

CSCI 5561: Assignment #4

Convolutional Neural Network

1 Submission

- Assignment due: Apr 17 (11:55pm)
- Individual assignment
- Up to 2 page summary write-up with resulting visualization (more than 2 page assignment will be automatically returned.).
- Submission through Canvas.
- Following skeletal functions are already included in the cnn.py file (https://www-users.cs.umn.edu/~hspark/csci5561_S2020/HW4.zip)

- main_slp_linear
 - main_slp
 - main_mlp
 - main_cnn

- List of function to submit:
 - get_mini_batch
 - fc
 - fc_backward
 - loss_euclidean
 - train_slp_linear
 - loss_cross_entropy_softmax
 - train_slp
 - relu
 - relu_backward
 - train_mlp
 - conv
 - conv_backward
 - pool2x2
 - pool2x2_backward
 - flattening
 - flattening_backward
 - trainCNN

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- A list of MAT files to submit that contain the following trained weights:
 - `slp_linear.mat`: `w`, `b`
 - `slp.mat`: `w`, `b`
 - `mlp.mat`: `w1`, `b1`, `w2`, `b2`
 - `cnn.mat`: `w_conv`, `b_conv`, `w_fc`, `b_fc`
- DO NOT SUBMIT THE PROVIDED IMAGE DATA
- The function that does not comply with its specification will not be graded.
- You are not allowed to use computer vision related package functions unless explicitly mentioned here. Please consult with TA if you are not sure about the list of allowed functions.

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2 Overview



Figure 1: You will implement (1) a multi-layer perceptron (neural network) and (2) convolutional neural network to recognize hand-written digit using the MNIST dataset.

The goal of this assignment is to implement neural network to recognize hand-written digits in the MNIST data.

MNIST Data You will use the MNIST hand written digit dataset to perform the first task (neural network). We reduce the image size ($28 \times 28 \rightarrow 14 \times 14$) and subsample the data. You can download the training and testing data from here:

http://www.cs.umn.edu/~hspark/csci5561_S2019/ReducedMNIST.zip

Description: The zip file includes two MAT files (`mnist_train.mat` and `mnist_test.mat`). Each file includes `im_*` and `label_*` variables:

- `im_*` is a matrix ($196 \times n$) storing vectorized image data ($196 = 14 \times 14$)
- `label_*` is $1 \times n$ vector storing the label for each image data.

n is the number of images. You can visualize the i^{th} image, e.g.,
`plt.imshow(mnist_train['im_train'][:, 0].reshape((14, 14), order='F'), cmap='gray').`

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3 Single-layer Linear Perceptron

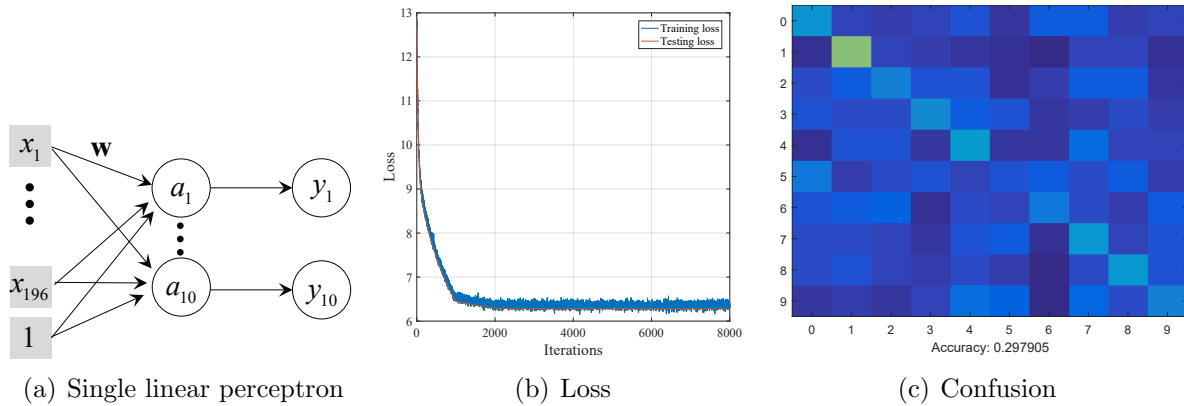


Figure 2: You will implement a single linear perceptron that produces accuracy near 30%. Random chance is 10% on testing data.

You will implement a single-layer *linear* perceptron (Figure 2(a)) with stochastic gradient descent method. We provide `main_slp_linear` where you will implement `get_mini_batch` and `train_slp_linear`.

```
def get_mini_batch(im_train, label_train, batch_size)
    ...
    return mini_batch_x, mini_batch_y
```

Input: `im_train` and `label_train` are a set of images and labels, and `batch_size` is the size of the mini-batch for stochastic gradient descent.

Output: `mini_batch_x` and `mini_batch_y` are cells that contain a set of batches (images and labels, respectively). Each batch of images is a matrix with size $196 \times \text{batch_size}$, and each batch of labels is a matrix with size $10 \times \text{batch_size}$ (one-hot encoding). Note that the number of images in the last batch may be smaller than `batch_size`.

Description: You should randomly permute the the order of images when building the batch, and whole sets of `mini_batch_*` must span all training data.

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```
def fc(x, w, b)
```

```
    ...
```

```
    return y
```

Input: $\mathbf{x} \in \mathbb{R}^{m \times 1}$ is the input to the fully connected layer, and $\mathbf{w} \in \mathbb{R}^{n \times m}$ and $\mathbf{b} \in \mathbb{R}^{n \times 1}$ are the weights and bias.

Output: $\mathbf{y} \in \mathbb{R}^{n \times 1}$ is the output of the linear transform (fully connected layer).

Description: FC is a linear transform of \mathbf{x} , i.e., $\mathbf{y} = \mathbf{w}\mathbf{x} + \mathbf{b}$.

```
def fc_backward(dl_dy, x, w, b, y)
```

```
    ...
```

```
    return dl_dx, dl_dw, dl_db
```

Input: $\mathbf{dl_dy} \in \mathbb{R}^{1 \times n}$ is the loss derivative with respect to the output \mathbf{y} .

Output: $\mathbf{dl_dx} \in \mathbb{R}^{1 \times m}$ is the loss derivative with respect the input \mathbf{x} , $\mathbf{dl_dw} \in \mathbb{R}^{1 \times (n \times m)}$ is the loss derivative with respect to the weights, and $\mathbf{dl_db} \in \mathbb{R}^{1 \times n}$ is the loss derivative with respect to the bias.

Description: The partial derivatives w.r.t. input, weights, and bias will be computed. $\mathbf{dl_dx}$ will be back-propagated, and $\mathbf{dl_dw}$ and $\mathbf{dl_db}$ will be used to update the weights and bias.

```
def loss_euclidean(y_tilde, y)
```

```
    ...
```

```
    return l, dl_dy
```

Input: $\mathbf{y_tilde} \in \mathbb{R}^m$ is the prediction, and $\mathbf{y} \in 0, 1^m$ is the ground truth label.

Output: $l \in \mathbb{R}$ is the loss, and $\mathbf{dl_dy}$ is the loss derivative with respect to the prediction.

Description: `loss_euclidean` measure Euclidean distance $L = \|\mathbf{y} - \tilde{\mathbf{y}}\|^2$.

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```
def train_slp_linear(mini_batch_x, mini_batch_y)
```

```
    ...
```

```
    return w, b
```

Input: `mini_batch_x` and `mini_batch_y` are cells where each cell is a batch of images and labels.

Output: $\mathbf{w} \in \mathbb{R}^{10 \times 196}$ and $\mathbf{b} \in \mathbb{R}^{10 \times 1}$ are the trained weights and bias of a single-layer perceptron.

Description: You will use `fc`, `fc_backward`, and `loss_euclidean` to train a single-layer perceptron using a stochastic gradient descent method where a pseudo-code can be found below. Through training, you are expected to see reduction of loss as shown in Figure 2(b). As a result of training, the network should produce more than 25% of accuracy on the testing data (Figure 2(c)).

Algorithm 1 Stochastic Gradient Descent based Training

```
1: Set the learning rate  $\gamma$ 
2: Set the decay rate  $\lambda \in (0, 1]$ 
3: Initialize the weights with a Gaussian noise  $\mathbf{w} \in \mathcal{N}(0, 1)$ 
4:  $k = 1$ 
5: for ilter = 1 : nIters do
6:   At every 1000th iteration,  $\gamma \leftarrow \lambda\gamma$ 
7:    $\frac{\partial L}{\partial \mathbf{w}} \leftarrow 0$  and  $\frac{\partial L}{\partial \mathbf{b}} \leftarrow 0$ 
8:   for Each image  $\mathbf{x}_i$  in  $k^{\text{th}}$  mini-batch do
9:     Label prediction of  $\mathbf{x}_i$ 
10:    Loss computation  $l$ 
11:    Gradient back-propagation of  $\mathbf{x}_i$ ,  $\frac{\partial l}{\partial \mathbf{w}}$  using back-propagation.
12:     $\frac{\partial L}{\partial \mathbf{w}} = \frac{\partial L}{\partial \mathbf{w}} + \frac{\partial l}{\partial \mathbf{w}}$  and  $\frac{\partial L}{\partial \mathbf{b}} = \frac{\partial L}{\partial \mathbf{b}} + \frac{\partial l}{\partial \mathbf{b}}$ 
13:   end for
14:    $k++$  (Set  $k = 1$  if  $k$  is greater than the number of mini-batches.)
15:   Update the weights,  $\mathbf{w} \leftarrow \mathbf{w} - \frac{\gamma}{R} \frac{\partial L}{\partial \mathbf{w}}$ , and bias  $\mathbf{b} \leftarrow \mathbf{b} - \frac{\gamma}{R} \frac{\partial L}{\partial \mathbf{b}}$ 
16: end for
```

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4 Single-layer Perceptron

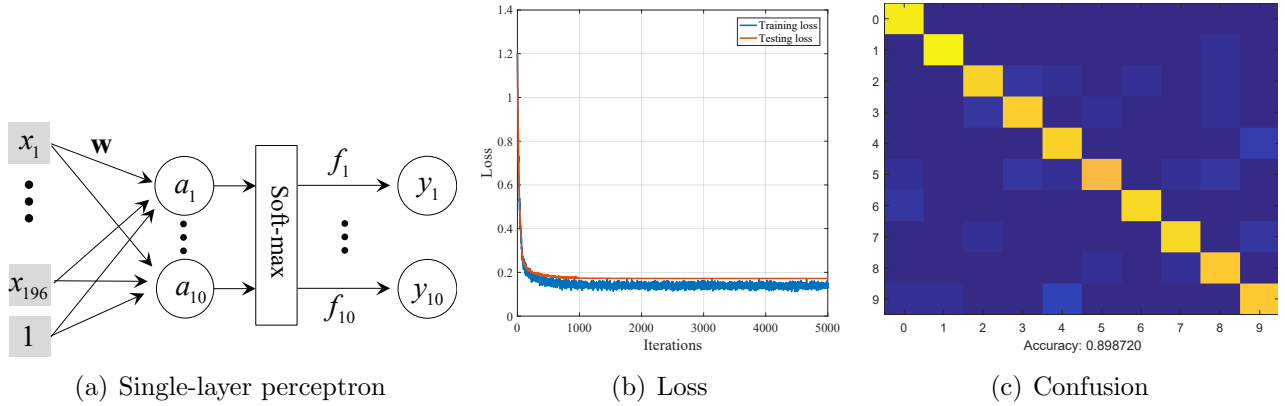


Figure 3: You will implement a single perceptron that produces accuracy near 90% on testing data.

You will implement a single-layer perceptron with *soft-max cross-entropy* using stochastic gradient descent method. We provide `main_slp` where you will implement `train_slp`. Unlike the single-layer linear perceptron, it has a soft-max layer that approximates a max function by clamping the output to $[0, 1]$ range as shown in Figure 3(a).

```
def loss_cross_entropy_softmax(x, y)
    ...
    return l, dl_dy
```

Input: $\mathbf{x} \in \mathbb{R}^{m \times 1}$ is the input to the soft-max, and $y \in 0, 1^m$ is the ground truth label.

Output: $L \in \mathbb{R}$ is the loss, and dl_dy is the loss derivative with respect to \mathbf{x} .

Description: `Loss_cross_entropy_softmax` measure cross-entropy between two distributions $L = \sum_i^m \mathbf{y}_i \log \tilde{\mathbf{y}}_i$ where $\tilde{\mathbf{y}}_i$ is the soft-max output that approximates the max operation by clamping \mathbf{x} to $[0, 1]$ range:

$$\tilde{\mathbf{y}}_i = \frac{e^{\mathbf{x}_i}}{\sum_i e^{\mathbf{x}_i}},$$

where \mathbf{x}_i is the i^{th} element of \mathbf{x} .

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```
def train_slp(mini_batch_x, mini_batch_y)
```

```
    ...
```

```
    return w, b
```

Output: $\mathbf{w} \in \mathbb{R}^{10 \times 196}$ and $\mathbf{b} \in \mathbb{R}^{10 \times 1}$ are the trained weights and bias of a single-layer perceptron.

Description: You will use the following functions to train a single-layer perceptron using a stochastic gradient descent method: `fc`, `fc_backward`, `loss_cross_entropy_softmax`

Through training, you are expected to see reduction of loss as shown in Figure 3(b). As a result of training, the network should produce more than 85% of accuracy on the testing data (Figure 3(c)).

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5 Multi-layer Perceptron

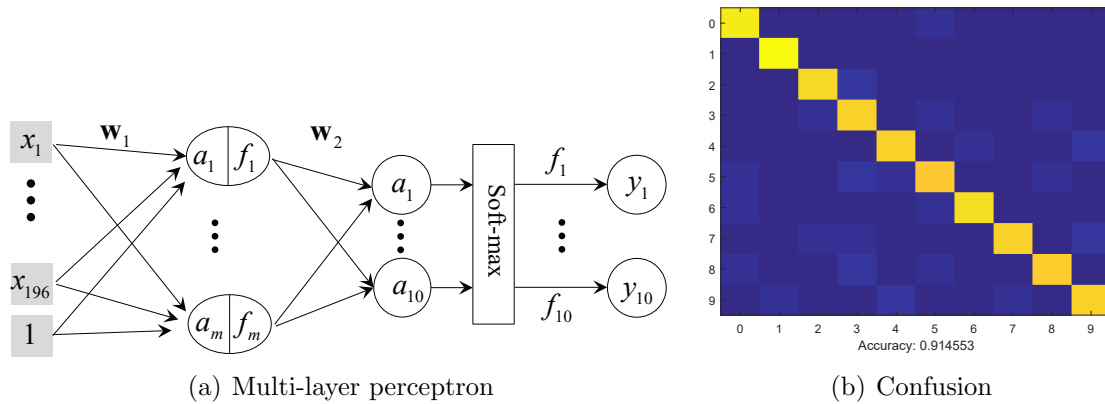


Figure 4: You will implement a multi-layer perceptron that produces accuracy more than 90% on testing data.

You will implement a multi-layer perceptron with a single hidden layer using a stochastic gradient descent method. We provide `main_mlp`. The hidden layer is composed of 30 units as shown in Figure 4(a).

```
def relu(x)
```

```
    ...
```

```
    return y
```

Input: x is a general tensor, matrix, and vector.

Output: y is the output of the Rectified Linear Unit (ReLU) with the same input size.

Description: ReLU is an activation unit ($y_i = \max(0, x_i)$). In some case, it is possible to use a Leaky ReLU ($y_i = \max(\epsilon x_i, x_i)$ where $\epsilon = 0.01$).

```
def relu_backward(dl_dy, x, y)
```

```
    ...
```

```
    return dl_dx
```

Input: $dl_dy \in \mathbb{R}^{1 \times z}$ is the loss derivative with respect to the output $y \in \mathbb{R}^z$ where z is the size of input (it can be tensor, matrix, and vector).

Output: $dl_dx \in \mathbb{R}^{1 \times z}$ is the loss derivative with respect to the input x .

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```
def train_mlp(mini_batch_x, mini_batch_y)
```

```
    ...
```

```
    return w1, b1, w2, b2
```

Output: $w1 \in \mathbb{R}^{30 \times 196}$, $b1 \in \mathbb{R}^{30 \times 1}$, $w2 \in \mathbb{R}^{10 \times 30}$, $b2 \in \mathbb{R}^{10 \times 1}$ are the trained weights and biases of a multi-layer perceptron.

Description: You will use the following functions to train a multi-layer perceptron using a stochastic gradient descent method: `fc`, `fc_backward`, `relu`, `relu_backward`, `loss_cross_entropy_softmax`. As a result of training, the network should produce more than 90% of accuracy on the testing data (Figure 4(b)).

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6 Convolutional Neural Network

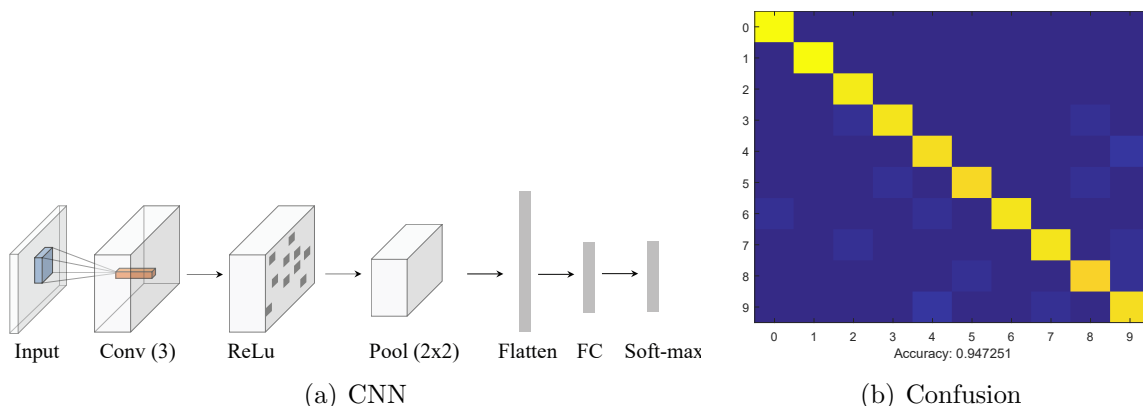


Figure 5: You will implement a convolutional neural network that produces accuracy more than 92% on testing data.

You will implement a convolutional neural network (CNN) using a stochastic gradient descent method. We provide `main_cnn`. As shown in Figure 4(a), the network is composed of: a single channel input ($14 \times 14 \times 1$) \rightarrow Conv layer (3×3 convolution with 3 channel output and stride 1) \rightarrow ReLu layer \rightarrow Max-pooling layer (2×2 with stride 2) \rightarrow Flattening layer (147 units) \rightarrow FC layer (10 units) \rightarrow Soft-max.

```
def conv(x, w_conv, b_conv)
```

```
    ...
    return y
```

Input: $\mathbf{x} \in \mathbb{R}^{H \times W \times C_1}$ is an input to the convolutional operation, $\mathbf{w_conv} \in \mathbb{R}^{h \times w \times C_1 \times C_2}$ and $\mathbf{b_conv} \in \mathbb{R}^{C_2 \times 1}$ are weights and bias of the convolutional operation.

Output: $\mathbf{y} \in \mathbb{R}^{H \times W \times C_2}$ is the output of the convolutional operation. Note that to get the same size with the input, you may pad zero at the boundary of the input image.

Description: You can use `np.pad` for padding 0s at boundary. Optionally, you may use `im2col`¹ to simplify convolutional operation.

¹https://leonardoaraujosantos.gitbooks.io/artificial-intelligence/content/making_faster.html

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```
def conv_backward(dl_dy, x, w_conv, b_conv, y)
```

```
    ...
```

```
    return dl_dw, dl_db
```

Input: dl_dy is the loss derivative with respect to y .

Output: dl_dw and dl_db are the loss derivatives with respect to convolutional weights and bias w and b , respectively.

Description: Note that for the single convolutional layer, $\frac{\partial L}{\partial x}$ is not needed. Optionally, you may use `im2col` to simplify convolutional operation.

```
def pool2x2(x)
```

```
    ...
```

```
    return y
```

Input: $x \in \mathbb{R}^{H \times W \times C}$ is a general tensor and matrix.

Output: $y \in \mathbb{R}^{\frac{H}{2} \times \frac{W}{2} \times C}$ is the output of the 2×2 max-pooling operation with stride 2.

```
def pool2x2_backward(dl_dy, x, y)
```

```
    ...
```

```
    return dl_dx
```

Input: dl_dy is the loss derivative with respect to the output y .

Output: dl_dx is the loss derivative with respect to the input x .

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```
def flattening(x)
```

```
    ...
```

```
    return y
```

Input: $x \in \mathbb{R}^{H \times W \times C}$ is a tensor.

Output: $y \in \mathbb{R}^{HWC}$ is the vectorized tensor (column major).

```
def flattening_backward(dl_dy, x, y)
```

```
    ...
```

```
    return dl_dx
```

Input: dl_dy is the loss derivative with respect to the output y .

Output: dl_dx is the loss derivative with respect to the input x .

```
function train_cnn(mini_batch_x, mini_batch_y)
```

```
    ...
```

```
    return w_conv, b_conv, w_fc, b_fc
```

Output: $w_conv \in \mathbb{R}^{3 \times 3 \times 1 \times 3}$, $b_conv \in \mathbb{R}^3$, $w_fc \in \mathbb{R}^{10 \times 147}$, $b_fc \in \mathbb{R}^{10 \times 1}$ are the trained weights and biases of the CNN.

Description: You will use the following functions to train a convolutional neural network using a stochastic gradient descent method: `conv`, `conv_backward`, `pool2x2`, `pool2x2_backward`, `Flattening`, `flattening_backward`, `fc`, `fc_backward`, `relu`, `relu_backward`, `loss_cross_entropy_softmax`. As a result of training, the network should produce more than 92% of accuracy on the testing data (Figure 5(b)).