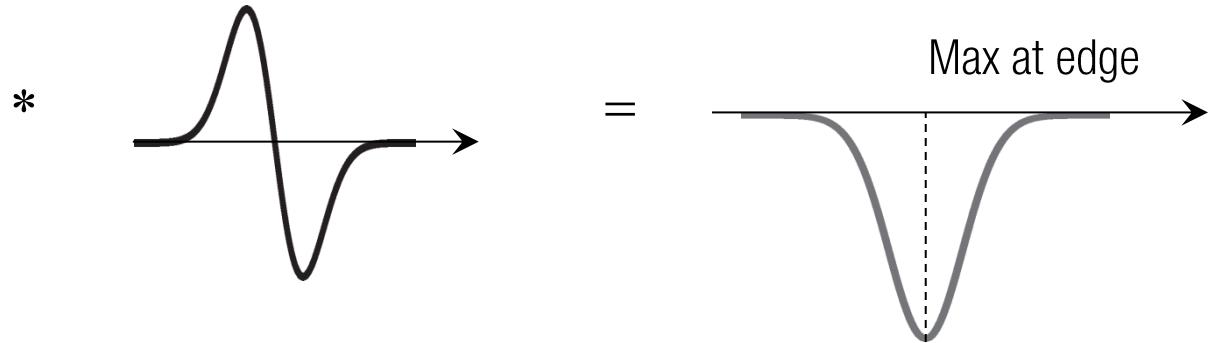


$$* \begin{array}{|c|c|c|} \hline & + & 0 & - \\ \hline \end{array} * \quad = \quad \text{Heatmap of Einstein's face}$$

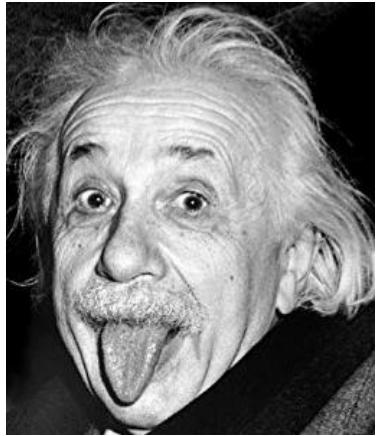
$$\frac{\partial G}{\partial u} = \frac{G(u+h) - G(u-h)}{2h}$$



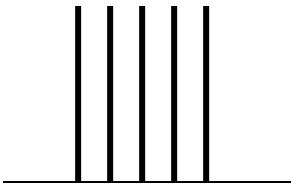
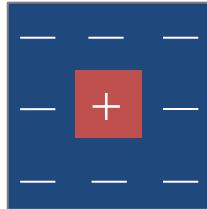
Edge



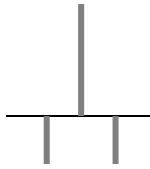
RECALL: IMAGE SHARPENING



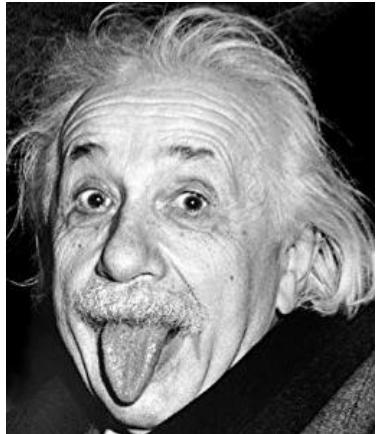
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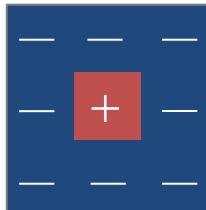
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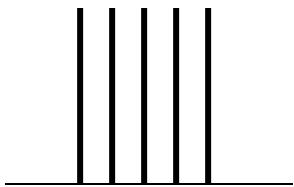
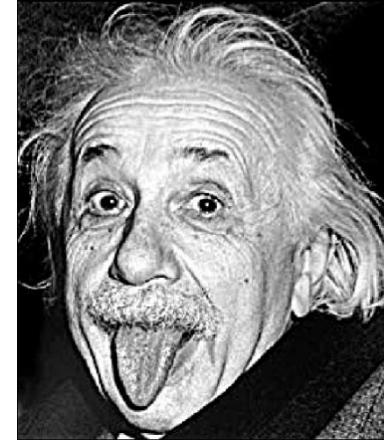
RECALL: IMAGE SHARPENING



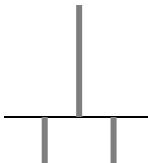
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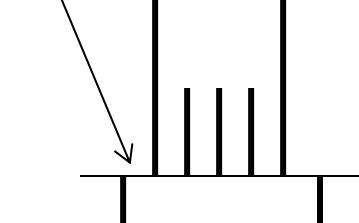
=



*

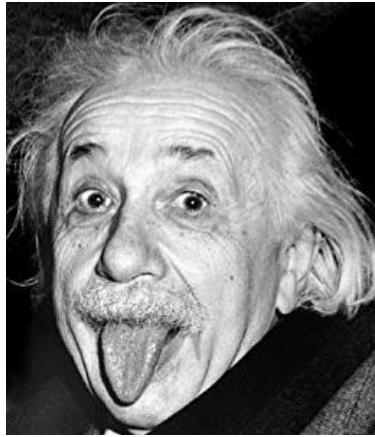


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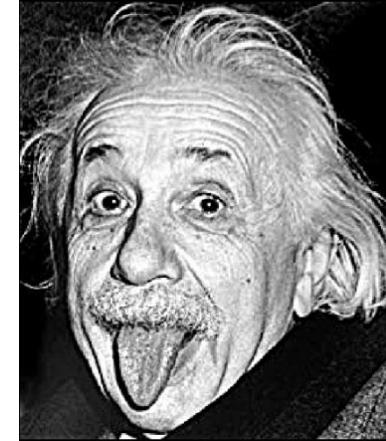


Zero crossing at the edge

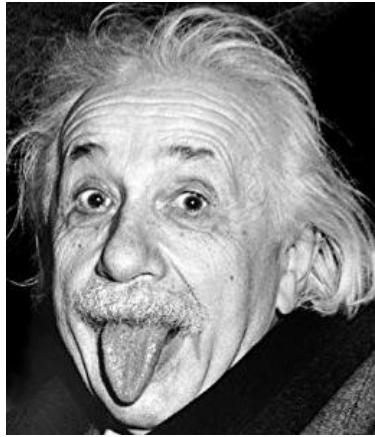
LAPLACIAN



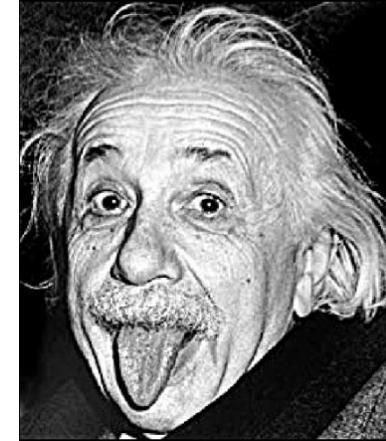
$$\begin{matrix} * & \begin{matrix} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{+} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{matrix} & * & = \\ & \begin{matrix} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{+} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{matrix} & \begin{matrix} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{+} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{matrix} & \end{matrix}$$
$$\frac{G(u+h) + G(u-h) - 2G(u)}{h}$$



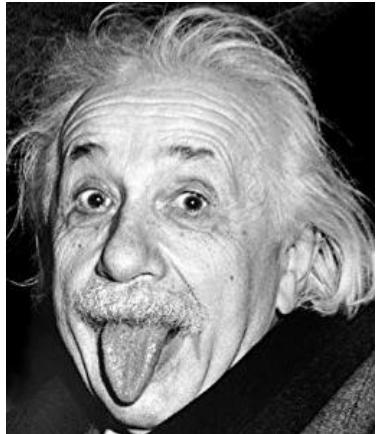
LAPLACIAN

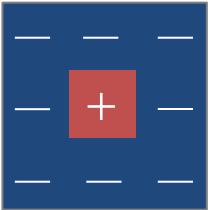


$$\begin{matrix} * & \begin{matrix} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{+} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{matrix} & * & = \\ \frac{G(u+h) - G(u)}{h} & - \frac{G(u) - G(u-h)}{h} \end{matrix}$$

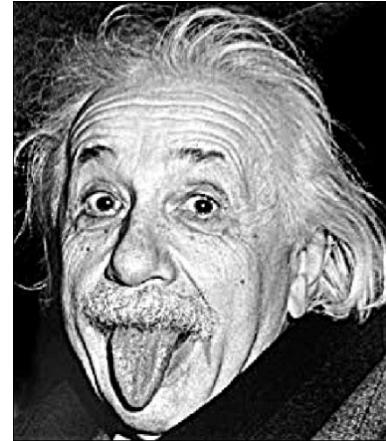


LAPLACIAN

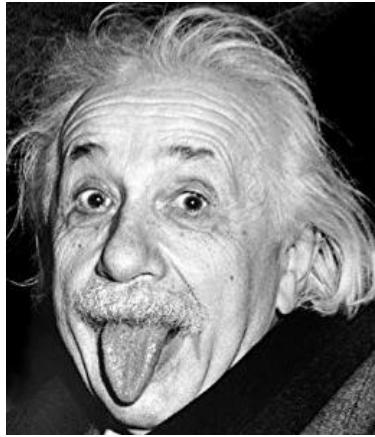


$$\begin{matrix} * & \begin{matrix} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{+} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{matrix} & * \end{matrix} =$$
A 3x3 matrix multiplication diagram. On the left is the input image of Einstein. In the center is a 3x3 kernel with a central red square containing a white plus sign (+), surrounded by a blue background. To the right of the kernel is an equals sign (=).
A black and white photograph of Albert Einstein, similar to the one on the left, but with a noticeable blurring effect applied across the entire face. His features are less sharp, and the overall image has a softer, more diffused appearance.

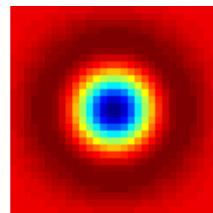
$$\lim_{h \rightarrow 0} \left(\frac{G(u+h) - G(u)}{h} - \frac{G(u) - G(u-h)}{h} \right) / h$$



LAPLACIAN



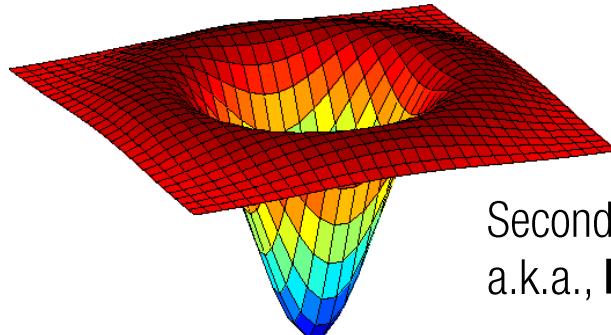
*



=

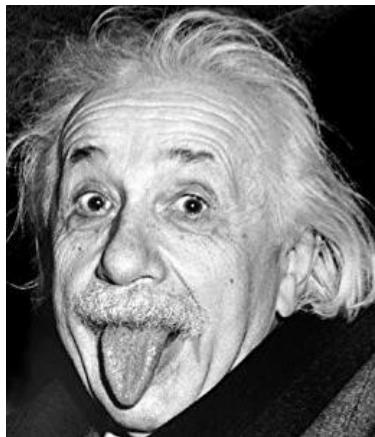


$$\nabla \cdot \nabla G = \nabla \left(\frac{\partial G}{\partial u} \mathbf{i} + \frac{\partial G}{\partial v} \mathbf{j} \right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2}$$



Second order derivative of Gaussian,
a.k.a., **Laplacian of Gaussian**

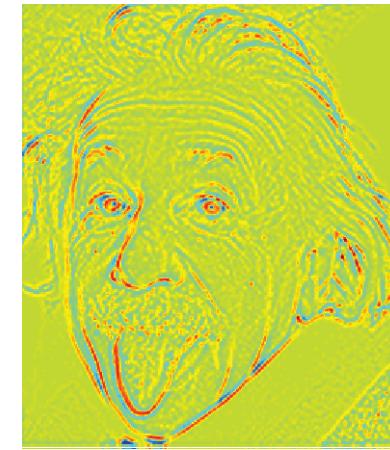
LAPLACIAN



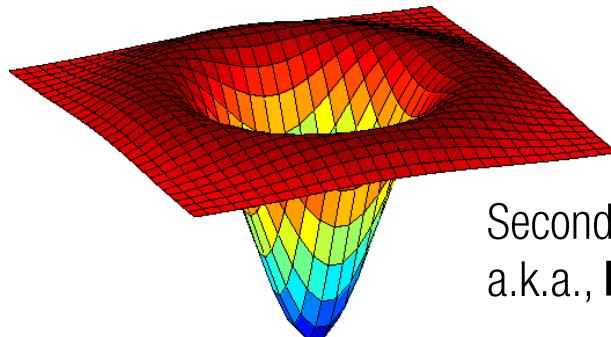
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$$\begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}$$

=

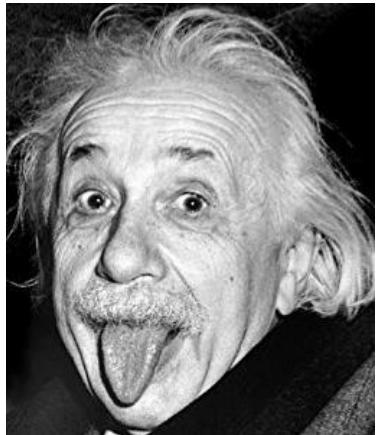


$$\nabla \cdot \nabla G = \nabla \left(\frac{\partial G}{\partial u} \mathbf{i} + \frac{\partial G}{\partial v} \mathbf{j} \right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2}$$



Second order derivative of Gaussian,
a.k.a., **Laplacian of Gaussian**

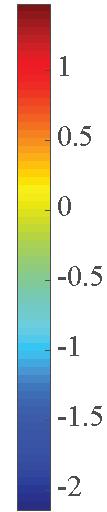
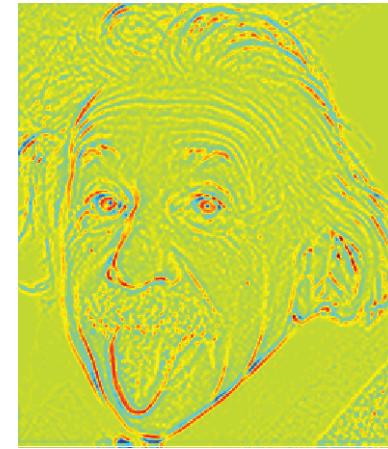
LAPLACIAN



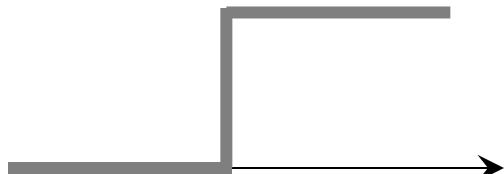
*

$$\begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}$$

=

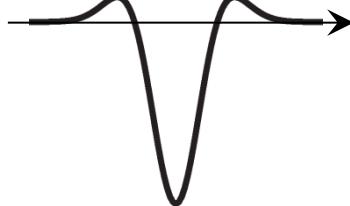


$$\nabla \cdot \nabla G = \nabla \left(\frac{\partial G}{\partial u} \mathbf{i} + \frac{\partial G}{\partial v} \mathbf{j} \right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2}$$

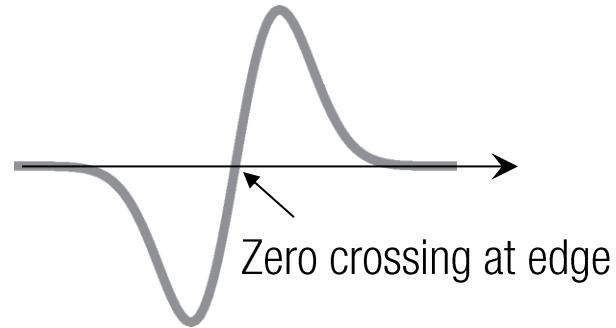


Edge

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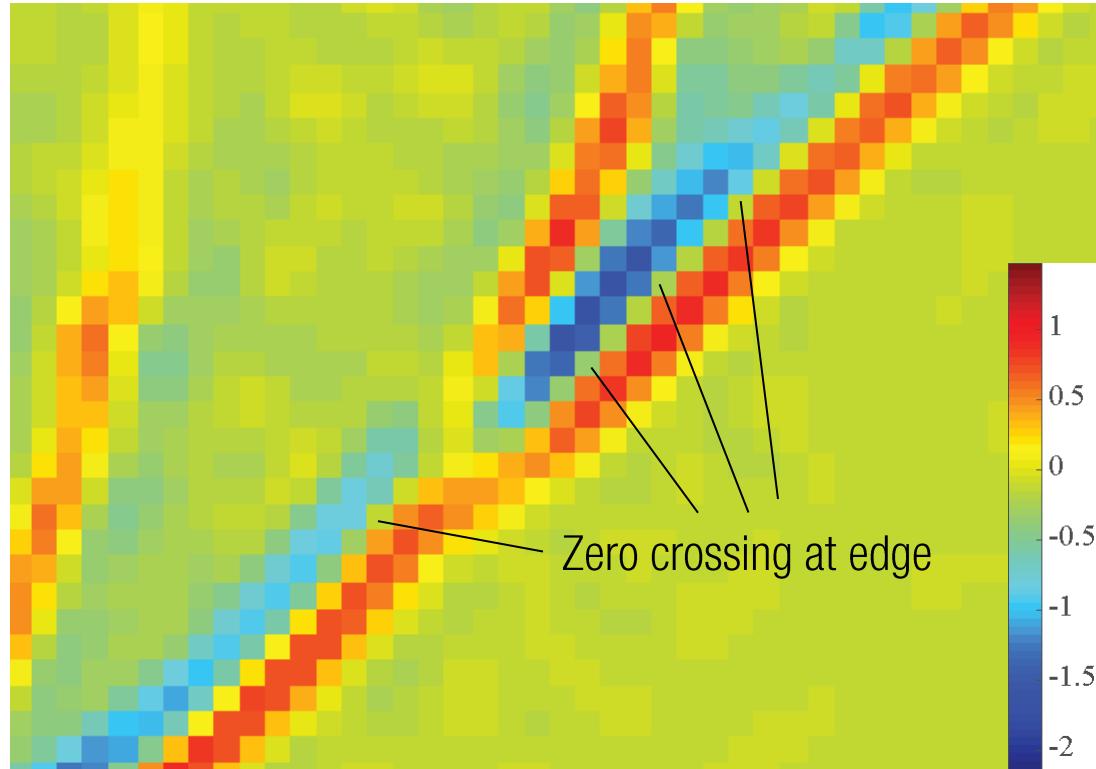
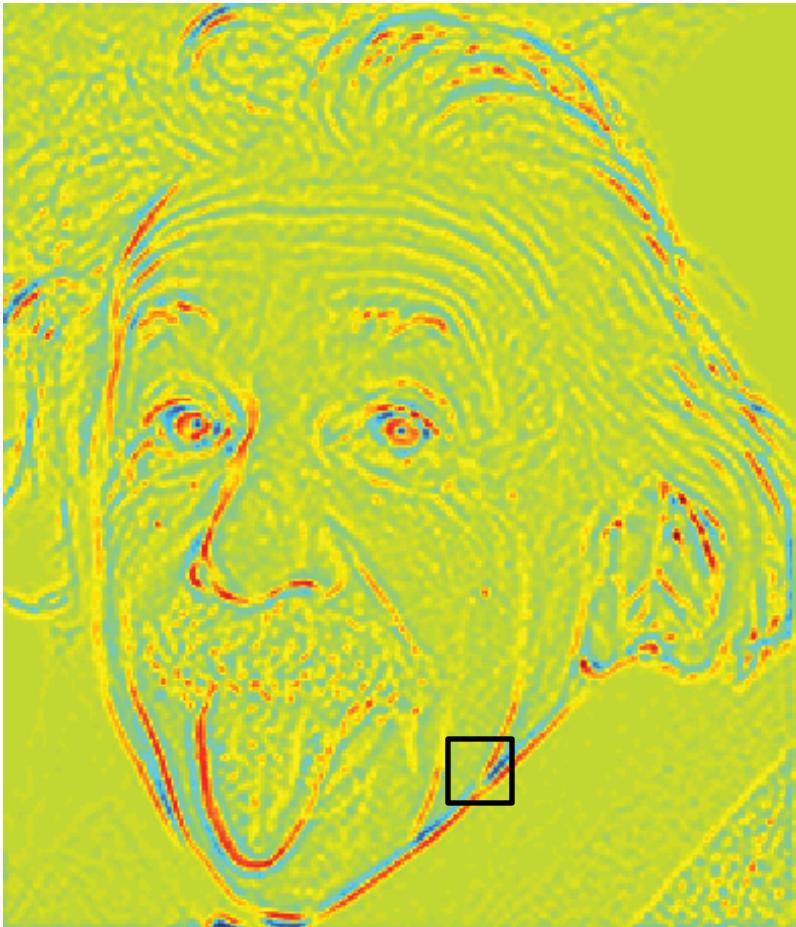


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Zero crossing at edge

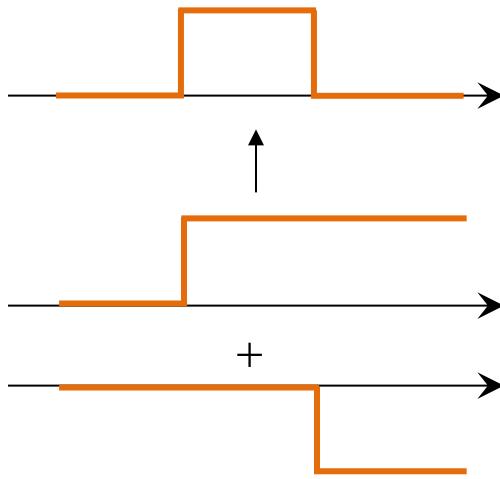
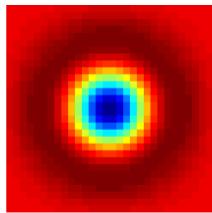
EDGE LOCALIZATION



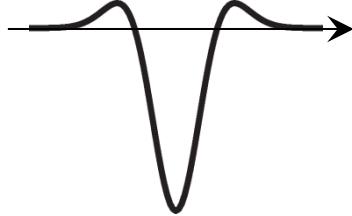
STRUCTURED EDGES (E.G., BLOB)



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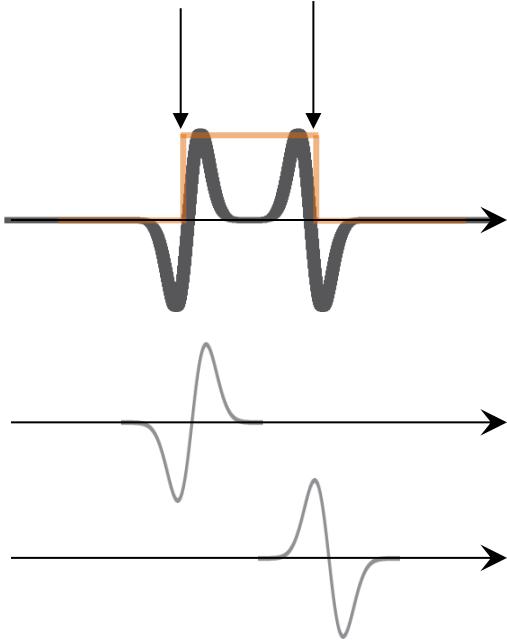


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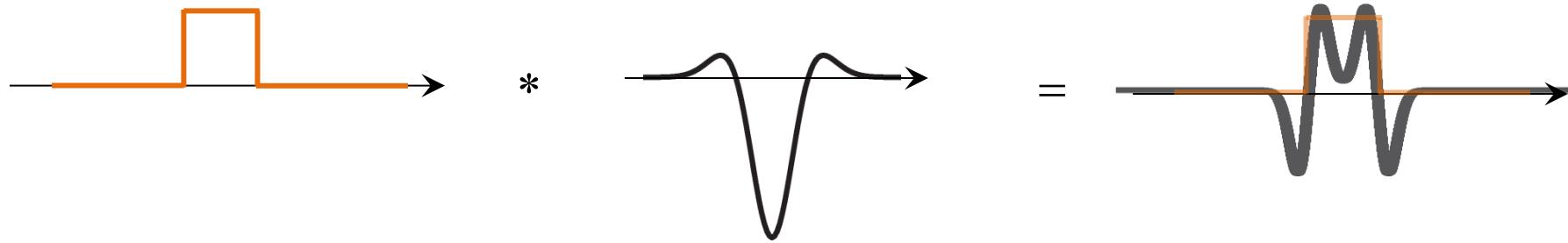
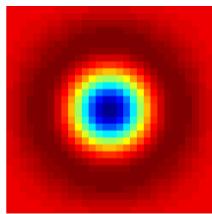
Zero crossing edges



STRUCTURED EDGES (E.G., BLOB)



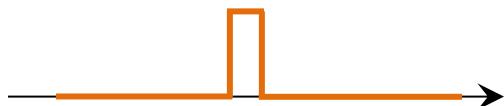
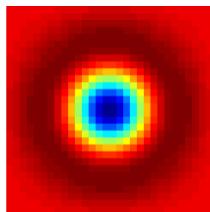
*



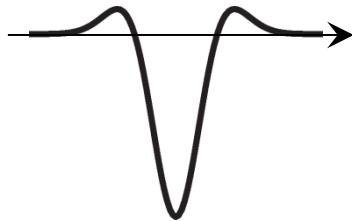
STRUCTURED EDGES (E.G., BLOB)



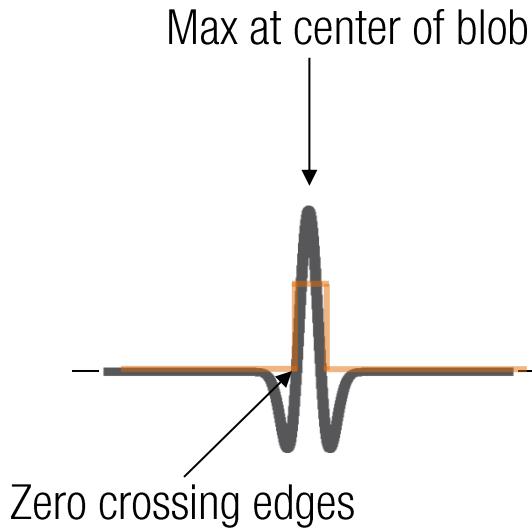
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STRUCTURED EDGES (E.G., BLOB)

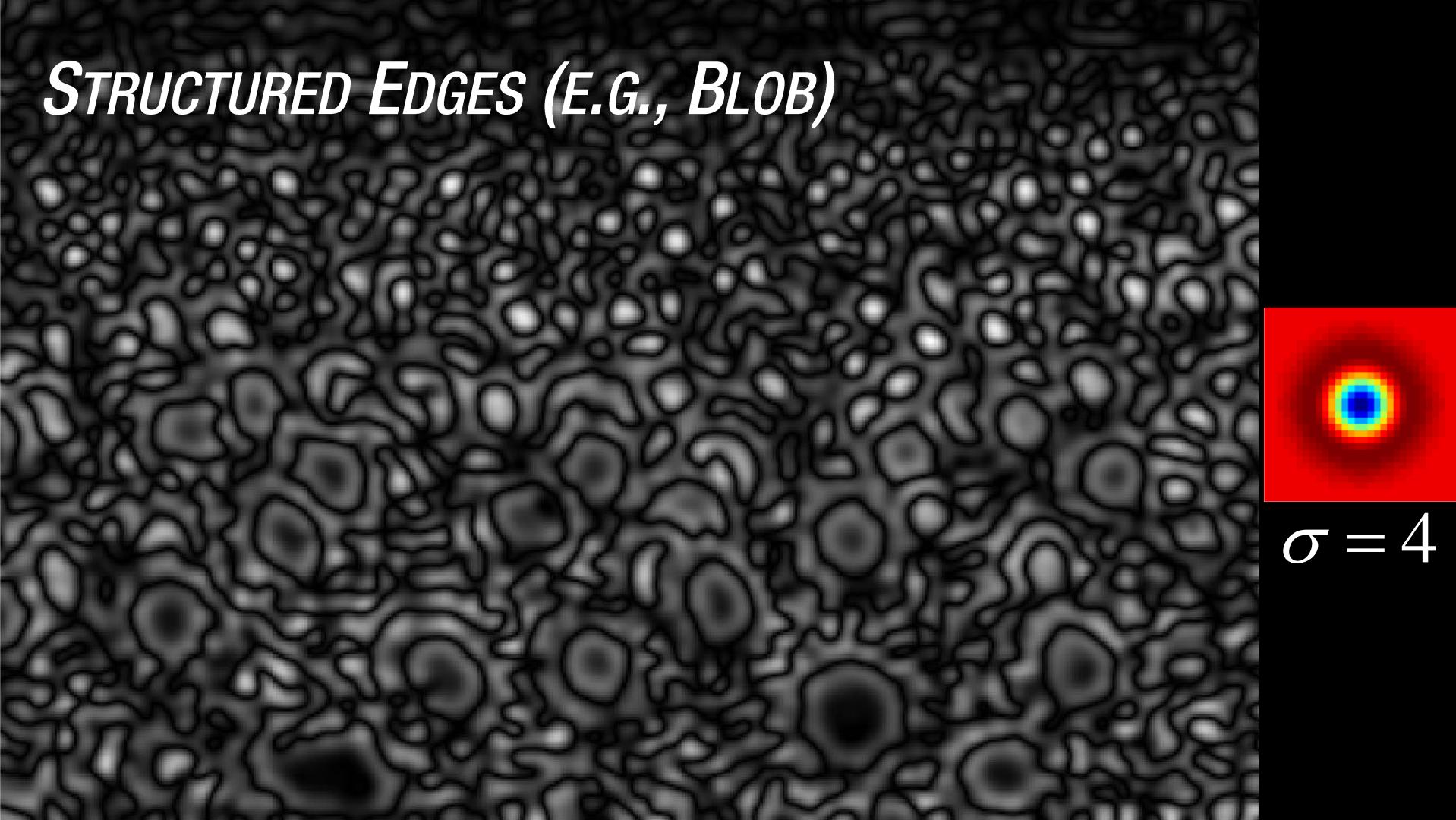


STRUCTURED EDGES (E.G., BLOB)



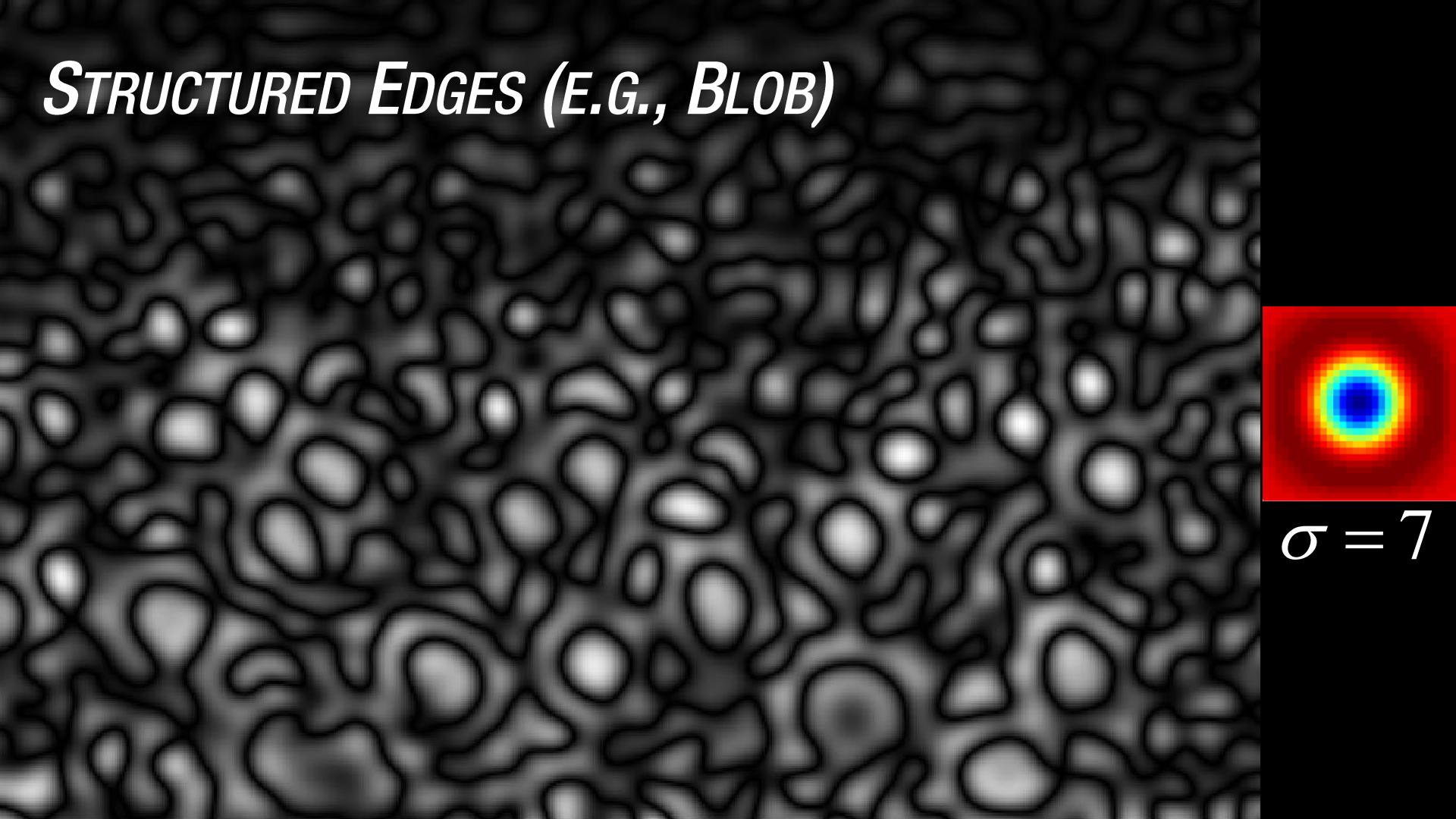
$$\sigma = 1$$

STRUCTURED EDGES (E.G., BLOB)

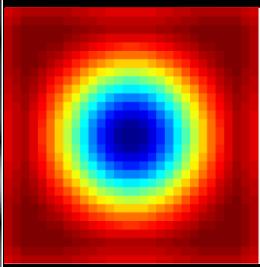


$$\sigma = 4$$

STRUCTURED EDGES (E.G., BLOB)



STRUCTURED EDGES (E.G., BLOB)



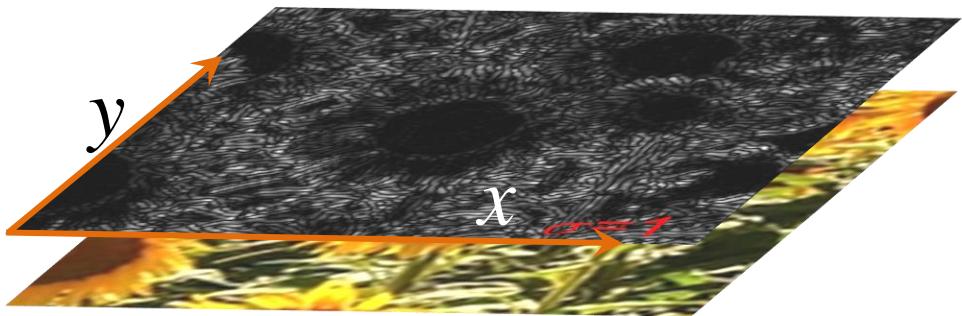
$\sigma = 10$

SCALE SPACE



$I(x, y)$

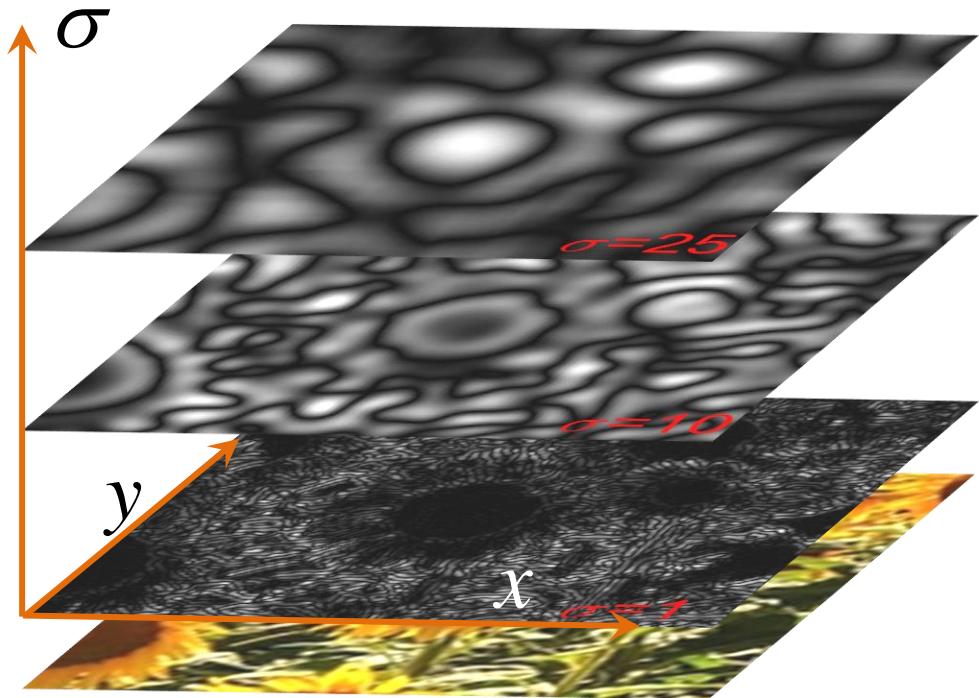
SCALE SPACE



$$I(x, y) * \underline{L(\sigma)}$$

Laplacian operator

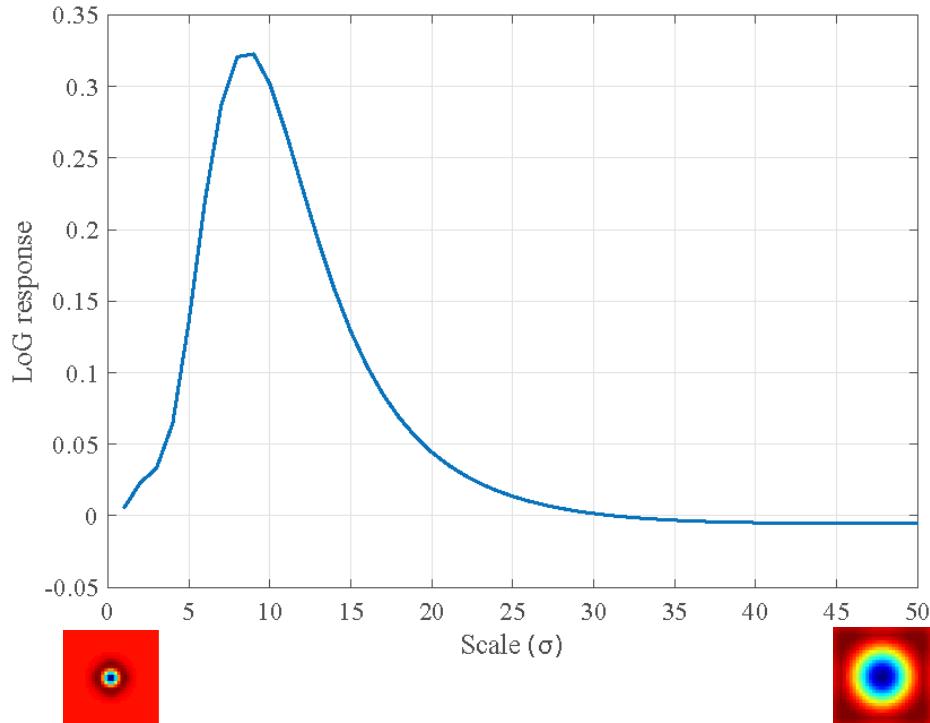
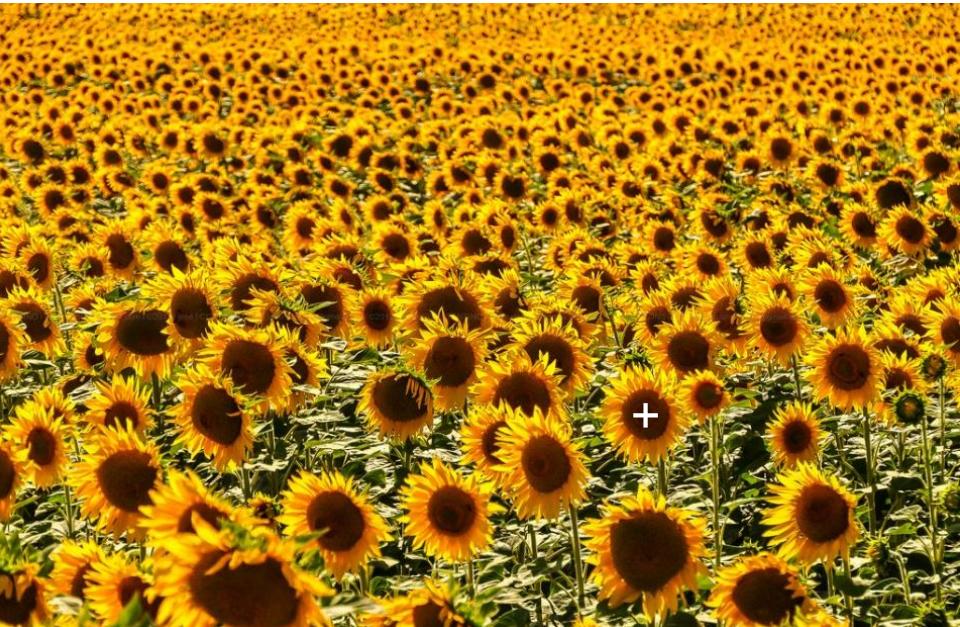
SCALE SPACE



$$D(x, y, \sigma) = I(x, y) * L(\sigma)$$

Scale space response

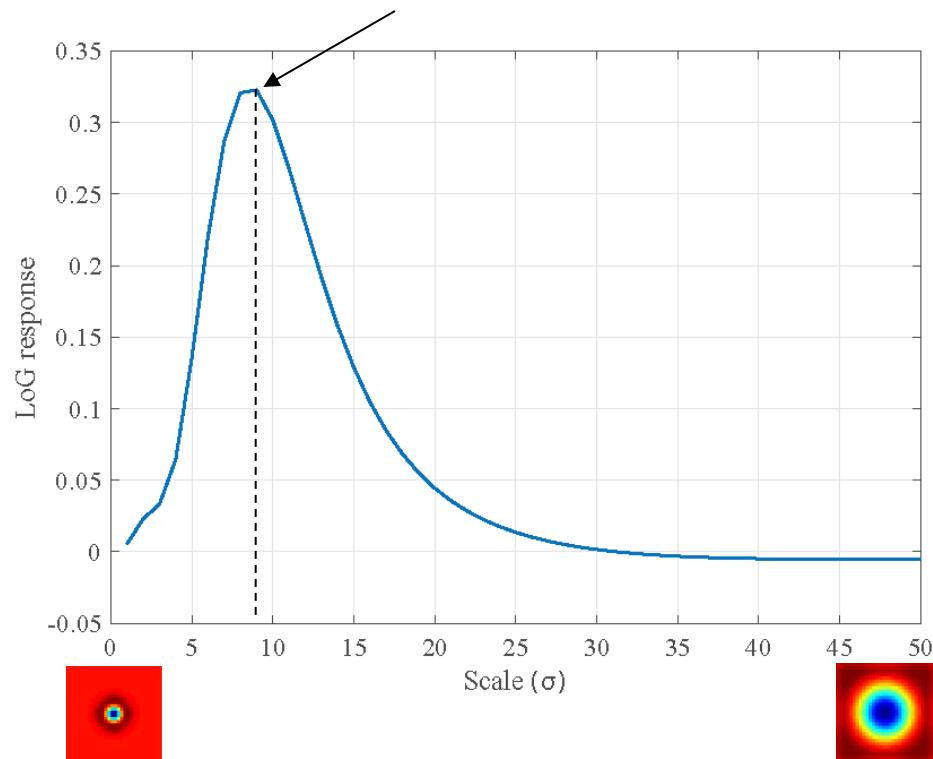
STRUCTURED EDGES (E.G., BLOB)



STRUCTURED EDGES (E.G., BLOB)



Characteristic scale

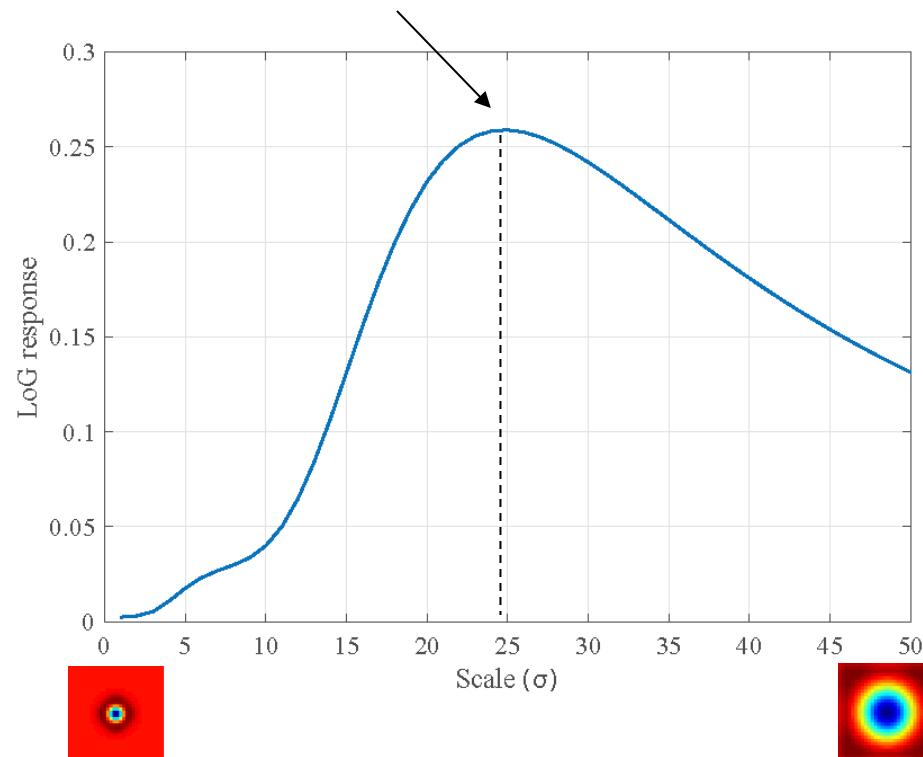


STRUCTURED EDGES (E.G., BLOB)

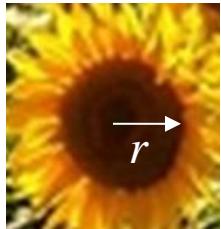


x3 bigger sunflower

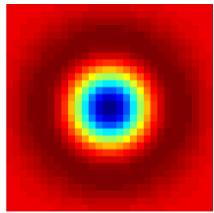
Characteristic scale



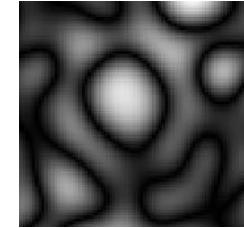
SCALE SELECTION



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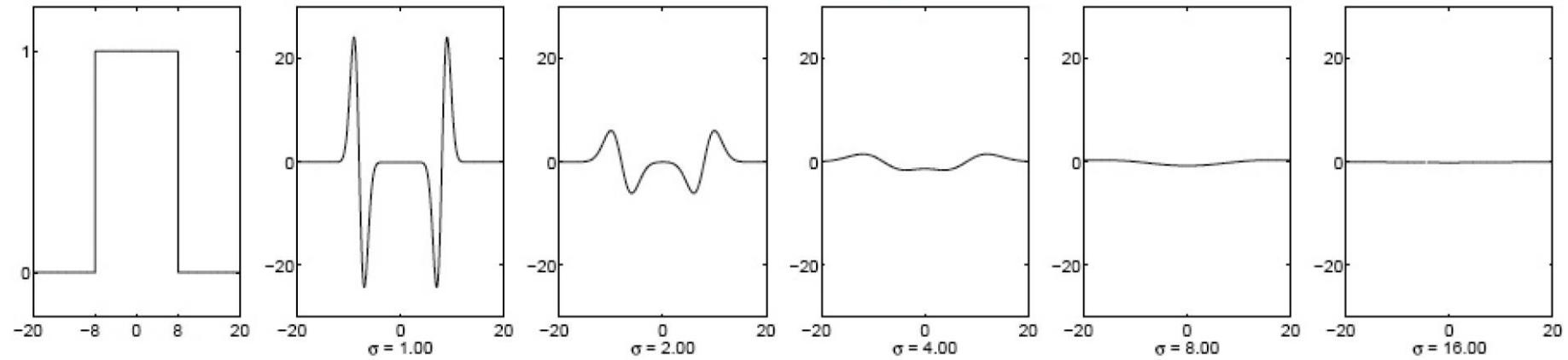
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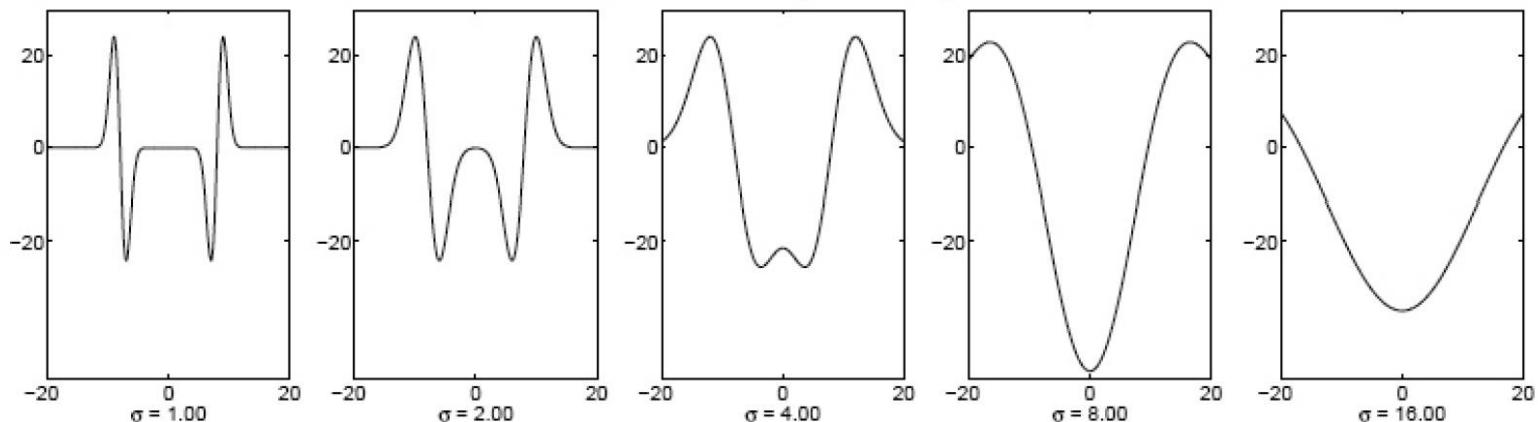
$$\sigma^2 \left(\frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2} \right)$$

Normalization factor

- The scale response (laplacian) is maximized when $\sigma = \frac{r}{\sqrt{2}}$
- The characteristic scale can be used to normalize the image, resulting in *scale invariant* image description.



Scale-normalized Laplacian response



Slide credit: Svetlana Lazebnik

SCALE NORMALIZATION



SCALE NORMALIZATION

