

The background features a grey and white checkered floor pattern that recedes into the distance. Overlaid on this are several large, expressive brushstrokes in various colors: a prominent blue stroke at the top, a green stroke to the right, a purple stroke in the center, a red stroke below it, and a thick yellow-green stroke at the bottom. The overall aesthetic is modern and artistic.

GOOD FEATURE TO TRACK

HYUN SOO PARK

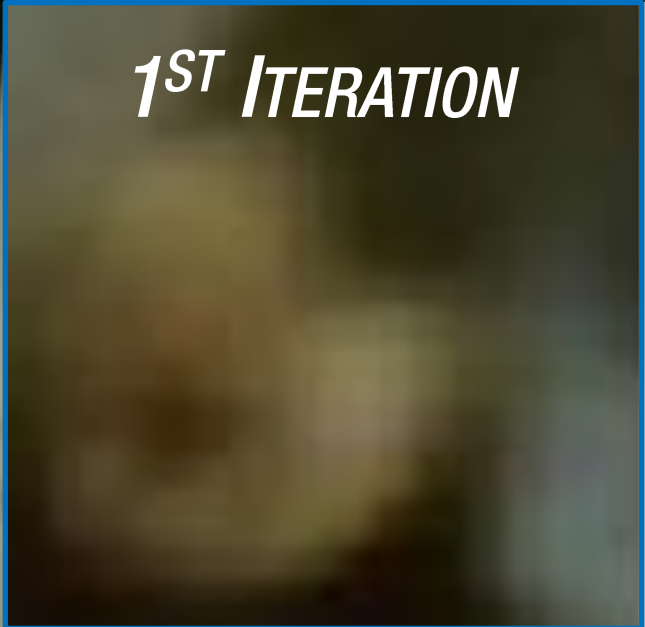
1ST ITERATION



$$x = (A^T A)^{-1} A^T b \quad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

$I(:, :, t)$

1ST ITERATION



$I(:, :, t')$

10TH ITERATION



$$x = (A^T A)^{-1} A^T b \quad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

$I(:, :, t)$

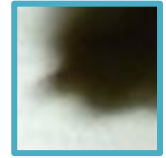
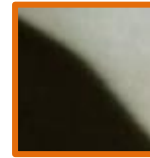
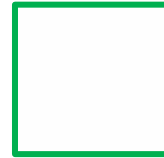
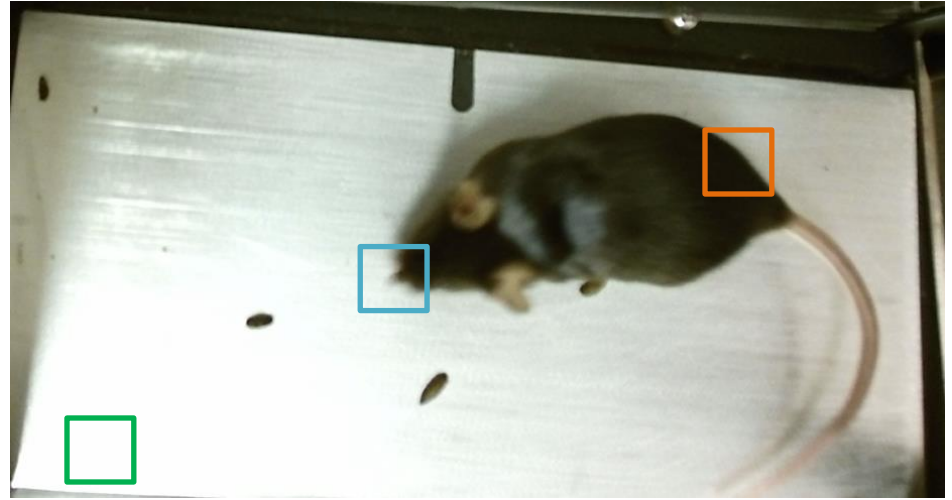
10TH ITERATION



$I(:, :, t')$

CONDITION NUMBER

Some patches are better tracked than others.



SOLVABILITY

$$\underbrace{\begin{bmatrix} I_{x|1} & I_{y|1} \\ \vdots & \vdots \\ I_{x|n} & I_{y|n} \end{bmatrix} A}_{nx2} \underbrace{\begin{bmatrix} u \\ x \\ v \end{bmatrix}}_{\text{Unknowns}} = \underbrace{\begin{bmatrix} I_{t|1} \\ \vdots \\ I_{t|n} \end{bmatrix} b}_{nx1}$$

$$\underline{x = (A^T A)^{-1} A^T b}$$

Least squares solution

SOLVABILITY

$$\underbrace{\begin{bmatrix} I_{x|1} & I_{y|1} \\ \vdots & \vdots \\ I_{x|n} & I_{y|n} \end{bmatrix}}_{n \times 2} \mathbf{A} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{\text{Unknowns}} = \underbrace{\begin{bmatrix} I_{t|1} \\ \vdots \\ I_{t|n} \end{bmatrix}}_{n \times 1} \mathbf{b}$$

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}$$

Least squares solution

Solvable if the inverse exists:

$$\mathbf{A}^T \mathbf{A} = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

SOLVABILITY

$$\begin{array}{c} \begin{array}{|c|c|} \hline I_x |_1 & I_y |_1 \\ \hline \vdots & \vdots \\ \hline I_x |_n & I_y |_n \\ \hline \end{array} \mathbf{A} \begin{array}{|c|} \hline u \\ \hline v \\ \hline \end{array} \mathbf{x} = \begin{array}{|c|} \hline I_t |_1 \\ \hline \vdots \\ \hline I_t |_n \\ \hline \end{array} \mathbf{b}$$

$$\begin{array}{c} \text{---} \\ \text{nx2} \\ \text{---} \end{array} \qquad \begin{array}{c} \text{---} \\ \text{nx1} \\ \text{---} \end{array}$$

Unknowns

$$\mathbf{x} = \left(\mathbf{A}^T \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{b}$$

Least squares solution

Numerical stability \sim condition number

$$\mathbf{A}^T \mathbf{A} = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

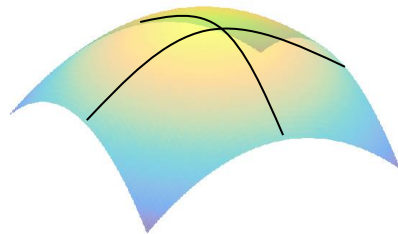
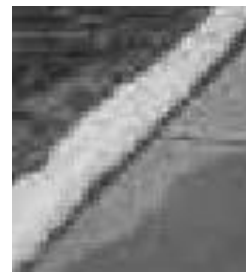
CONDITION NUMBER

$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

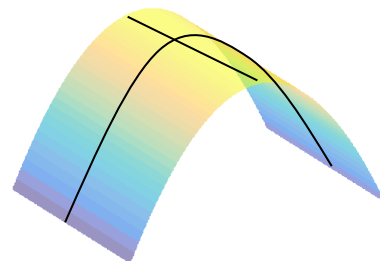
Condition number of matrix: $\frac{\lambda_1}{\lambda_2}$

where λ_1, λ_2 are singular values of $A^T A$
and $\lambda_1 \geq \lambda_2$

RECALL: EDGE THRESHOLDING



$$\lambda_1 \approx \lambda_2$$



$$\lambda_1 \gg \lambda_2$$

Principal curvatures are eigenvalues of Hessian matrix:

$$H = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix}$$

CONDITION NUMBER

$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

Condition number of matrix: $\frac{\lambda_1}{\lambda_2}$

where λ_1, λ_2 are eigenvalues of $A^T A$
and $\lambda_1 \geq \lambda_2$

$Ax = b$ is well-conditioned if $\frac{\lambda_1}{\lambda_2} \approx 1$

CONDITION NUMBER

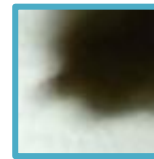
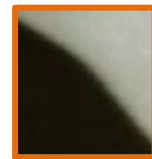
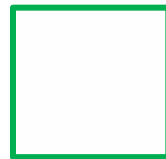
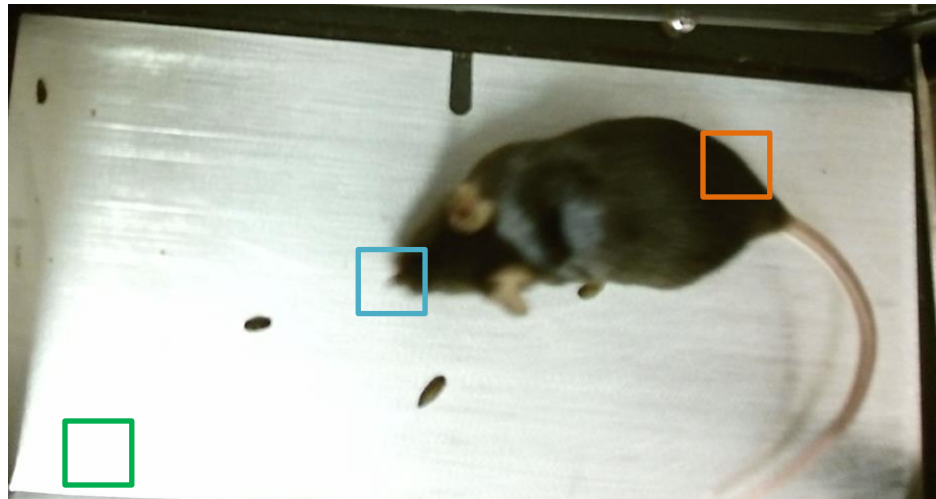
$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

Condition number of matrix: $\frac{\lambda_1}{\lambda_2}$

where λ_1, λ_2 are eigenvalues of $A^T A$
and $\lambda_1 \geq \lambda_2$

$Ax = b$ is well-conditioned if $\frac{\lambda_1}{\lambda_2} \approx 1$

Some patches are better tracked than others.



CONDITION NUMBER

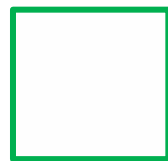
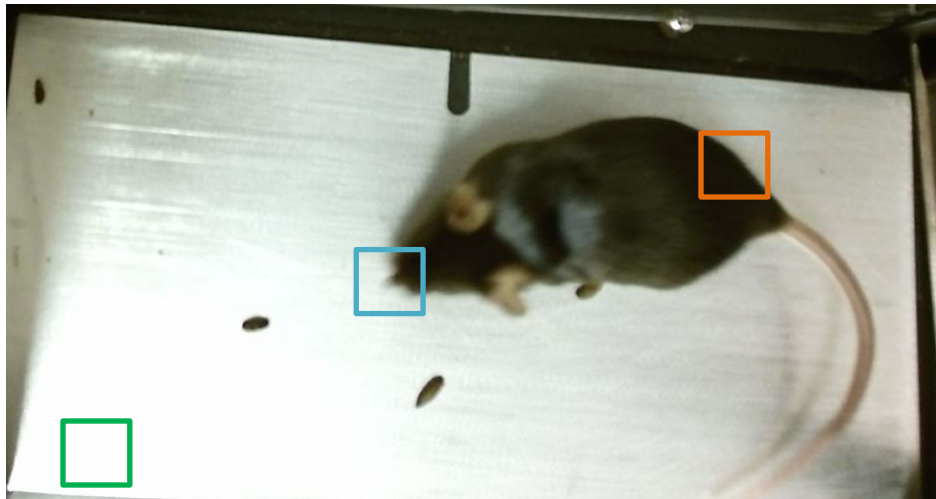
$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

Condition number of matrix: $\frac{\lambda_1}{\lambda_2}$

where λ_1, λ_2 are eigenvalues of $A^T A$
and $\lambda_1 \geq \lambda_2$

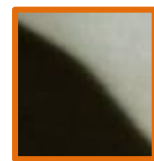
$Ax = b$ is well-conditioned if $\frac{\lambda_1}{\lambda_2} \approx 1$

Some patches are better tracked than others.



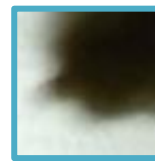
$$\lambda_2 \approx 0$$

\gg



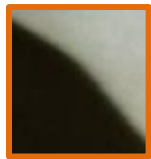
$$\lambda_1 \gg \lambda_2$$

$>$

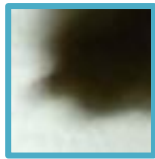


$$\lambda_1 \approx \lambda_2$$

APERTURE PROBLEM



>



$$\lambda_1 \gg \lambda_2$$

$$\lambda_1 \approx \lambda_2$$

There exists an approximate nullspace z , i.e.,

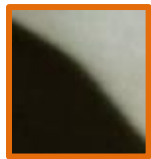
$$Az = 0$$

$$A(x + z) = b \quad A = \begin{bmatrix} I_x & I_y \end{bmatrix}$$

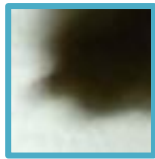
$$\begin{bmatrix} I_x \\ I_y \end{bmatrix} \perp z$$

z is perpendicular to the image gradient.

APERTURE PROBLEM



>



$$\lambda_1 \gg \lambda_2$$

$$\lambda_1 \approx \lambda_2$$

There exists an approximate nullspace z , i.e.,

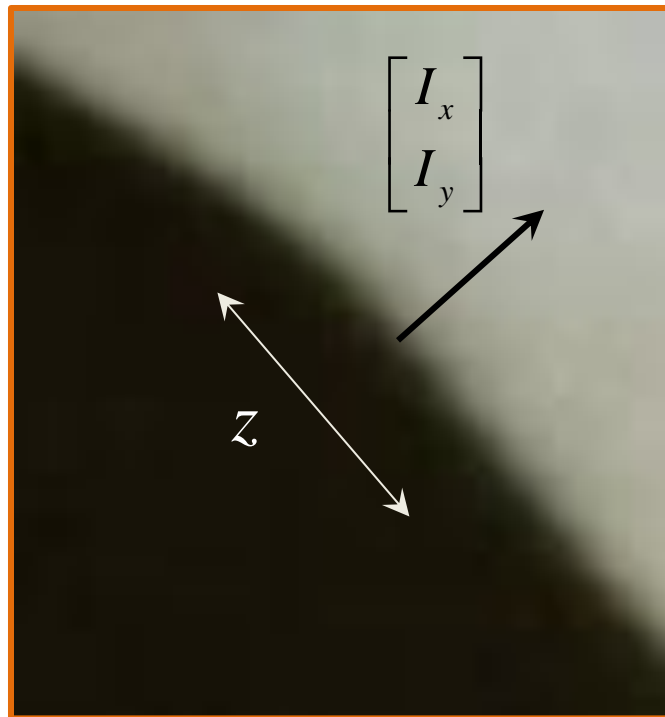
$$Az = 0$$

$$A(x + z) = b \quad A = \begin{bmatrix} I_x & I_y \end{bmatrix}$$

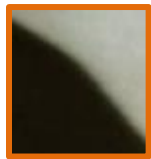
$$\begin{bmatrix} I_x \\ I_y \end{bmatrix} \perp z$$

z is perpendicular to the image gradient.

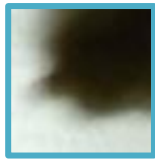
Any motion perpendicular to the dominant image gradient cannot be recovered.



APERTURE PROBLEM



>



$$\lambda_1 \gg \lambda_2$$

$$\lambda_1 \approx \lambda_2$$

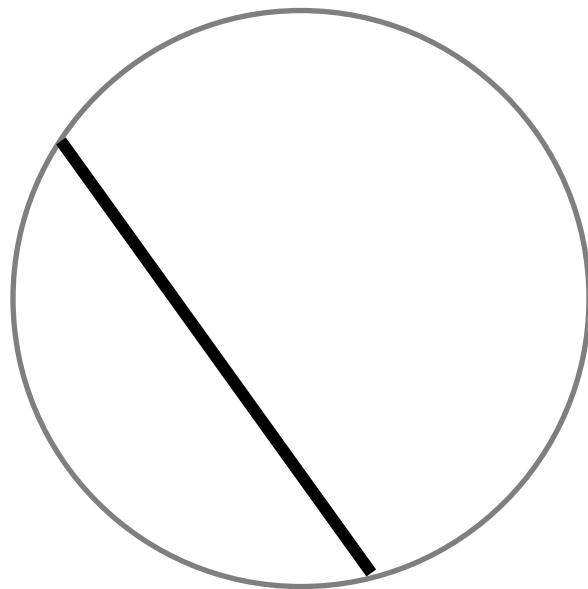
There exists an approximate nullspace z , i.e.,

$$Az = 0$$

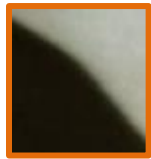
$$A(x + z) = b$$

$$\begin{bmatrix} I_x \\ I_y \end{bmatrix} \perp z$$

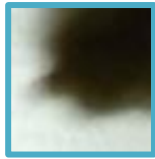
z is perpendicular to the image gradient.



APERTURE PROBLEM



>



$$\lambda_1 \gg \lambda_2$$

$$\lambda_1 \approx \lambda_2$$

There exists an approximate nullspace z , i.e.,

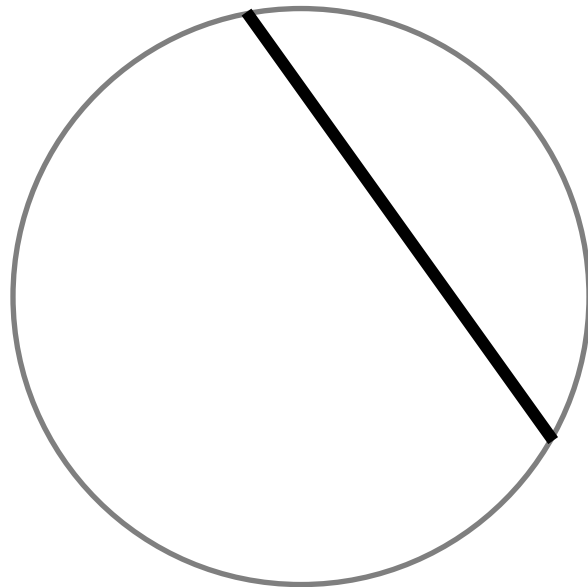
$$Az = 0$$

$$A(x + z) = b$$

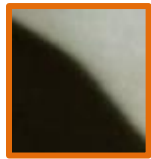
$$\begin{bmatrix} I_x \\ I_y \end{bmatrix} \perp z$$

z is perpendicular to the image gradient.

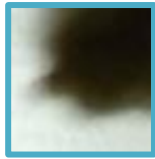
Perceived motion



APERTURE PROBLEM



>



$$\lambda_1 \gg \lambda_2$$

$$\lambda_1 \approx \lambda_2$$

There exists an approximate nullspace z , i.e.,

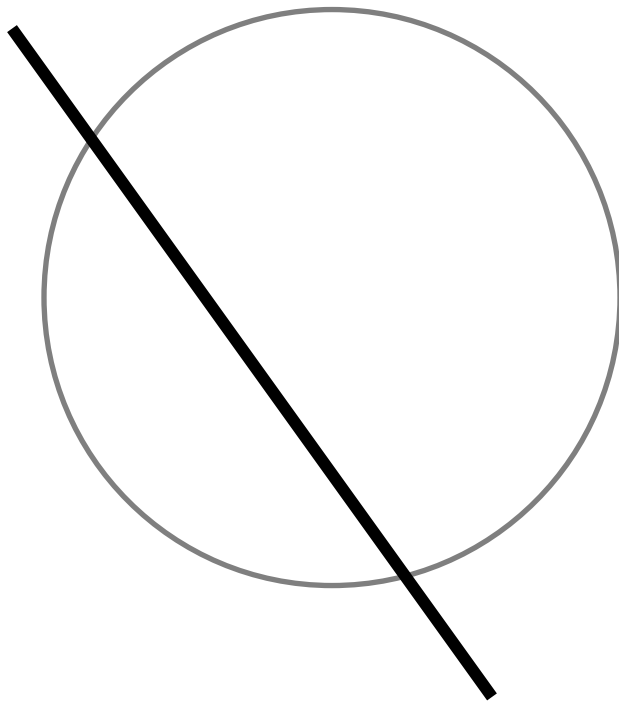
$$Az = 0$$

$$A(x + z) = b$$

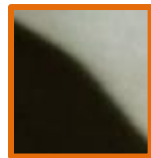
$$\begin{bmatrix} I_x \\ I_y \end{bmatrix} \perp z$$

z is perpendicular to the image gradient.

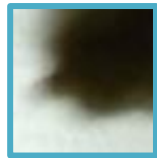
Perceived motion



APERTURE PROBLEM



>



$$\lambda_1 \gg \lambda_2$$

$$\lambda_1 \approx \lambda_2$$

There exists an approximate nullspace z , i.e.,

$$Az = 0$$

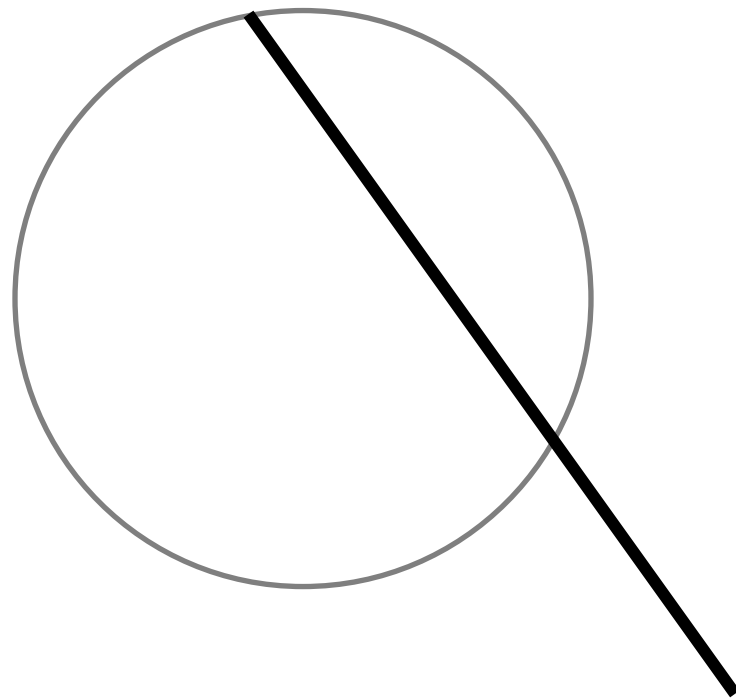
$$A(x + z) = b$$

$$\begin{bmatrix} I_x \\ I_y \end{bmatrix} \perp z$$

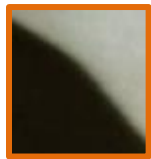
z is perpendicular to the image gradient.

↖
Perceived motion

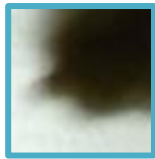
→
Actual motion



APERTURE PROBLEM



>



$$\lambda_1 \gg \lambda_2$$

$$\lambda_1 \approx \lambda_2$$

There exists an approximate nullspace z , i.e.,

$$Az = 0$$

$$A(x + z) = b$$

$$\begin{bmatrix} I_x \\ I_y \end{bmatrix} \perp z$$

z is perpendicular to the image gradient.



GOOD FEATURES TO TRACK



$$\lambda_1 \approx \lambda_2$$

$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

Shi-Tomasi feature criterion:

$$\lambda_{\min} > \lambda_{\text{threshold}}$$

GOOD FEATURES TO TRACK



$$\lambda_1 \approx \lambda_2$$

$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

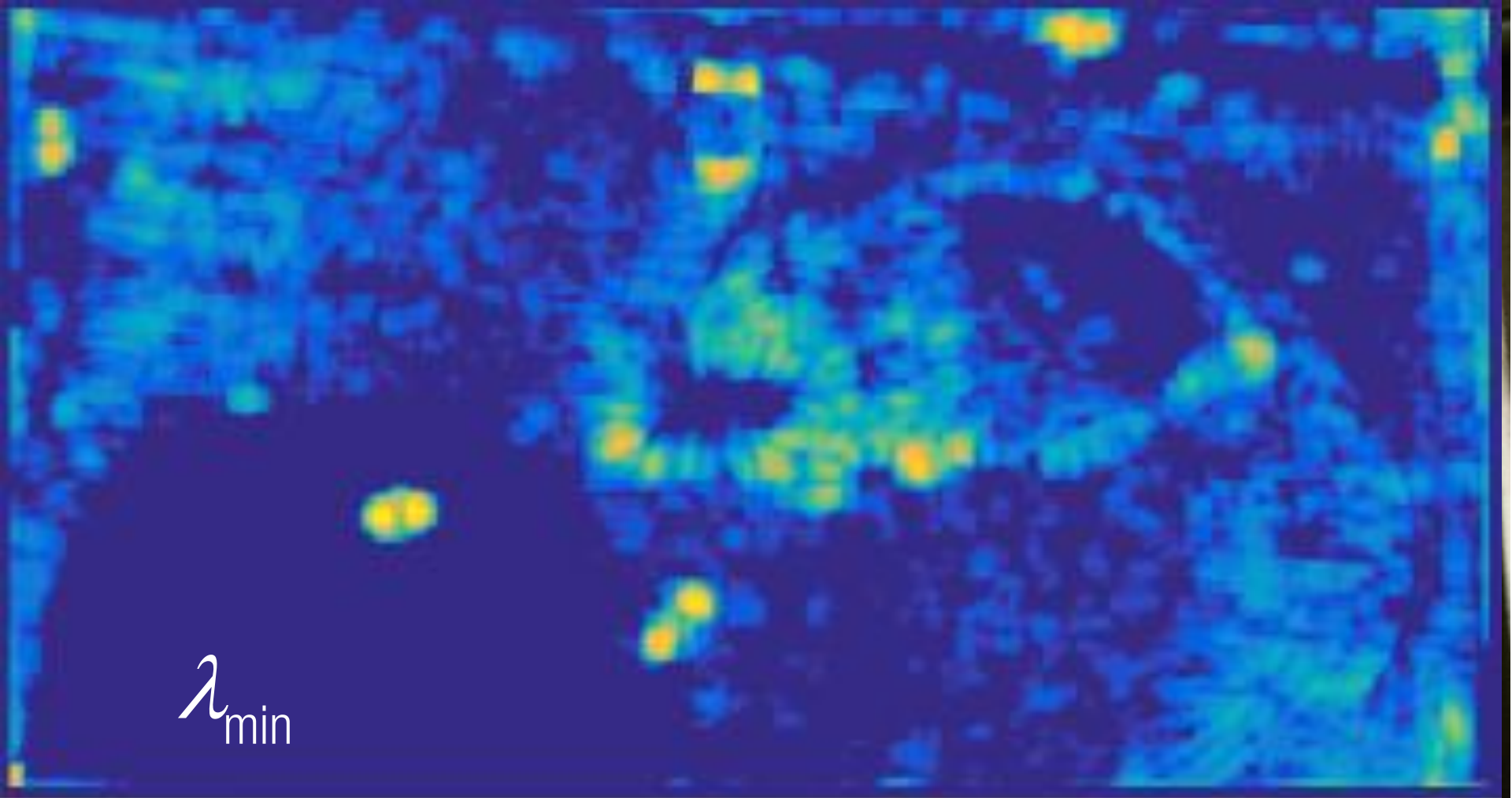
Shi-Tomasi feature criterion:

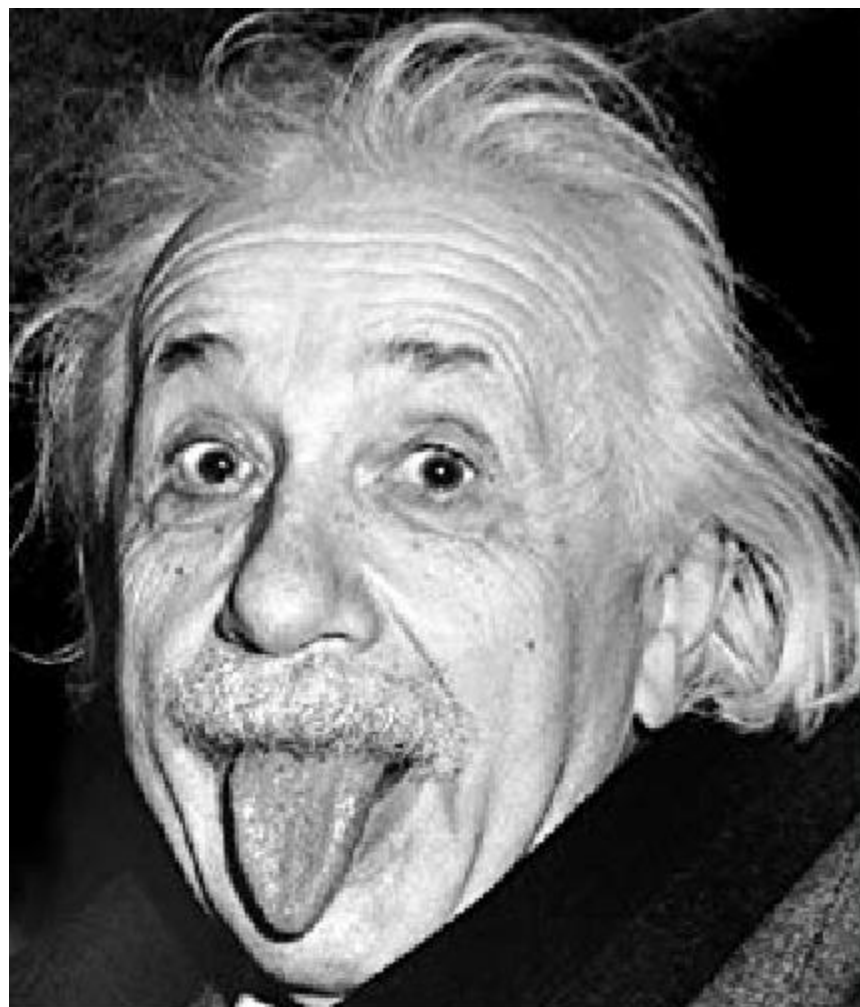
$$\lambda_{\min} > \lambda_{\text{threshold}}$$

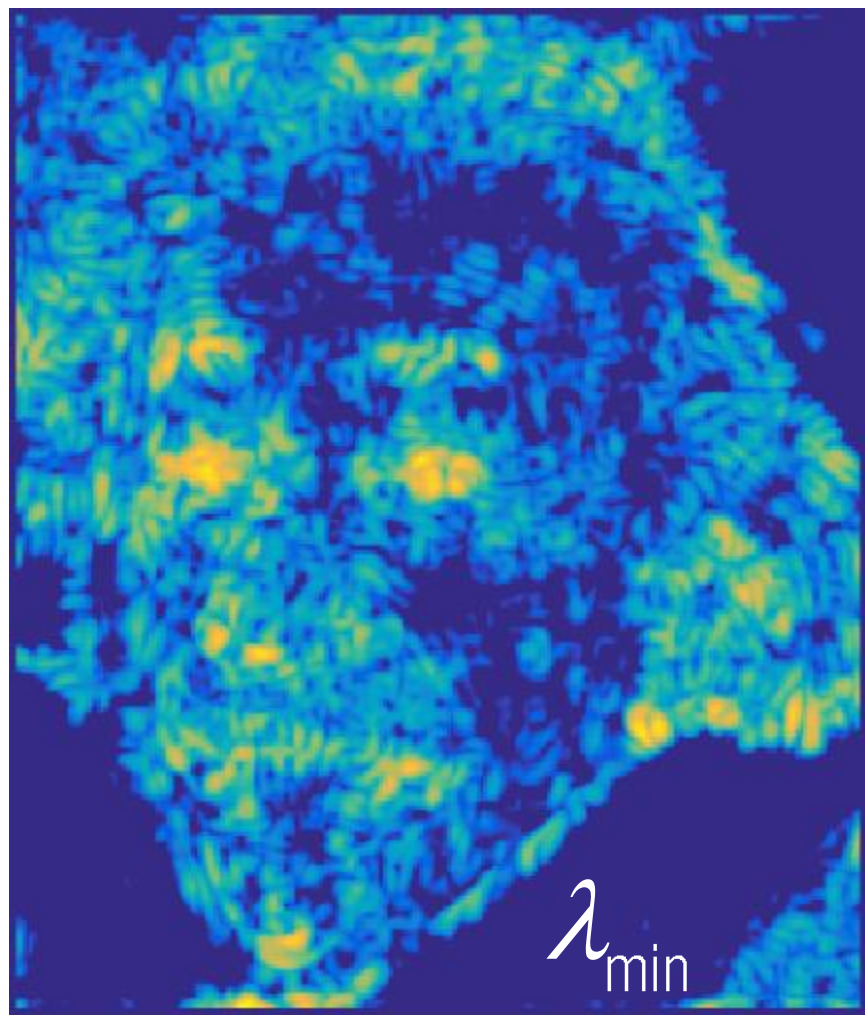
Harris corner criterion:

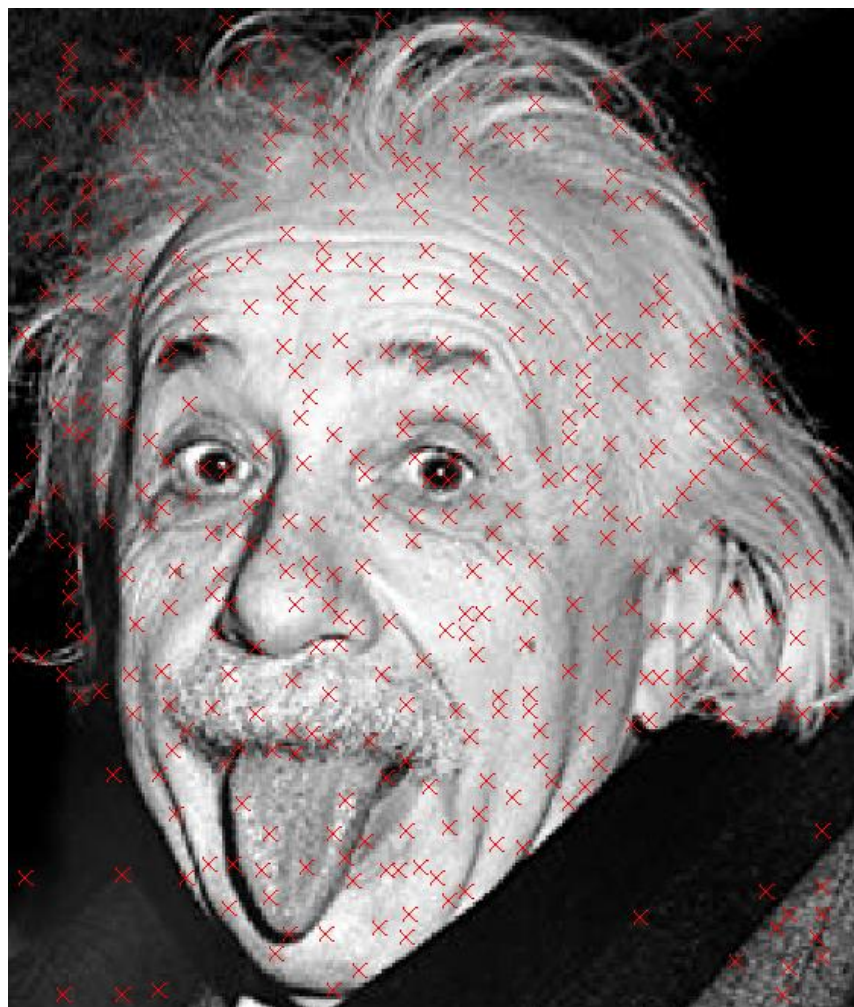
$$\lambda_{\min} \approx \frac{\lambda_{\max} \lambda_{\min}}{(\lambda_{\max} + \lambda_{\min})} = \frac{\det(A^T A)}{\text{trace}(A^T A)} > \lambda_{\text{threshold}}$$











Non-maximum suppression



<https://www.youtube.com/watch?v=JOKTbR-sd6s>