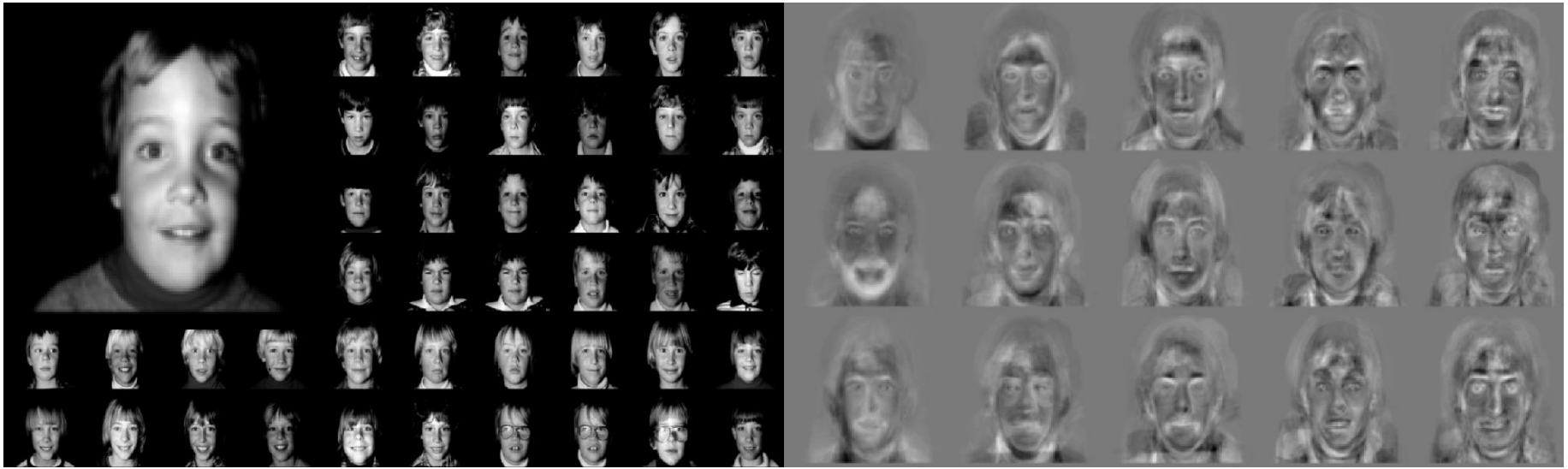

FISHER FACE
HYUN SOO PARK

RECALL: EIGENFACES



Reconstruction

$$\alpha^* = \underset{\alpha}{\text{minimize}} \left\| y - m - B\alpha \right\|^2$$

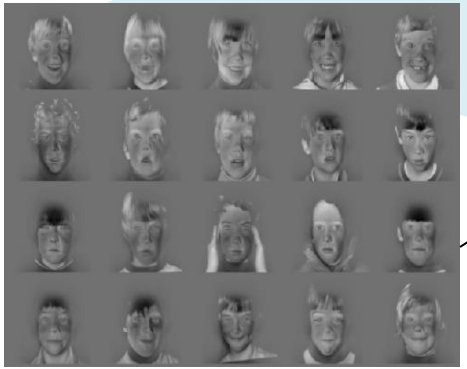
FACE DETECTION

Residual

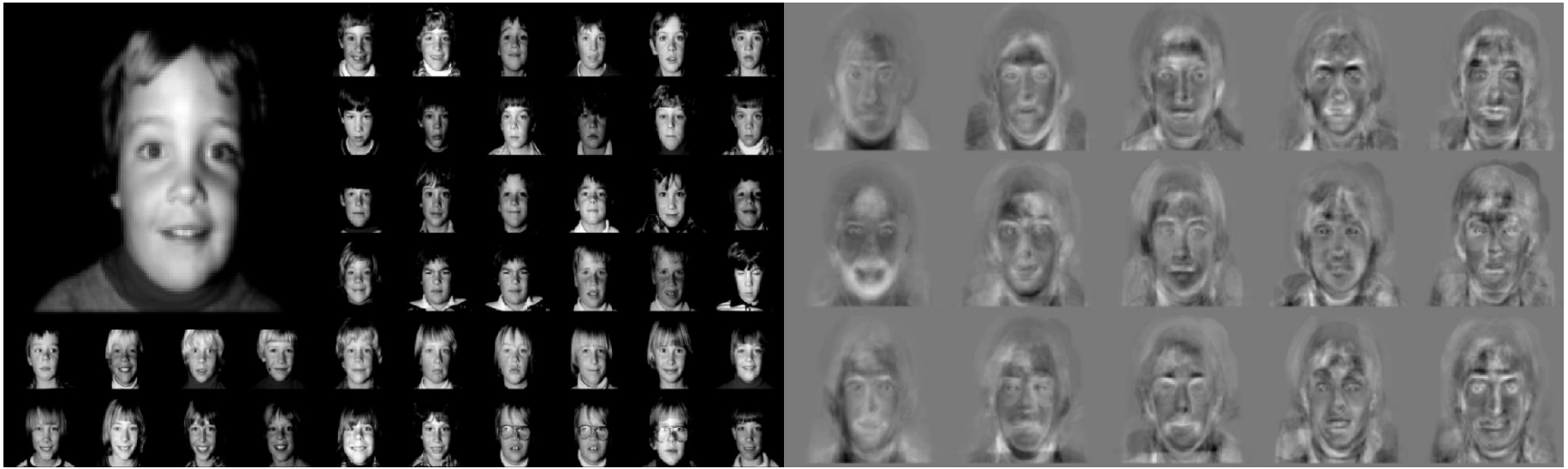
Residual

$$\left\| \overline{x} - \frac{\overline{x} \cdot b}{\|b\|^2} b \right\|$$

Face subspace



RECALL: EIGENFACES



Reconstruction

$$\alpha^* = \underset{\alpha}{\text{minimize}} \left\| y - m - B\alpha \right\|^2$$

Detection

$$\left\| \overline{x} - \frac{\overline{x} \cdot b}{\left\| b \right\|^2} b \right\|$$

FACE RECOGNITION WITH EIGENFACE

Who is this guy?

Subject 1

Subject 2

Subject 3

FACE RECOGNITION WITH EIGENFACE COEFFICIENT

Who is this guy?

α

Subject 1

$$\alpha_1 = \left\{\alpha_1^1, \cdots, \alpha_1^n\right\}$$

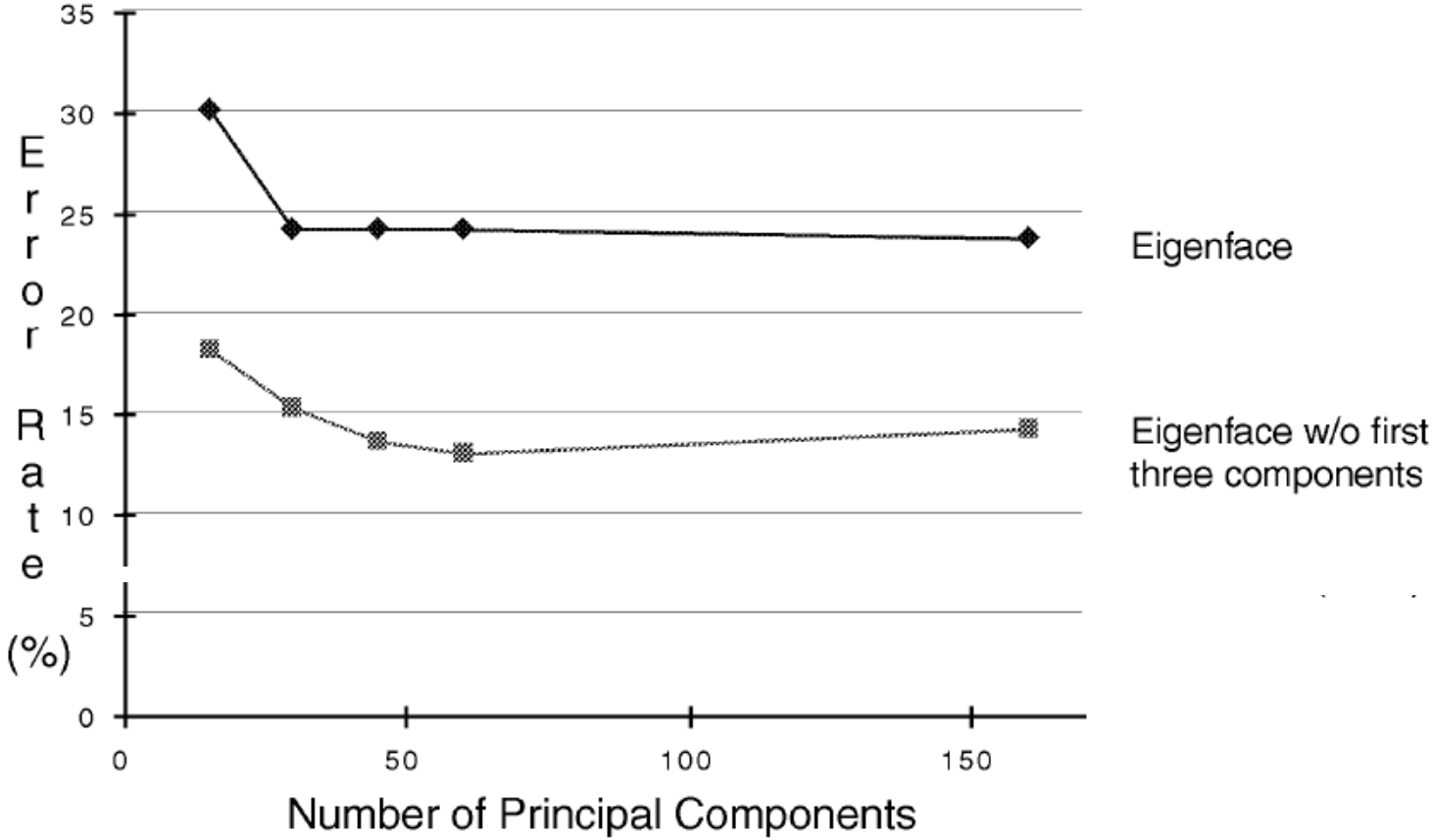
Subject 2

$$\alpha_2 = \left\{\alpha_2^1, \cdots, \alpha_2^n\right\}$$

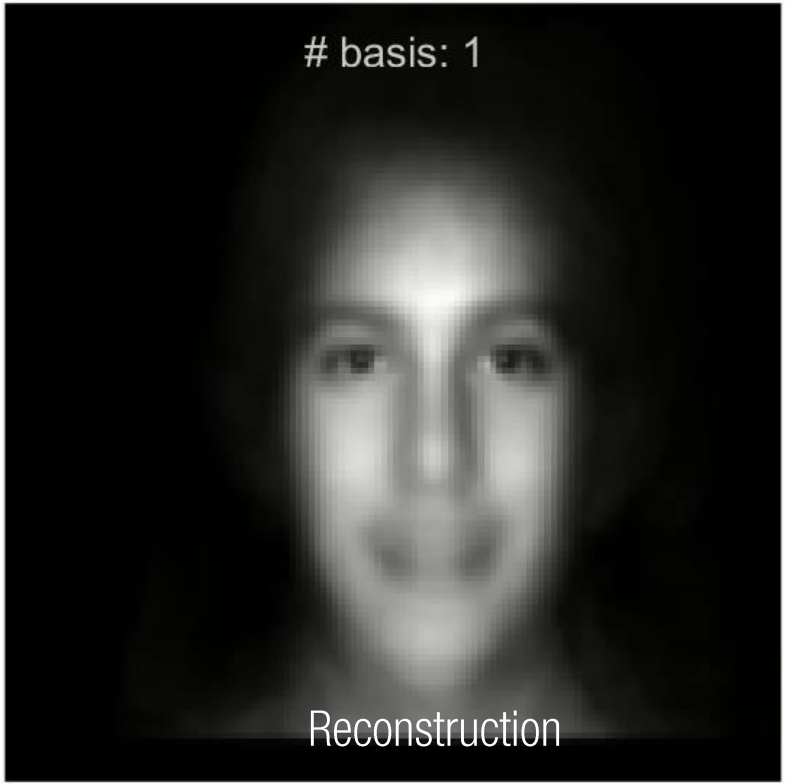
Subject 3

$$\alpha_3 = \left\{\alpha_3^1, \cdots, \alpha_3^n\right\}$$

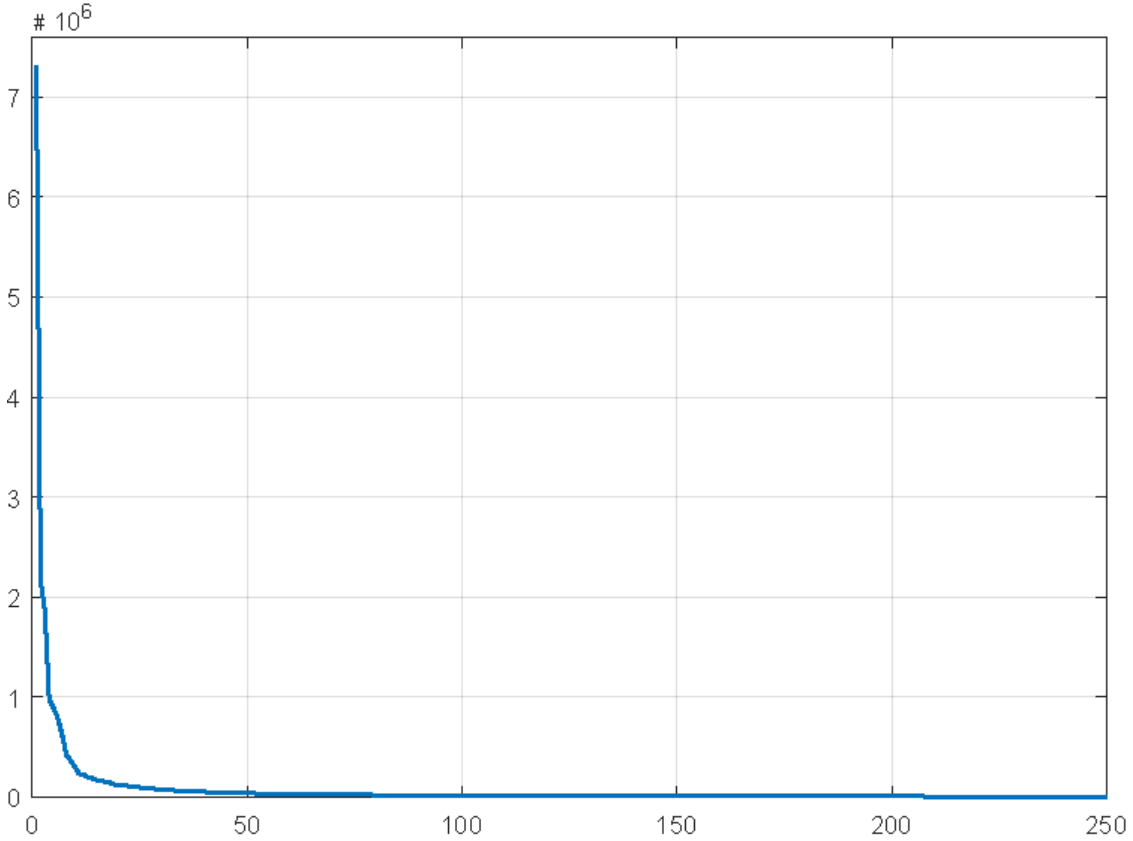
FACE RECOGNITION WITH EIGENFACES



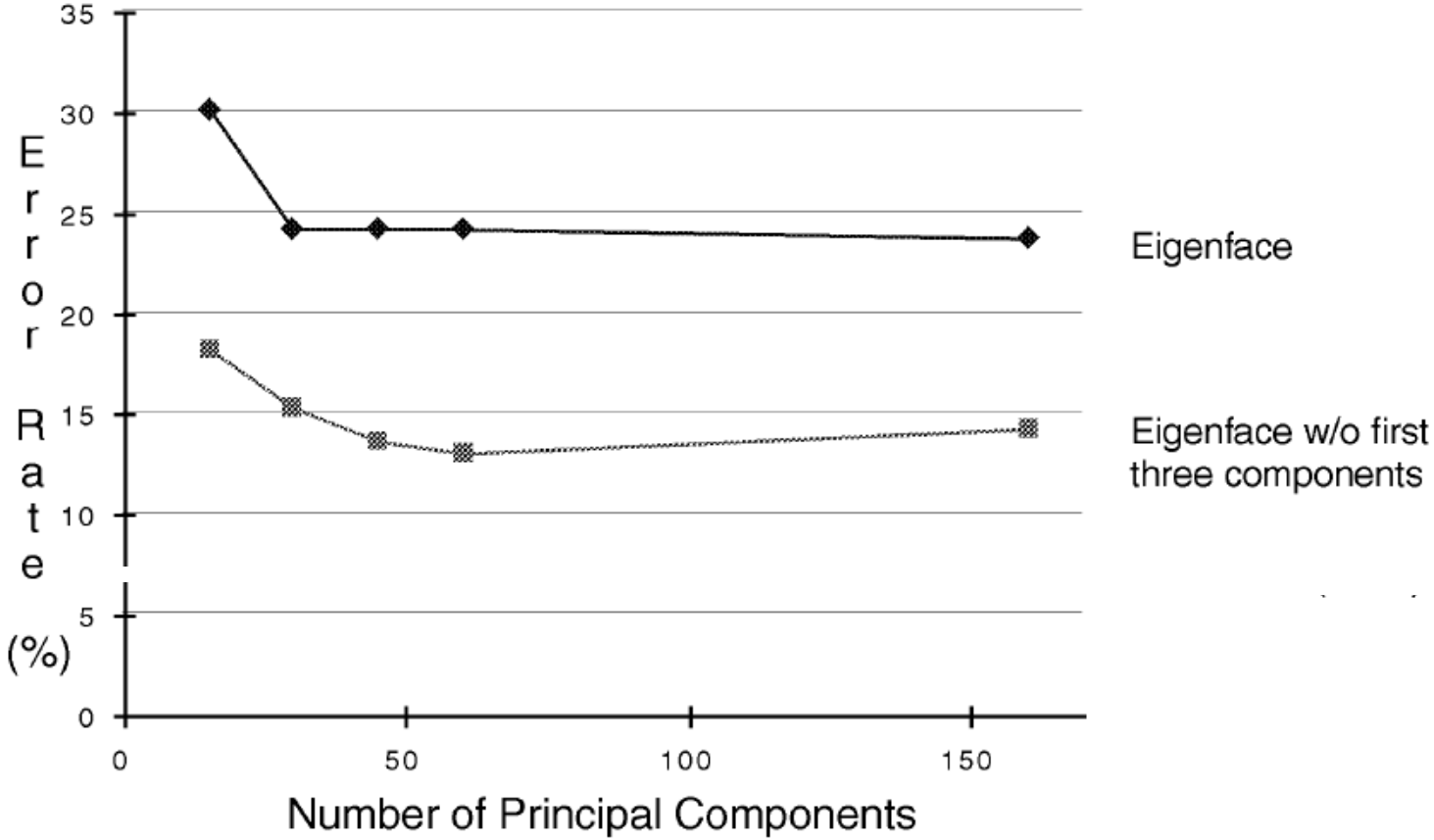
RECONSTRUCTION EXPRESSIBILITY

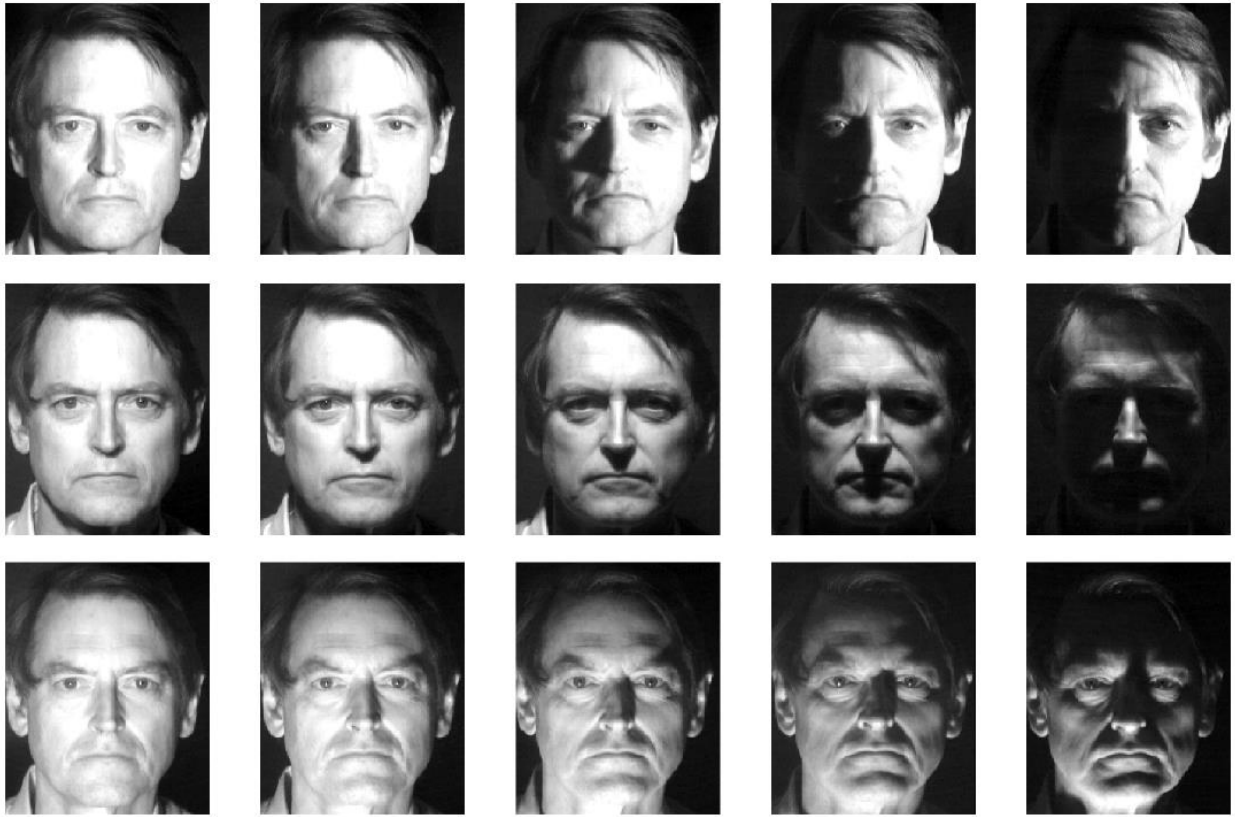


How to Choose # of Basis Vectors?



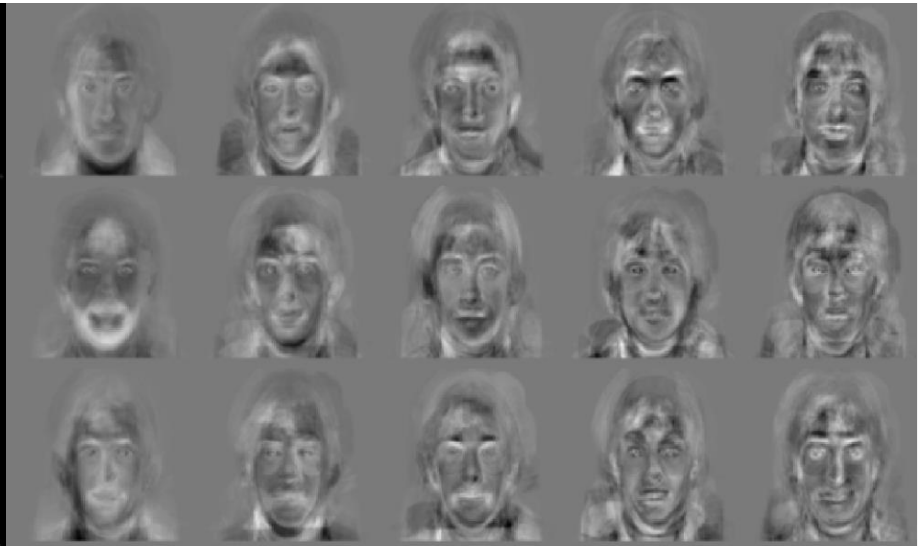
FACE RECOGNITION WITH EIGENFACES





“The variations between the images of the same face due to illumination and viewing direction are almost always larger than image variations due to change in face identity.”

RECALL: EIGENFACES



Reconstruction

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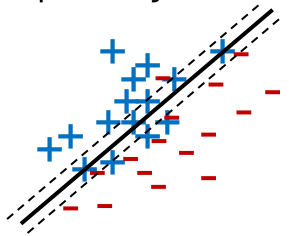
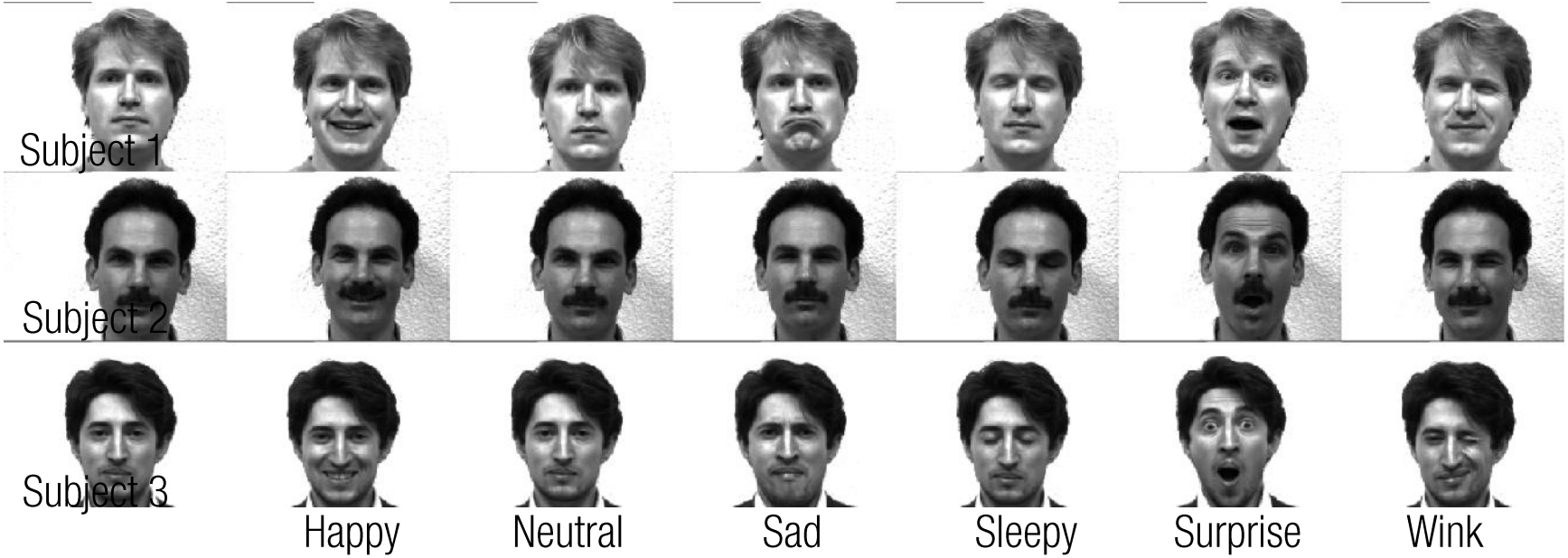
Detection

$$\left\| \overline{x} - \frac{\overline{x} \cdot b}{\|b\|^2} b \right\|$$

Not optimally discriminative.

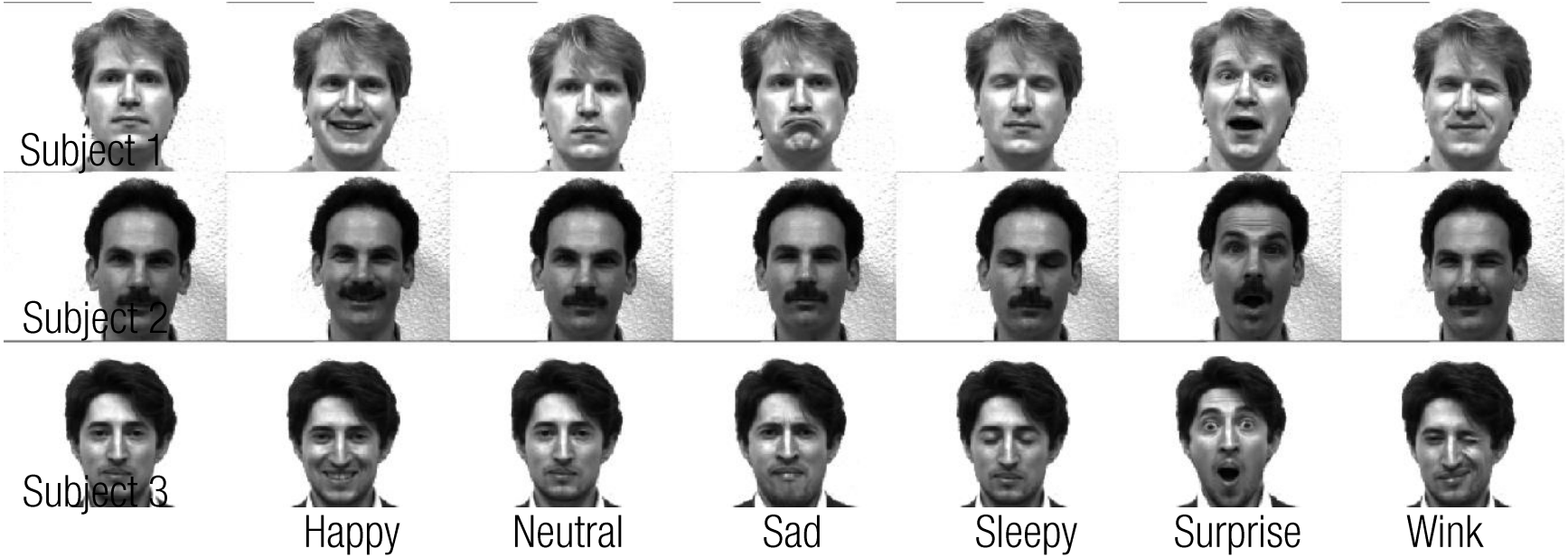


LINEAR DISCRIMINANT ANALSYS



LDA ~ PCA + Class label ~ *Discriminative PCA*

LINEAR DISCRIMINANT ANALSYS

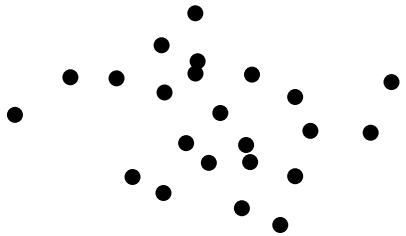


LDA ~ PCA + Class label ~ *Discriminative PCA*

Find projection that maximizes scatter between classes and minimizes scatter within classes

PCA vs. LDA

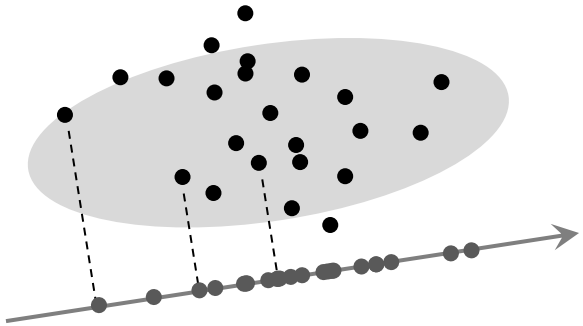
PCA



Maximize covariance

PCA vs. LDA

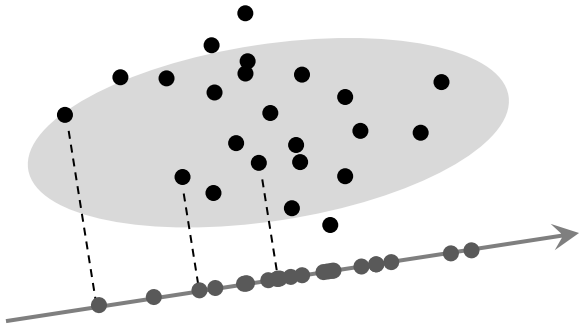
PCA



Maximize covariance

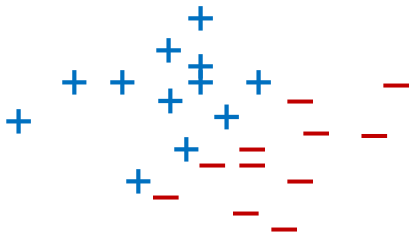
PCA vs. LDA

PCA



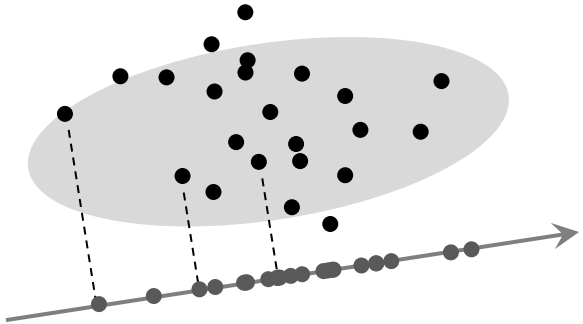
Maximize covariance

LDA



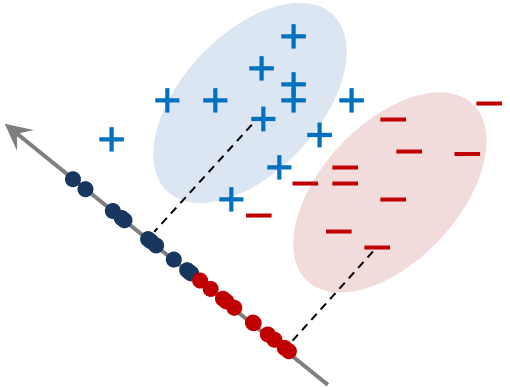
PCA vs. LDA

PCA

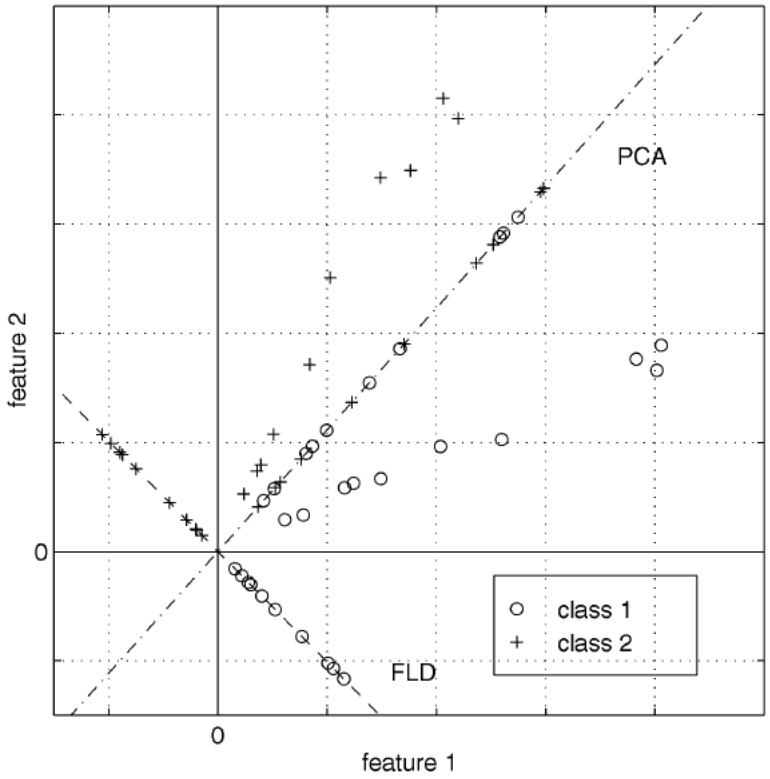


Maximize covariance

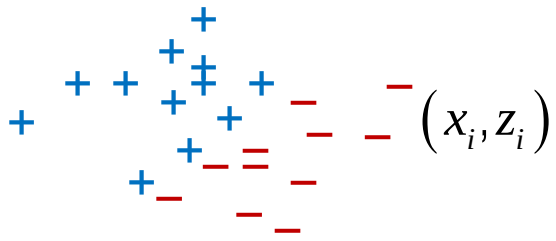
LDA



Maximize discriminance


PCA
FLD
class 1
class 2
feature 2
feature 1
0
0

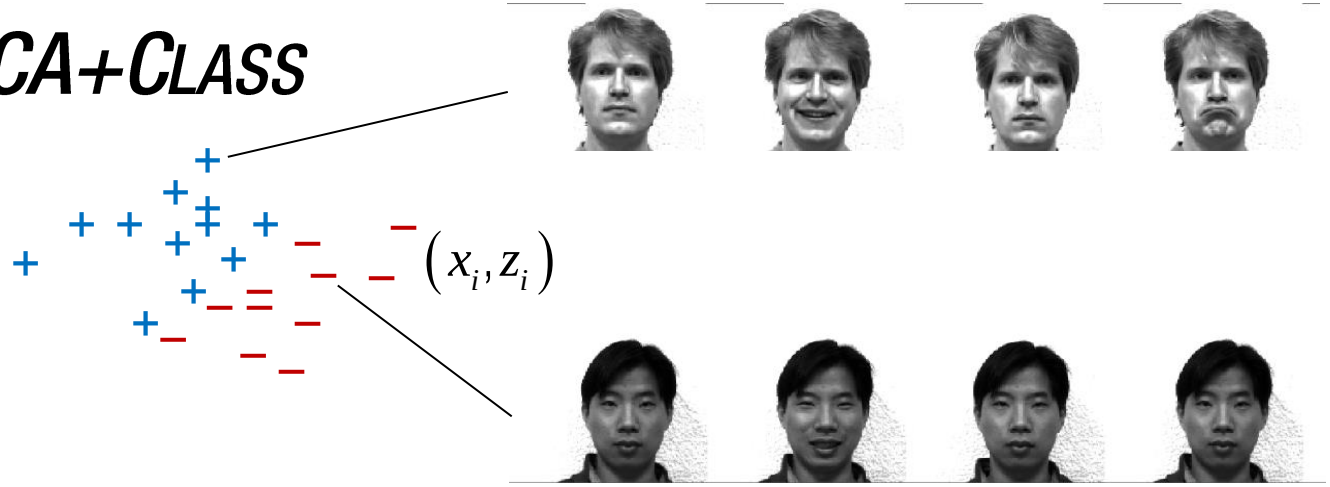
PCA+Class



(x_i, z_i)

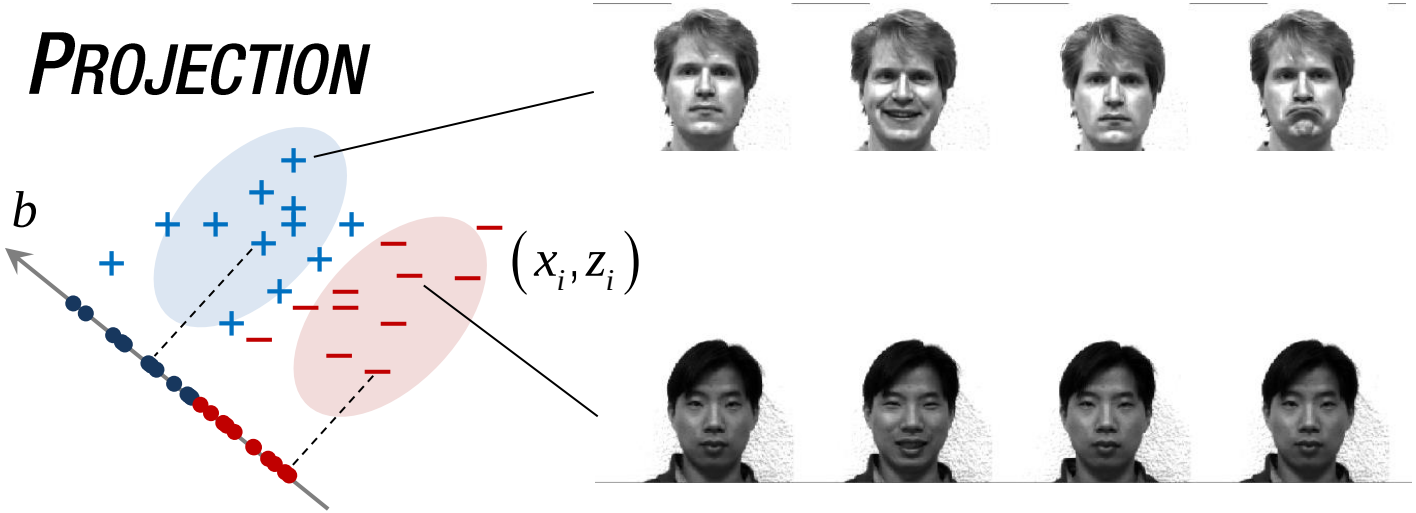
(x_i, z_i): image x with label z

PCA+CLASS



(x_i, z_i): image x with label z

PROJECTION

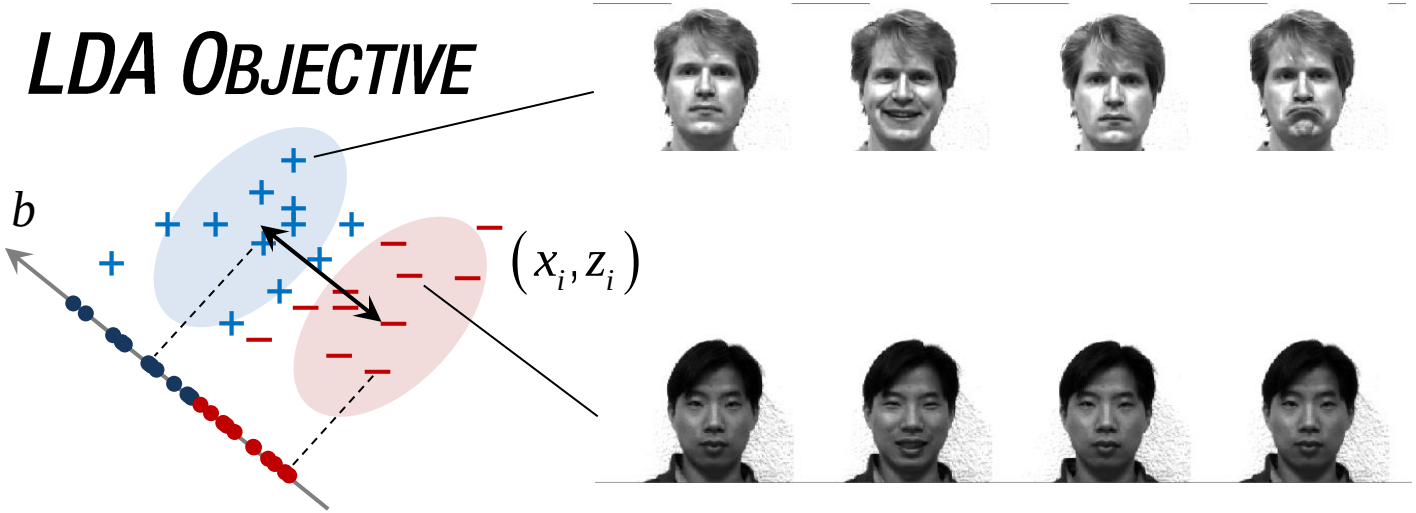


(x_i, z_i): image x with label z

Recall: Eigen basis

Projection onto basis set: $y = bx \qquad y \in \mathbb{R}^d$

LDA Objective



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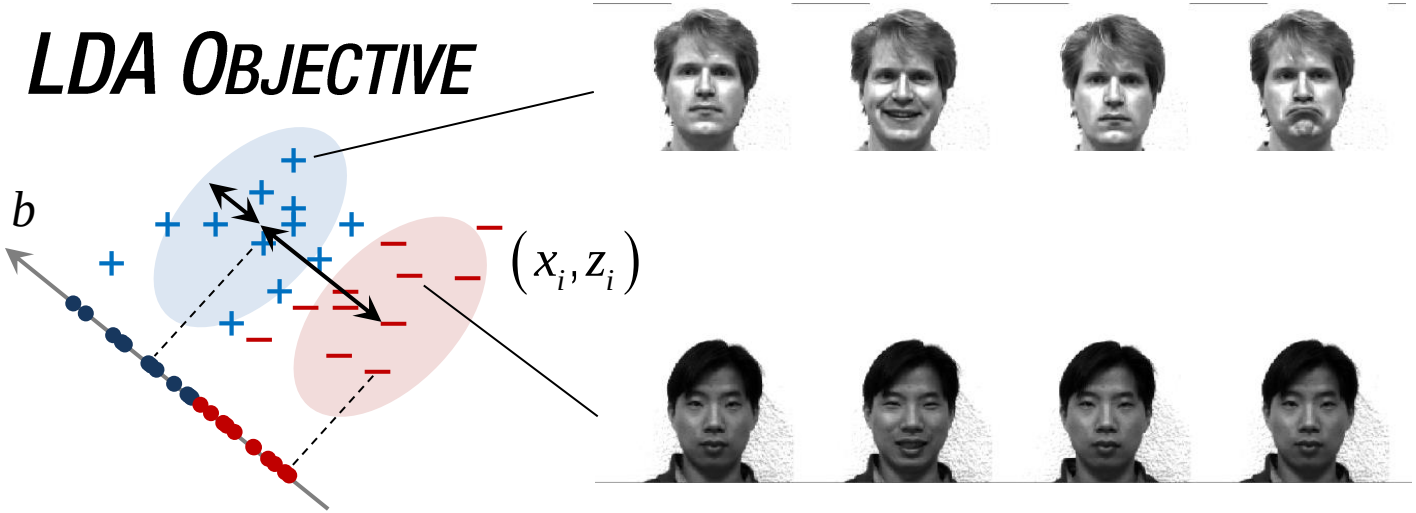
Objective:

- Maximize variance between classes

LDA Objective



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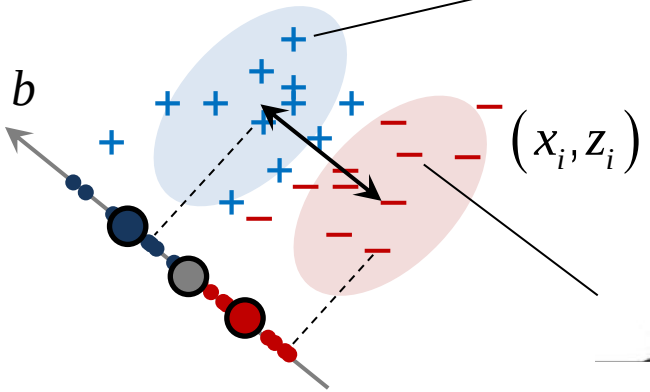
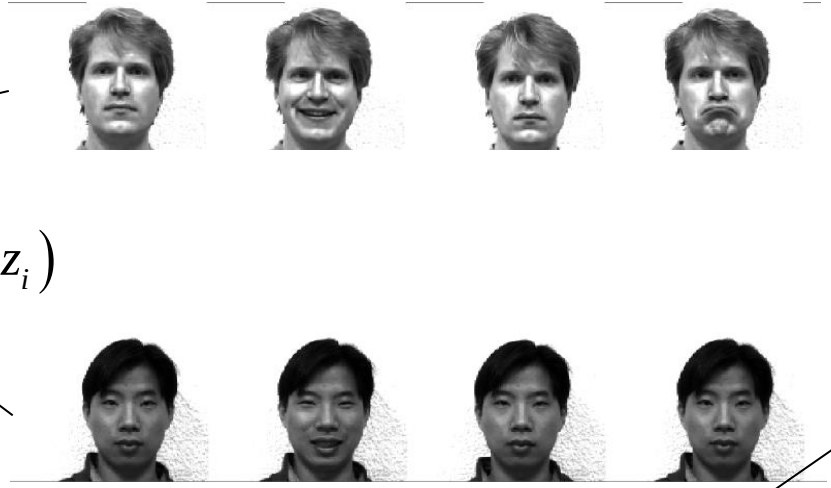
Objective:

- Maximize variance between classes
- Minimize variance within class

LDA Objective





Number of data for each class

(x_i, z_i): image x with label z

$$\tilde{S}_{\text{inter}} = \sum_{j \in C} n_j \left(\bar{y} - \bar{y}_j\right)\left(\bar{y} - \bar{y}_j\right)^T$$

Recall: Eigen basis

Projection onto basis set: $y = bx \qquad y \in \mathbb{R}^d$

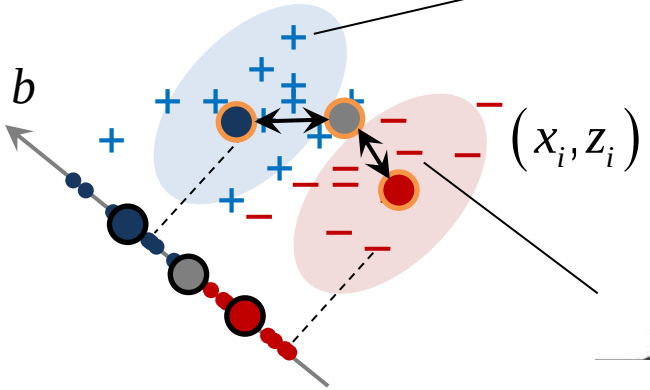
Objective:

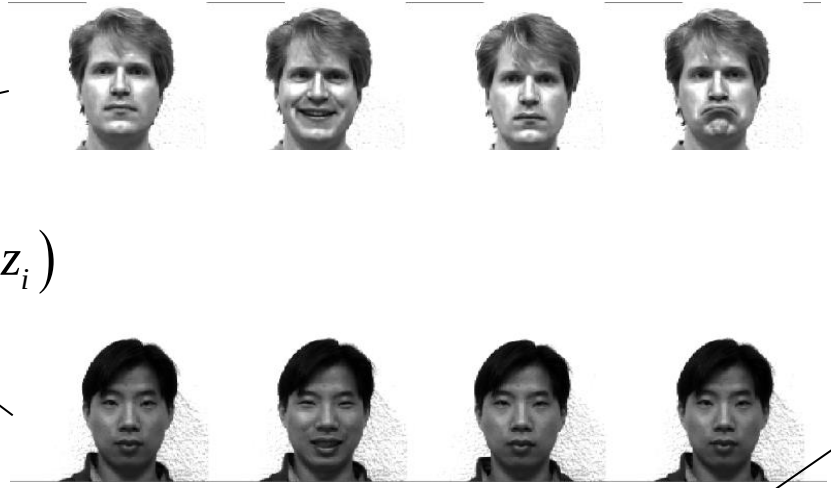
- **Maximize variance between classes**
- Minimize variance within class

$\bar{y}$: mean of projected features

$\bar{y}_j$: class mean of projected features (C is class label set)

LDA Objective





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- Minimize variance within class

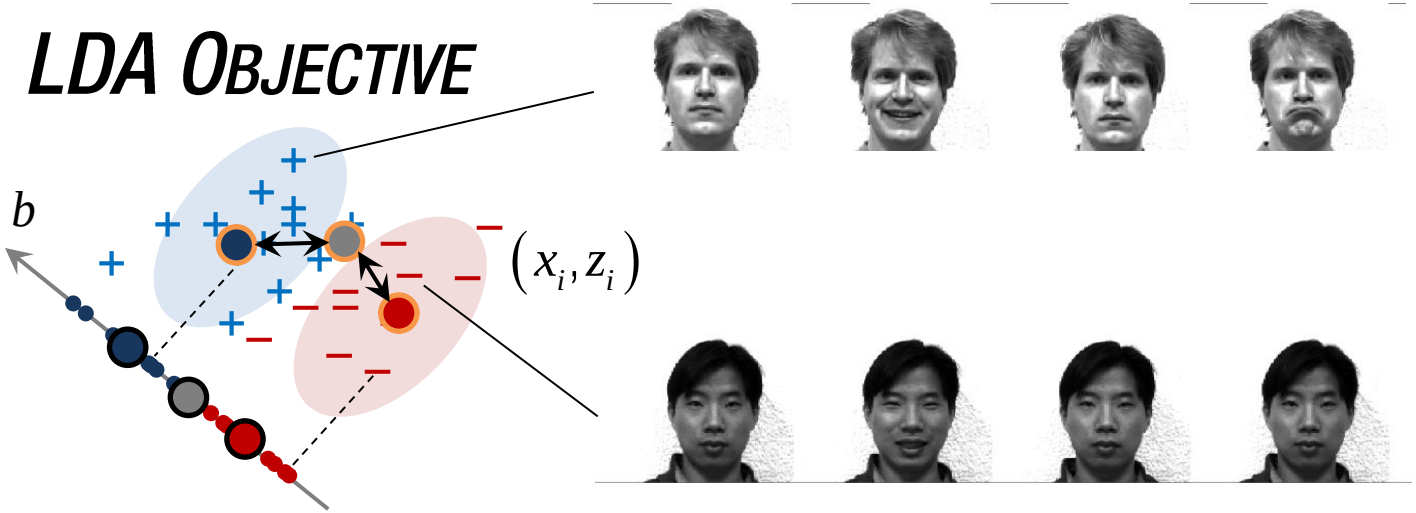
$$\tilde{S}_{\text{inter}} = \sum_{j \in C} n_j \left(\bar{y} - \bar{y}_j\right)\left(\bar{y} - \bar{y}_j\right)^T$$

$$= \sum_{j \in C} n_j b^T \left(\bar{x} - \bar{x}_j\right)\left(\bar{x} - \bar{x}_j\right)^T b$$

$\bar{x}$: mean of features

$\bar{x}_j$: class mean of features (C is class label set)

LDA Objective



(x_i, z_i): image x with label z

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Objective:

- **Maximize variance between classes**
- Minimize variance within class

$$\tilde{S}_{\text{inter}} = b^T S_{\text{inter}} b$$

where $S_{\text{inter}} = \sum_{j \in C} n_j \left(\bar{x} - \bar{x}_j\right)\left(\bar{x} - \bar{x}_j\right)^T$

- $\bar{x}$: mean of features
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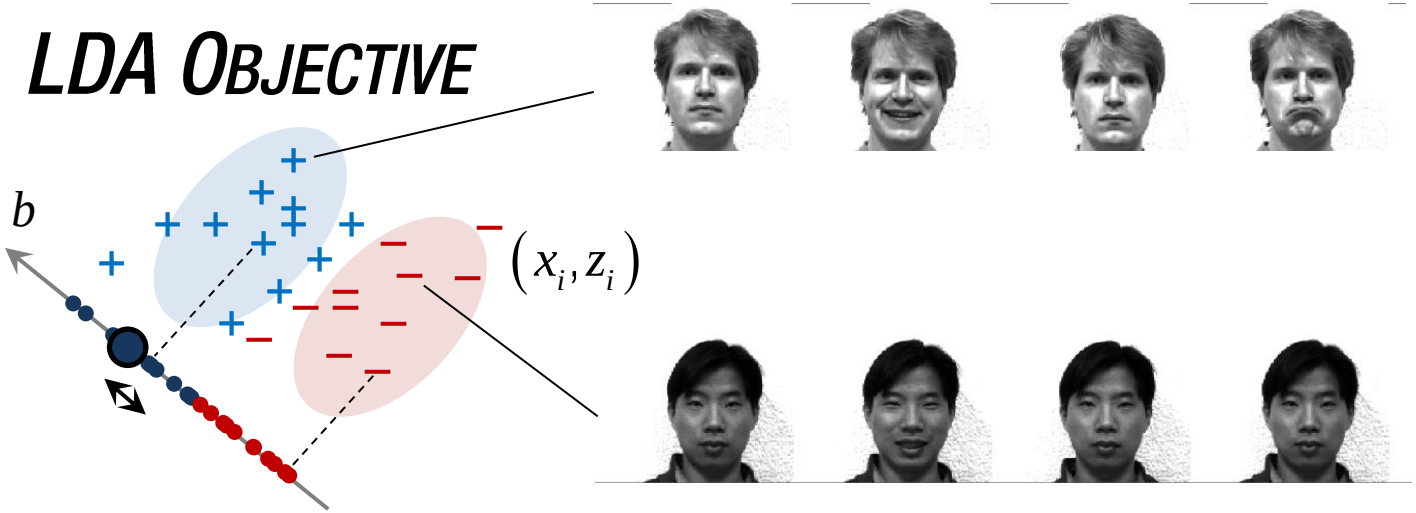

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Recall: Eigen basis

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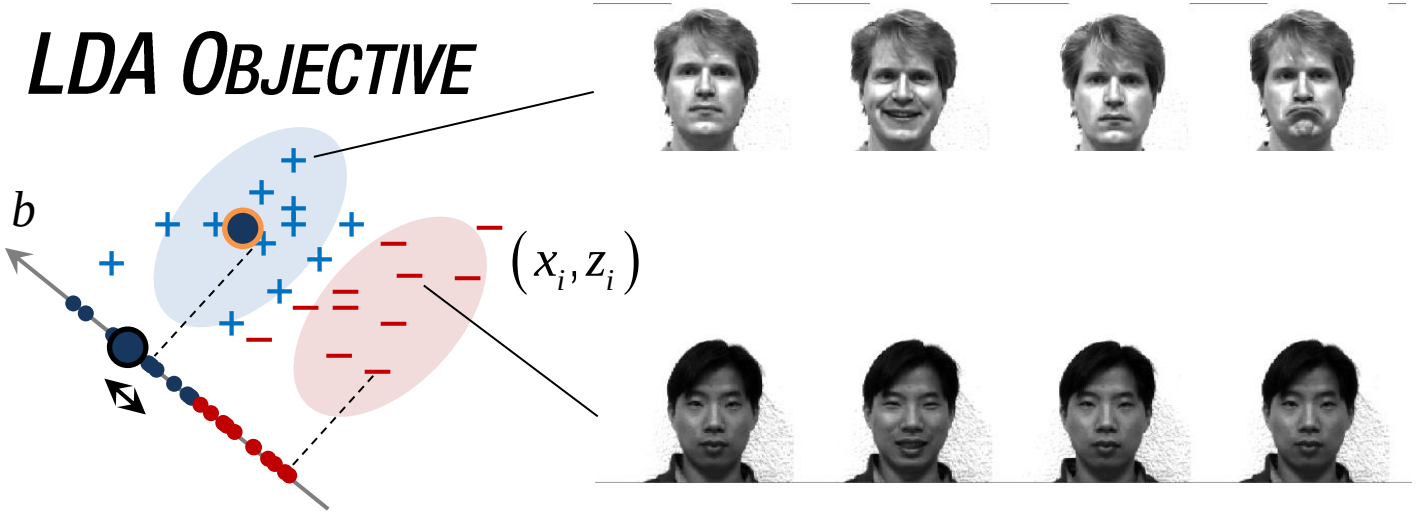
Objective:

- Maximize variance between classes
- **Minimize variance within class**

$$\tilde{S}_{\text{intra}} = \sum_{j \in C} \sum_{i \in C_j} \left(\bar{y}_j - y_j^i\right)\left(\bar{y}_j - y_j^i\right)^T$$

- $\bar{y}_j$: class mean of projected features
- y_j^i : projected features
- C_j : class index set

LDA Objective



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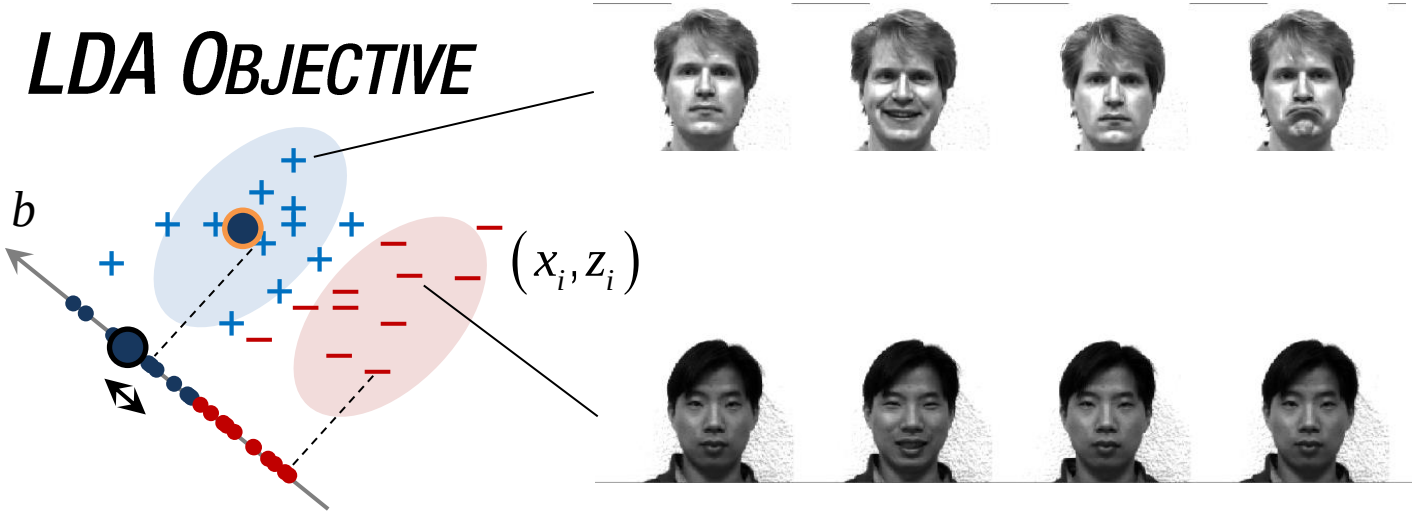
$$= \sum_{j \in C} \sum_{i \in C_j} b^T \left(\bar{x}_j - x_j^i\right)\left(\bar{x}_j - x_j^i\right)^T b$$

$\bar{x}_j$: class mean of features

x_j^i : features

C_j : class index set

LDA Objective



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$$\tilde{S}_{\text{intra}} = b^T S_{\text{intra}} b$$

where $S_{\text{intra}} = \sum_{j \in C} \sum_{i \in C_j} \left(\bar{x}_j - x_j^i\right)\left(\bar{x}_j - x_j^i\right)^T$

- $\bar{x}_j$: class mean of features
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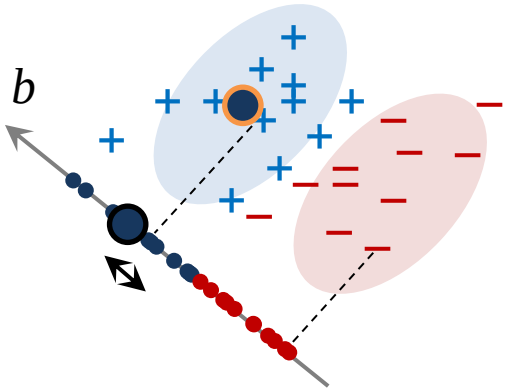

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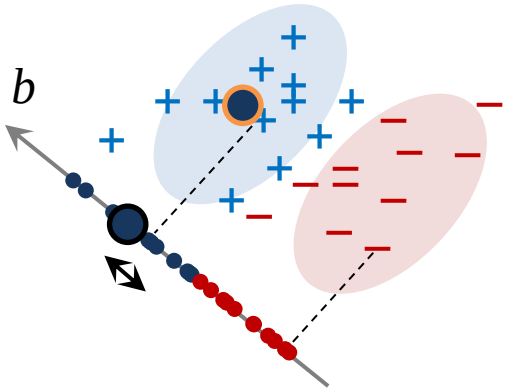
- Minimize variance within class $\underset{b}{\text{minimize}} \left| \tilde{S}_{\text{intra}} \right|$

LDA Objective



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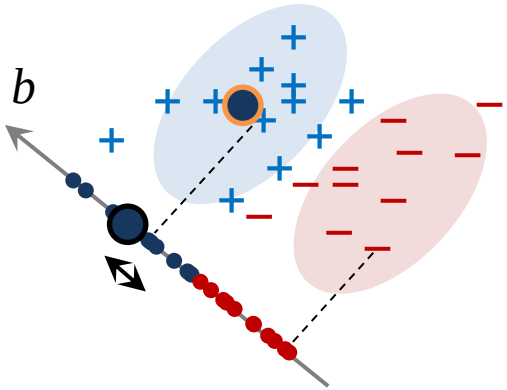
- Minimize variance within class $\underset{b}{\text{minimize}} \left|\tilde{S}_{\text{intra}}\right|$

$$\longrightarrow b^* = \underset{b}{\text{argmax}} \frac{\left|\tilde{S}_{\text{inter}}\right|}{\left|\tilde{S}_{\text{intra}}\right|}$$

$$= \underset{b}{\text{argmax}} \frac{\left|b^T S_{\text{inter}} b\right|}{\left|b^T S_{\text{intra}} b\right|}$$

LDA Objective



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$$= \underset{b}{\text{argmax}} \frac{\left|b^T S_{\text{inter}} b\right|}{\left|b^T S_{\text{intra}} b\right|}$$

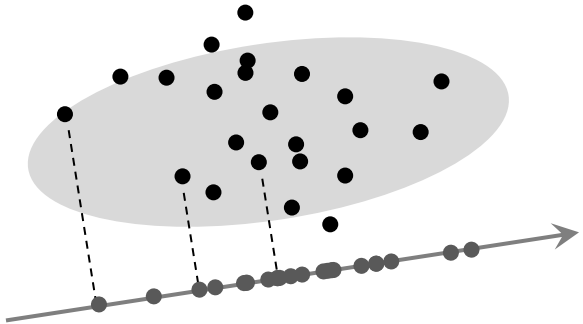
~ Solution of a generalized Eigenvalue problem:

$$S_{\text{inter}} b^* = \lambda S_{\text{intra}} b^*$$

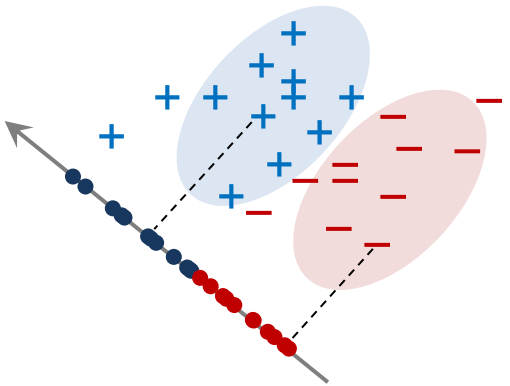
LEARNED BASIS

EIGENFACES VS. FISHER FACES

PCA



LDA



$$b^* = \underset{b}{\operatorname{argmax}}\, b^T X^T X b$$

$$X^T X b^* = \lambda b^*$$

$$b^* = \underset{b}{\operatorname{argmax}} \frac{\left| b^T S_{\text{inter}} b \right|}{\left| b^T S_{\text{intra}} b \right|}$$

$$S_{\text{inter}} b^* = \lambda S_{\text{intra}} b^*$$

FISHER FACES RECOGNITION

Residual

Residual

$$\left\| \overline{x} - \frac{\overline{x} \cdot b}{\|b\|^2} b \right\|$$

Face subspace

FACE RECOGNITION WITH FISHER COEFFICIENT

Who is this guy?

α

Subject 1

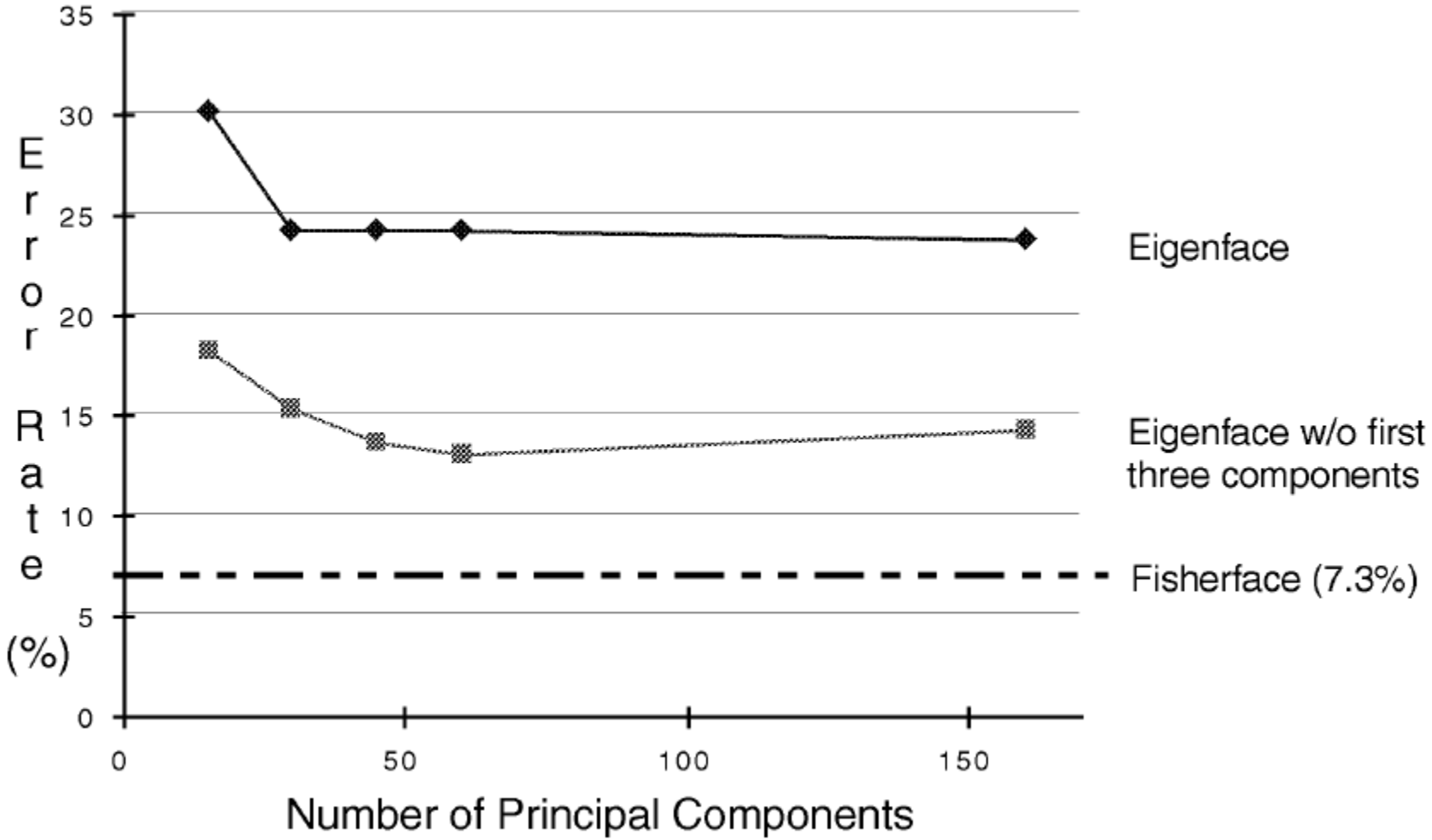
$$\alpha_1 = \left\{\alpha_1^1, \cdots, \alpha_1^n\right\}$$

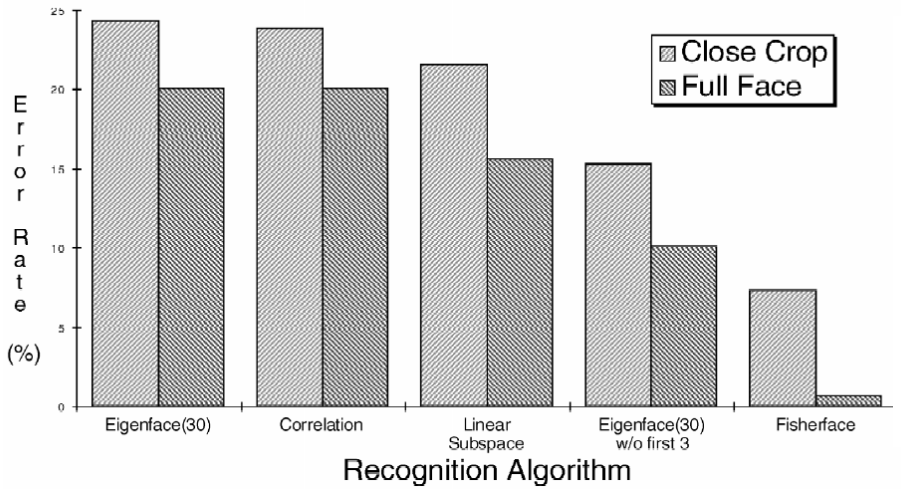
Subject 2

$$\alpha_2 = \left\{\alpha_2^1, \cdots, \alpha_2^n\right\}$$

Subject 3

$$\alpha_3 = \left\{\alpha_3^1, \cdots, \alpha_3^n\right\}$$


35
30
25
20
15
10
5
0
Error Rate (%)
0
50
100
150
Number of Principal Components
Eigenface
Eigenface w/o first three components
Fisherface (7.3%)



"Leaving-One-Out" of Yale Database			
Method	Reduced Space	Error Rate (%)	
		Close Crop	Full Face
Eigenface	30	24.4	19.4
Eigenface w/o 1st 3	30	15.3	10.8
Correlation	160	23.9	20.0
Linear Subspace	48	21.6	15.6
Fisherface	15	7.3	0.6

GLASS RECOGNITION