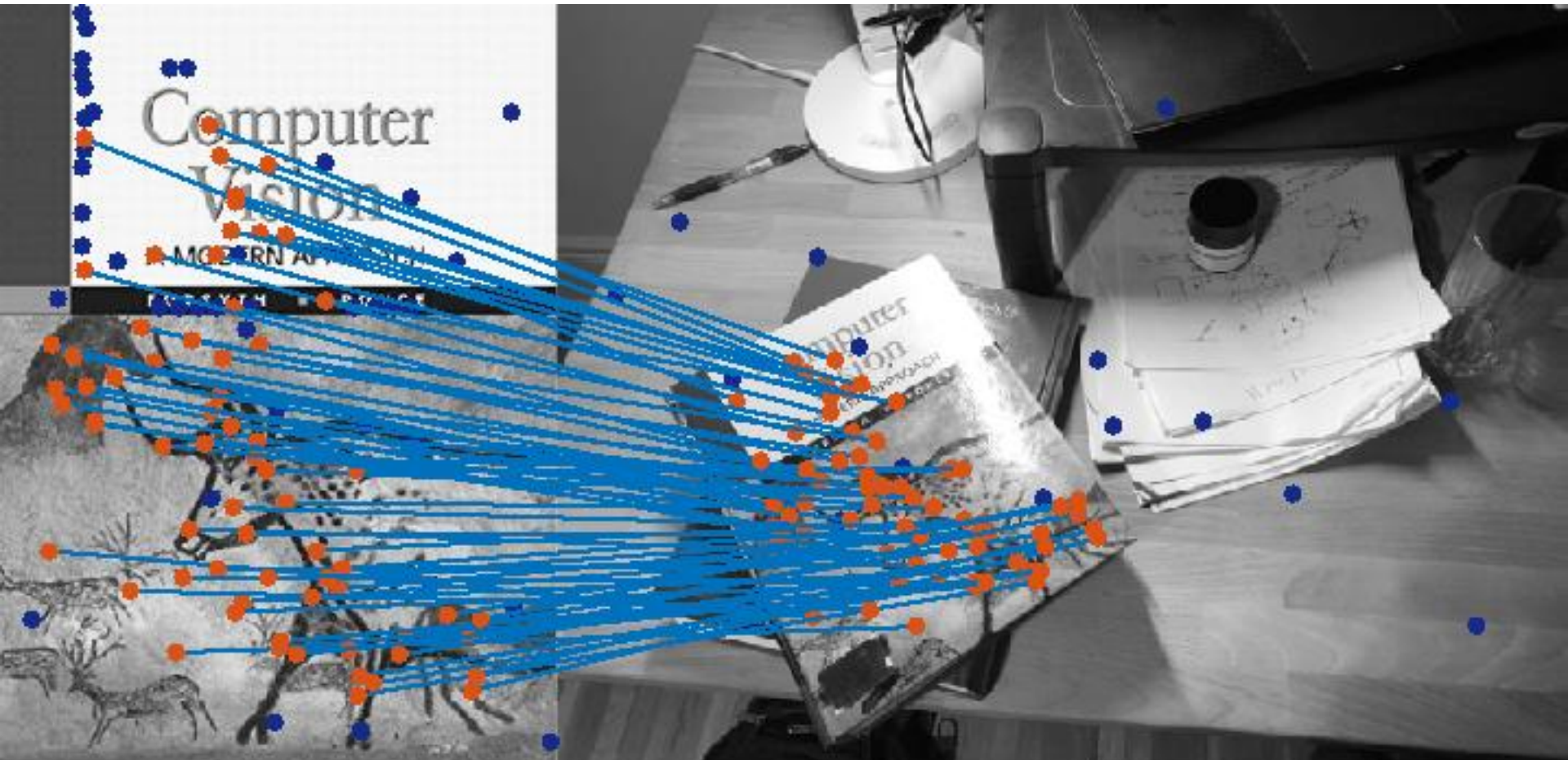


IMAGE WARPING

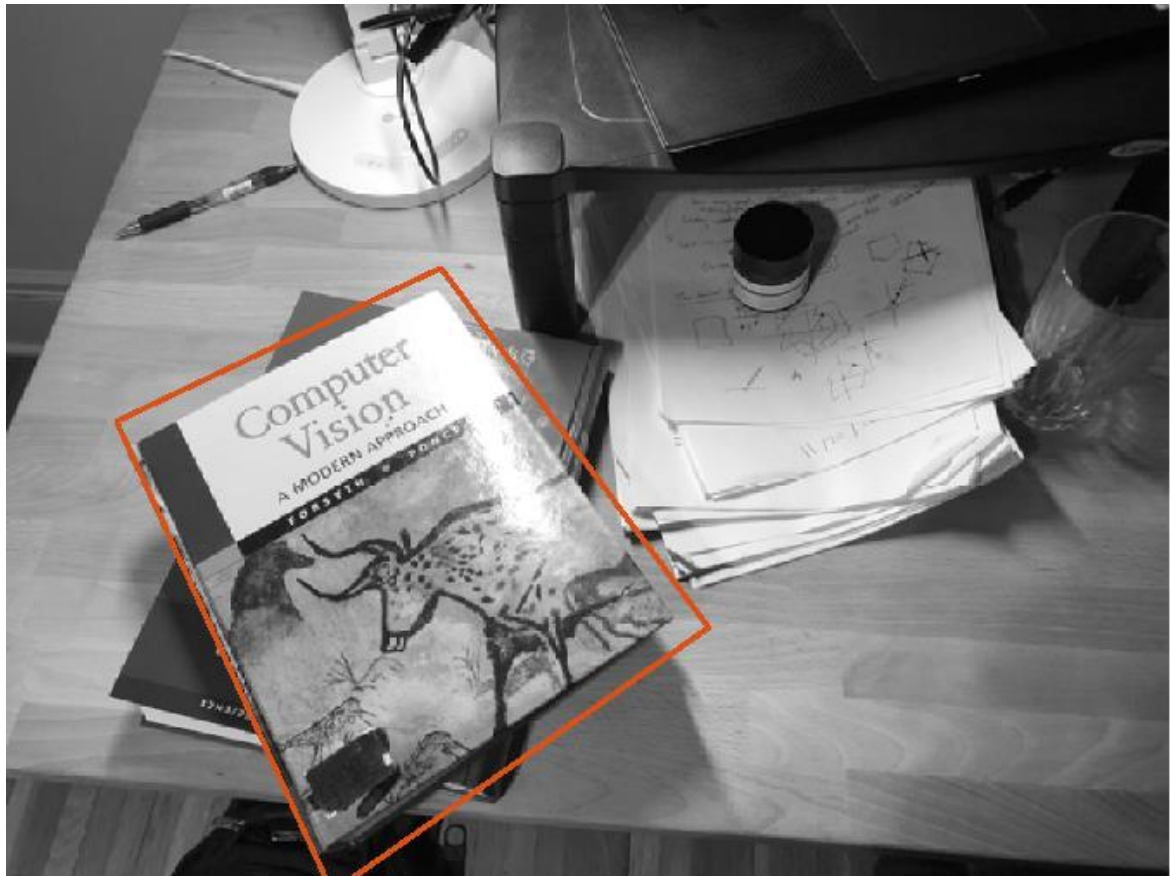
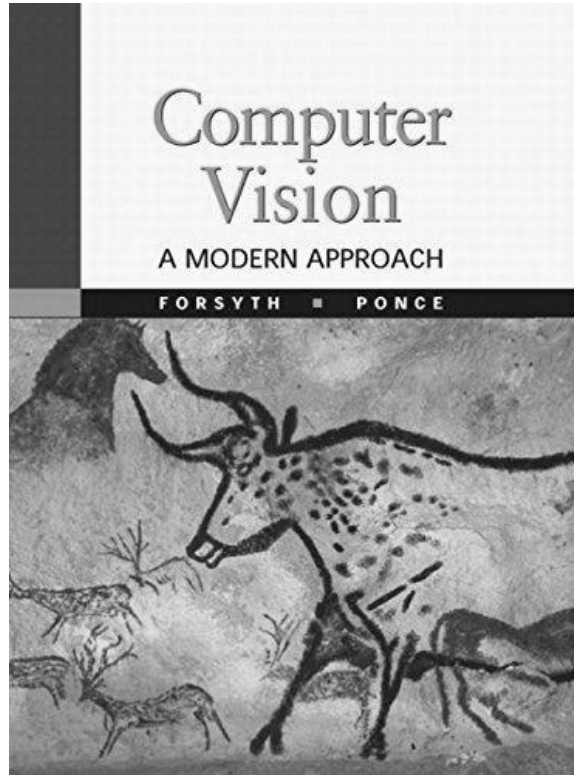
HYUN SOO PARK



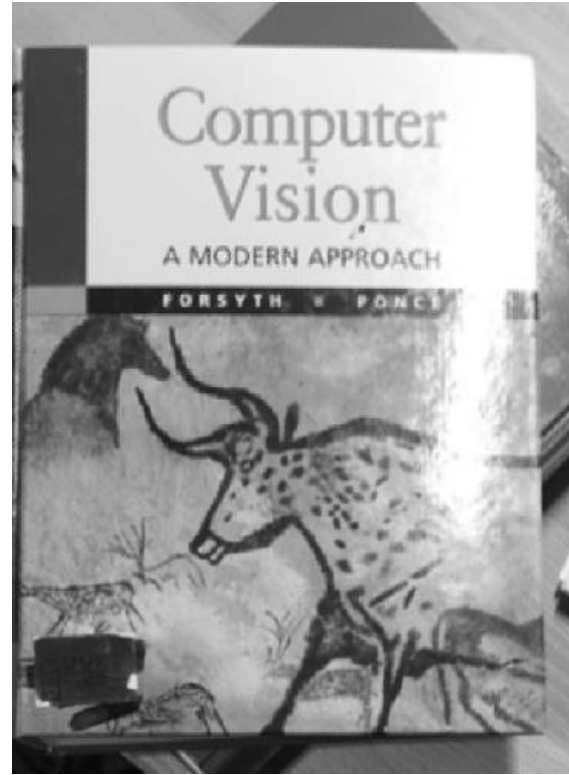
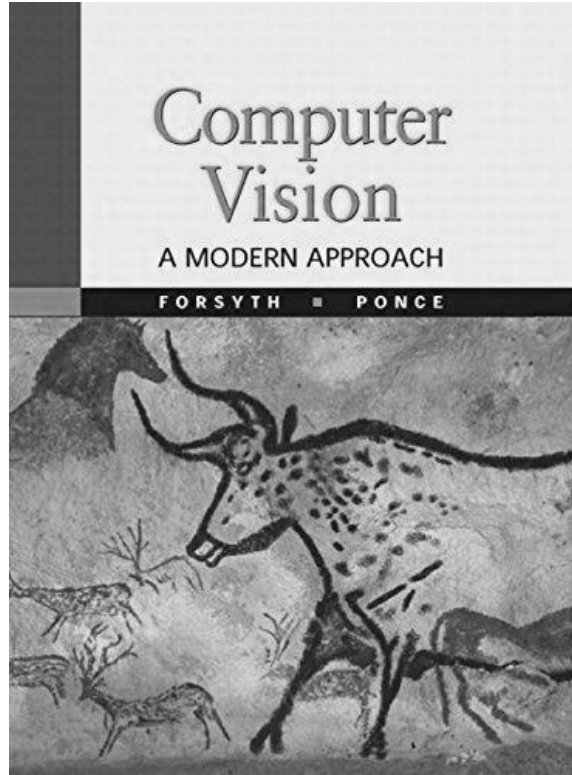
RECALL: ROBUST FILTERING



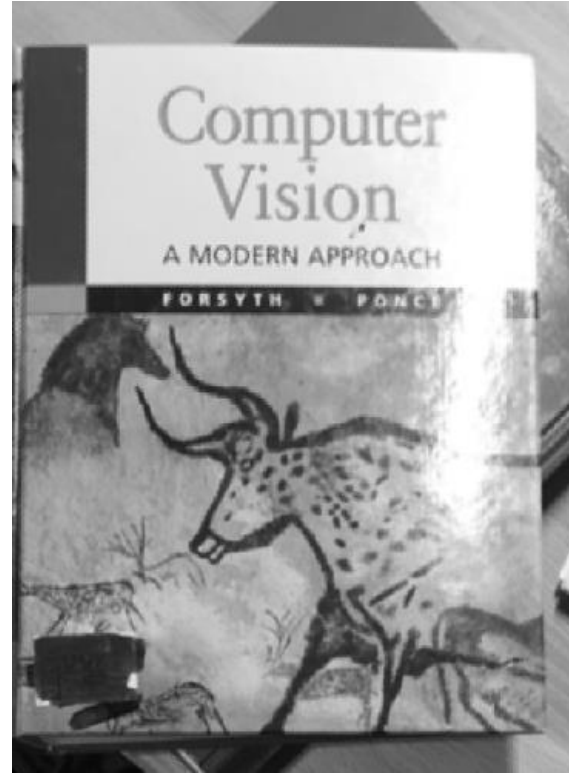
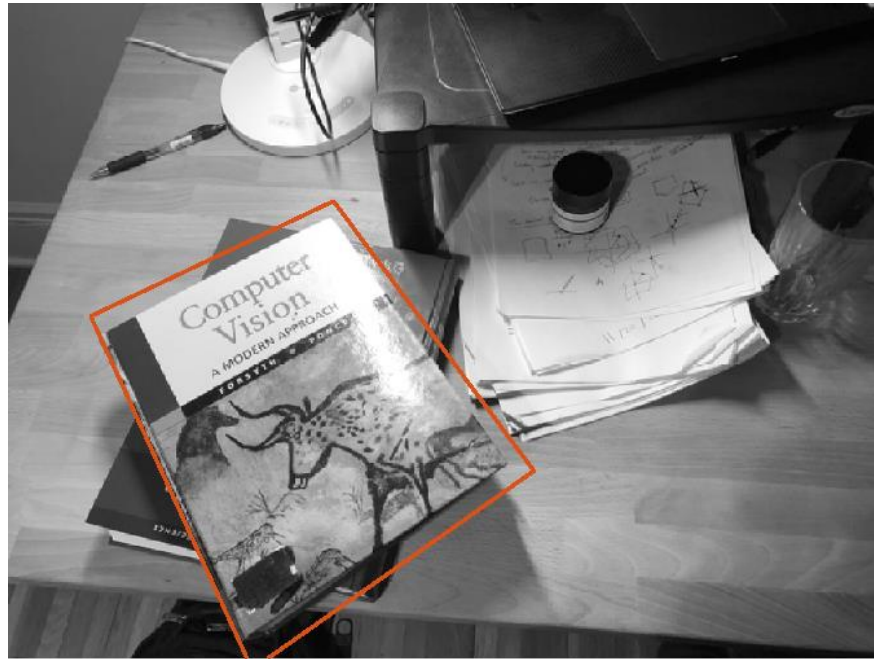
RECALL: PARAMETRIC MODEL



RECALL: IMAGE WARPING



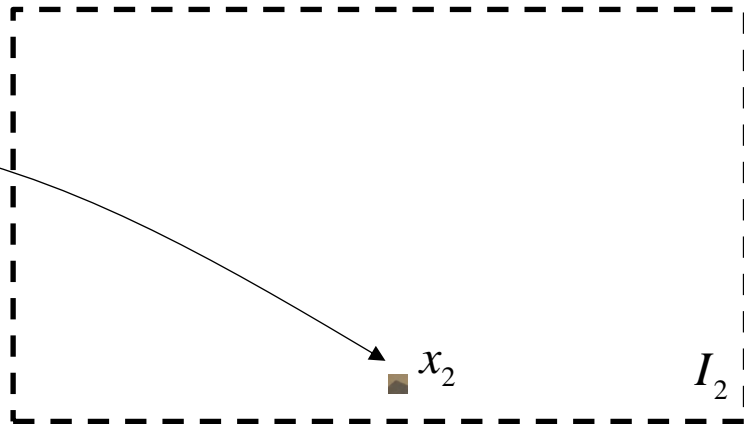
RECALL: IMAGE WARPING



PIXEL TRANSPORT



Source image



Target image

$$\frac{x_2 = f(x_1)}{\text{Coordinate transform}}$$

PIXEL TRANSPORT



Source image

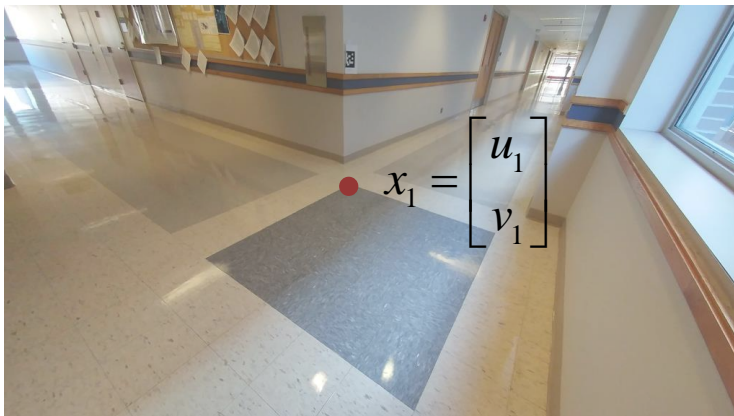


Target image

$$\frac{x_2 = f(x_1)}{\text{Coordinate transform}}$$

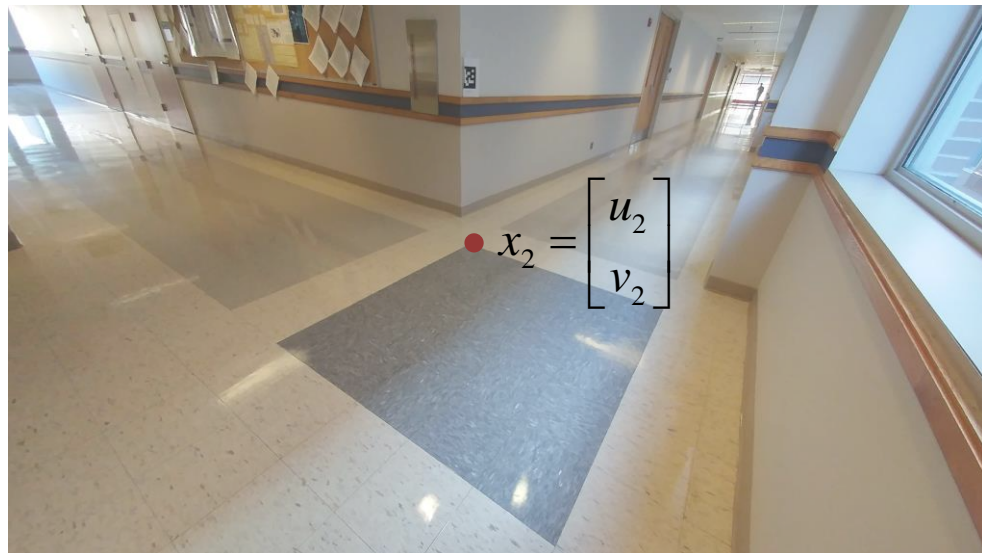
$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

UNIFORM SCALING

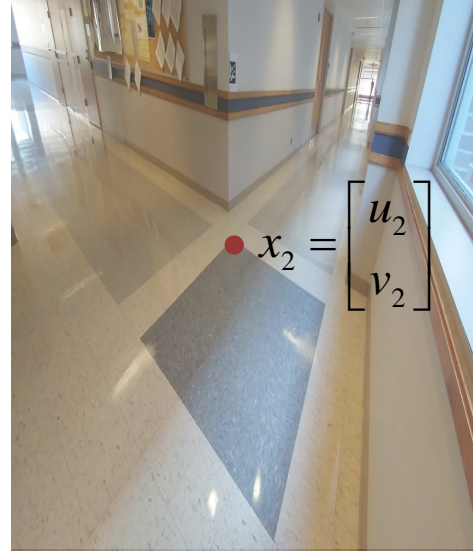
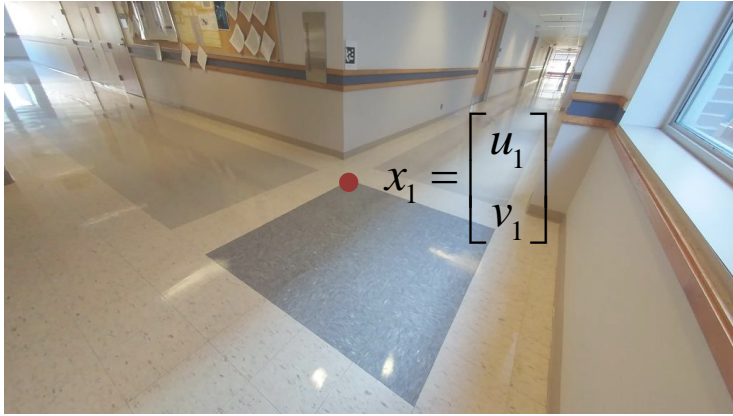


$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

$$x_2 = f(x_1) \longrightarrow \begin{aligned} u_2 &= su_1 \\ v_2 &= sv_1 \end{aligned}$$



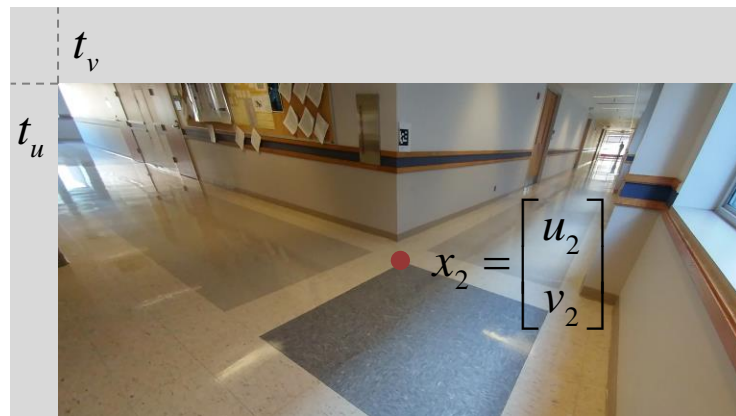
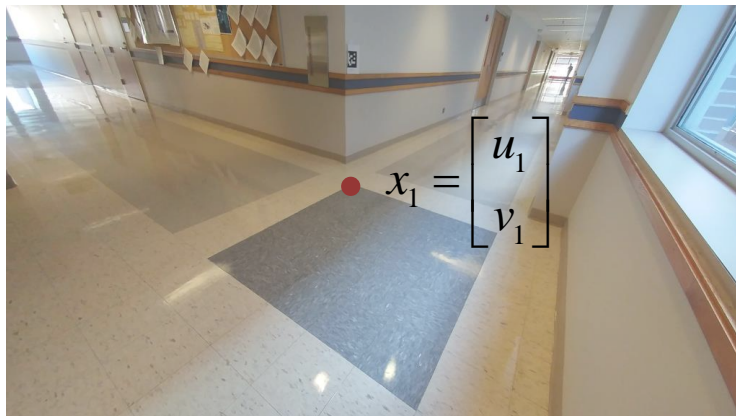
NON-UNIFORM SCALING (ASPECT RATIO CHANGE)



$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

$$x_2 = f(x_1) \rightarrow \begin{aligned} u_2 &= s_u u_1 \\ v_2 &= s_v v_1 \end{aligned}$$

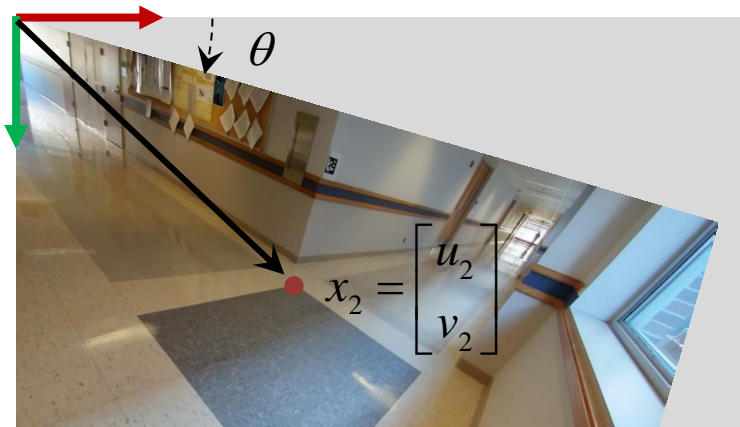
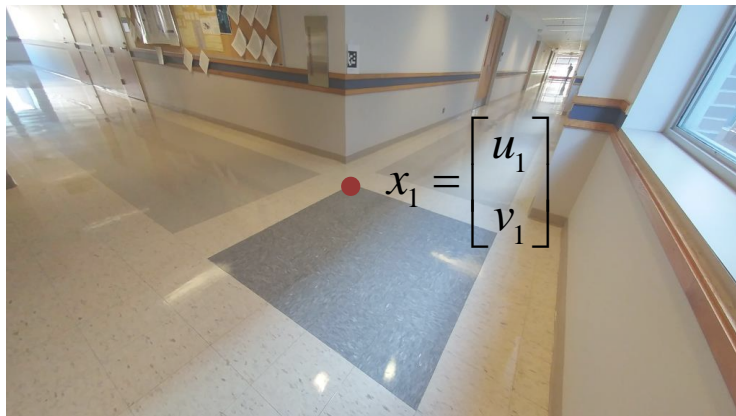
TRANSLATION



$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

$$x_2 = f(x_1) \rightarrow \begin{aligned} u_2 &= u_1 + t_u \\ v_2 &= v_1 + t_v \end{aligned}$$

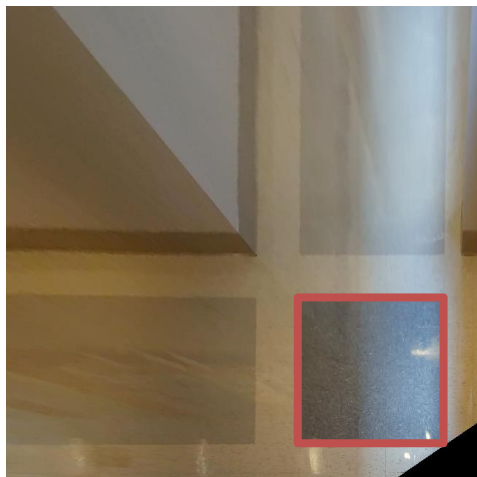
ROTATION



$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

$$x_2 = f(x_1) \rightarrow \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \end{bmatrix}$$

EUCLIDEAN TRANSFORM SE(2) ~ TRANSLATION+ROTATION



Invariant properties

- Length
- Angle
- Area

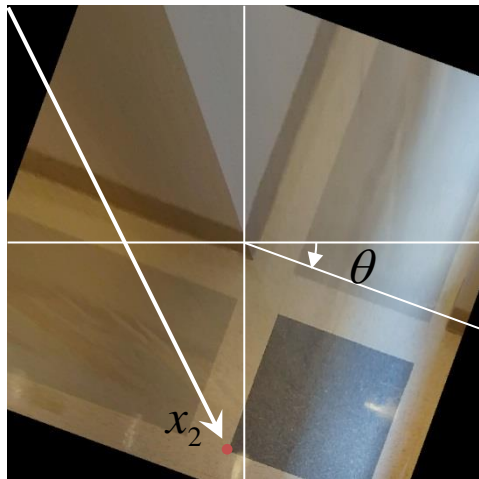
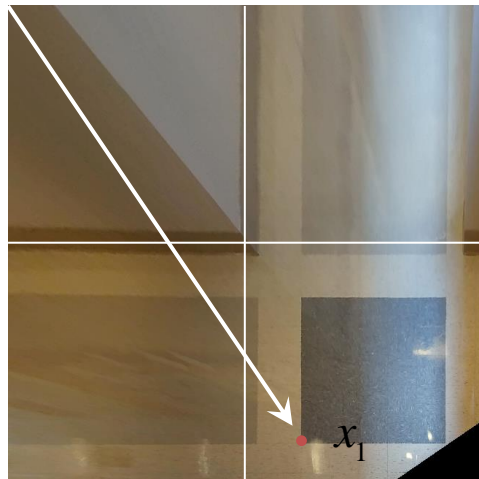
Degree of freedom

3 (2 translation+1 rotation)

$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

$$x_2 = f(x_1) \rightarrow \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & t_u \\ \sin \theta & \cos \theta & t_v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

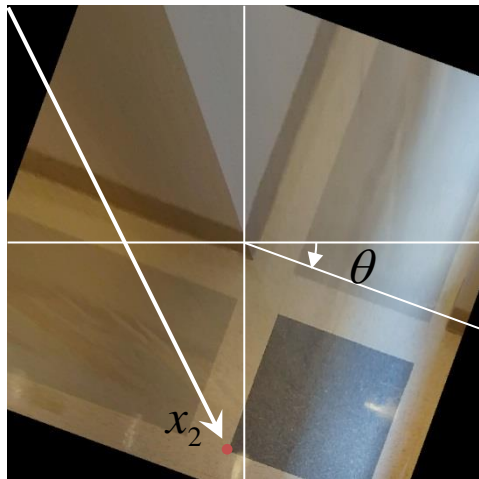
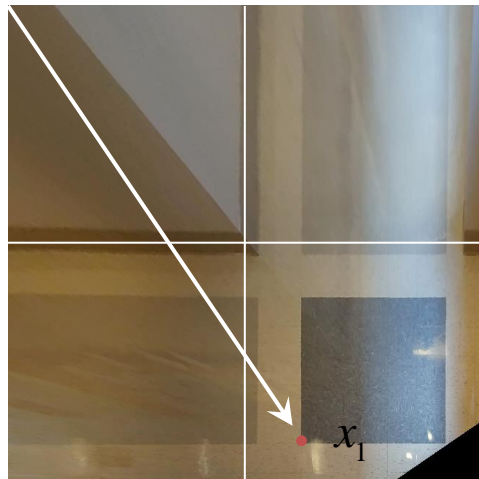
EXAMPLE OF EUCLIDEAN TRANSFORM



Rotate about the image center

$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & t_u \\ \sin \theta & \cos \theta & t_v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

EXAMPLE OF EUCLIDEAN TRANSFORM

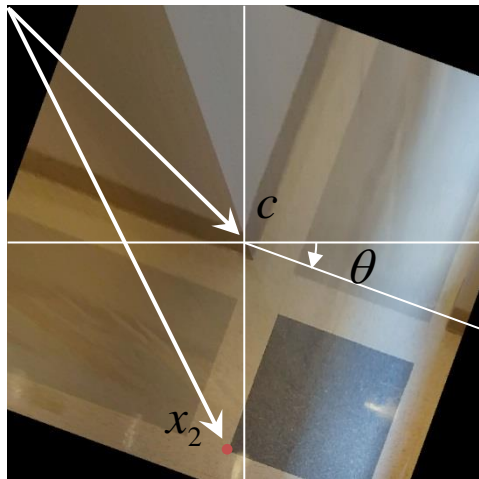
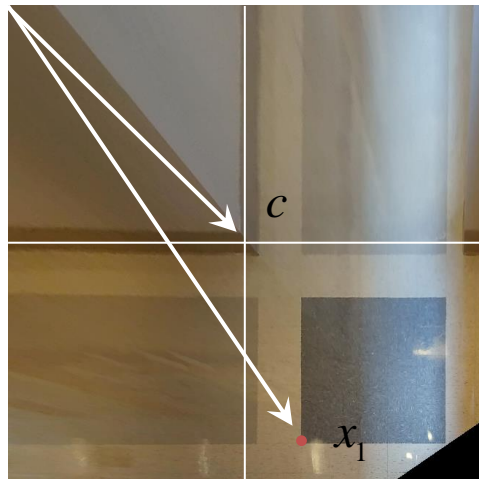


Rotate about the image center

$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix} \quad t = ?$$

$$x_2 = Rx_1 + t$$

EXAMPLE OF EUCLIDEAN TRANSFORM



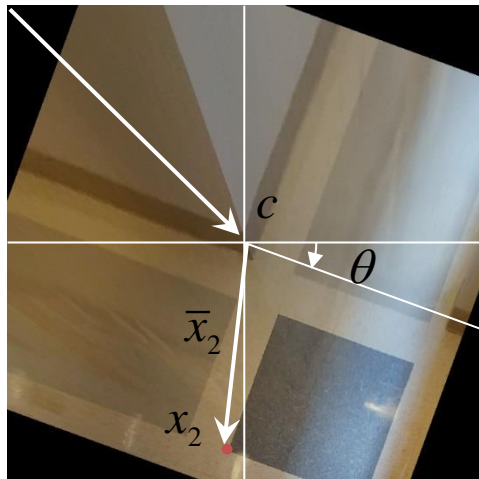
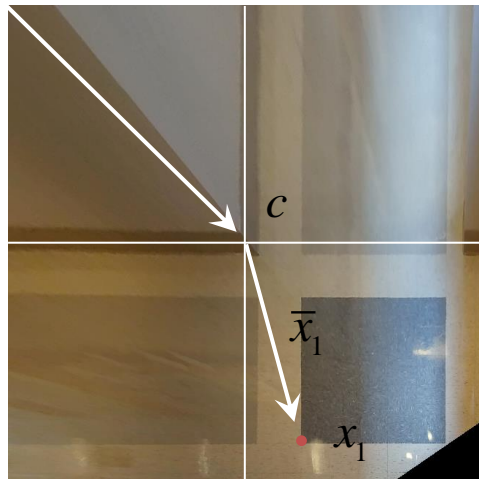
c : image center

Rotate about the image center

$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix} \quad t = ?$$

$$x_2 = Rx_1 + t$$

EXAMPLE OF EUCLIDEAN TRANSFORM



c : image center

Rotate about the image center

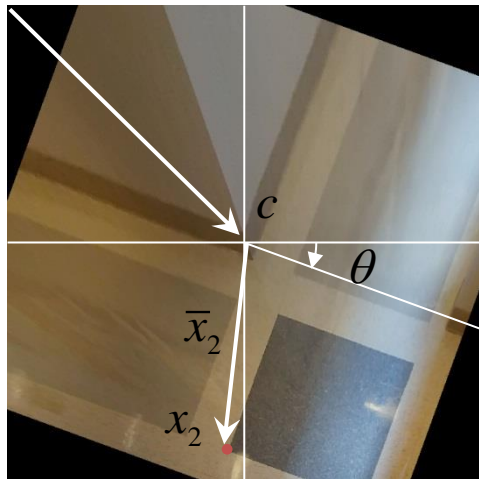
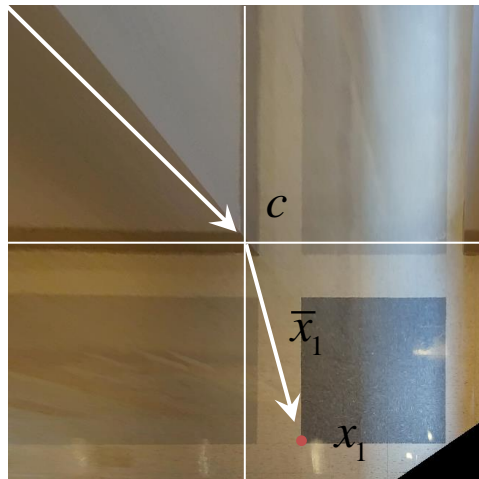
$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix} \quad t = ?$$

$$x_2 = Rx_1 + t$$

$$x_1 = c + \bar{x}_1$$

$$x_2 = c + \bar{x}_2$$

EXAMPLE OF EUCLIDEAN TRANSFORM



c : image center

Rotate about the image center

$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix} \quad t = ?$$

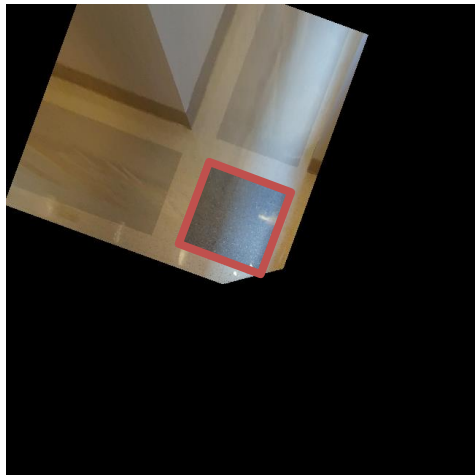
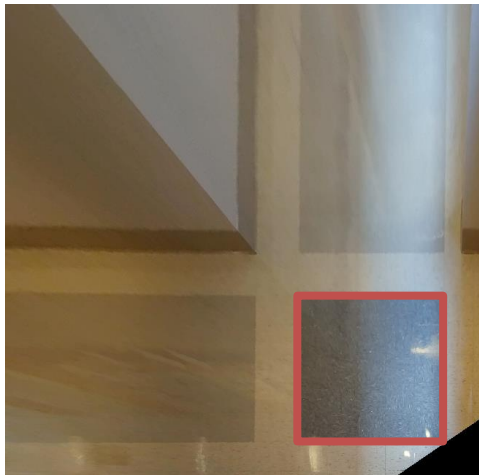
$$x_2 = Rx_1 + t$$

$$x_1 = c + \bar{x}_1$$

$$x_2 = c + \bar{x}_2$$

$$\longrightarrow t = -Rc + c$$

SIMILARITY TRANSFORM ~ EUCLIDEAN + UNIFORM SCALING



$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} sR & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix}$$

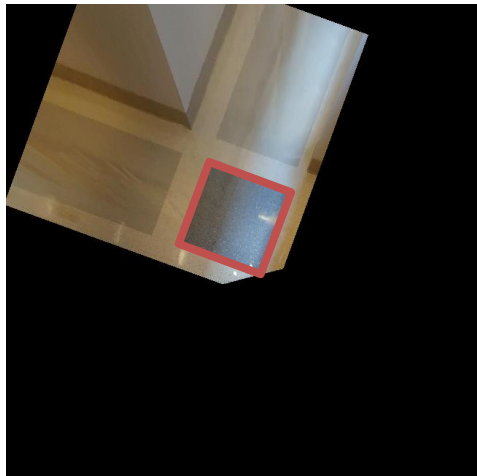
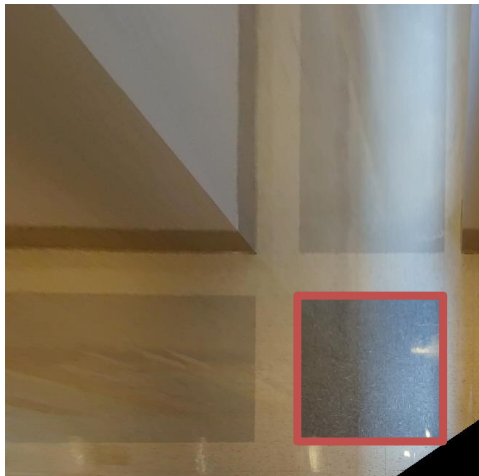
Invariant properties

- Length ratio
- Angle

Degree of freedom

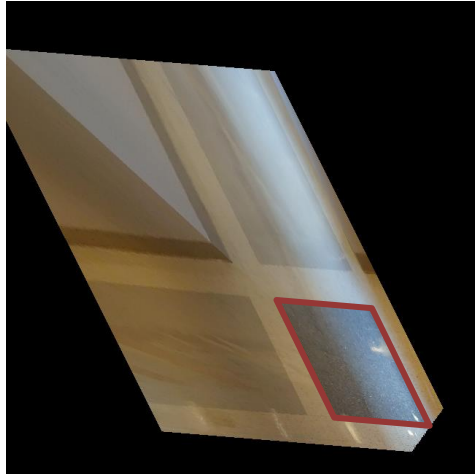
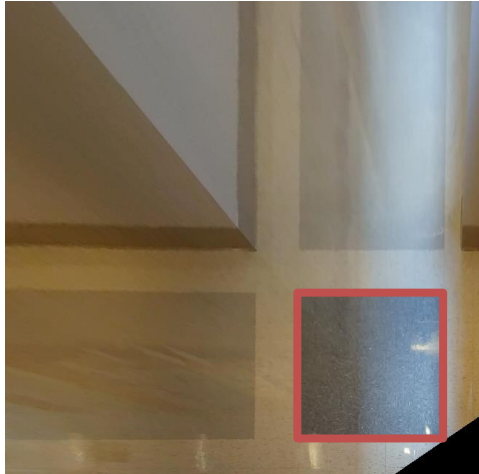
4 (2 translation+1 rotation+1 scale)

SIMILARITY TRANSFORM ~ EUCLIDEAN + UNIFORM SCALING



$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} s \cos \theta & -s \sin \theta & t_u \\ s \sin \theta & s \cos \theta & t_v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

AFFINE TRANSFORM



$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

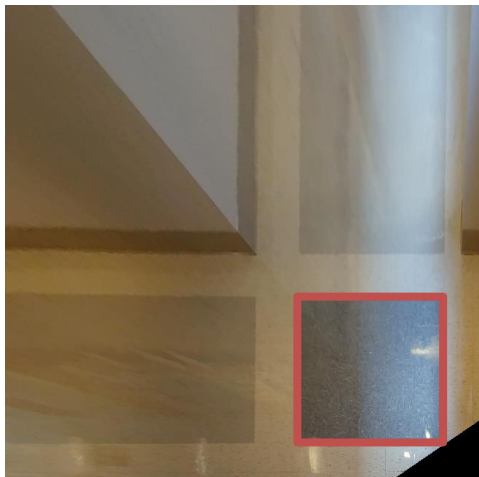
Invariant properties

- Parallelism
- Ratio of area
- Ratio of length

Degree of freedom

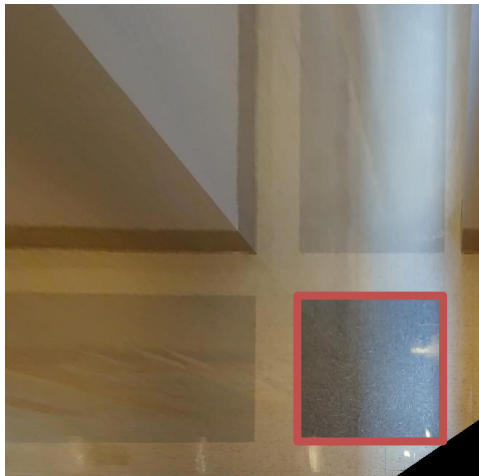
6

PERSPECTIVE TRANSFORM ~ HOMOGRAPHY



$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

PERSPECTIVE TRANSFORM ~ HOMOGRAPHY



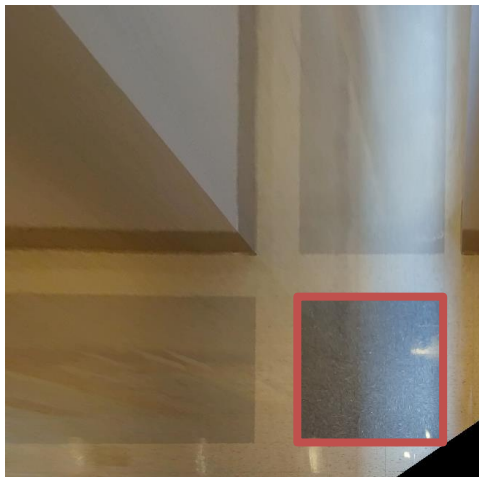
$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

$$u_2 = \frac{h_{11}u_1 + h_{12}v_1 + h_{13}}{h_{31}u_1 + h_{32}v_1 + h_{33}}$$

$$v_2 = \frac{h_{21}u_1 + h_{22}v_1 + h_{23}}{h_{31}u_1 + h_{32}v_1 + h_{33}}$$

: General form of plane to plane
linear mapping

PERSPECTIVE TRANSFORM ~ HOMOGRAPHY



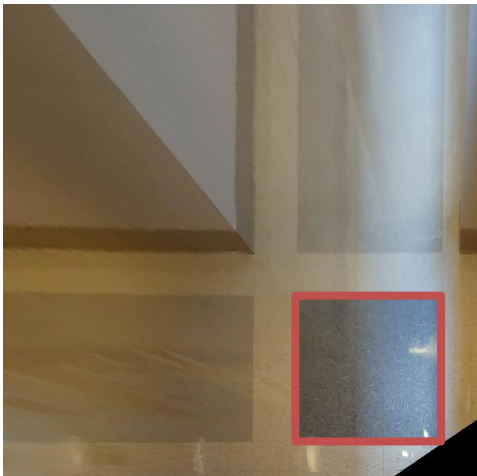
$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

$$u_2 = \frac{h_{11}u_1 + h_{12}v_1 + h_{13}}{h_{31}u_1 + h_{32}v_1 + 1}$$

$$v_2 = \frac{h_{21}u_1 + h_{22}v_1 + h_{23}}{h_{31}u_1 + h_{32}v_1 + 1}$$

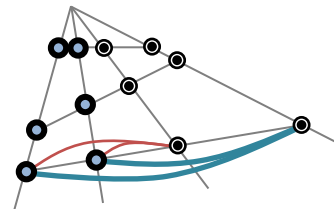
: General form of plane to plane
linear mapping

PERSPECTIVE TRANSFORM ~ HOMOGRAPHY



Invariant properties

- Cross ratio



- Concurrency



- Colinearity



Degree of freedom

8 (9 variables – 1 scale)

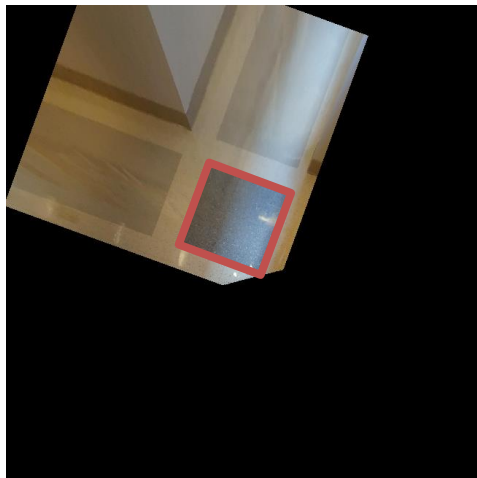
HIERACHY OF TRANSFORMATIONS



Euclidean (3 dof)

- Length
- Angle
- Area

$$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix}$$



Similarity (4 dof)

- Length ratio
- Angle

$$\begin{bmatrix} \alpha \cos\theta & -\alpha \sin\theta & t_x \\ \alpha \sin\theta & \alpha \cos\theta & t_y \\ & & 1 \end{bmatrix}$$



Affine (6 dof)

- Parallelism
- Ratio of area
- Ratio of length

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

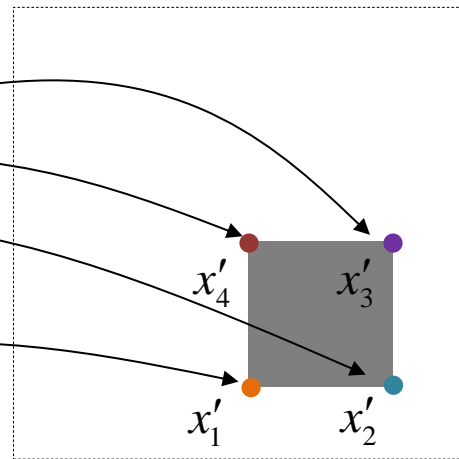
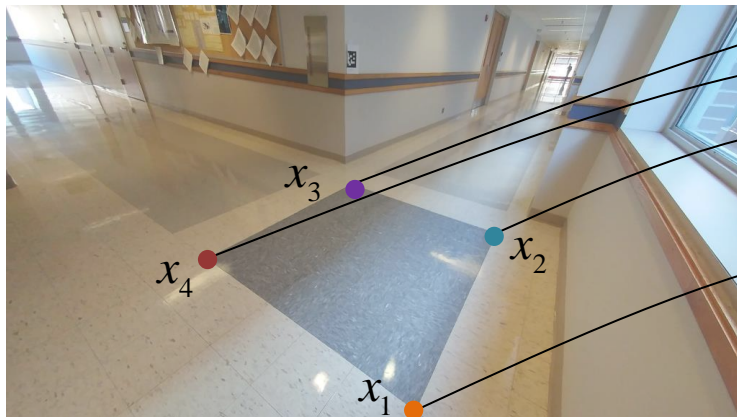


Projective (8 dof)

- Cross ratio
- Concurrency
- Colinearity

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

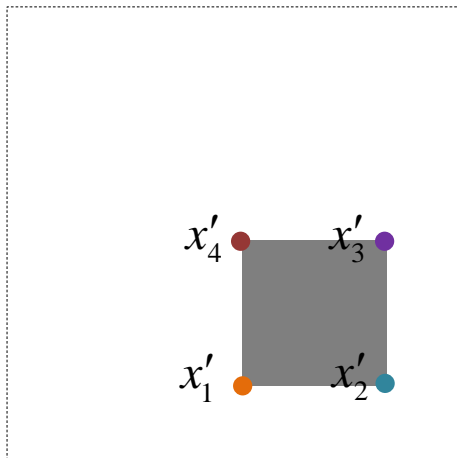
HOMOGRAPHY COMPUTATION



$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

The image can be rectified as if it is seen from top view.

HOMOGRAPHY COMPUTATION

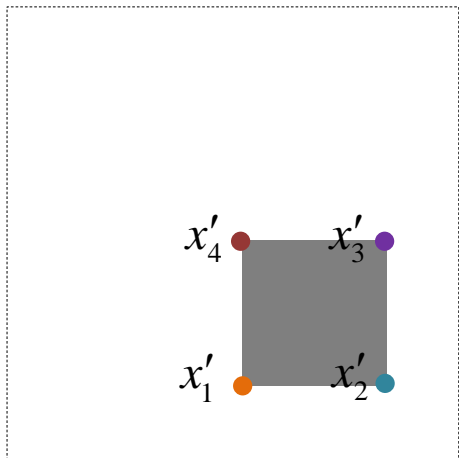


$$u'_1 = \frac{h_{11}u_1 + h_{12}v_1 + h_{13}}{h_{31}u_1 + h_{32}v_1 + 1}$$

$$v'_1 = \frac{h_{21}u_1 + h_{22}v_1 + h_{23}}{h_{31}u_1 + h_{32}v_1 + 1}$$

$$\begin{aligned} \rightarrow \quad & h_{11}u_1 + h_{12}v_1 + h_{13} - h_{31}u_1u'_1 - h_{32}v_1u'_1 - u'_1 = 0 \\ & h_{21}u_1 + h_{22}v_1 + h_{23} - h_{31}u_1v'_1 - h_{32}v_1v'_1 - v'_1 = 0 \end{aligned}$$

HOMOGRAPHY COMPUTATION



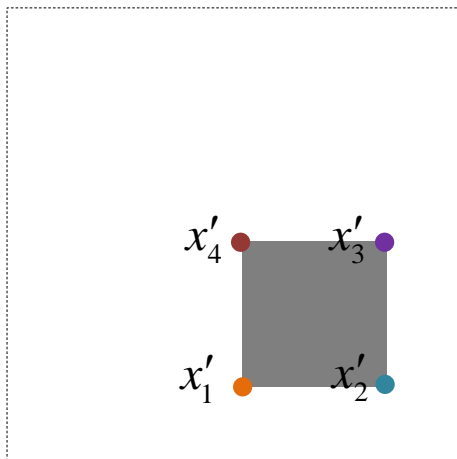
$$\begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1u' & -v_1u' \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1v' & -v_1v' \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} = \begin{bmatrix} u'_1 \\ v'_1 \end{bmatrix}$$

of equations: 2

of unknowns: 8

How many correspondences
are needed?

HOMOGRAPHY COMPUTATION

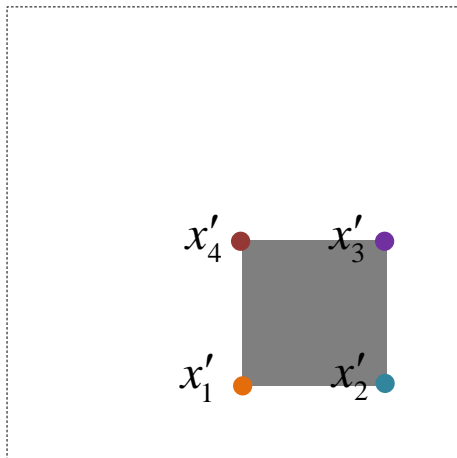


$$\begin{bmatrix}
 u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1u'_1 & -v_1u'_1 \\
 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1v'_1 & -v_1v'_1 \\
 & & & & & \vdots & & \\
 u_4 & v_4 & 1 & 0 & 0 & 0 & -u_4u'_4 & -v_4u'_4 \\
 0 & 0 & 0 & u_4 & v_4 & 1 & -u_4v'_4 & -v_4v'_4
 \end{bmatrix}
 \begin{bmatrix}
 h_{11} \\
 h_{12} \\
 h_{13} \\
 h_{21} \\
 h_{22} \\
 h_{23} \\
 h_{31} \\
 h_{32}
 \end{bmatrix}
 =
 \begin{bmatrix}
 u'_1 \\
 v'_1 \\
 \vdots \\
 u'_4 \\
 v'_4
 \end{bmatrix}$$

of equations: 2

of unknowns: 8

HOMOGRAPHY COMPUTATION



$$\begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1u'_1 & -v_1u'_1 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1v'_1 & -v_1v'_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_4 & v_4 & 1 & 0 & 0 & 0 & -u_4u'_4 & -v_4u'_4 \\ 0 & 0 & 0 & u_4 & v_4 & 1 & -u_4v'_4 & -v_4v'_4 \end{bmatrix} \mathbf{x} = \mathbf{b}$$

The matrix A is a 8×8 matrix. The vector \mathbf{x} is a column vector of size 8, and the vector \mathbf{b} is a column vector of size 8.

$$Ax = b \longrightarrow x = (A^T A)^{-1} A^T b$$

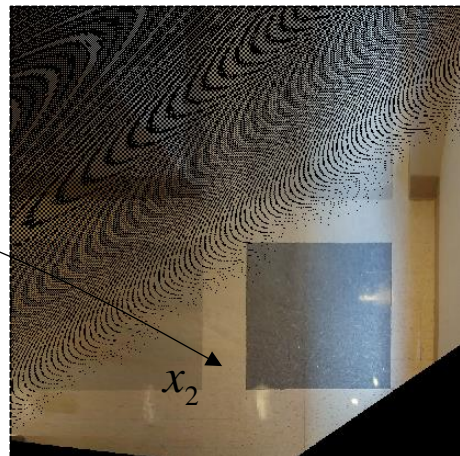
IMAGE WARPING



$$x_2 = f(x_1) \quad I_2(x_2) = I_1(f(x_1))$$

```
for i = 1 : size(im,1)
  for j = 1 : size(im,2)
    x1 = [j, i];
    x2 = floor(mapping(x1));
    I2(x2) = I1(x1);
  end
end
```

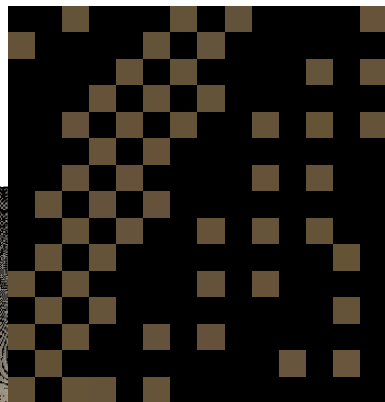
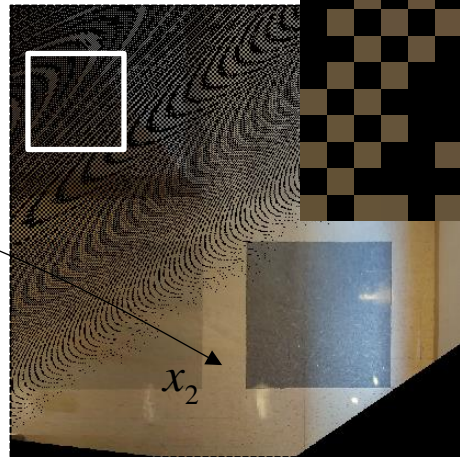
IMAGE WARPING



$$x_2 = f(x_1) \quad I_2(x_2) = I_1(f(x_1))$$

```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x1 = [j, i];
        x2 = floor(mapping(x1));
        I2(x2) = I1(x1);
    end
end
```


IMAGE WARPING

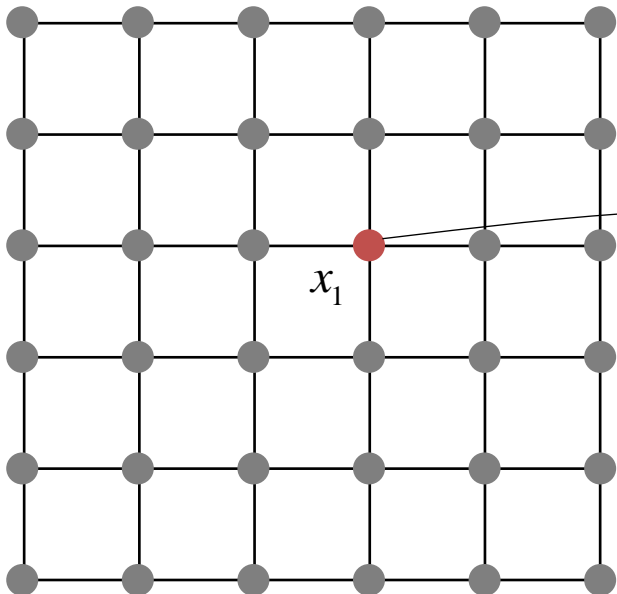


$$x_2 = f(x_1) \quad I_2(x_2) = I_1(f(x_1))$$

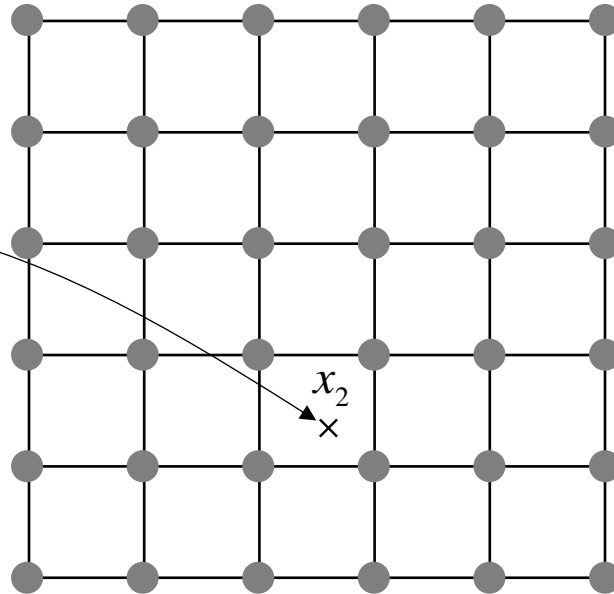
```
for i = 1 : size(im,1)
  for j = 1 : size(im,2)
    x1 = [j, i];
    x2 = floor(mapping(x1));
    I2(x2) = I1(x1);
  end
end
```

IMAGE WARPING

Source



Target

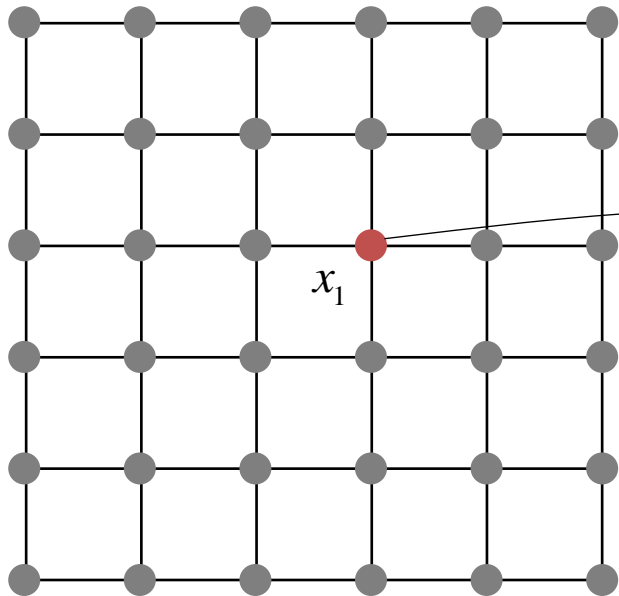


$$x_2 = f(x_1)$$

```
for i = 1 : size(im,1)
  for j = 1 : size(im,2)
    x1 = [j, i];
    x2 = floor(mapping(x1));
    I2(x2) = I1(x1);
  end
end
```

IMAGE WARPING

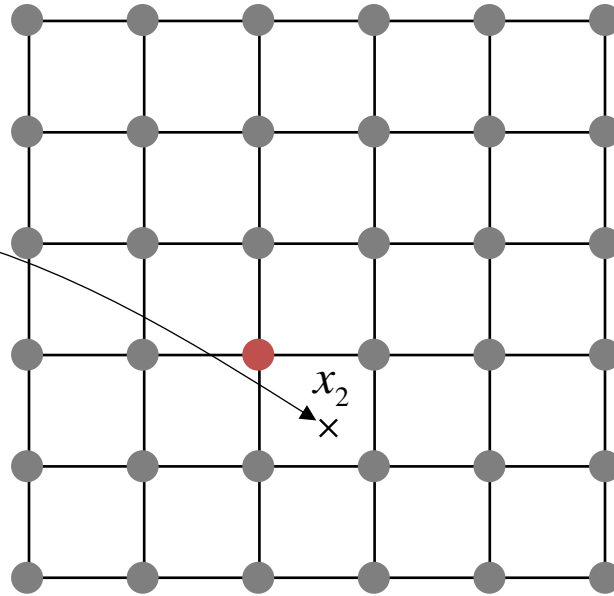
Source



Source

$$x_2 = f(x_1)$$

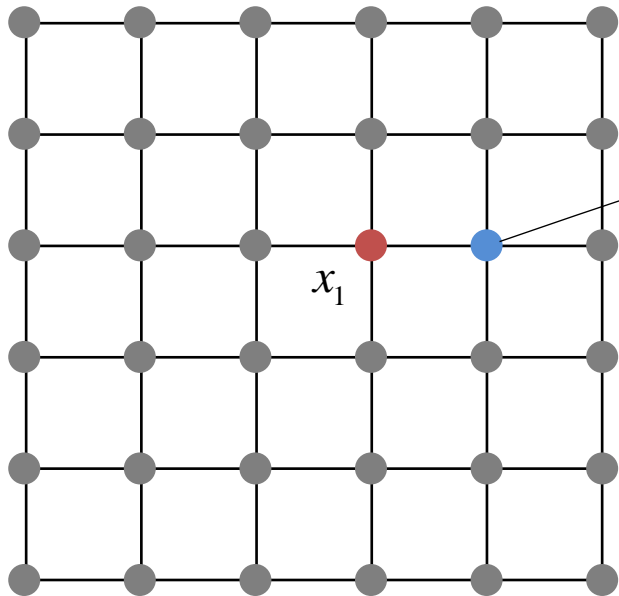
Target



```
for i = 1 : size(im,1)
  for j = 1 : size(im,2)
    x1 = [j, i];
    x2 = floor(mapping(x1));
    I2(x2) = I1(x1);
  end
end
```

IMAGE WARPING

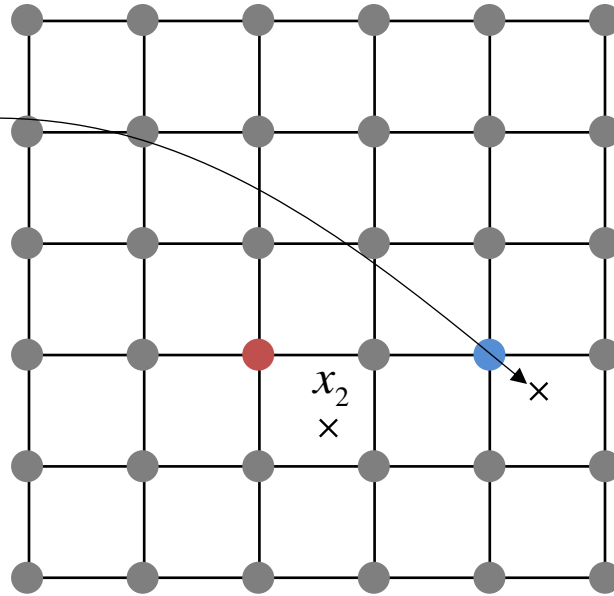
Source



Source

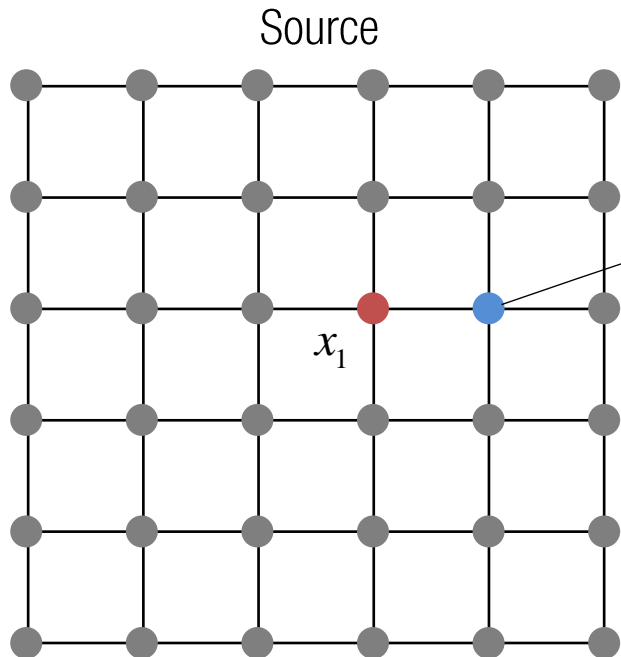
$$x_2 = f(x_1)$$

Target

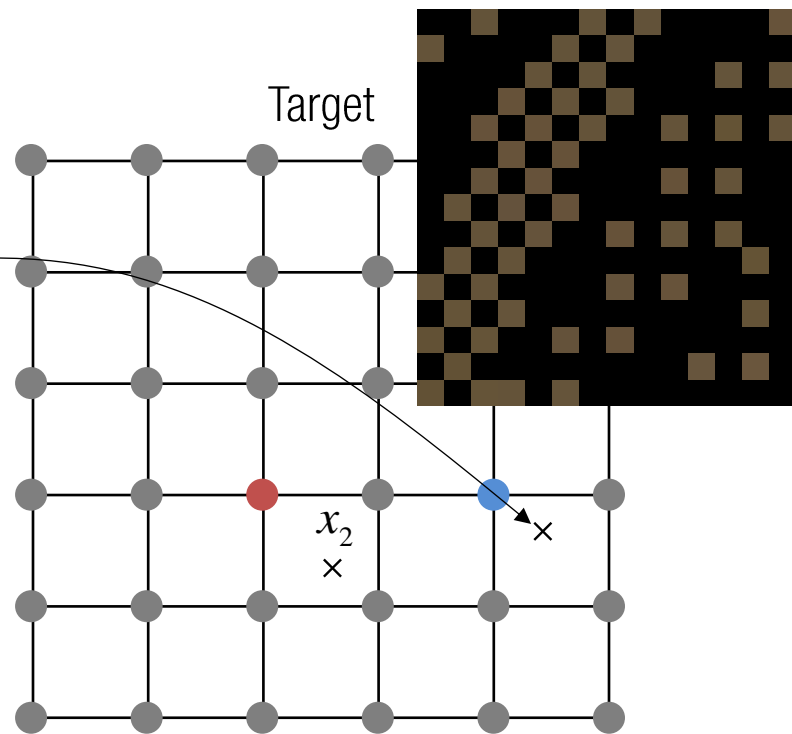


```
for i = 1 : size(im,1)
  for j = 1 : size(im,2)
    x1 = [j, i];
    x2 = floor(mapping(x1));
    I2(x2) = I1(x1);
  end
end
```

IMAGE WARPING



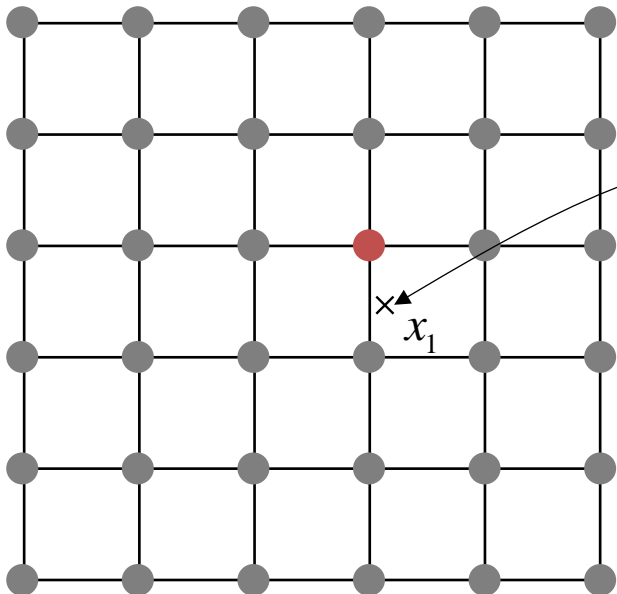
$x_2 = f(x_1)$
Forward warping



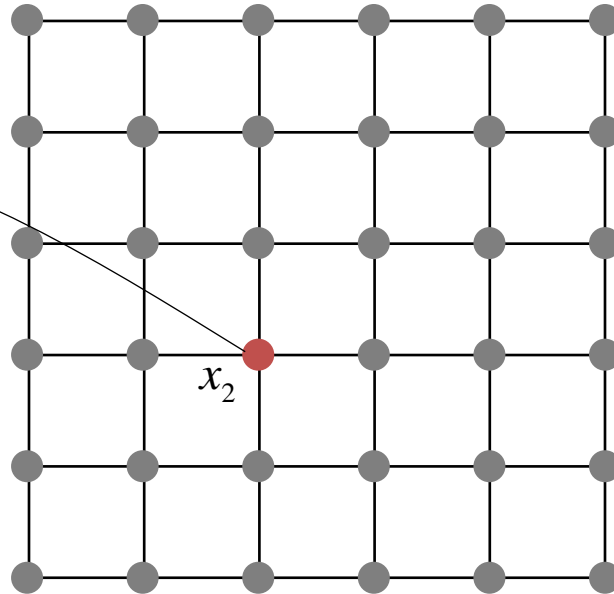
```
for i = 1 : size(im,1)
  for j = 1 : size(im,2)
    x1 = [j, i];
    x2 = floor(mapping(x1));
    I2(x2) = I1(x1);
  end
end
```

INVERSE WARPING

Source



Target

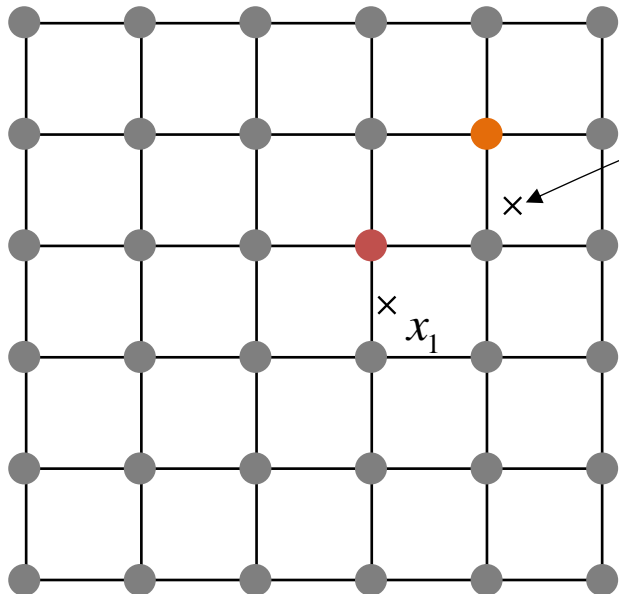


$x_1 = f^{-1}(x_2)$
Backward warping

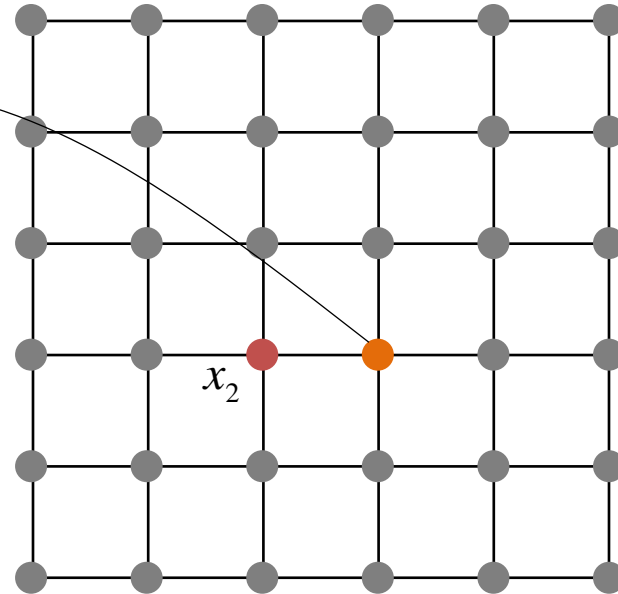
```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x2 = [j, i];
        x1 = floor(inverse_mapping(x2));
        I2(x2) = I1(x1);
    end
end
```

INVERSE WARPING

Source



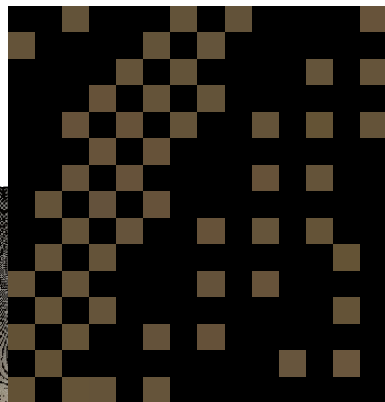
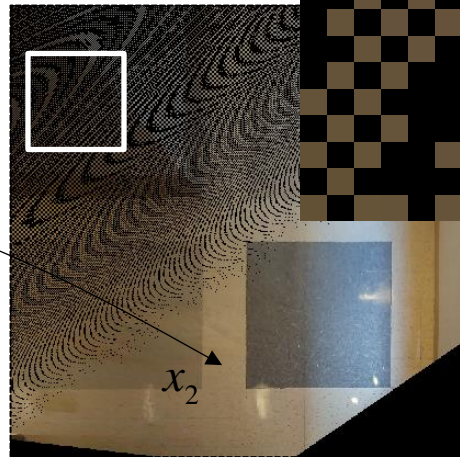
Target



$x_1 = f^{-1}(x_2)$
Backward warping

```
for i = 1 : size(im,1)
  for j = 1 : size(im,2)
    x2 = [j, i];
    x1 = floor(inverse_mapping(x2));
    I2(x2) = I1(x1);
  end
end
```

IMAGE WARPING

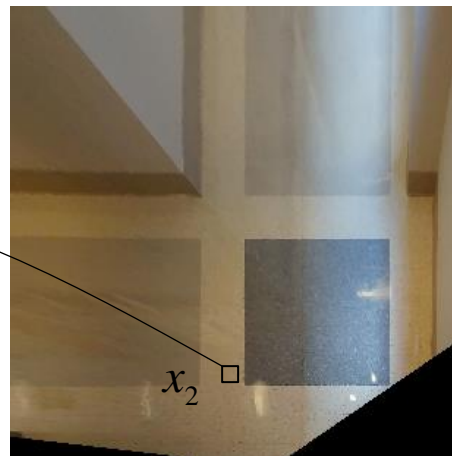


Forward warping

$$x_2 = f(x_1) \quad I_2(x_2) = I_1(f(x_1))$$

```
for i = 1 : size(im,1)
  for j = 1 : size(im,2)
    x1 = [j, i];
    x2 = floor(mapping(x1));
    I2(x2) = I1(x1);
  end
end
```


IMAGE WARPING



Backward warping

$$x_1 = f^{-1}(x_2) \quad I_2(x_2) = I_1(f(x_1))$$

```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x2 = [j, i];
        x1 = floor(inverse_mapping(x2));
        I2(x2) = I1(x1);
    end
end
```

IMAGE WARPING

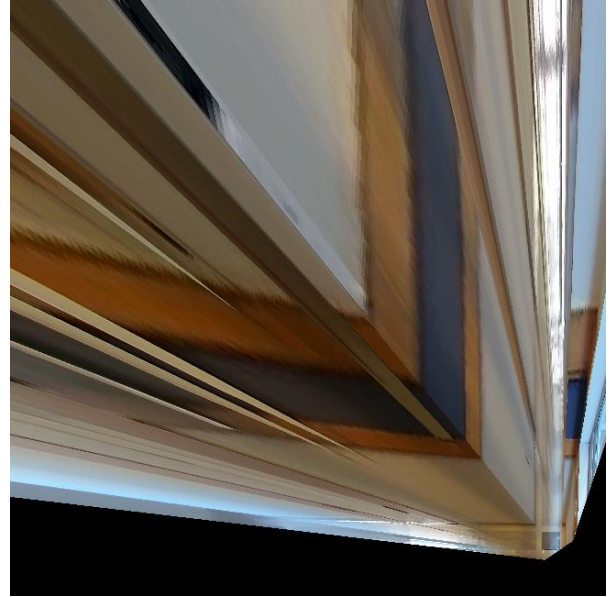


IMAGE WARPING



IMAGE WARPING

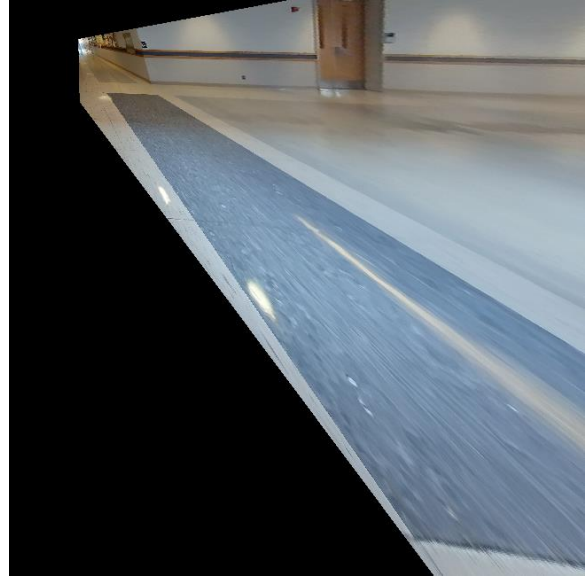


IMAGE WARPING







PSV 0 - 0 AJA | 01:46

HRM - Payroll

driessen

HRM - Payroll

driessen

HRM



Virtual Advertisement

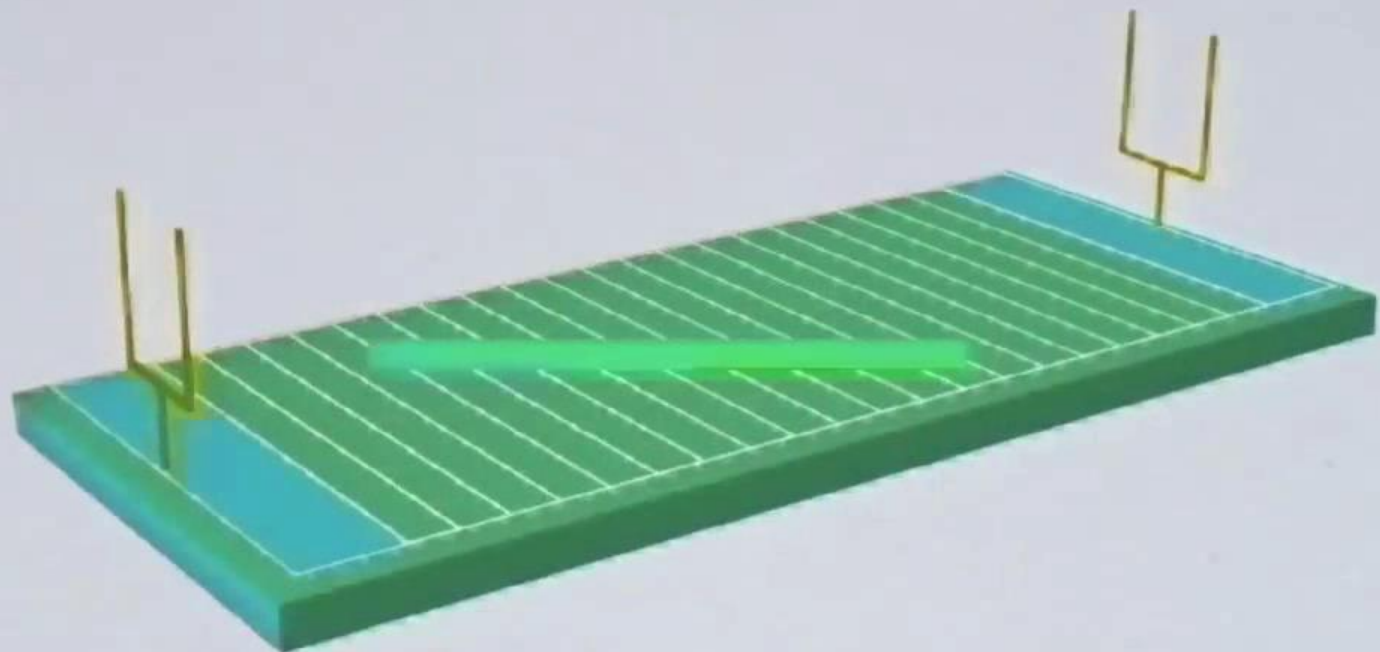


IMAGE WARPING



Keller entrance left

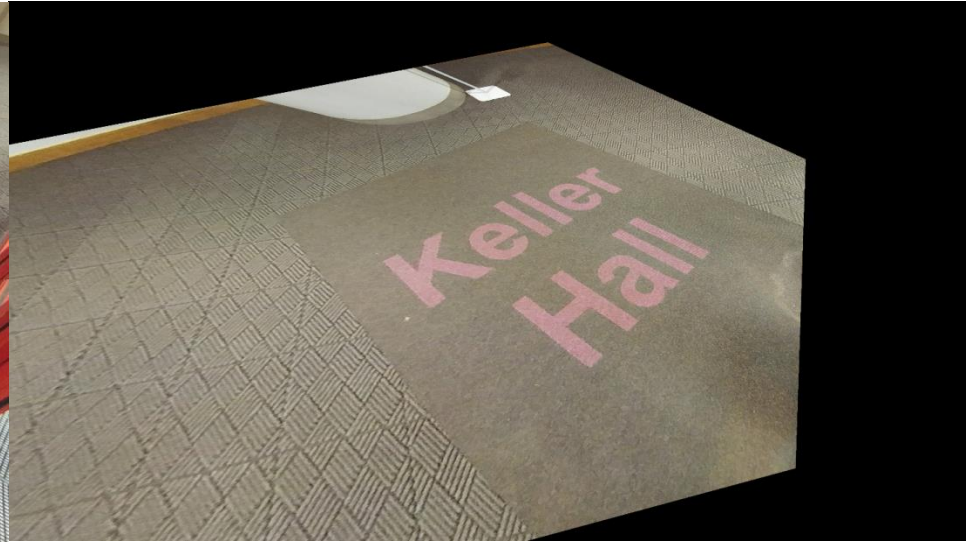


Keller entrance right

IMAGE WARPING



Keller entrance left



Right image to left

IMAGE WARPING



IMAGE WARPING



Left image to right



Right image to left

IMAGE WARPING

*Keller
Hall*



$$\lambda u = h\nu$$



Lind Hall Left



Lind Hall Right



Euclidean Transform (Translation)



Homography





WOMEN

E8

← Gates 8B-8C →

E8





W
O
M
E
N

ES

Hertz

Starbucks

Passenger Information