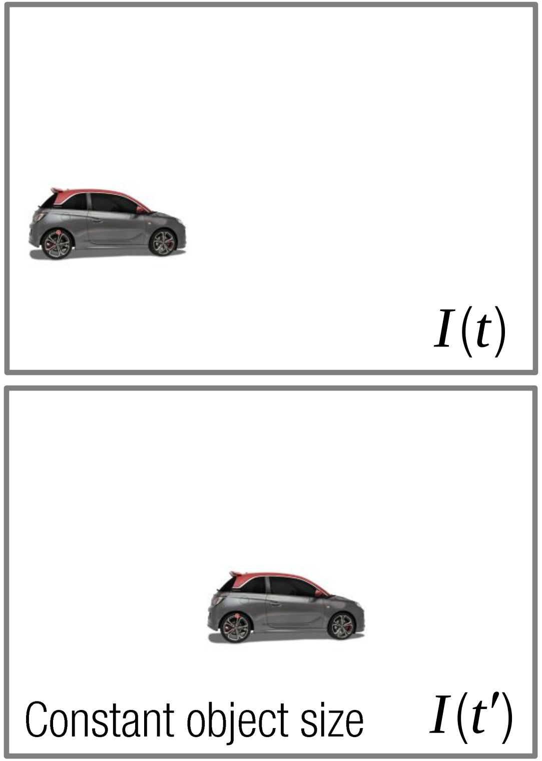
Image Alignment

Hyun Soo Park

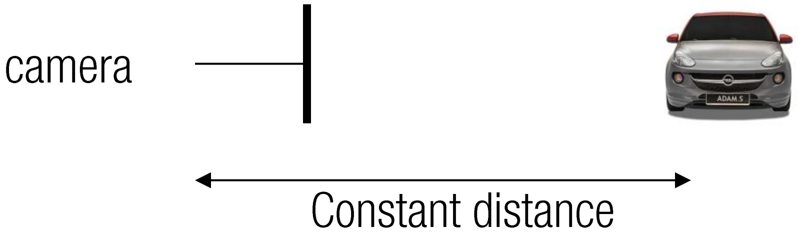
When (u,v) Flow Makes Sense?



$$\frac{\partial I}{\partial x}u+\frac{\partial I}{\partial y}v+\frac{\partial I}{\partial t}=0$$

$$x' = x + u$$
$$y' = y + v$$

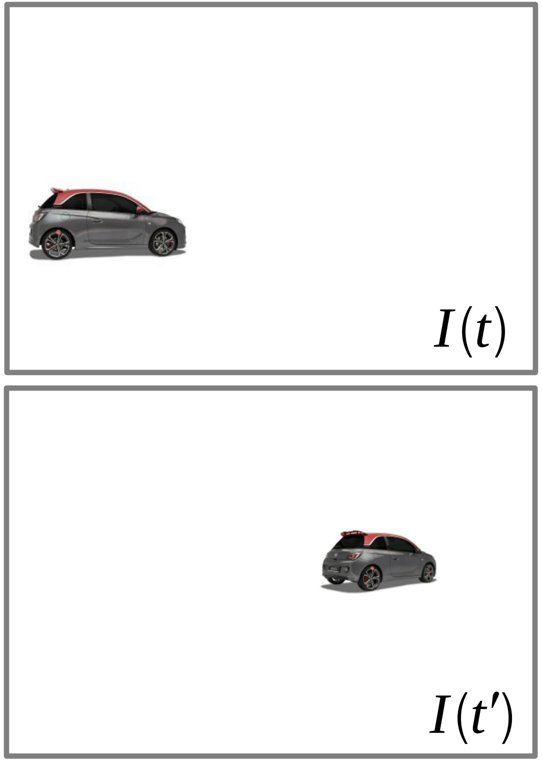
Side view



General Object Motion

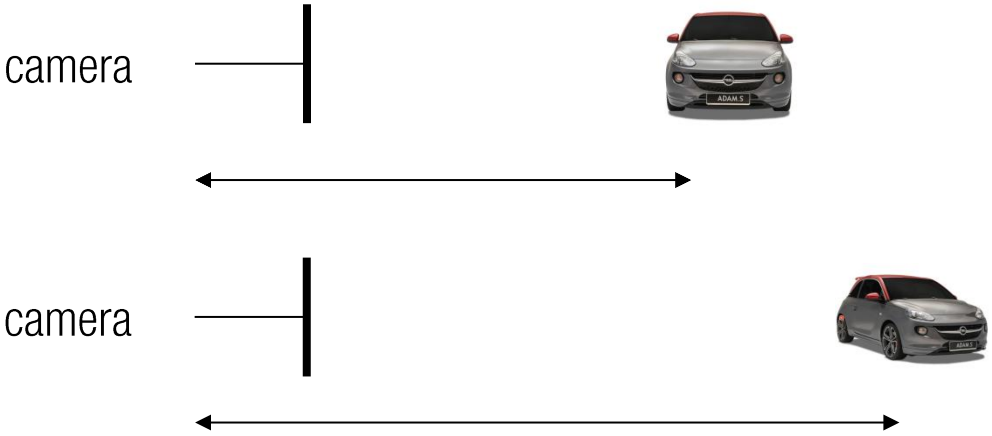
$$\frac{\partial I}{\partial x}u+\frac{\partial I}{\partial y}v+\frac{\partial I}{\partial t}=0$$

$$x' \neq x+u$$
$$y' \neq y+v$$

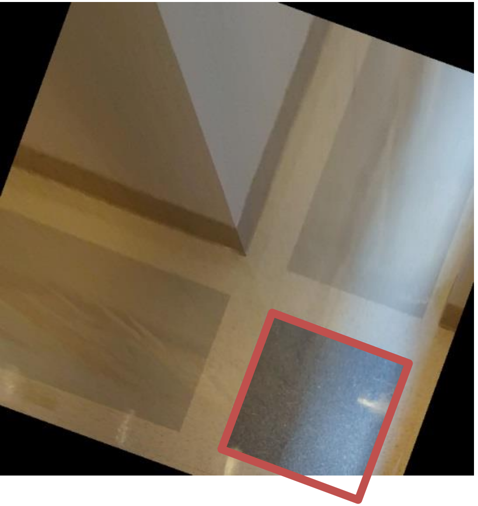


Different depth/orientation

Side view



RECALL: PARAMETRIC TRANSFORMATIONS



Euclidean (3 dof)

- Length
- Angle
- Area

$$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix}$$

Similarity (4 dof)

- Length ratio
- Angle

$$\begin{bmatrix} \alpha\cos\theta & -\alpha\sin\theta & t_x \\ \alpha\sin\theta & \alpha\cos\theta & t_y \\ & & 1 \end{bmatrix}$$

Affine (6 dof)

- Parallelism
- Ratio of area
- Ratio of length

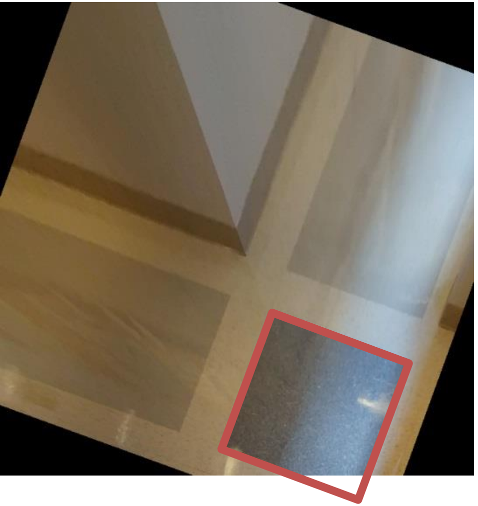
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Projective (8 dof)

- Cross ratio
- Concurrency
- Colinearity

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

Recall: Parametric Transformations



Euclidean (3 dof)

- Length
- Angle
- Area

Ex) Aerial images

Similarity (4 dof)

- Length ratio
- Angle

Change of depth

Affine (6 dof)

- Parallelism
- Ratio of area
- Ratio of length

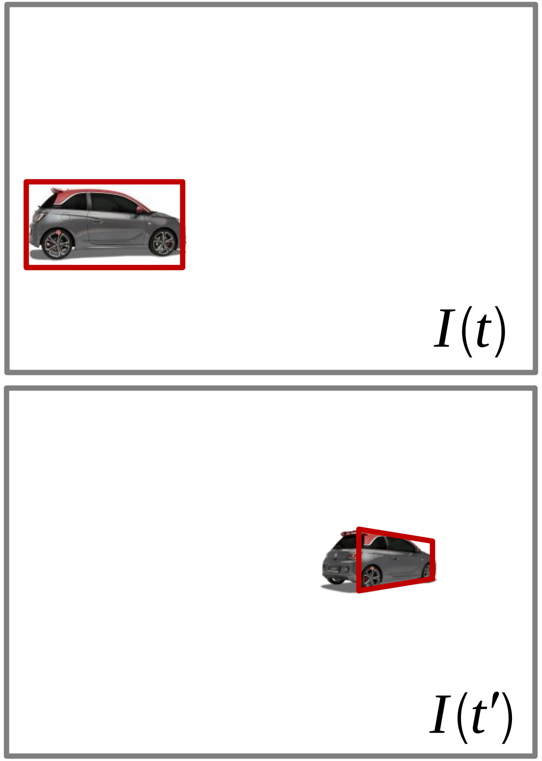
Far objects

Projective (8 dof)

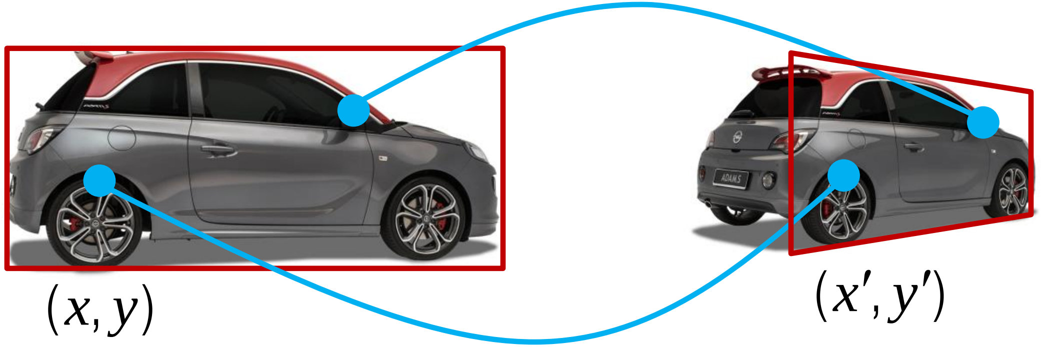
- Cross ratio
- Concurrency
- Colinearity

Planar objects

Parametric Transformation



Different depth/orientation



$$\mathbf{x}' = W(\mathbf{x}; p)$$

Unknowns: p

OPTICAL FLOW

Template
$T(x)$

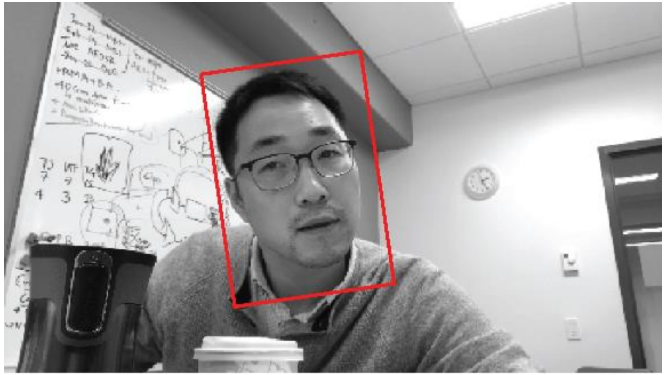
Target image
$I(x)$

Brightness constancy

$$I(x+\Delta x) \approx T(x)$$

IMAGE ALIGNMENT

Template
$T(x)$



Target image
$I(x)$

Brightness constancy

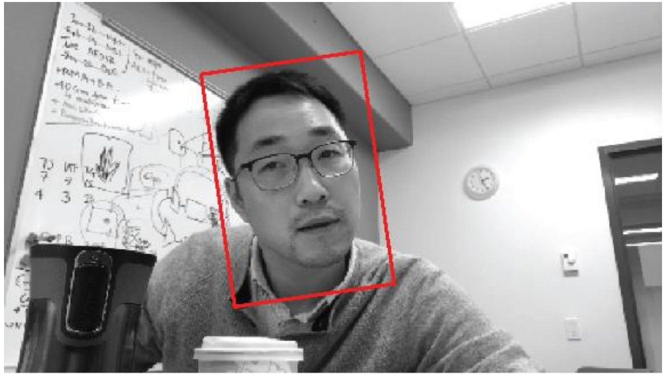
$$I(W(x;p)) \approx T(x)$$

ex) affine transform

$$W(x;p) = \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

Image Alignment

Template
$T(x)$



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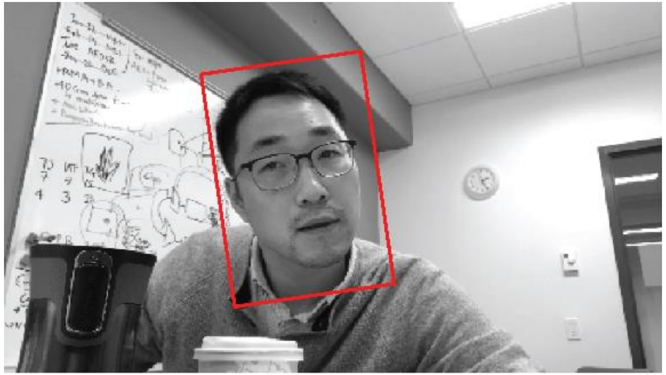
$$W(x;p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

$$= \begin{bmatrix} (p_1 + 1)u + p_2 v + p_3 \\ p_4 u + (p_5 + 1)v + p_6 \end{bmatrix}$$

IMAGE ALIGNMENT

Template
$T(x)$



Target image
$I(x)$

Brightness constancy

$$I(W(x;p)) \approx T(x)$$

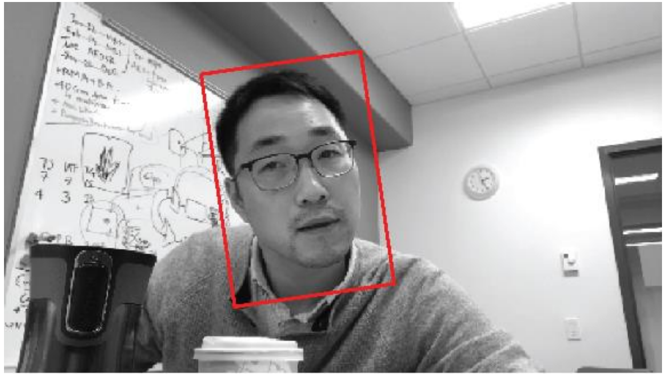
Objective: to find the optimal warping parameter *p* that minimizes warping error.

$$p* = \underset{p}{\text{minimize}} \sum_{x} \left(I(W(x;p)) - \underline{T(x)} \right)^2$$

Template

Image Alignment

Template
$T(x)$



Target image
$I(x)$

Brightness constancy

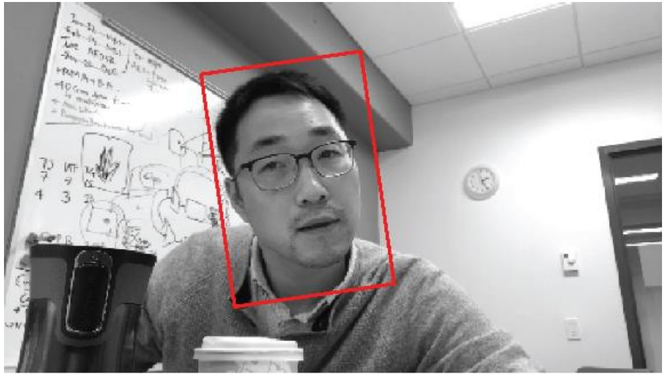
$$I(W(x;p)) \approx T(x)$$

Objective: to find the optimal warping parameter *p* that minimizes warping error.

$$p* = \underset{p}{\text{minimize}} \sum_{x} \Big(\underbrace{I(W(x;p))}_{\text{Warped image}} - \underbrace{T(x)}_{\text{Template}} \Big)^2$$

Image Alignment

Template
$T(x)$



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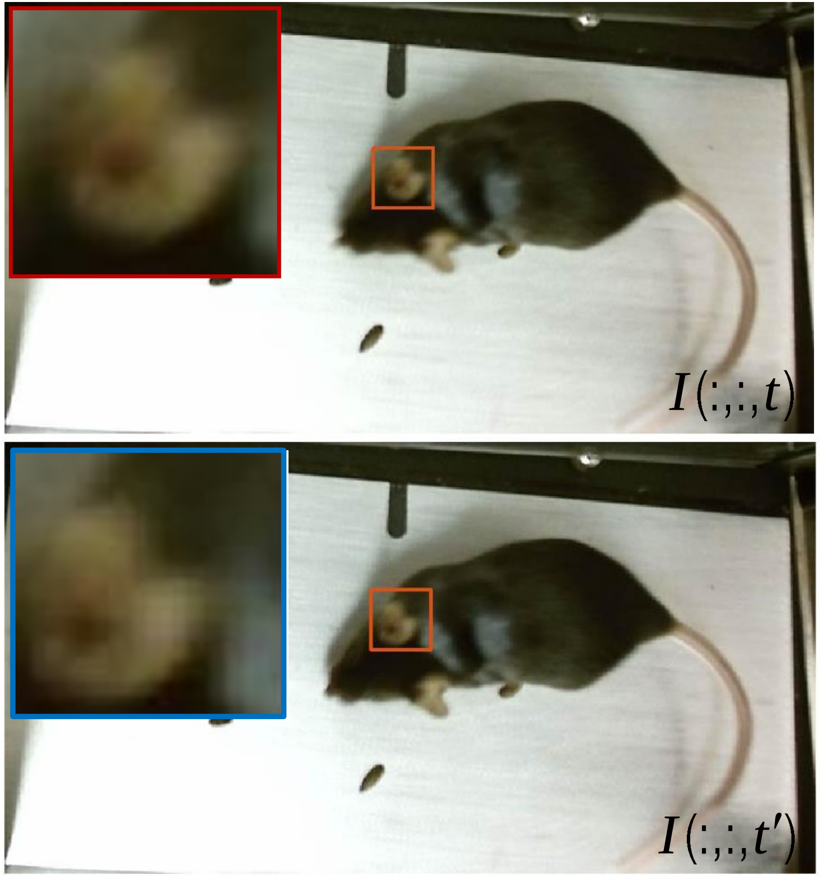
$$p* = \underset{p}{\text{minimize}} \sum_{x} \underbrace{\left(I(W(x;p)) - T(x) \right)^2}_{\text{Error image}}$$

RECALL: LOCAL PATCH TRACKING



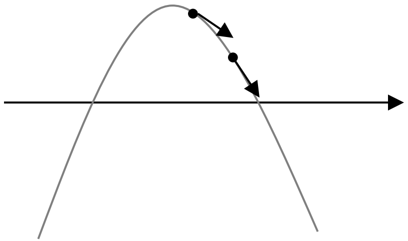
NCC: 0.923520; SSD: 3.854872

$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

First order approximation

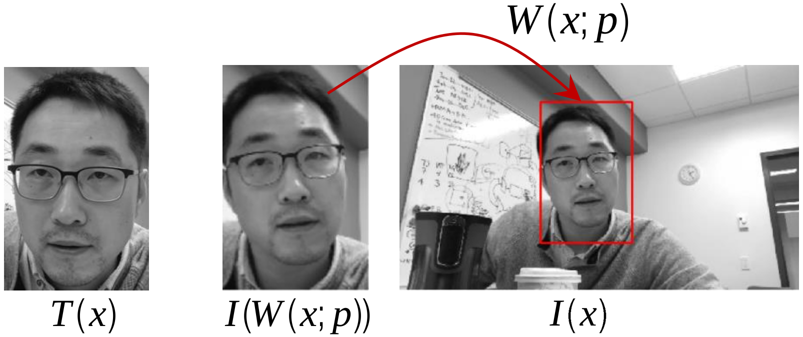
$$I(x + u\delta t, y + v\delta t, t + \delta t)$$

$$\approx I(x, y, t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$



Guass-Newton's method

Image Alignment Objective



$$p* = \underset{p}{\text{minimize}} \sum_{x} \left(I(W(x;p) - T(x) \right)^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x;p+\Delta p)) \approx I(W(x;p)) + \frac{\partial I}{\partial p} \Delta p$$

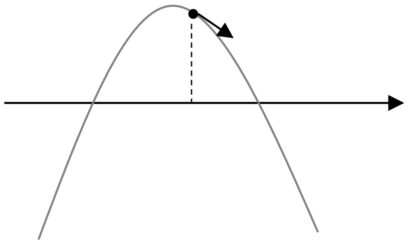
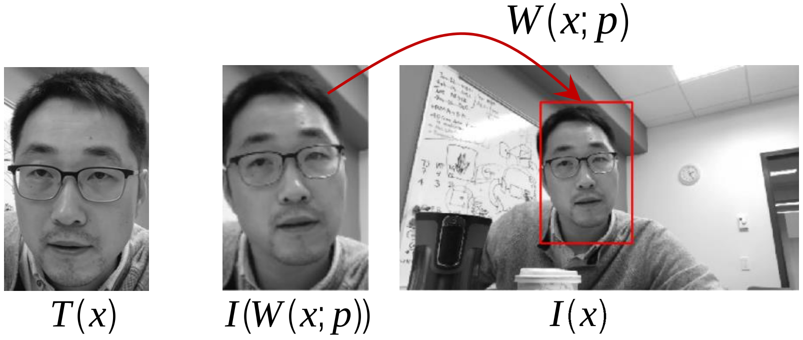


IMAGE ALIGNMENT OBJECTIVE



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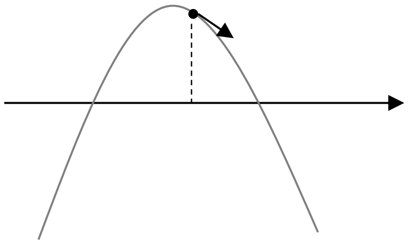


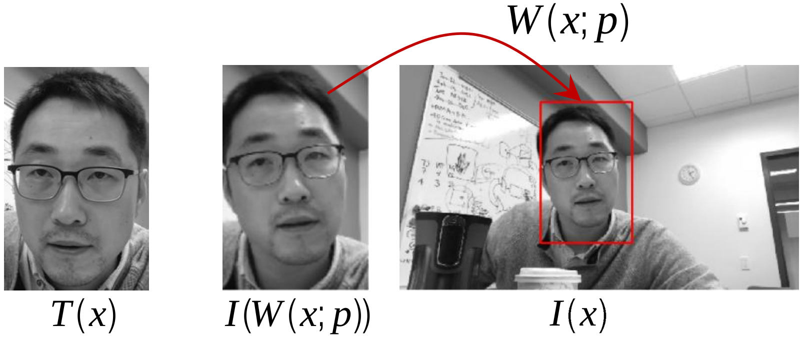
Ex) optical flow (translation)

$$W(x;p) = x + \Delta x$$

$$\longrightarrow \frac{\partial I}{\partial p}\Delta p = \frac{\partial I}{\partial x}\Delta x = \nabla I \Delta x$$

$=$  $\Delta u +$  Δv

Image Alignment Objective



$$p* = \underset{p}{\text{minimize}} \sum_{x} \left(I(W(x;p) - T(x) \right)^2$$

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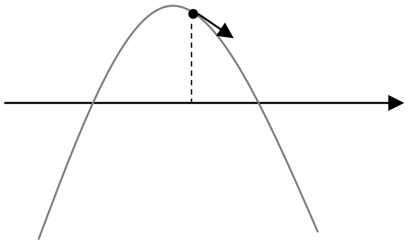
$$I(W(x;p+\Delta p)) \approx I(W(x;p)) + \frac{\partial I}{\partial p} \Delta p$$



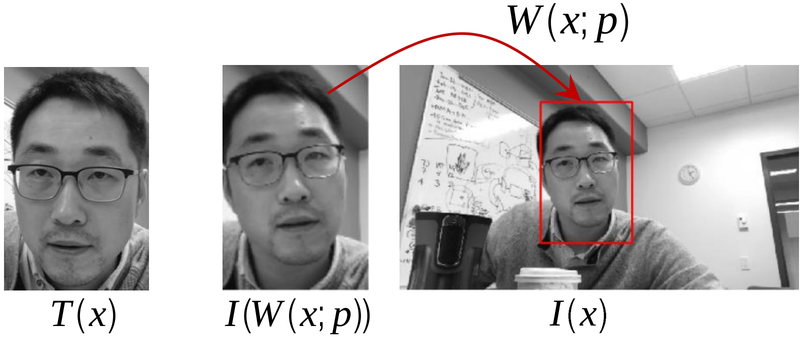
Ex) affine transform

$$W(x;p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

$$\rightarrow \frac{\partial I}{\partial p} \Delta p$$

What does the gradient image w.r.t. the affine parameters mean?

Image Alignment Objective



$$p* = \underset{p}{\text{minimize}} \sum_x \left(I(W(x;p) - T(x)\right)^2$$

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$$W(x;p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

$$\rightarrow \frac{\partial I}{\partial p}\Delta p = \underline{\frac{\partial I}{\partial x}\frac{\partial W}{\partial p}}\Delta p$$

Chain rule

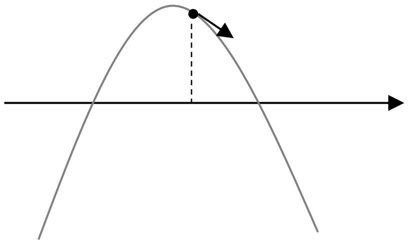
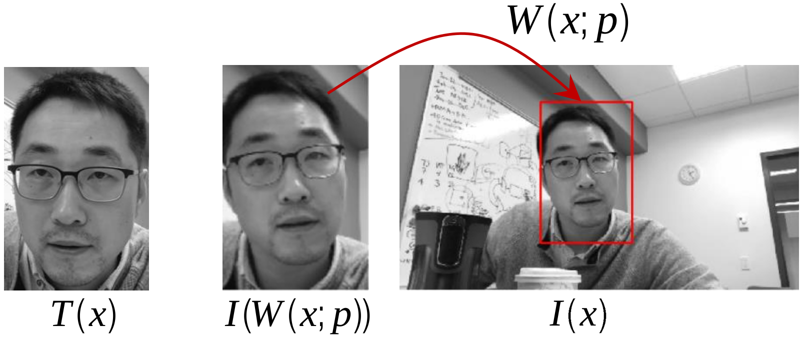


IMAGE ALIGNMENT OBJECTIVE



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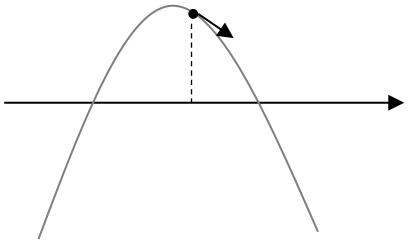


Ex) affine transform

$$W(x;p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

$$\rightarrow \frac{\partial I}{\partial p}\Delta p = \frac{\partial I}{\partial x}\frac{\partial W}{\partial p}\Delta p = \underline{\nabla I}\frac{\partial W}{\partial p}\Delta p$$

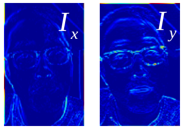
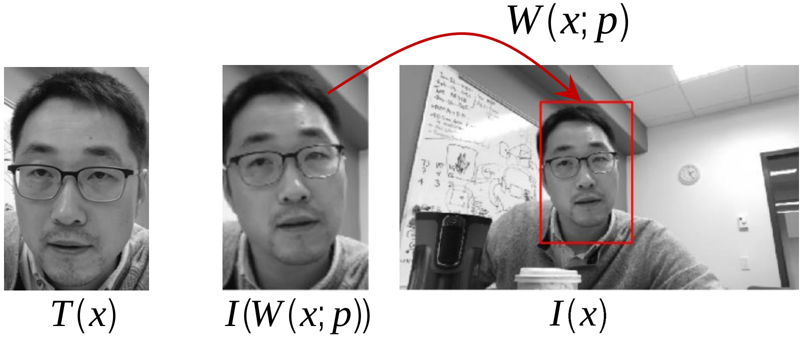


IMAGE ALIGNMENT OBJECTIVE



$$p* = \underset{p}{\text{minimize}} \sum_x \left(I(W(x;p) - T(x) \right)^2$$

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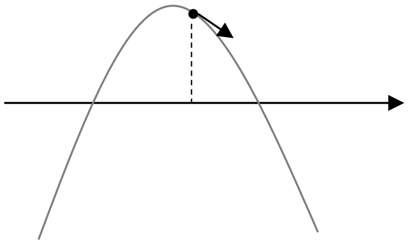


Ex) affine transform

$$W(x;p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

$$\rightarrow \frac{\partial I}{\partial p}\Delta p = \frac{\partial I}{\partial x}\frac{\partial W}{\partial p}\Delta p = \nabla I \frac{\partial W}{\partial p}\Delta p$$

Jacobian: $\frac{\partial W}{\partial p} = \begin{bmatrix} \frac{\partial u}{\partial p_1} & \cdots & \frac{\partial u}{\partial p_6} \\ \frac{\partial v}{\partial p_1} & \cdots & \frac{\partial v}{\partial p_6} \end{bmatrix}$

IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x;p)=\begin{bmatrix}u\\v\end{bmatrix}+\begin{bmatrix}p_1u+p_2v+p_3\\p_4u+p_5v+p_6\end{bmatrix} \longrightarrow \frac{\partial I}{\partial p}\Delta p=\nabla I\frac{\partial W}{\partial p}\Delta p$$

$$\begin{bmatrix}I_x & I_y\end{bmatrix}$$

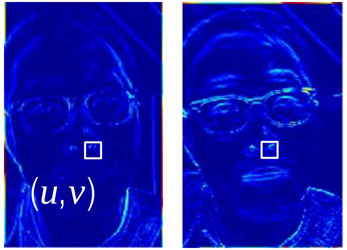
1 x 2

$$\begin{bmatrix}\frac{\partial u}{\partial p_1} & \cdots & \frac{\partial u}{\partial p_6}\\ \frac{\partial v}{\partial p_1} & \cdots & \frac{\partial v}{\partial p_6}\end{bmatrix}$$

2 x 6

$$\begin{bmatrix}\Delta p_1\\ \vdots \\ \Delta p_6\end{bmatrix}$$

1 x 6



$$W(x;p)=\begin{bmatrix}u\\v\end{bmatrix}+\begin{bmatrix}p_1u+p_2v+p_3\\p_4u+p_5v+p_6\end{bmatrix}$$

Image Jacobian (Steepest Descent Image)

$$W(x;p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \longrightarrow \frac{\partial I}{\partial p}\Delta p = \nabla I \frac{\partial W}{\partial p}\Delta p$$

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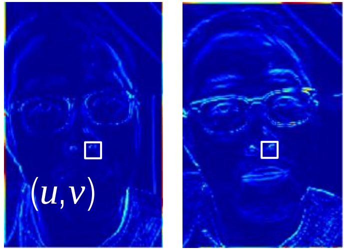
1 x 2

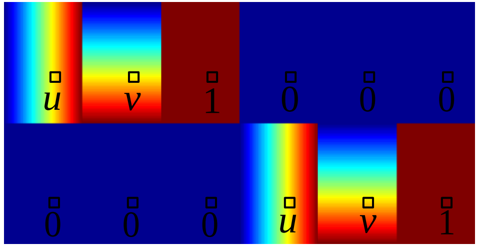
$$\begin{bmatrix} u & v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & v & 1 \end{bmatrix}$$

2 x 6

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6





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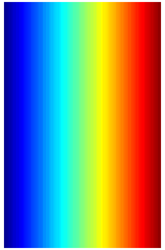

$$\nabla I \frac{\partial W}{\partial p}$$

Image Jacobian (Steepest Descent Image)

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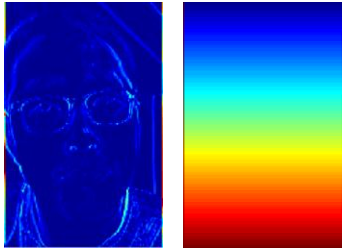

$$\nabla I \frac{\partial W}{\partial p}$$

IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x;p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \longrightarrow \frac{\partial I}{\partial p}\Delta p = \nabla I \frac{\partial W}{\partial p}\Delta p$$

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$$\nabla I \frac{\partial W}{\partial p}$$

IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

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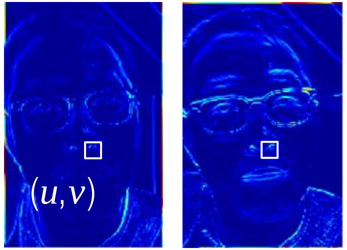

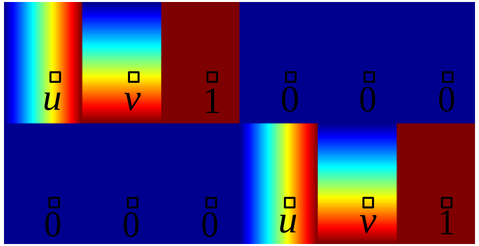

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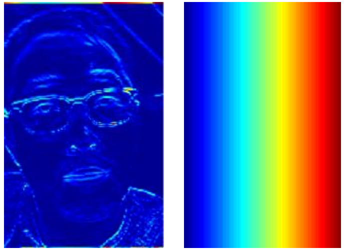

$$\nabla I \frac{\partial W}{\partial p}$$

Image Jacobian (Steepest Descent Image)

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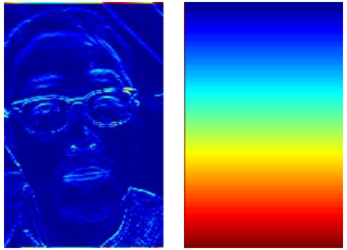
$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6



$$\nabla I \frac{\partial W}{\partial p}$$

IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x;p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \longrightarrow \frac{\partial I}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

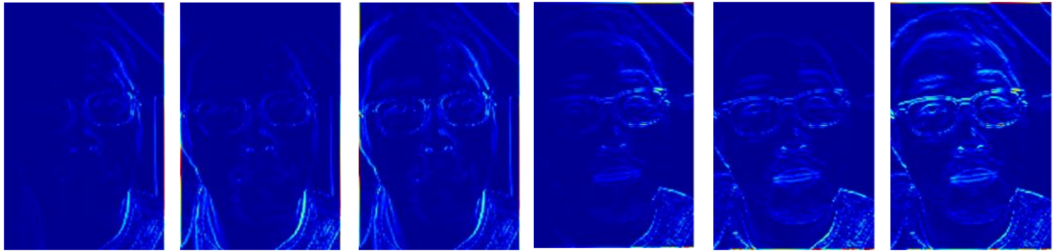
1 x 2

$$\begin{bmatrix} u & v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & v & 1 \end{bmatrix}$$

2 x 6

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6



$$\nabla I \frac{\partial W}{\partial p}$$

Image Jacobian (Steepest Descent Image)

$$W(x;p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \longrightarrow \frac{\partial I}{\partial p}\Delta p = \nabla I \frac{\partial W}{\partial p}\Delta p$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

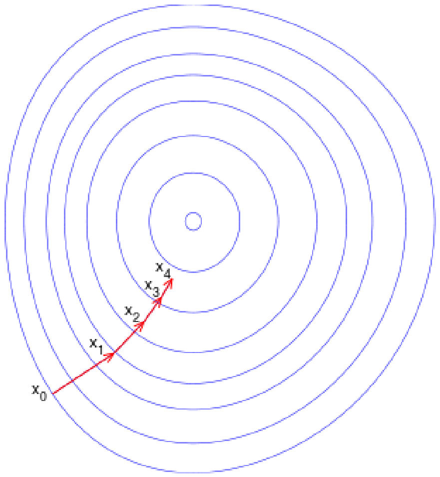
1 x 2

$$\begin{bmatrix} u & v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & v & 1 \end{bmatrix}$$

2 x 6

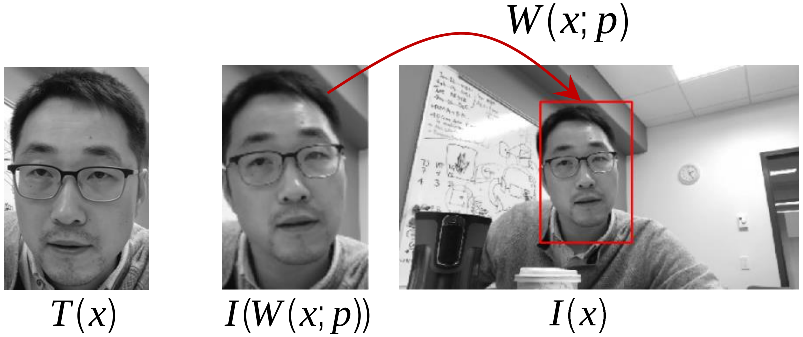
$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6



$\nabla I \frac{\partial W}{\partial p}$: steepest decent images

IMAGE ALIGNMENT OBJECTIVE



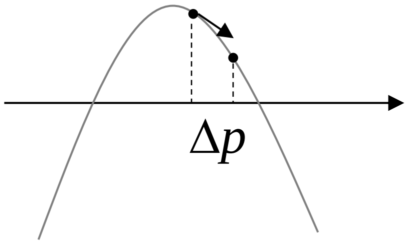
$$p* = \underset{p}{\text{minimize}} \sum_{x} \left(I(W(x;p) - T(x) \right)^2$$

Guass-Newton's method

1. Linearize the obj. function at p

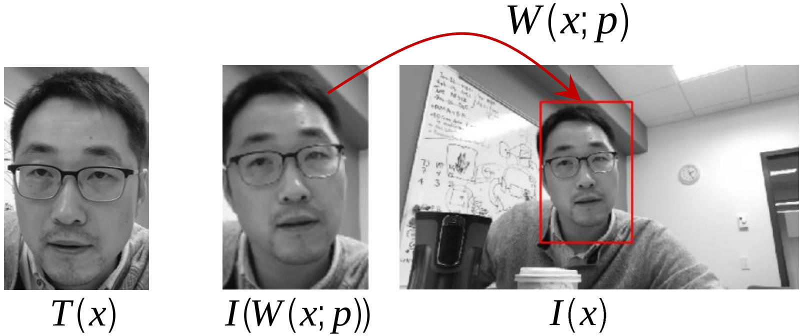
$$I(W(x;p+\Delta p)) \approx I(W(x;p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$

2. Find Δp that minimizes the obj. function at p



$$I(W(x;p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x) = 0 \longrightarrow \nabla I \frac{\partial W}{\partial p} \Delta p = T(x) - I(W(x;p))$$

Image Alignment Objective



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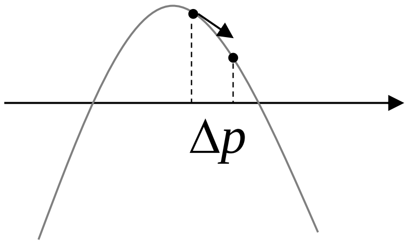
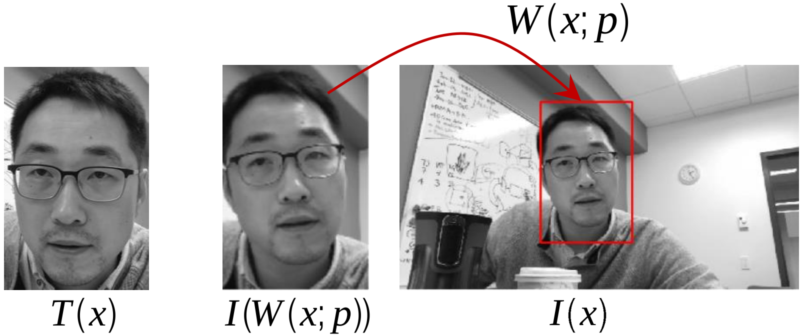


Image Alignment Objective



$$p* = \underset{p}{\text{minimize}} \sum_x \left(I(W(x;p) - T(x) \right)^2$$

Guass-Newton's method

1.Linearize the obj. function at p

$$I(W(x;p+\Delta p)) \approx I(W(x;p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$

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$$I(W(x;p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x) = 0 \longrightarrow \nabla I \frac{\partial W}{\partial p} \Delta p = T(x) - I(W(x;p))$$

$$\begin{bmatrix} \nabla I \left.\frac{\partial W}{\partial p}\right|_{x_1} \\ \vdots \\ \nabla I \left.\frac{\partial W}{\partial p}\right|_{x_n} \end{bmatrix} \Delta p = -\begin{bmatrix} I(W(x_1;p)) - T(x_1) \\ \\ I(W(x_n;p)) - T(x_n) \end{bmatrix}$$

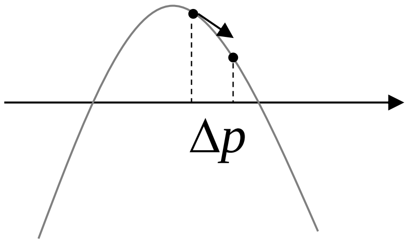


Image Alignment Objective

$W(x;p)$

$T(x)$ $\quad I(W(x;p))$ $\quad I(x)$

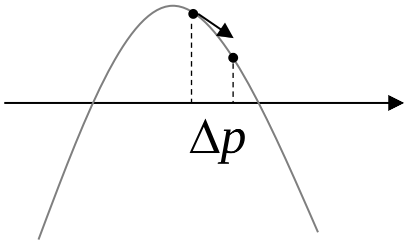
$$p* = \underset{p}{\text{minimize}} \sum_{x} \left(I(W(x;p) - T(x) \right)^2$$

Guass-Newton's method

1.Linearize the obj. function at p

$$I(W(x;p+\Delta p)) \approx I(W(x;p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$

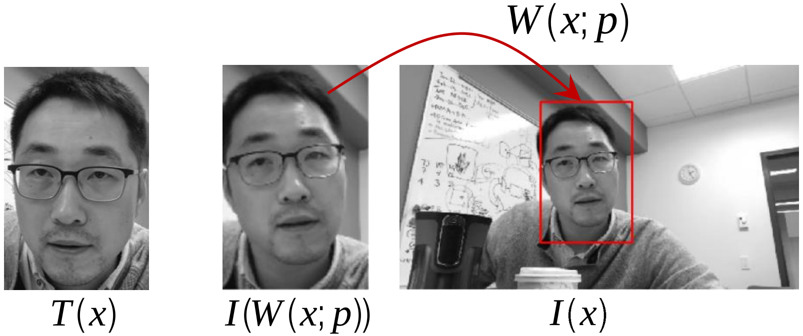
2. Find Δp that minimizes the obj. function at p



$$I(W(x;p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x) = 0 \longrightarrow \nabla I \frac{\partial W}{\partial p} \Delta p = T(x) - I(W(x;p))$$

$$\underbrace{\begin{bmatrix} \nabla I \frac{\partial W}{\partial p}\Big|_{x_1} \\ \vdots \\ \nabla I \frac{\partial W}{\partial p}\Big|_{x_n} \end{bmatrix}}_{A} \underbrace{\Delta p}_{x} = -\underbrace{\begin{bmatrix} I(W(x_1;p)) - T(x_1) \\ \vdots \\ I(W(x_n;p)) - T(x_n) \end{bmatrix}}_{b}$$

Image Alignment Objective



$$p* = \underset{p}{\text{minimize}} \sum_x \left(I(W(x;p) - T(x) \right)^2$$

Guass-Newton's method

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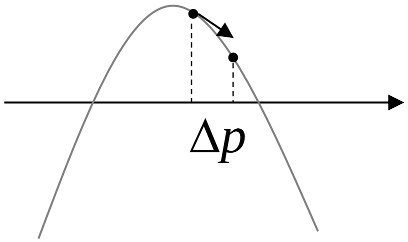
$$I(W(x;p+\Delta p)) \approx I(W(x;p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$

2. Find Δp that minimizes the obj. function at p

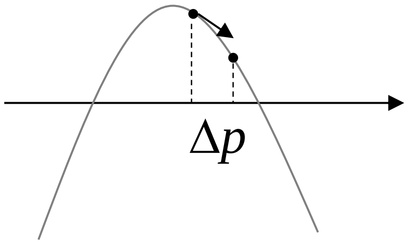


$$I(W(x;p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x) = 0 \longrightarrow \nabla I \frac{\partial W}{\partial p} \Delta p = T(x) - I(W(x;p))$$

$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x;p)))$$

where $H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$

IMAGE ALIGNMENT OBJECTIVE



$$p* = \underset{p}{\text{minimize}} \sum_x \left(I(W(x;p) - T(x) \right)^2$$

Guass-Newton's method

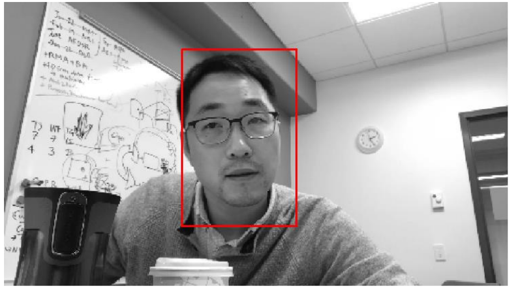
1.Linearize the obj. function at p

$$I(W(x;p+\Delta p)) \approx I(W(x;p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$

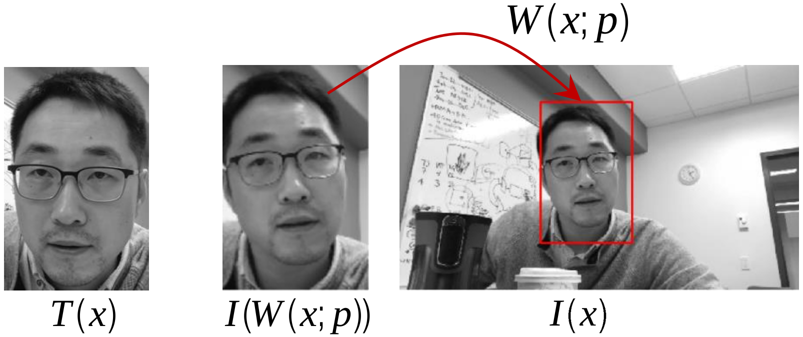
2. Find Δp that minimizes the obj. function at p

$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x;p)))$$

3. Update $p \leftarrow p + \Delta p$



Optical Flow Derivation



$$\Delta x^* = \underset{\Delta x}{\text{minimize}} \sum_x \left(I(x;\Delta x) - T(x) \right)^2$$

Guass-Newton's method

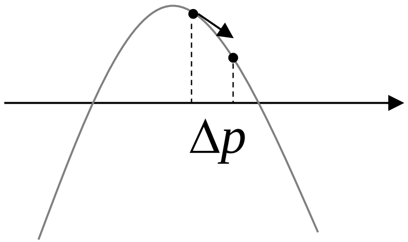
1. Linearize the obj. function at x

$$I(x+\Delta x) \approx I(x) + \nabla I \Delta x$$

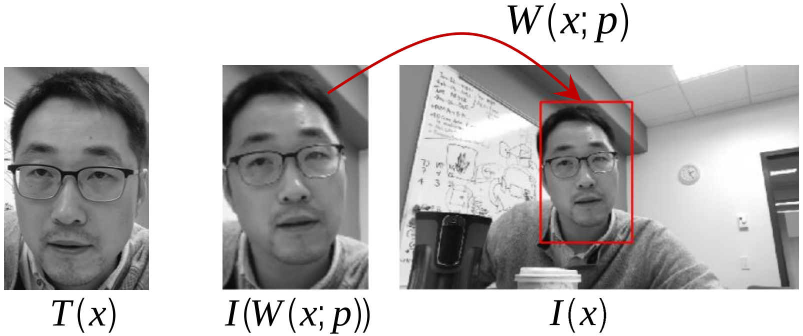
: steepest descent images

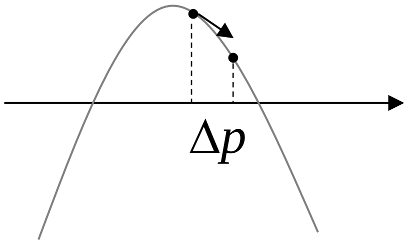
2. Find Δx that minimizes the obj. function at x

$$\nabla I \Delta x = T(x) - I(x) = -(I(x) - T(x)) \quad \triangleq \quad I_x \Delta u + I_y \Delta v = -I_t$$



Optical Flow Derivation





$$\Delta x* = \underset{\Delta x}{\text{minimize}} \sum_x \left(I(x;\Delta x) - T(x) \right)^2$$

Guass-Newton's method

1.Linearize the obj. function at x

$$I(x+\Delta x) \approx I(x) + \nabla I \Delta x$$

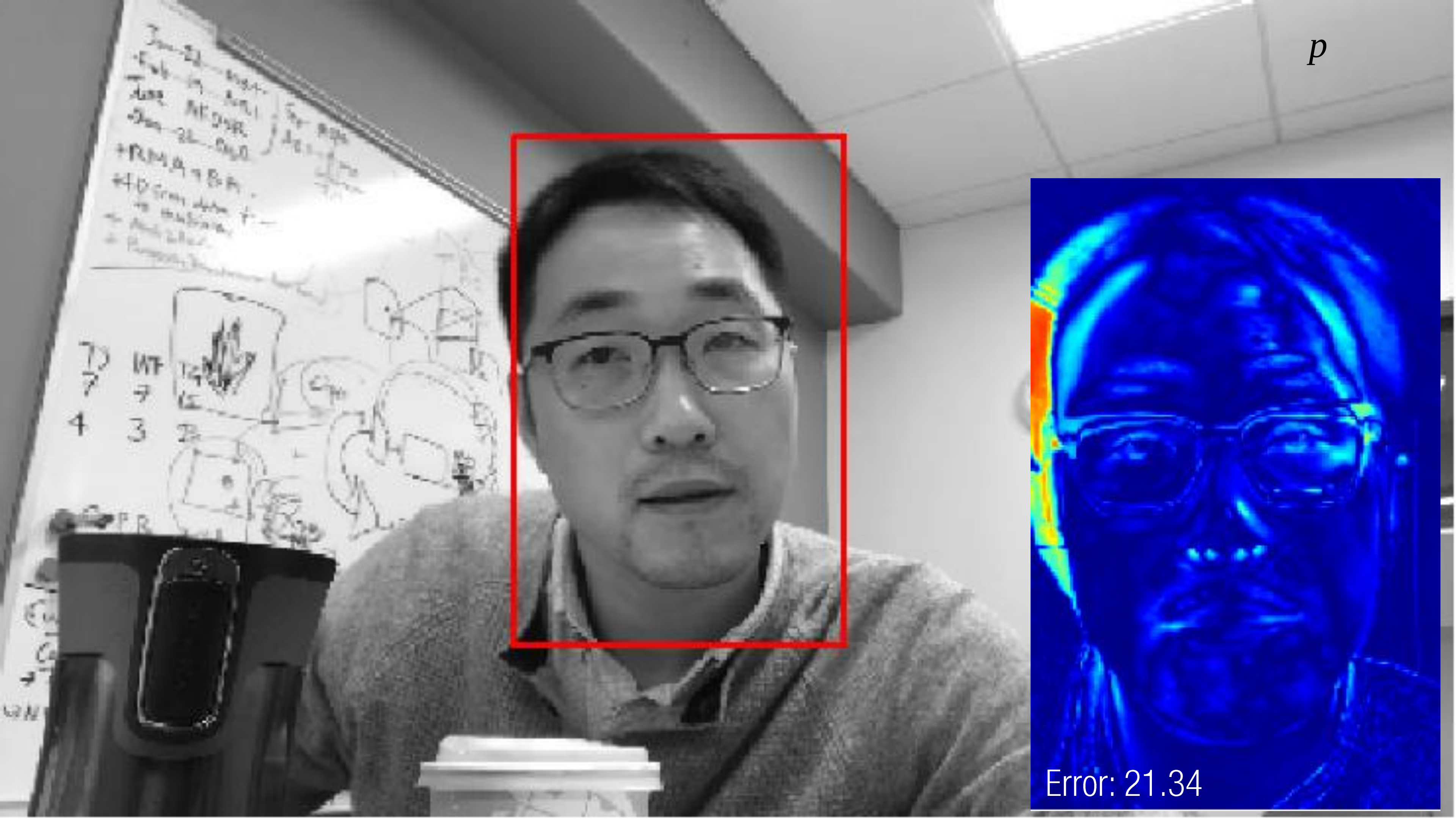
: steepest descent images

2. Find Δx that minimizes the obj. function at x

$$\nabla I \Delta x = T(x) - I(x) = -(I(x) - T(x)) \quad \triangleq \quad I_x \Delta u + I_y \Delta v = -I_t$$

$$\Delta x = H^{-1} \sum_x (\nabla I)^T (T(x) - I(W(x;p)))$$

where $H = \sum_x \nabla I^T \nabla I$


p
Error: 21.34

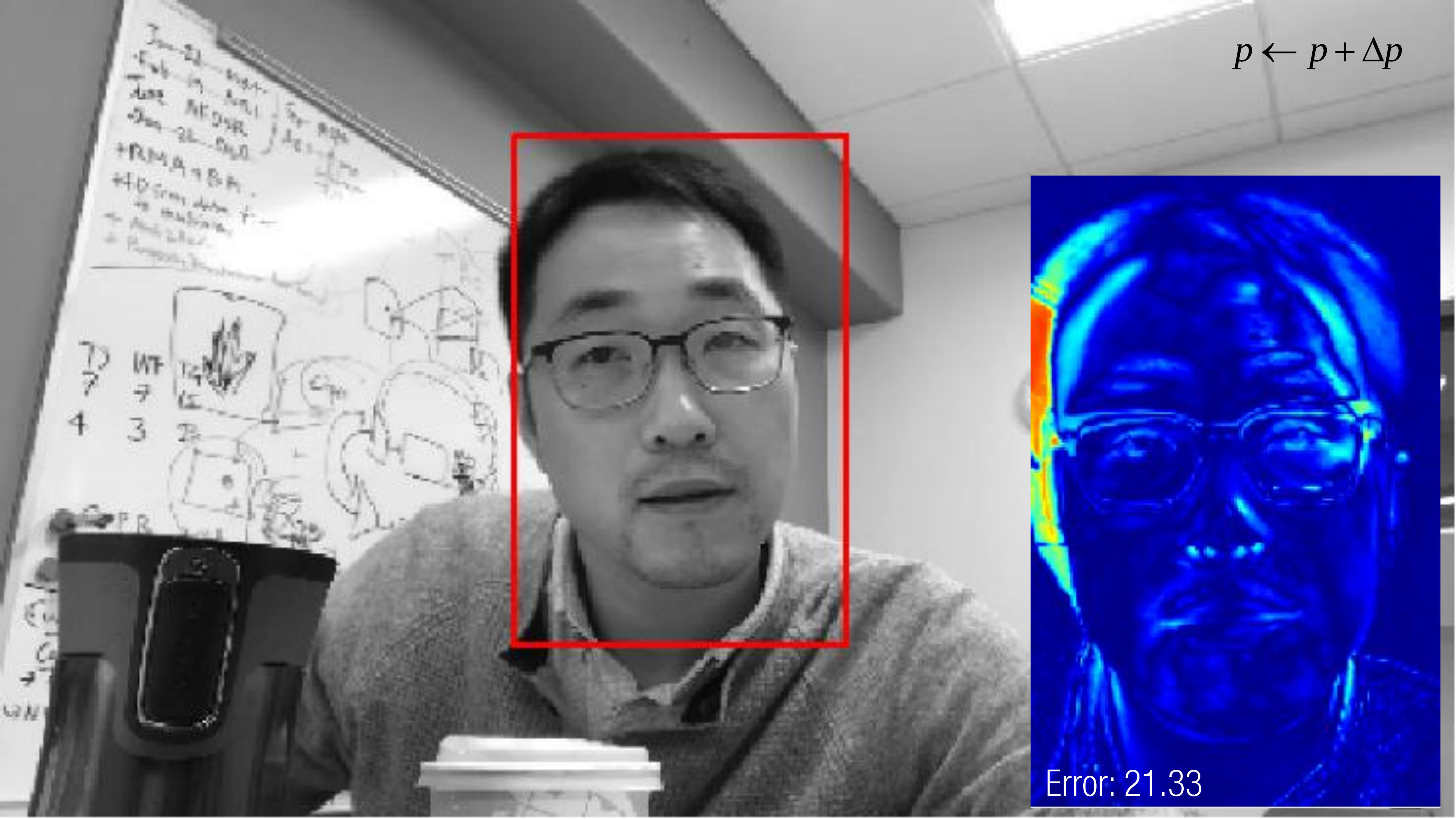
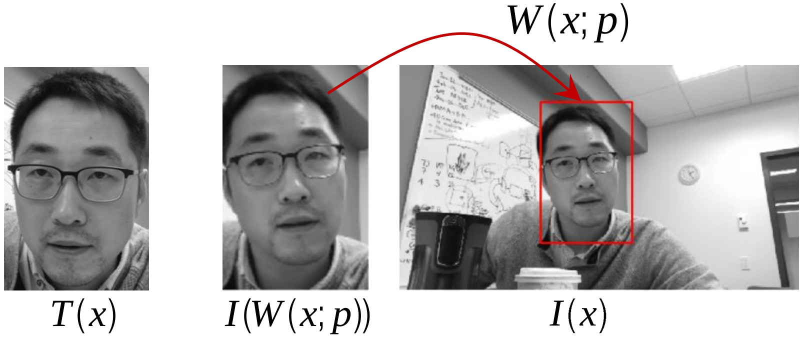

$p \leftarrow p + \Delta p$
Error: 21.33

IMAGE ALIGNMENT



$$p* = \underset{p}{\text{minimize}} \sum_{x} \left(I(W(x;p) - T(x) \right)^2$$

Guass-Newton's method

1. Linearize the obj. function at p

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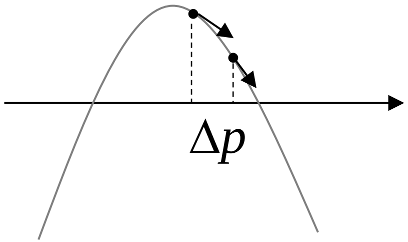
2. Find Δp that minimizes the obj. function at p

$$\Delta p = H^{-1} \sum_{x} \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x;p)))$$



3. Update $p \leftarrow p + \Delta p$

4. Goto 1

$I(x)$

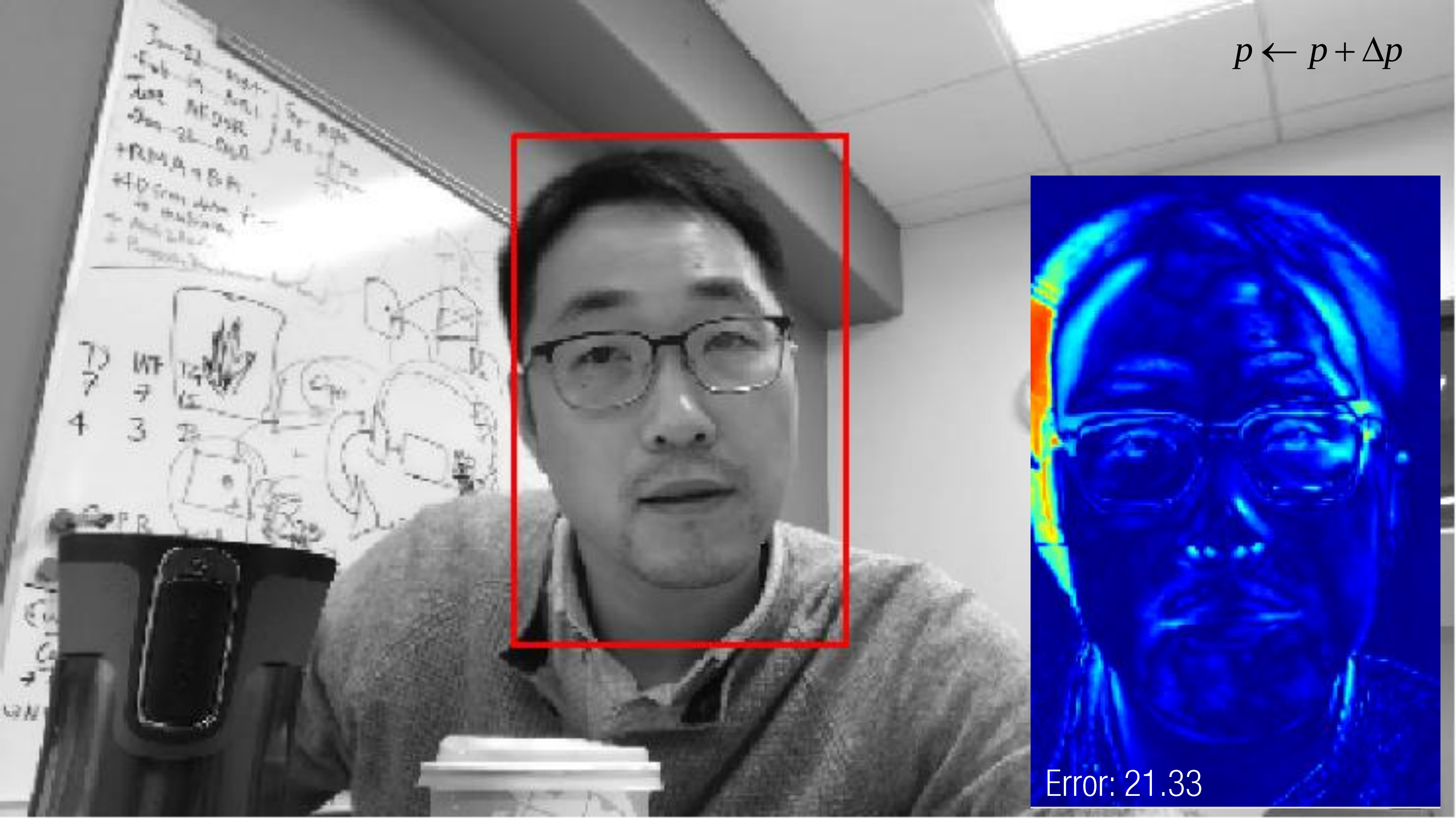
$T(x)$

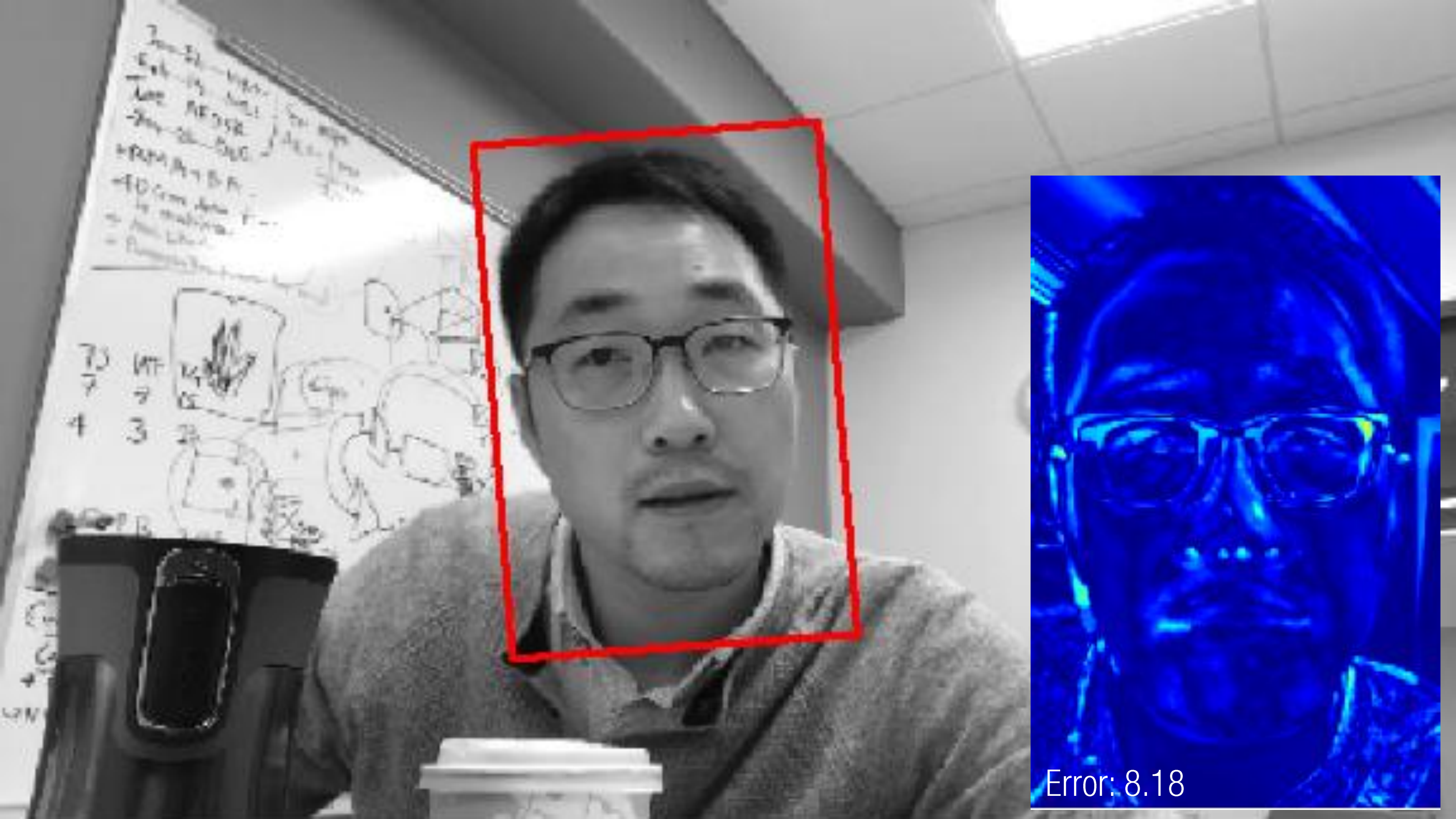
$I(W(x;p))$

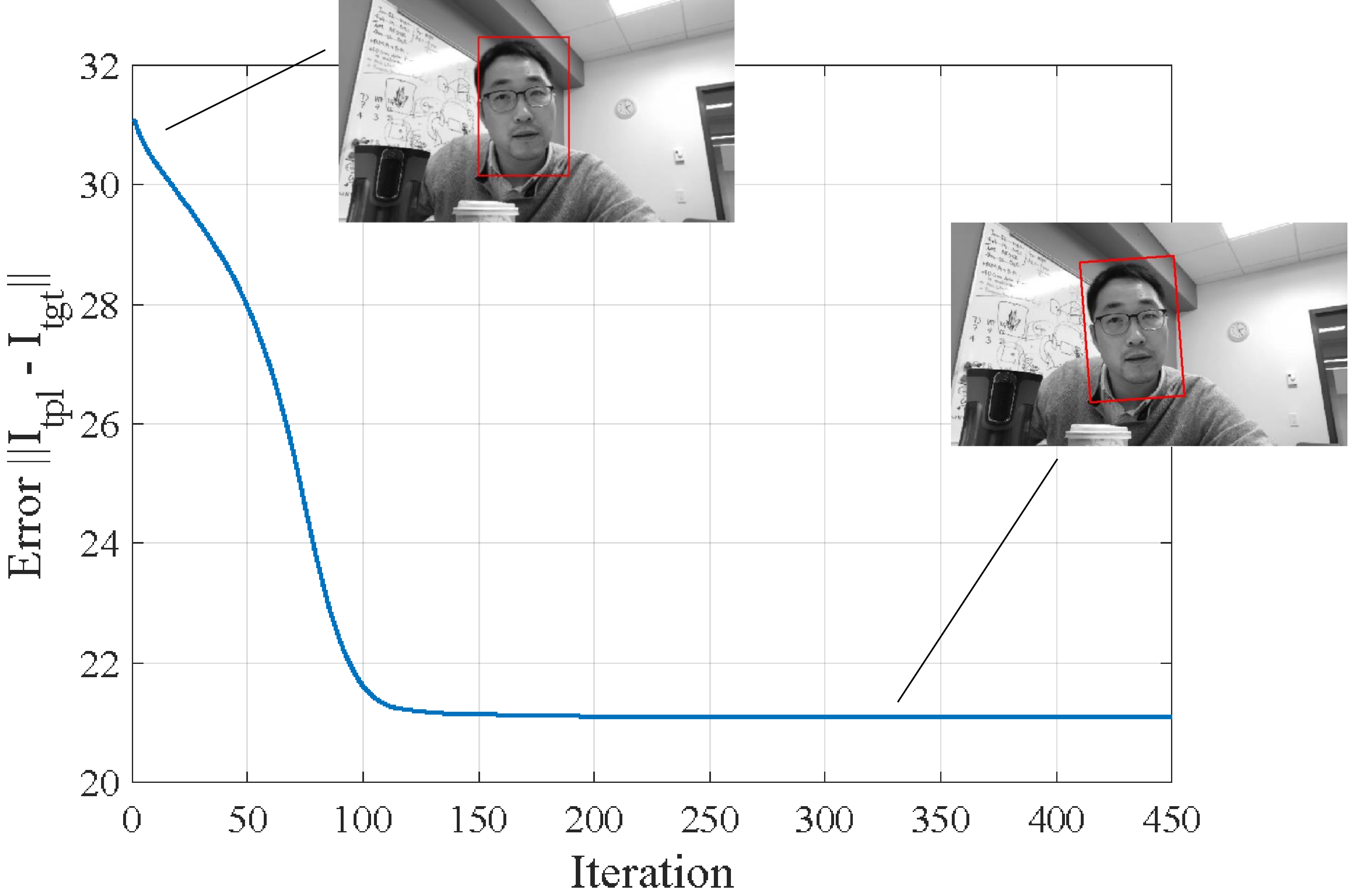
$0.5T(x)+0.5I(W(x;p))$

$\left|T(x)-I(W(x;p))\right|$

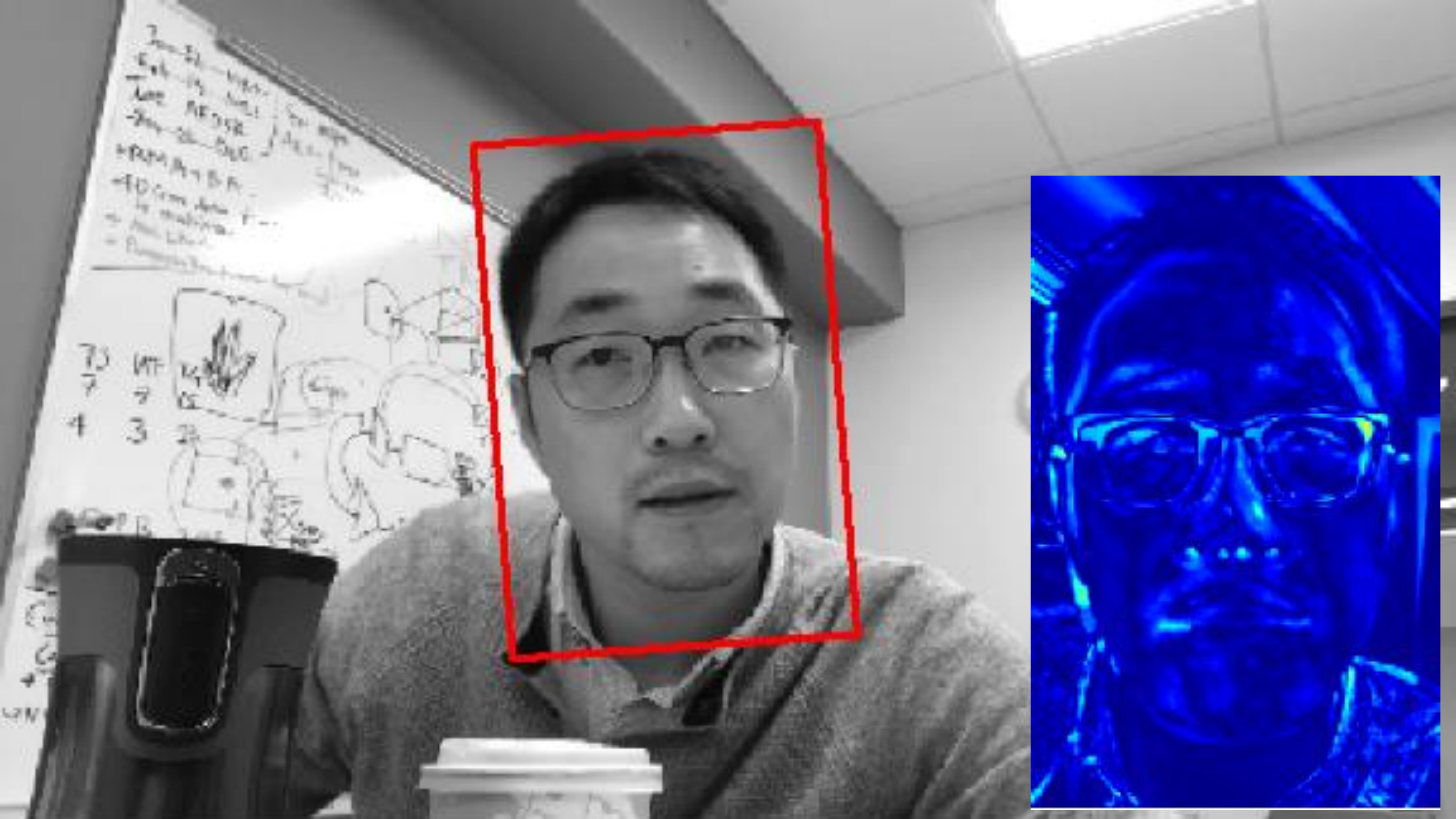
$$p \leftarrow p + \Delta p$$

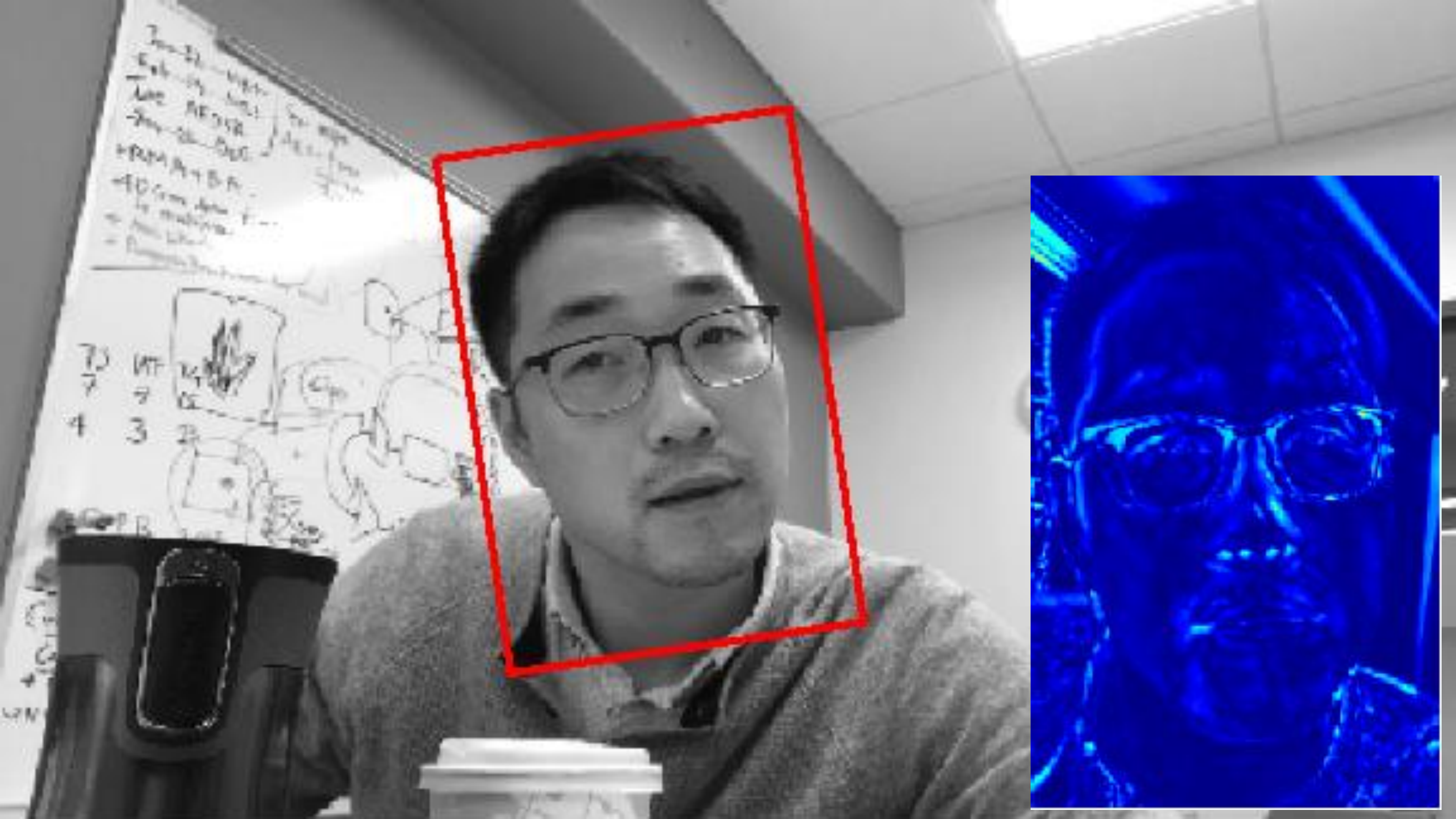


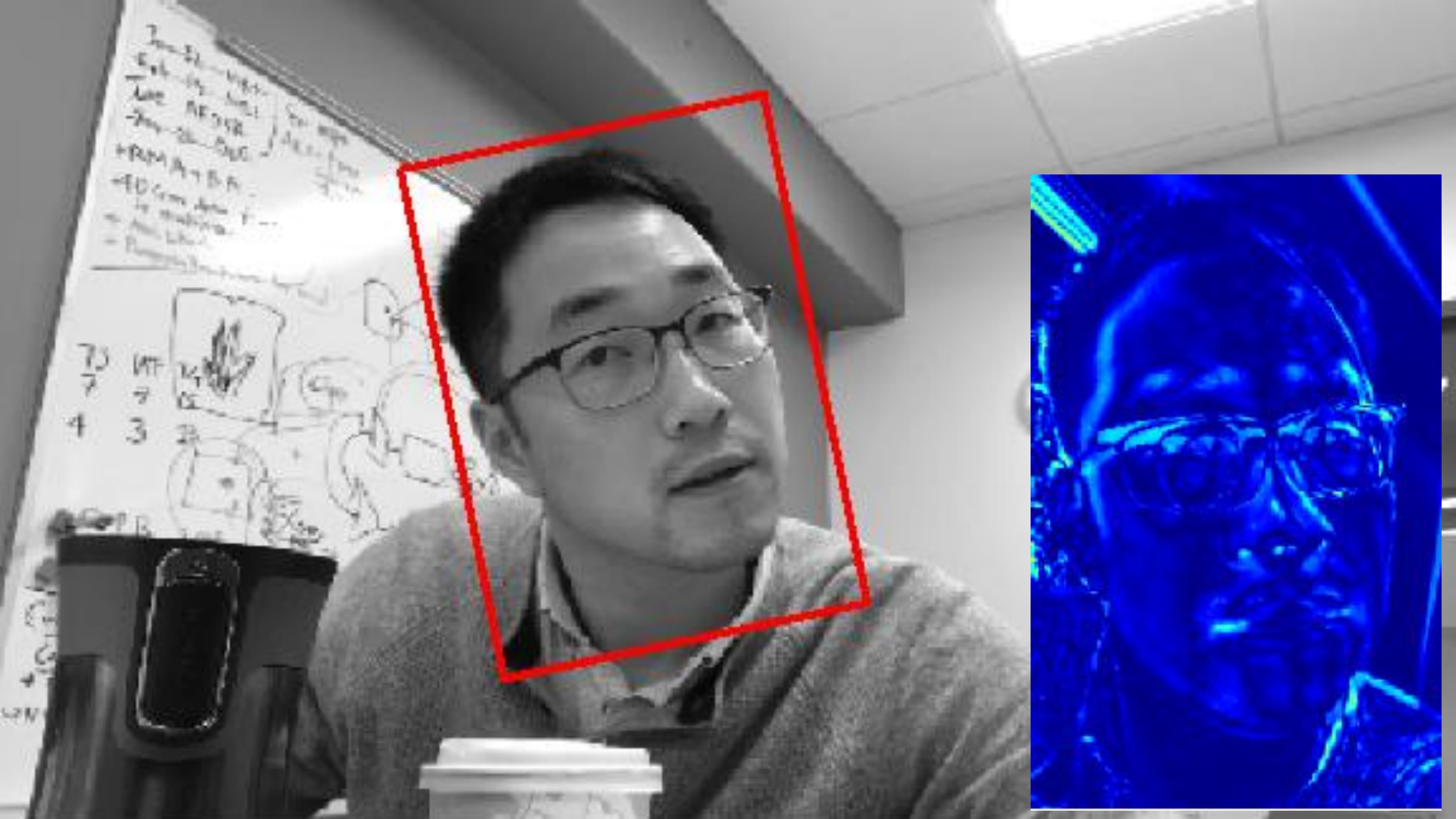

Error: 8.18

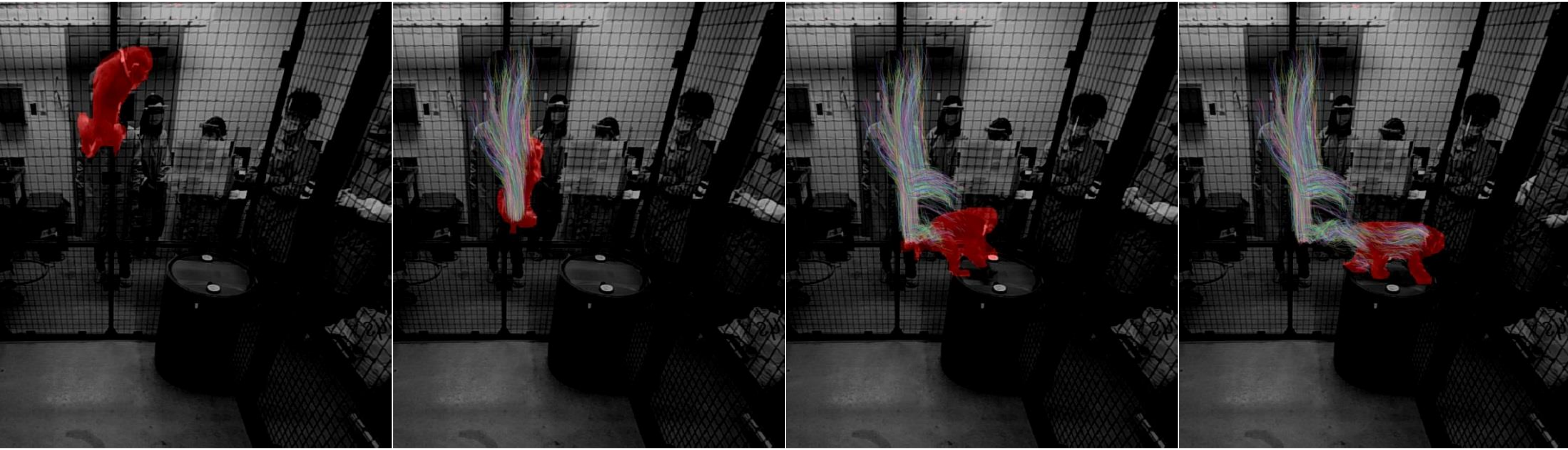


32
30
28
26
24
22
20
0
50
100
150
200
250
300
350
400
450
Iteration
Error ||I_tpl - I_tgt||







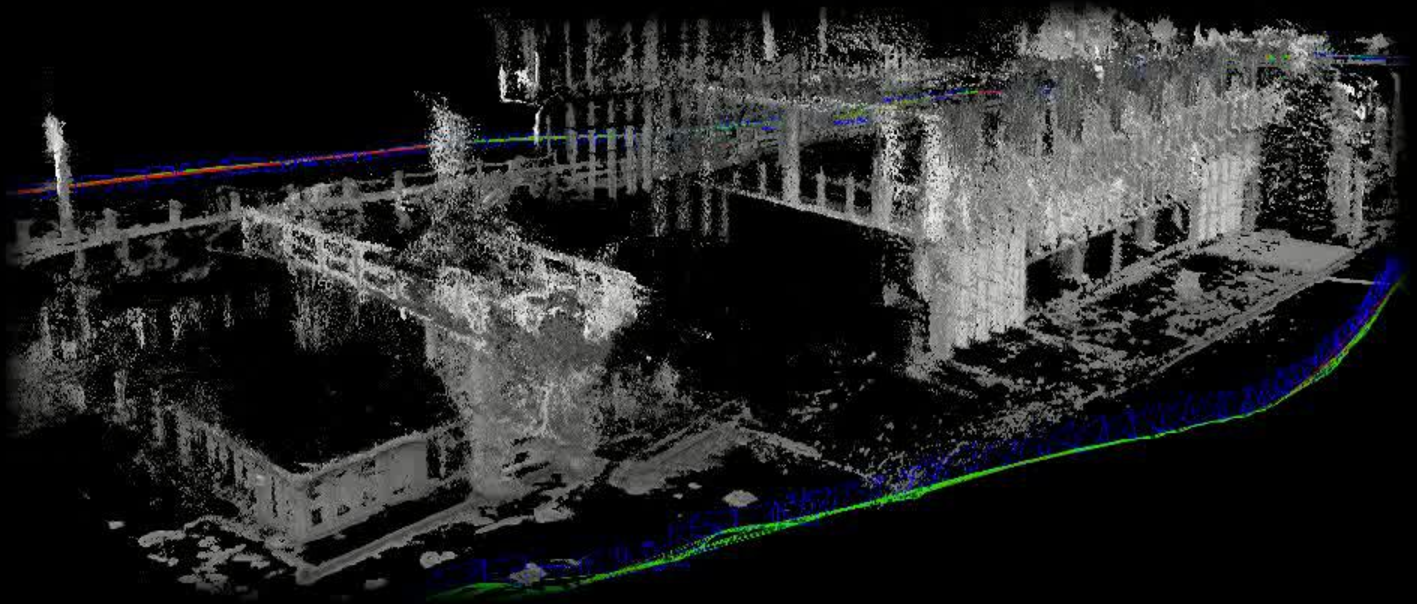


Monkey interest point tracking

LSD-SLAM: Large-Scale Direct Monocular SLAM

Jakob Engel, Thomas Schöps, Daniel Cremers

ECCV 2014, Zurich



Computer Vision Group
Department of Computer Science
Technical University of Munich