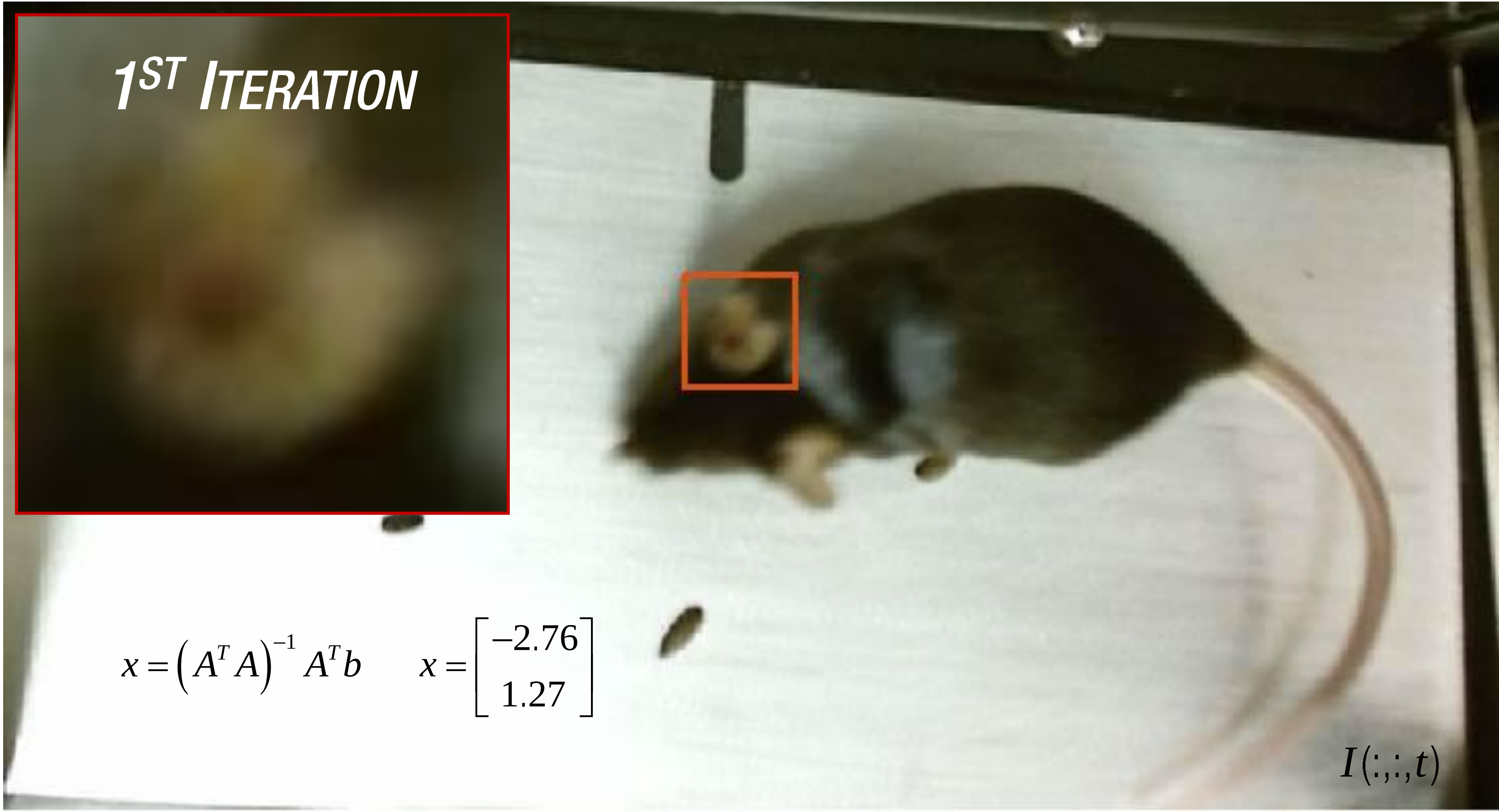
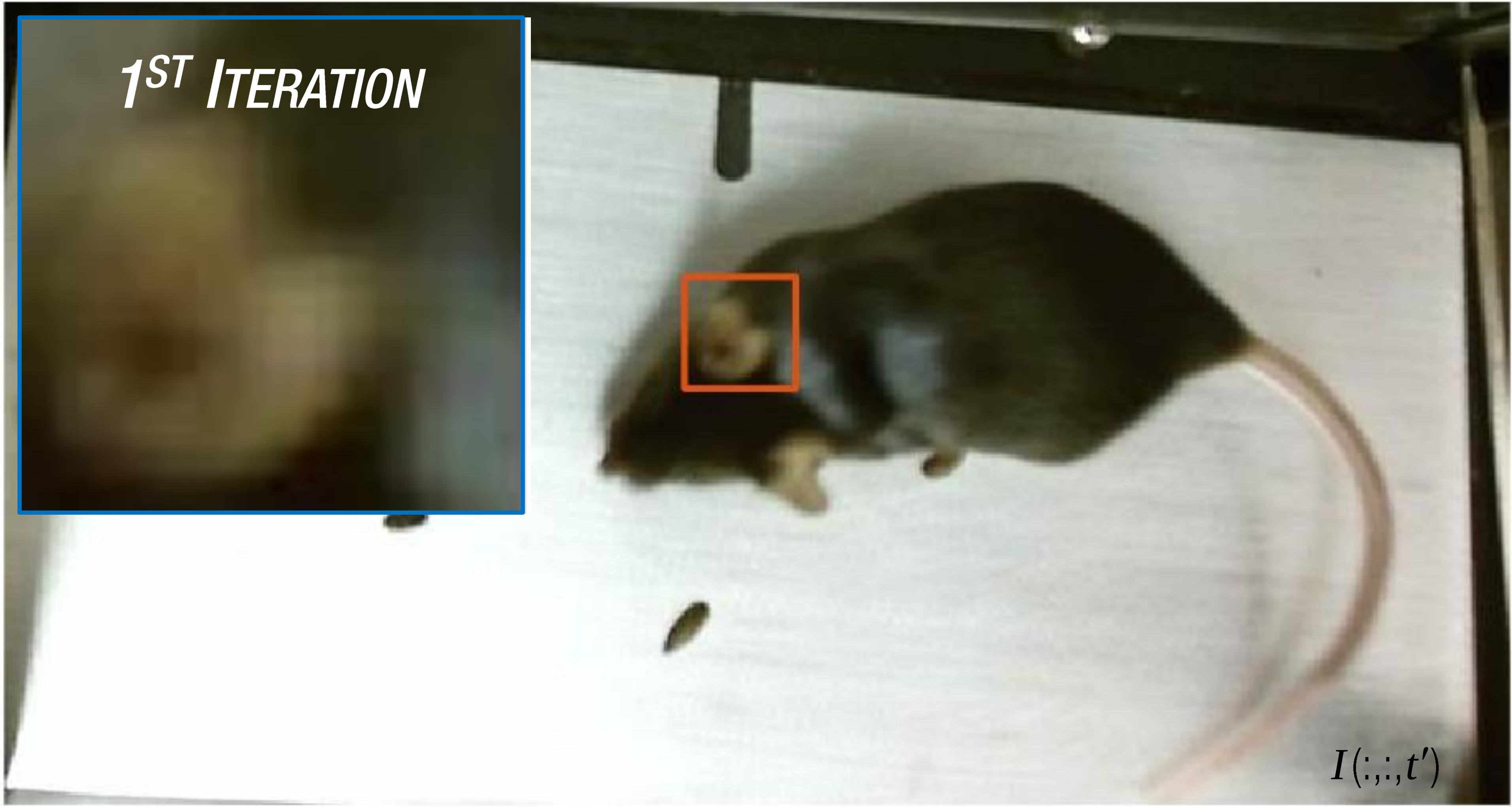
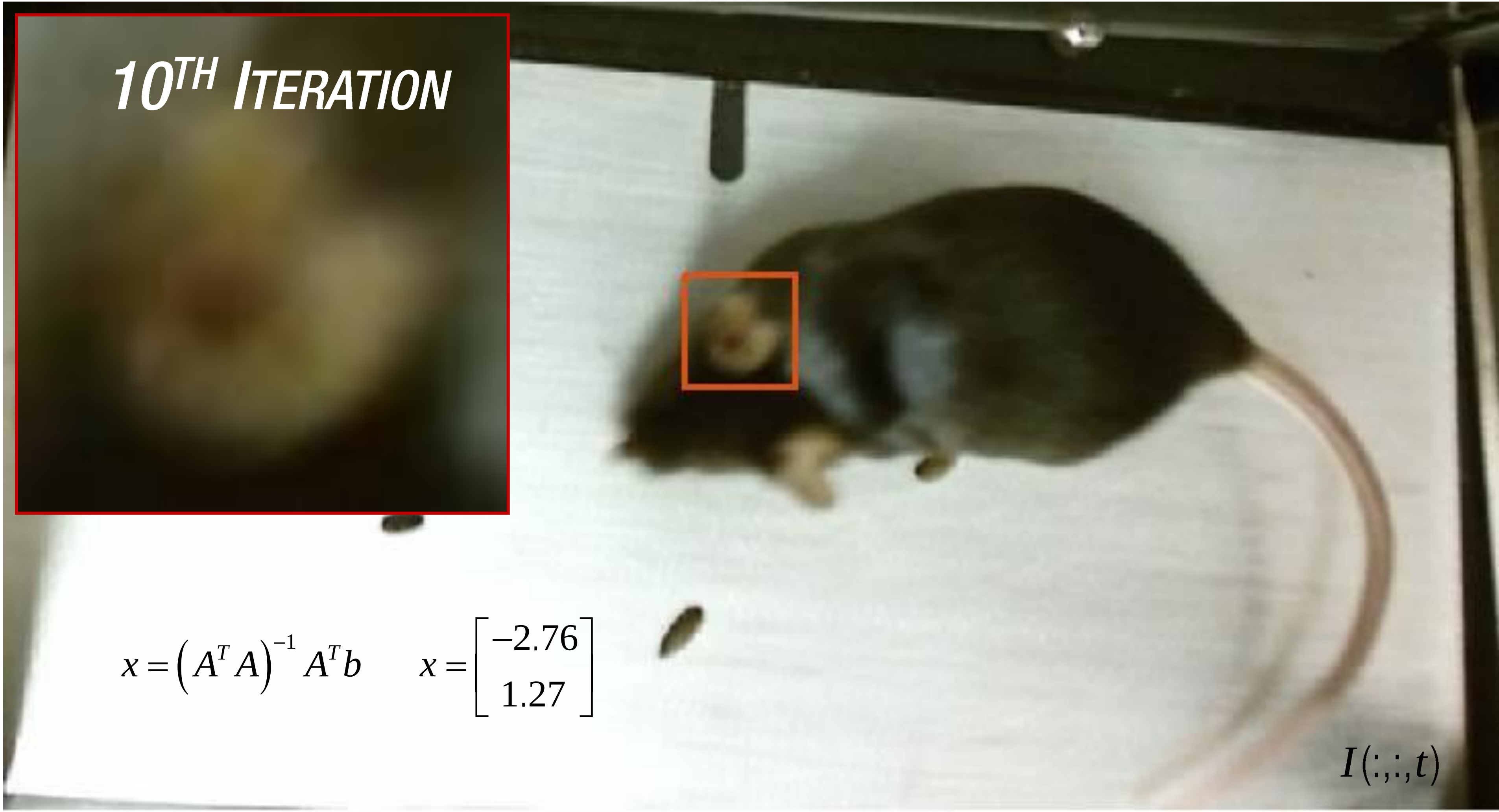
Good Feature To Track

Hyun Soo Park


1ST ITERATION
$x = \left(A^T A\right)^{-1} A^T b$
$x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$
$I(:,:,t)$


1ST ITERATION
$I(:,:,t')$


10TH ITERATION
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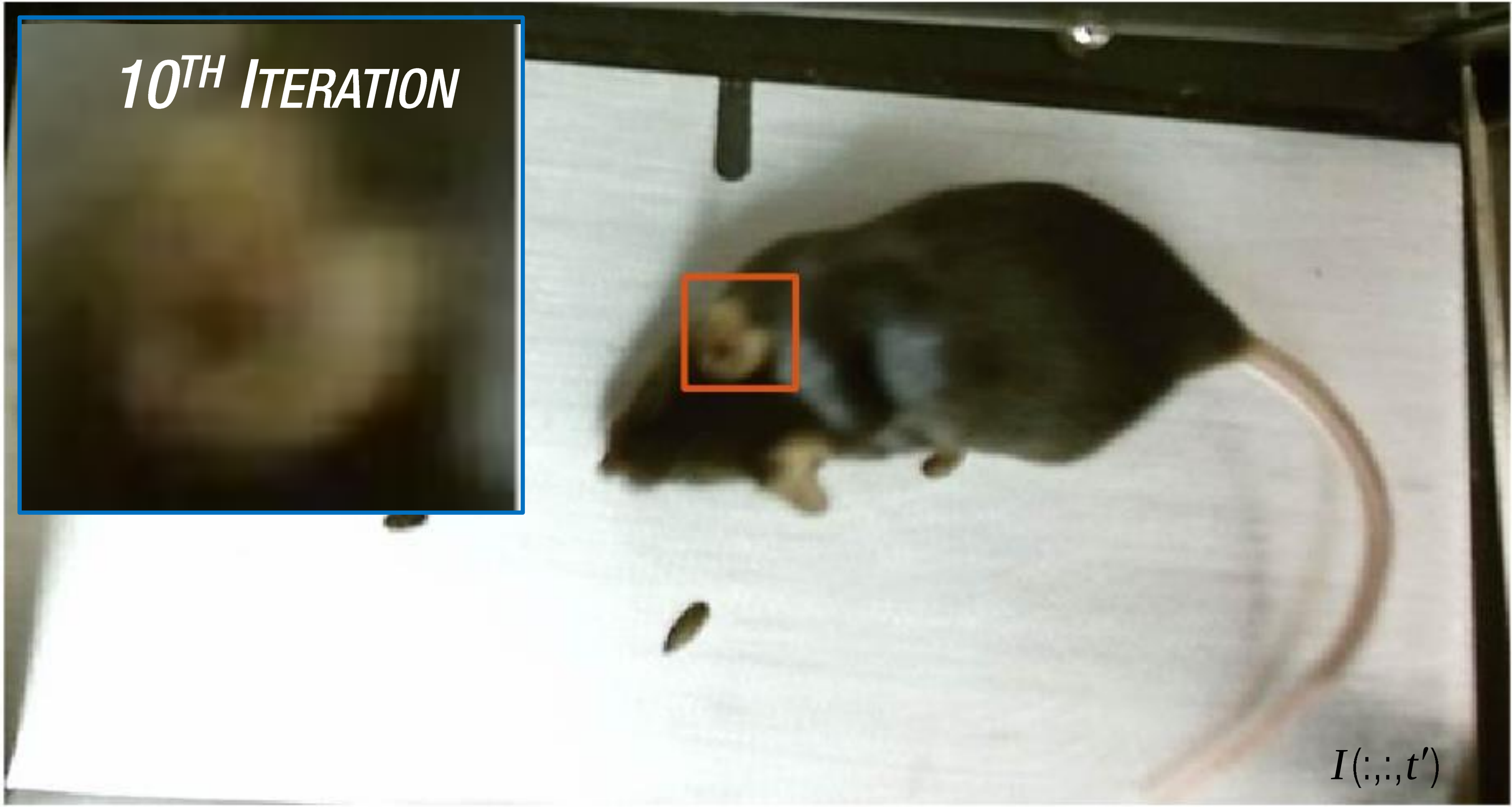
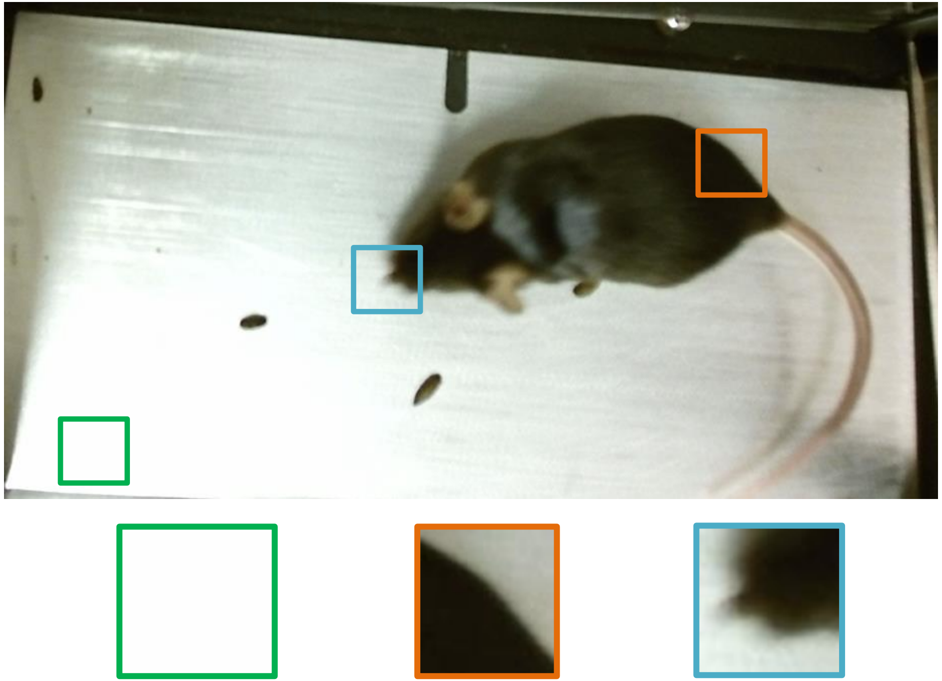

10TH ITERATION
$I(:,:,t')$

Condition Number

Some patches are better tracked than others.



Solvability

$$\underbrace{\begin{bmatrix} I_x|_1 & I_y|_1 \\ \vdots & \vdots \\ I_x|_n & I_y|_n \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{x} = -\underbrace{\begin{bmatrix} I_t|_1 \\ \vdots \\ I_t|_n \end{bmatrix}}_{b}$$

nx2

Unknowns

nx1

$$x = \left(A^T A\right)^{-1} A^T b$$

Least squares solution

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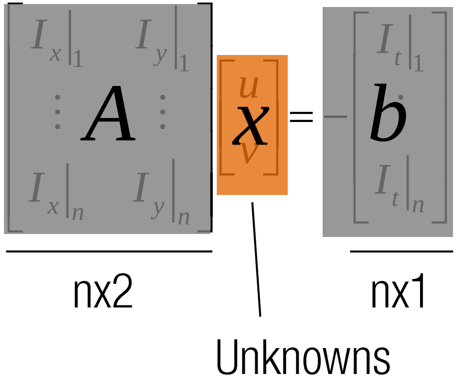
Least squares solution

Solvable if the inverse exists:

$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

Solvability



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Least squares solution

Numerical stability ~ condition number

$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

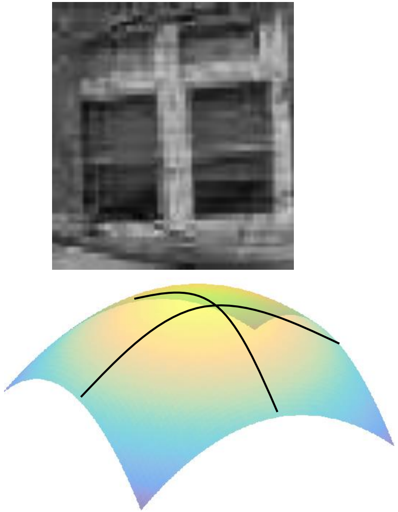
Condition Number

$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

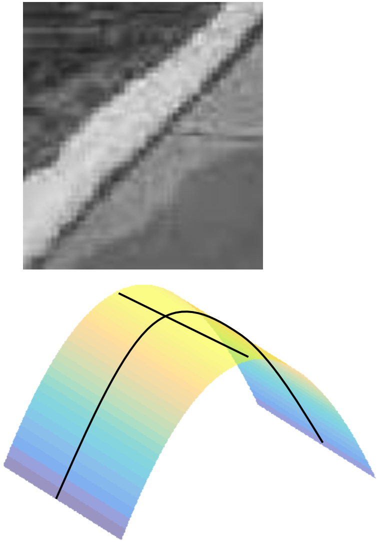
Condition number of matrix: $\frac{\lambda_1}{\lambda_2}$

where λ_1, λ_2 are singular values of $A^T A$
and $\lambda_1 \geq \lambda_2$

Recall: Edge Thresholding



$\lambda_1 \approx \lambda_2$



$\lambda_1 \gg \lambda_2$

Principal curvatures are eigen values of Hessian matrix:

$$H = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix}$$

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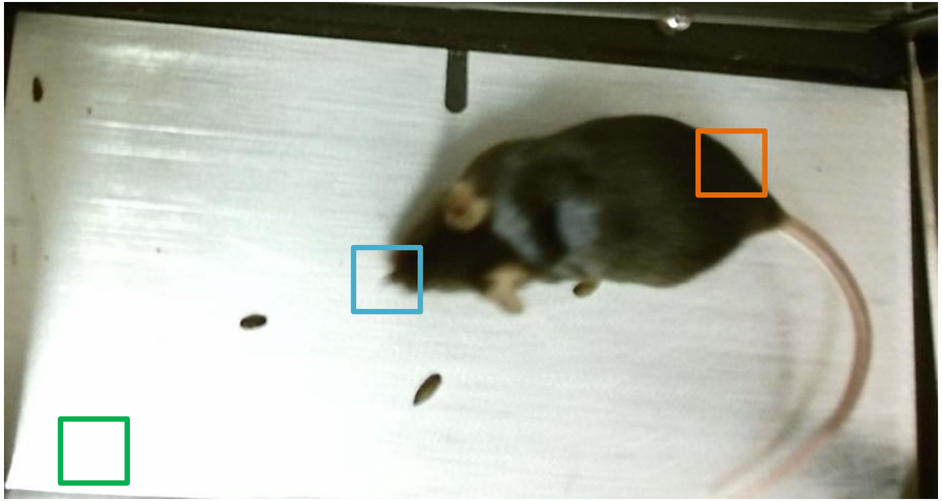

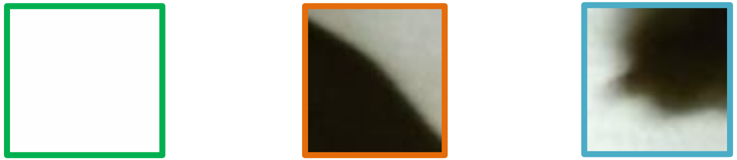

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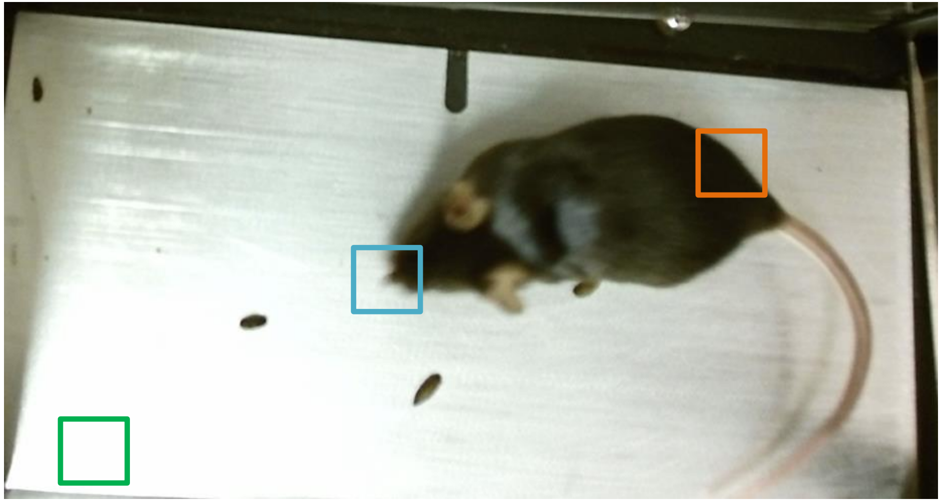
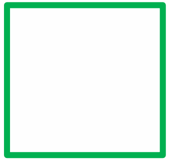
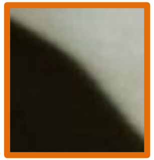




$\gg$



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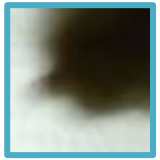
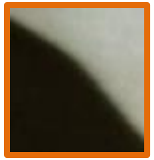


$\lambda_2 \approx 0$ $\lambda_1 \gg \lambda_2$ $\lambda_1 \approx \lambda_2$

Aperture Problem



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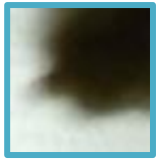


$\lambda_1 \gg \lambda_2$ $\lambda_1 \approx \lambda_2$

There exists an approximate nullspace *z*, i.e.,

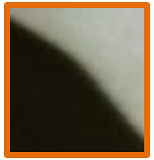
$$Az = 0$$

$$A(x+z) = b \qquad A = \begin{bmatrix} I_x & I_y \end{bmatrix}$$

$$\begin{bmatrix} I_x \\ I_y \end{bmatrix} \perp z$$

z is perpendicular to the image gradient.

Aperture Problem



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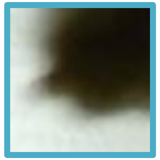

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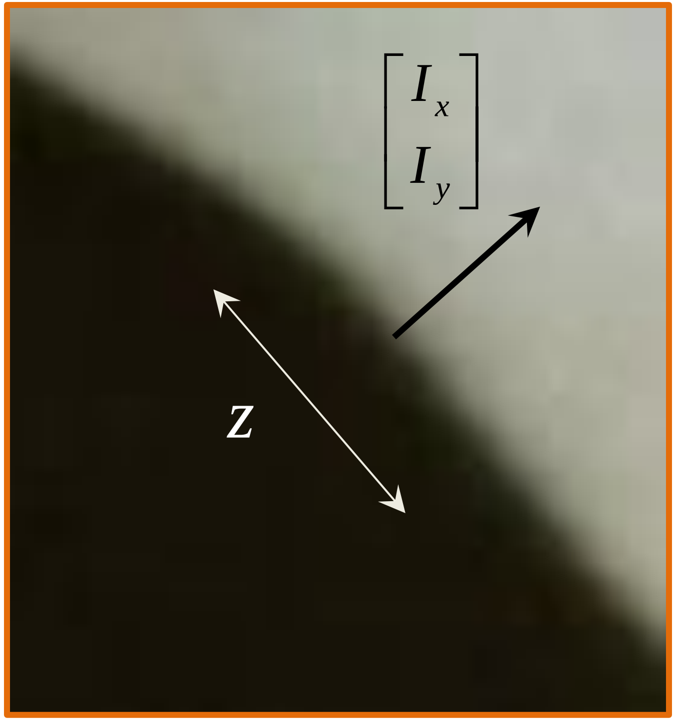
Any motion perpendicular to the dominant image gradient cannot be recovered.



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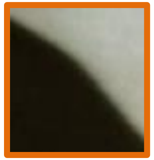

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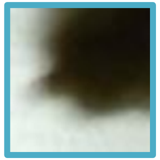

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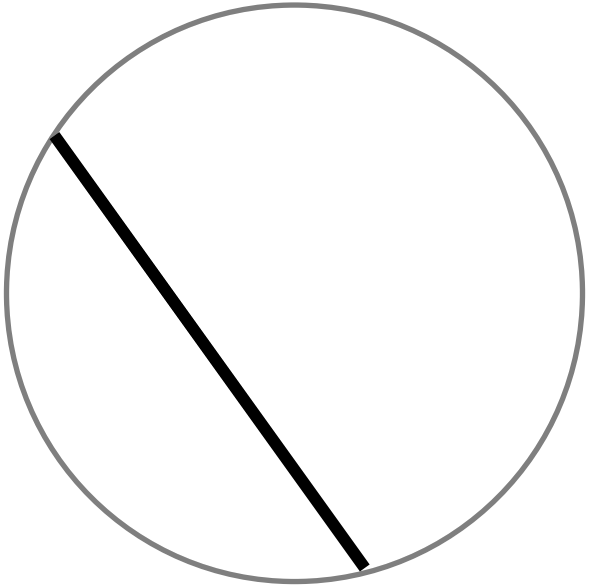

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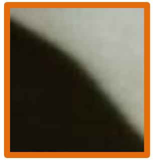
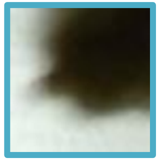
 $>$ 

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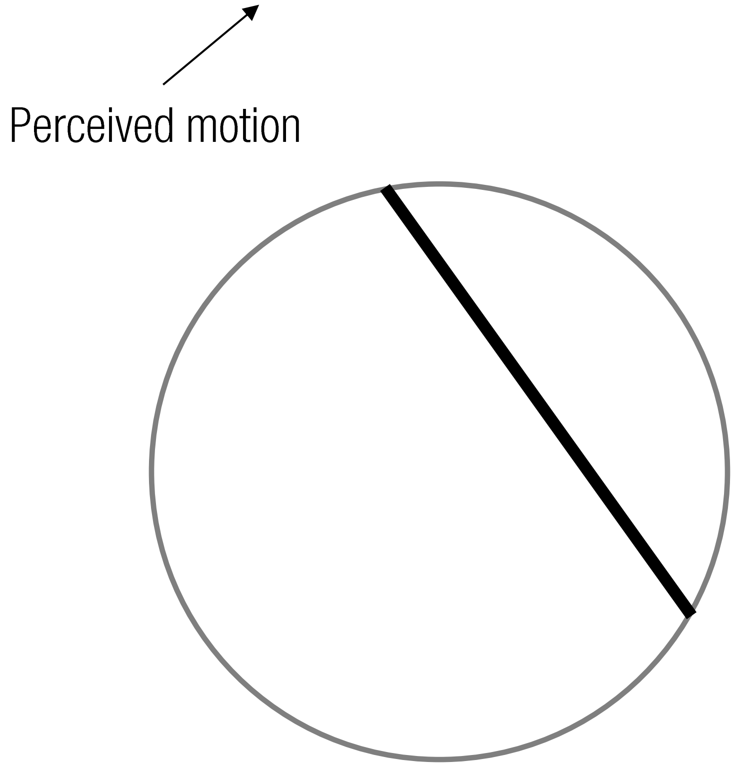

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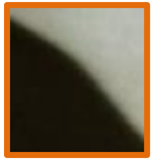

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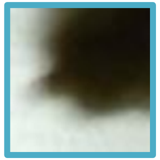

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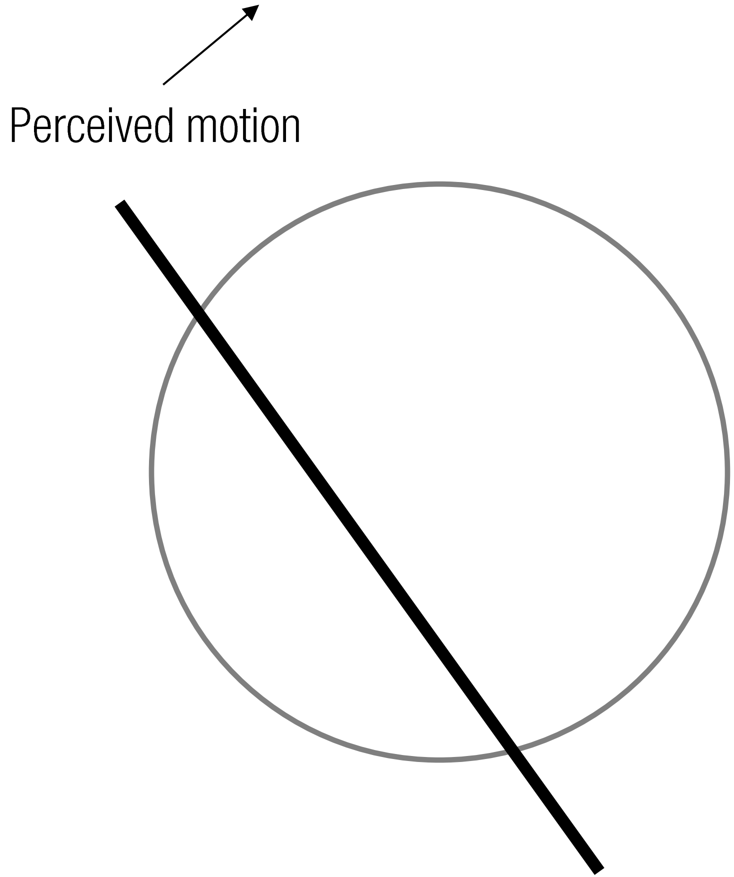

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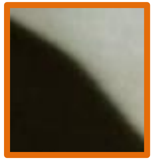

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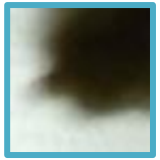

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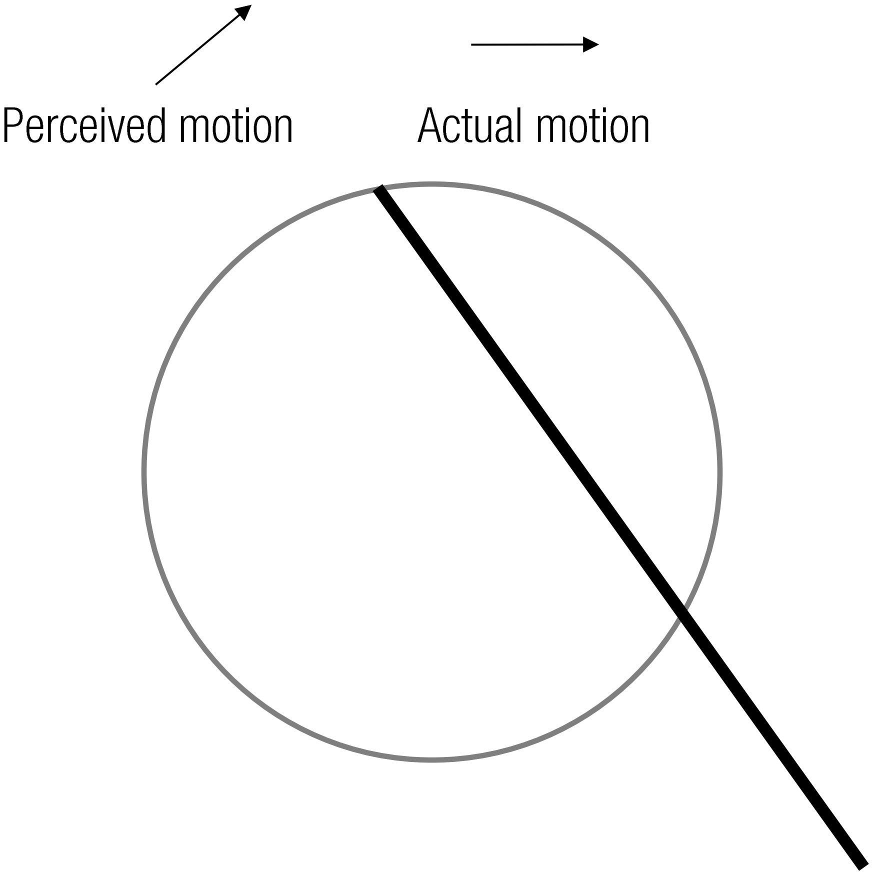

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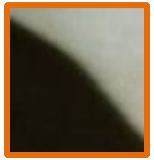

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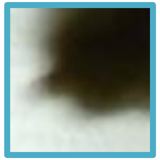

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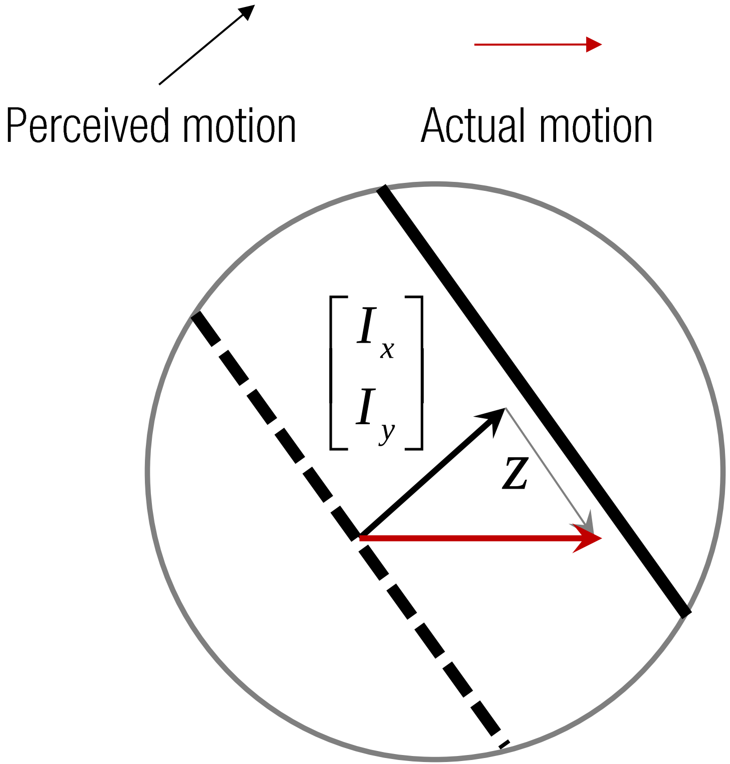

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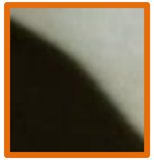

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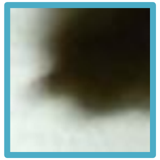

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Shi-Tomasi feature criterion:

$$\lambda_{\text{min}} > \lambda_{threshold}$$

Good Features To Track

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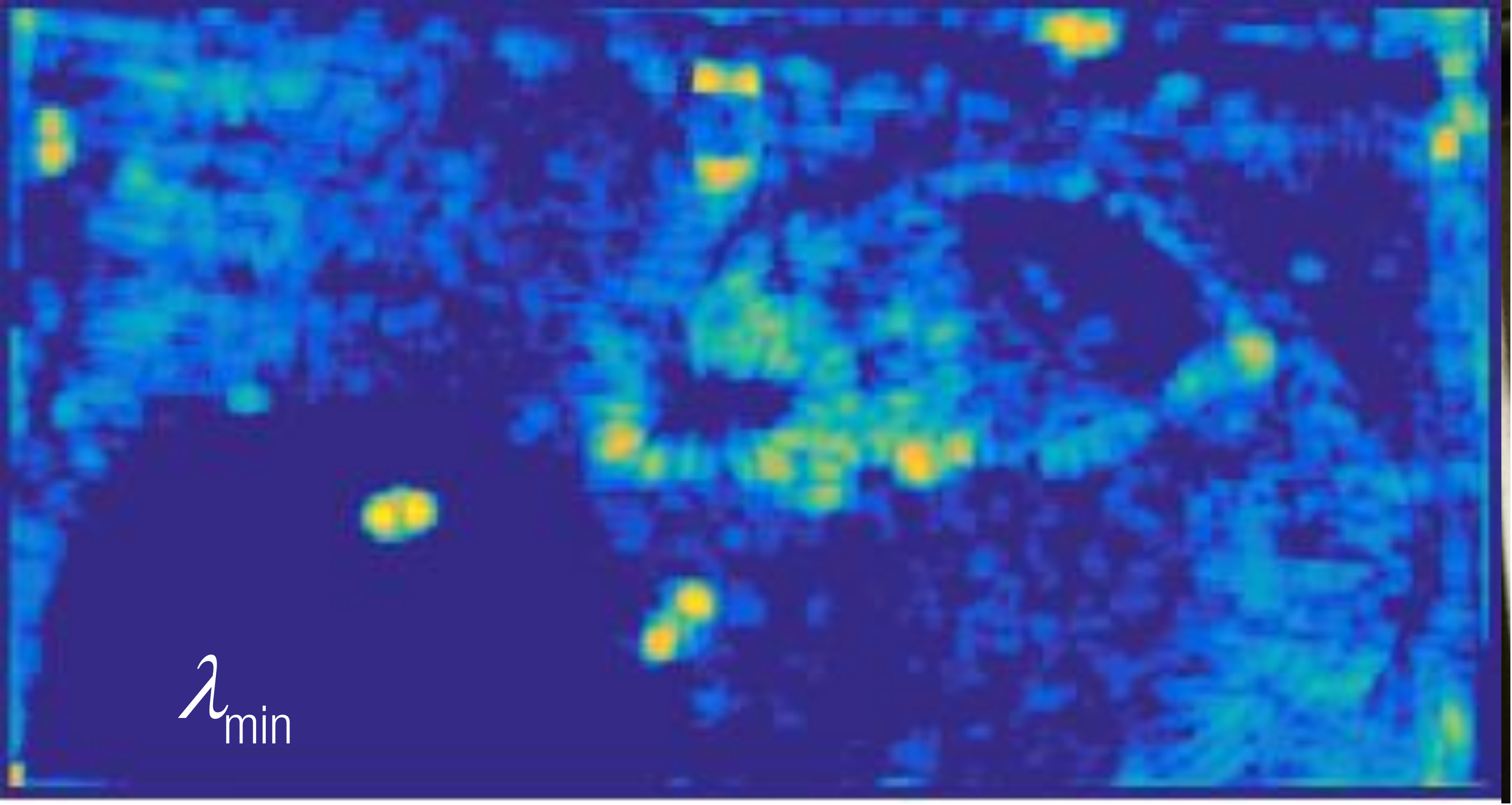
Shi-Tomasi feature criterion:

$$\lambda_{\min} > \lambda_{threshold}$$

Harris corner criterion:

$$\lambda_{\min} \approx \frac{\lambda_{\max}\lambda_{\min}}{(\lambda_{\max} + \lambda_{\min})} = \frac{det(A^T A)}{trace(A^T A)} > \lambda_{threshold}$$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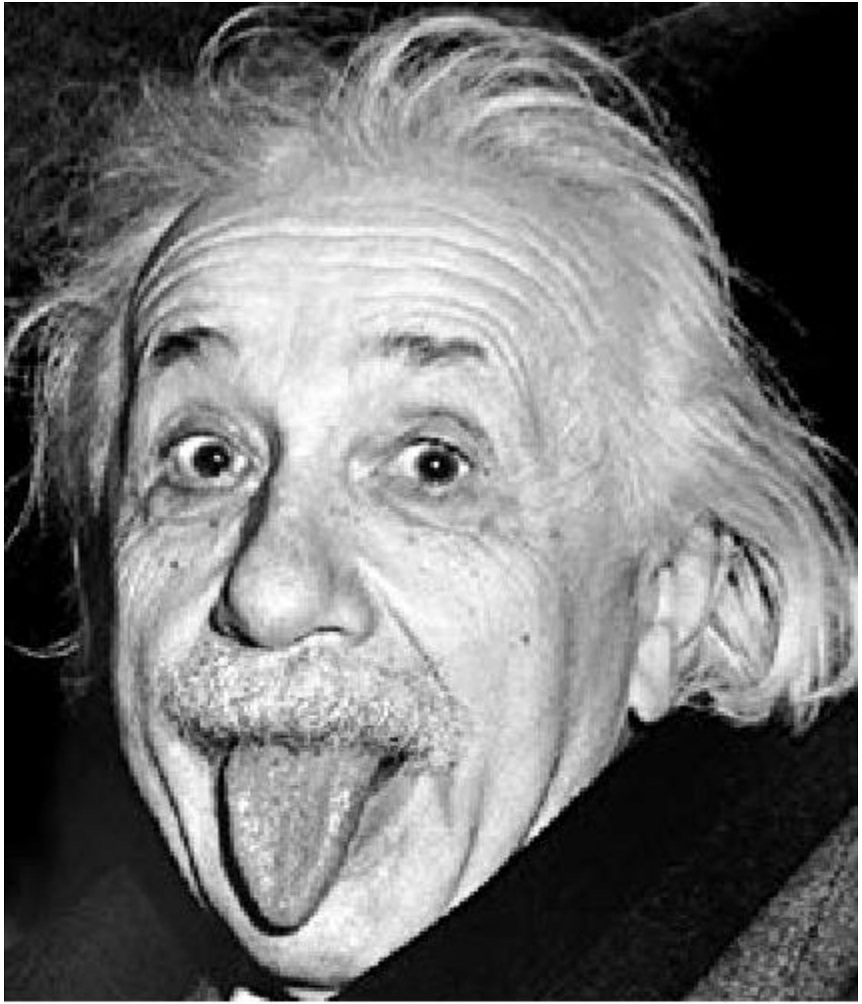

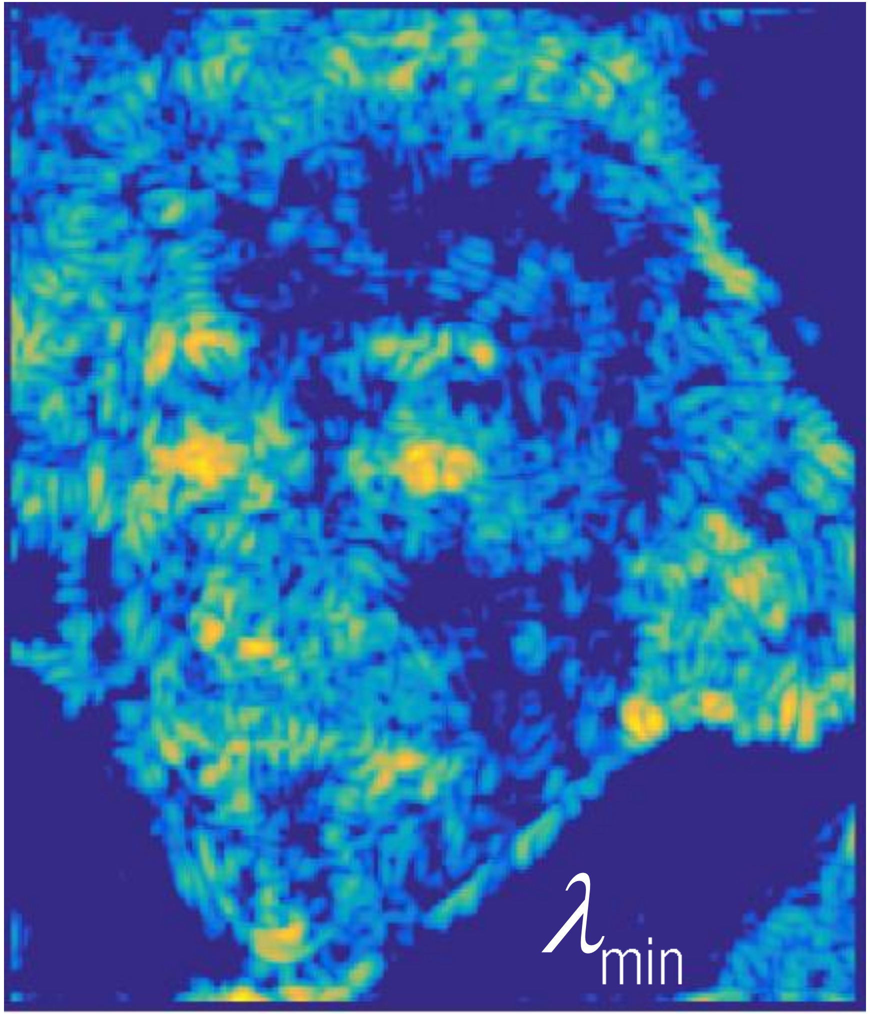
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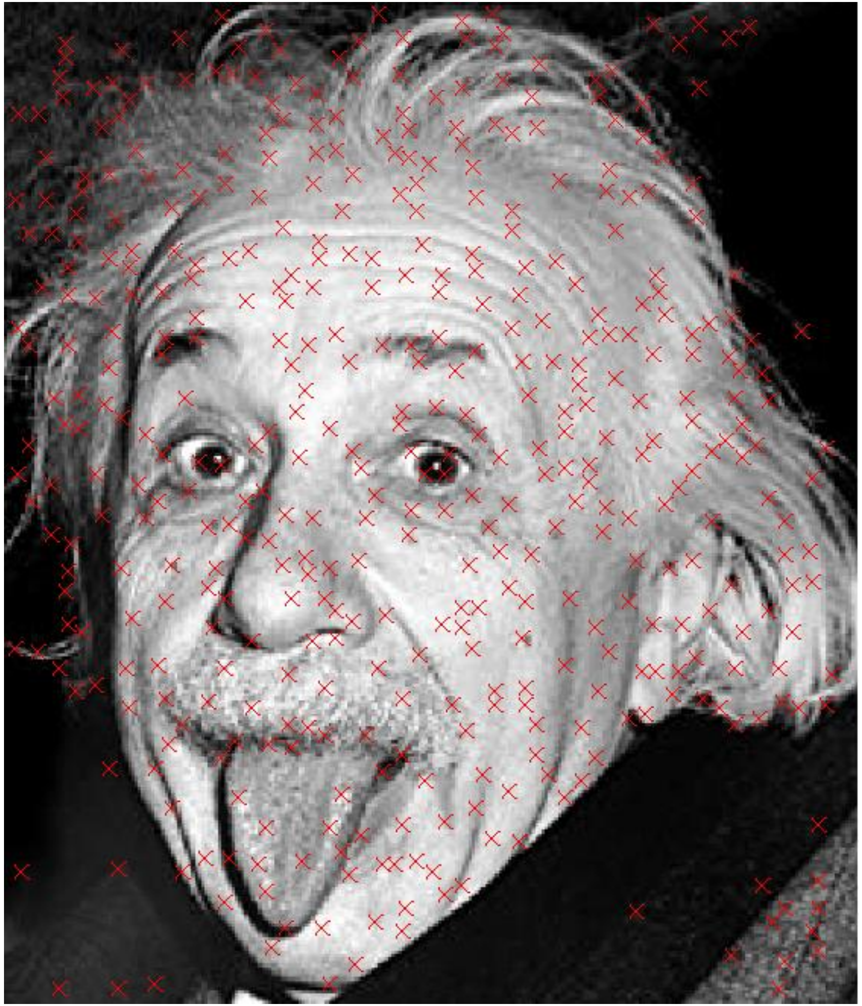

Non-maximum suppression

https://www.youtube.com/watch?v=JOKTbR-sd6s