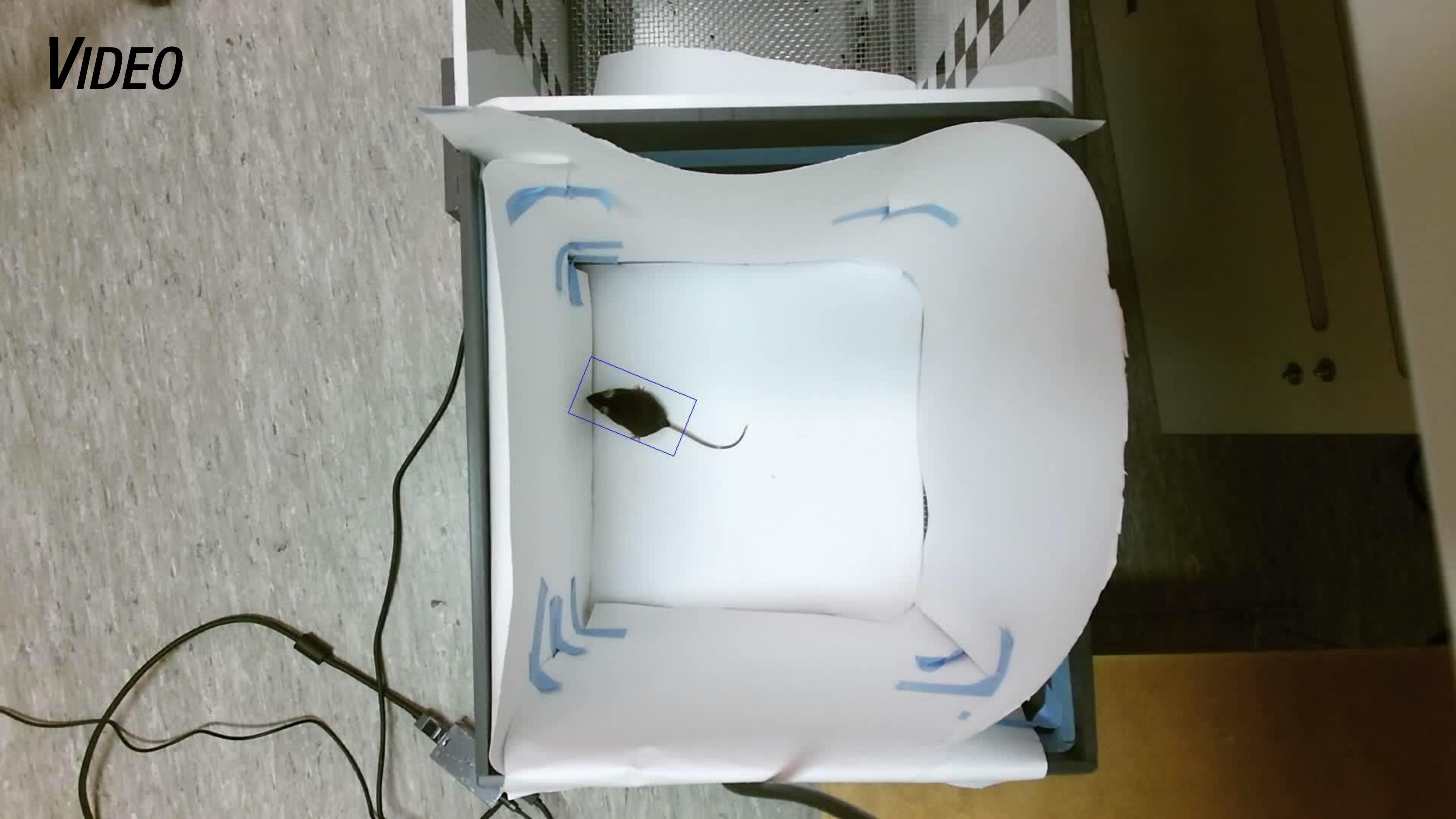
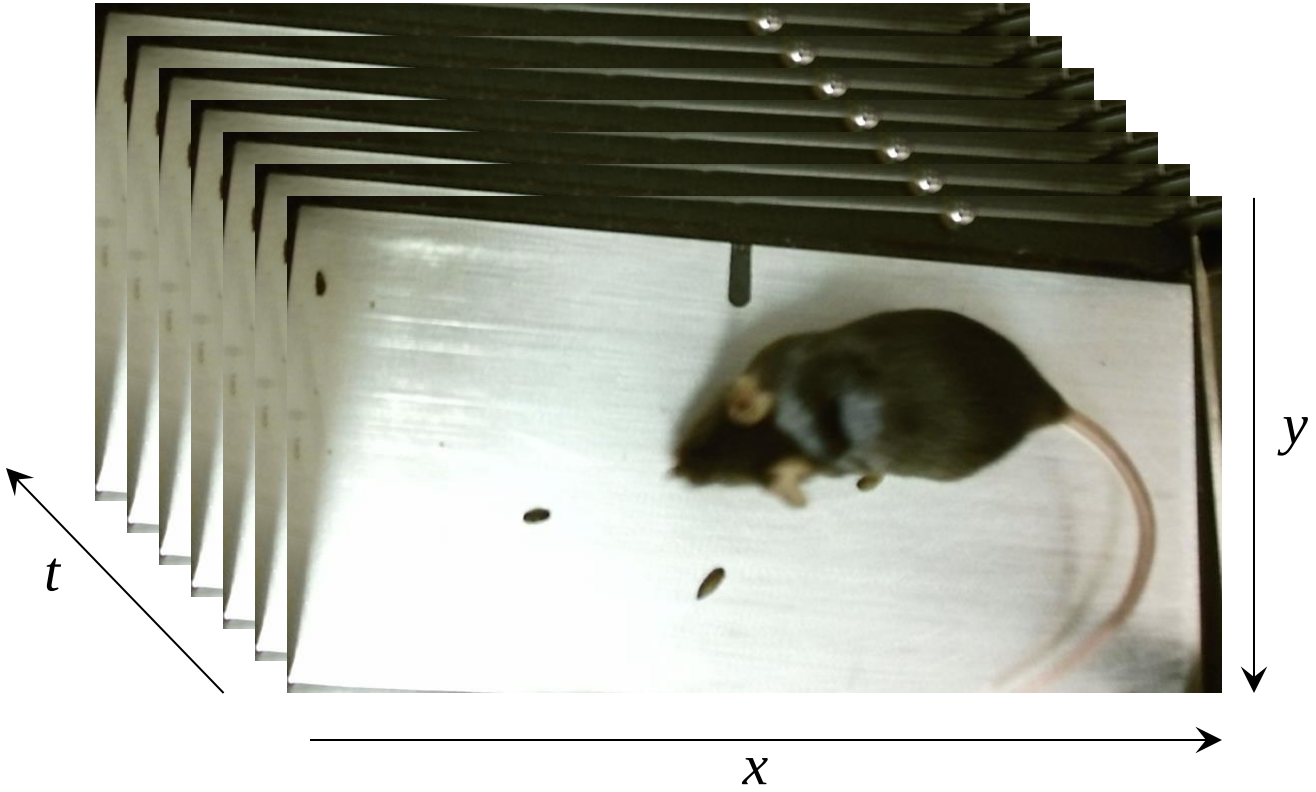
Patch Tracking

Hyun Soo Park


VIDEO

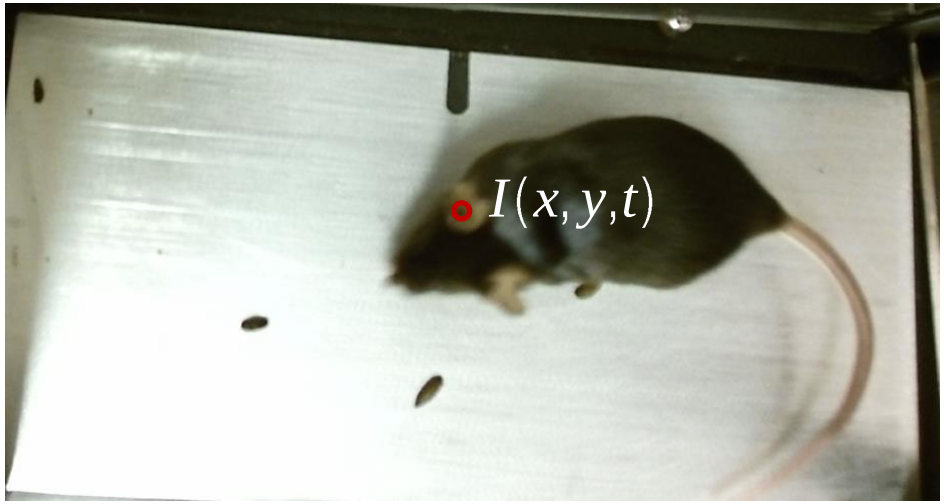
Continous Video Representation



Video as a function over spacetime volume

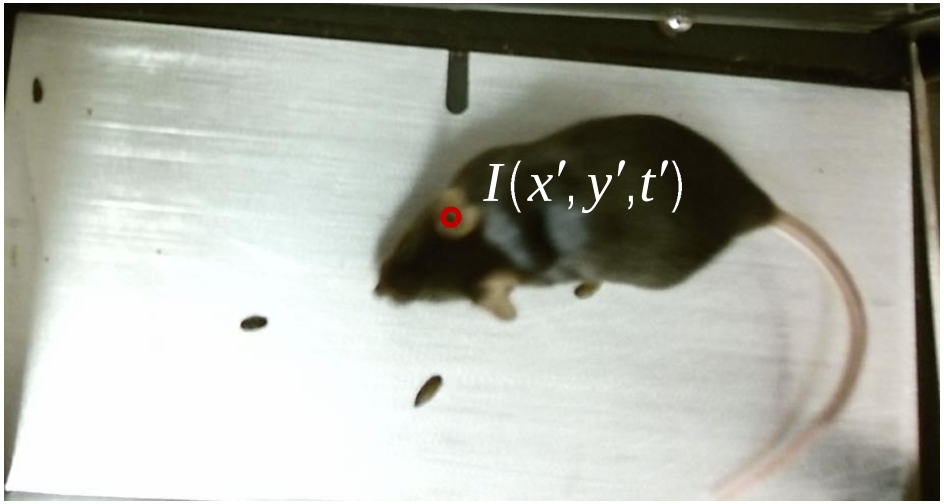
$$I(x, y, t)$$

Local Point Tracking



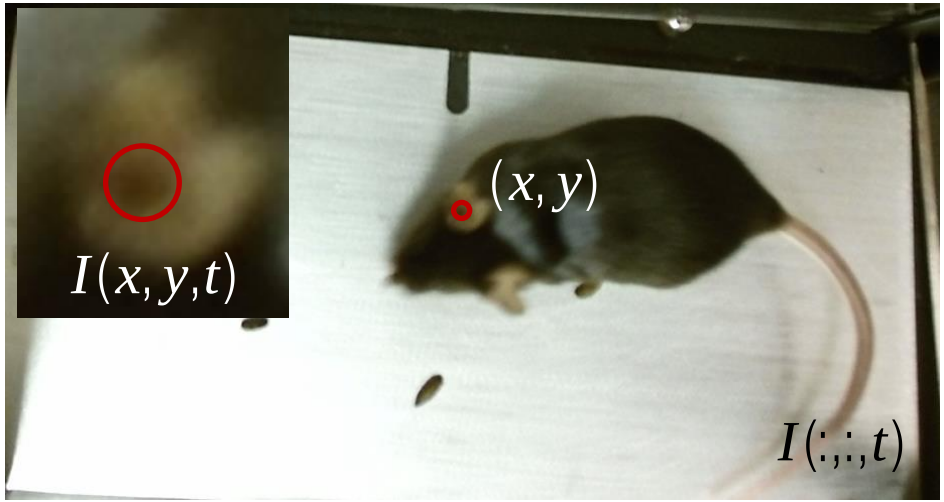
$I(x,y,t)$

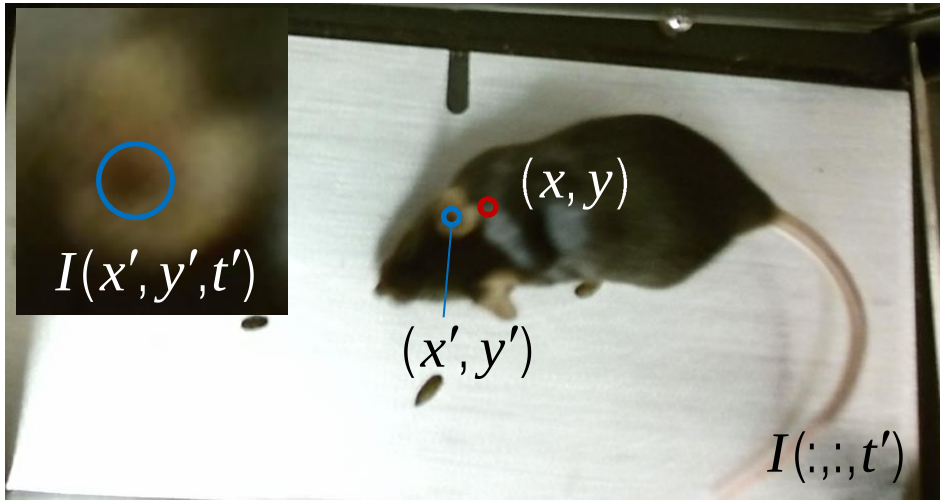
Local Point Tracking



$I(x',y',t')$

Assumption 1: Brightness Constancy

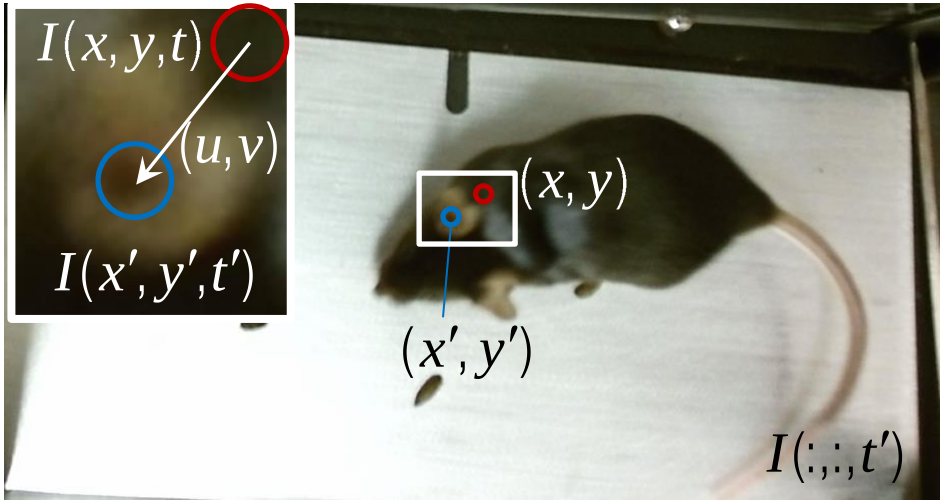




Brightness constancy: $I(x,y,t) \approx I(x',y',t')$

e.g., RGB value must be similar.

Assumption 2: Small Motion



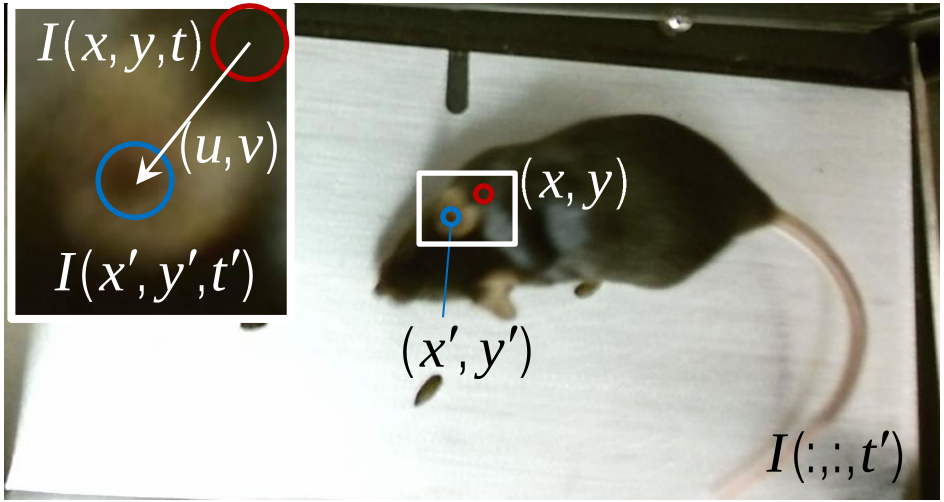
First order approximation must hold.

$$x' \approx x + u\delta t$$

$$y' \approx y + v\delta t$$

$$t' \approx t + \delta t$$

Assumption 2: Small Motion



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$$x' \approx x + u\delta t$$

$$y' \approx y + v\delta t$$

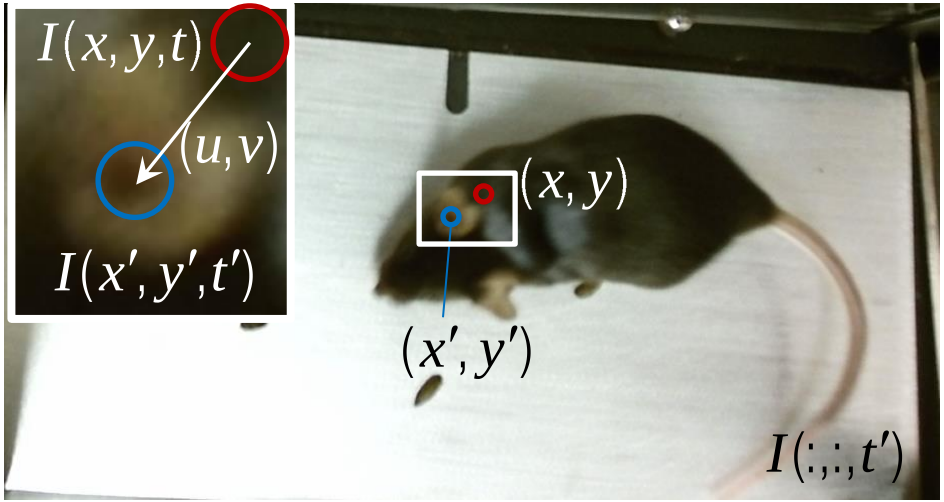
$$t' \approx t + \delta t$$

First order Taylor expansion:

$$I(x',y',t') \approx I(x + u\delta t, y + v\delta t, t + \delta t)$$

$$\approx I(x,y,t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$

Optical Flow



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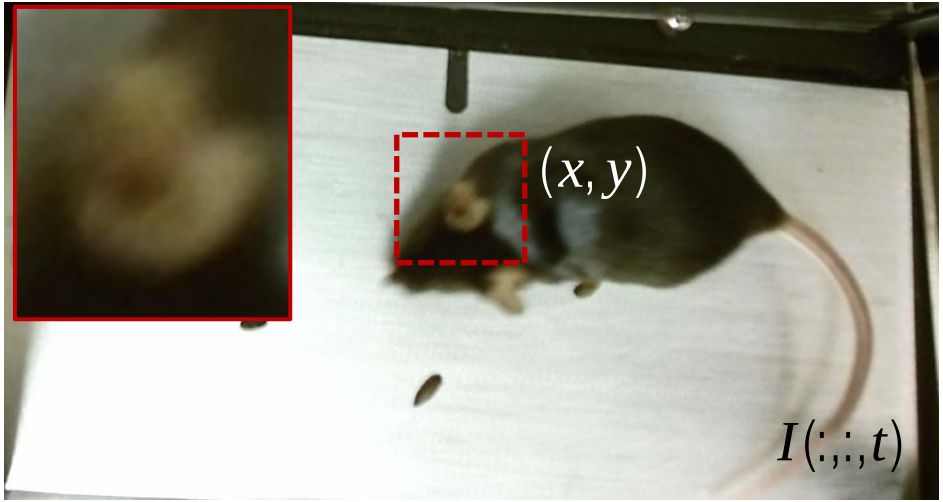
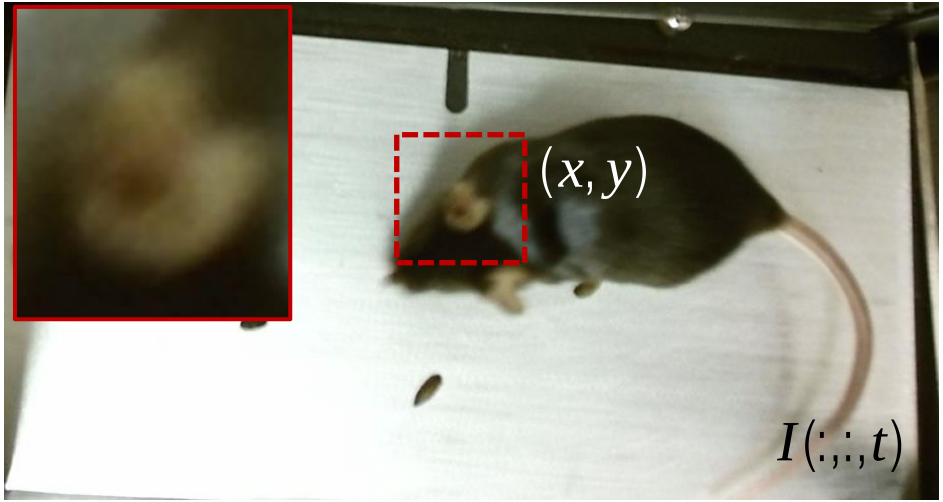
$$\approx I(x,y,t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$

$$\approx \underline{I(x,y,t)}$$

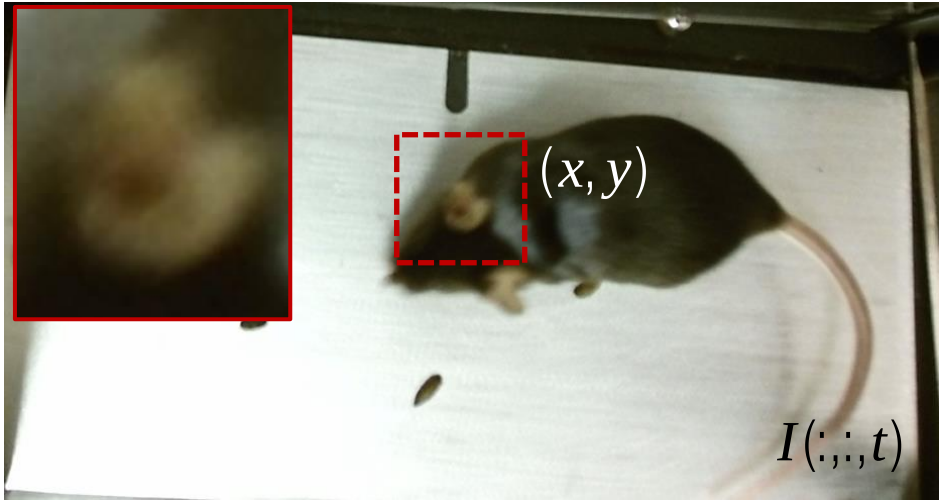
Brightness constancy



$$\frac{\partial I}{\partial x}u\delta t + \frac{\partial I}{\partial y}v\delta t + \frac{\partial I}{\partial t}\delta t \approx 0$$

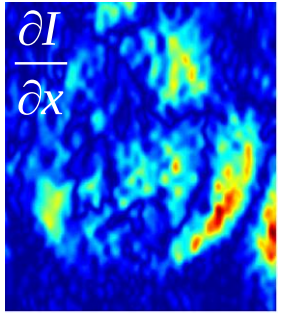


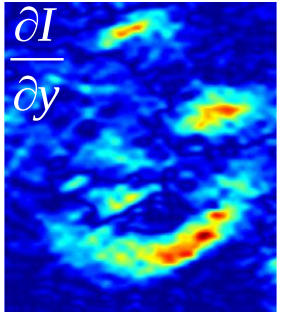
$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \approx -\frac{\partial I}{\partial t}$$

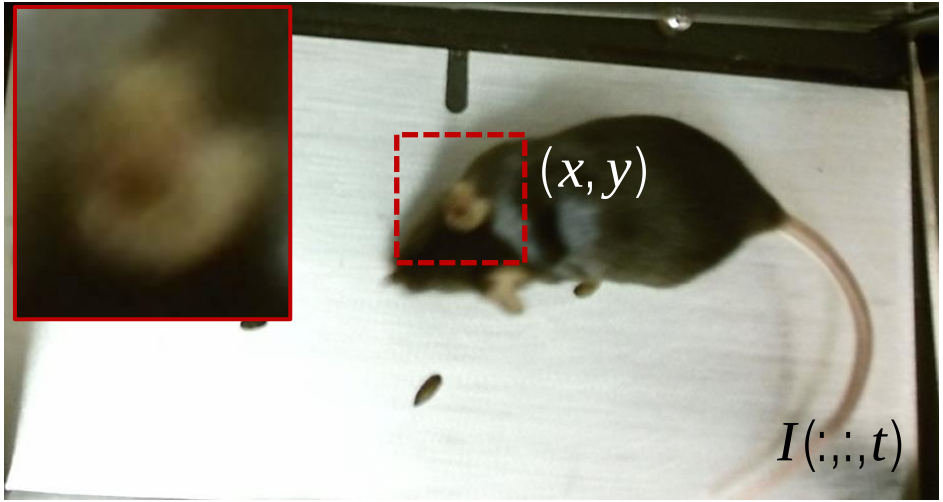


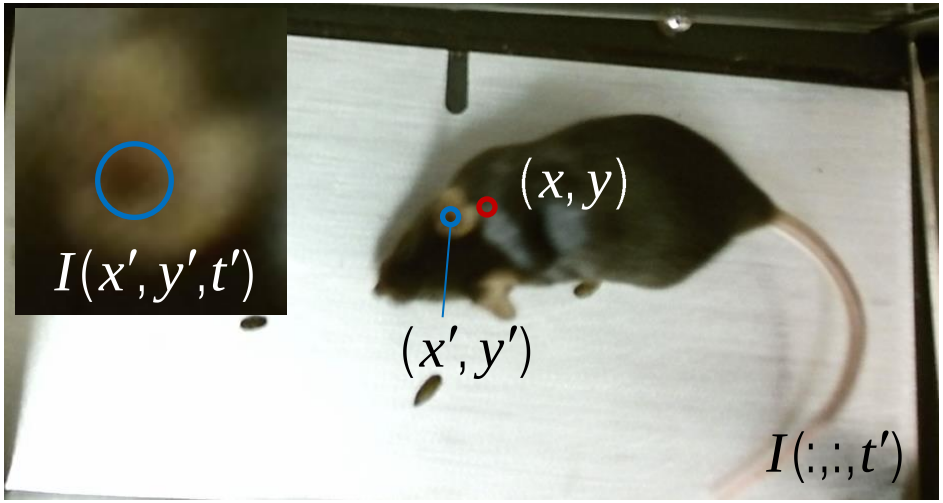
$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \approx -\frac{\partial I}{\partial t}$$

Spatial derivatives:



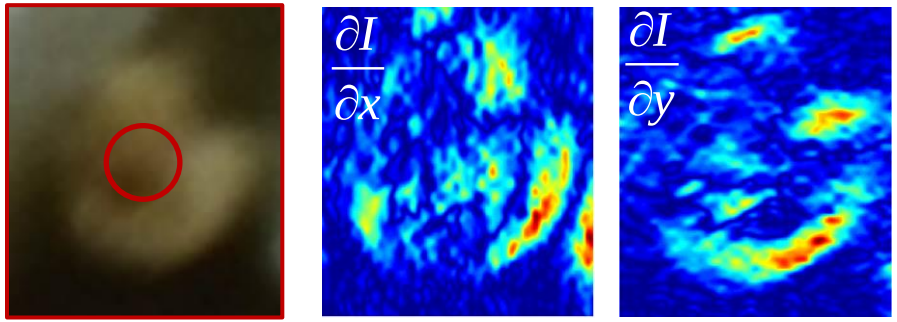




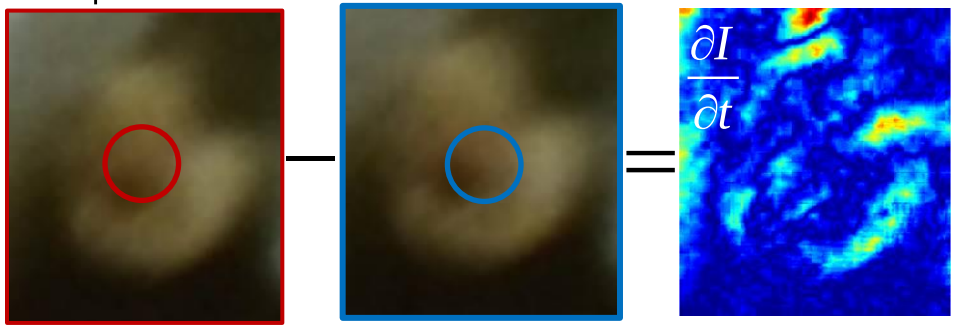


$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \approx -\frac{\partial I}{\partial t}$$

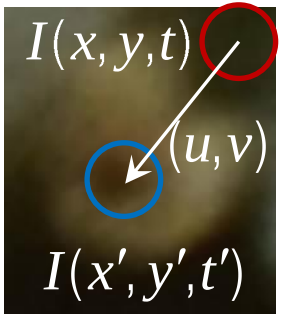
Spatial derivatives:



Temporal derivatives:



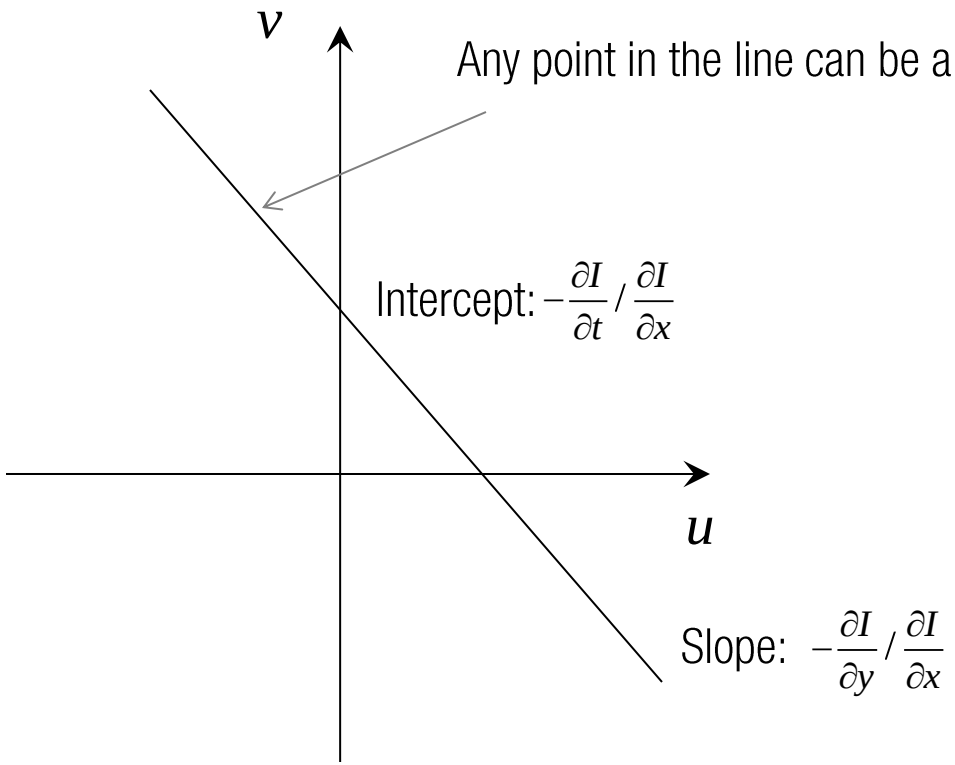
Ambiguity For Single Pixel Tracking



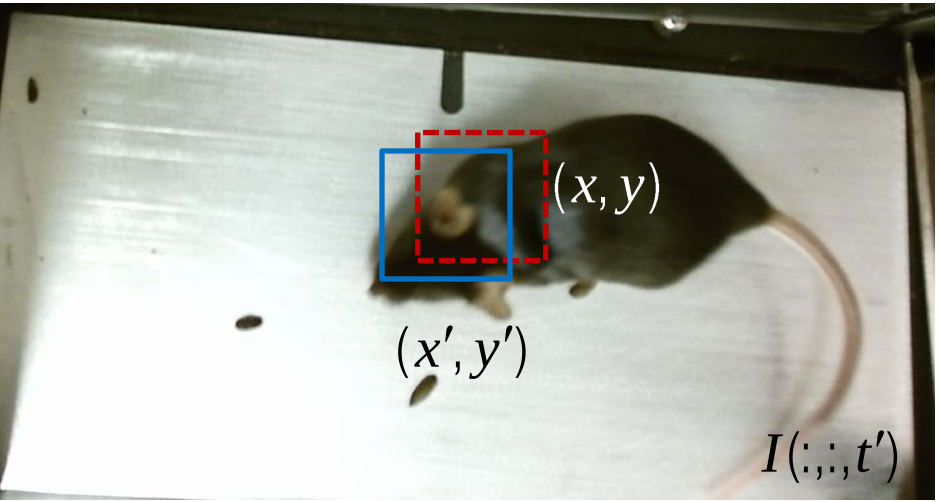
$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \approx -\frac{\partial I}{\partial t}$$

# of unknowns: 2

# of equations: 1



Local Patch Tracking



Neighboring pixels move similarly.

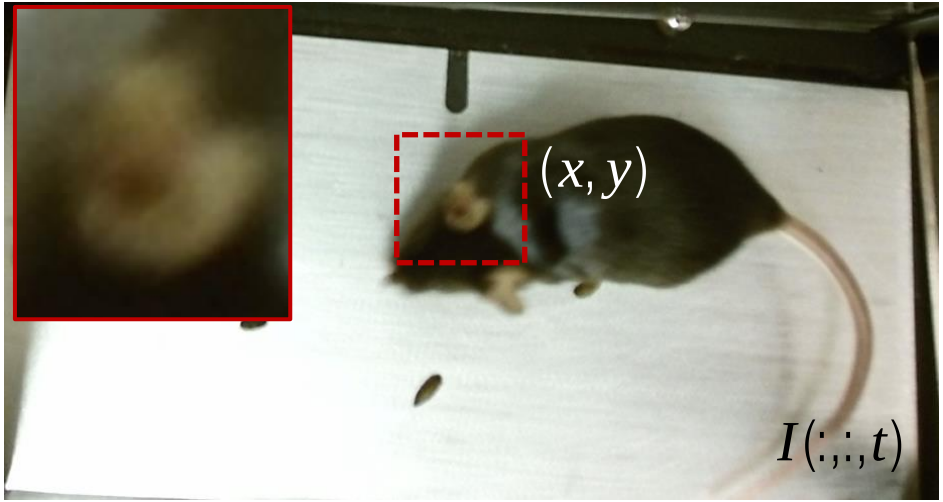
$$\{I(x_i, y_i, t)\}_{(x_i, y_i) \in N(x,y)} \qquad \{I(x_i', y_i', t)\}_{(x_i', y_i') \in N(x', y')}$$

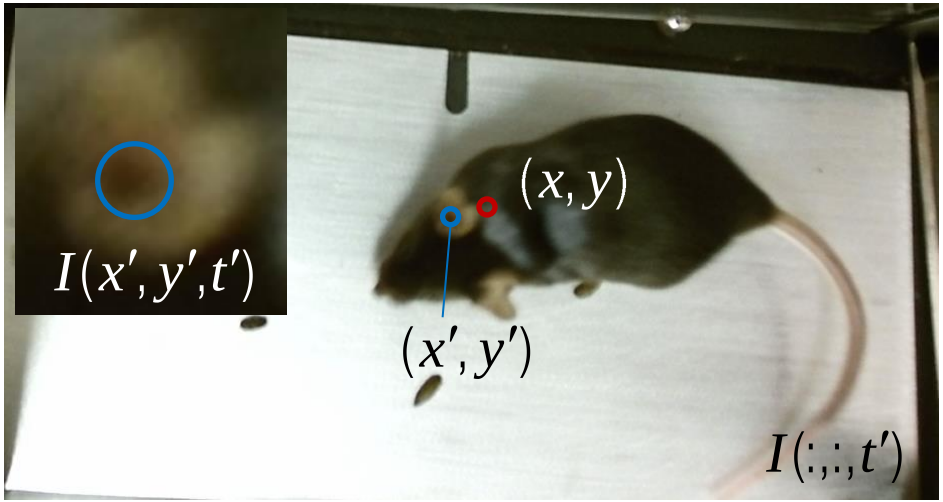
$$\left.\begin{aligned} &\frac{\partial I}{\partial x}\Big|_1 u + \frac{\partial I}{\partial y}\Big|_1 v = -\frac{\partial I}{\partial t}\Big|_1 \\ &\vdots \\ &\frac{\partial I}{\partial x}\Big|_n u + \frac{\partial I}{\partial y}\Big|_n v = -\frac{\partial I}{\partial t}\Big|_n \end{aligned}\right\}$$

# of unknowns: 2

# of equations: # of pixels in the local patch

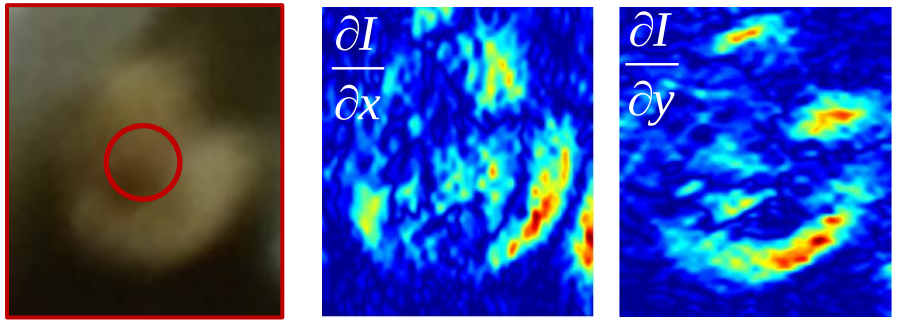
Can be solved via linear least squares.



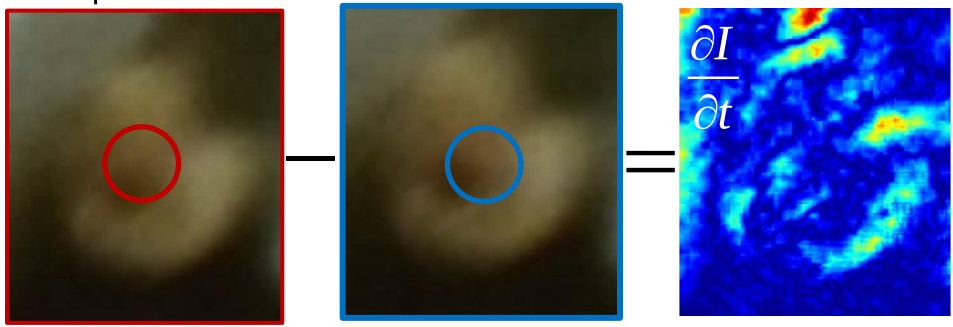


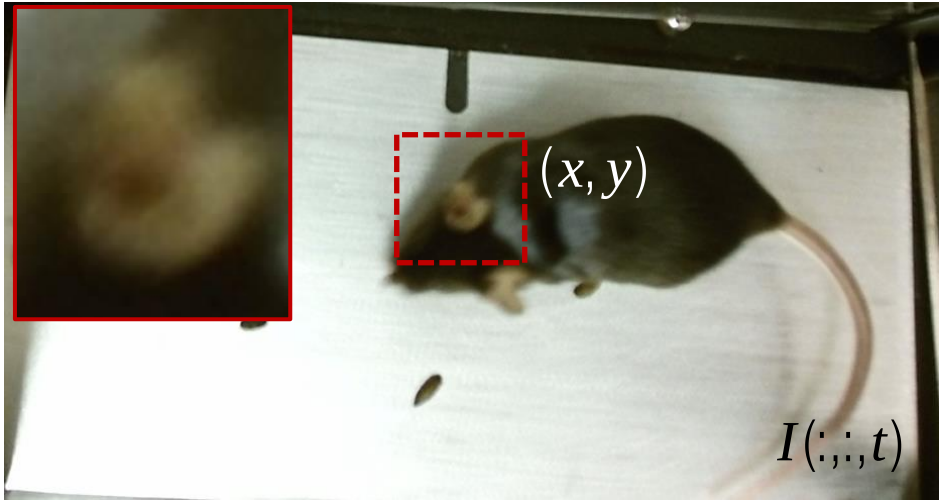
$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \approx -\frac{\partial I}{\partial t}$$

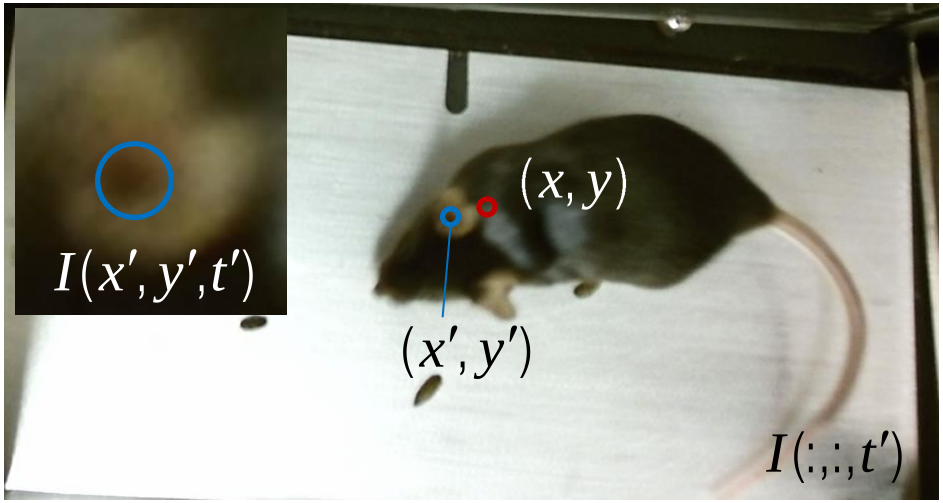
Spatial derivatives:



Temporal derivatives:





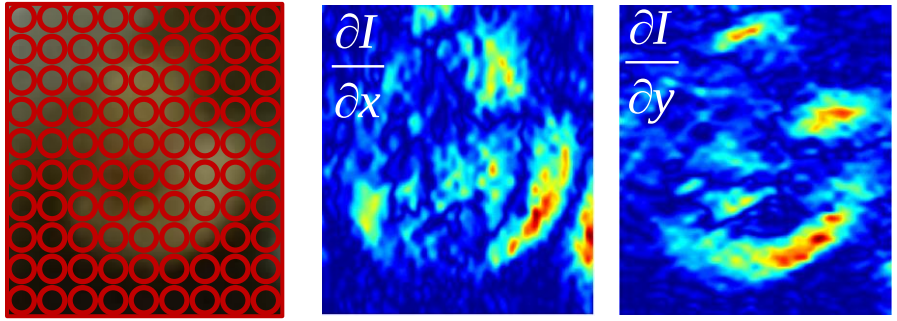


$$\left.\frac{\partial I}{\partial x}\right|_1 u + \left.\frac{\partial I}{\partial y}\right|_1 v = -\left.\frac{\partial I}{\partial t}\right|_1$$

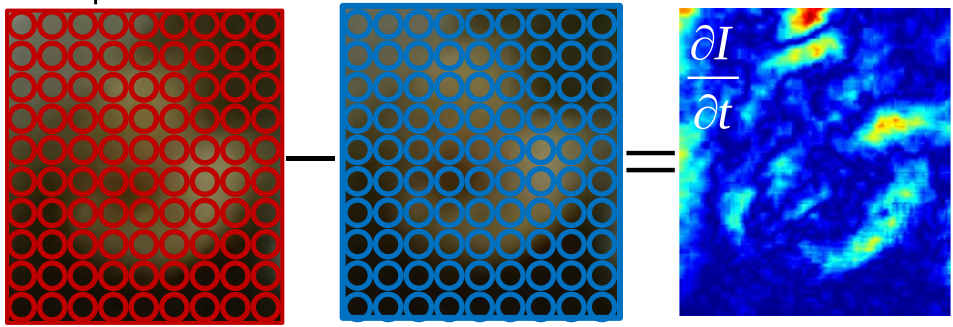
$$\vdots$$

$$\left.\frac{\partial I}{\partial x}\right|_n u + \left.\frac{\partial I}{\partial y}\right|_n v = -\left.\frac{\partial I}{\partial t}\right|_n$$

Spatial derivatives:



Temporal derivatives:



LOCAL PATCH TRACKING

$$\left.\frac{\partial I}{\partial x}\right|_1 u + \left.\frac{\partial I}{\partial y}\right|_1 v = -\left.\frac{\partial I}{\partial t}\right|_1$$

$$\vdots$$

$$\left.\frac{\partial I}{\partial x}\right|_n u + \left.\frac{\partial I}{\partial y}\right|_n v = -\left.\frac{\partial I}{\partial t}\right|_n$$

$\longrightarrow$

$$I_x|_1 u + I_y|_1 v = -I_t|_1$$

$$\vdots$$

$$I_x|_n u + I_y|_n v = -I_t|_n$$

$\longrightarrow$

$$\underbrace{\begin{bmatrix} I_x|_1 & I_y|_1 \\ \vdots & \vdots \\ I_x|_n & I_y|_n \end{bmatrix}}_{\text{nx2}} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{\text{Unknowns}} = -\underbrace{\begin{bmatrix} I_t|_1 \\ \vdots \\ I_t|_n \end{bmatrix}}_{\text{nx1}}$$

LOCAL PATCH TRACKING

$$\left.\frac{\partial I}{\partial x}\right|_1 u + \left.\frac{\partial I}{\partial y}\right|_1 v = -\left.\frac{\partial I}{\partial t}\right|_1$$

$$\vdots$$

$$\left.\frac{\partial I}{\partial x}\right|_n u + \left.\frac{\partial I}{\partial y}\right|_n v = -\left.\frac{\partial I}{\partial t}\right|_n$$

$$\longrightarrow$$

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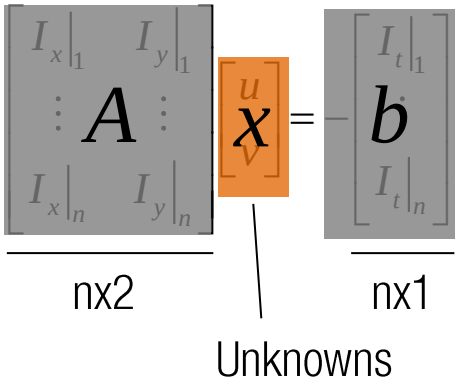
$$\vdots$$

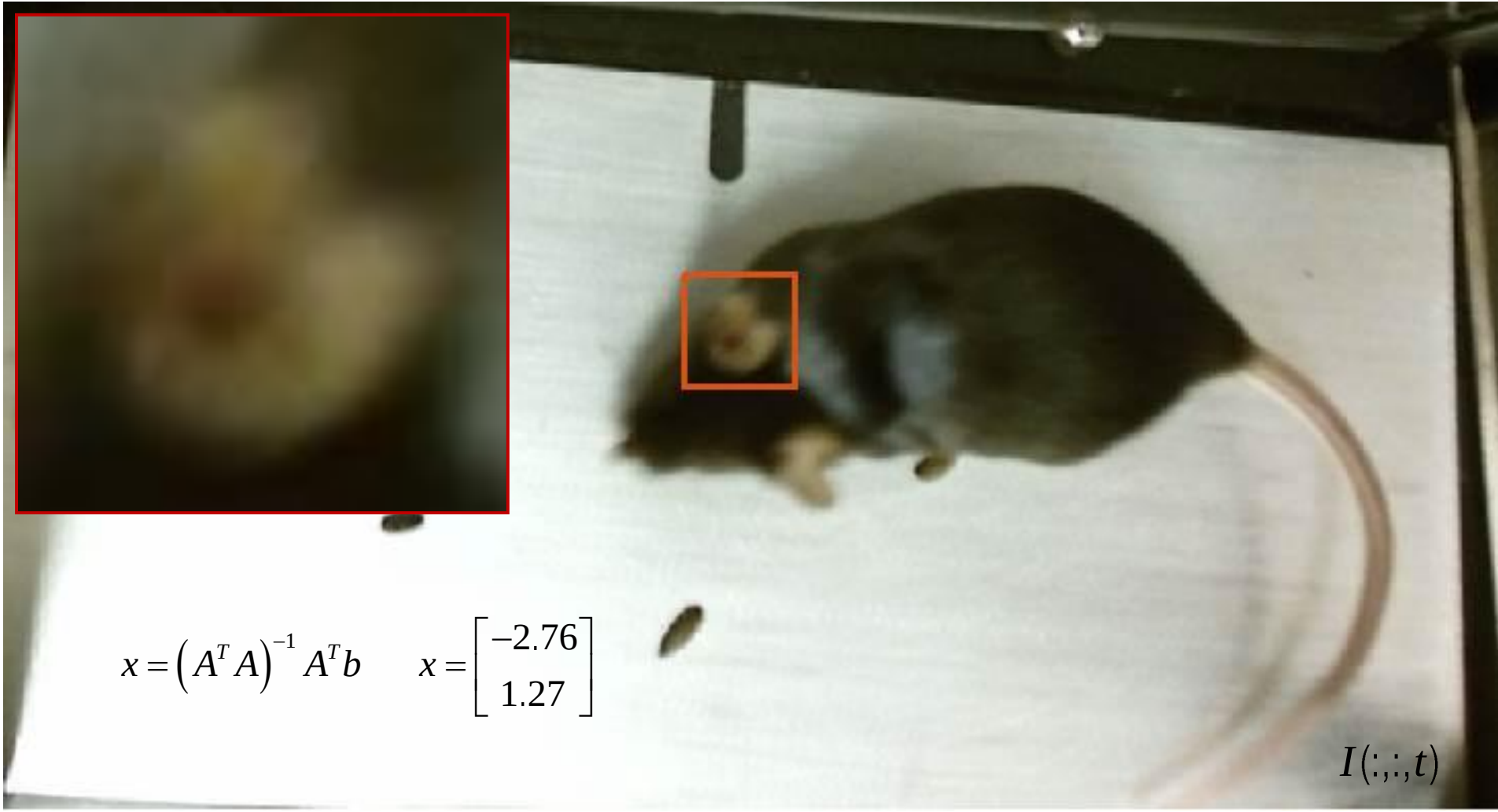
$$I_x|_n u + I_y|_n v = -I_t|_n$$

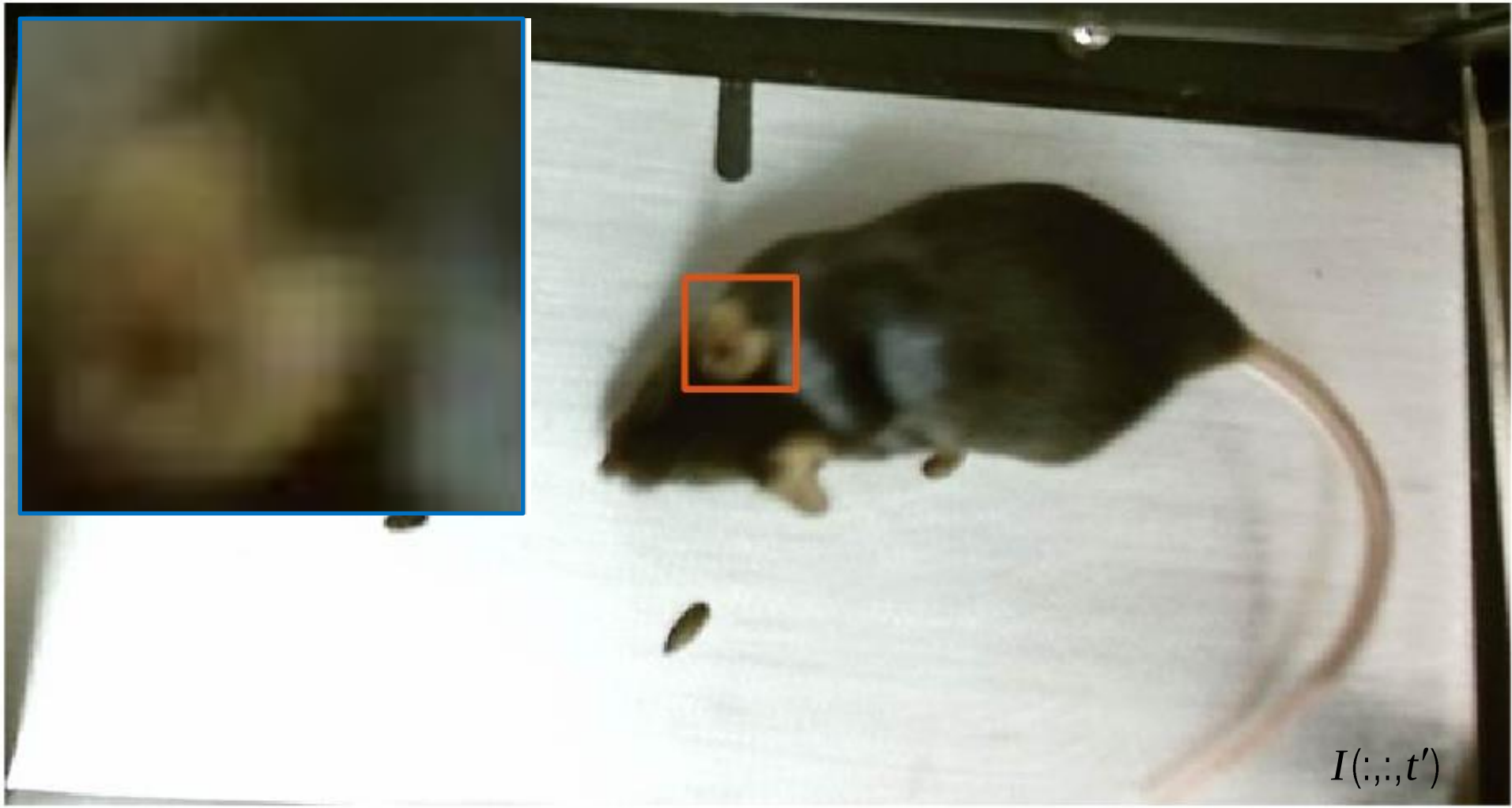
$$\longrightarrow$$



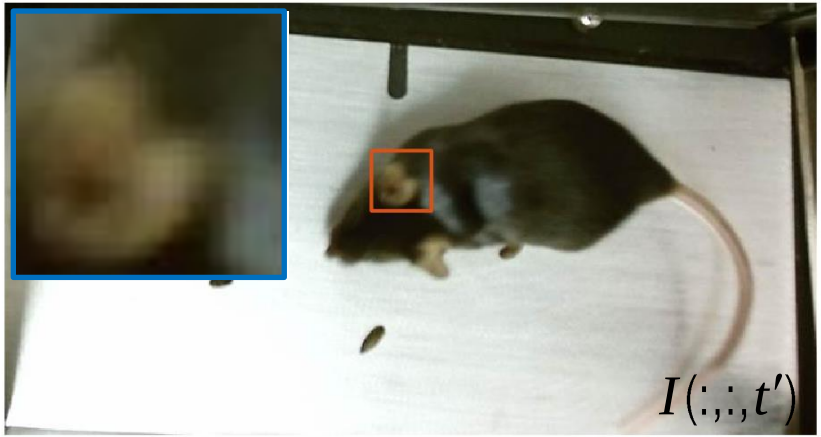
$$x = \left(A^T A\right)^{-1} A^T b$$

Least squares solution


$x = \left(A^T A\right)^{-1} A^T b$
$x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$
$I(:,:,t)$

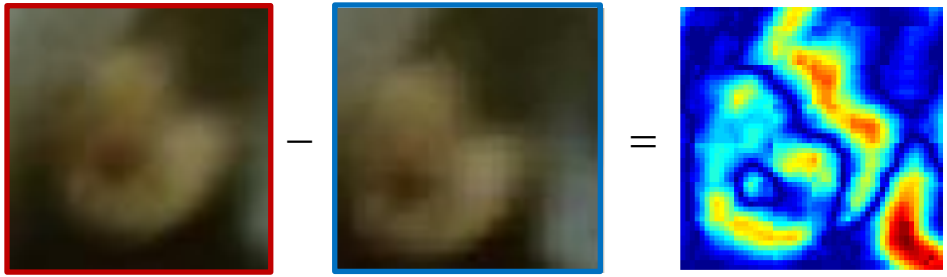

$I(:,:,t')$

LOCAL PATCH TRACKING



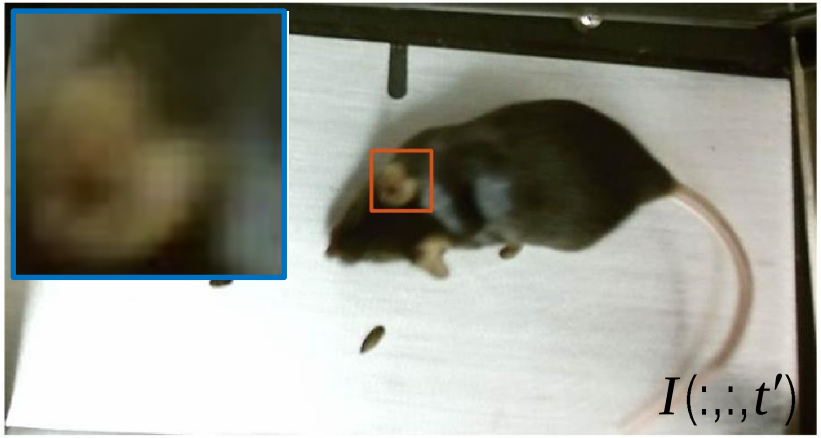
NCC: 0.923520; SSD: 3.854872

$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$



SSD: 3.85

LOCAL PATCH TRACKING



NCC: 0.923520; SSD: 3.854872

$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

First order approximation

$$I(x + u\delta t, y + v\delta t, t + \delta t)$$

$$\approx I(x, y, t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$

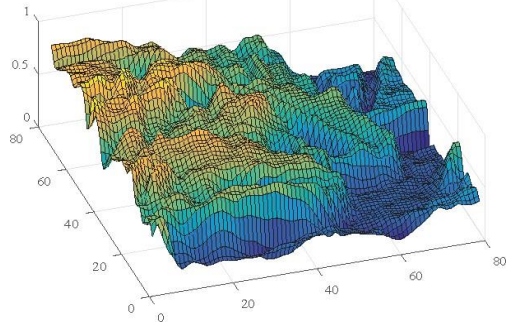
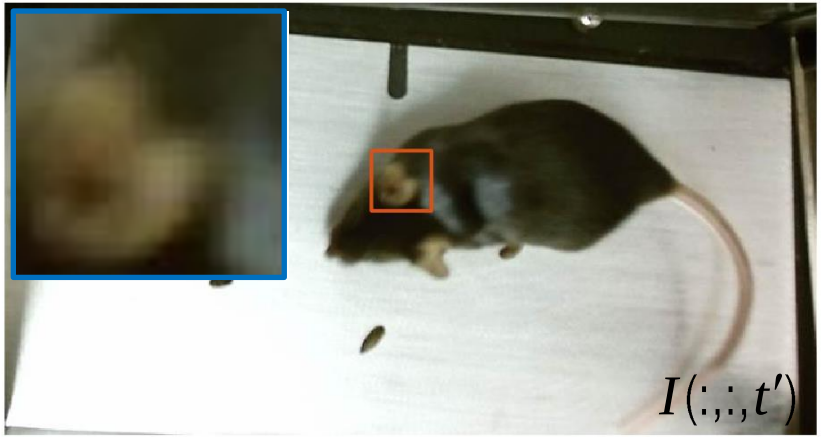


Image is nonlinear function!

LOCAL PATCH TRACKING



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First order approximation

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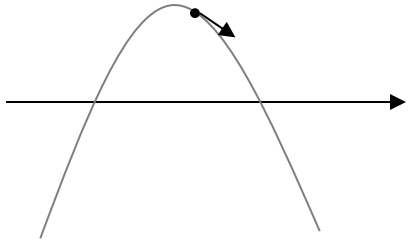
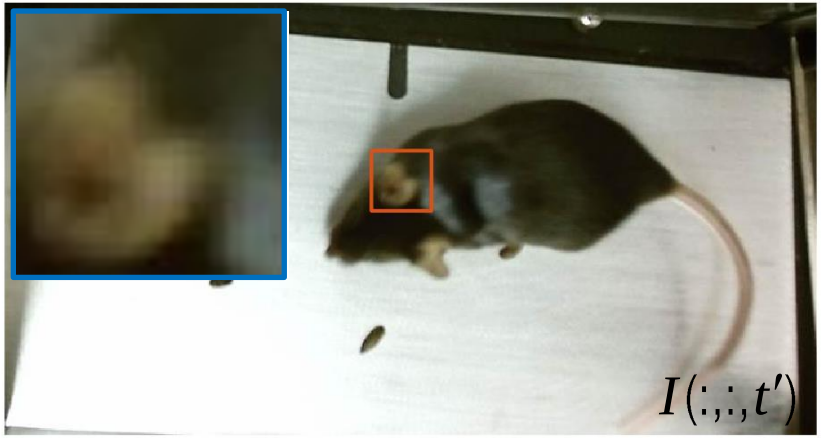
$$\approx I(x, y, t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$



1.Linearize

Image is nonlinear function!

LOCAL PATCH TRACKING



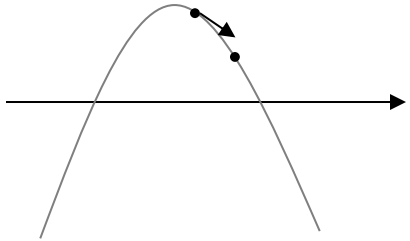
NCC: 0.923520; SSD: 3.854872

$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

First order approximation

$$I(x + u\delta t, y + v\delta t, t + \delta t)$$

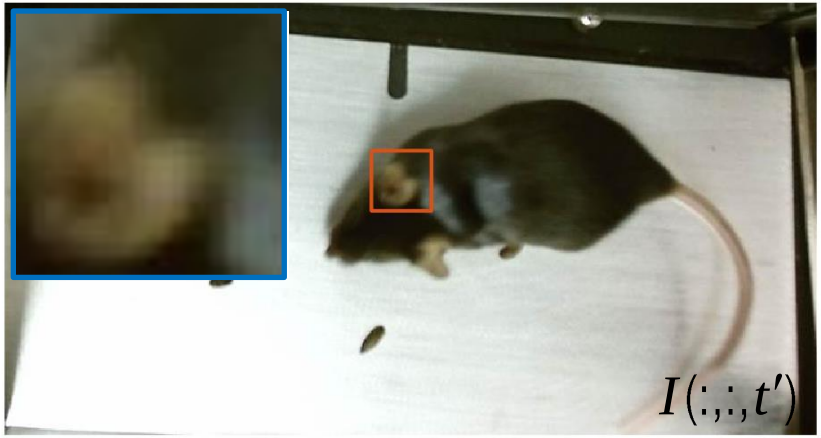
$$\approx I(x,y,t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$



1. Linearize
2. Move $x \leftarrow x + (u,v)$

Image is nonlinear function!

LOCAL PATCH TRACKING



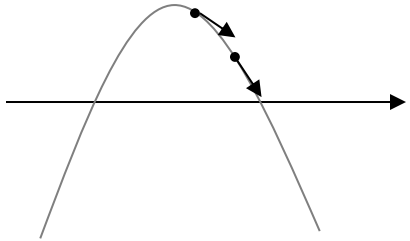
NCC: 0.923520; SSD: 3.854872

$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

First order approximation

$$I(x + u\delta t, y + v\delta t, t + \delta t)$$

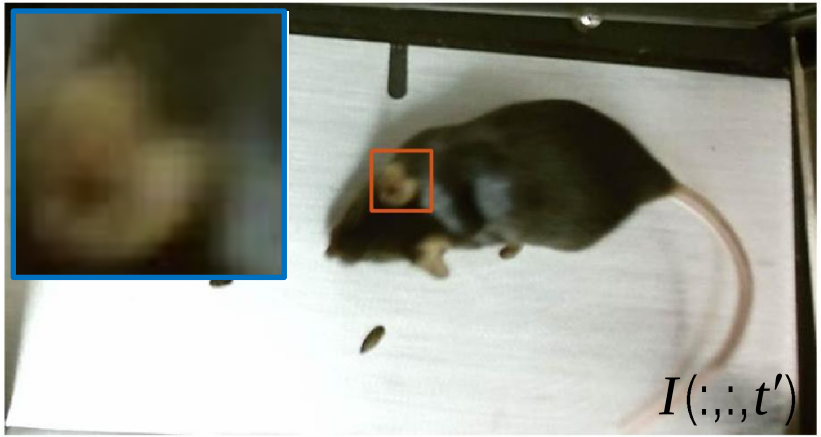
$$\approx I(x,y,t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$



1. Linearize
2. Move $x \leftarrow x + (u,v)$
3. Goto 1

Image is nonlinear function!

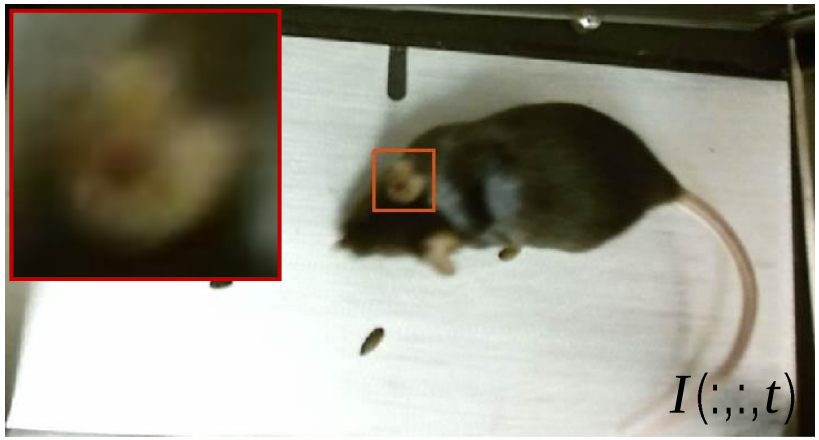
Local Patch Tracking

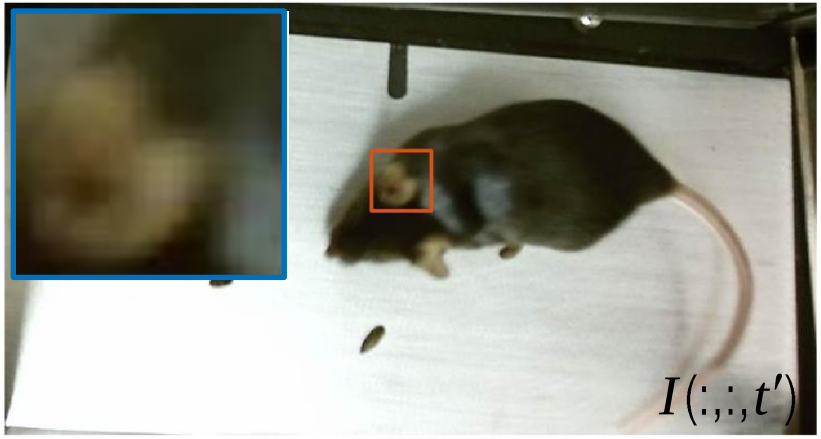


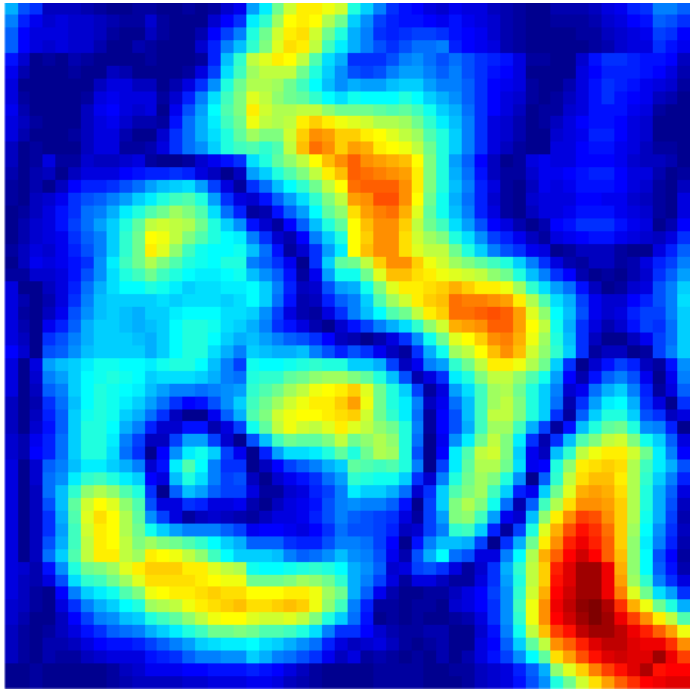
$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

NCC: 0.923520; SSD: 3.854872

1ST ITERATION

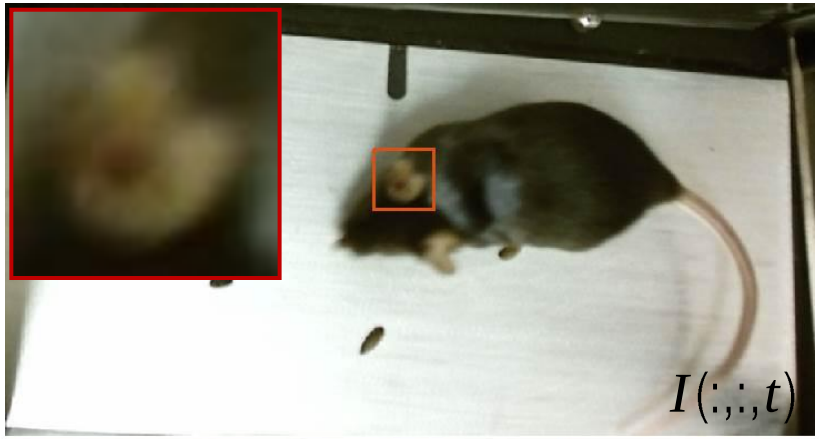


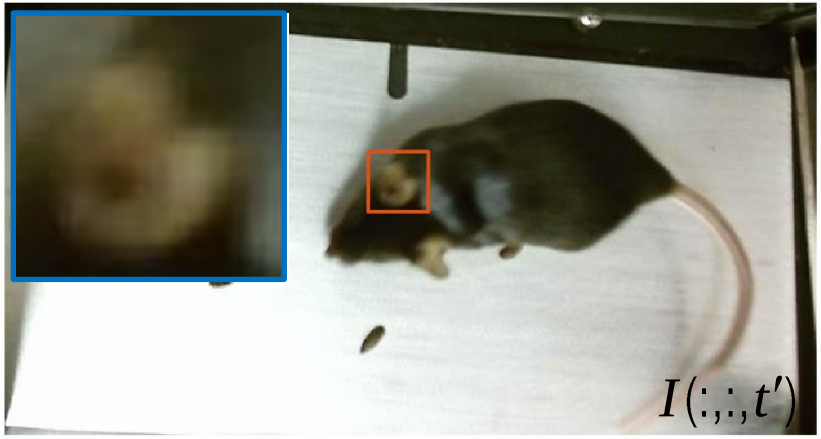


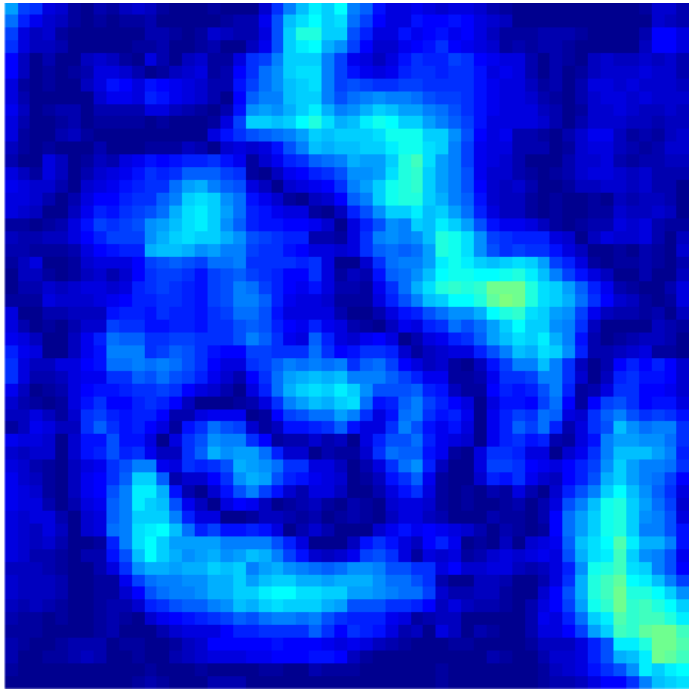


NCC: 0.923520; SSD: 3.854872

3RD ITERATION

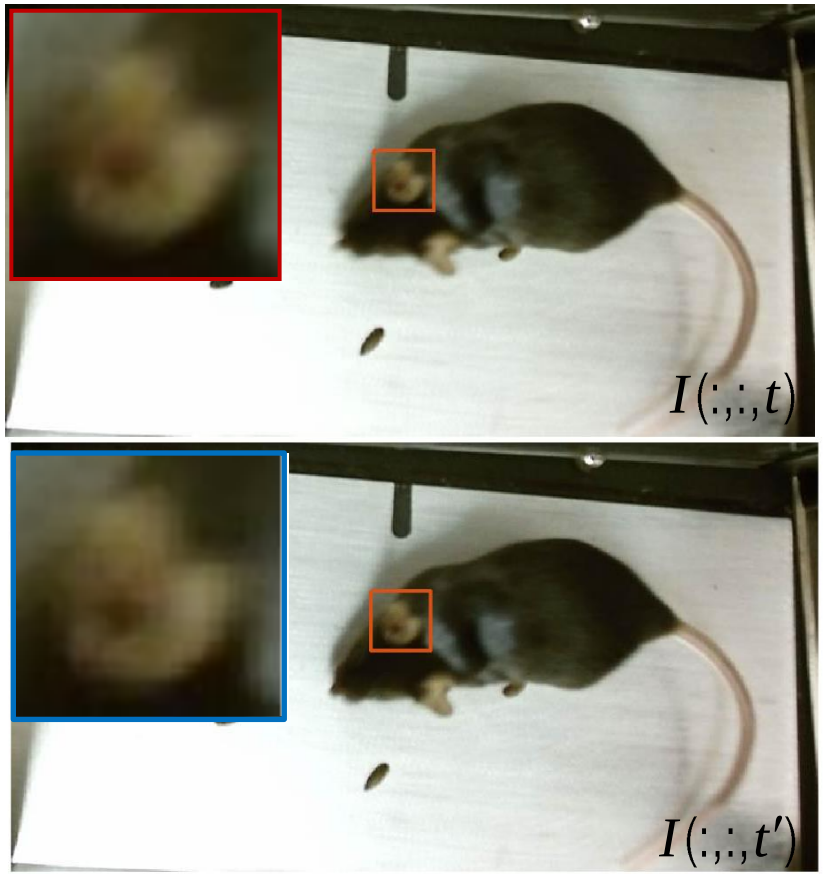


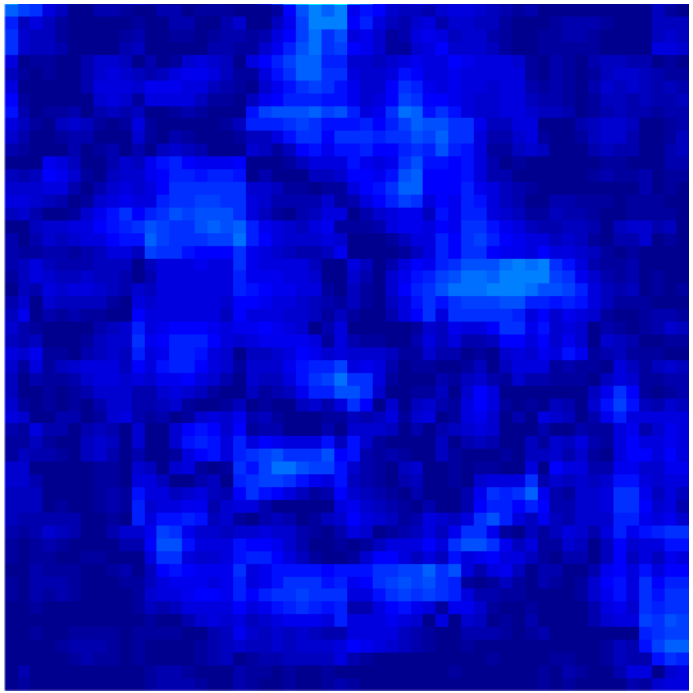




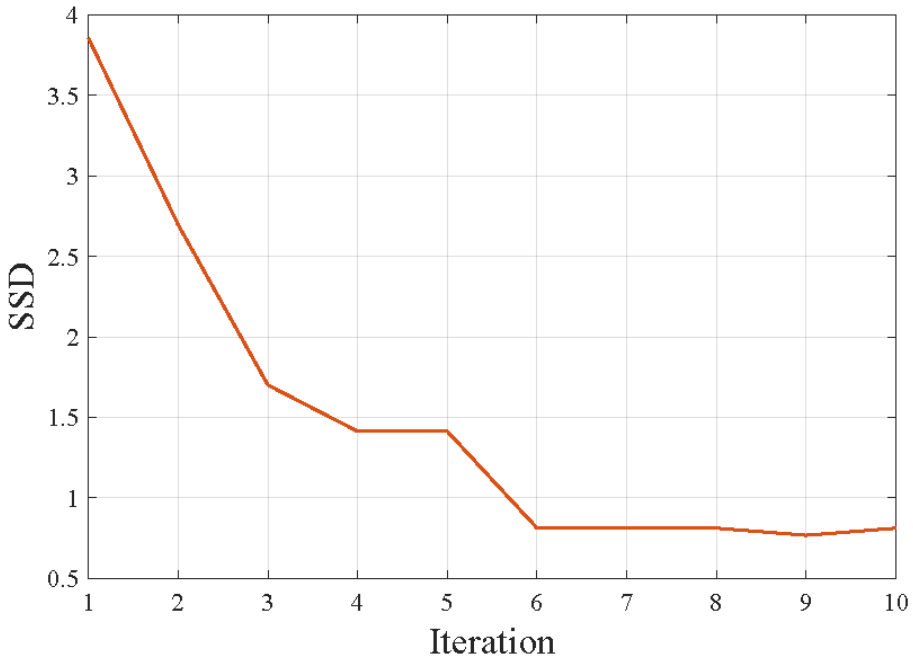
NCC: 0.982435; SSD: 1.700352

6TH ITERATION





NCC: 0.995528; SSD: 0.811369


SSD
Iteration
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2
1.5
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0.5
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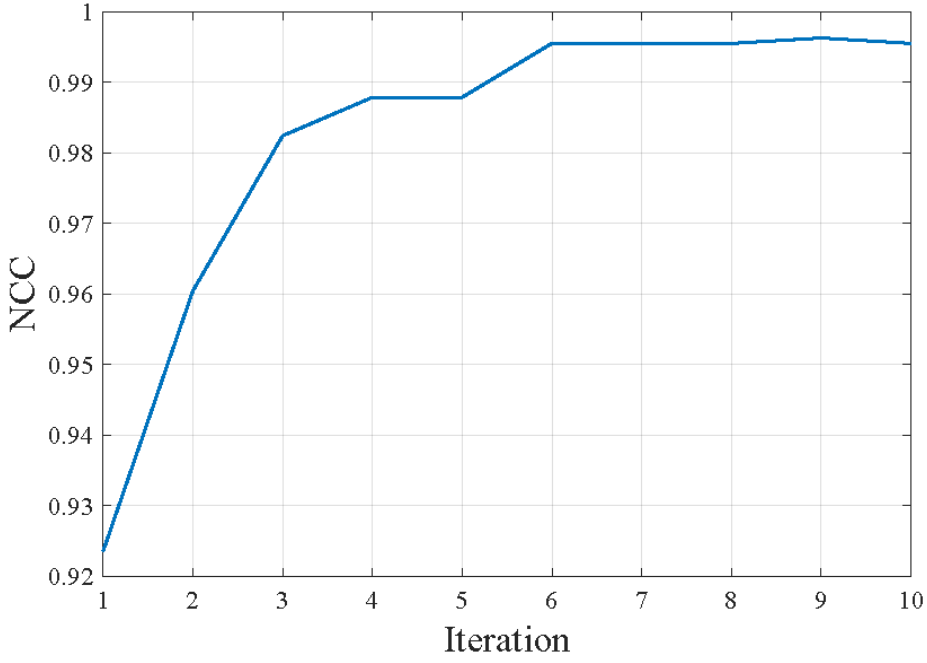
2
3
4
5
6
7
8
9
10


NCC
Iteration
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0.98
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0.94
0.93
0.92
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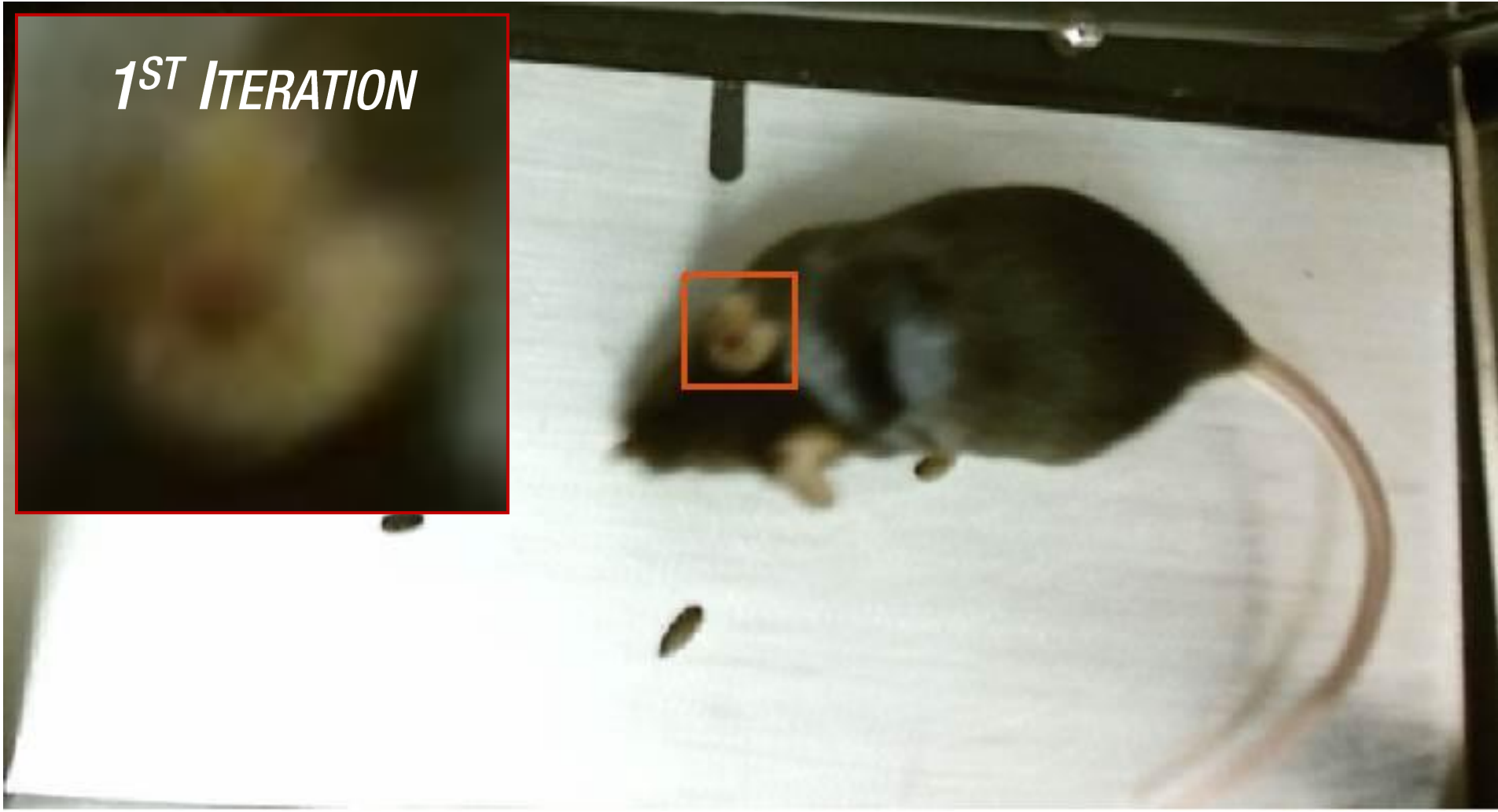
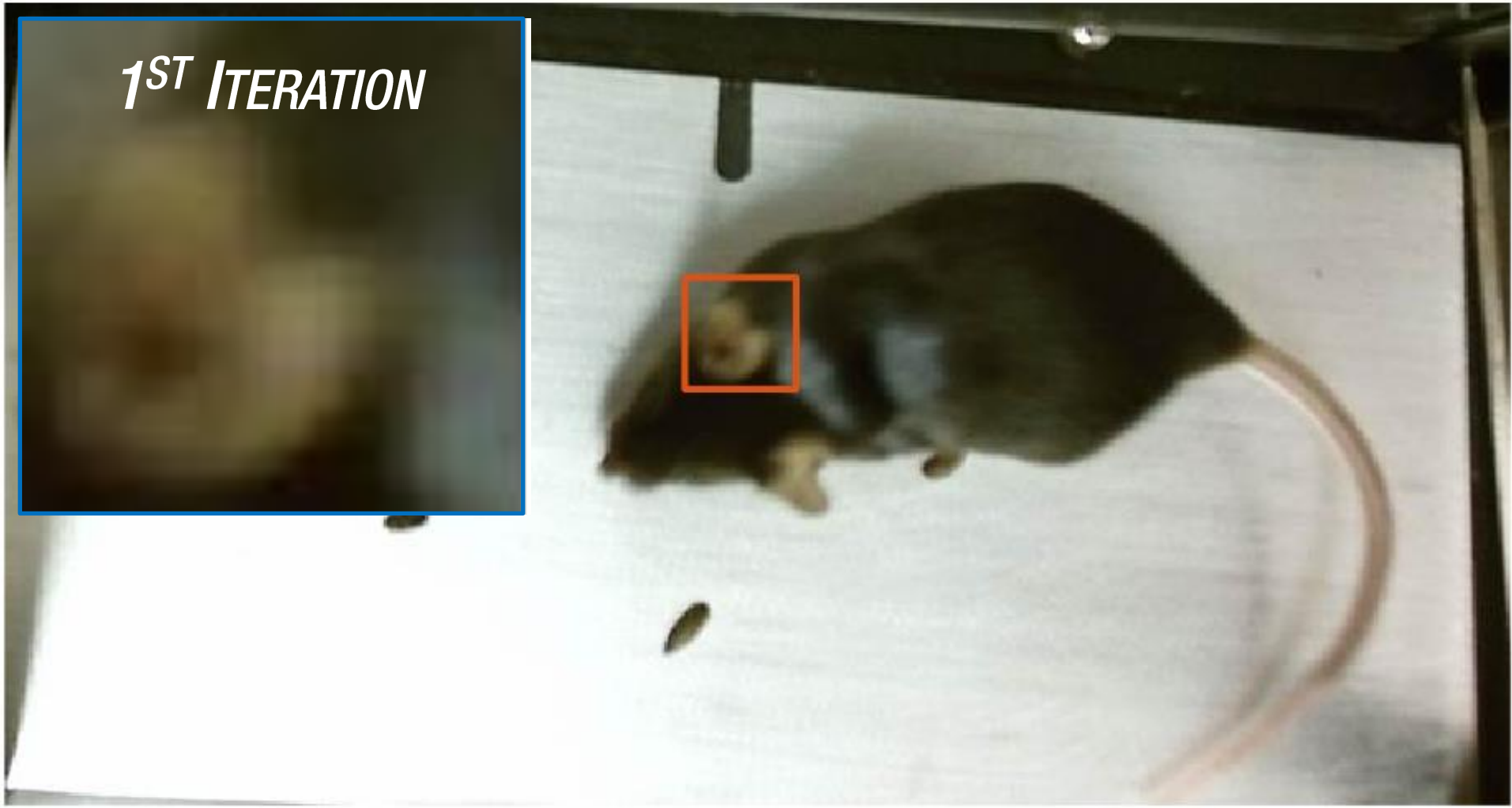
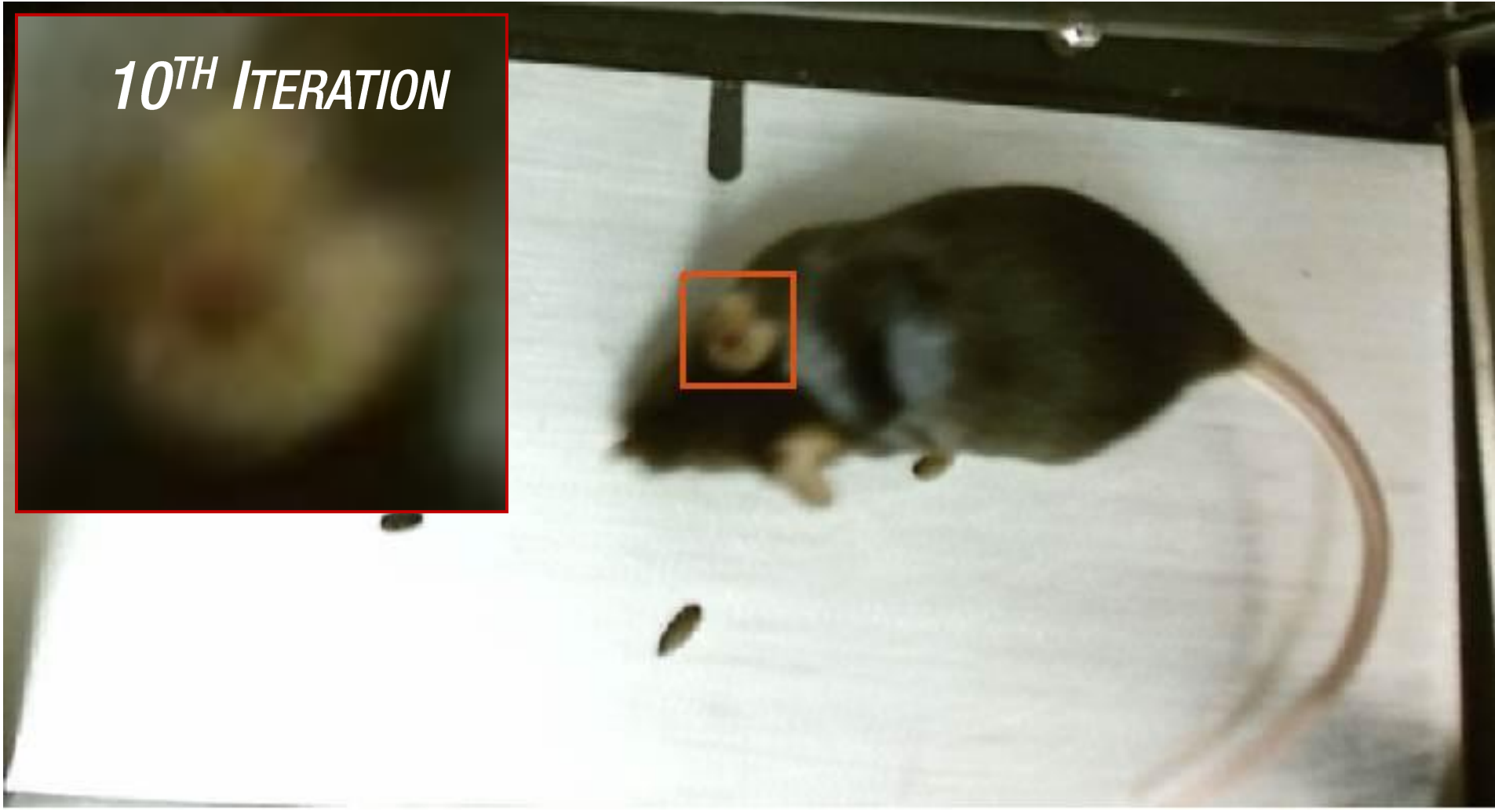
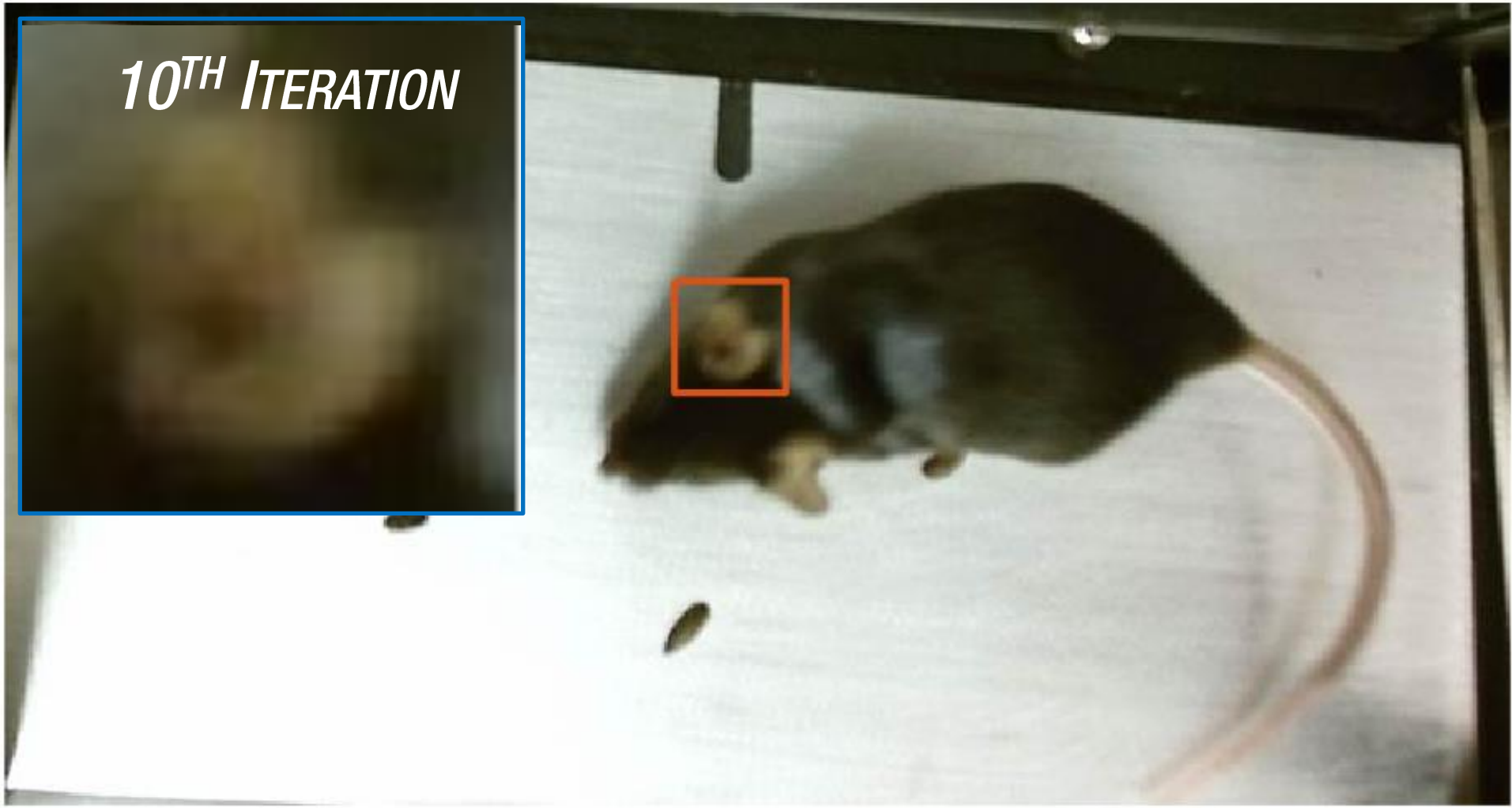
2
3
4
5
6
7
8
9
10


1ST ITERATION


1ST ITERATION


10TH ITERATION


10TH ITERATION