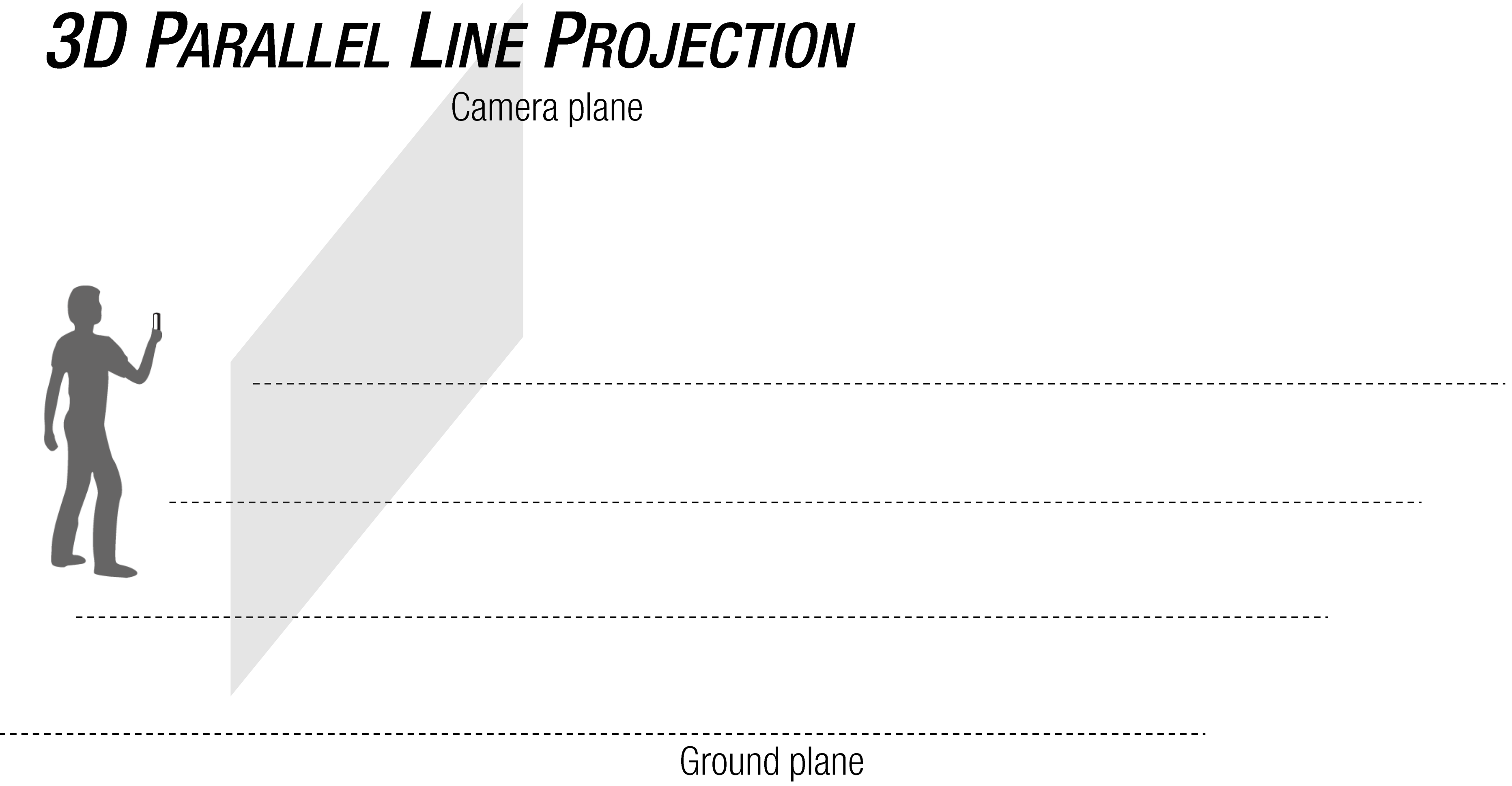
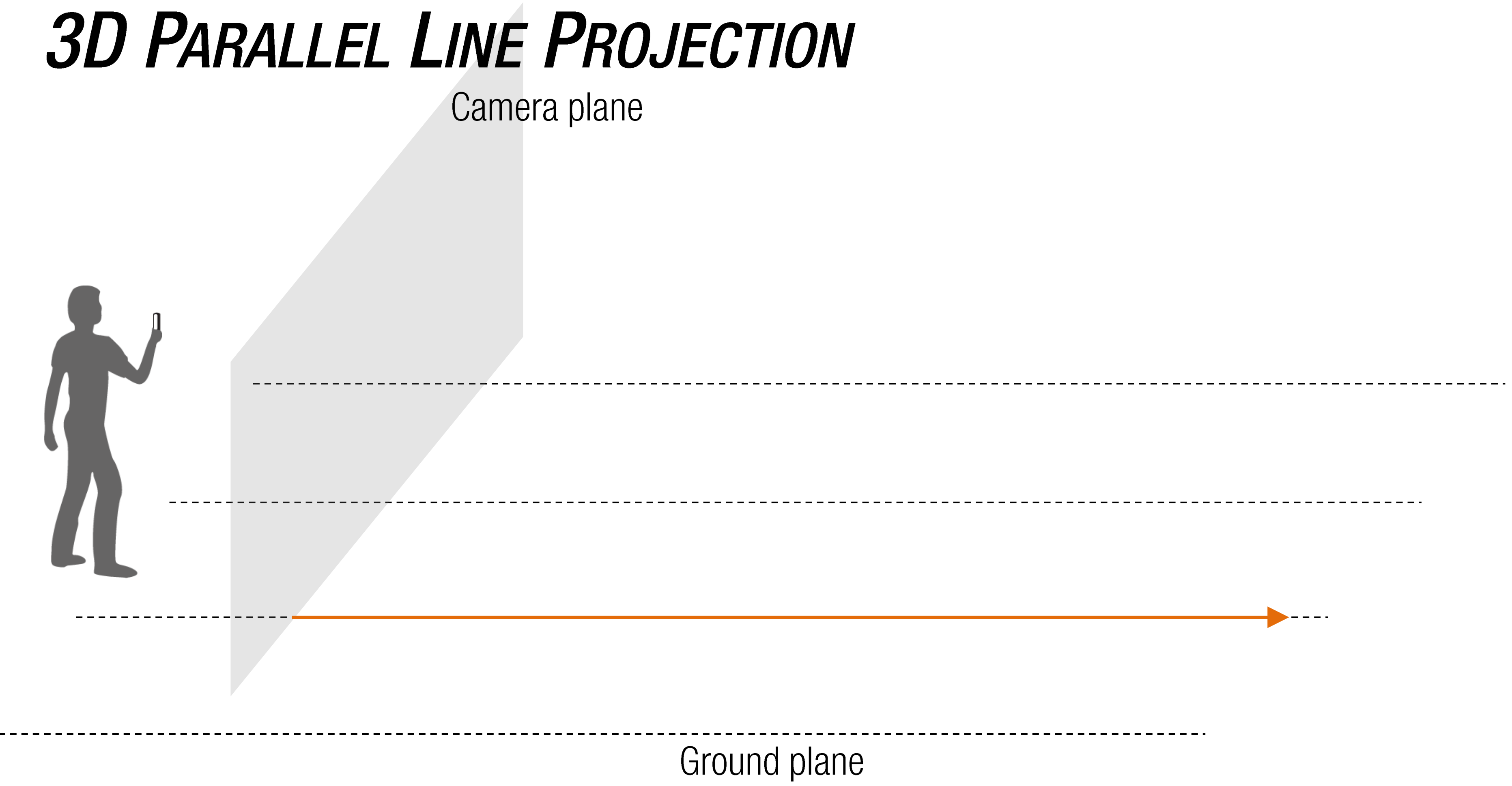
PROJECTIVE LINE

HYUN SOO PARK

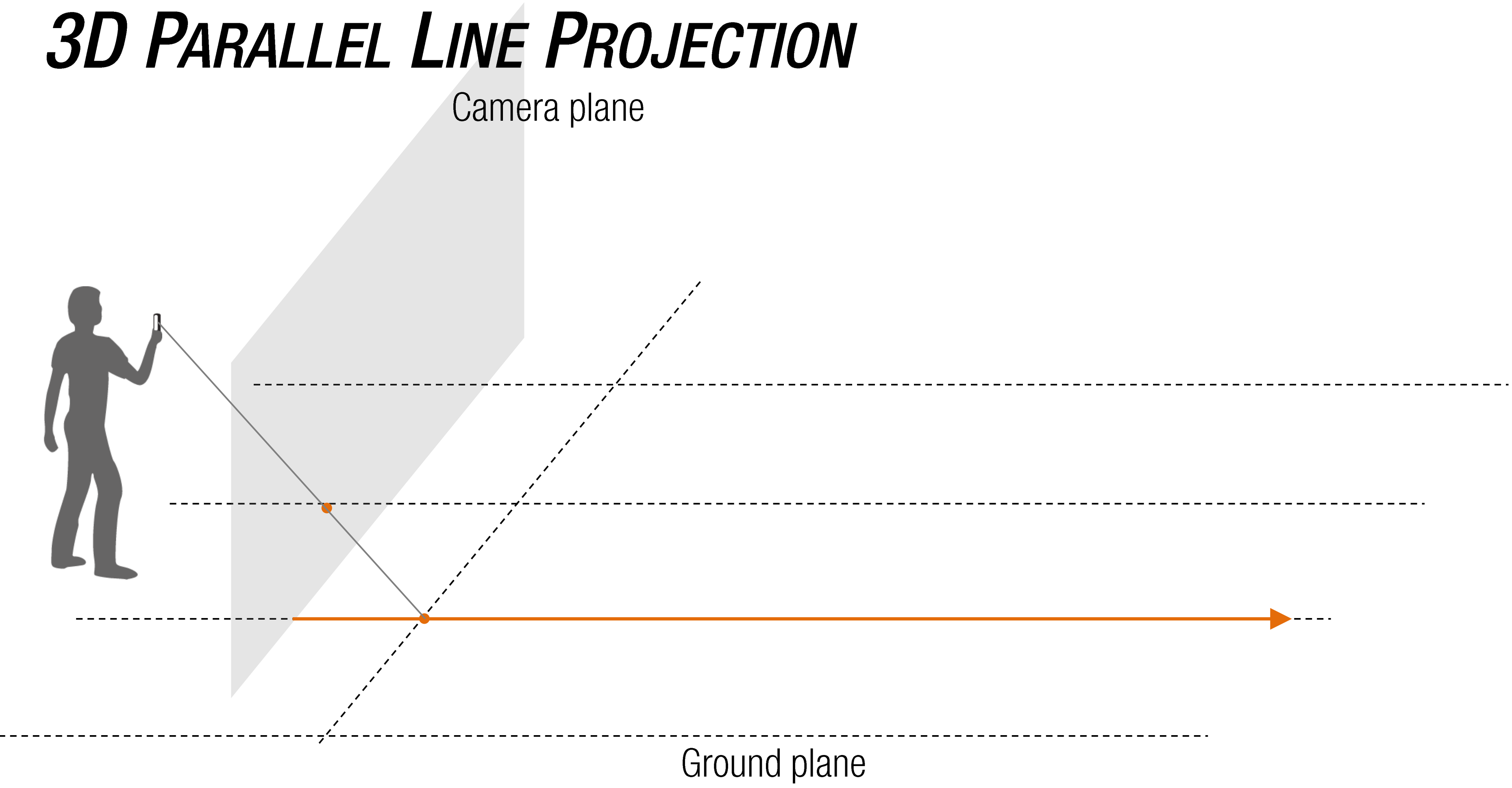
3D Parallel Line Projection



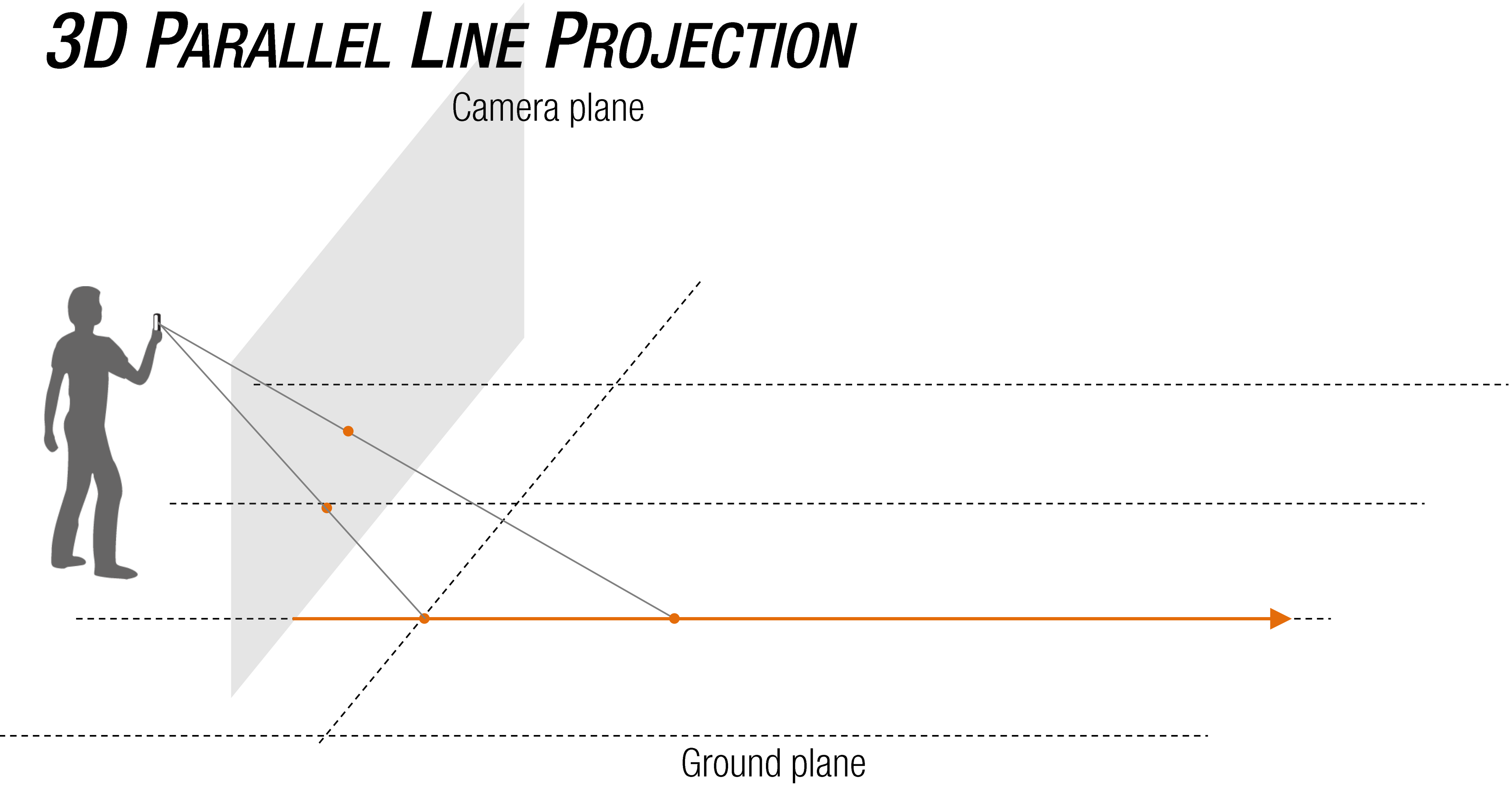
3D Parallel Line Projection



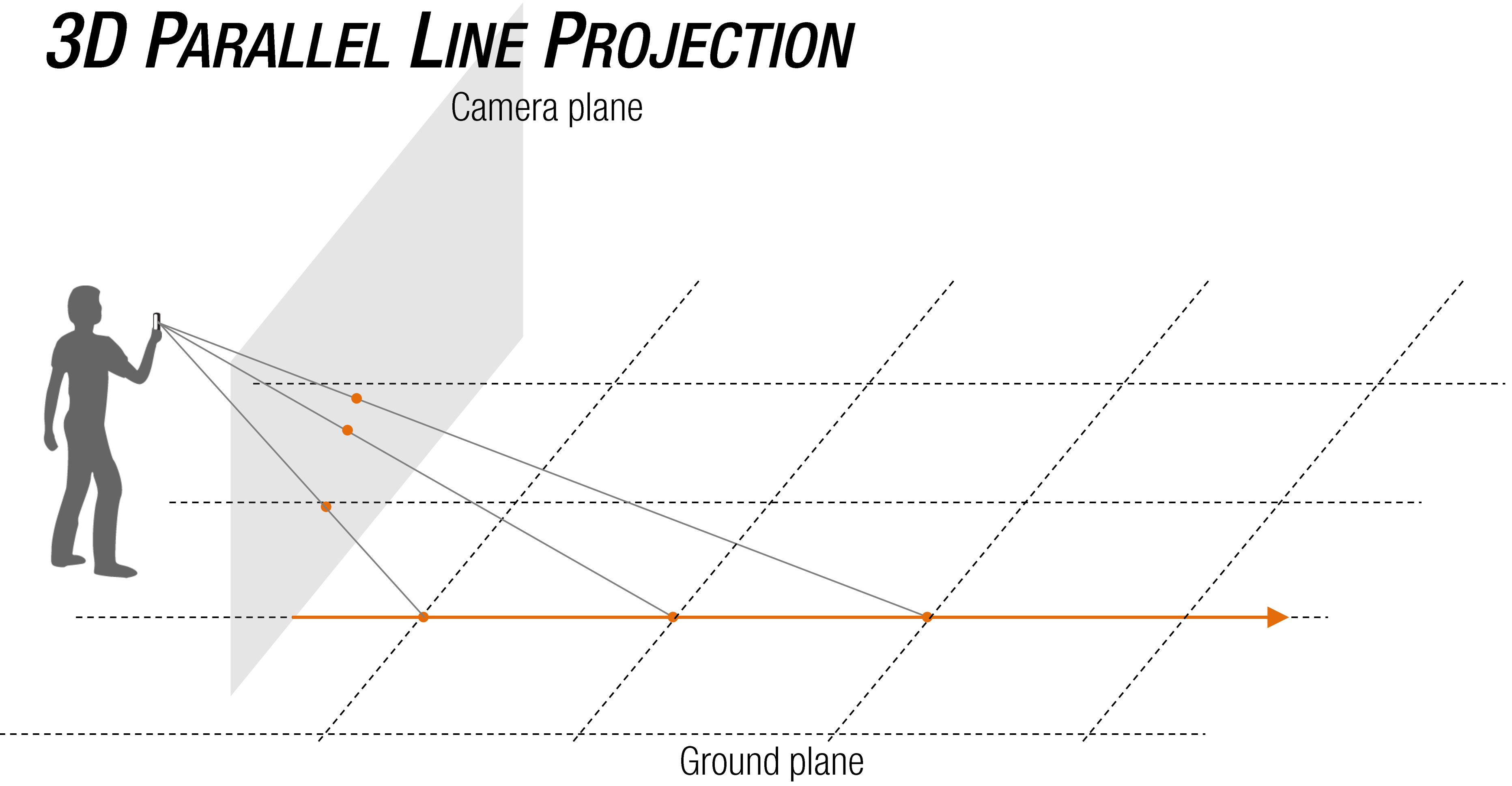
3D Parallel Line Projection



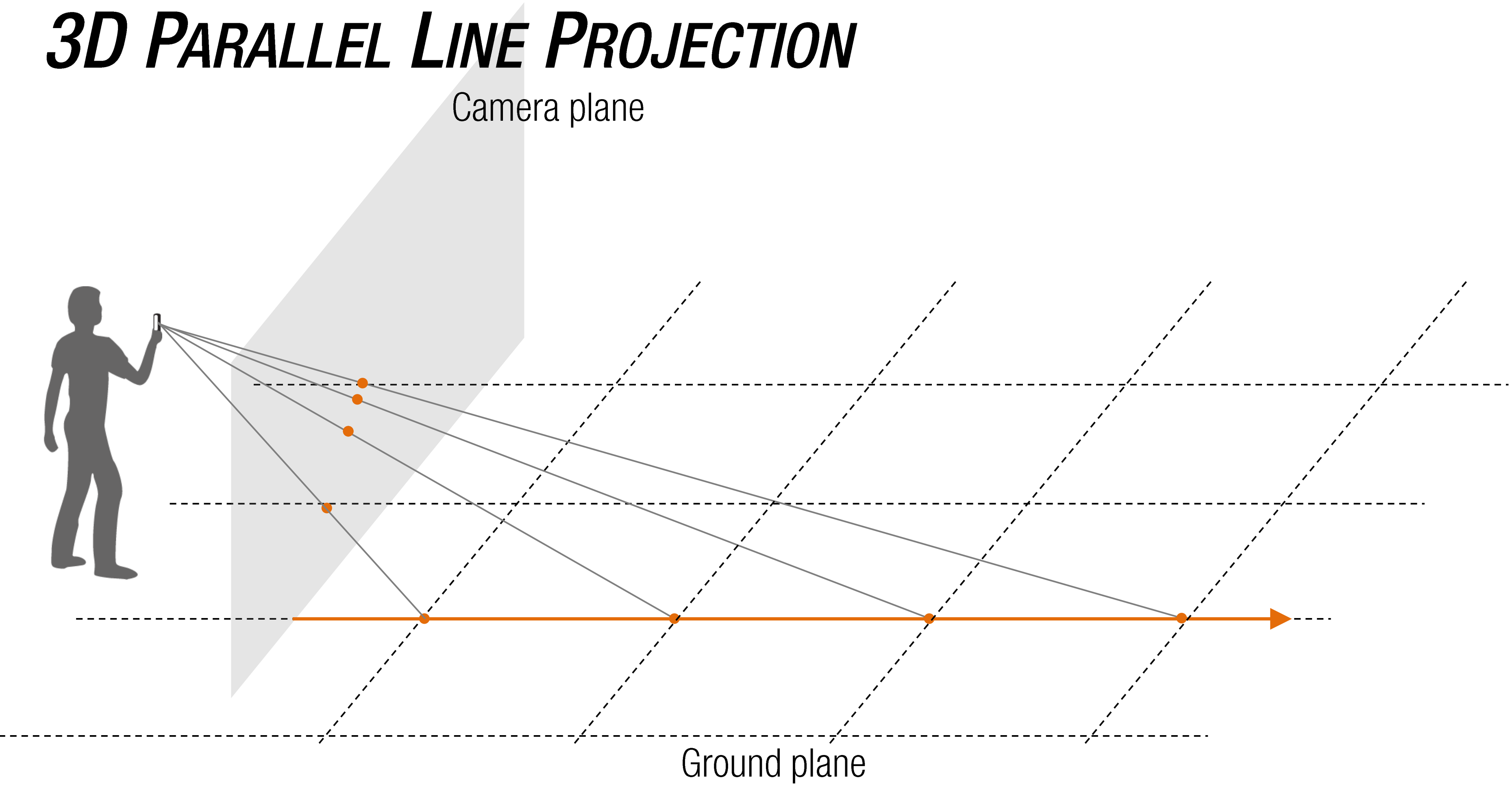
3D Parallel Line Projection



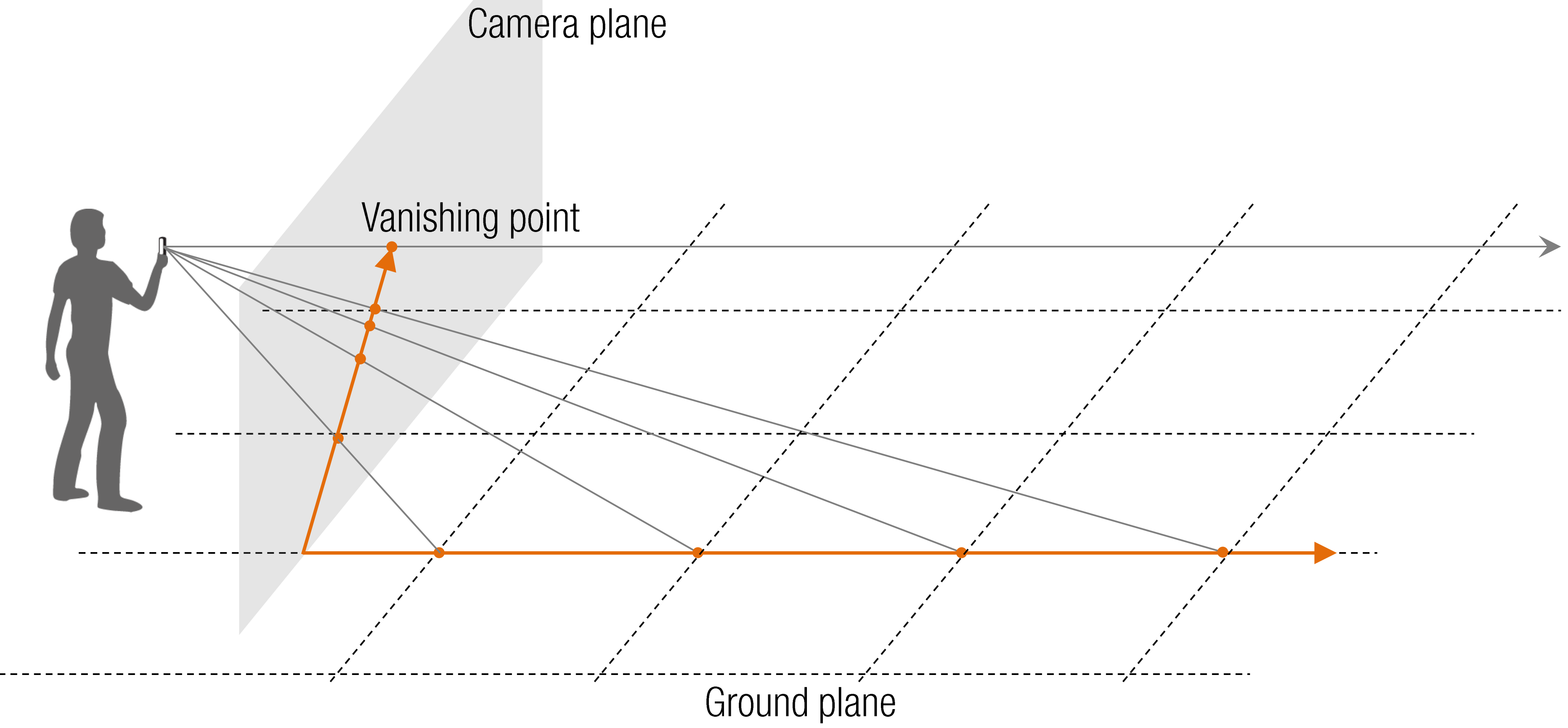
3D Parallel Line Projection



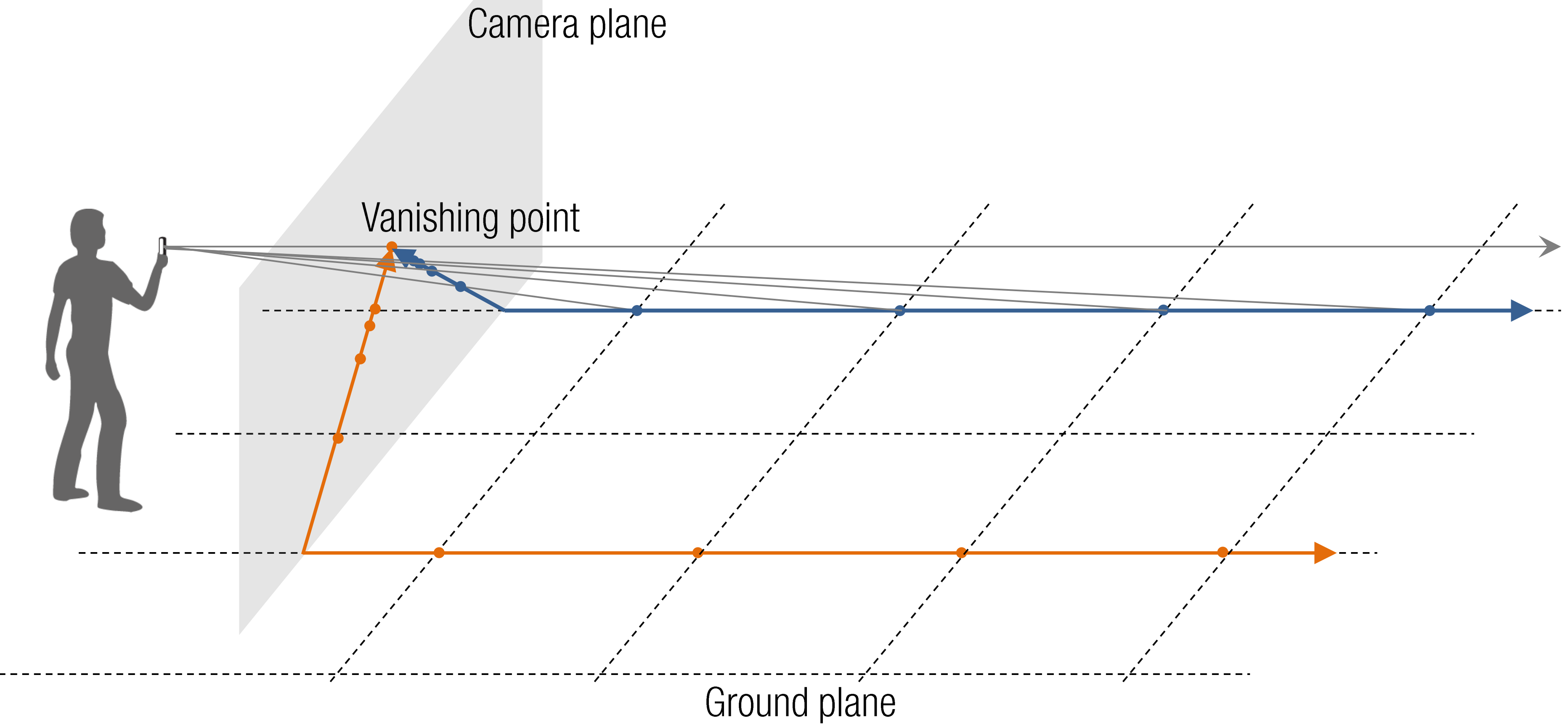
3D PARALLEL LINE PROJECTION



3D Parallel Line Projection

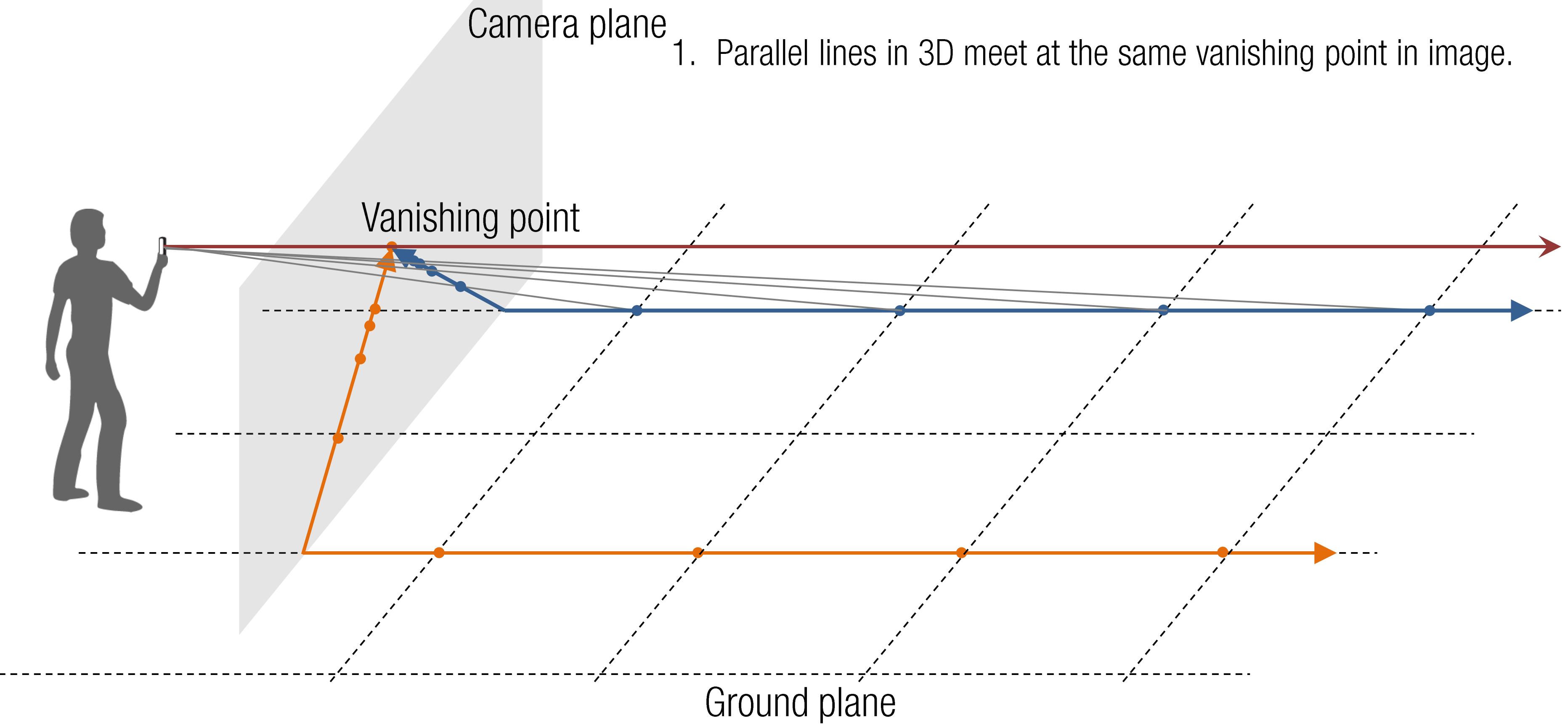


3D Parallel Line Projection



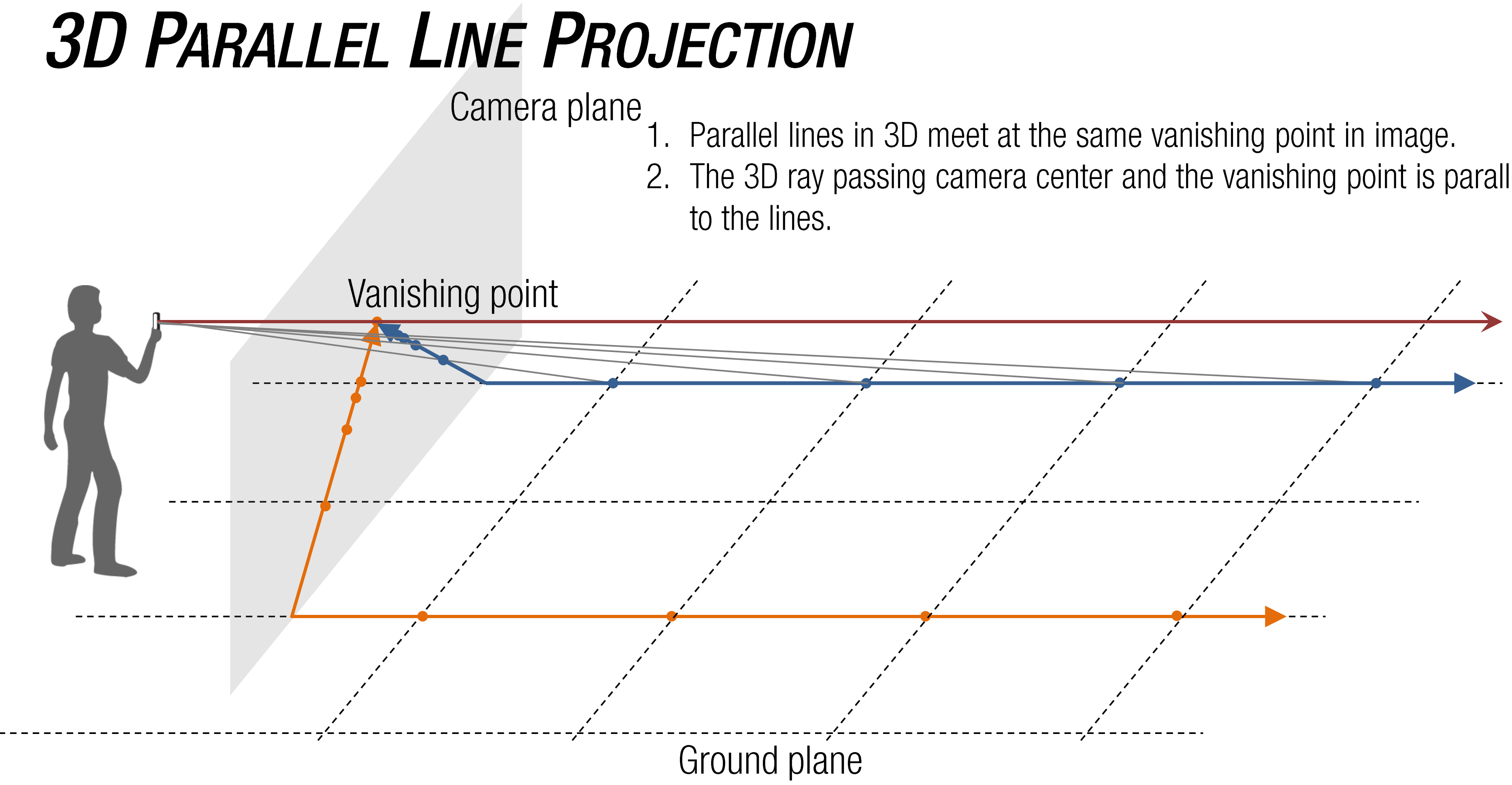
3D PARALLEL LINE PROJECTION

1. Parallel lines in 3D meet at the same vanishing point in image.



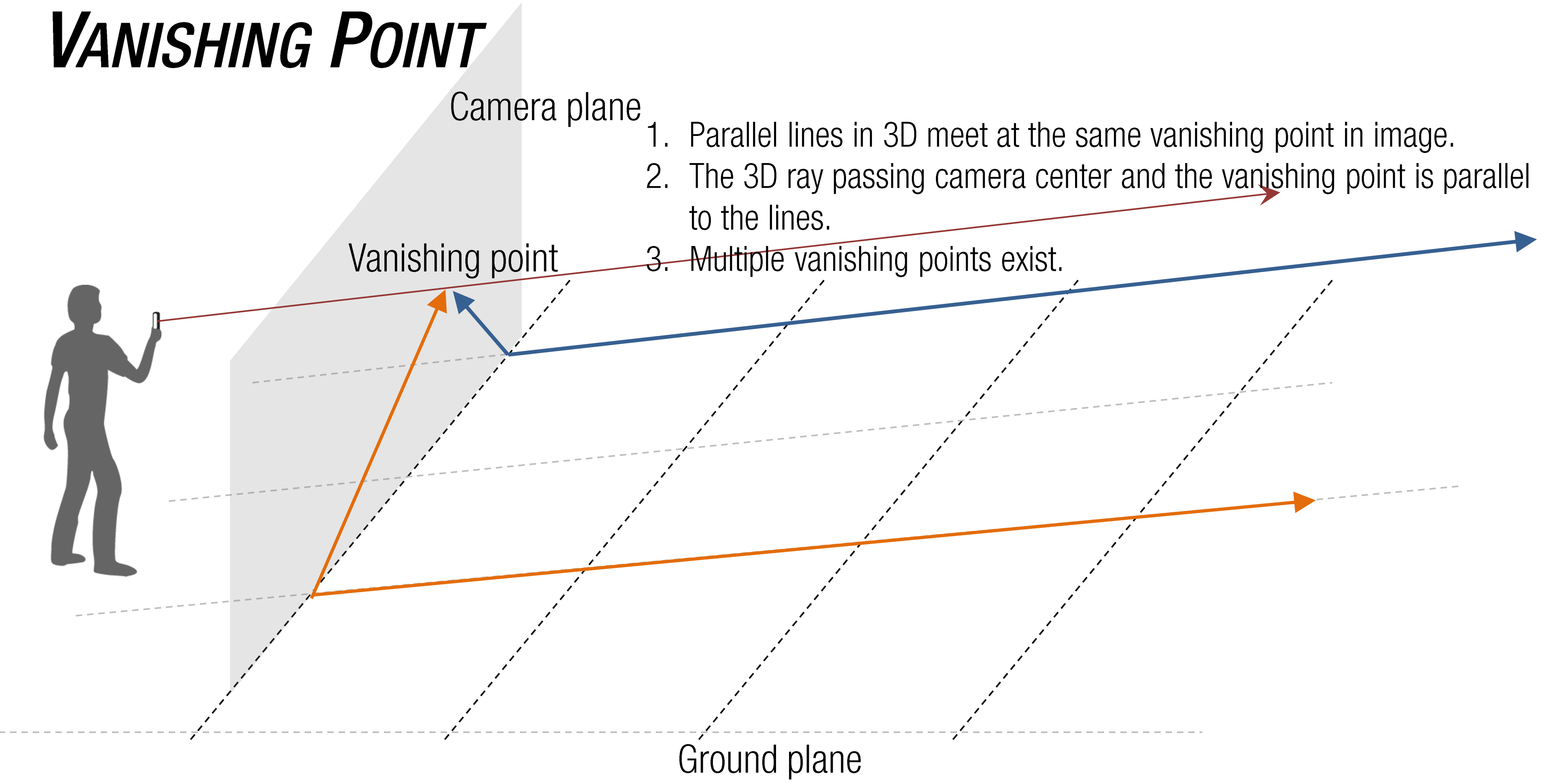
3D Parallel Line Projection

1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.



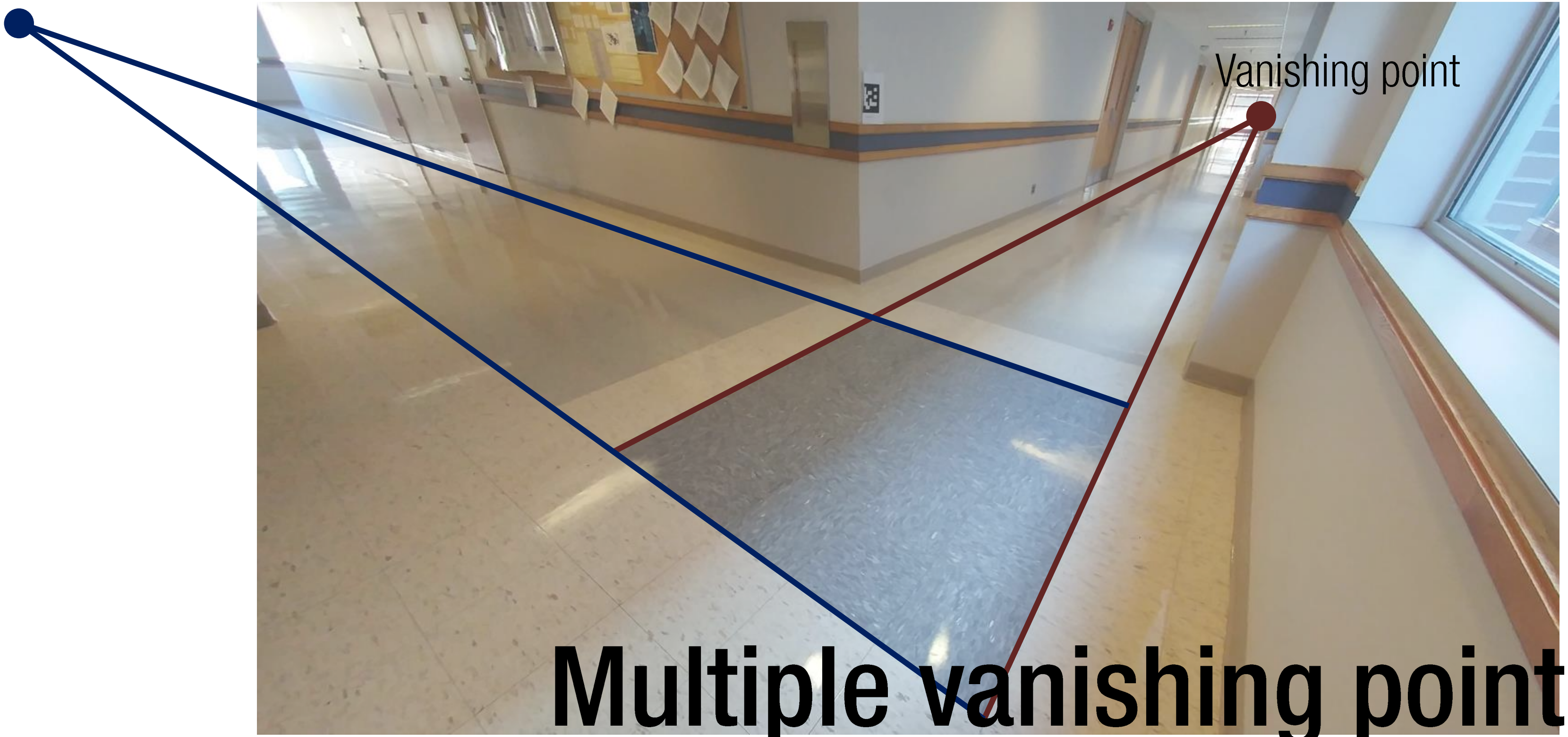
VANISHING POINT

1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.
3. Multiple vanishing points exist.




Vanishing point

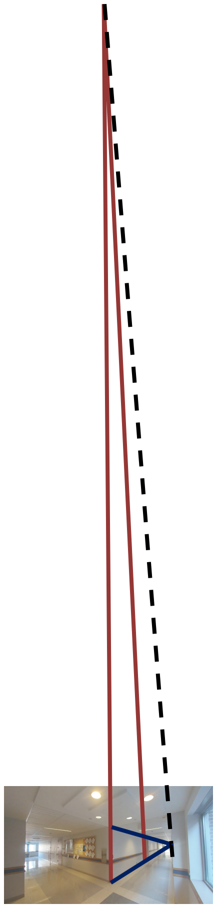
Multiple vanishing point



Vanishing line: Horizon

Parallel 3D planes share the vanishing line.

Different plane produces different vanishing line.

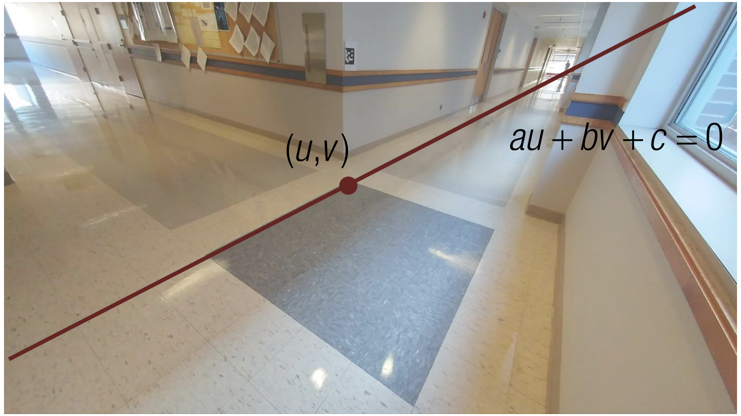


Different plane produces different vanishing line.

How to compute a vanishing point?

Different plane produces different vanishing line.

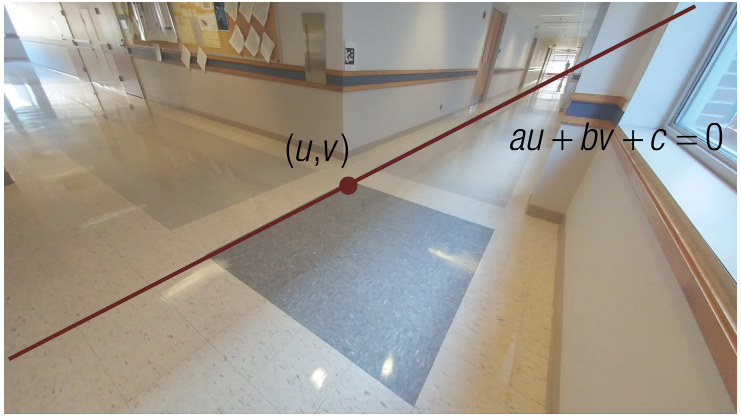
Point-Line



A 2D line passing through 2D point (u,v):
$au + bv + c = 0$

Line parameter: (a,b,c)

POINT-LINE

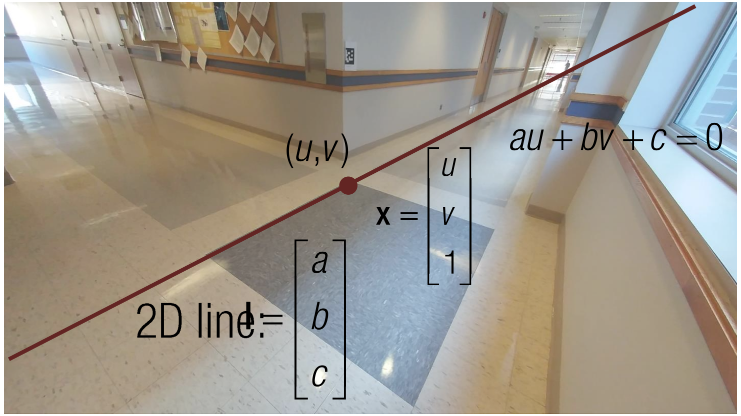


A 2D line passing through 2D point (u,v):
$au + bv + c = 0$

Line parameter: (a,b,c)

$$au + bv + c = 0 \rightarrow \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{l}^{\mathsf{T}} \mathbf{x} = 0$$

Point-Line



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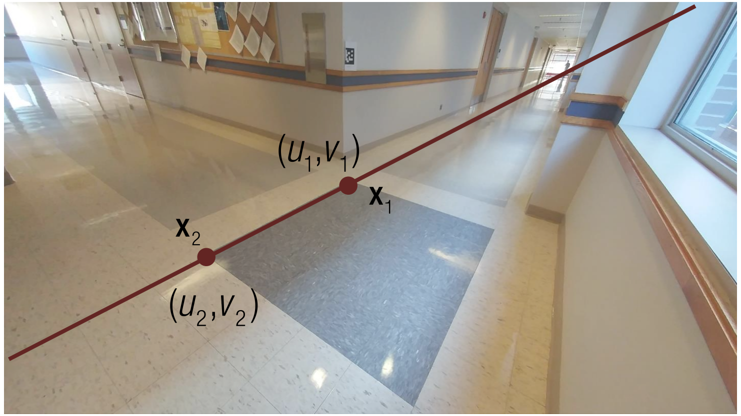
where $\underbrace{\mathbf{x}=\begin{bmatrix}u\\v\\1\end{bmatrix}}_{\text{2D point}}$ and $\underbrace{\mathbf{l}=\begin{bmatrix}a\\b\\c\end{bmatrix}}_{\text{Line parameter}}$

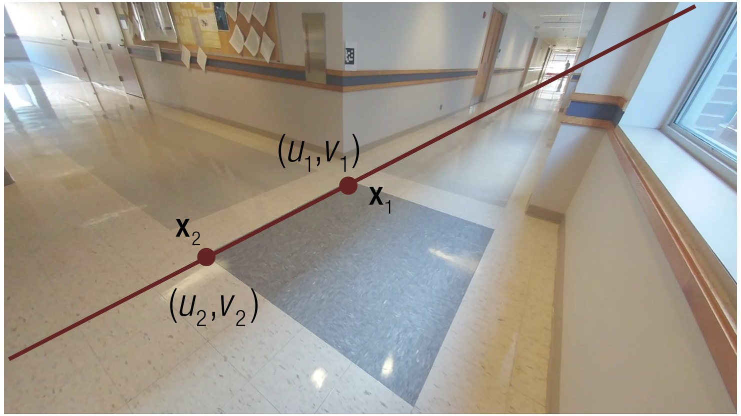
Point-Line



A 2D line passing through two 2D points:

$au_1 + bv_1 + c = 0 \qquad au_2 + bv_2 + c = 0$

POINT-LINE



A 2D line passing through two 2D points:

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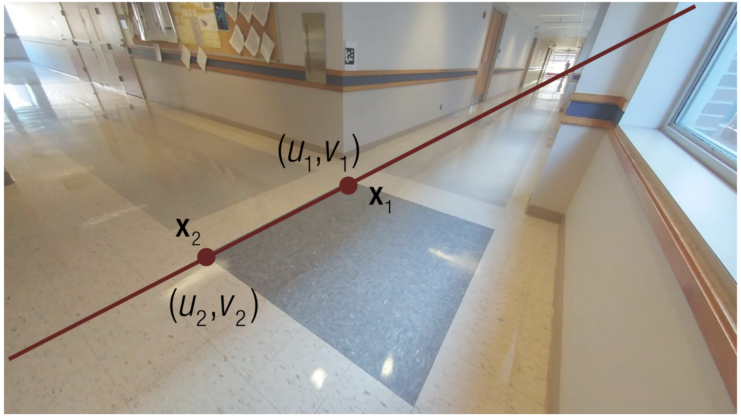
$$\mathbf{x}_1^{\mathsf{T}}\mathbf{l} = 0 \qquad \mathbf{x}_2^{\mathsf{T}}\mathbf{l} = 0$$

where $\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} \quad \mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} \quad \mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

Point-Line



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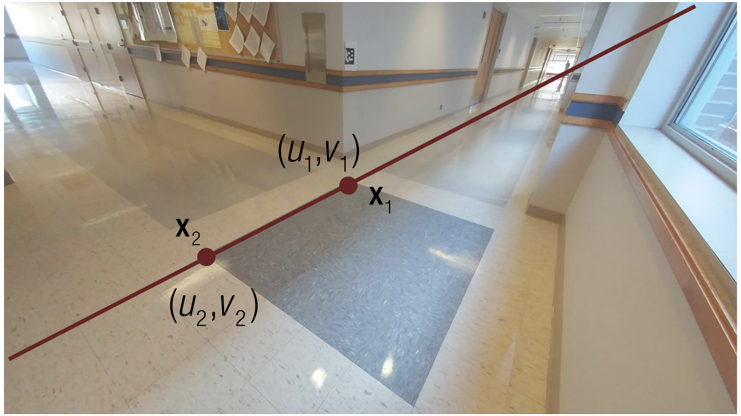
$$\rightarrow \begin{bmatrix} \mathbf{x}_1^{\mathsf{T}} \\ \mathbf{x}_2^{\mathsf{T}} \end{bmatrix} \mathbf{l} = \mathbf{0}$$

$$\underbrace{\mathbf{A}}_{2\text{x}3} \; \blacksquare = \begin{matrix} 0 \\ 0 \end{matrix}$$

Point-Line



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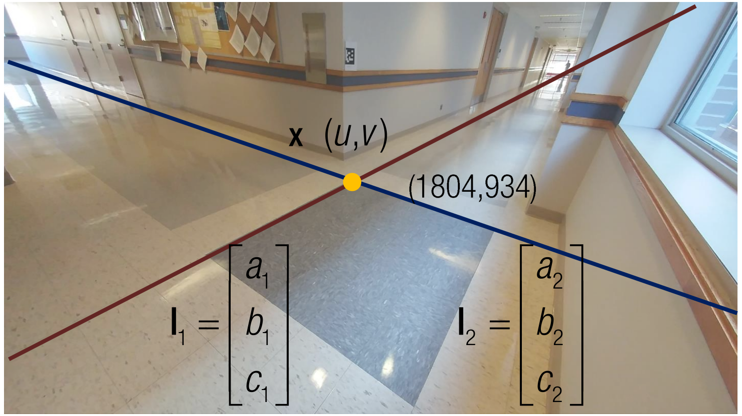
$$\underbrace{\mathbf{A}}_{2\text{x}3}\;\square = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \rightarrow \quad \square = \text{null}\left(\mathbf{A}\right) \quad \text{or} \quad \mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$$

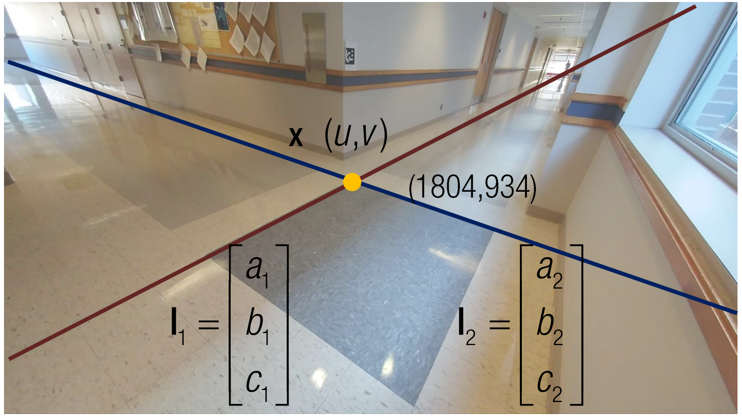
LINE-LINE



Two 2D lines in an image intersect at a 2D point:

$$a_1 u + b_1 v + c_1 = 0 \qquad a_2 u + b_2 v + c_2 = 0$$

LINE-LINE



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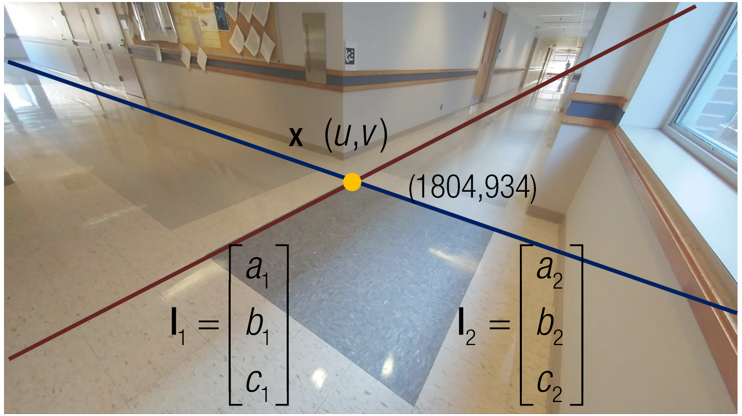
$$\mathbf{l}_1^{\mathsf{T}} \mathbf{x} = 0 \qquad \mathbf{l}_2^{\mathsf{T}} \mathbf{x} = 0$$

where $\mathbf{x} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \quad \mathbf{l}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \quad \mathbf{l}_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$

LINE-LINE



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$\rightarrow \begin{bmatrix} \mathbf{l}_1^{\mathsf{T}} \\ \mathbf{l}_2^{\mathsf{T}} \end{bmatrix} \mathbf{x} = \mathbf{0}$

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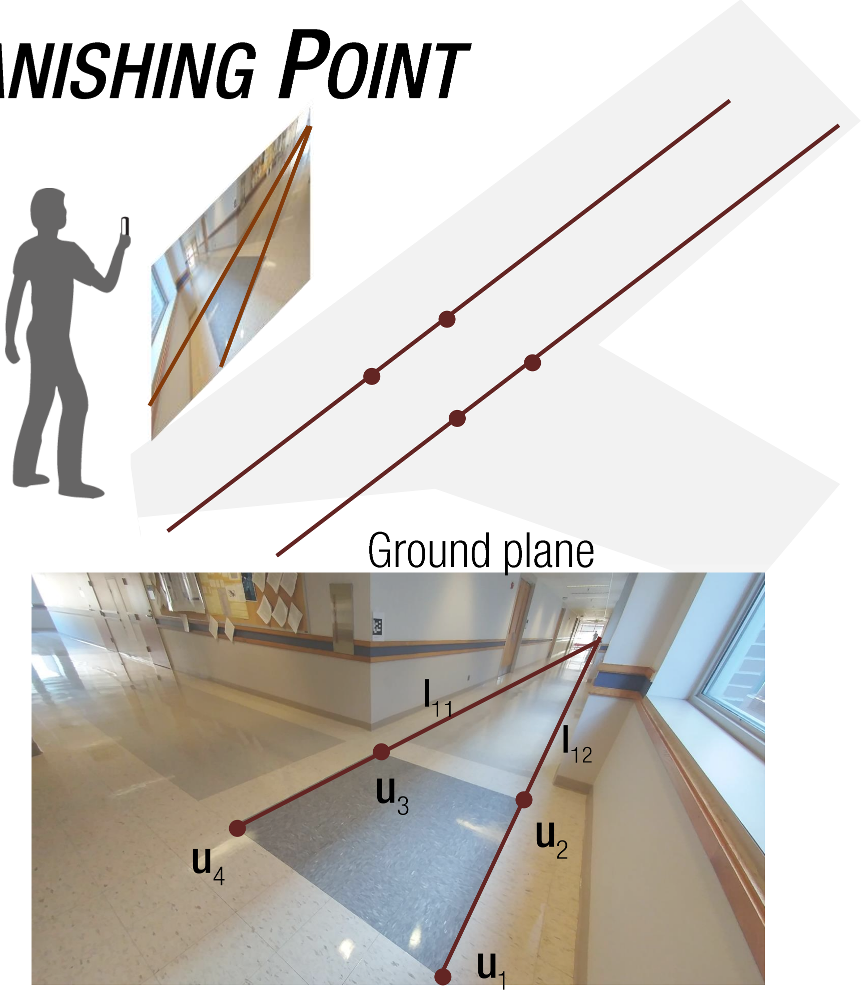
or $\mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2$

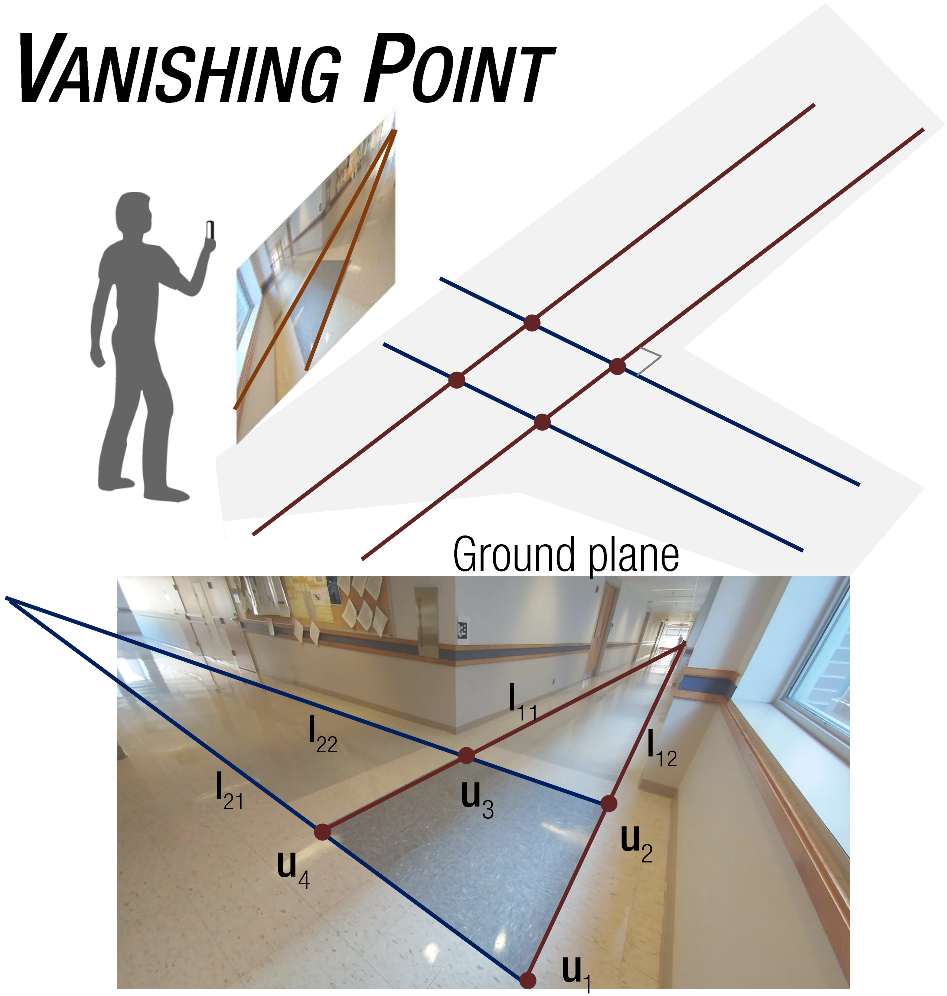
VANISHING POINT



Parallel lines:

$$\mathbf{l}_{11} = \mathbf{u}_4 \times \mathbf{u}_3 \qquad \mathbf{l}_{12} = \mathbf{u}_1 \times \mathbf{u}_2$$

Vanishing Point

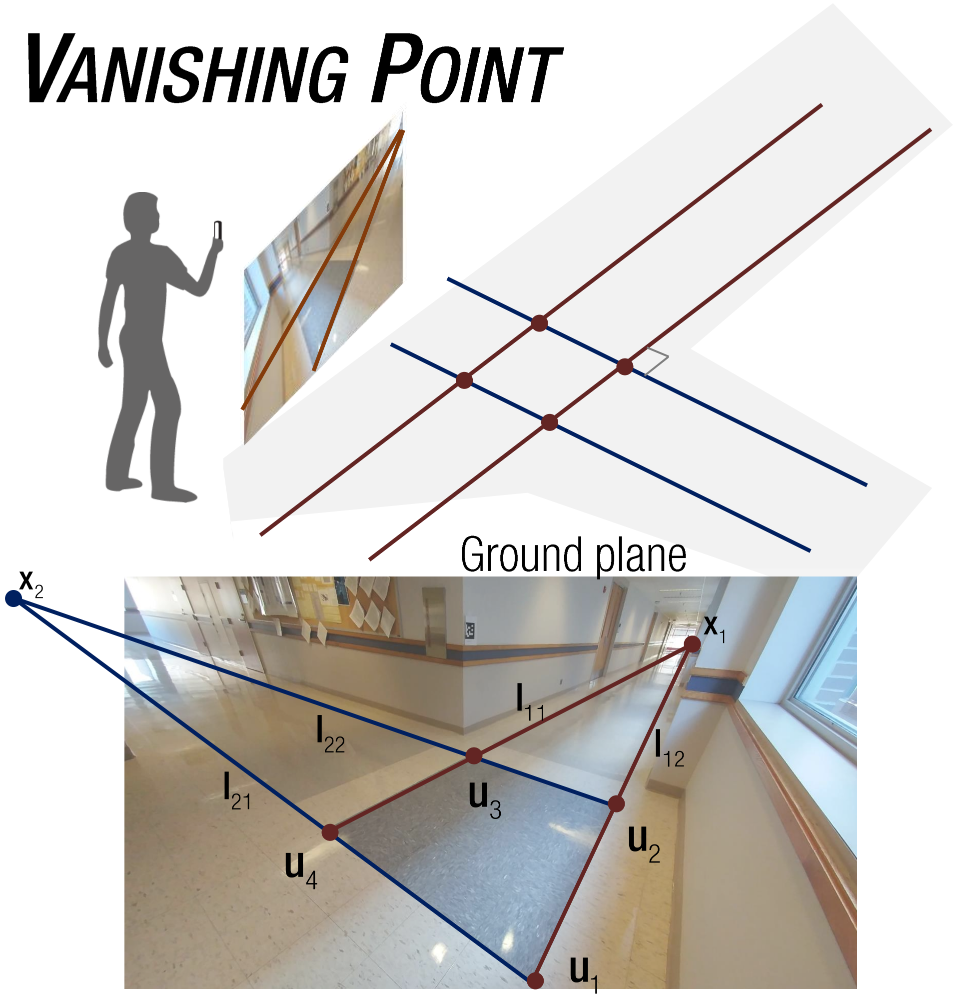


Parallel lines:

$$\mathbf{l}_{11} = \mathbf{u}_4 \times \mathbf{u}_3 \qquad \mathbf{l}_{12} = \mathbf{u}_1 \times \mathbf{u}_2$$

$$\mathbf{l}_{21} = \mathbf{u}_4 \times \mathbf{u}_1 \qquad \mathbf{l}_{22} = \mathbf{u}_3 \times \mathbf{u}_4$$

Vanishing Point



Parallel lines:

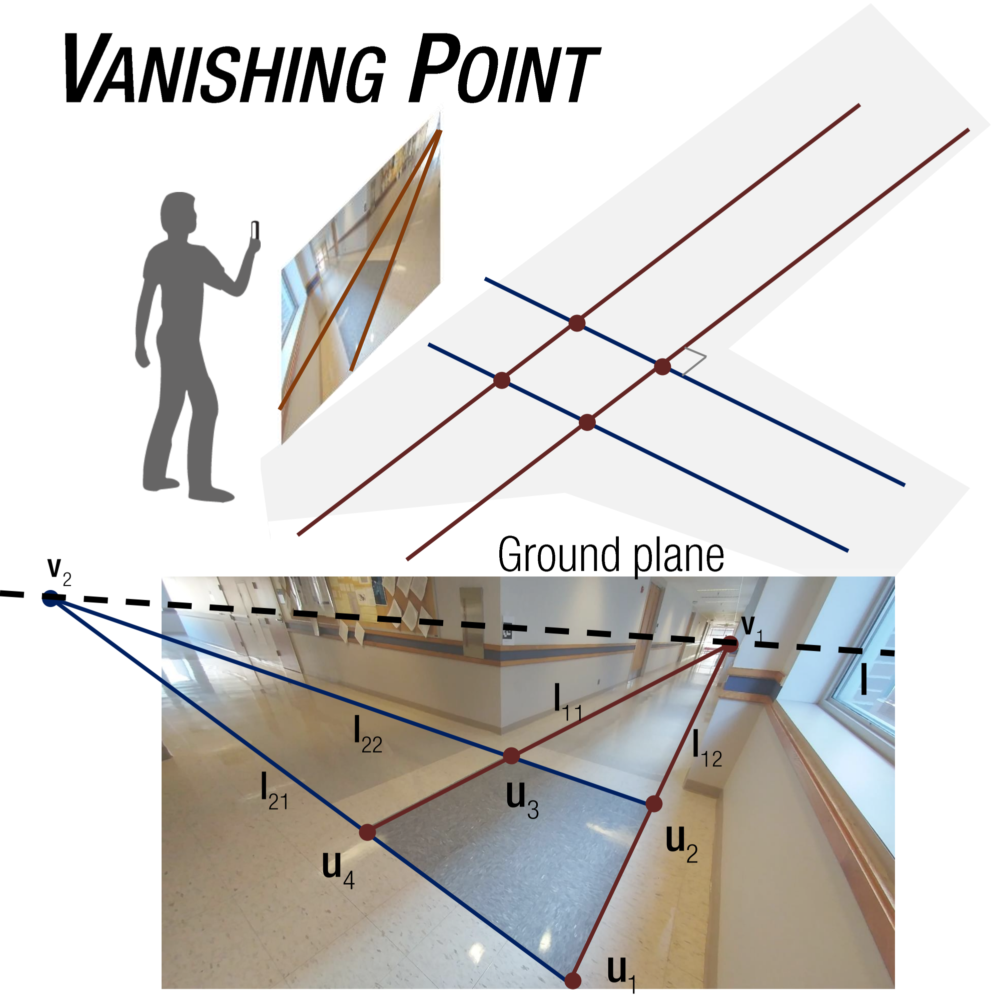
$$\mathbf{l}_{11} = \mathbf{u}_4 \times \mathbf{u}_3 \qquad \mathbf{l}_{12} = \mathbf{u}_1 \times \mathbf{u}_2$$

$$\mathbf{l}_{21} = \mathbf{u}_4 \times \mathbf{u}_1 \qquad \mathbf{l}_{22} = \mathbf{u}_3 \times \mathbf{u}_4$$

Vanishing points:

$$\mathbf{x}_1 = \mathbf{l}_{11} \times \mathbf{l}_{12} \qquad \mathbf{x}_2 = \mathbf{l}_{21} \times \mathbf{l}_{22}$$

Vanishing Point



Parallel lines:

$\mathbf{l}_{11} = \mathbf{u}_4 \times \mathbf{u}_3$ $\quad$ $\mathbf{l}_{12} = \mathbf{u}_1 \times \mathbf{u}_2$

$\mathbf{l}_{21} = \mathbf{u}_4 \times \mathbf{u}_1$ $\quad$ $\mathbf{l}_{22} = \mathbf{u}_3 \times \mathbf{u}_4$

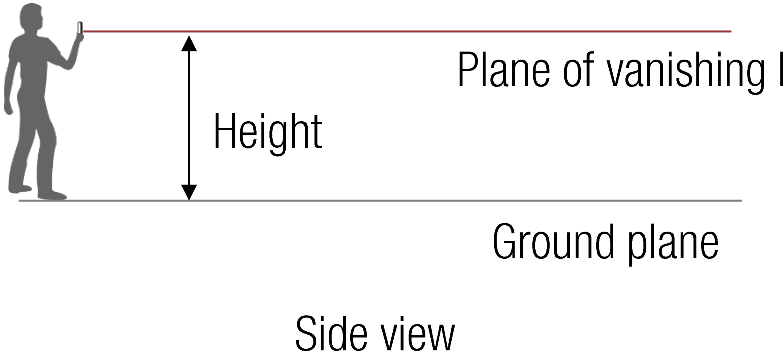
Vanishing points:

$\mathbf{v}_1 = \mathbf{l}_{11} \times \mathbf{l}_{12}$ $\quad$ $\mathbf{v}_2 = \mathbf{l}_{21} \times \mathbf{l}_{22}$

Vanishing line:

$\mathbf{l} = \mathbf{v}_1 \times \mathbf{v}_2$

Geometric Interpretation of Vanishing Line



WHERE WAS I?

WHERE WAS I?

Taken from my hotel room (6^{th} floor)

Taken from beach