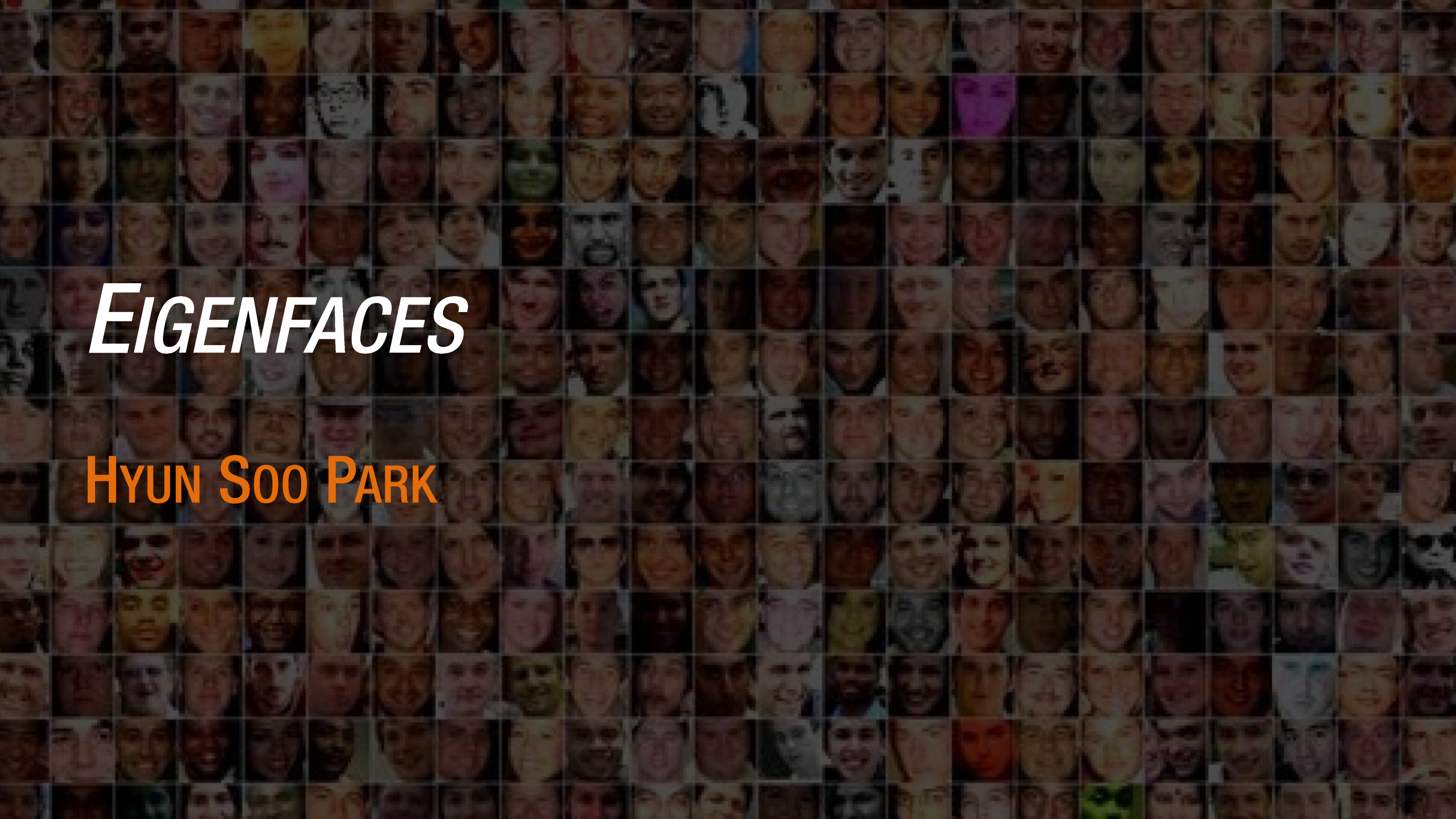

EIGENFACES

HYUN SOO PARK

Challenges of Visual Recognition

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- Appearance
    - DOF: texture, illumniation, material, shading, …
- Shape
    - DOF: object category, geometric pose, viewpoint, …

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- **Appearance**
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SPACE OF APPEARANCE (FIXED SHAPE)

Template

$x \in \mathbb{R}^D$

High dimension (D)

e.g., D: 10,000 = 100 x 100

Space of Appearance (Fixed Shape)

Template

$x \in \mathbb{R}^D$

High dimension (D)

e.g., D: 10,000 = 100 x 100

Naïve face detection algorithm:

x

x

y

Use NCC or SSD to measure similarity.

$$\text{maximize } corr(x, y)$$

$$\text{minimize} \|x - y\|^2$$

Why not working?

SPACE OF FACE APPEARANCE

SPACE OF FACE APPEARANCE

Miss Korea Contestants

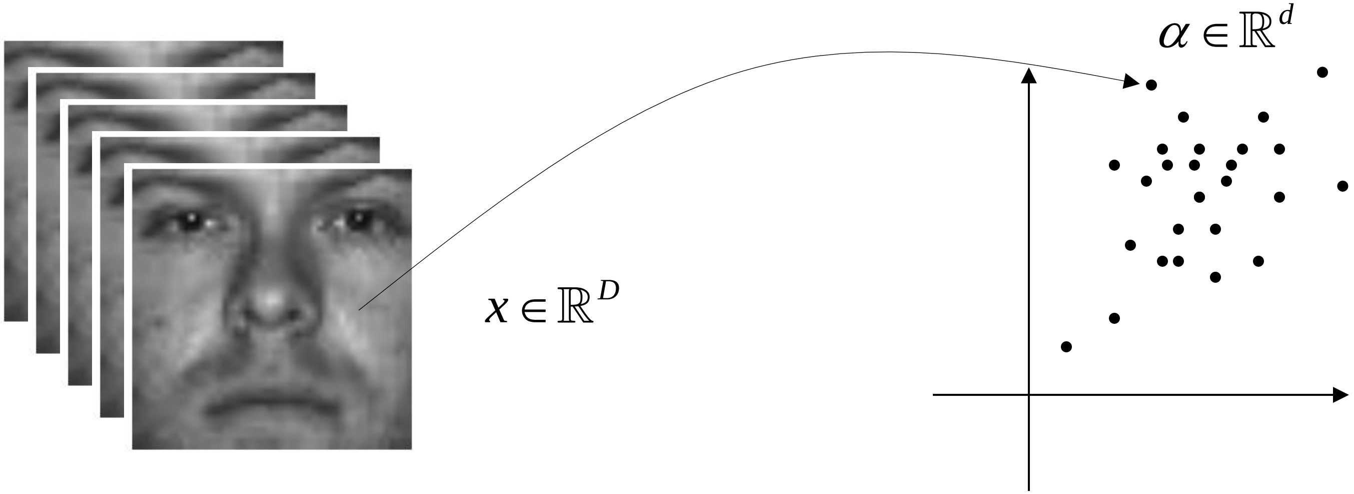
Observation: not all pixels are equally informative to detect a face

MISS KOREA CONTESTANTS

Observation: not all pixels are equally informative to detect a face

Structured Appearance

Idea: face images are highly correlated and can be represented in a low-dimensional subspace.

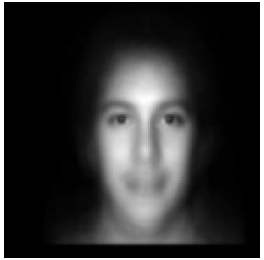


High dimension (D) Low dimension (d)

Linear Basis


Face

$=$


Mean face

$+\alpha_1$


Basis 1

$+\alpha_2$


Basis 2

$+\alpha_3$


Basis 3

$+\cdots$

$$y = m + \alpha_1 b_1 + \alpha_2 b_2 + \alpha_3 b_3 + \cdots$$

Linear Basis

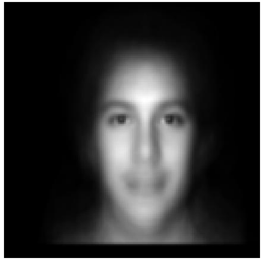
Face $=$ Mean face $+\alpha_1$ Basis 1 $+\alpha_2$ Basis 2 $+\alpha_3$ Basis 3 $+\cdots$

$$y = m + \alpha_1 b_1 + \alpha_2 b_2 + \alpha_3 b_3 + \cdots$$

Linear Basis

Face

$=$



Mean face

$+\alpha_1$

Basis 1

$+\alpha_2$

Basis 2

$+\alpha_3$

Basis 3

$+\cdots$

$$y = m + \alpha_1 b_1 + \alpha_2 b_2 + \alpha_3 b_3 + \cdots$$

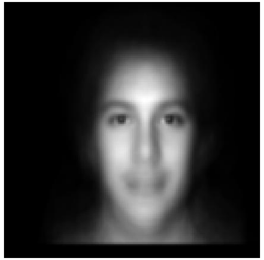
$$y \approx m + B \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_d \end{bmatrix}$$

$$y \approx m + B\alpha$$

RECONSTRUCTION FROM LINEAR BASIS

Face

$=$



Mean face

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Basis 1

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Basis 2

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Basis 3

$+\cdots$

$$y = m + \alpha_1 b_1 + \alpha_2 b_2 + \alpha_3 b_3 + \cdots$$

$$\begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_d \end{bmatrix}$$

$$y \approx m + B \alpha$$

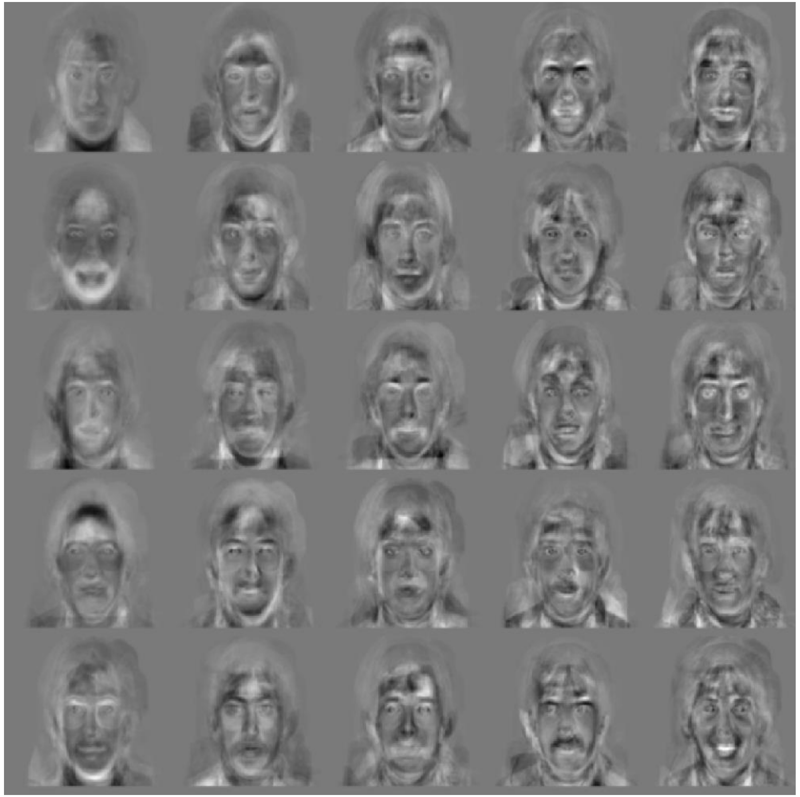
Mean, Basis, Coefficient

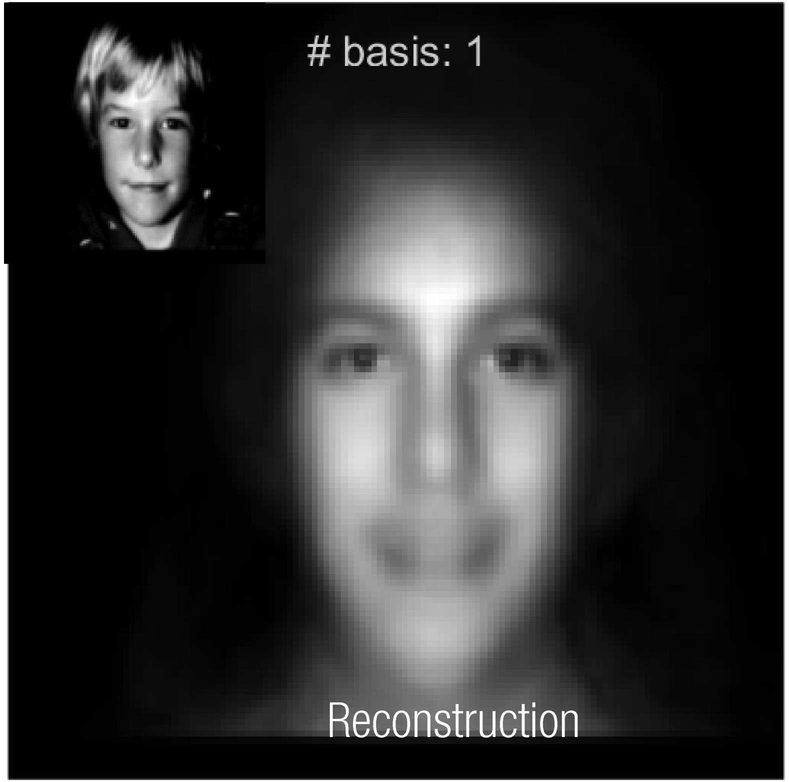
$$\alpha^* = \underset{\alpha}{\text{minimize}} \|y - m - B\alpha\|^2$$

Cf) $\text{minimize} \|x - y\|^2$

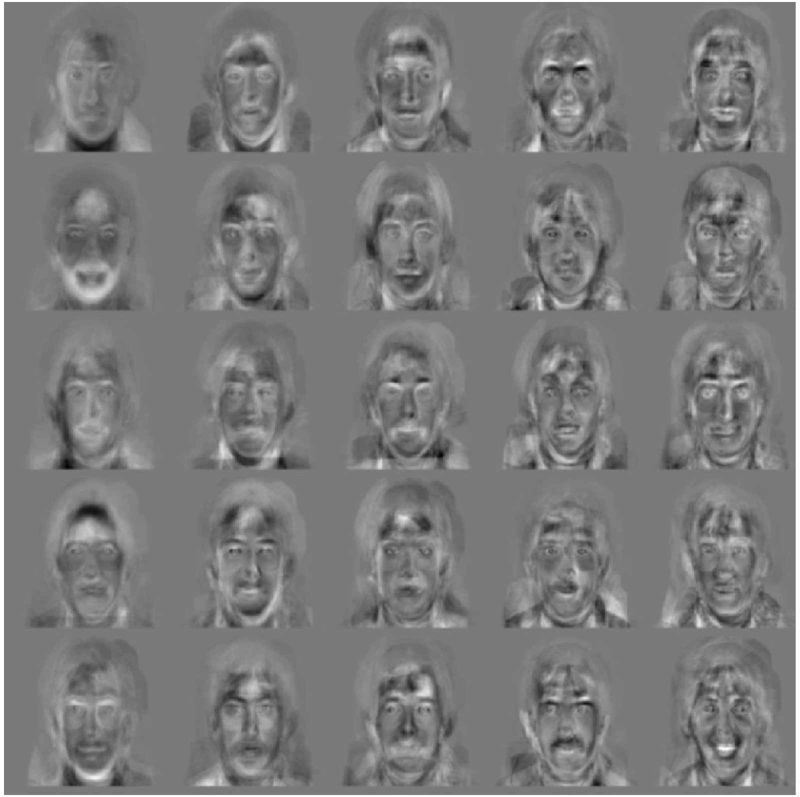
Template matching

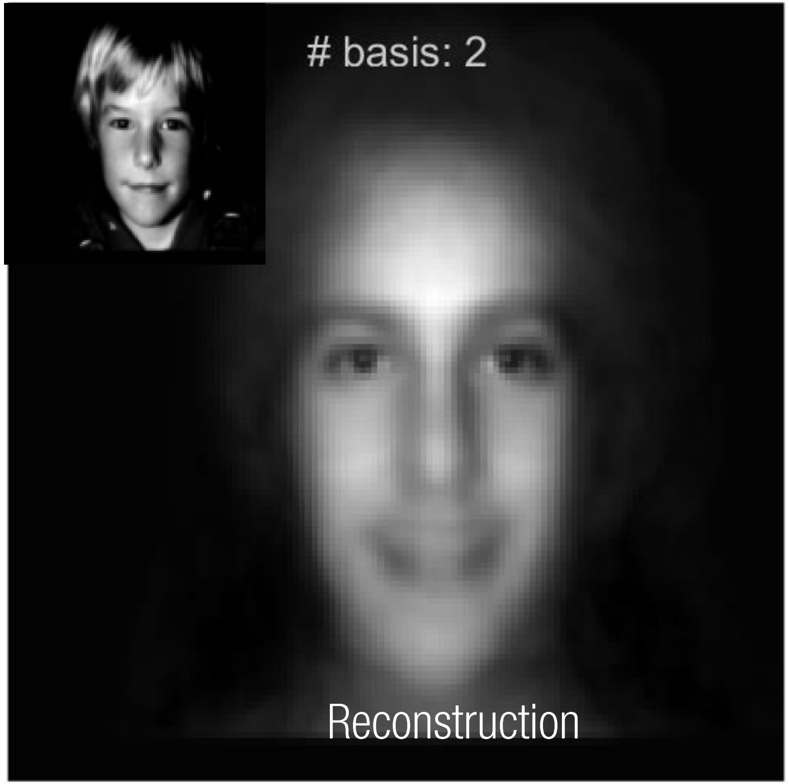
Reconstruction Expressibility



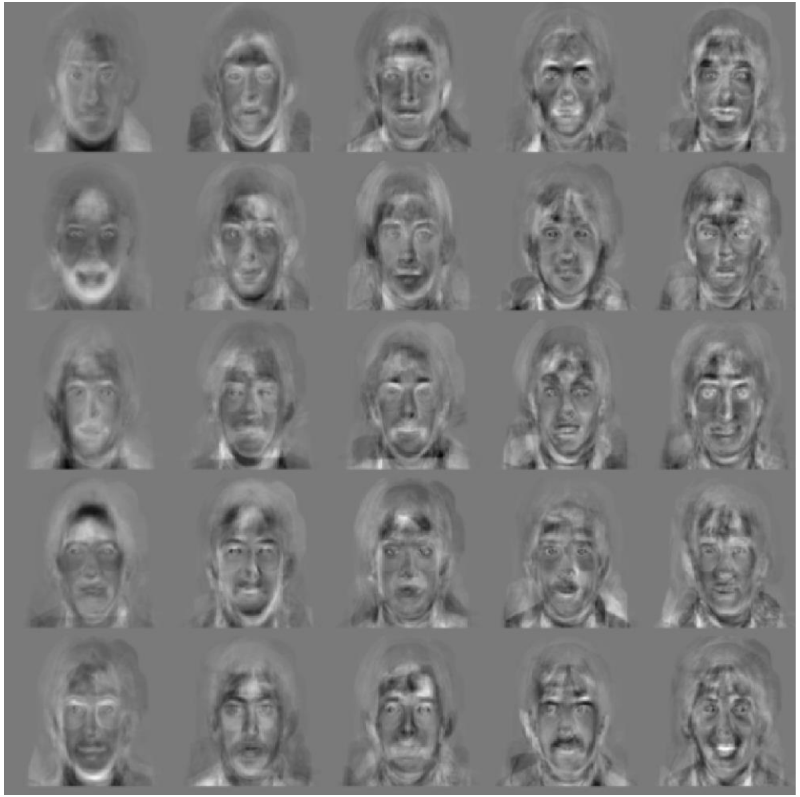


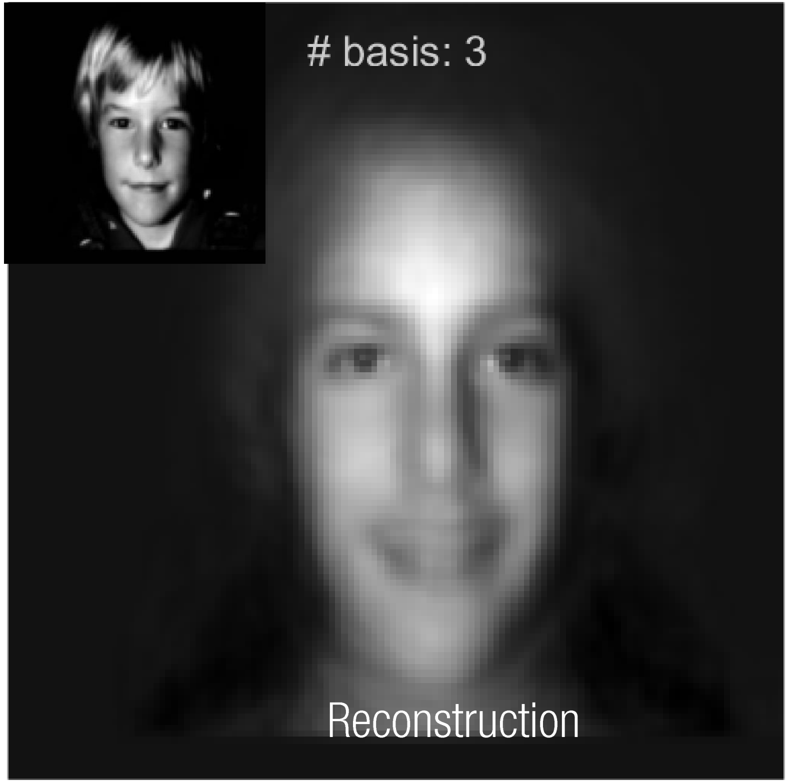
RECONSTRUCTION EXPRESSIBILITY





Reconstruction Expressibility





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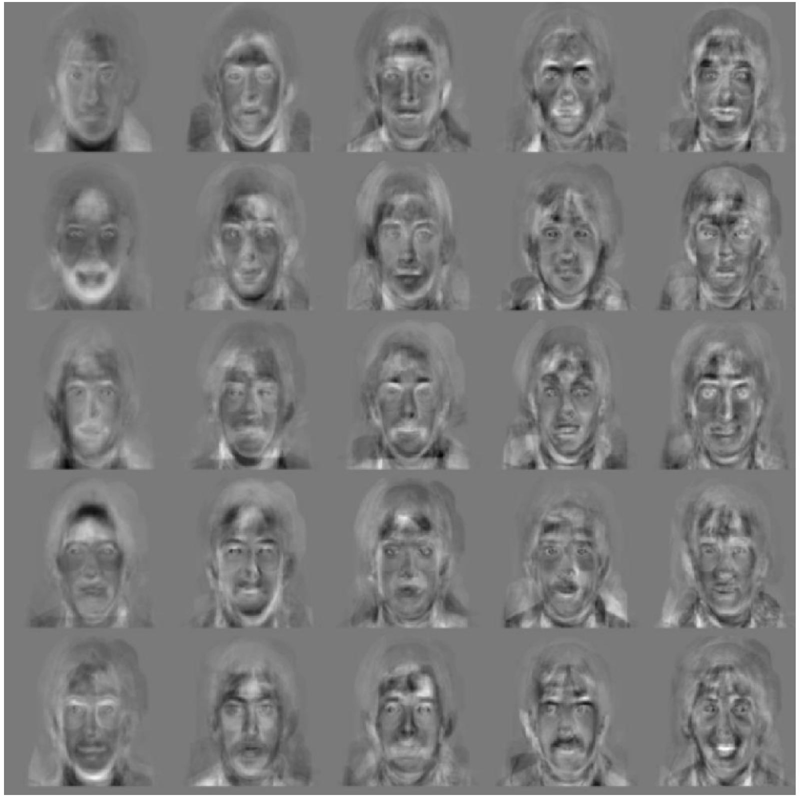
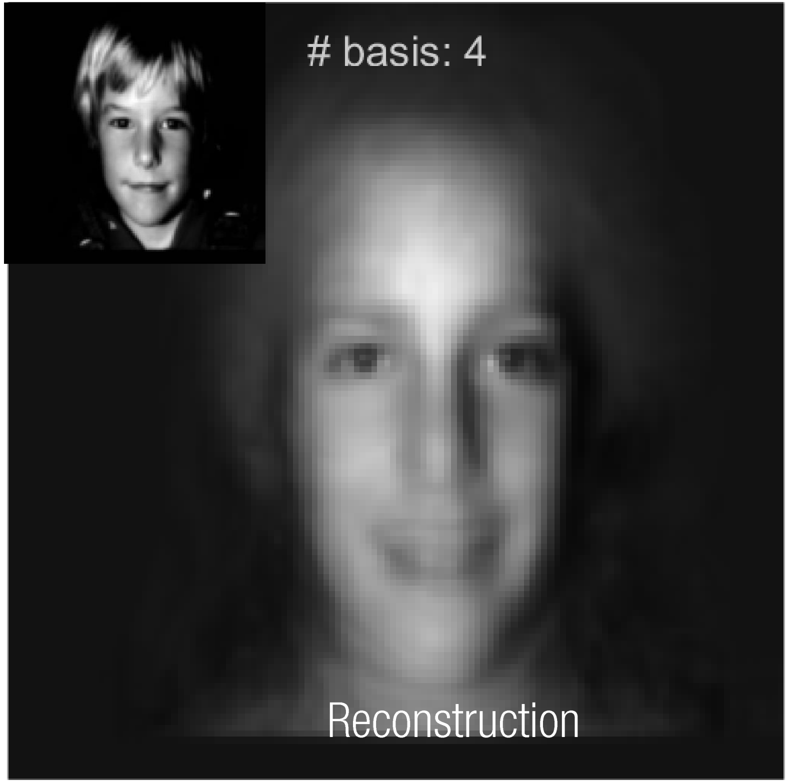
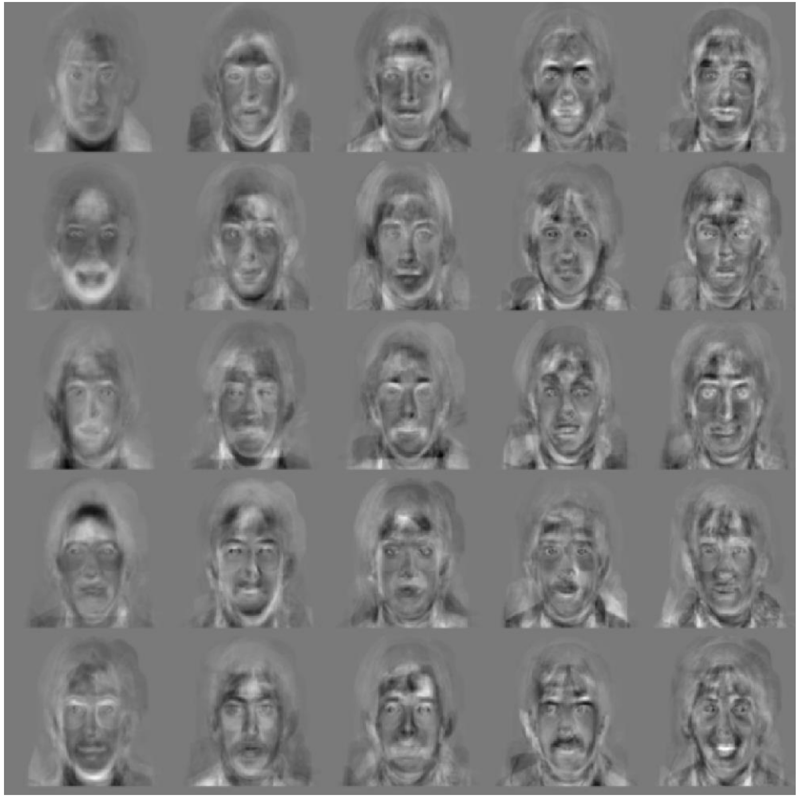
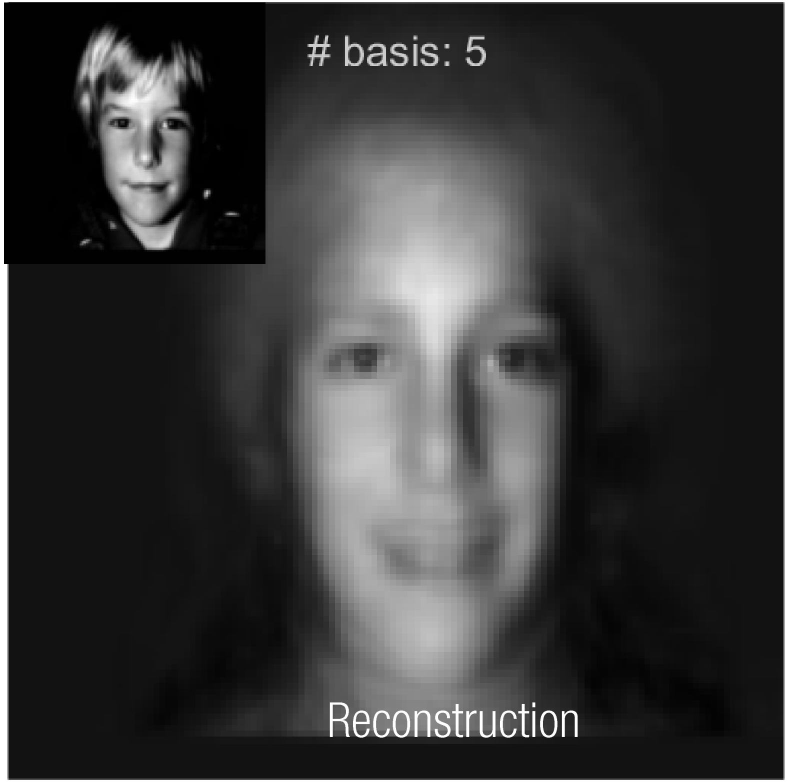




Reconstruction Expressibility





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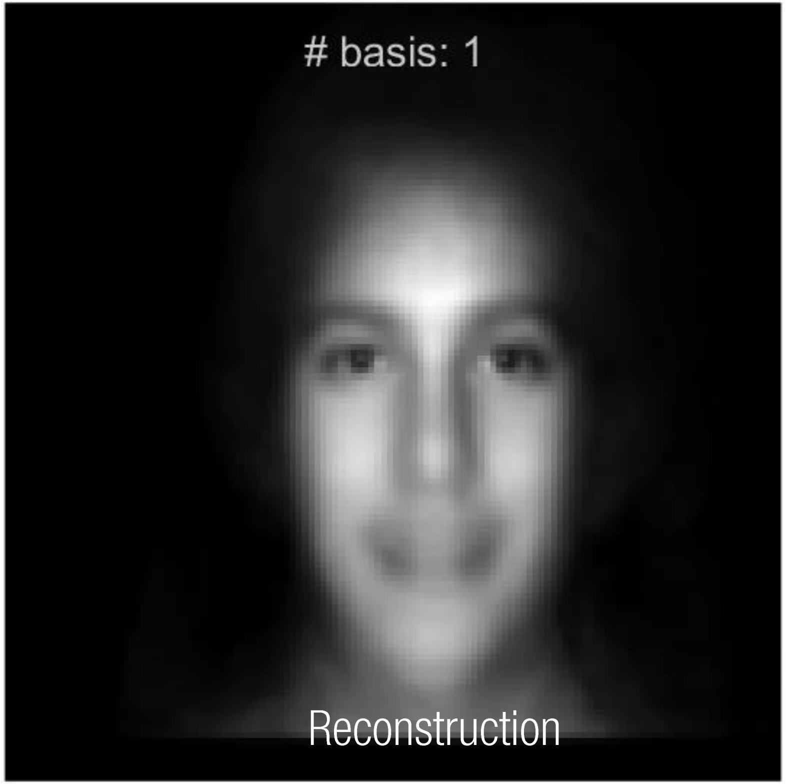


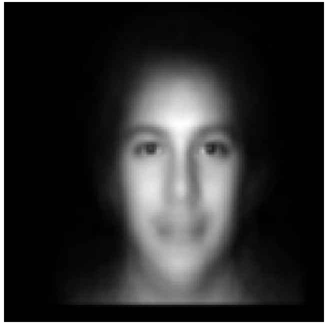
How to Compute Mean and Basis from Database?

$$m^*, B^*, \alpha^* = \underset{m,B,\alpha}{\text{minimize}} \left\| y - m - B\alpha \right\|^2$$

How to Compute Mean?

$$m^*, B^*, \alpha^* = \underset{m,B,\alpha}{\text{minimize}} \left\| y - m - B\alpha \right\|^2$$

$$m = \frac{1}{n} \sum_{i}^{n} x_i$$



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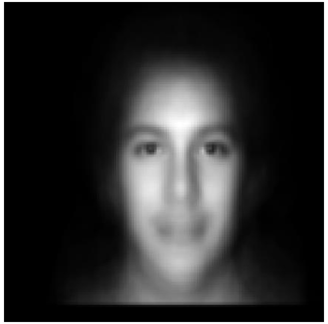
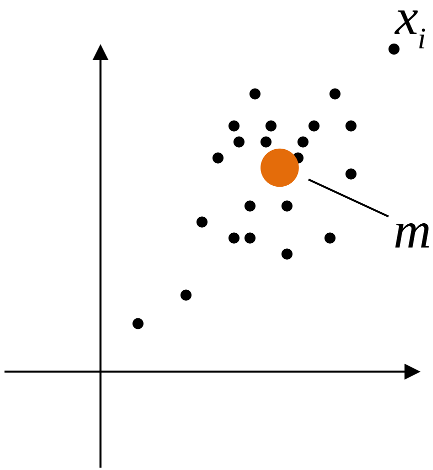
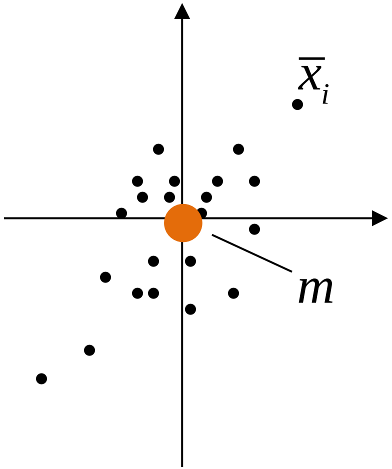


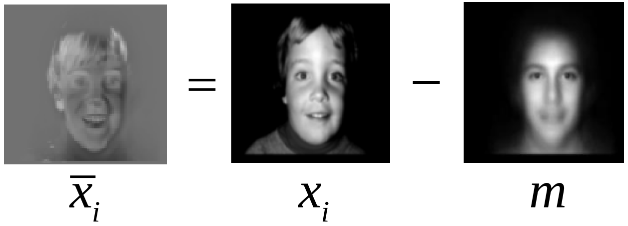


Mean Subtraction

$$m^*, B^*, \alpha^* = \underset{m,B,\alpha}{\text{minimize}} \left\| y - m - B\alpha \right\|^2$$

$$m = \frac{1}{n} \sum_{i}^{n} x_i$$





How to Computer Basis?

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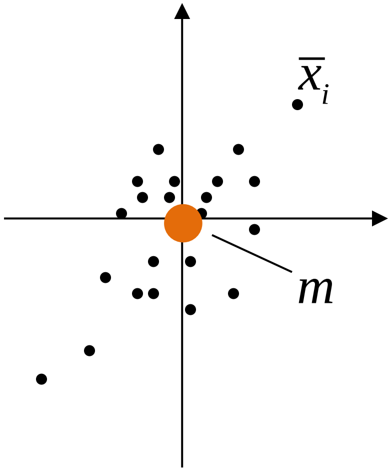
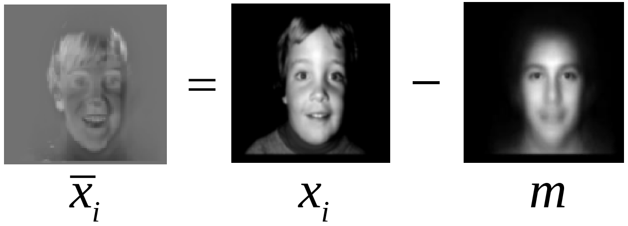
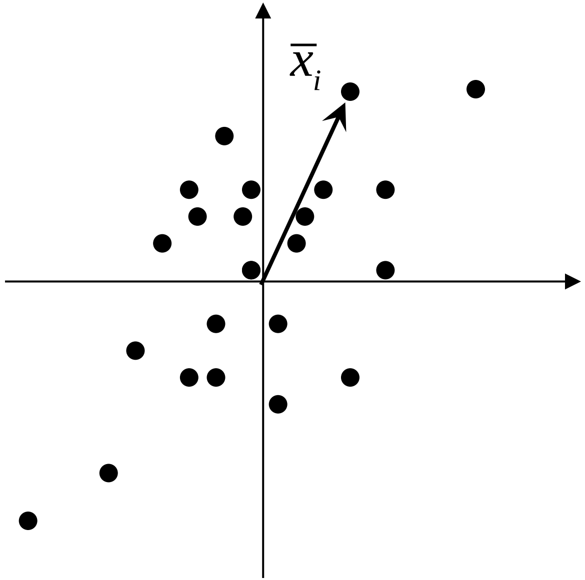




PRINCIPAL AXIS



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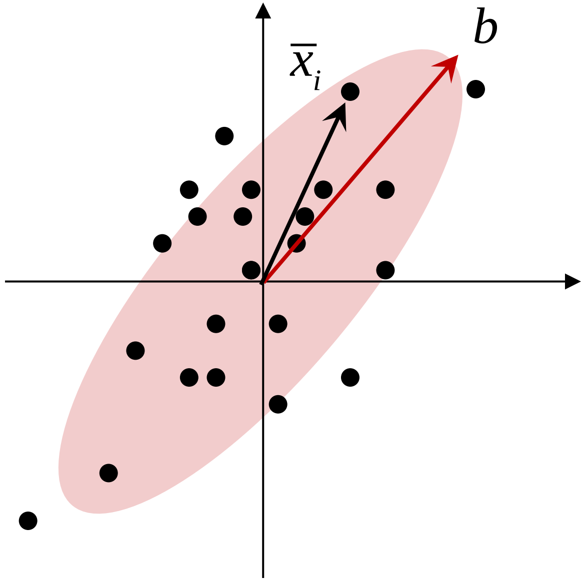
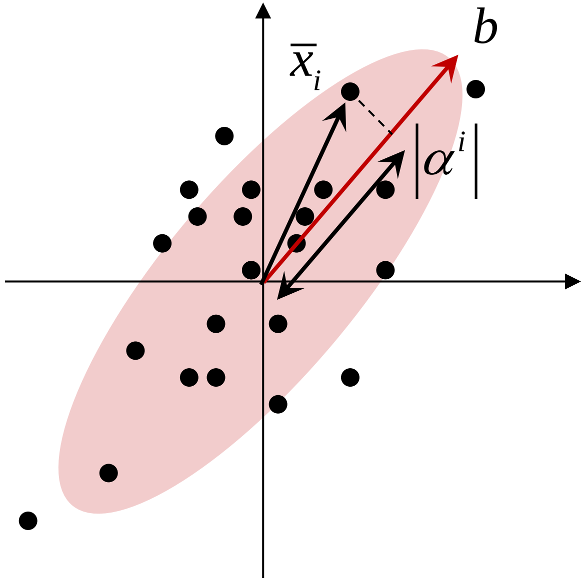


Basis is the axis that represents the maximum data covariance.

Principal Axis



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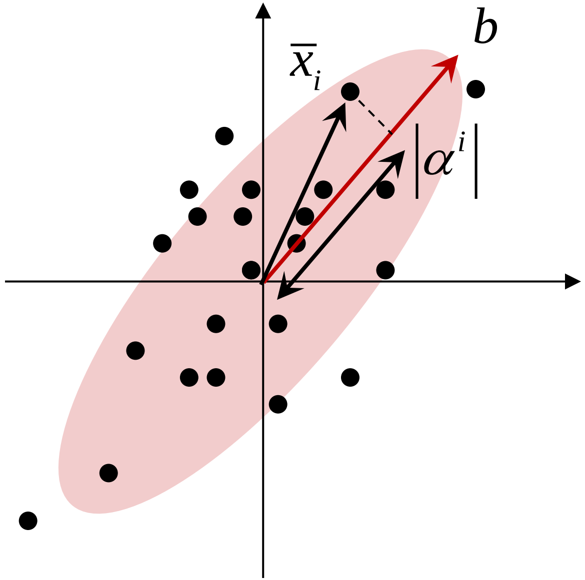
Coefficient $$\alpha^i = \frac{b \cdot \overline{x}_i}{\|b\|}$$

Principal Axis



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$$b^* = \underset{b}{\text{maximize}} \sum_{i=1}^{n} \left(\alpha^i\right)^2$$

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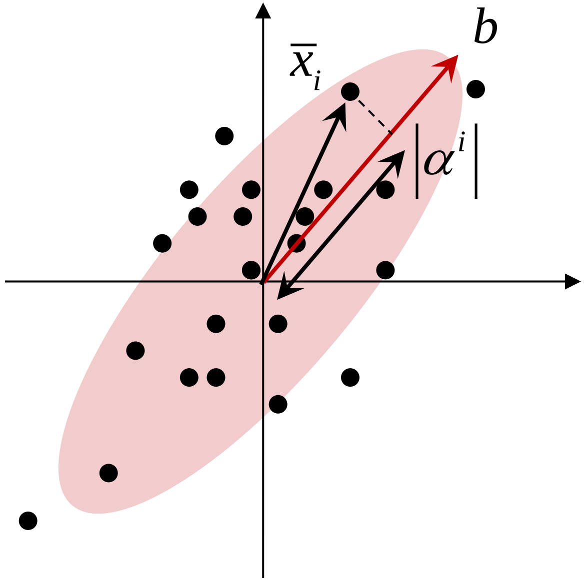

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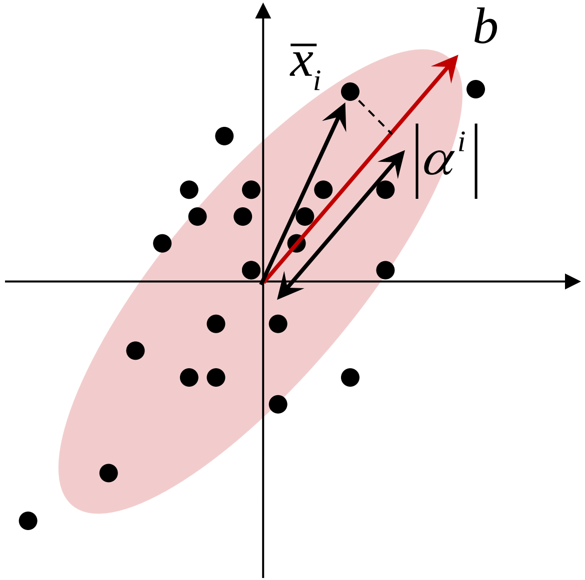

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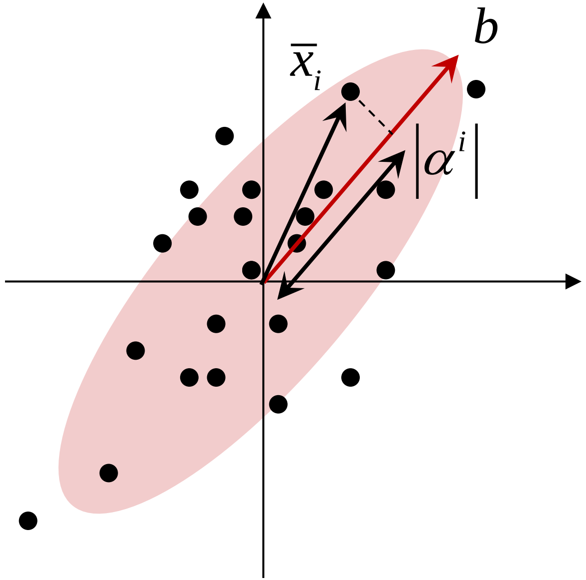
$$= \underset{b}{\text{maximize}} \sum_{i=1}^{n} \left(\frac{b \cdot \overline{x}_i}{\|b\|}\right)^2$$

$$= \underset{b}{\text{maximize}}\, b^T \underbrace{X^T X}_{\text{Covariance matrix}} b$$

where $X = \begin{bmatrix} x_1^T \\ \vdots \\ x_n^T \end{bmatrix}$

PRINCIPAL AXIS



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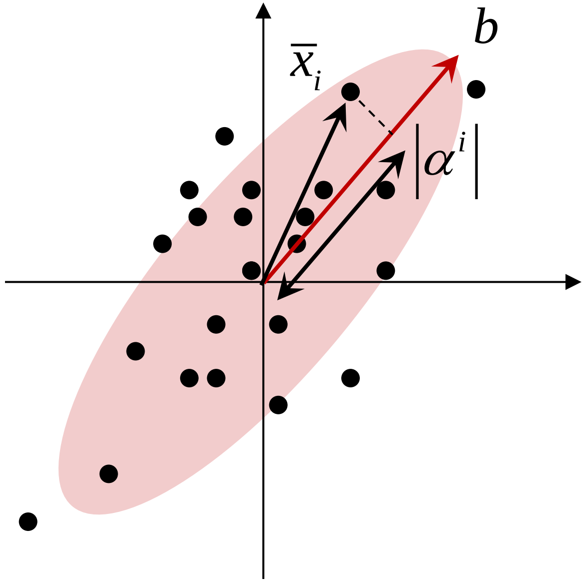
$$= \underset{b}{\text{maximize}}\, b^T \underbrace{X^T X}_{\text{Covariance matrix}} b$$

Solution is the eigenvector corresponding to the largest eigenvalue: $\quad b^* = \lambda_{\max}(X^T X)$

PRINCIPAL AXIS



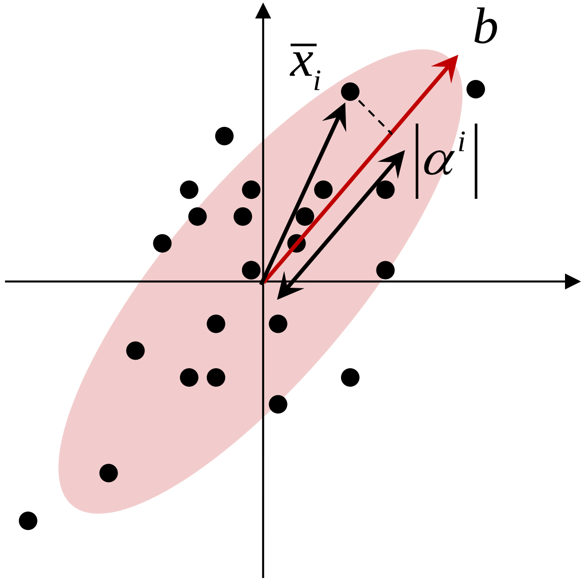
Basis is the axis that represents the maximum data covariance.

Coefficient $\alpha^i = \dfrac{b \cdot \overline{x}_i}{\|b\|}$

$$\begin{bmatrix} \alpha^1 \\ \vdots \\ \alpha^n \end{bmatrix} = \begin{bmatrix} \overline{x}_1^T \\ \vdots \\ \overline{x}_n^T \end{bmatrix} \begin{bmatrix} b \end{bmatrix}$$

nx1 nxD Dx1

PRINCIPAL AXES



Basis is the axis that represents the maximum data covariance.

Coefficient $\alpha_j^i = \dfrac{b_j \cdot \bar{x}_i}{\|b_j\|}$

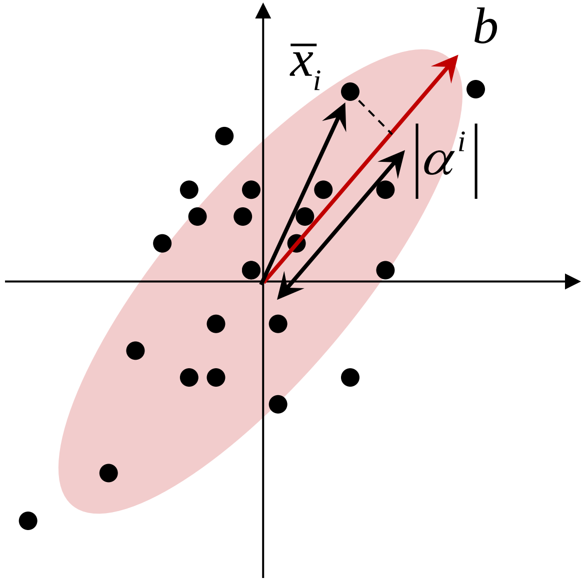
Orthogonal principal axes: first *d* largest eigenvectors

$$\begin{bmatrix} \alpha_1^1 & \alpha_d^1 \\ \vdots & \vdots \\ \alpha_n^1 & \alpha_n^d \end{bmatrix} = \begin{bmatrix} \bar{x}_1^T \\ \vdots \\ \bar{x}_n^T \end{bmatrix} \begin{bmatrix} b_1 & \cdots & b_d \end{bmatrix}$$

nxd nxD Dxd

$$d \ll D$$

PCA: Dimensional Reduction

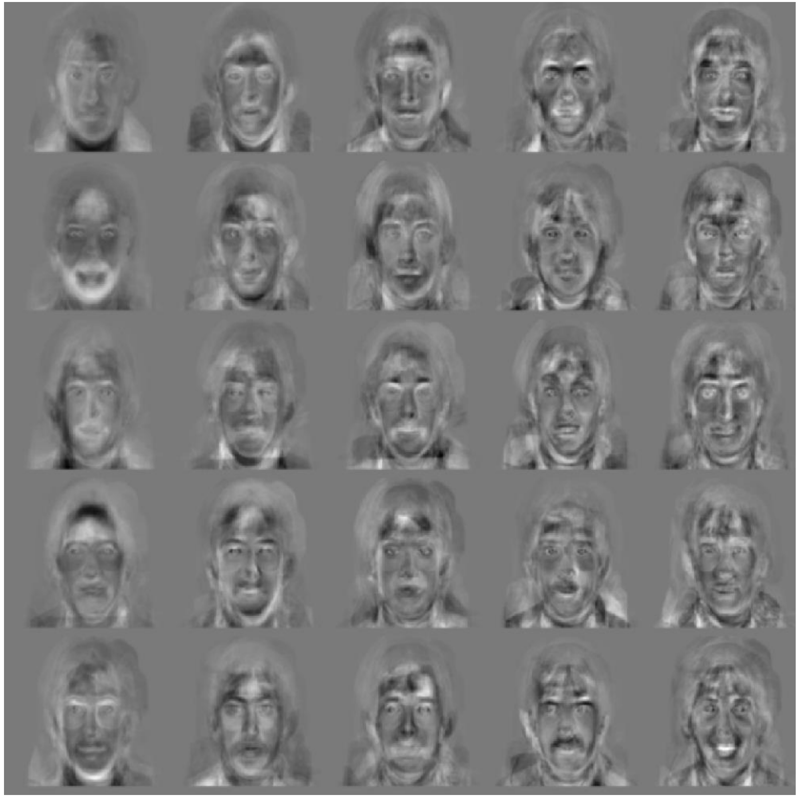


$$A = X \; B$$

nxd	nxD	Dxd

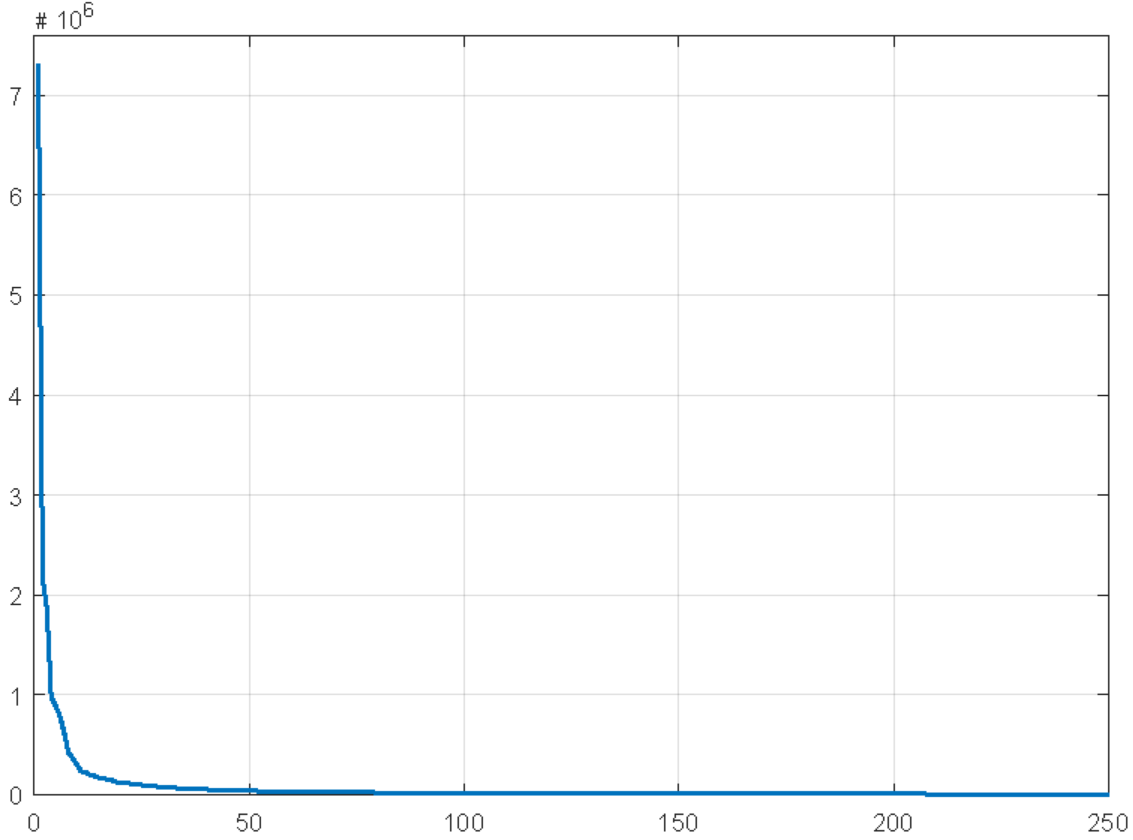
$d \ll D$

How to Compute Basis?

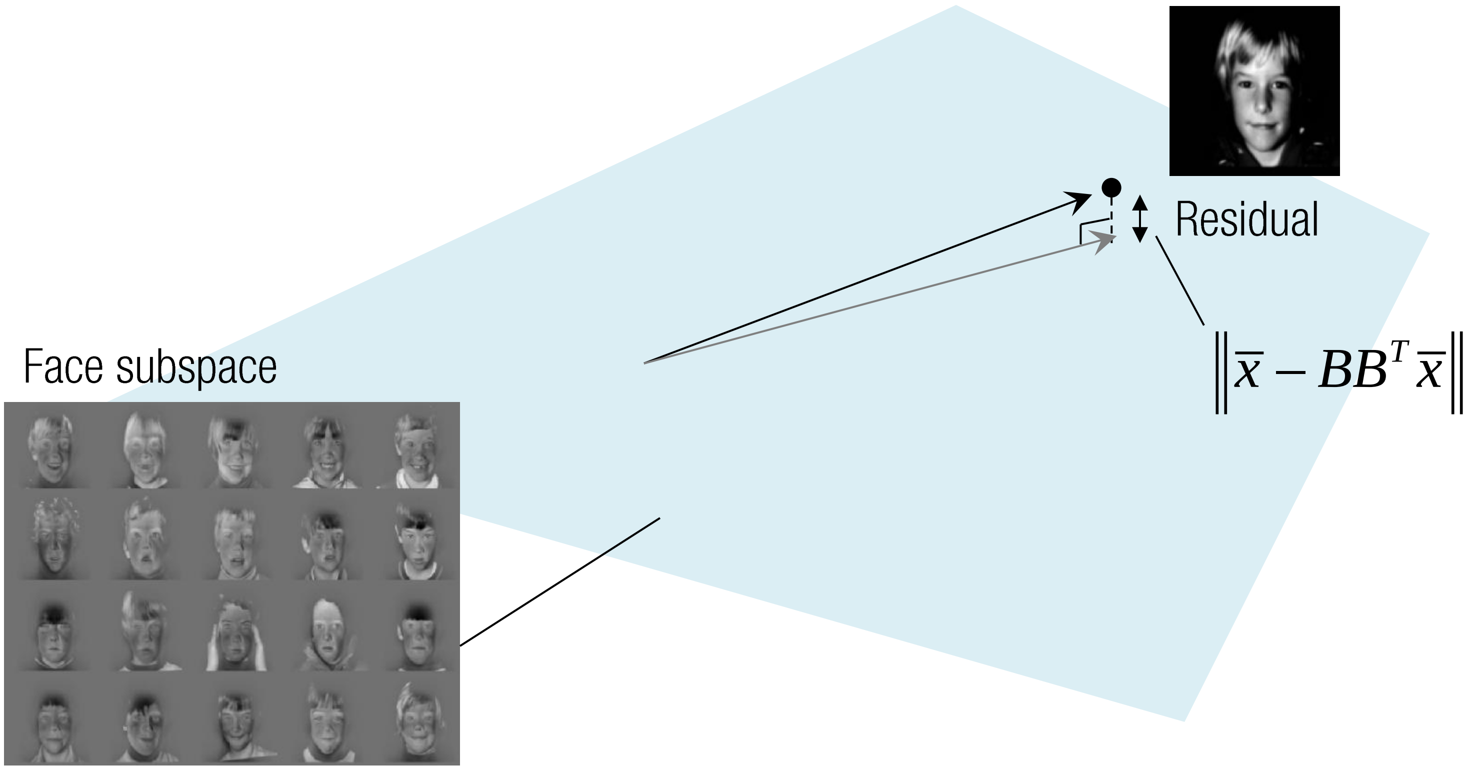


Set of basis vectors

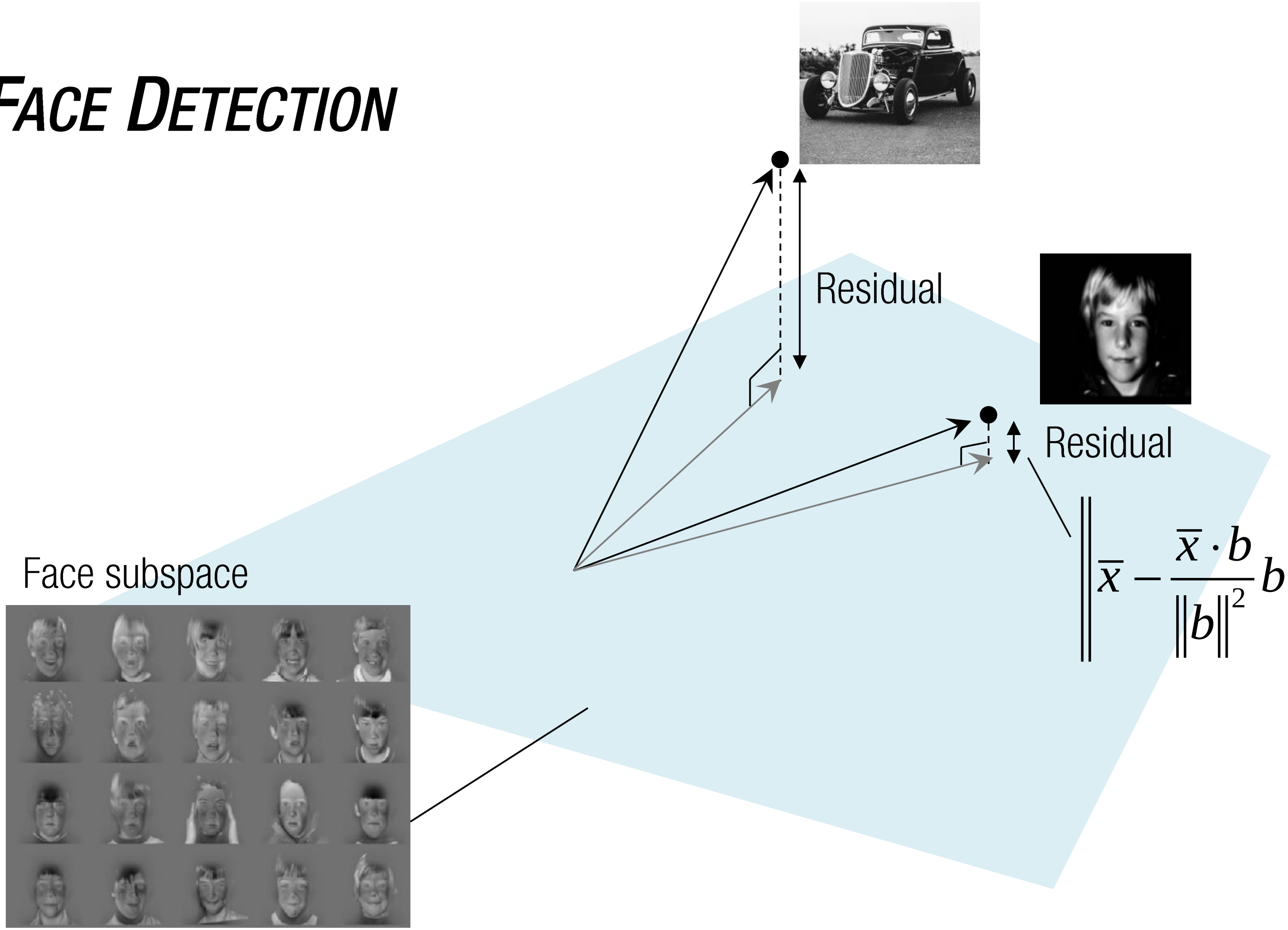
How to Choose # of Basis Vectors?



Face Detection

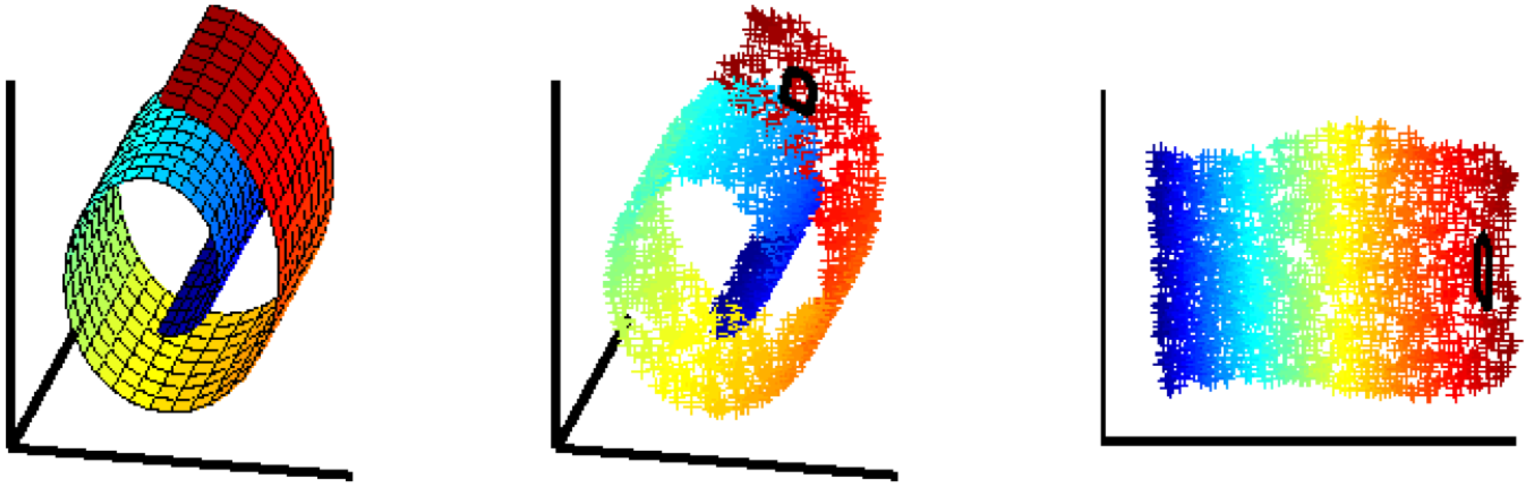


FACE DETECTION



$$\left\| \bar{x} - \frac{\bar{x} \cdot b}{\|b\|^2} b \right\|$$

LIMITATION



Object distribution does not follow Guassian!

https://www.youtube.com/watch?v=J0arU2PAMIs