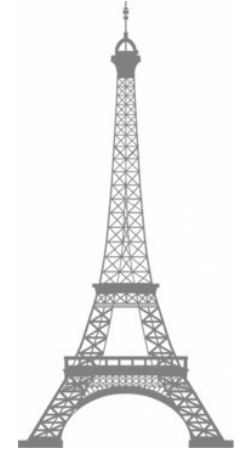


# *CAMERA MODEL*

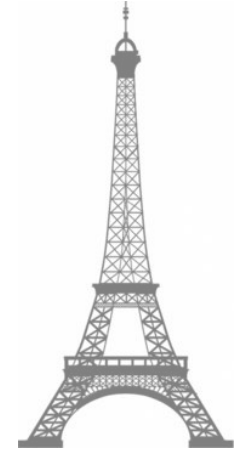
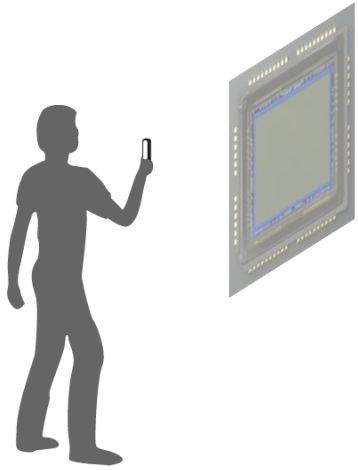
HYUN SOO PARK



# *CAMERA ANATOMY*



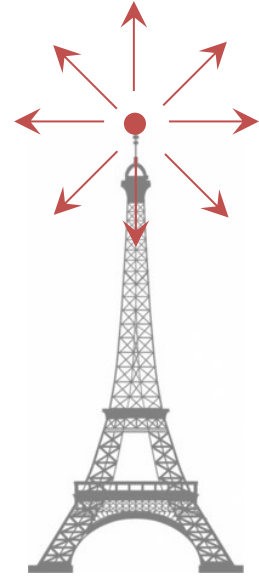
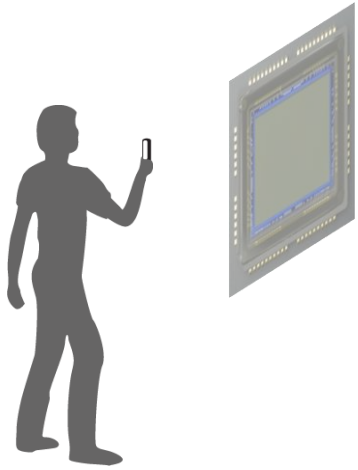
# *CAMERA ANATOMY*



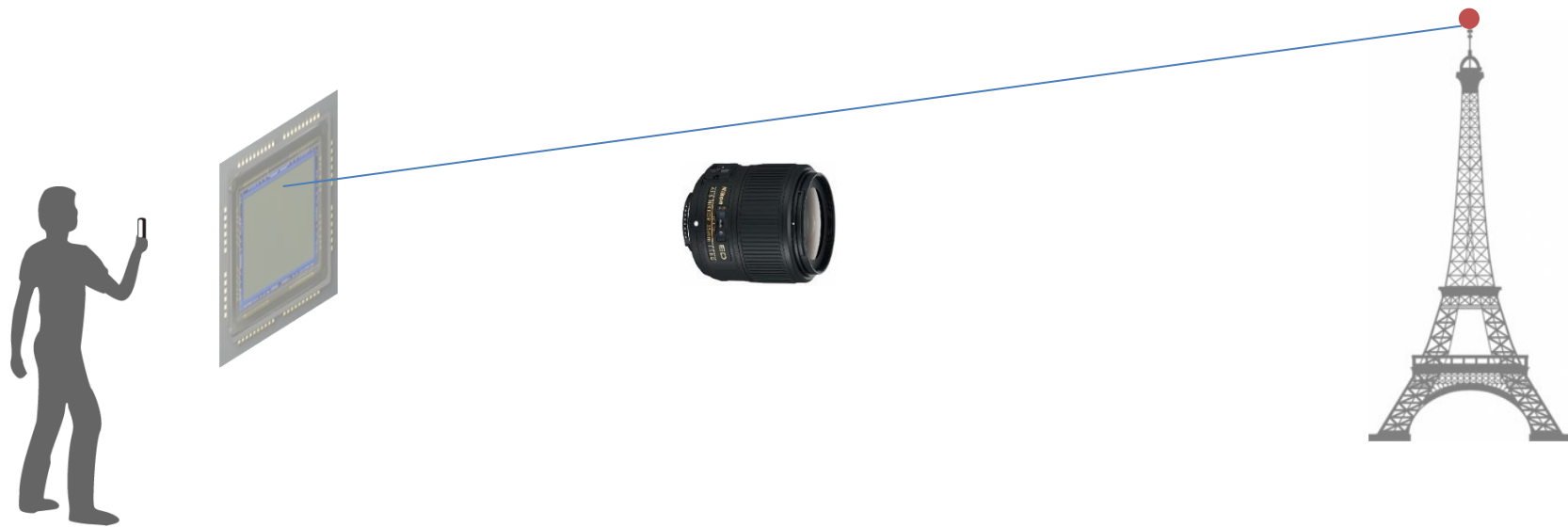
# *CAMERA ANATOMY*



# *CAMERA ANATOMY*



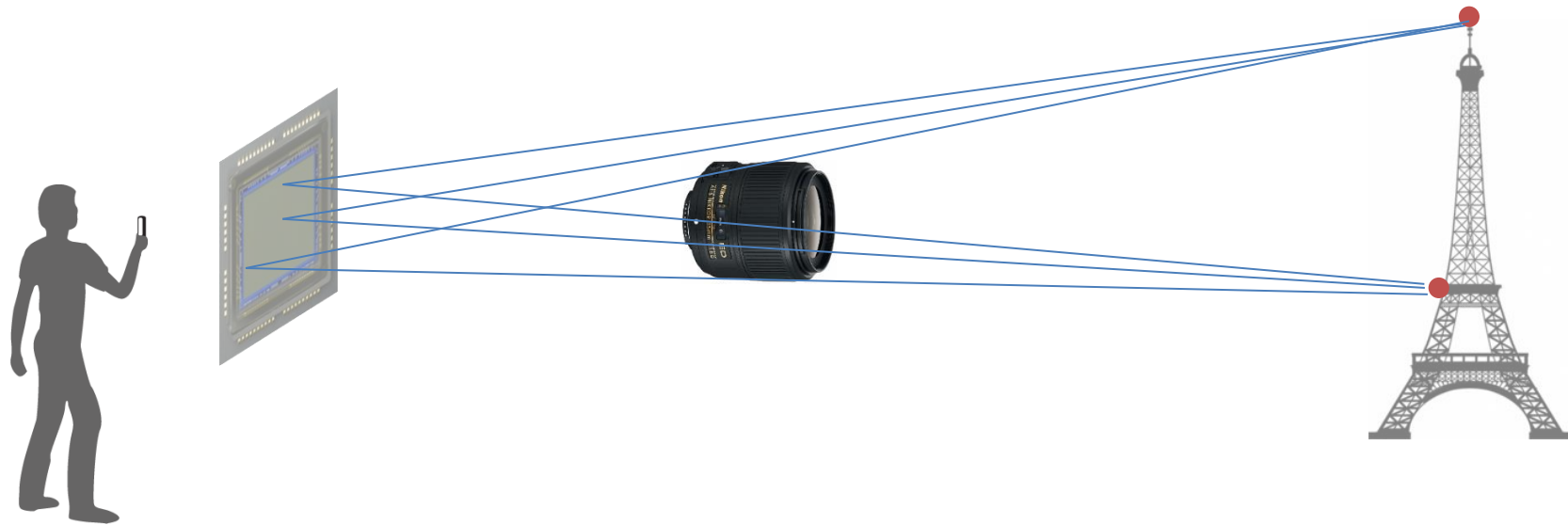
# *CAMERA ANATOMY*



# *CAMERA ANATOMY*

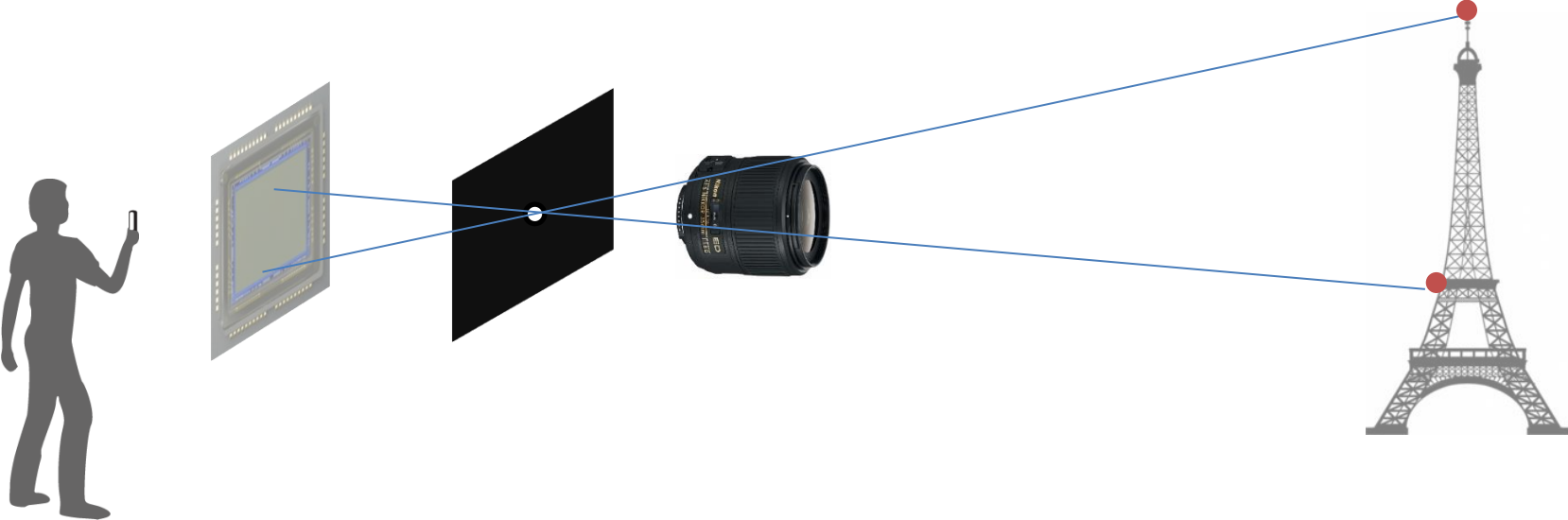


# *CAMERA ANATOMY*

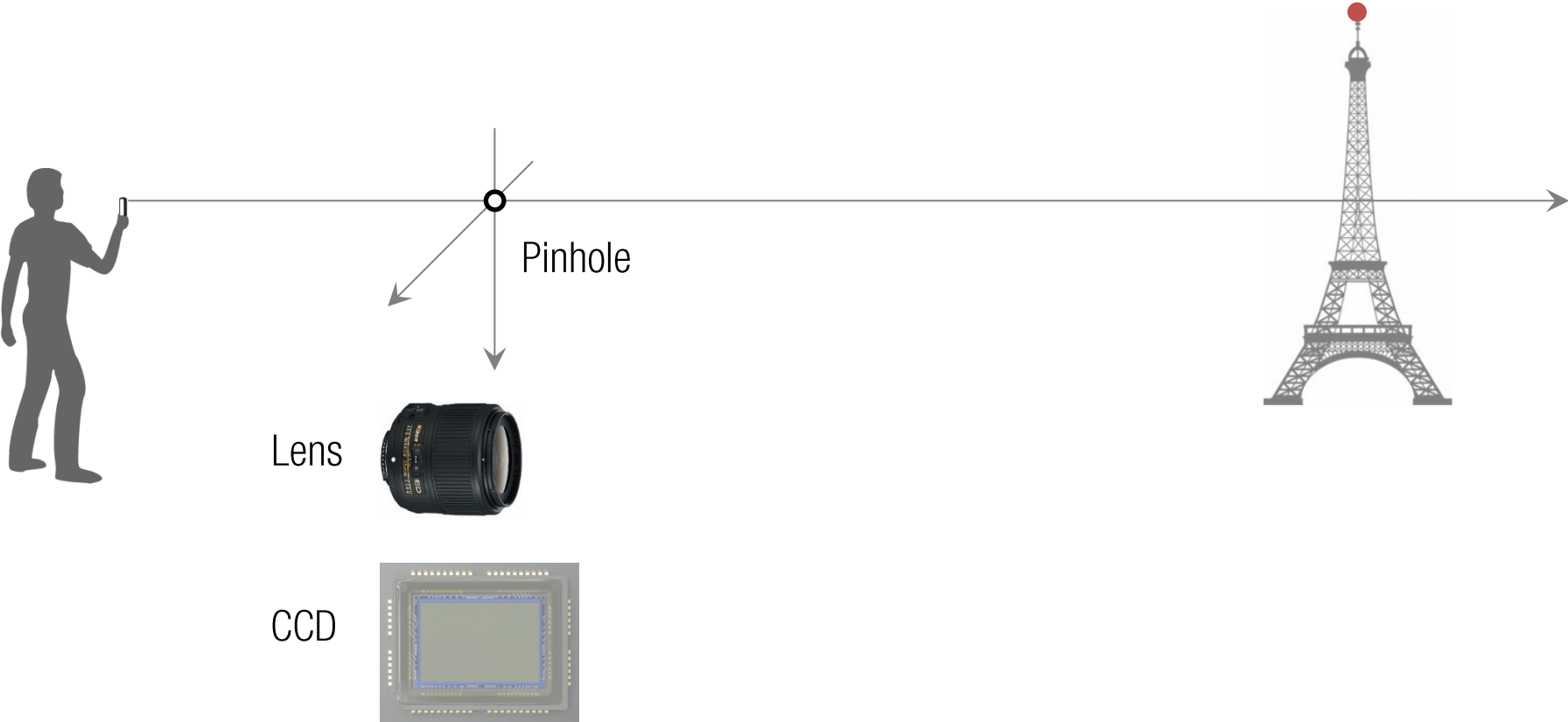




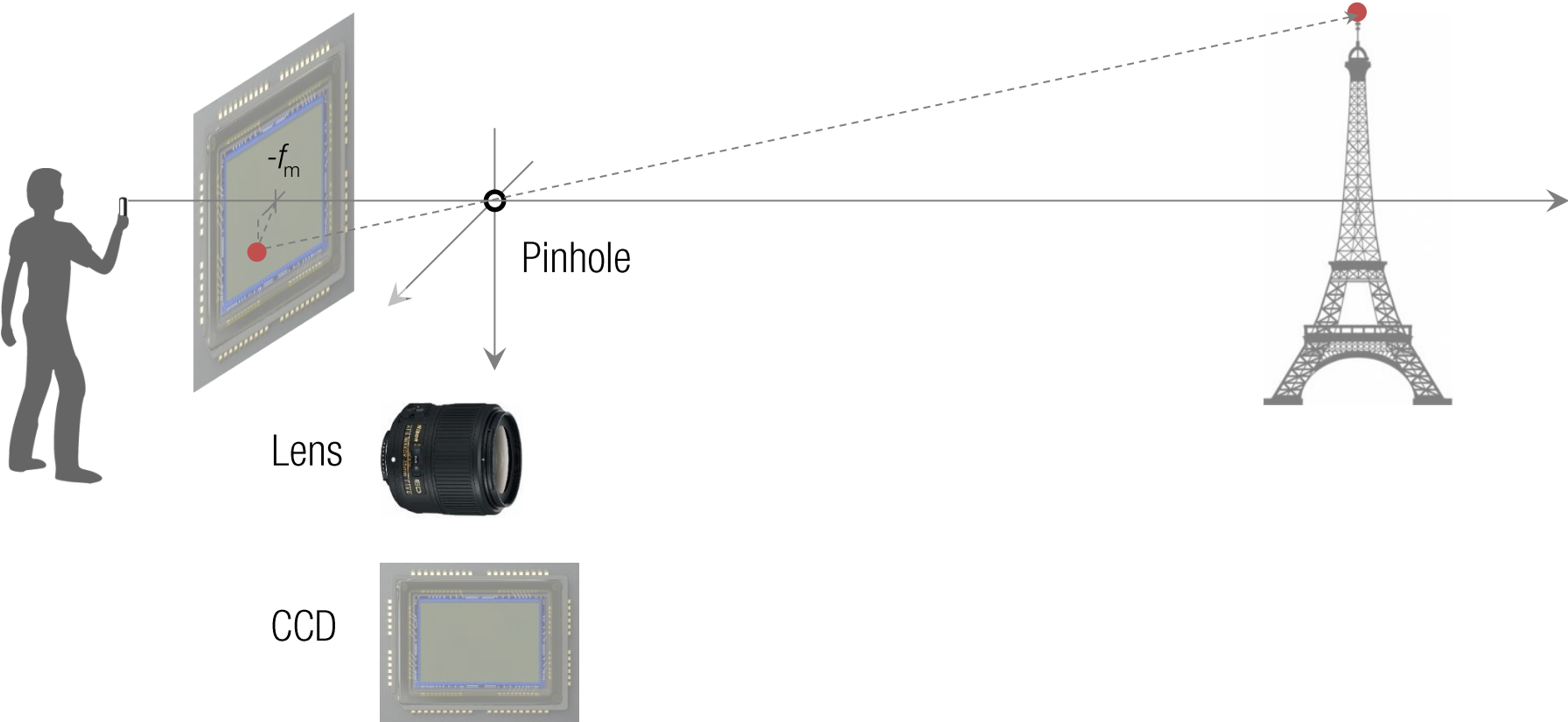
# *CAMERA ANATOMY*



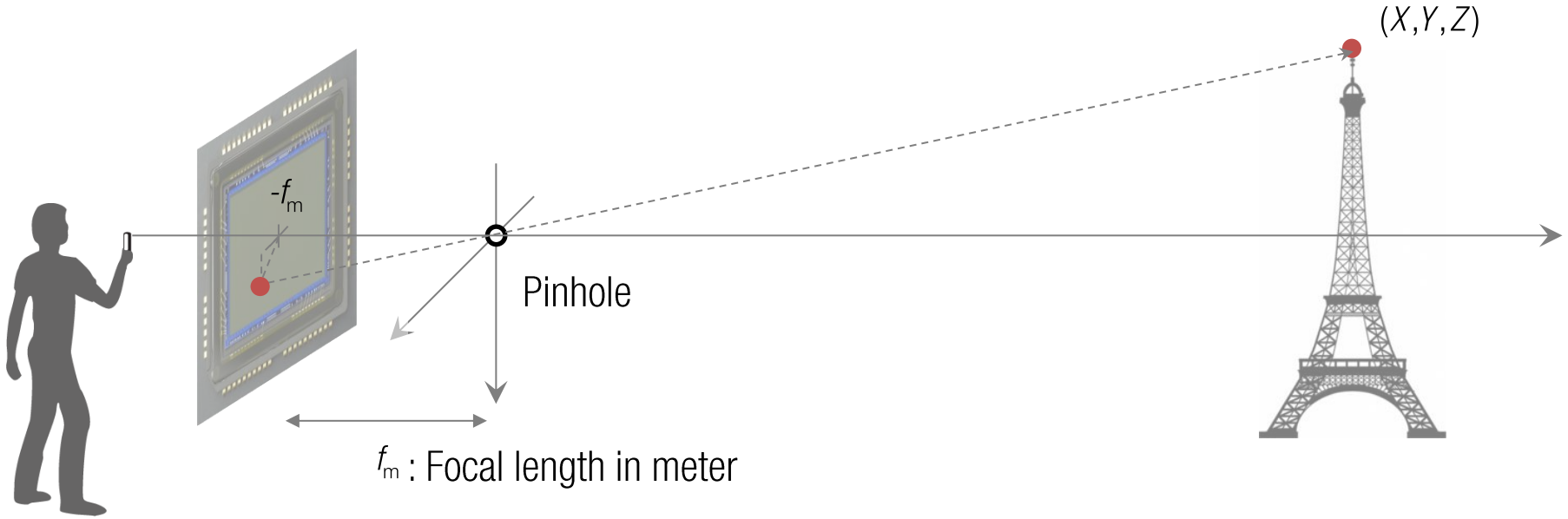
# CAMERA ANATOMY



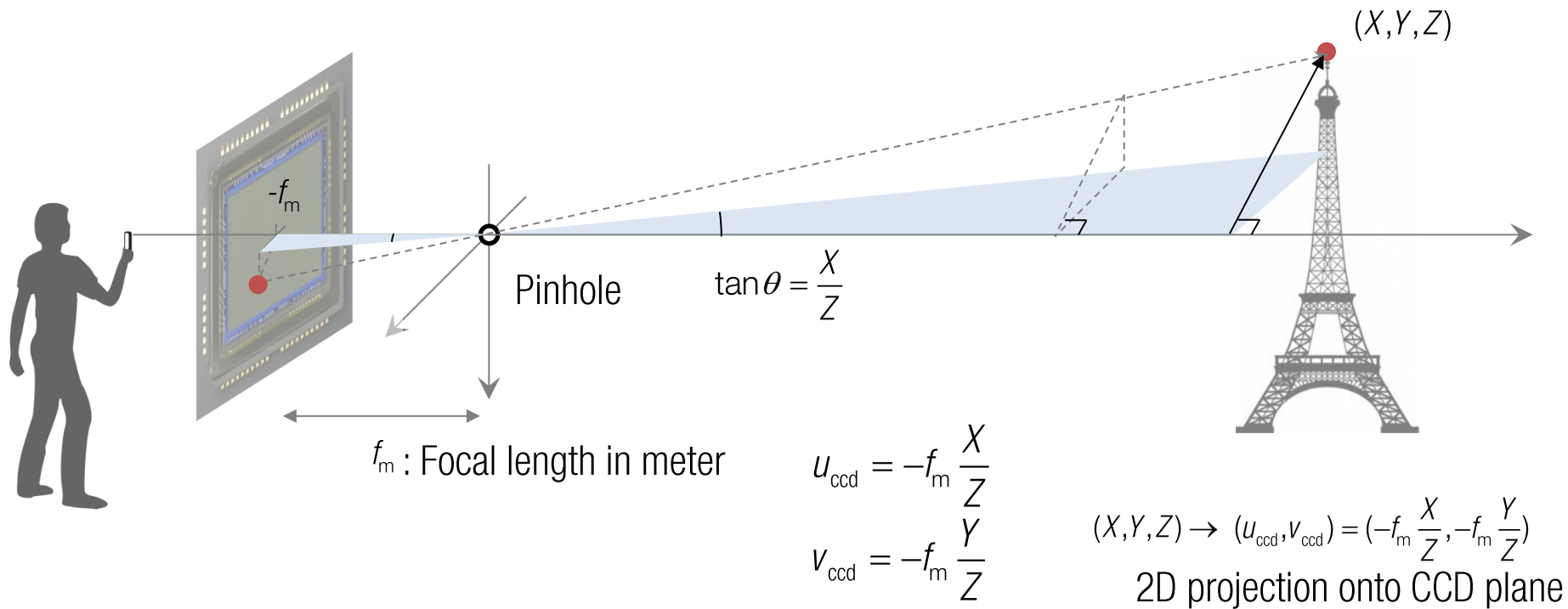
# CAMERA ANATOMY



# CAMERA ANATOMY

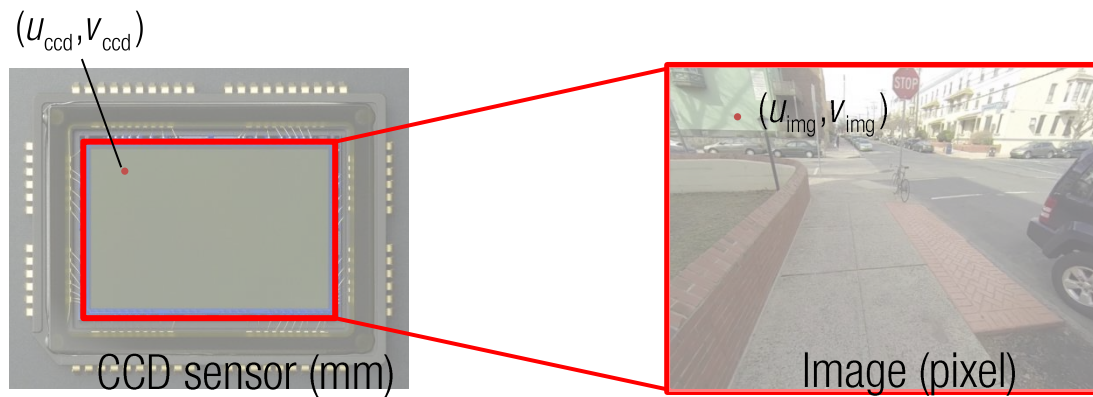


# CAMERA ANATOMY

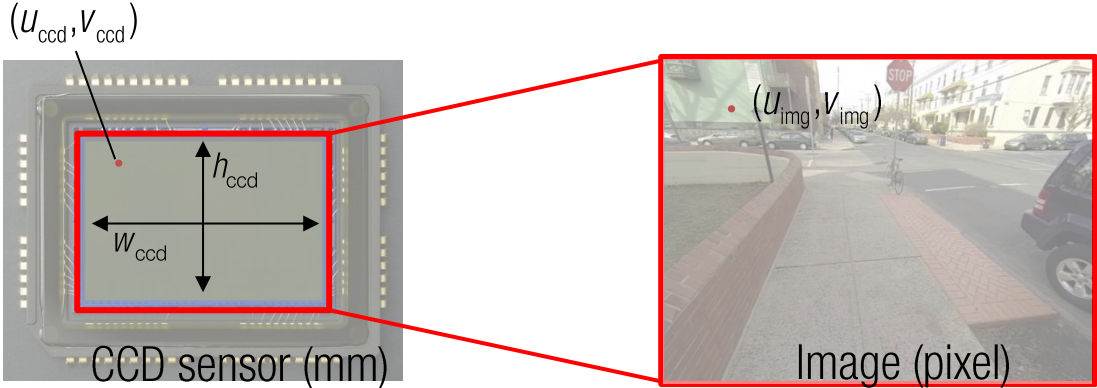


















# *CCD TO IMAGE*




# CCD TO IMAGE

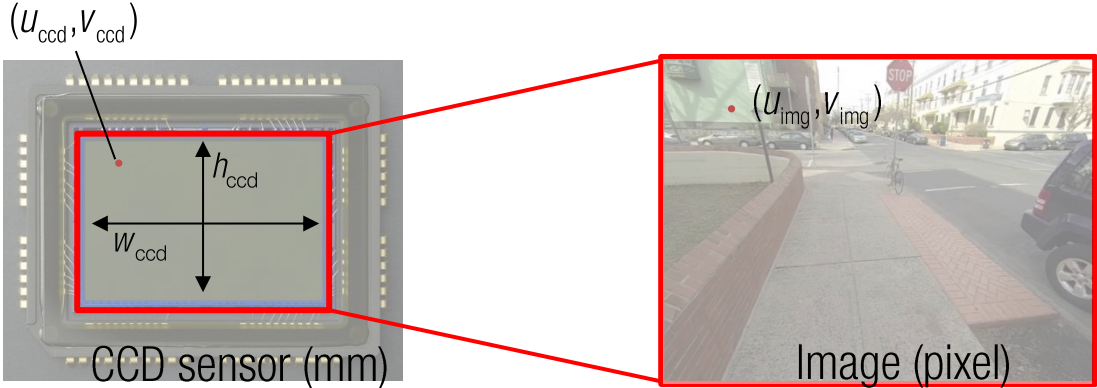




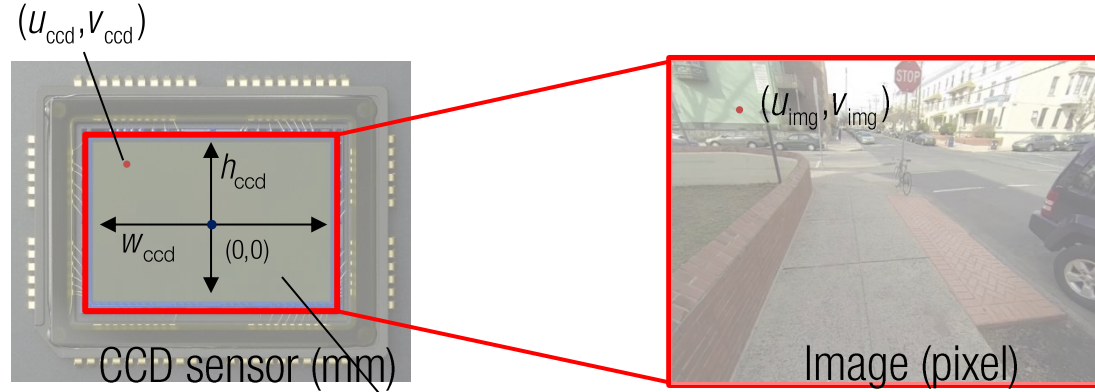
Imager Sizes	Formats (Type)	~Diag.	Uses
	1/7" - 1.85 x 1.39mm	2.3	Cell phones, web cams, etc....
	1/6" - 2.15 x 1.61mm	2.7	Cell phones, web cams, etc....
	1/5" - 2.55 x 1.91mm	3.2	Cell phones, web cams, etc....
	1/4" - 3.2 x 2.4mm	4.0	Cell phones, web cams, etc....
	1/3.6" - 4.0 x 3.0mm	5.0	P&S DSC
	1/3.2" - 4.536 x 3.416mm	5.678	P&S DSC
	1/3" - 4.8 x 3.6mm	6.0	Casio QV-8000SX (1.2MP), Epson PhotoPC 700 (1.2MP)
	1/2.7" - 5.27 x 3.96mm	6.592	Canon PowerShot A20 (1.92MP), HP PhotoSmart C618 (1.92)
	1/2" - 6.4 x 4.8mm	8.0	Olympus C-2100Z (1.92MP), Epson PhotoPC 850Z (1.92)
	1/1.8" - 7.176 x 5.319mm	8.932	Nikon Coolpix 995 (3.14MP), Olympus C-4040Z (3.9MP), Canon PowerShot G2 (3.8MP), Sony DSC-S85 (3.8MP)
	2/3" - 8.8 x 6.6mm	11.0	Nikon Coolpix 5000 (4.92MP), Sony DSC-F707 (4.92MP), Olympus E-10 (3.7MP), Minolta DiMAGE 7 (4.92MP)
	1" - 12.8 x 9.6mm	16.0	Not used in DSCs. Used in some high-end video cameras
	Kodak KAF-5100CE CCD 17.8 x 13.4mm (4/3")	22.28	Olympus announced development of a new camera and new lenses for this 4/3" size. 2614 x 1966 - 5.1MP - 6.8µm pixel
	Foveon X3 F7-35X3-A25B 20.7 x 13.8mm	24.9	Sigma SD9 (X3) 2268 x 1512 = 3.43MP - 9.12µm pixel 1.74x Focal Length Multiplier (35mm film)

	Canon D30 CMOS 21.8 x 14.5mm	26.2	Canon D30 2160 x 1440 = 3.11MP - 10.1µm pixel 1.65x Focal Length Multiplier (35mm film)
	Canon D60 CMOS 22.7 x 15.1mm	27.3	Canon D60 3072 x 2048 = 6.3MP - 7.4µm pixel 1.59x Focal Length Multiplier (35mm film)
	Nikon D100 CCD Nikon D1x CCD 23.7 x 15.6mm	28.2	Nikon D100 - 3008 x 2000 = 6.1MP - 7.8µm pixel Nikon D1x - 4024 x 1324 = 5.24MP - 5.9 x 11.7µm pixel 1.52x Focal Length Multiplier (35mm film)
	APS Film 25.1 x 16.7mm	30.148	APS cameras 1.44x Focal Length Multiplier (35mm film)
	Canon EOS-1D CCD 27.0 x 17.8mm	32.3	Canon EOS-1D 2464 x 1648 = 4.06MP - 10.8µm pixel 1.34x Focal Length Multiplier (35mm Film)
	Kodak KAF-6303CE CCD 27.8 x 18.5mm	33.4	Kodak 760 3088 x 2056 = 6.35MP - 9.0µm pixel 1.30x Focal Length Multiplier (35mm film)
	35mm Film Canon 1Ds Kodak 14n 36.0 x 24.0mm	43.27	35mm film cameras Canon 1Ds - 4064 x 2704 = 10.99MP - 8.85µm pixel Kodak DCS Pro 14n - 4536 x 3024 = 13.7MP - 7.94µm pixel

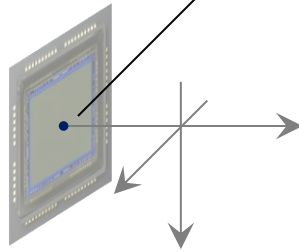
# CCD TO IMAGE



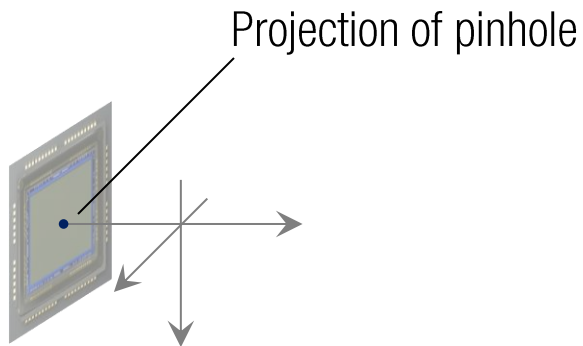
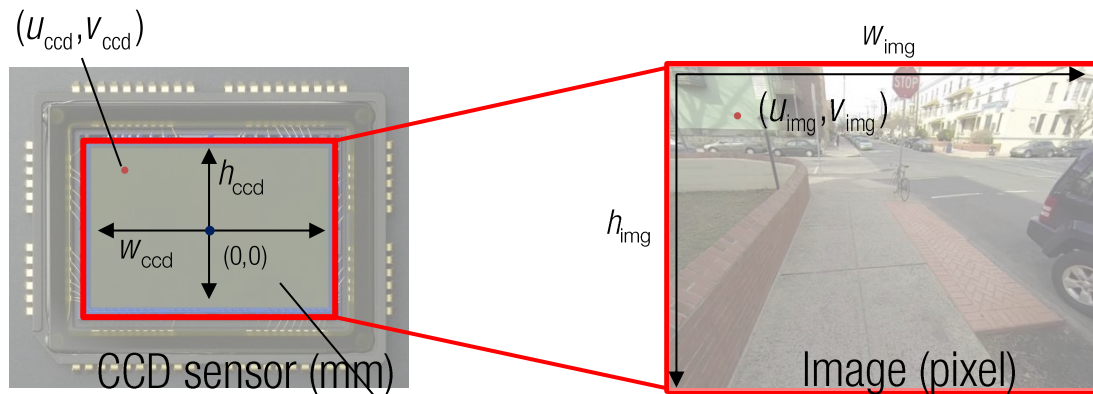
# CCD TO IMAGE



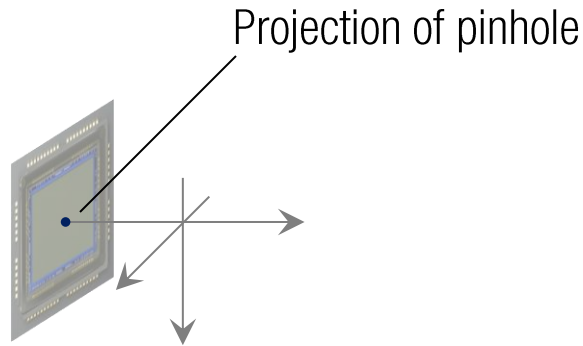
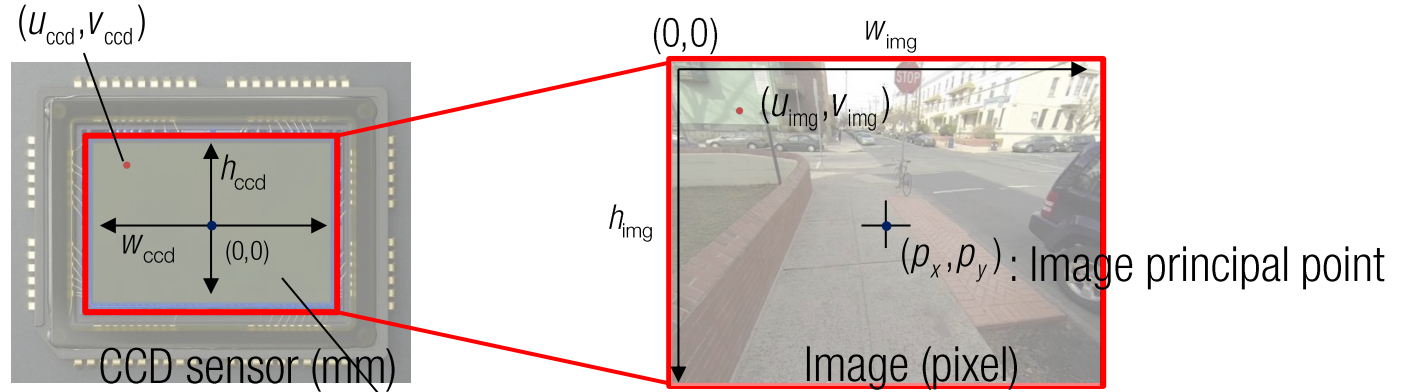
Projection of pinhole



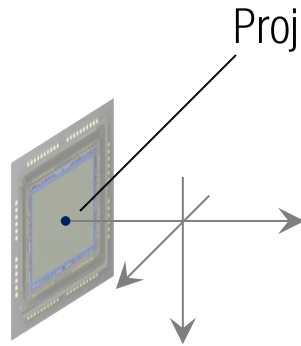
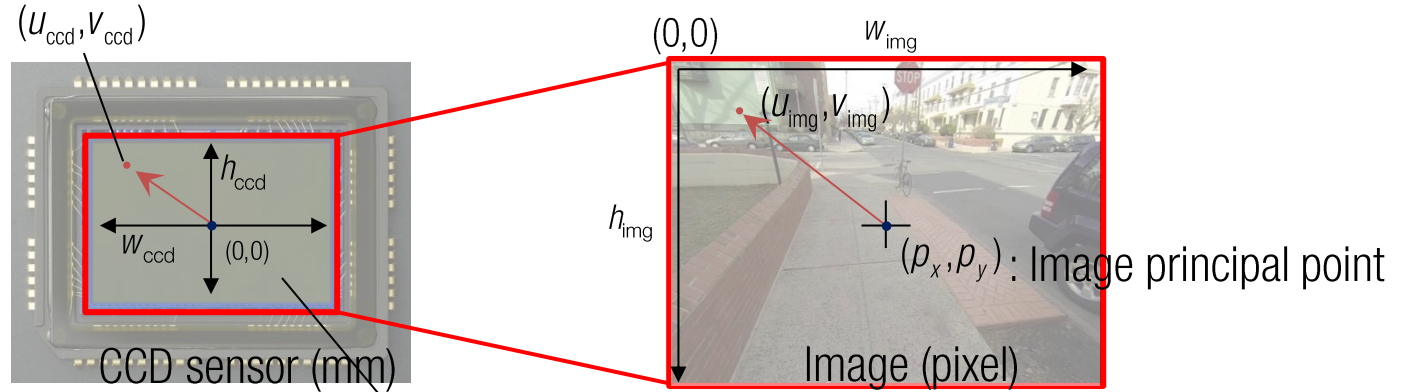
# CCD TO IMAGE



# CCD TO IMAGE

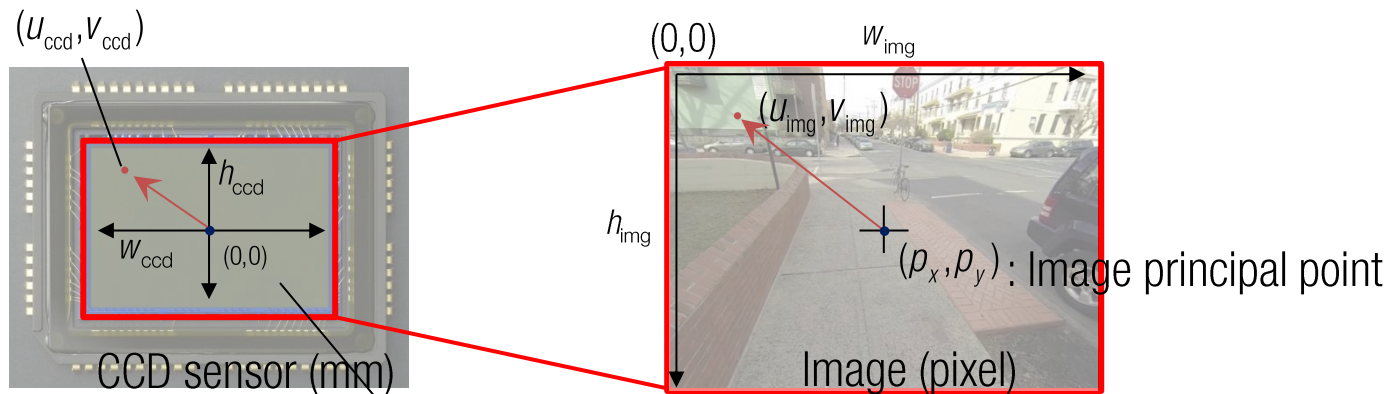


# CCD TO IMAGE

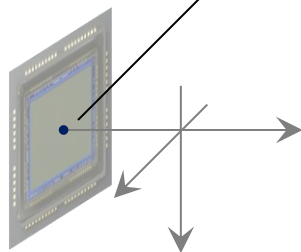


$$\frac{u_{\text{ccd}}}{w_{\text{ccd}}} = \frac{u_{\text{img}} - p_x}{w_{\text{img}}}$$

# CCD TO IMAGE



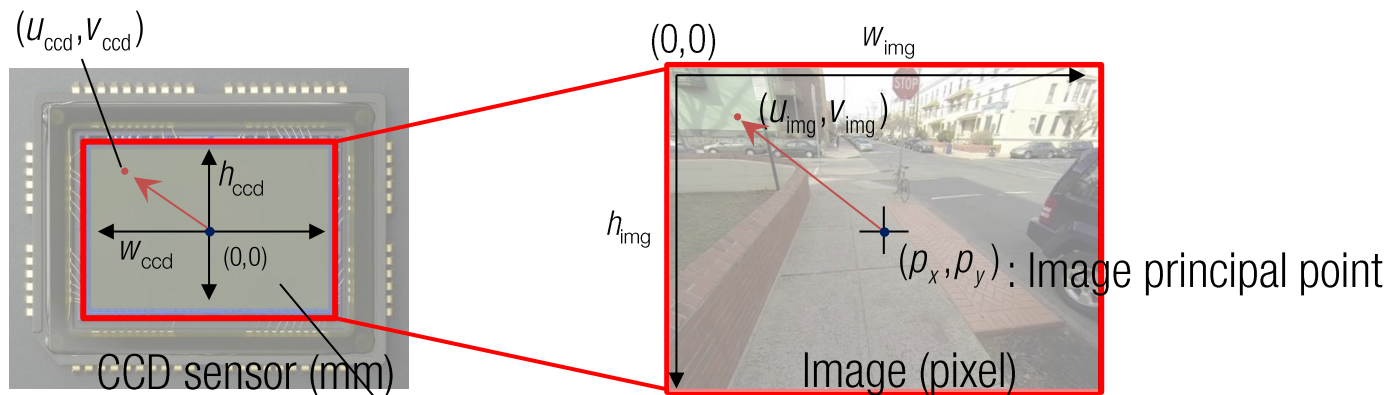
Projection of pinhole



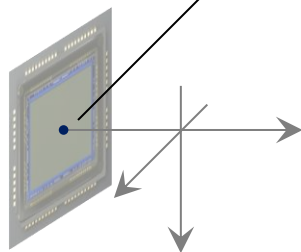
$$\frac{u_{\text{ccd}}}{w_{\text{ccd}}} = \frac{u_{\text{img}} - p_x}{w_{\text{img}}} \quad \frac{v_{\text{ccd}}}{h_{\text{ccd}}} = \frac{v_{\text{img}} - p_y}{h_{\text{img}}}$$



# CCD TO IMAGE



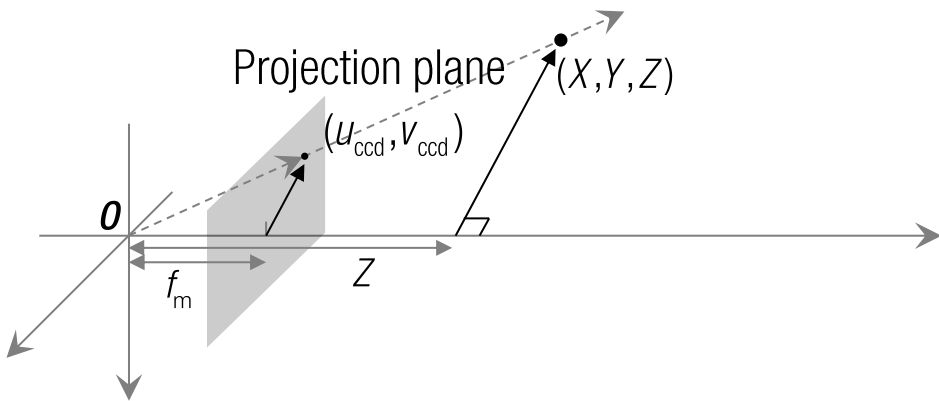
Projection of pinhole



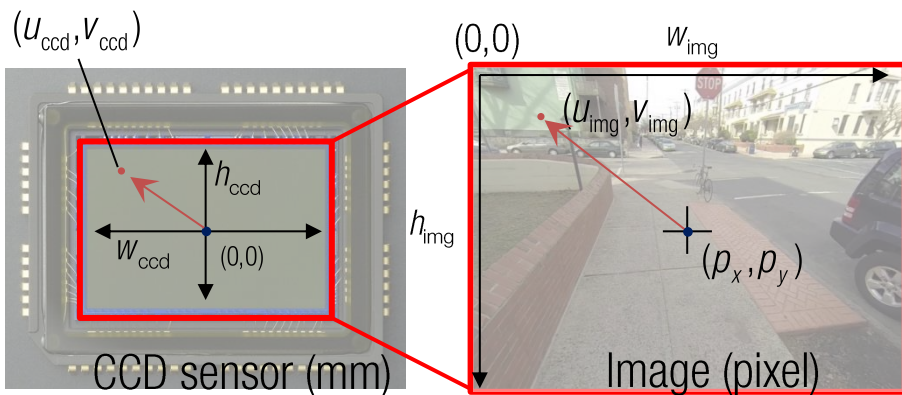
$$\frac{u_{\text{ccd}}}{w_{\text{ccd}}} = \frac{u_{\text{img}} - p_x}{w_{\text{img}}} \quad \frac{v_{\text{ccd}}}{h_{\text{ccd}}} = \frac{v_{\text{img}} - p_y}{h_{\text{img}}}$$

$$\rightarrow u_{\text{img}} = u_{\text{ccd}} \frac{w_{\text{img}}}{w_{\text{ccd}}} + p_x \quad v_{\text{img}} = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y$$

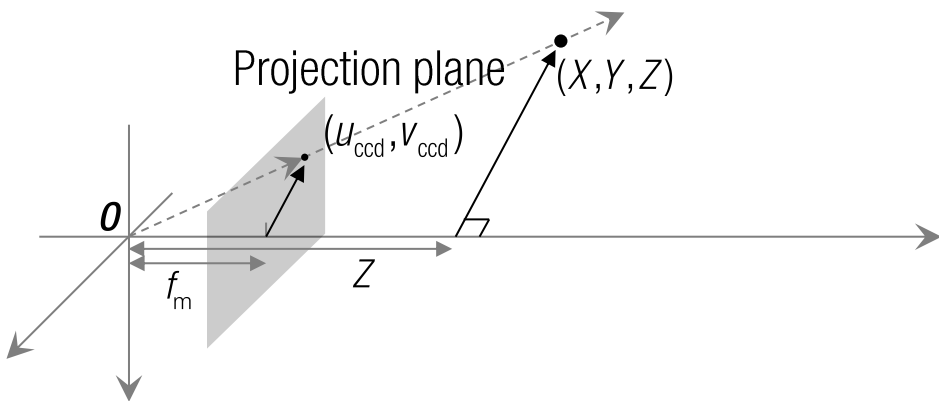
# IMAGE PROJECTION



$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left( f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$



# IMAGE PROJECTION

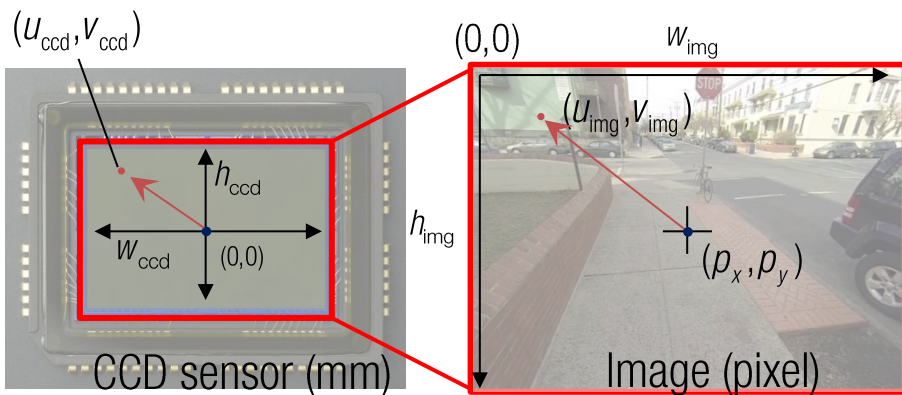


$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left( f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$

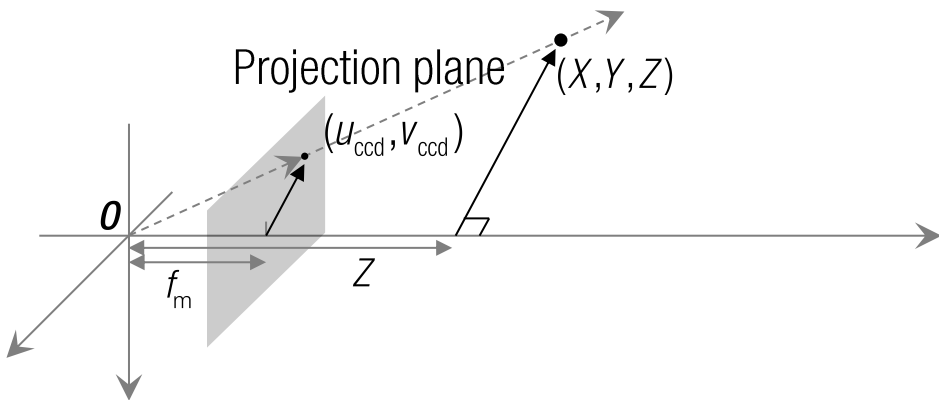
Pixel projection

$$\rightarrow u_{\text{img}} = u_{\text{ccd}} \frac{w_{\text{img}}}{w_{\text{ccd}}} + p_x$$

$$v_{\text{img}} = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y$$



# IMAGE PROJECTION

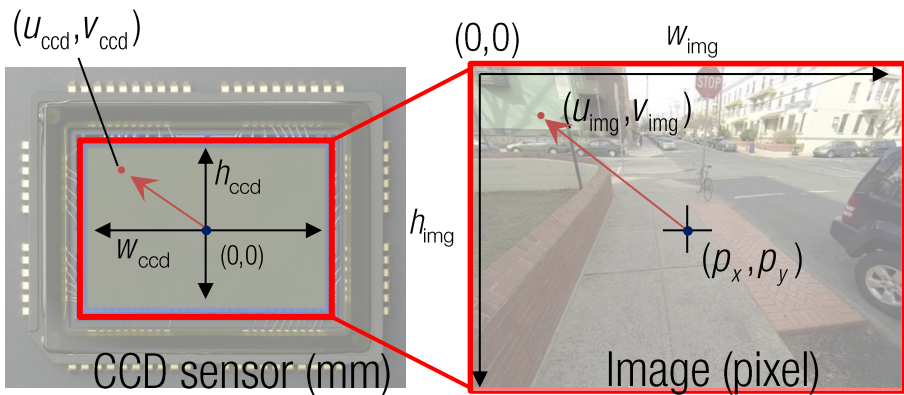


$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left( f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$

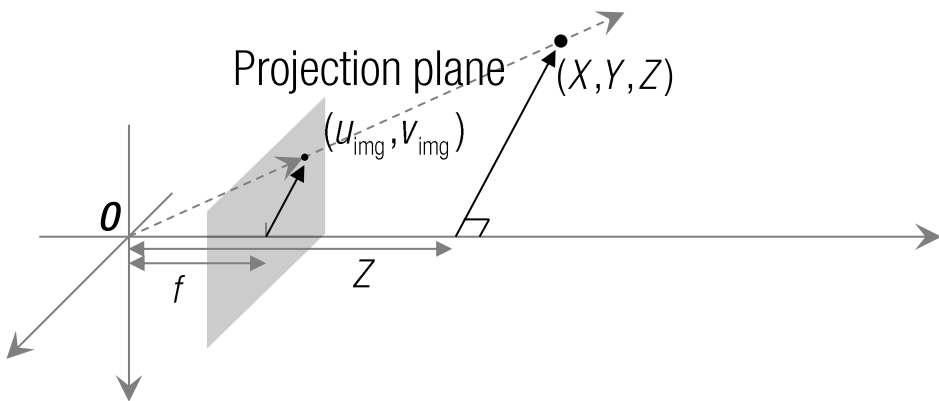
Pixel projection

$$\rightarrow u_{\text{img}} = u_{\text{ccd}} \frac{W_{\text{img}}}{W_{\text{ccd}}} + p_x = f_m \frac{W_{\text{img}}}{W_{\text{ccd}}} \frac{X}{Z} + p_x$$

$$v_{\text{img}} = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z} + p_y$$



# IMAGE PROJECTION



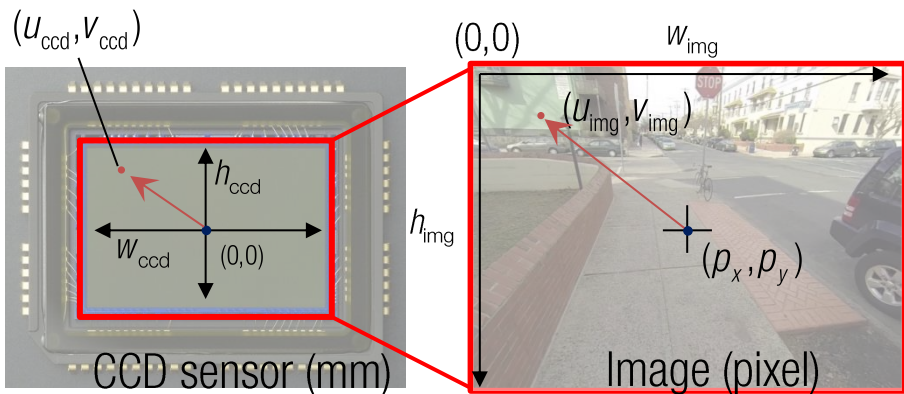
$$(u_{ccd}, v_{ccd}) = \left( f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$

Pixel projection

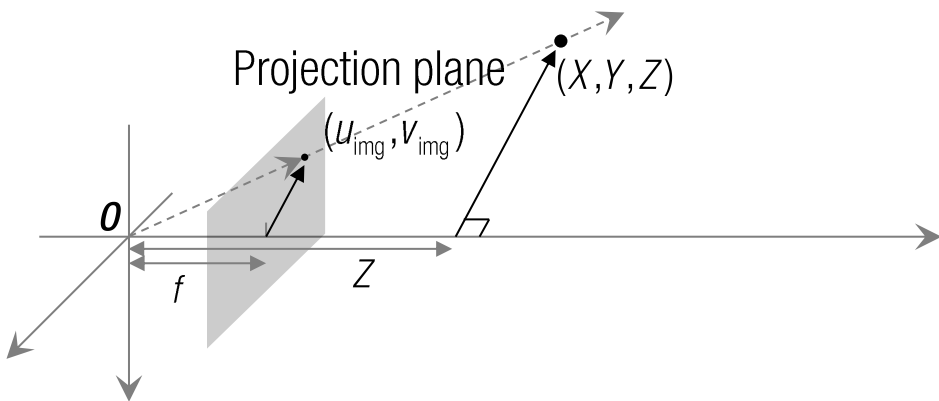
$$\rightarrow u_{img} = u_{ccd} \frac{W_{img}}{W_{ccd}} + p_x = f_m \frac{W_{img}}{W_{ccd}} \frac{X}{Z} + p_x$$

$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_m \frac{h_{img}}{h_{ccd}} \frac{Y}{Z} + p_y$$

Focal length in pixel



# IMAGE PROJECTION



$$(u_{ccd}, v_{ccd}) = \left( f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$

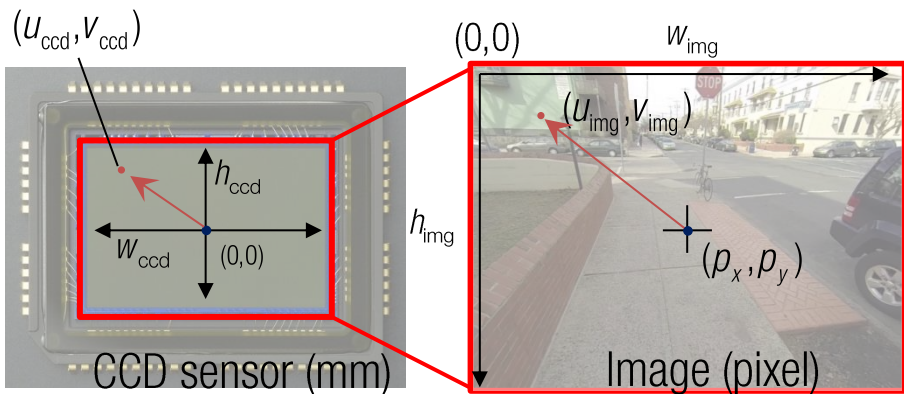
Pixel projection

$$\rightarrow u_{img} = u_{ccd} \frac{W_{img}}{W_{ccd}} + p_x = f_x \frac{X}{Z} + p_x$$

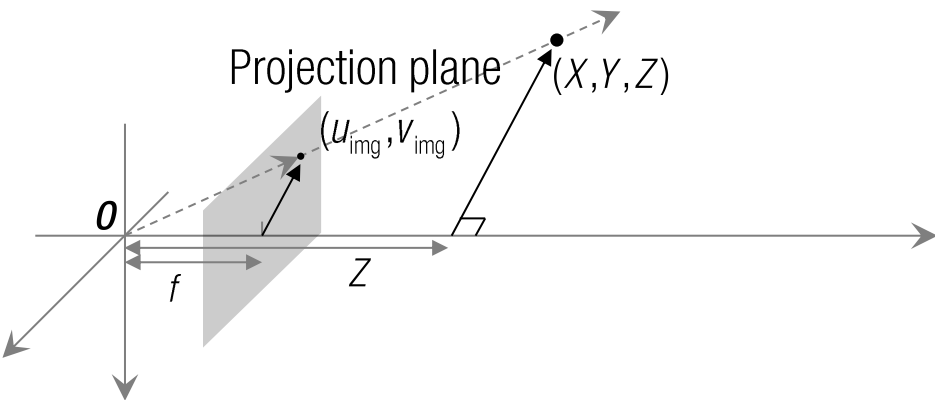
$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_y \frac{Y}{Z} + p_y$$

Focal length in pixel

$$\text{where } f_x = f_m \frac{W_{img}}{W_{ccd}} \quad f_y = f_m \frac{h_{img}}{h_{ccd}}$$



# IMAGE PROJECTION



$$(u_{ccd}, v_{ccd}) = \left( f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$

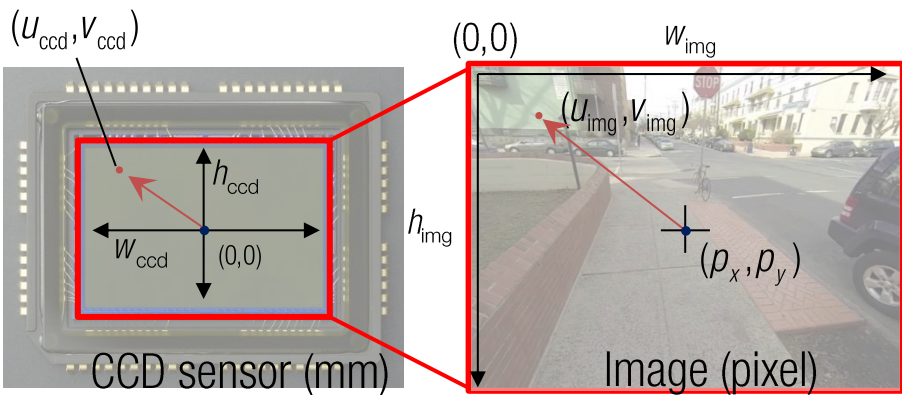
Pixel projection

$$\rightarrow u_{img} = u_{ccd} \frac{W_{img}}{W_{ccd}} + p_x = \boxed{f} \frac{X}{Z} + p_x$$

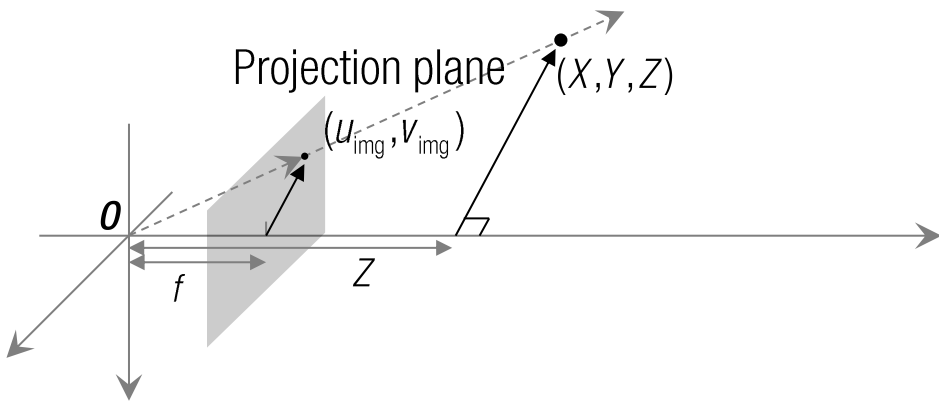
$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = \boxed{f} \frac{Y}{Z} + p_y$$

Focal length in pixel

$$\text{where } \bar{f} = f_m \frac{W_{img}}{W_{ccd}} = f_m \frac{h_{img}}{h_{ccd}} \quad \text{if } \frac{W_{img}}{W_{ccd}} = \frac{h_{img}}{h_{ccd}}$$

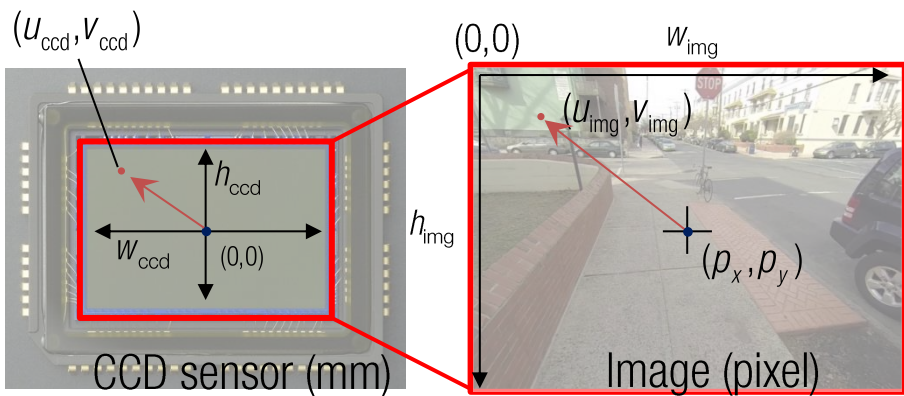


# IMAGE PROJECTION



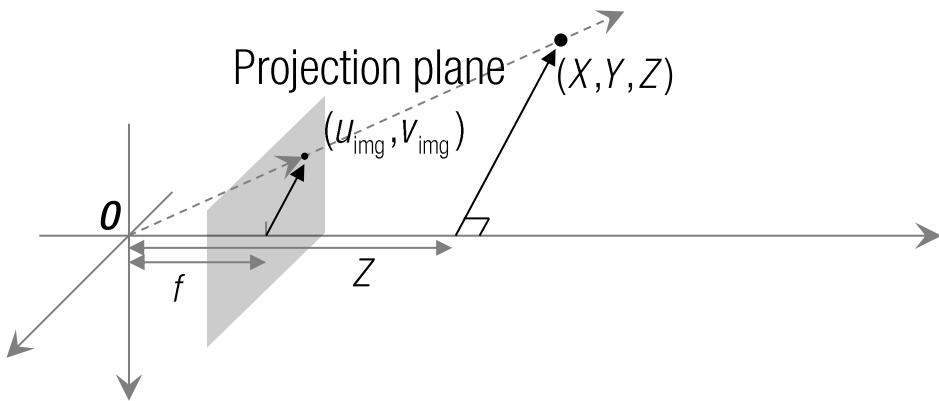
$$u_{\text{img}} = f \frac{X}{Z} + p_x$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y$$



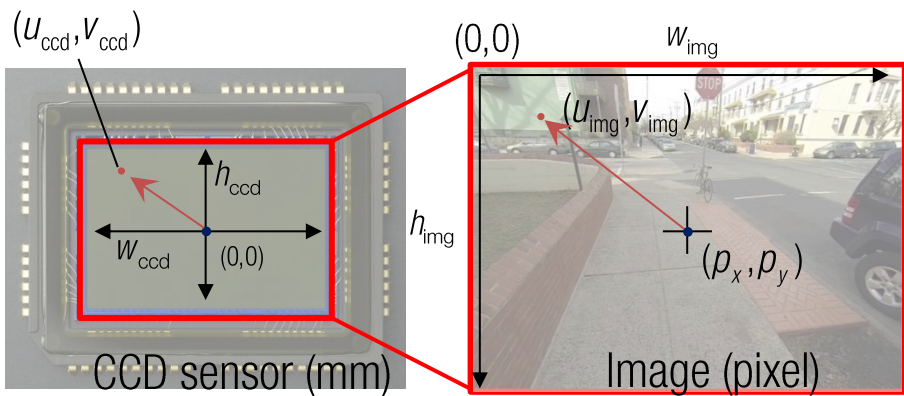


# IMAGE PROJECTION

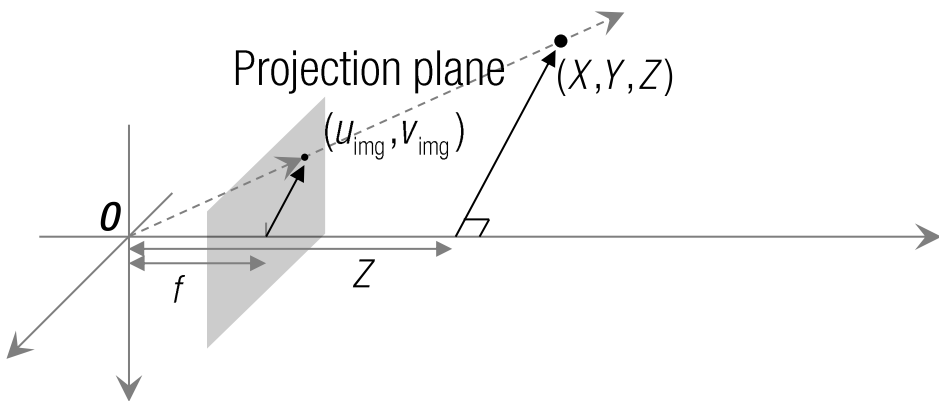


$$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{\text{img}} = fY + p_y Z$$



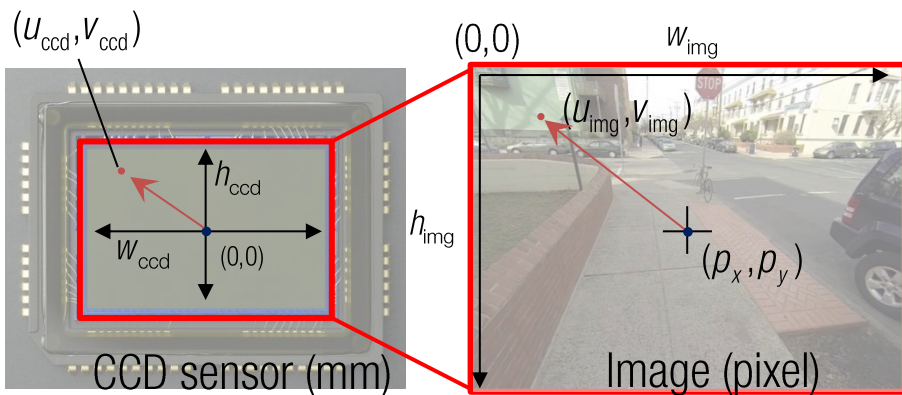
# IMAGE PROJECTION



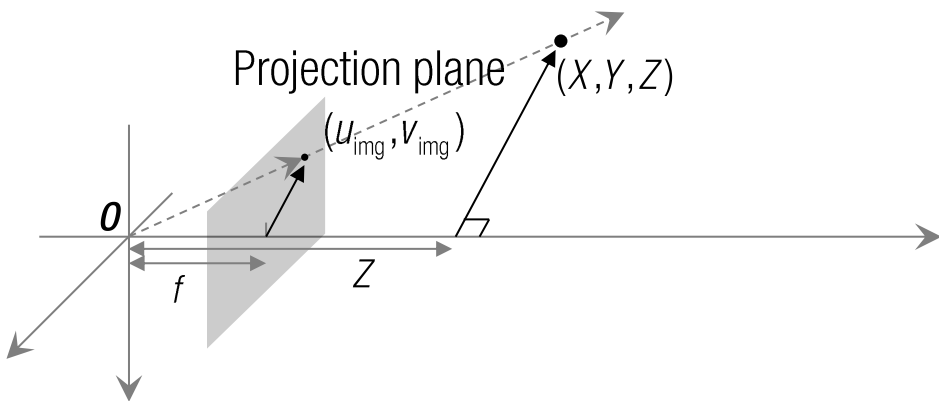
$$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{\text{img}} = fY + p_y Z$$

$$Z \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



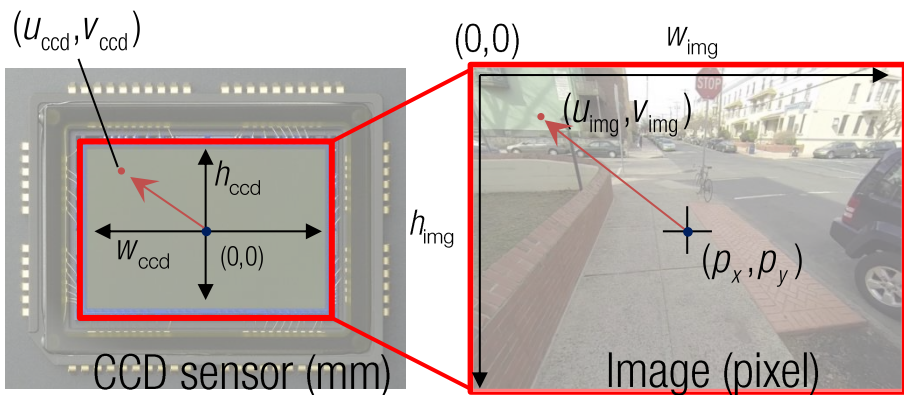
# IMAGE PROJECTION



$$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{\text{img}} = fY + p_y Z$$

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



# RENDERING



$$u_{\text{img}} = f \frac{X}{Z} + p_x \longrightarrow Z u_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \longrightarrow Z v_{\text{img}} = fY + p_y Z$$

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

←  
Graphics

# 3D RECONSTRUCTION ~ INVERSE RENDERING



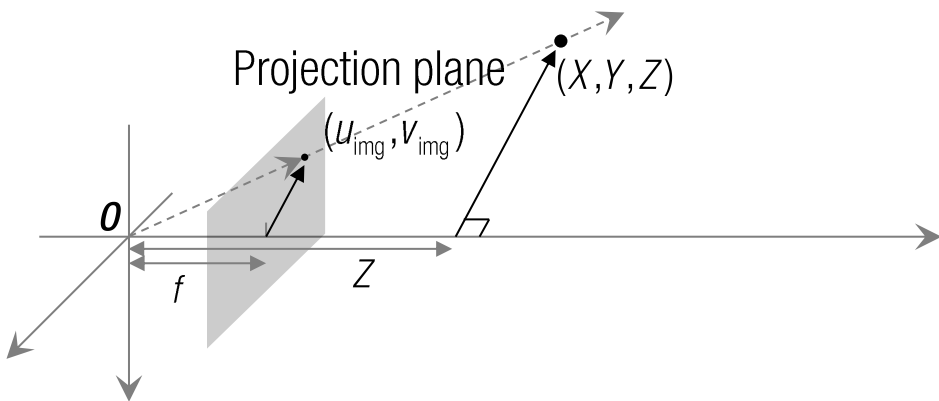
$$u_{\text{img}} = f \frac{X}{Z} + p_x \longrightarrow Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \longrightarrow Zv_{\text{img}} = fY + p_y Z$$

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

← Graphics  
→ Vision

# IMAGE PROJECTION



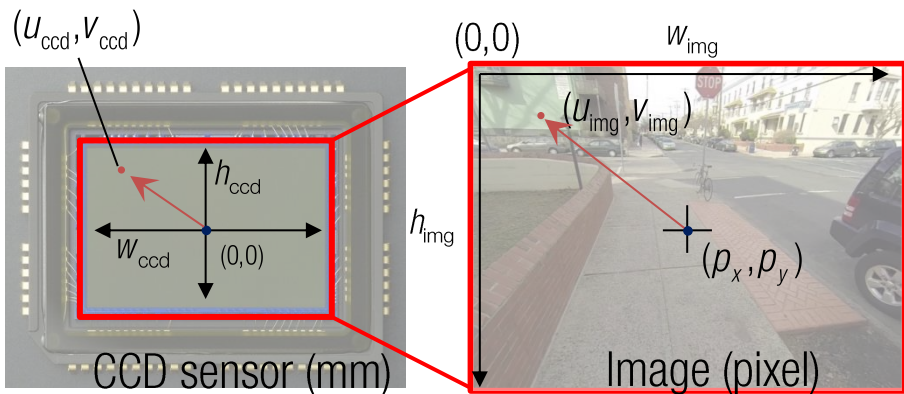
$$u_{img} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{img} = fX + p_x Z$$

$$v_{img} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{img} = fY + p_y Z$$

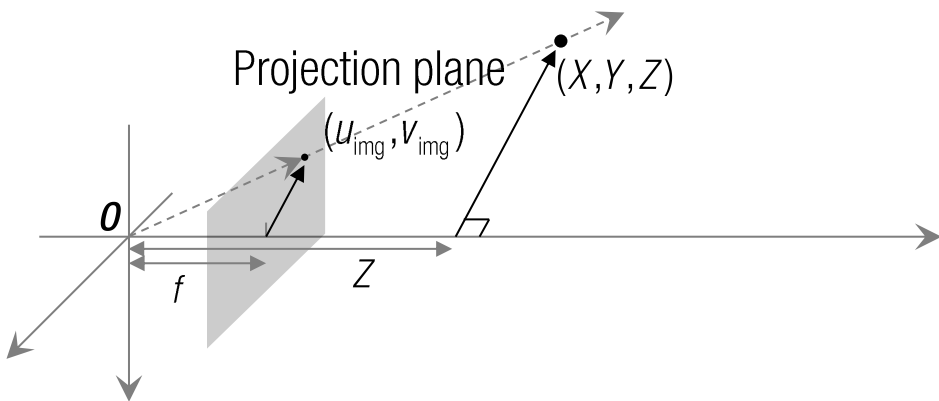
Pixel space

Metric space

$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ f \\ 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



# IMAGE PROJECTION

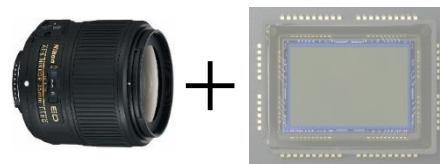
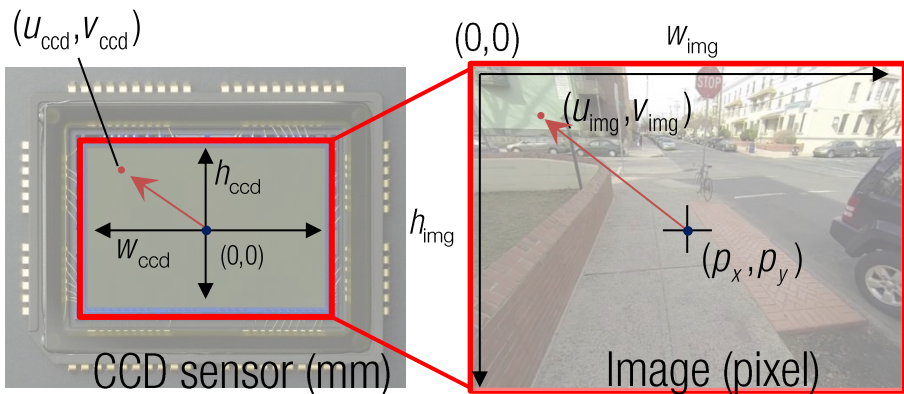


$$u_{img} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{img} = fX + p_x Z$$

$$v_{img} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{img} = fY + p_y Z$$

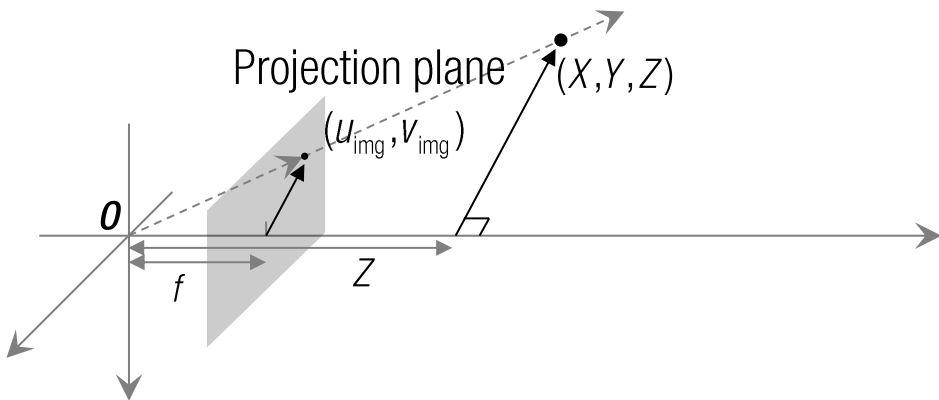
Pixel space                      Metric space

$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Camera intrinsic parameter  
: metric space to pixel space

# IMAGE PROJECTION

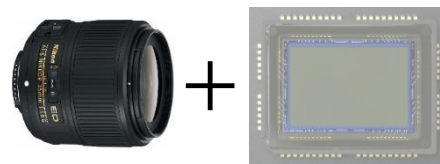
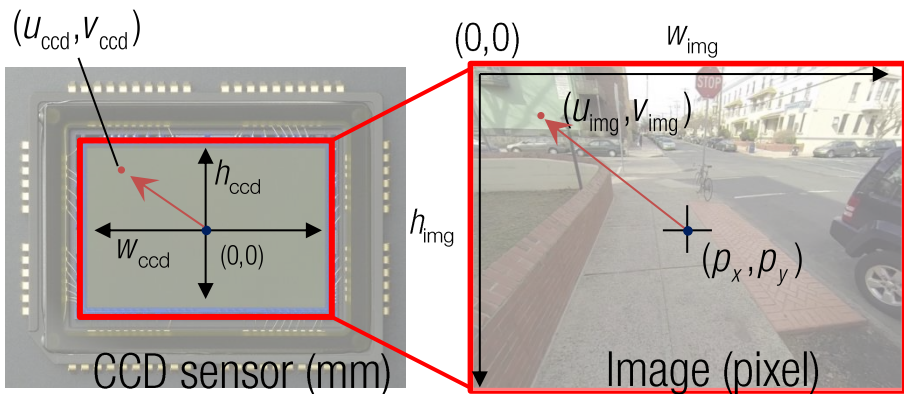


$$u_{img} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{img} = fX + p_x Z$$

$$v_{img} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{img} = fY + p_y Z$$

Pixel space                      Metric space

$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & p_y \\ & 1 \end{bmatrix} \mathbf{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



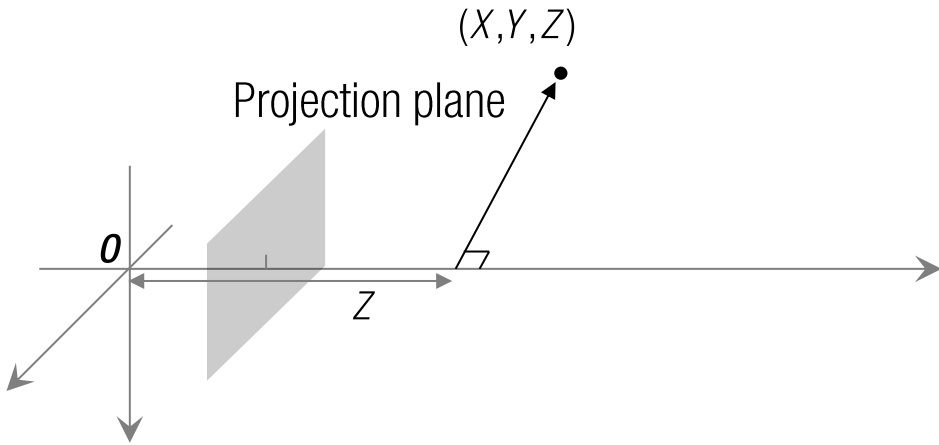
Camera intrinsic parameter  
: metric space to pixel space



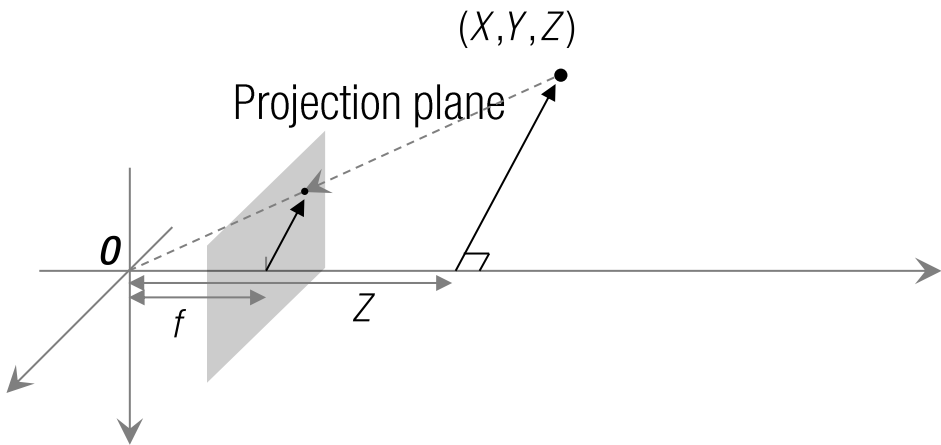
# ***INTRINSIC PARAMETER***

Metric space

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



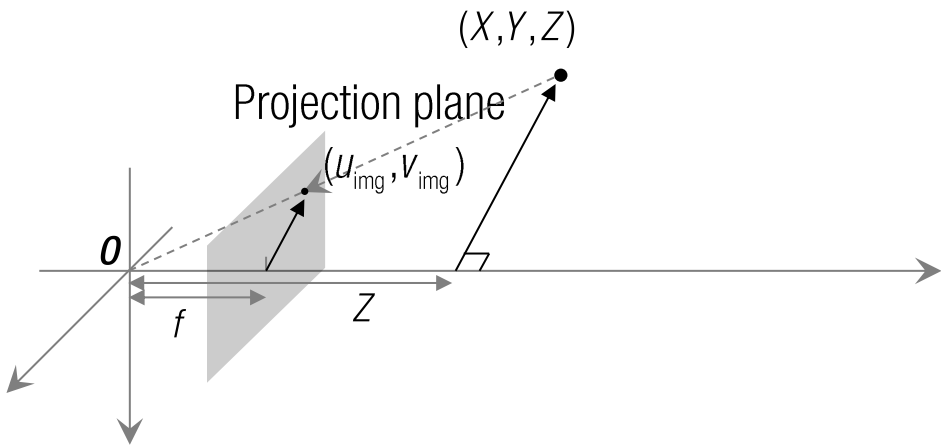
# ***INTRINSIC PARAMETER***



$$\mathbf{K} \begin{bmatrix} f \\ \rho_x \\ \rho_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Metric space

# ***INTRINSIC PARAMETER***



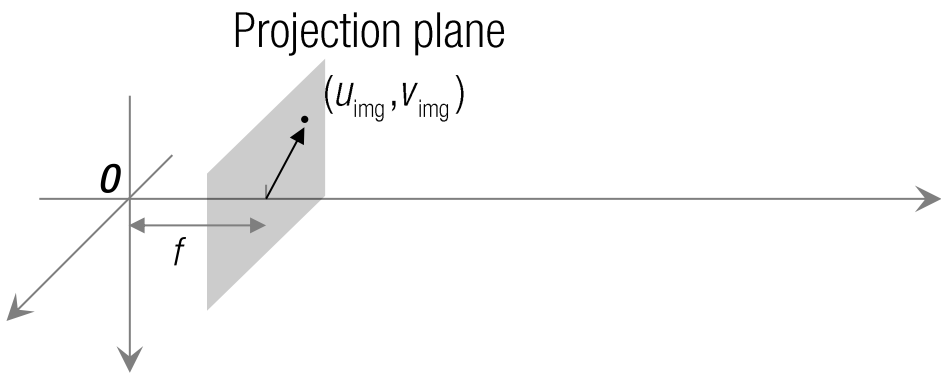
Pixel space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \end{bmatrix}$$

Metric space

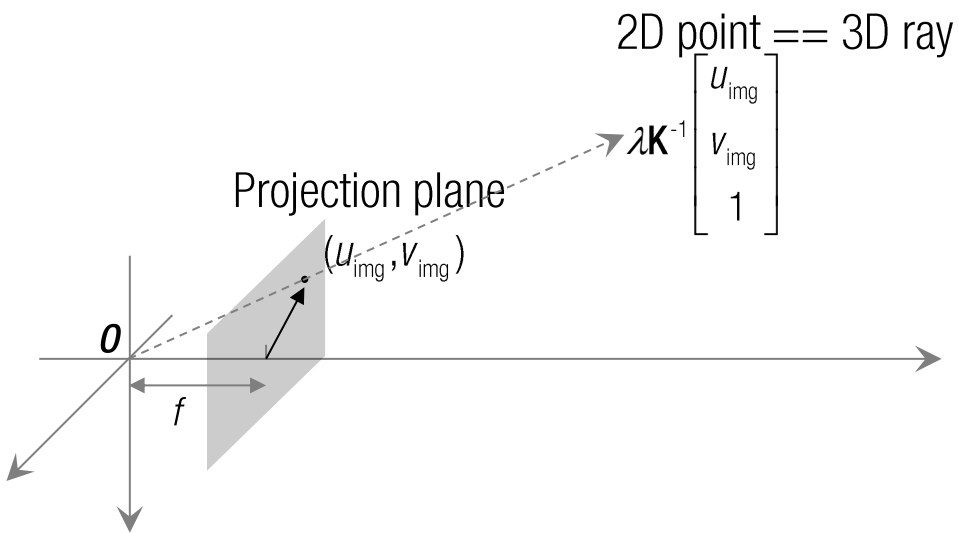
$$\mathbf{K} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

# ***INVERSE PROJECTION***



$$\begin{array}{l} \text{Pixel space} \\ \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \end{array} \quad \mathbf{K} \quad \begin{array}{l} \text{Metric space} \\ \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \end{array}$$
$$\begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix}$$

# ***INVERSE PROJECTION***

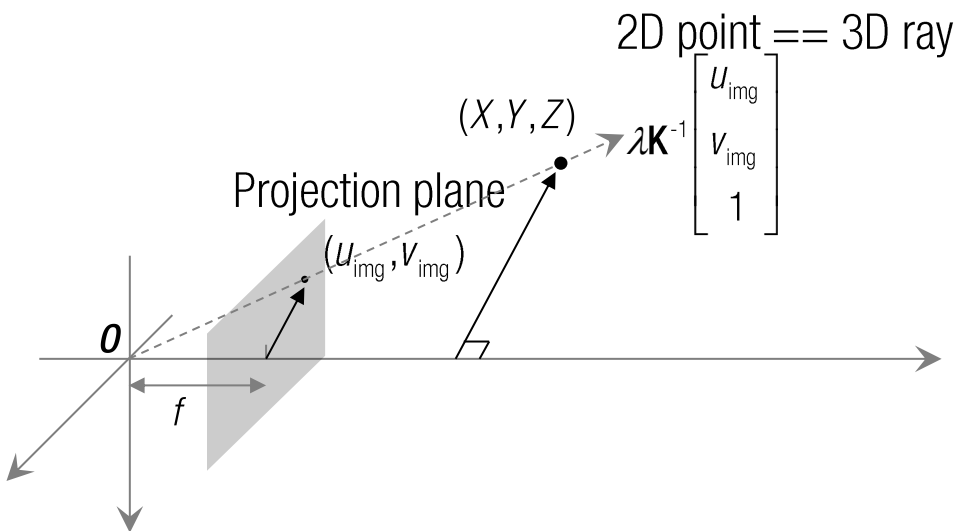


2D point == 3D ray

$$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix}$$

$$\begin{array}{c} \text{Pixel space} \\ \lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f \end{bmatrix} \\ \mathbf{K} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \\ \text{Metric space} \end{array}$$
$$\frac{\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix}}{\text{3D ray}}$$

# ***INVERSE PROJECTION***



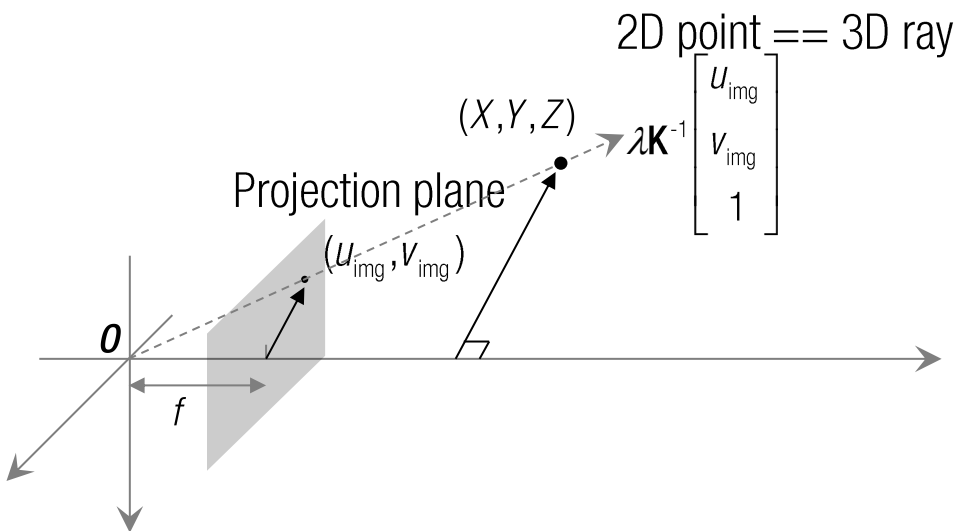
2D point == 3D ray

$$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix}$$

$$\begin{array}{c} \text{Pixel space} \\ \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ \\ \end{bmatrix} \end{array} \quad \mathbf{K} \quad \begin{array}{c} \text{Metric space} \\ \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \end{array}$$
$$\underline{\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix}} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

3D ray

# ***INVERSE PROJECTION***



$$\begin{array}{c}
 \text{Pixel space} \\
 \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ \mathbf{K} \begin{bmatrix} \rho_x \\ \rho_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \\
 \text{Metric space} \\
 \lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \\
 \text{3D ray}
 \end{array}$$

The 3D point must lie in the 3D ray passing through the origin and 2D image point.