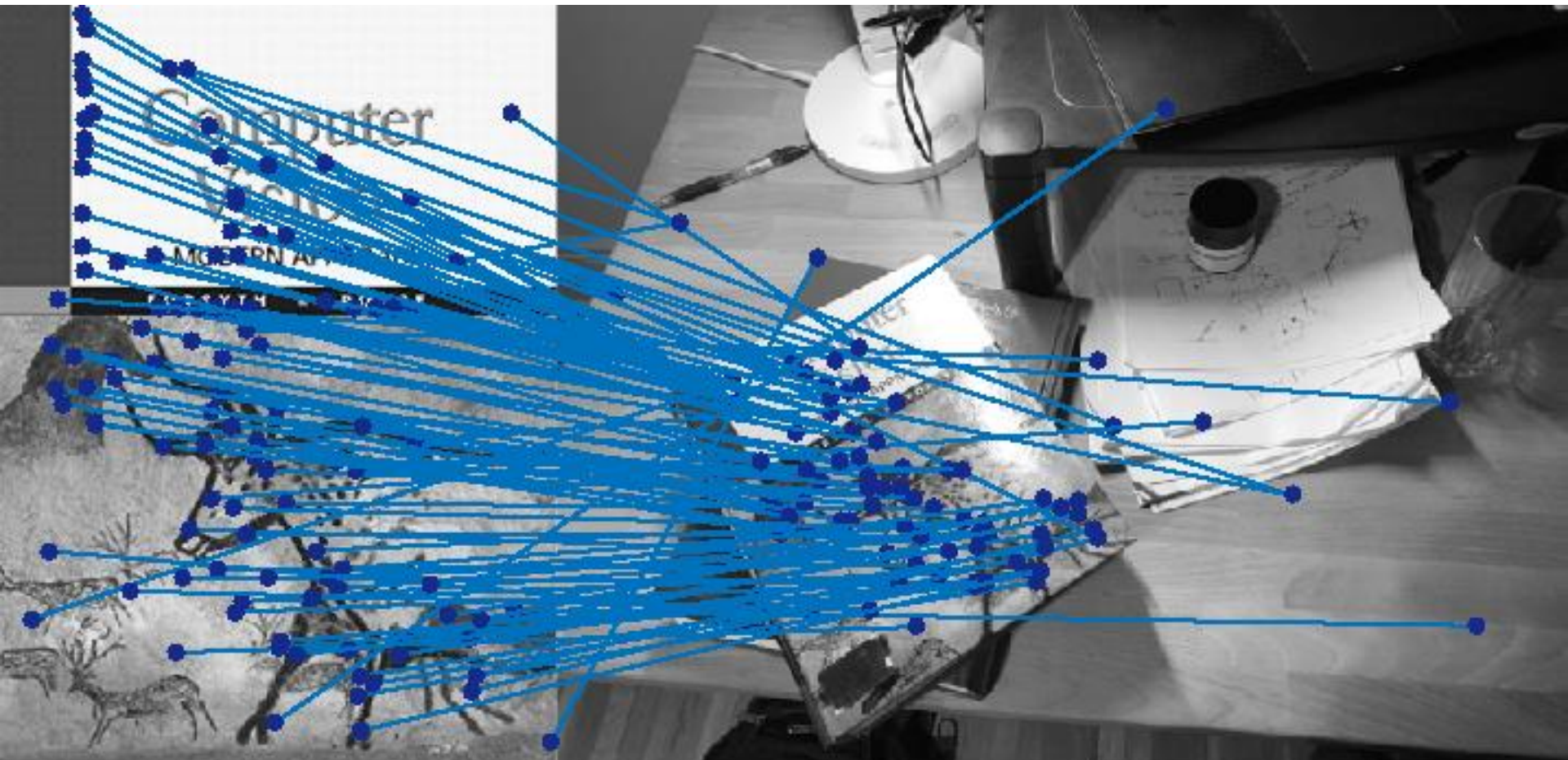


A photograph of a street scene with a brick building and trees in the background. The image is overlaid with a dense network of colorful lines (red, green, blue, purple, orange) that all converge at a single point in the center of the frame. Small colored dots are scattered along these lines, representing feature points in a computer vision context. The overall scene is dimly lit, suggesting an overcast day.

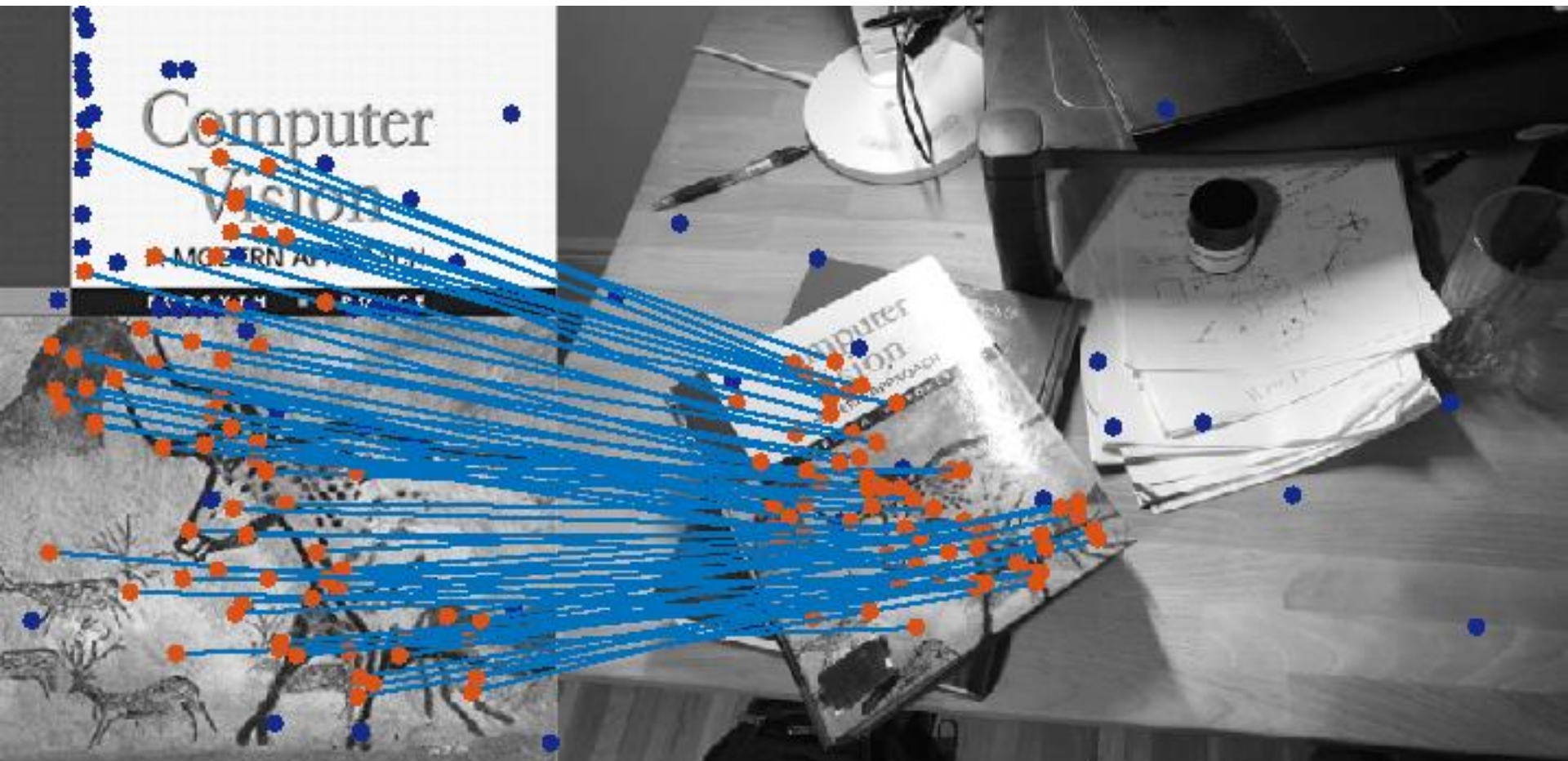
RANSAC

HYUN SOO PARK

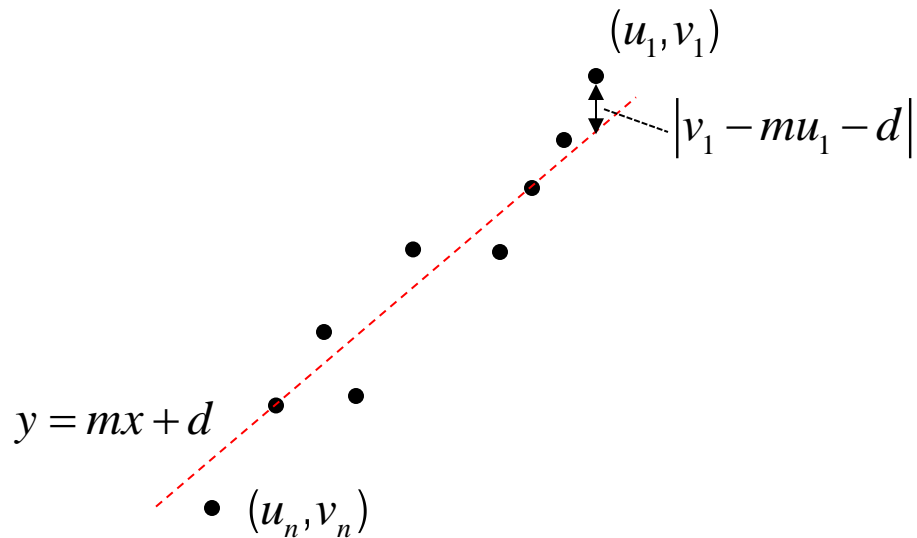
RECALL: LOCAL FEATURE MATCHING



RECALL: ROBUST FILTERING



LINE FITTING



Given points: $(u_1, v_1), \dots, (u_n, v_n)$

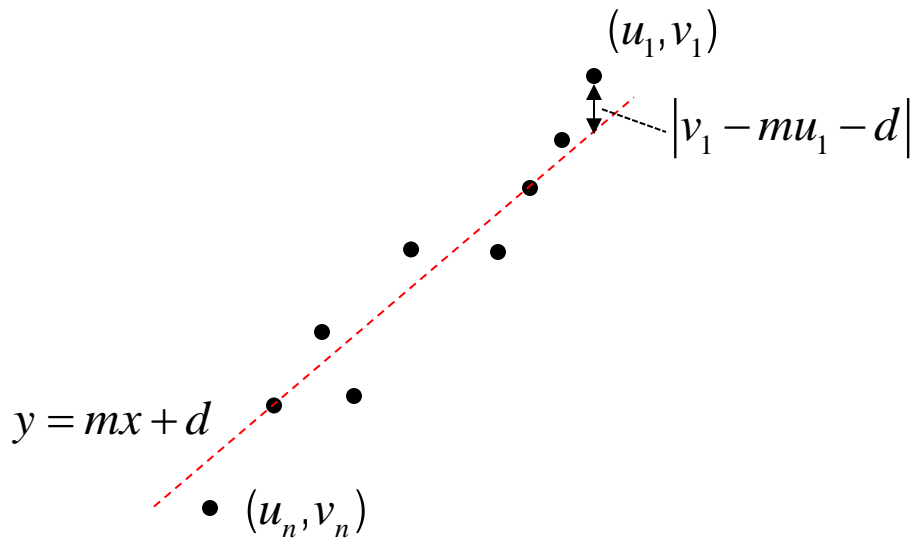
Find the best line: $v_1 \approx mu_1 + d$

\vdots

$v_n \approx mu_n + d$

$$\longrightarrow \begin{bmatrix} u_1 & 1 \\ \vdots & \vdots \\ u_n & 1 \end{bmatrix} \begin{bmatrix} m \\ d \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

LINE FITTING



Given points: $(u_1, v_1), \dots, (u_n, v_n)$

Find the best line: $v_1 \approx mu_1 + d$

\vdots

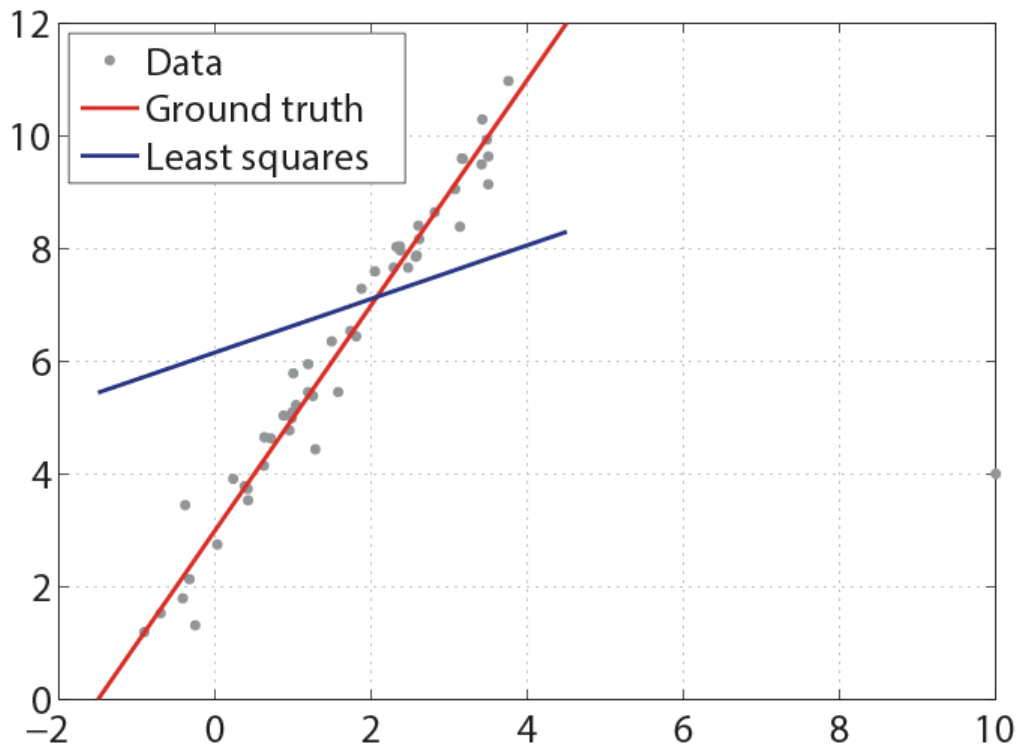
$v_n \approx mu_n + d$

$$\begin{matrix} u_1 & 1 \\ \vdots & \vdots \\ u_n & 1 \end{matrix} \begin{matrix} m \\ x \\ d \end{matrix} = \begin{matrix} v_1 \\ \vdots \\ v_n \end{matrix}$$

The matrix $\begin{bmatrix} u_1 & 1 \\ \vdots & \vdots \\ u_n & 1 \end{bmatrix}$ is labeled A . The vector $\begin{bmatrix} m \\ x \\ d \end{bmatrix}$ is labeled x . The vector $\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ is labeled b .

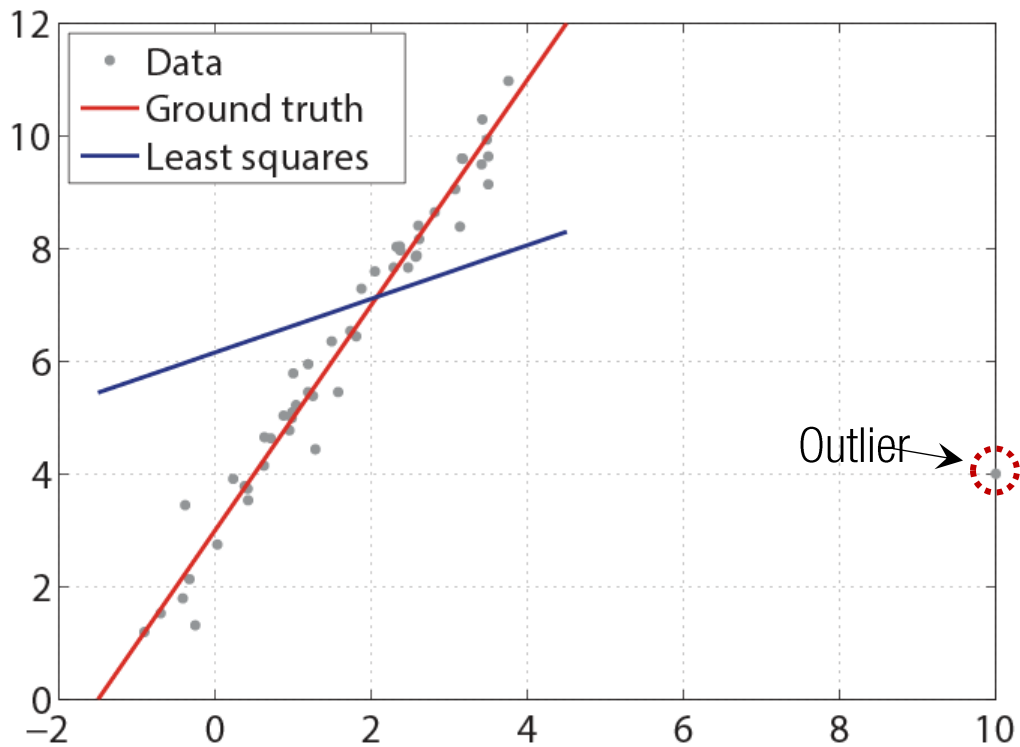
Least squares solution: $x = (A^T A)^{-1} A^T b$

OUTLIER



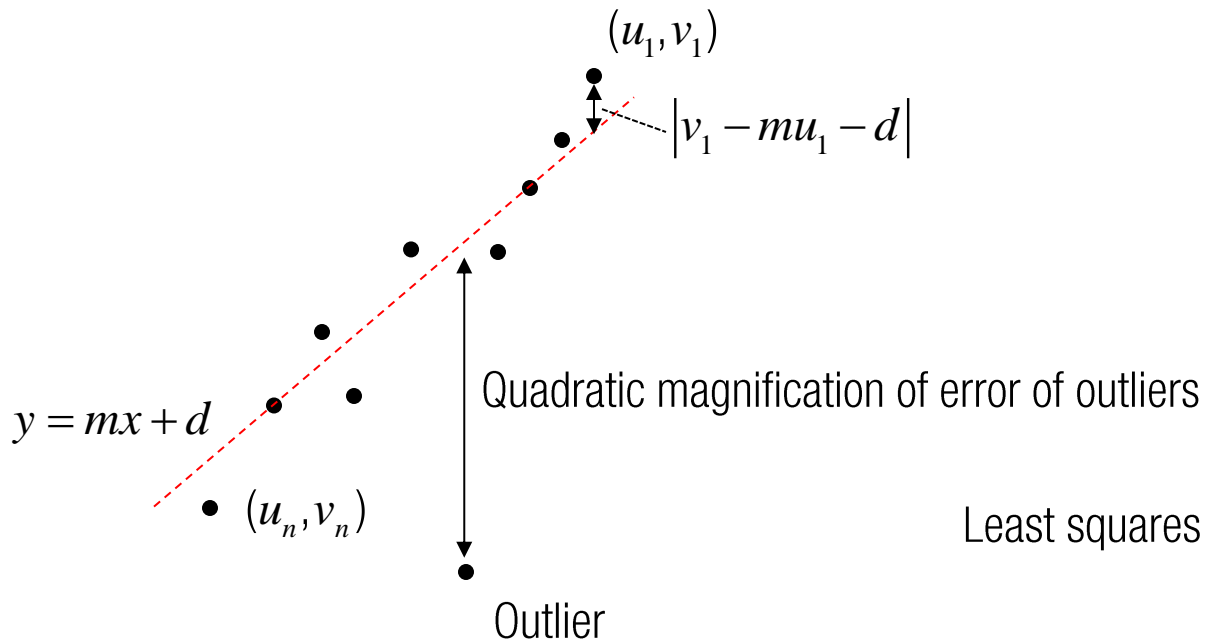
$$\begin{bmatrix} u_1 & 1 \\ \vdots & \vdots \\ A & \vdots \\ u_n & 1 \end{bmatrix} \begin{bmatrix} m \\ x \\ d \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ b \\ v_n \end{bmatrix}$$

OUTLIER



$$\begin{bmatrix} u_1 & 1 \\ \vdots & \vdots \\ A & \vdots \\ u_n & 1 \end{bmatrix} \begin{bmatrix} m \\ x \\ d \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ b \\ v_n \end{bmatrix}$$

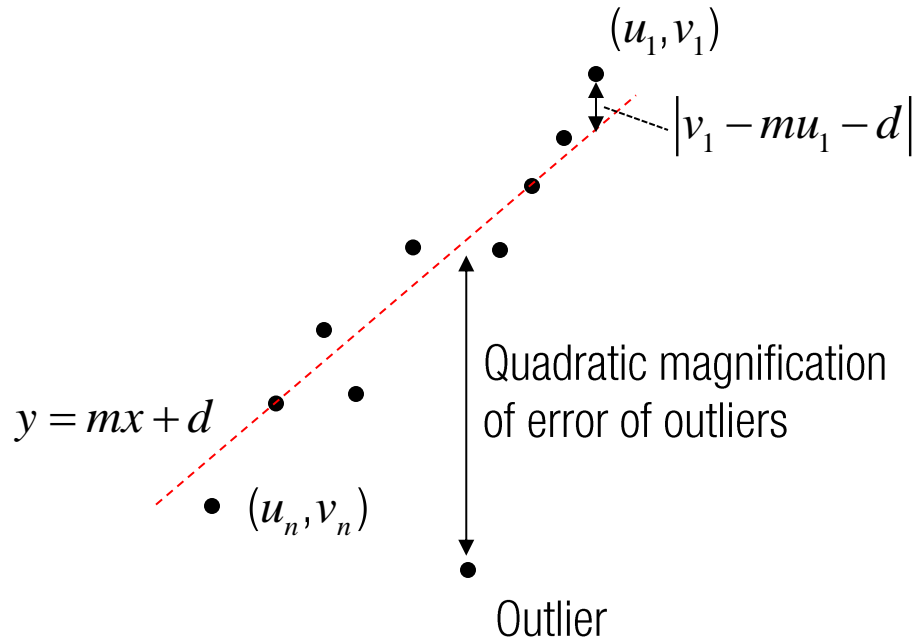
OUTLIER SENSITIVITY



$$\begin{bmatrix} u_1 & 1 \\ \vdots & \vdots \\ u_n & 1 \end{bmatrix} \begin{bmatrix} m \\ d \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

Least squares solution: $x = (A^T A)^{-1} A^T b$

OUTLIER REJECTION STRATEGY

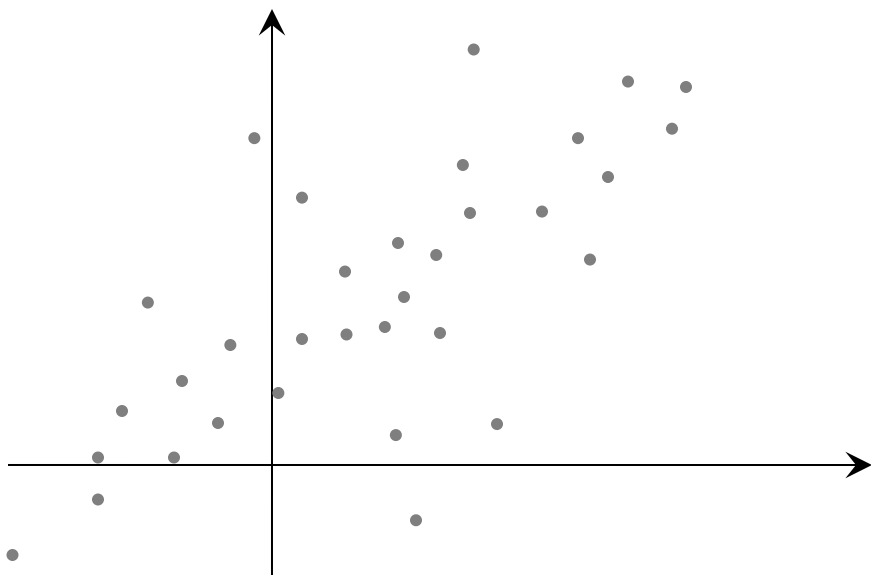


Outlier rejection strategy:

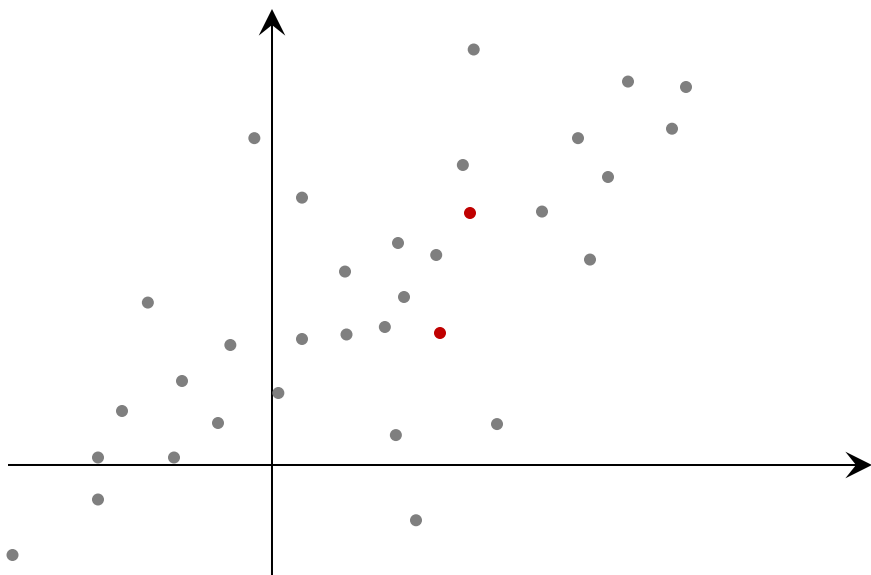
To find the best line that explains the maximum number of points.

Assumptions:

1. Majority of good samples agree with the underlying model.
2. Bad samples does not consistently agree with a single model.

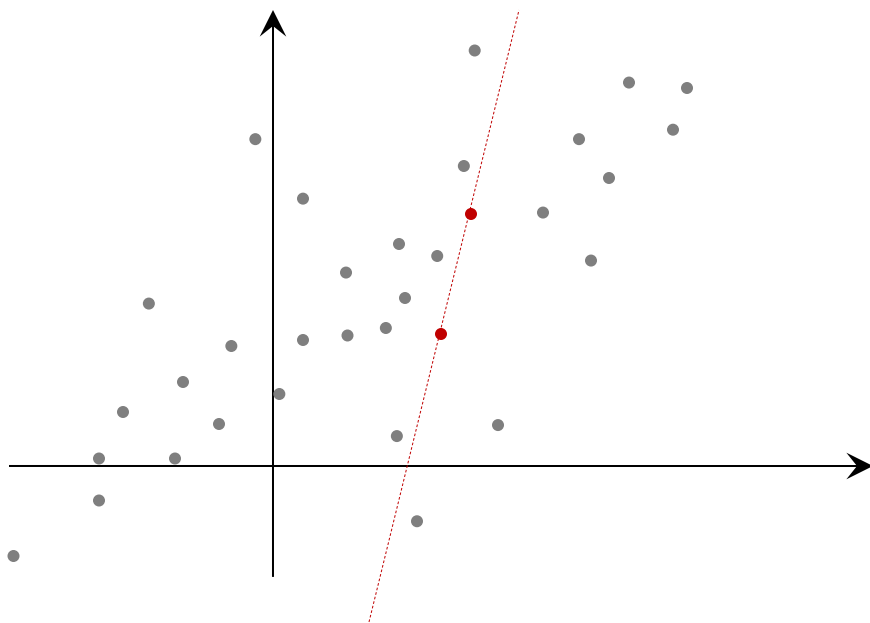


RANSAC: RANdom SAmple Consensus



1. Random sampling

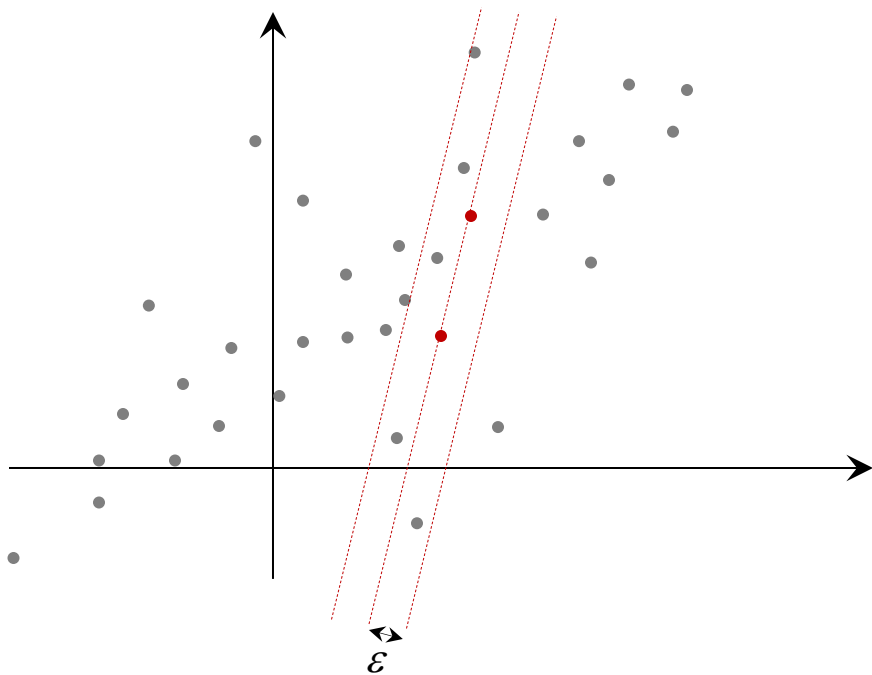
RANSAC: RANdom SAmple Consensus



1. Random sampling

2. Model building

RANSAC: RANdom SAMple Consensus

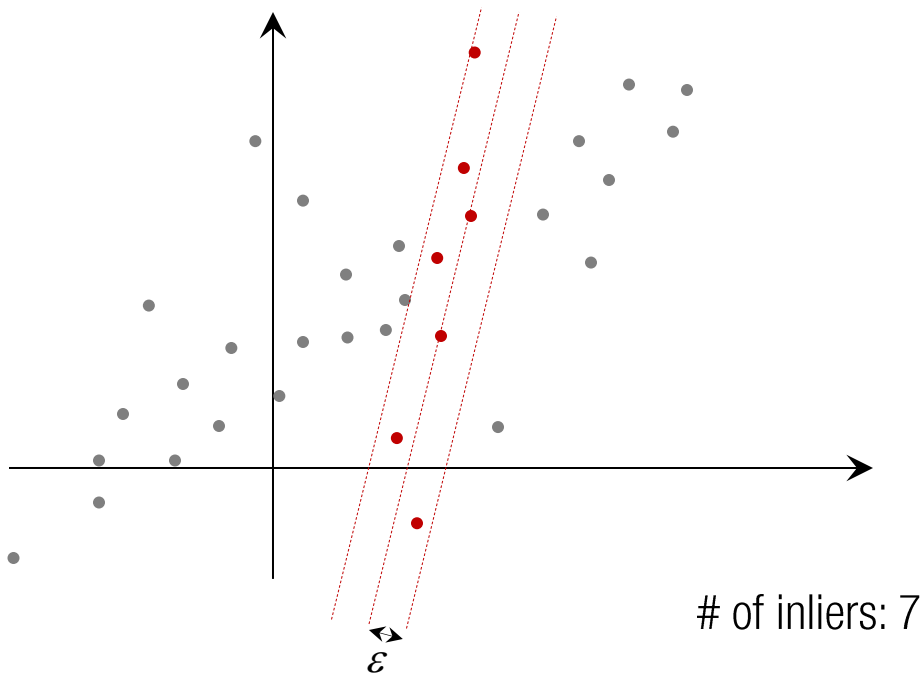


1. Random sampling

2. Model building

3. Thresholding

RANSAC: RANdom SAmple Consensus



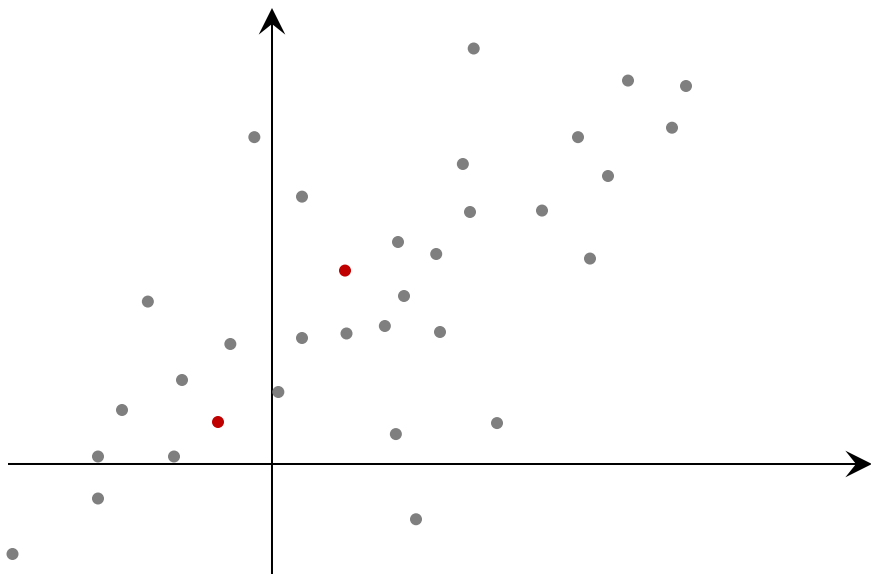
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

RANSAC: RANdom SAMple Consensus



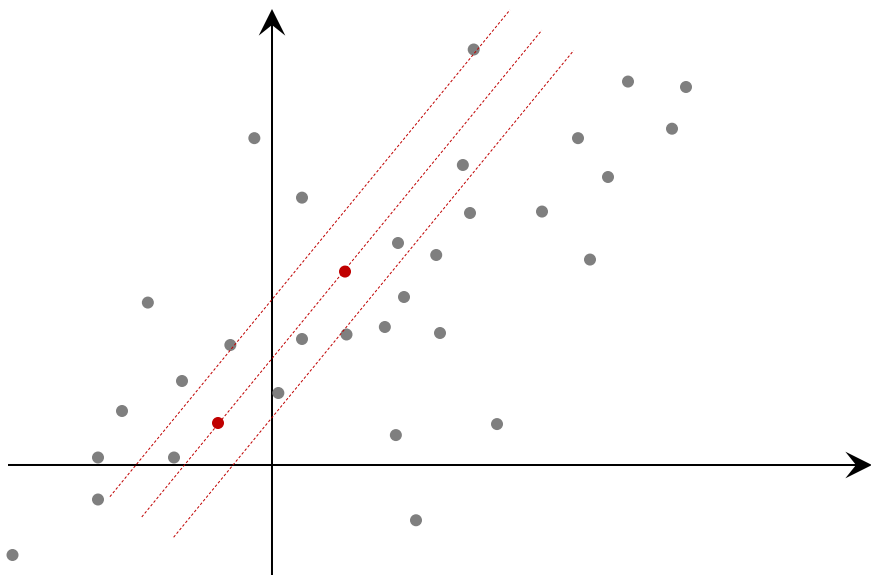
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

RANSAC: RANdom SAmple Consensus



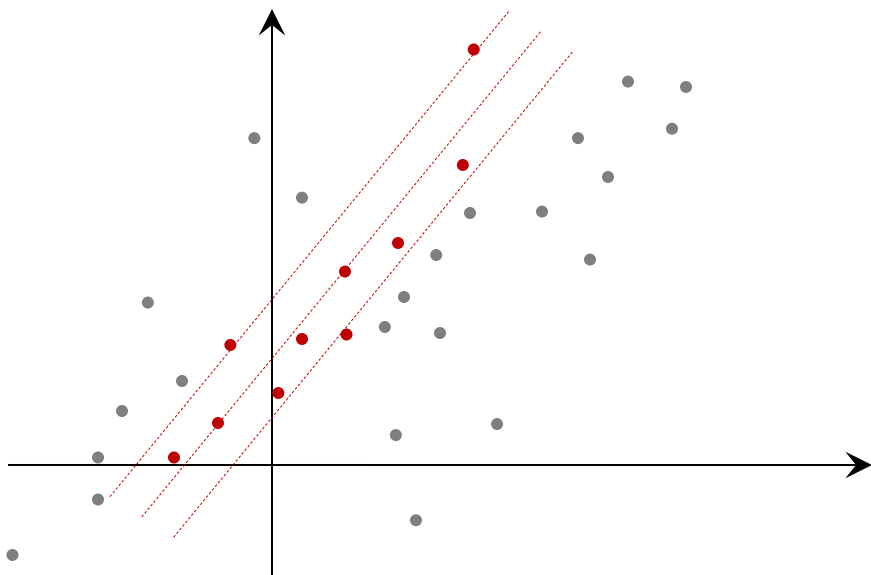
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

RANSAC: RANdom SAmple Consensus



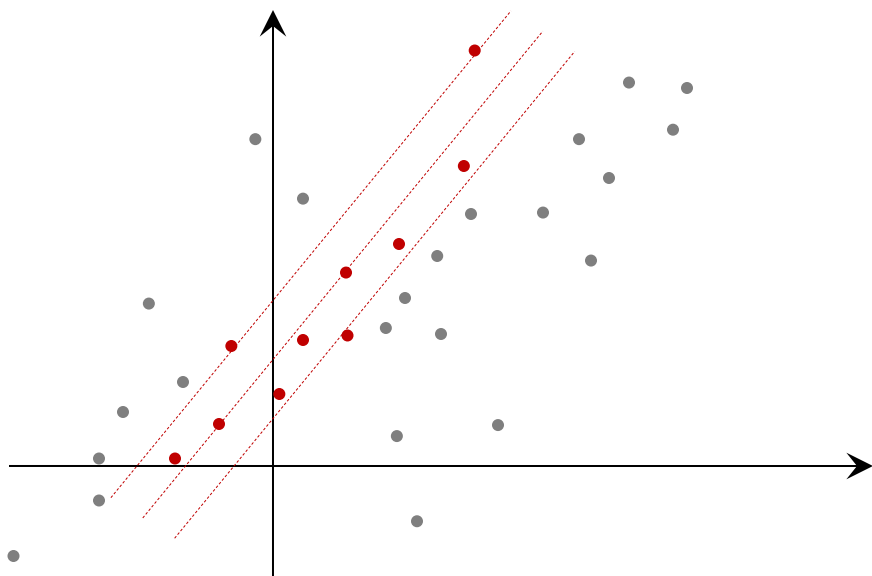
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

RANSAC: RANdom SAmple Consensus



of inliers: 10

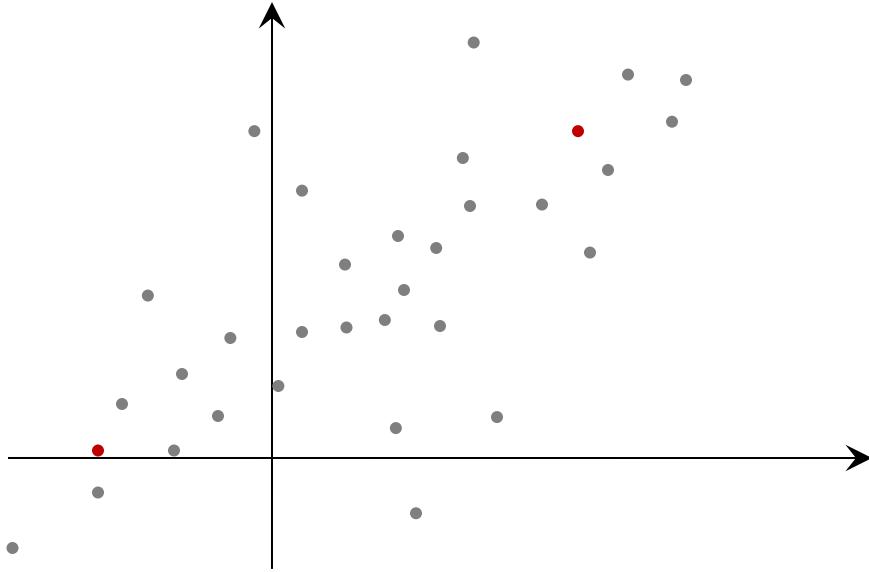
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

RANSAC: RANdom SAMple Consensus



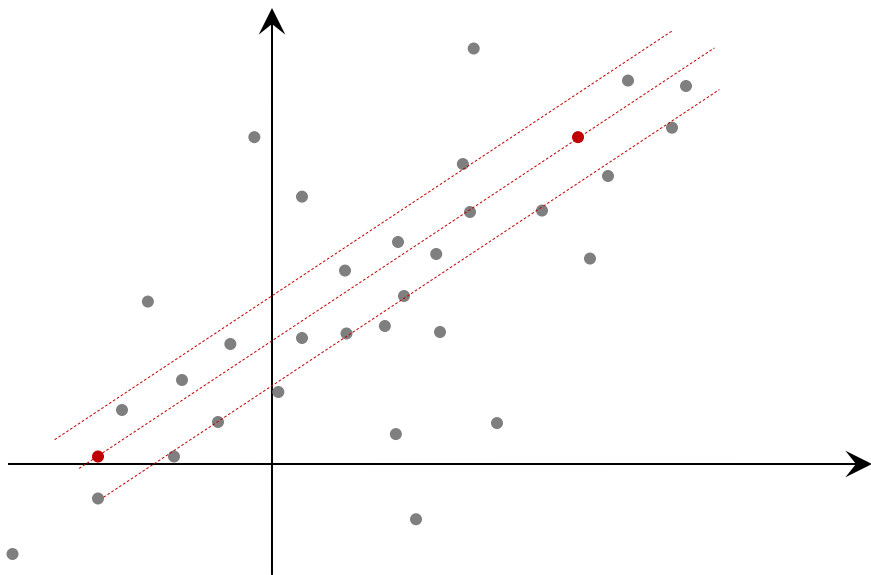
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

RANSAC: RANdom SAmple Consensus



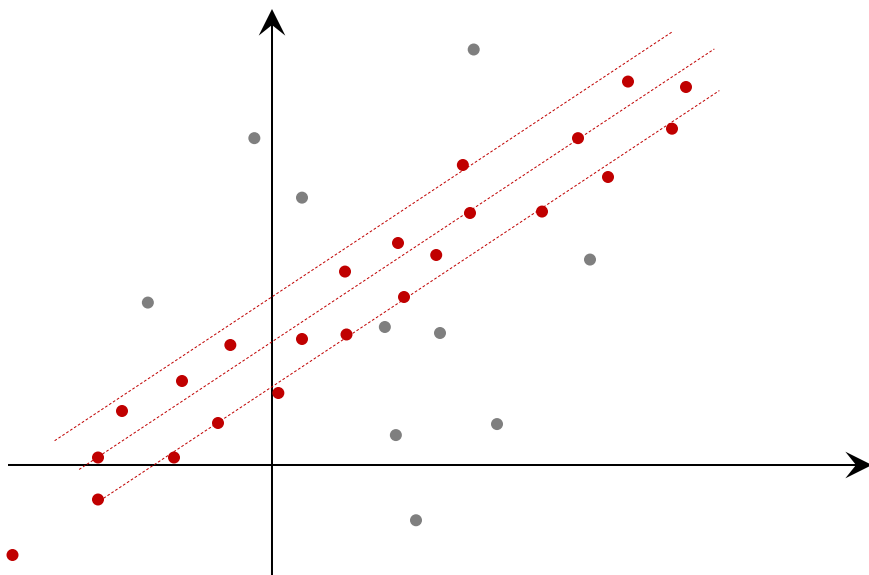
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

RANSAC: RANdom SAMple Consensus



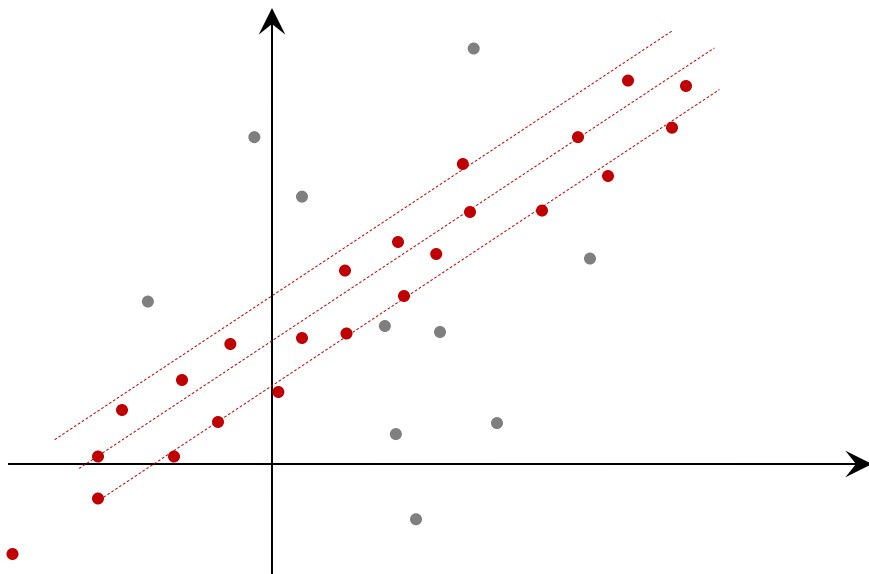
1. Random sampling

2. Model building

3. Thresholding

4. Inlier counting

RANSAC: RANdom SAmple Consensus



1. Random sampling

2. Model building

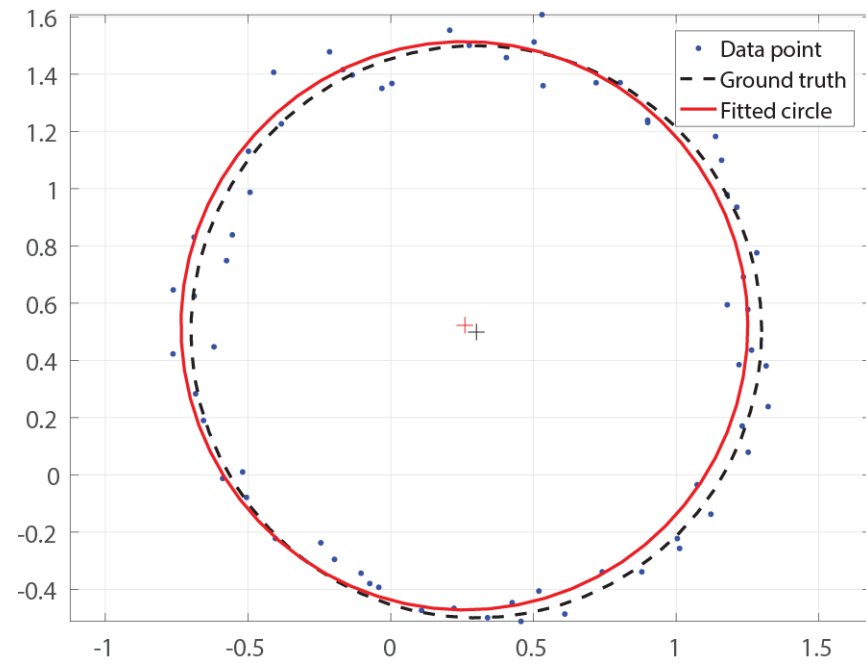
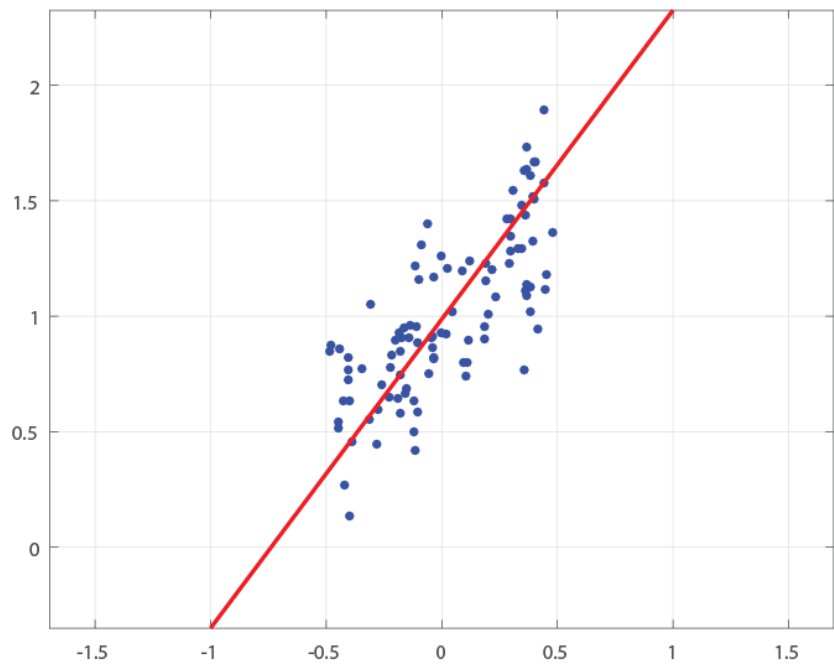
3. Thresholding

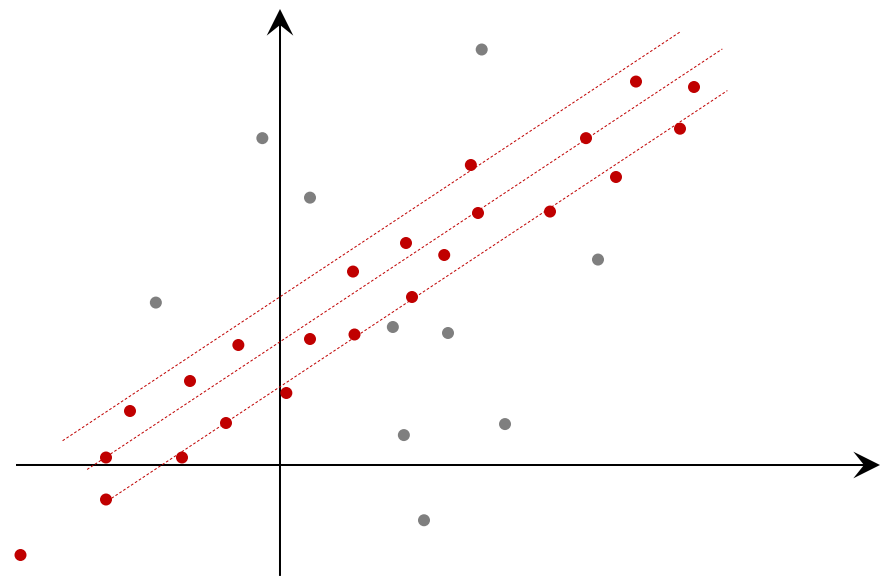
4. Inlier counting

of inliers: 23

Maximum number of inliers

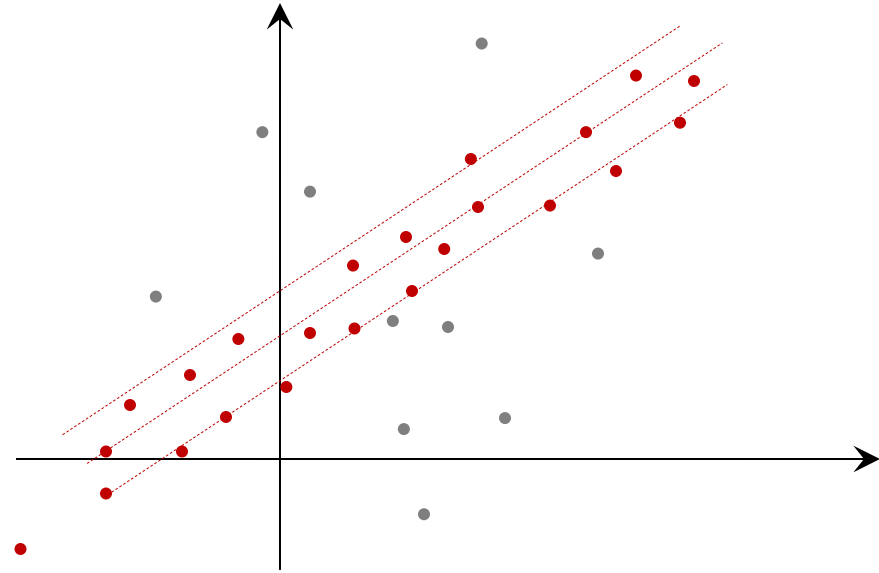
RANSAC: RANdom SAmple Consensus





Required number of iterations with p success rate:

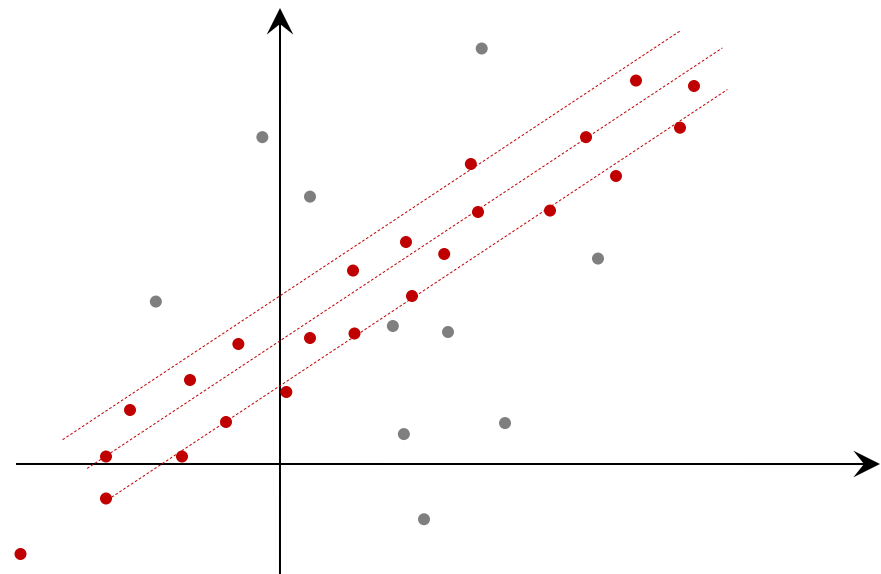
Prob. of success $>$ Prob. of desired success p



Required number of iterations with p success rate:

Prob. of success $>$ Prob. of desired success p

Prob. of success: $1 - (1 - \text{prob. of success per trial})^k$



Required number of iterations with p success rate:

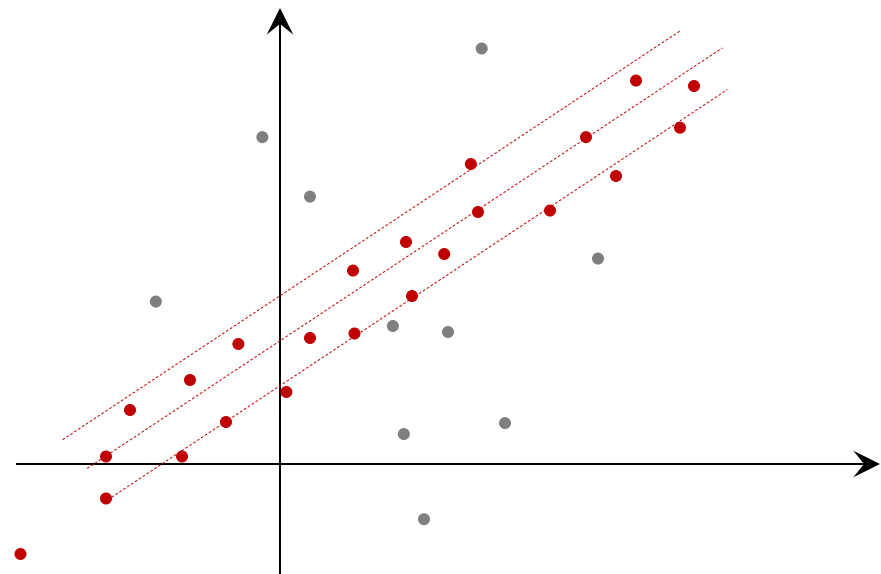
Prob. of success $>$ Prob. of desired success p

Prob. of success: $1 - (1 - \text{prob. of success per trial})^k$

Prob. of success per trial: w^n

where $w = \frac{\text{\# of inliers}}{\text{\# of samples}}$

and n is the number of samples to build a model.



Required number of iterations with p success rate:

Prob. of success $>$ Prob. of desired success p

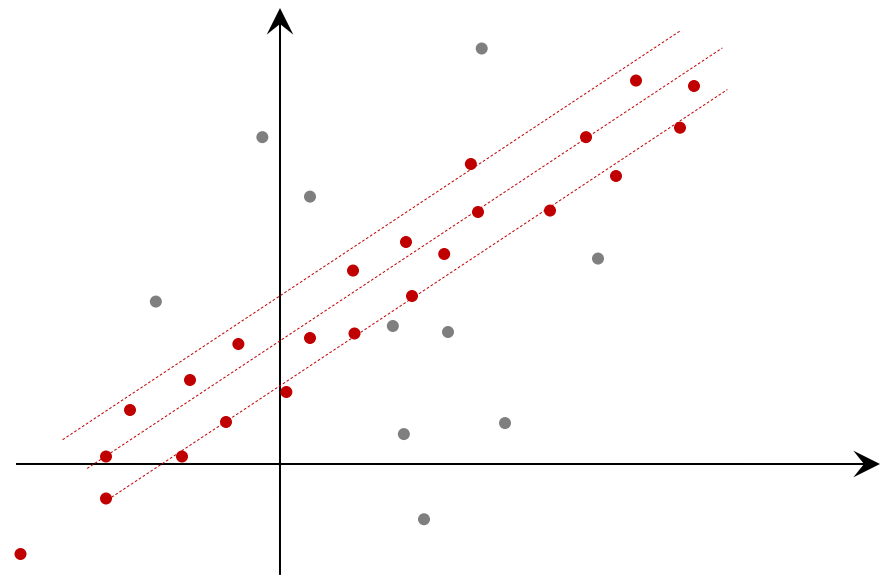
Prob. of success: $1 - (1 - \text{prob. of success per trial})^k$

Prob. of success per trial: w^n

where $w = \frac{\text{\# of inliers}}{\text{\# of samples}}$

and n is the number of samples to build a model.

Prob. of success: $1 - (1 - w^n)^k$



Required number of iterations with p success rate:

Prob. of success $>$ Prob. of desired success p

Prob. of success: $1 - (1 - \text{prob. of success per trial})^k$

Prob. of success per trial: w^n

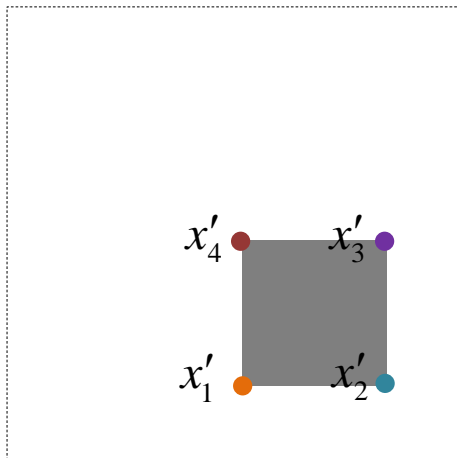
where $w = \frac{\text{\# of inliers}}{\text{\# of samples}}$

and n is the number of samples to build a model.

Prob. of success: $1 - (1 - w^n)^k$

$$k = \frac{\log(1-p)}{\log(1-w^n)}$$

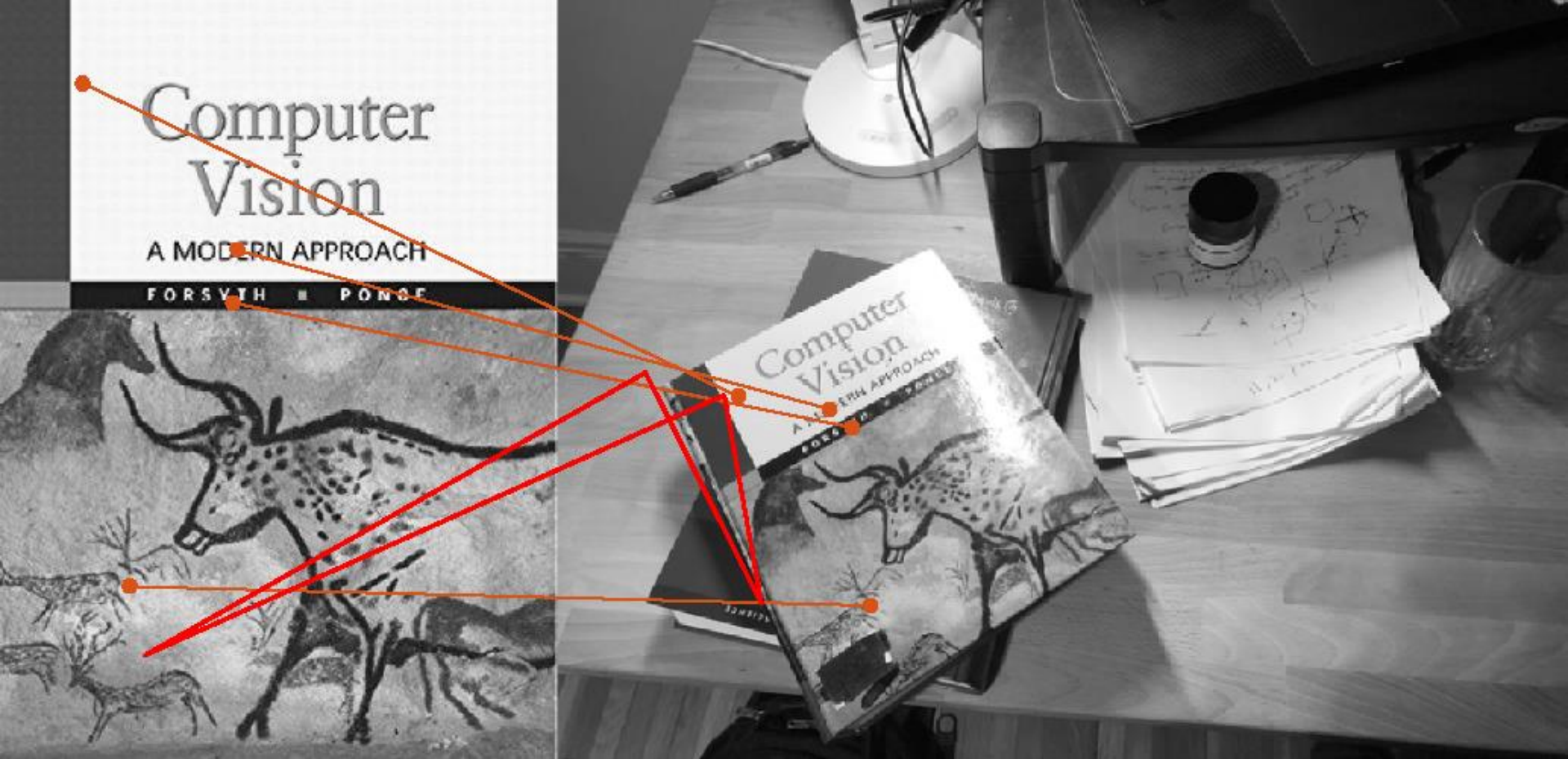
RECALL: HOMOGRAPHY COMPUTATION



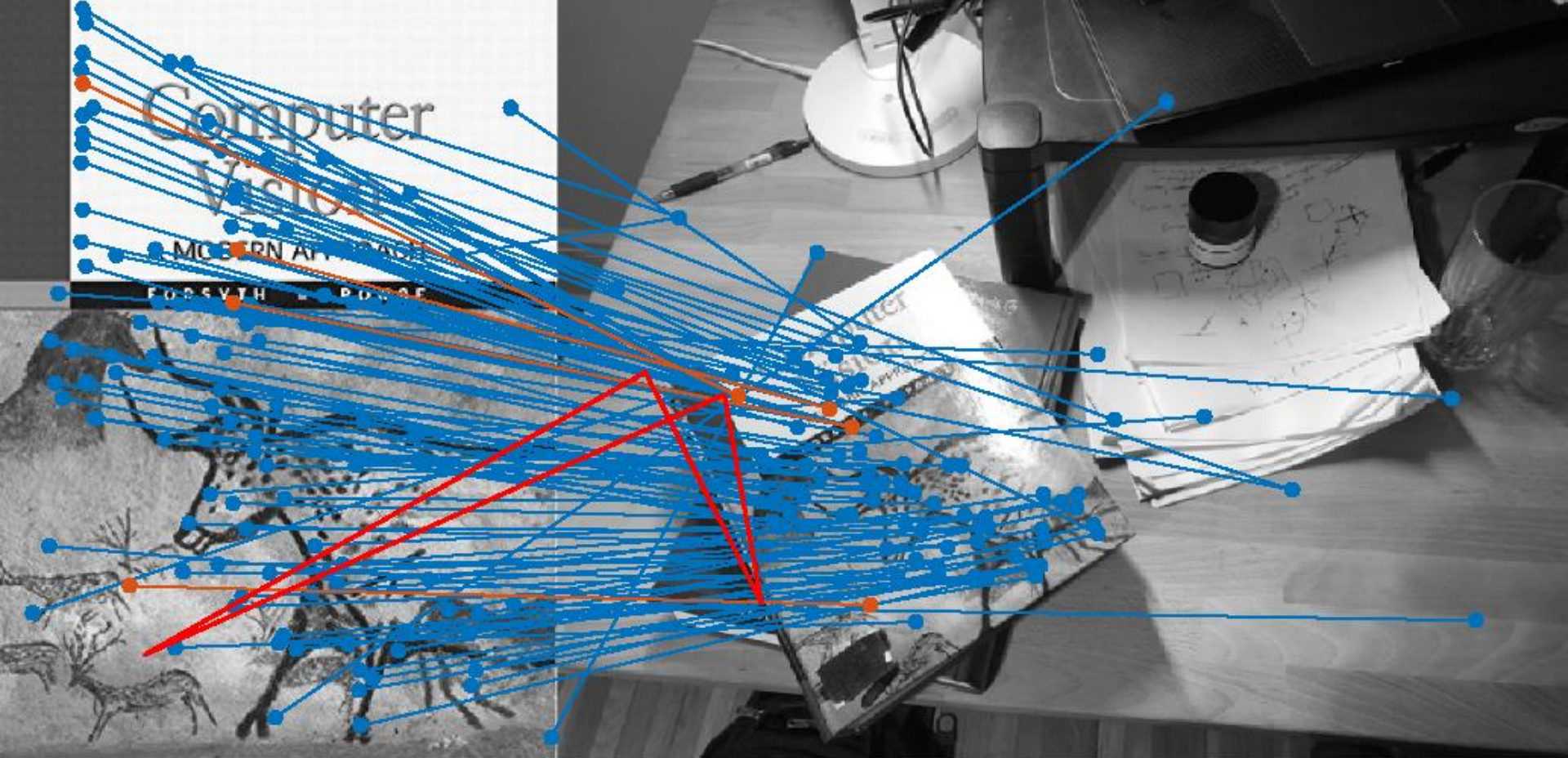
$$\begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1u'_1 & -v_1u'_1 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1v'_1 & -v_1v'_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_4 & v_4 & 1 & 0 & 0 & 0 & -u_4u'_4 & -v_4u'_4 \\ 0 & 0 & 0 & u_4 & v_4 & 1 & -u_4v'_4 & -v_4v'_4 \end{bmatrix} \mathbf{x} = \mathbf{b}$$

The matrix A is a 8×8 matrix. The vector \mathbf{x} is a column vector of size 8, and the vector \mathbf{b} is a column vector of size 8.

$$Ax = b \longrightarrow x = (A^T A)^{-1} A^T b$$

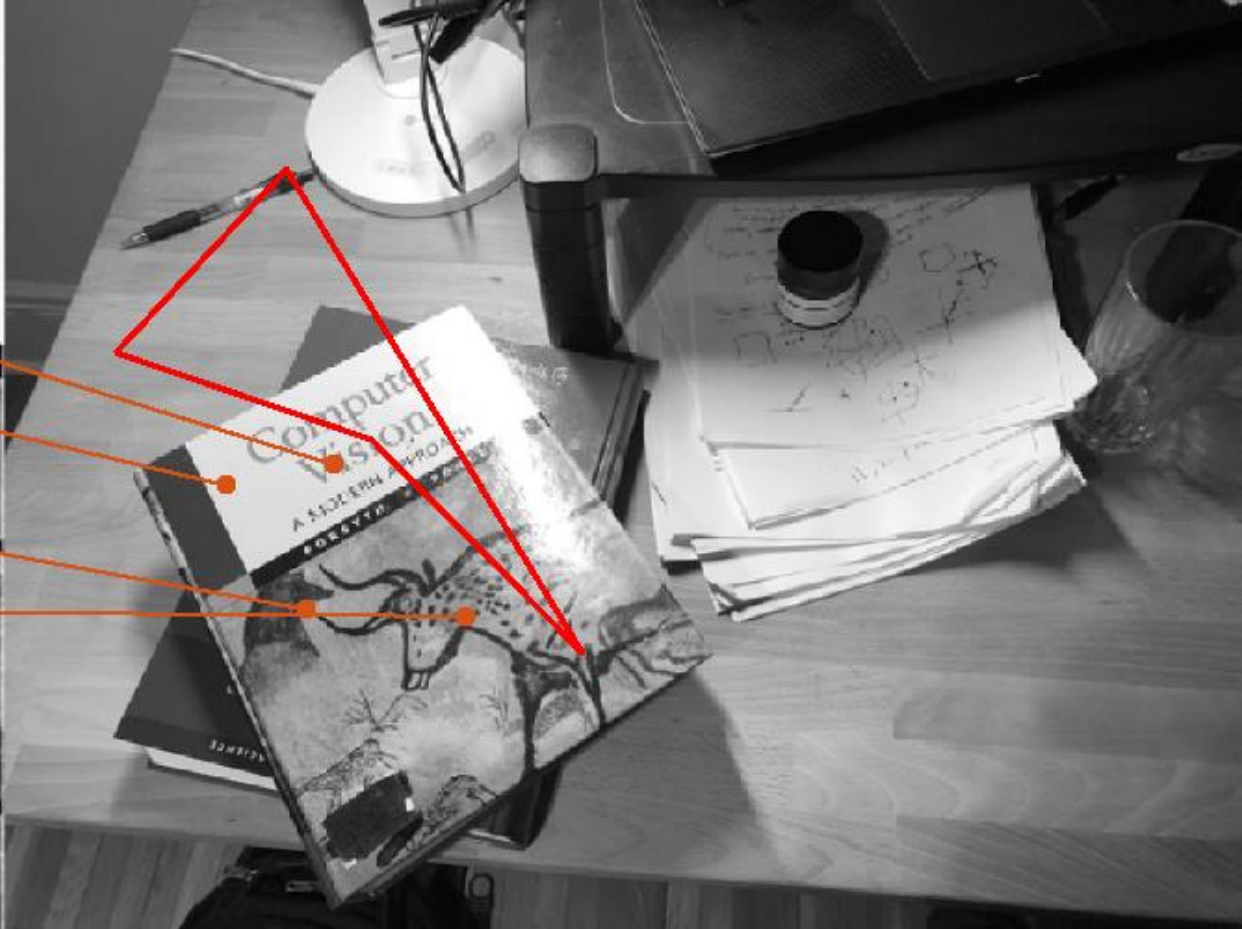


Homography from 4 random correspondences

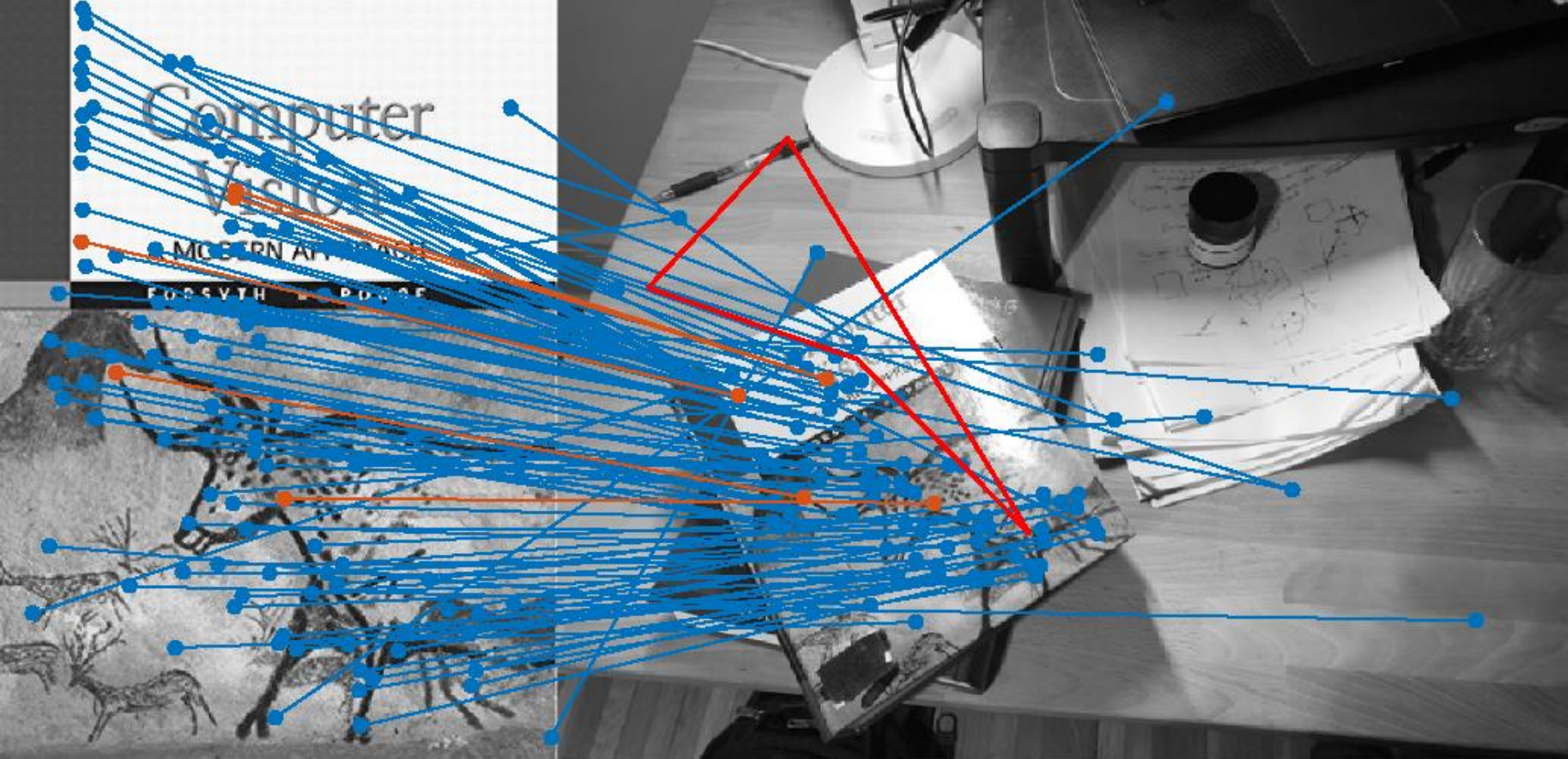


Inlier counting

Number of inliers: 5

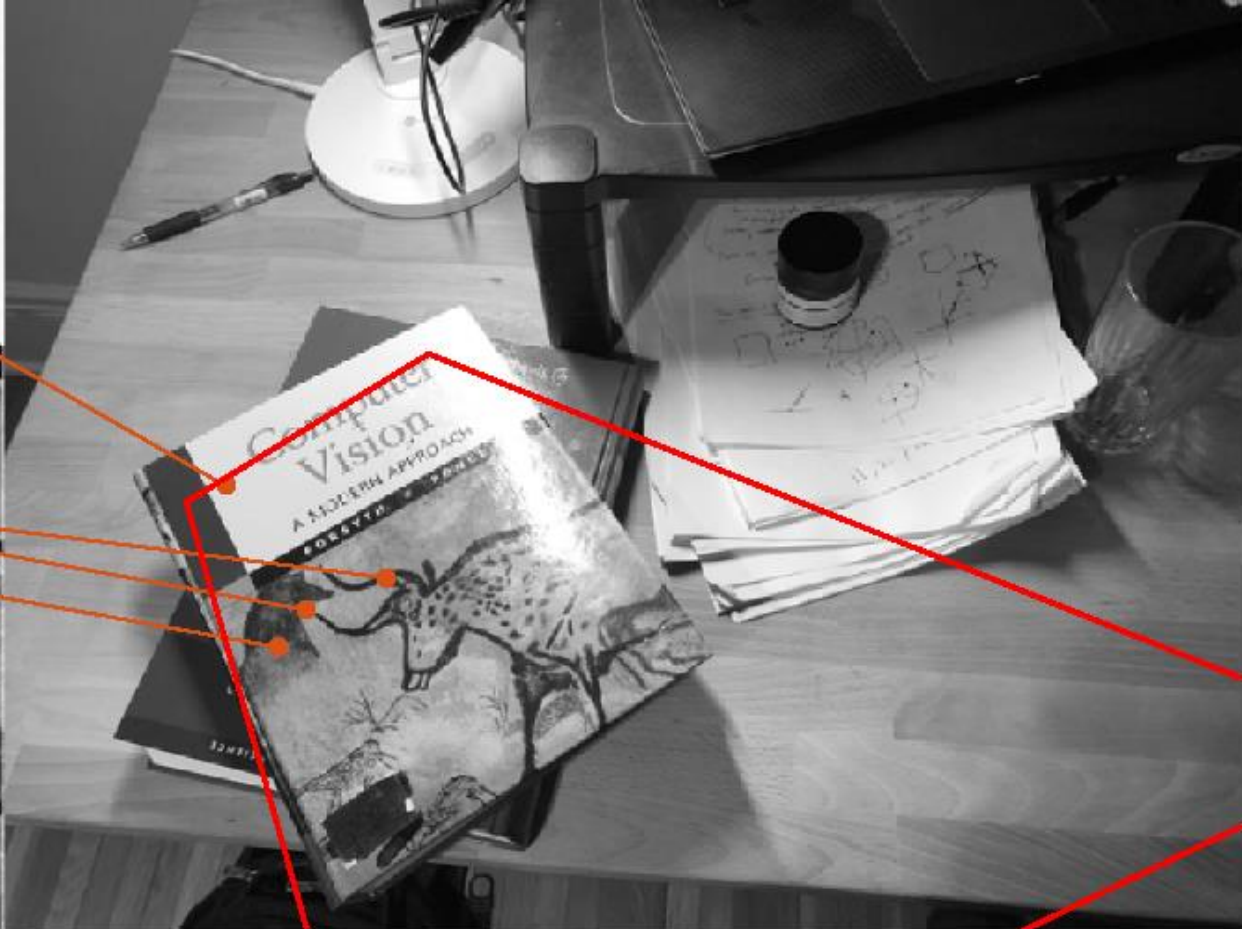
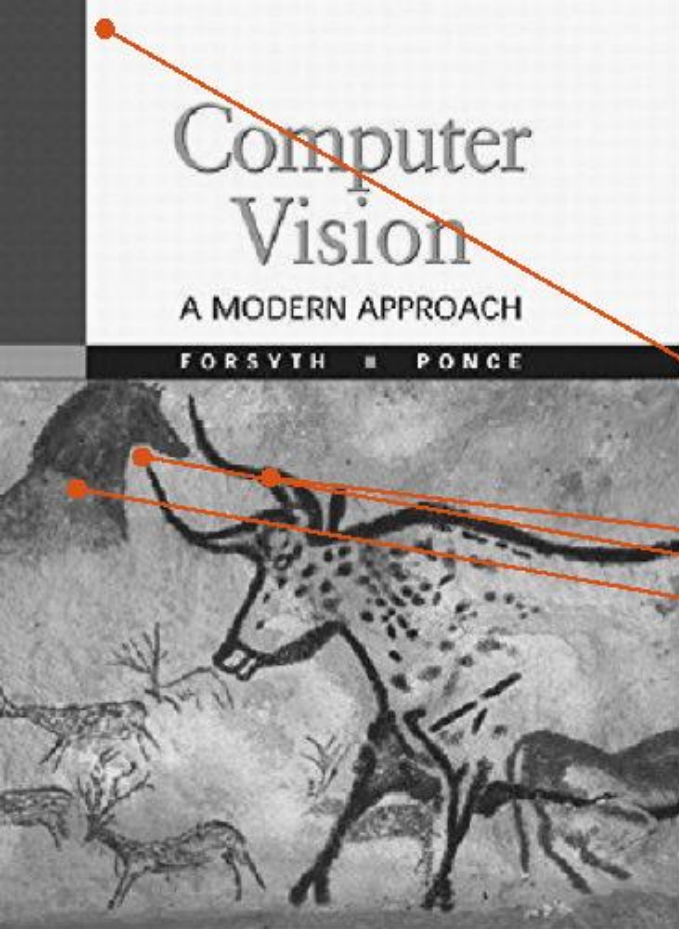


Homography from 4 random correspondences

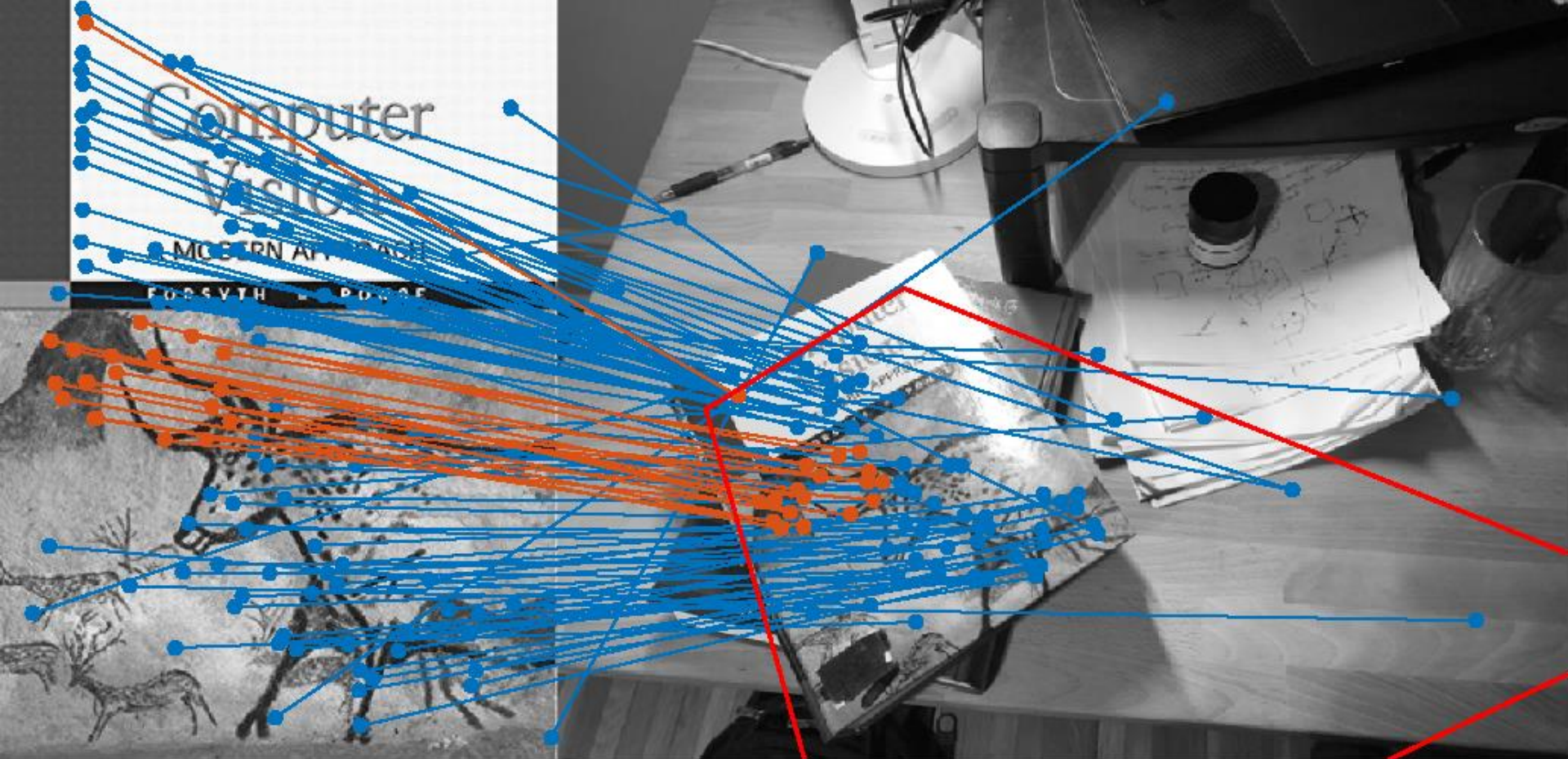


Inlier counting

Number of inliers: 8



Homography from 4 random correspondences



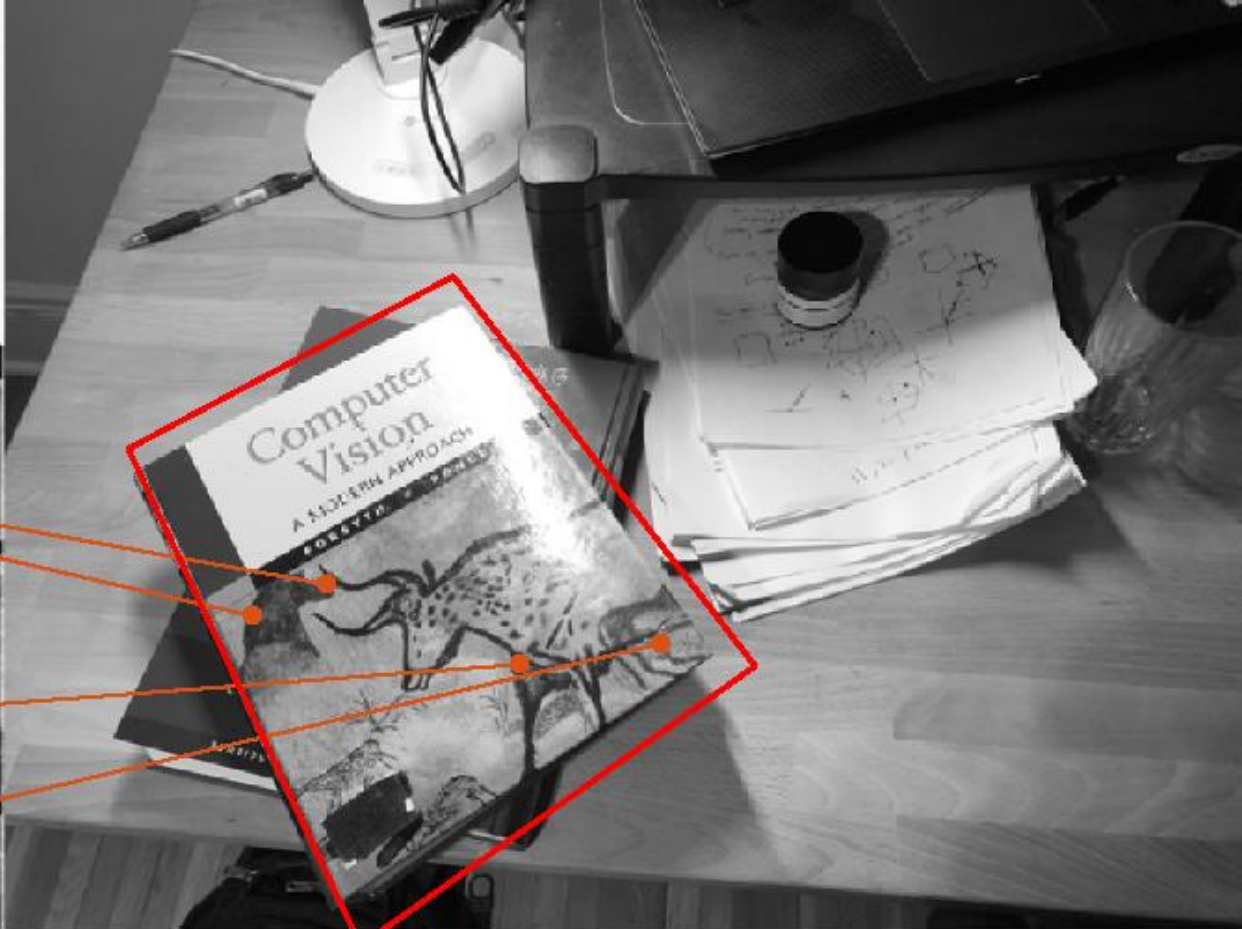
Inlier counting

Number of inliers: 25

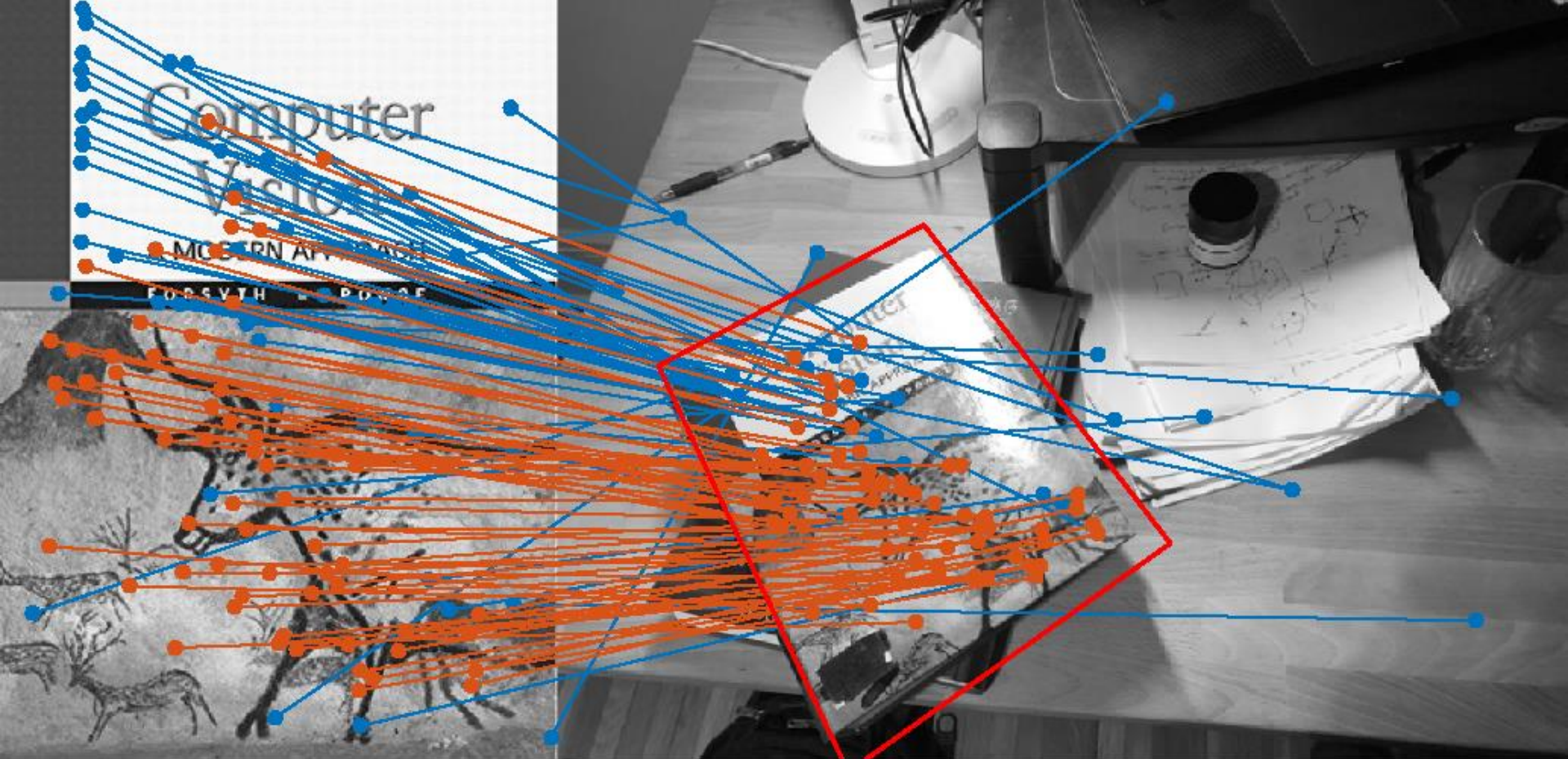
Computer Vision

A MODERN APPROACH

FORSYTH ■ PONCE



Homography from 4 random correspondences



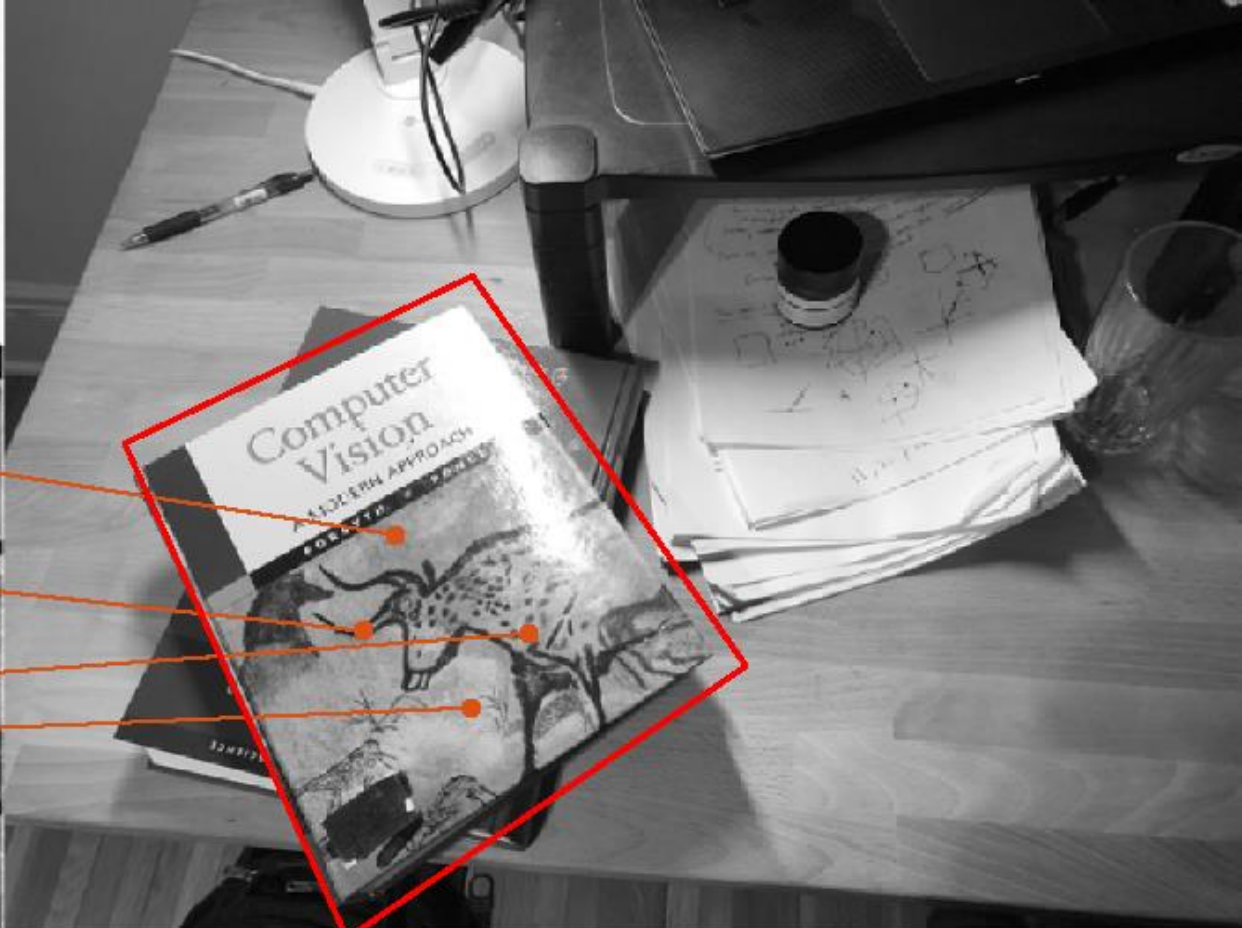
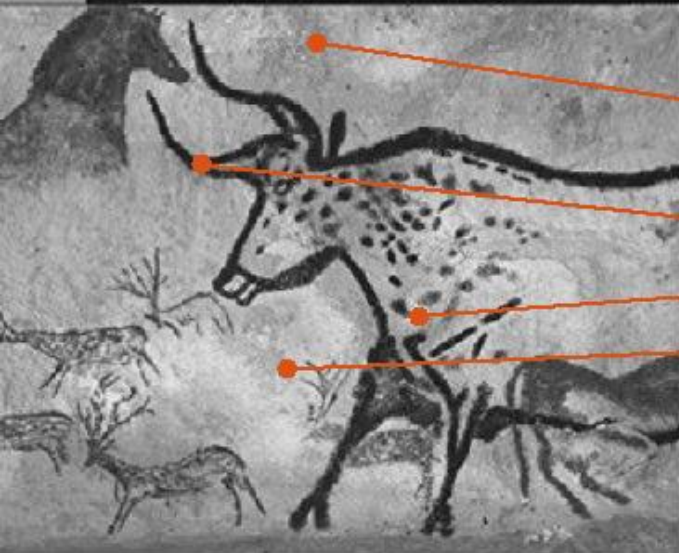
Inlier counting

Number of inliers: 76

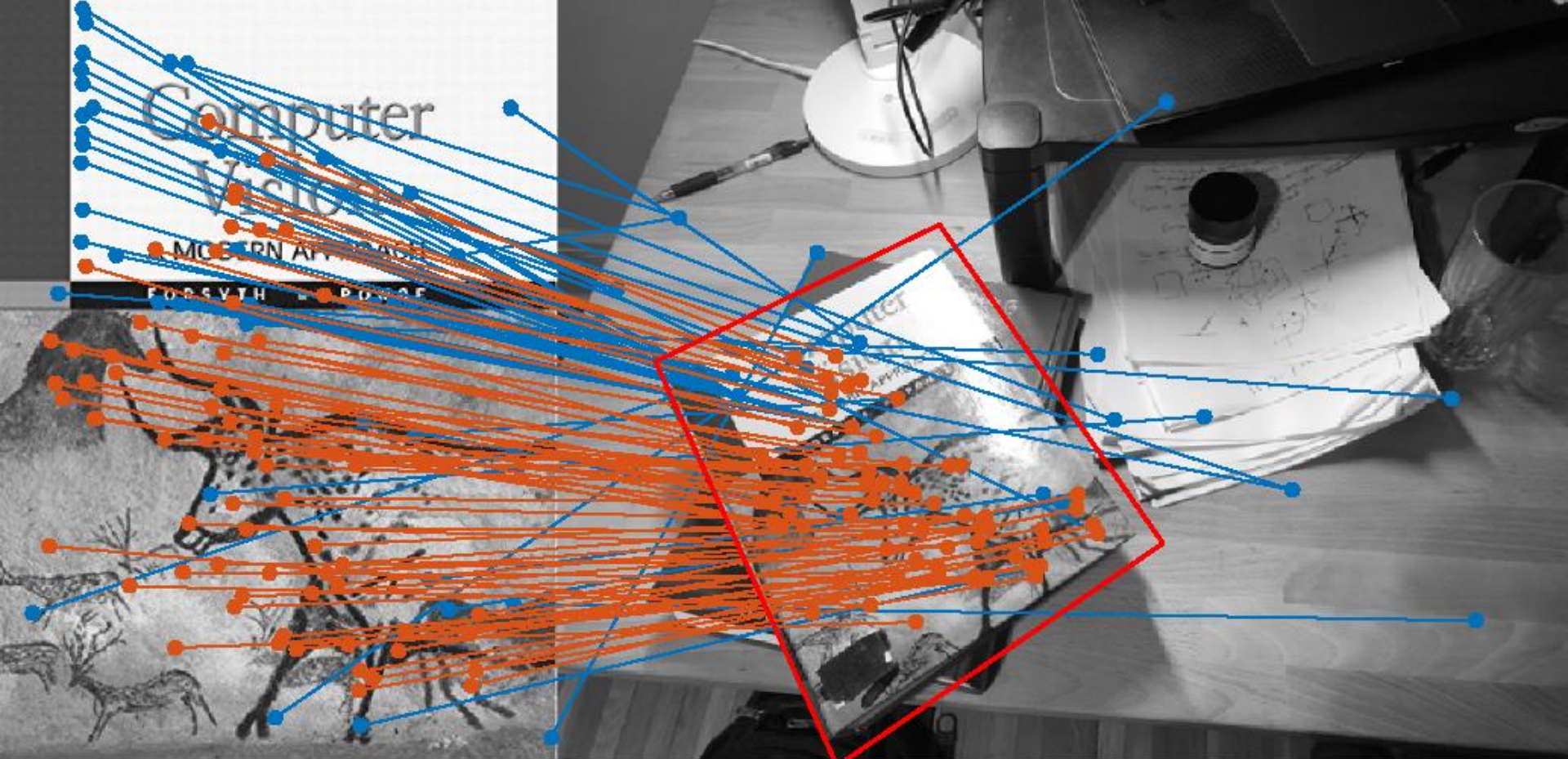
Computer Vision

A MODERN APPROACH

FORSYTH ■ PONCE



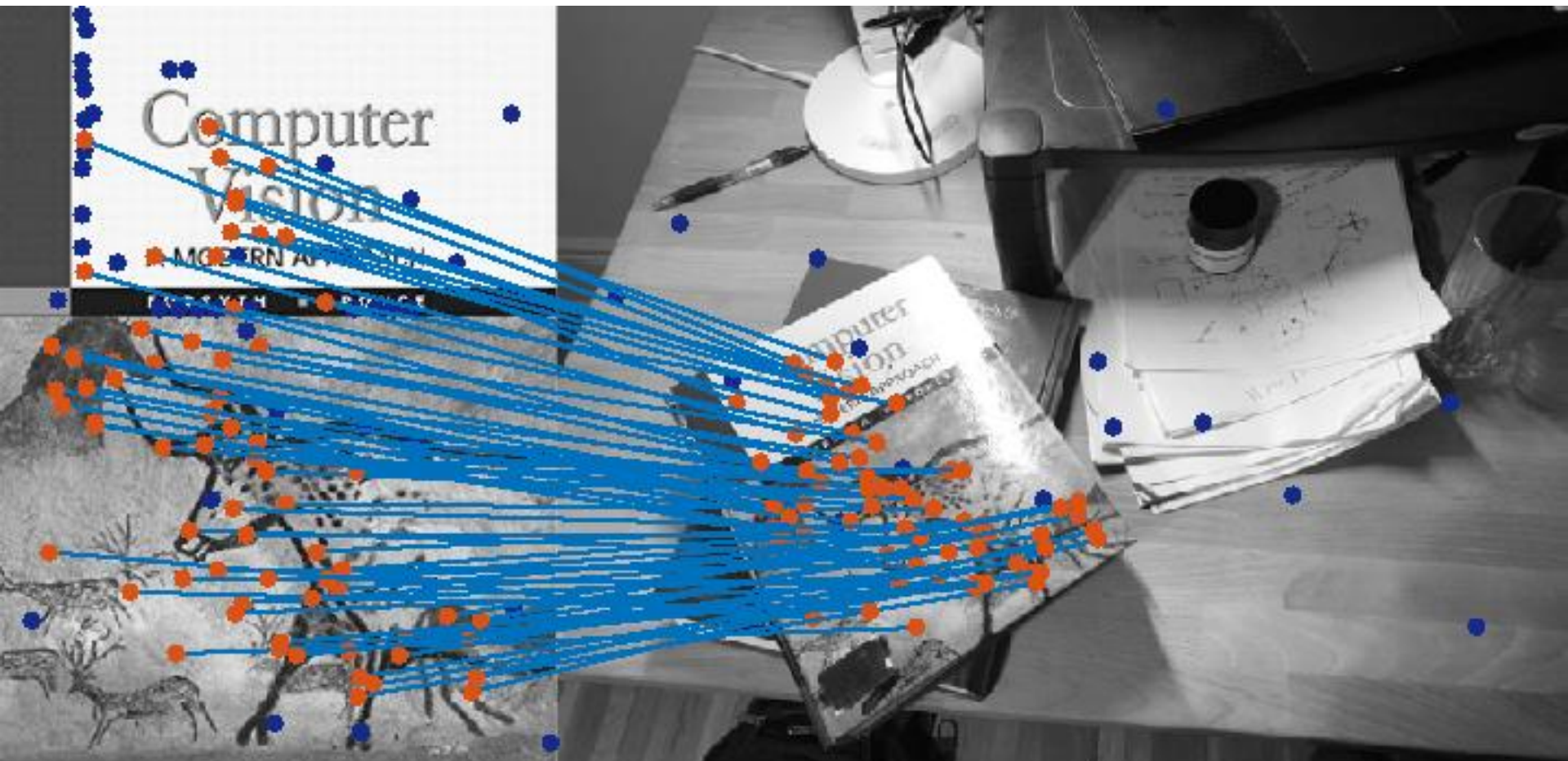
Homography from 4 random correspondences



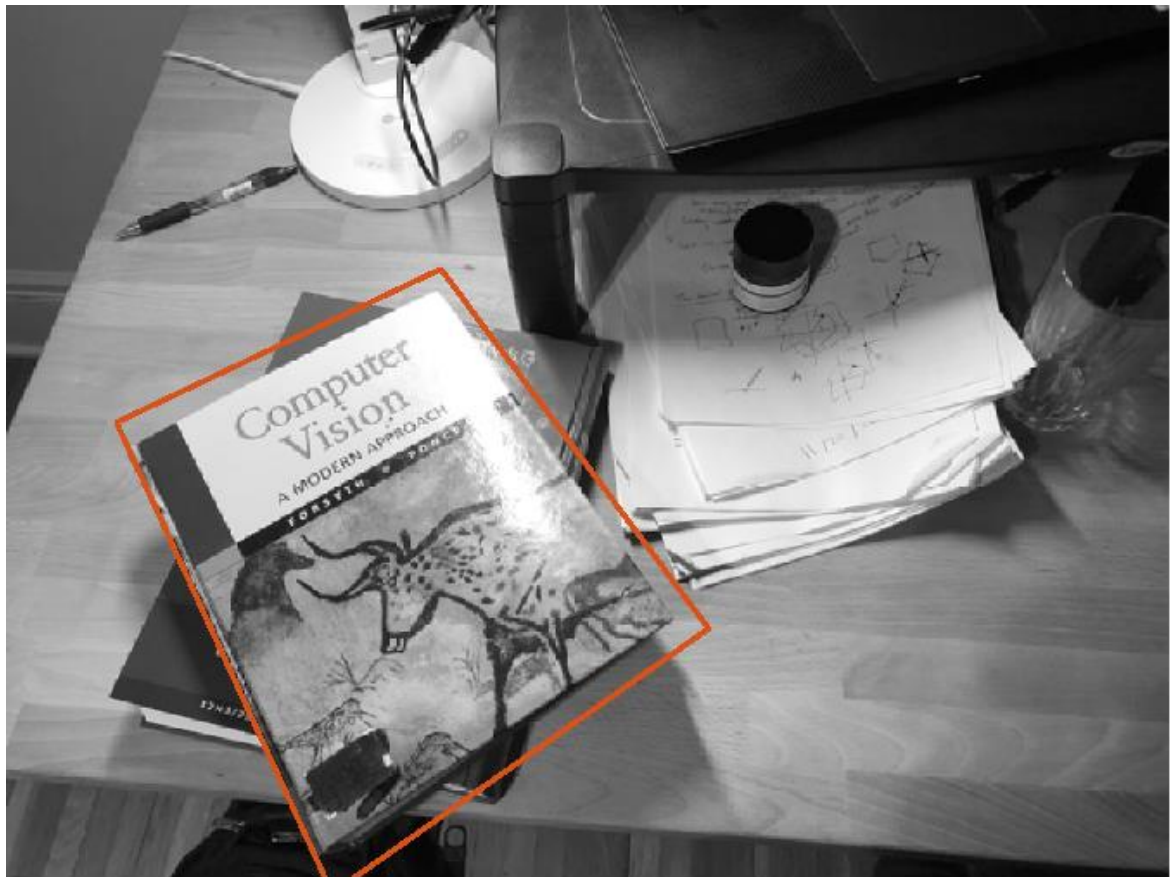
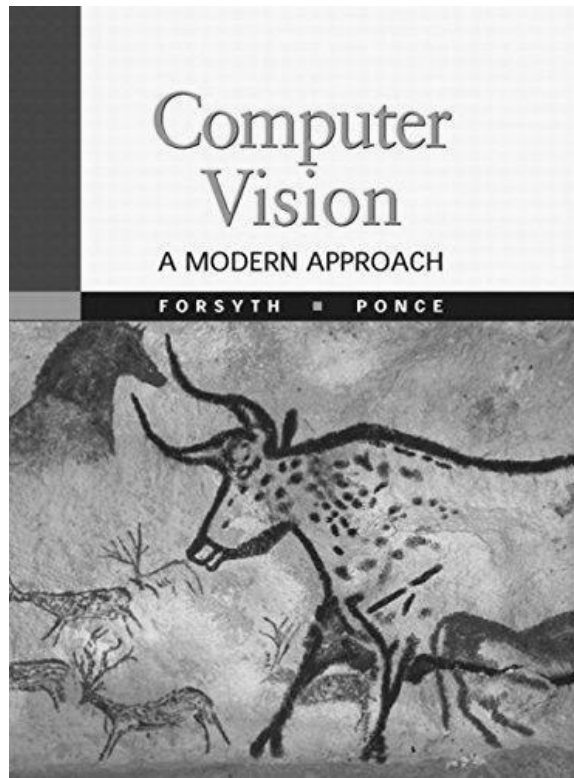
Inlier counting

Number of inliers: 83

RECALL: ROBUST FILTERING



RECALL: PARAMETRIC MODEL



RECALL: IMAGE WARPING

