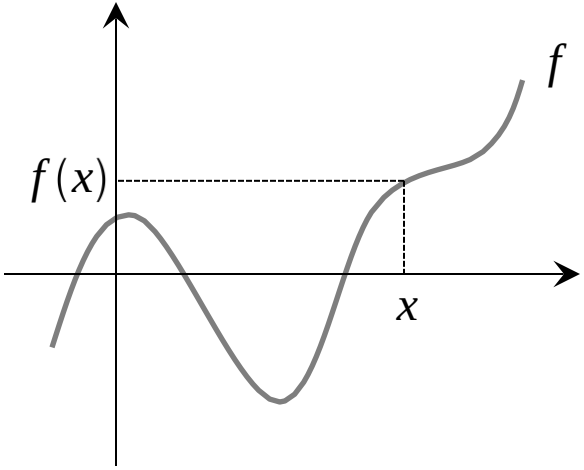
Image Filtering

Hyun Soo Park


OLO PAPAYA 6 FOR $2

$I(x_1)$

Pixel value at x



$I(x_1)$

Pixel value at x

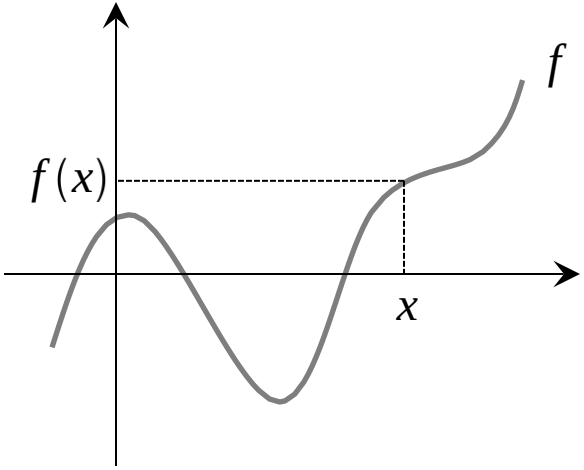
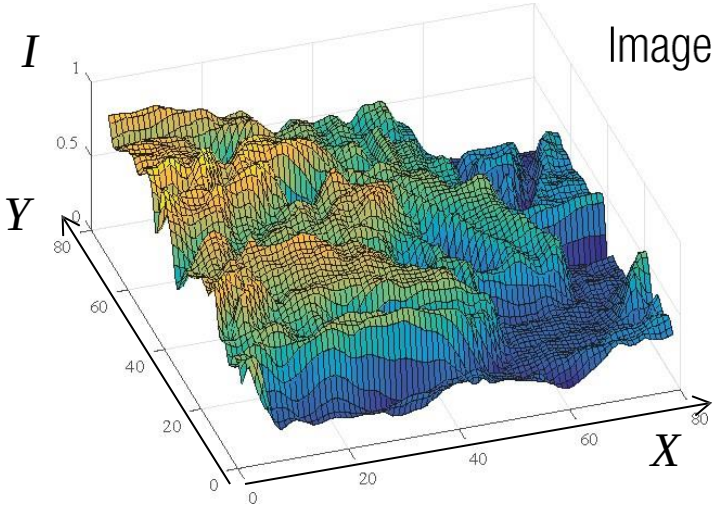
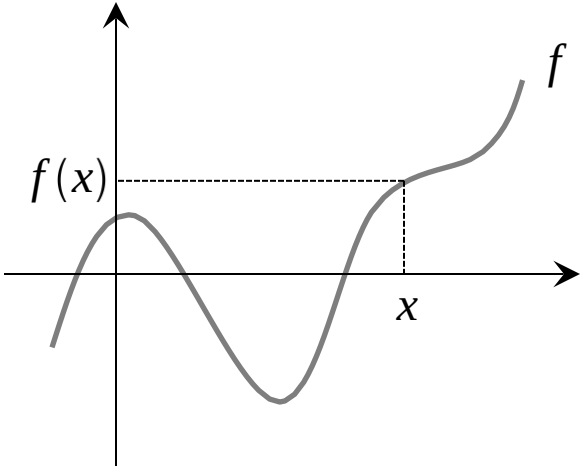


Image as a 2D function



$I(x_1) = (212, 200, 221)$

Pixel value at x



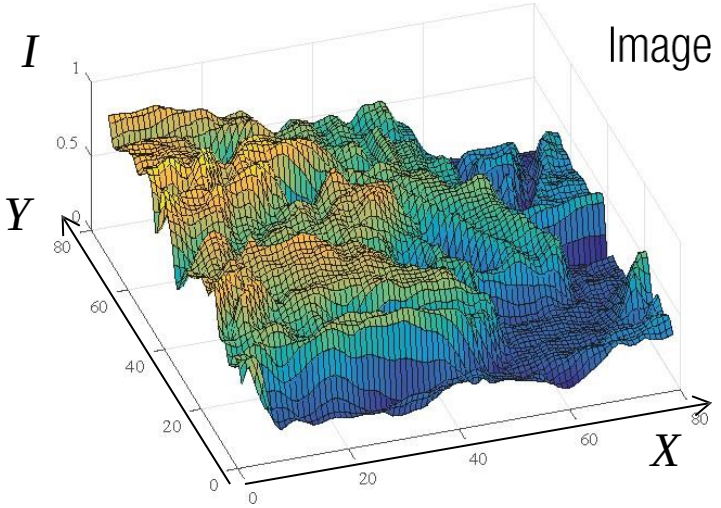
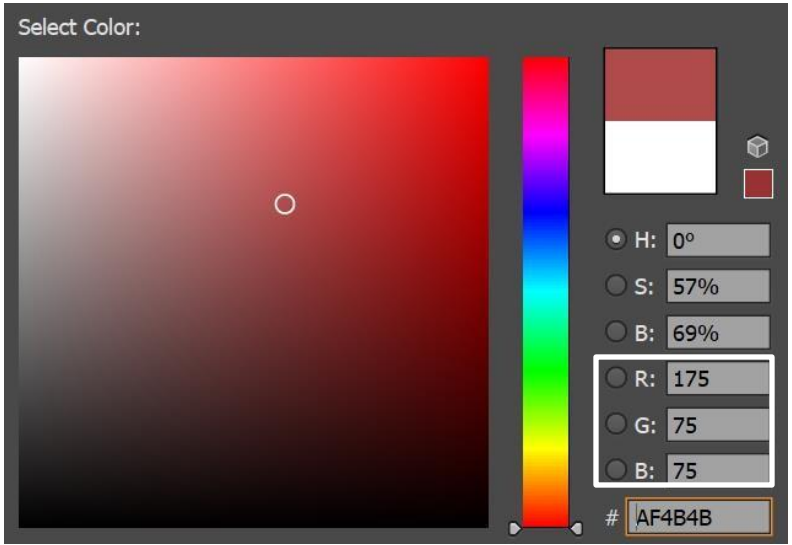


Image as a 2D function

$I(x_1) = (212, 200, 221)$

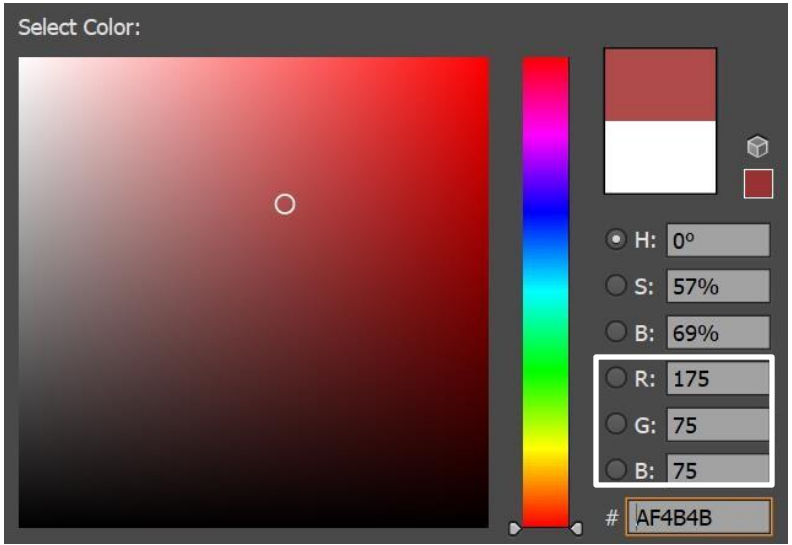
Pixel value at x



$I(x_1) = (212, 200, 221)$

Pixel value at x

$I(x_2) = (212, 20, 51)$



$I(x_1) = (212, 200, 221)$

Pixel value at x

$I(x_2) = (212, 20, 51)$

$I(x) \in 2^8 \times 2^8 \times 2^8$

8-bit image

$I(x_1) = (212, 200, 221)$

Pixel value at x

$I(x_2) = (212, 20, 51)$

$I(x) \in 2^8 \times 2^8 \times 2^8$

8-bit image

IMAGE FILTERING

$$f(I) = I_f$$

IMAGE FILTERING

$$f(I) = I_f$$

$$I(x_1) = (212,200,221)$$

$$f(I(x_1)) = (212,200,221)$$
$$= (242,152,135)$$

IMAGE FILTERING (PIXEL-WISE FILTERING)

$$I_f = I + 100$$

$$I_f = I - 100$$

$$I_f = 255 - I$$

IMAGE FILTERING (PIXEL-WISE FILTERING)

$$I_f = I + 100$$

$$I_f = I - 100$$

$$I_f = 255 - I$$

$$I_r > 100 \;\&\; I_g < 100 \;\&\; I_b < 100$$

IMAGE FILTERING (PIXEL-WISE FILTERING)

$$I_r > 100 \;\&\; I_g < 100 \;\&\; I_b < 100$$

Blue/black

White/gold

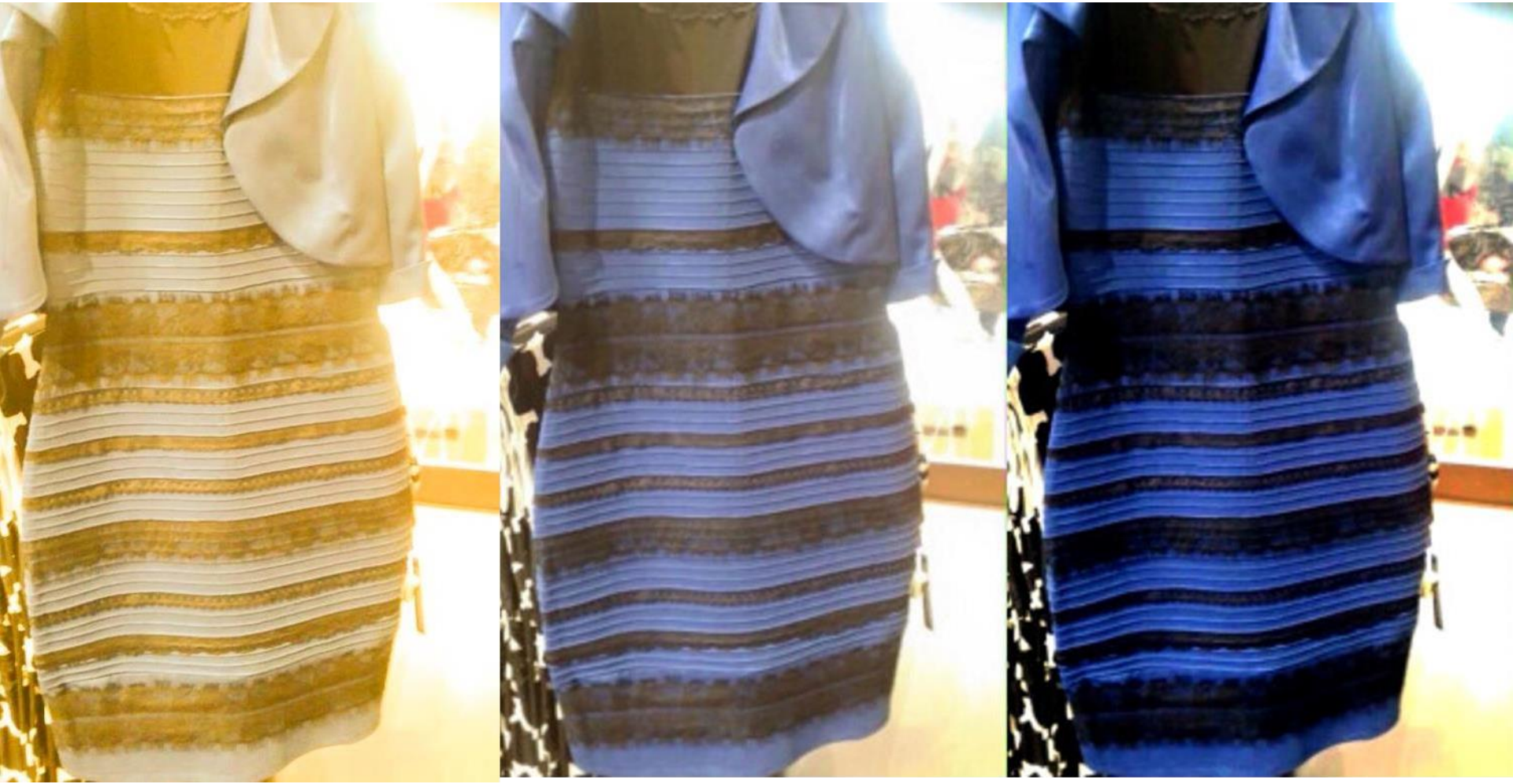
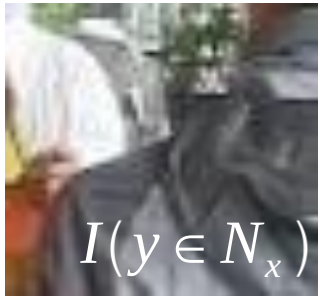


IMAGE FILTERING (PIXEL-WISE FILTERING)

$$I_r > 100 \;\&\; I_g < 100 \;\&\; I_b < 100$$

IMAGE SPATIAL FILTERING

$$I_f(x) = f(I(y \in N_x))$$



Local patch around x

N_x : Neighboring pixels of x

IMAGE SPATIAL FILTERING

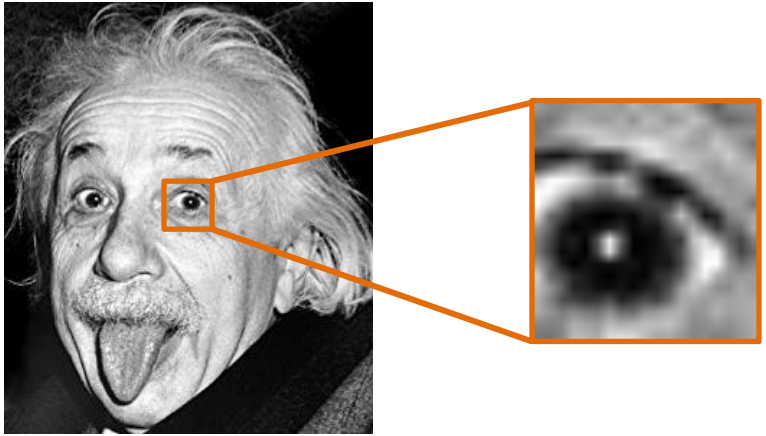
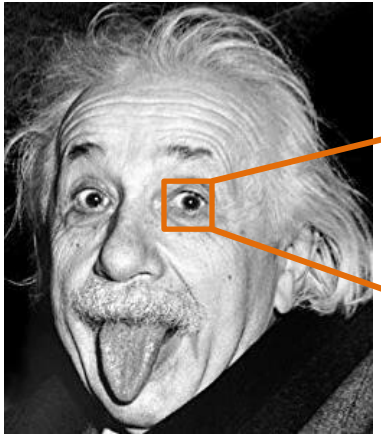


IMAGE SPATIAL FILTERING



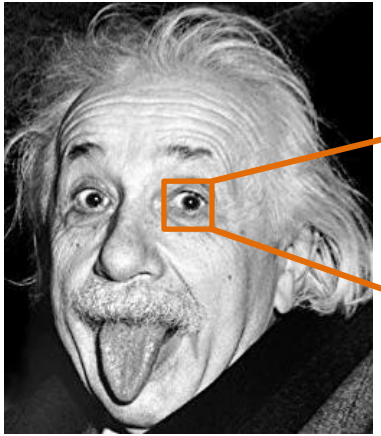
2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

I

Image

IMAGE SPATIAL FILTERING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$



2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

I

Image

$\otimes$

a	b	c
d	e	f
g	h	i

z

Filter

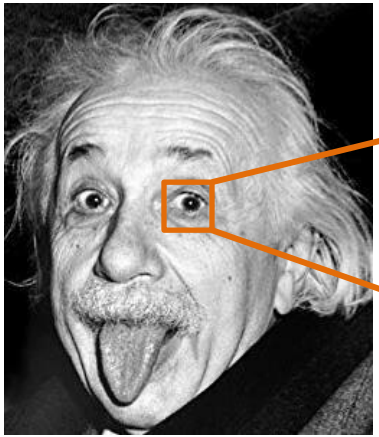
$=$

I_f

Filtered image

IMAGE SPATIAL FILTERING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$



2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

Image

$\otimes$

a	*b*	*c*
d	*e*	*f*
g	*h*	*i*

Filter

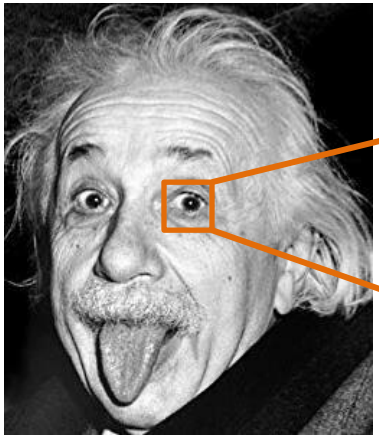
$=$

	y			

Filtered image

IMAGE SPATIAL FILTERING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$



2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$

a	b	c
d	e	f
g	h	i

$=$

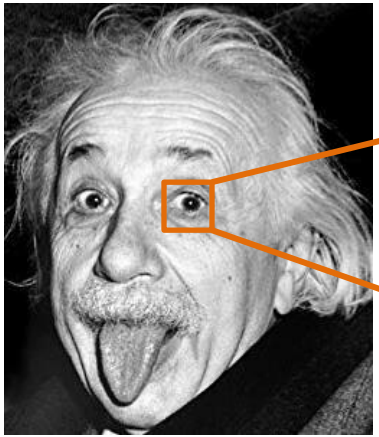
	y			

Image Filter Filtered image

$$y = 2a + b + 4c + d + 2e + 2f + 3g + 3h + 5i$$

IMAGE SPATIAL FILTERING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$



2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$

a	*b*	*c*
d	*e*	*f*
g	*h*	*i*

$=$

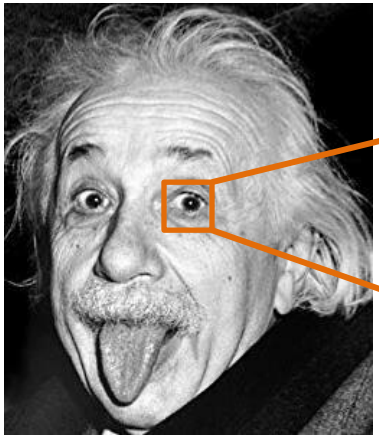
		y		

Image Filter Filtered image

$$y = a + 4b + 4c + 2d + 2e + 3f + 3g + 5h + 8i$$

IMAGE SPATIAL FILTERING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$



2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$

a	*b*	*c*
d	*e*	*f*
g	*h*	*i*

$=$

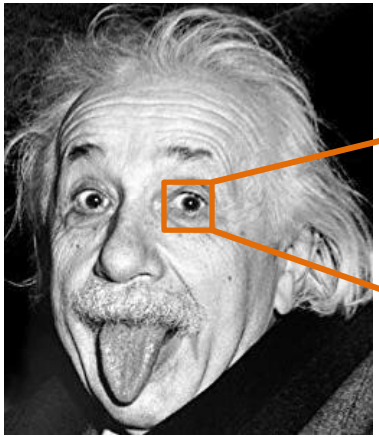
			y	

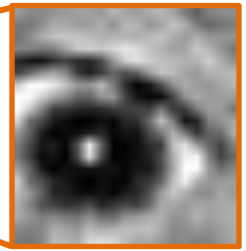
Image Filter Filtered image

$$y = 5a + 8b + 9c + 2d + 6e + 7f + 2g + h + 3i$$

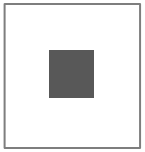
IDENTITY

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$





$\otimes$



$=$

2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

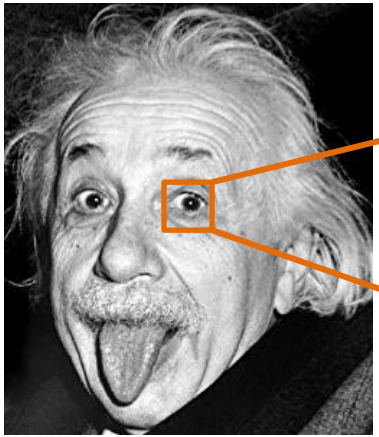
$\otimes$

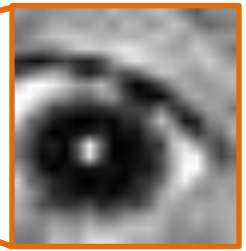
0	0	0
0	1	0
0	0	0

$=$

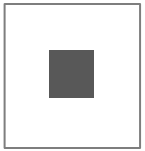
IDENTITY

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$





$\otimes$



$=$

2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

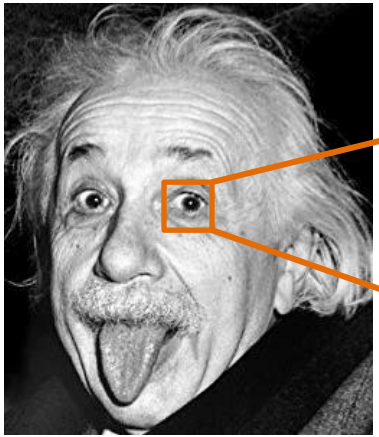
$\otimes$

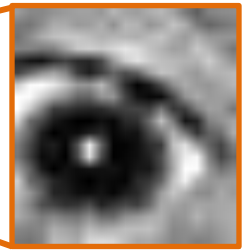
0	0	0
0	1	0
0	0	0

$=$

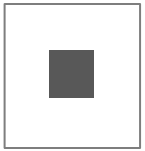
IDENTITY

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$





$\otimes$



$=$

2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$

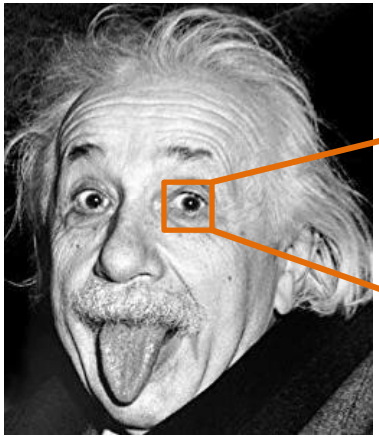
0	0	0
0	1	0
0	0	0

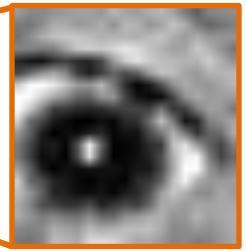
$=$

	2	2	3	
	3	5	8	
	2	2	6	

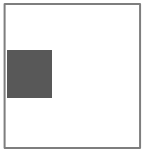
SHIFTING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$





$\otimes$



$=$

2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

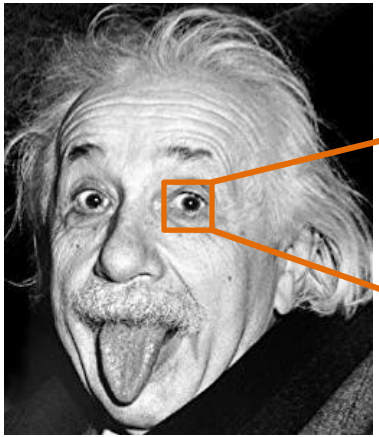
$\otimes$

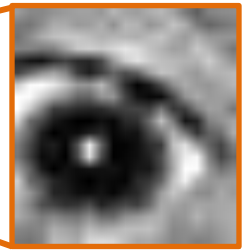
0	0	0
1	0	0
0	0	0

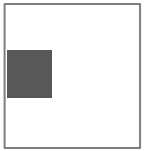
$=$

SHIFTING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$







2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

⊗

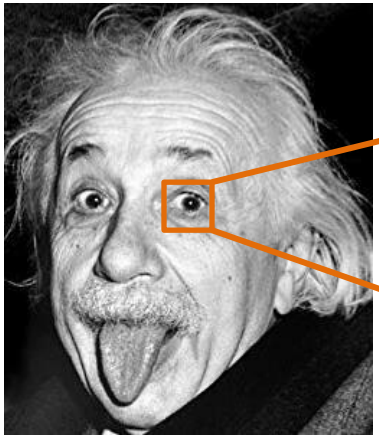
0	0	0
1	0	0
0	0	0

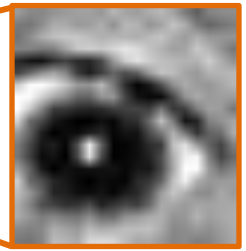
=

	1	2	2	
	3	3	5	
	5	2	2	

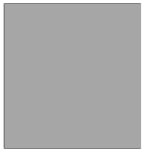
Box Filter (Moving Averaging)

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$





$\otimes$



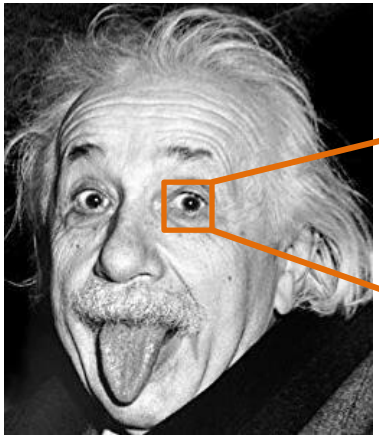
2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

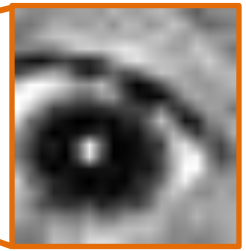
$\otimes$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

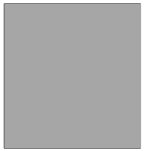
Box Filter (Moving Averaging)

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$

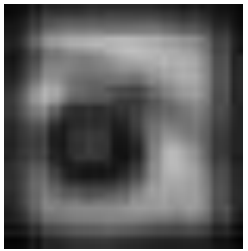




$\otimes$



$=$



2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

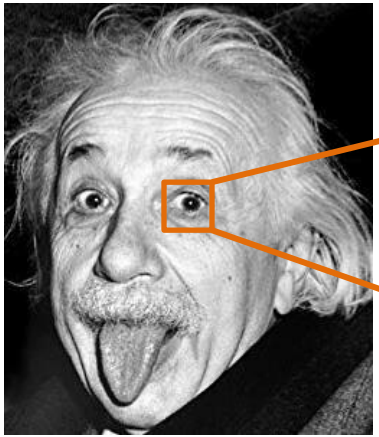
$\otimes$ $\frac{1}{9}$

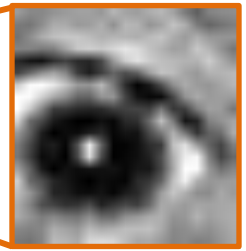
1	1	1
1	1	1
1	1	1

Average

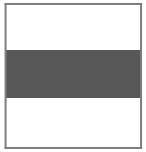
DIRECTIONAL BLUR

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$





$\otimes$



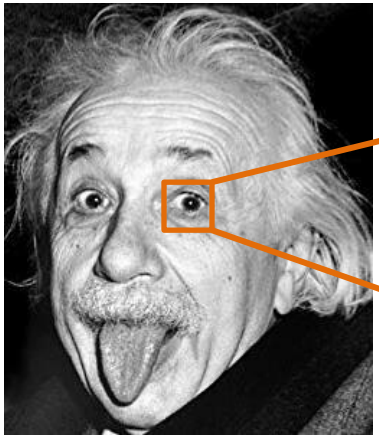
2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

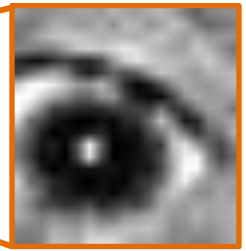
$\otimes$ $\frac{1}{3}$

0	0	0
1	1	1
0	0	0

DIRECTIONAL BLUR

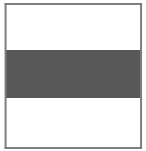
$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$



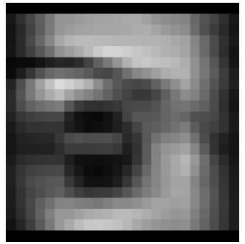


2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$



$=$



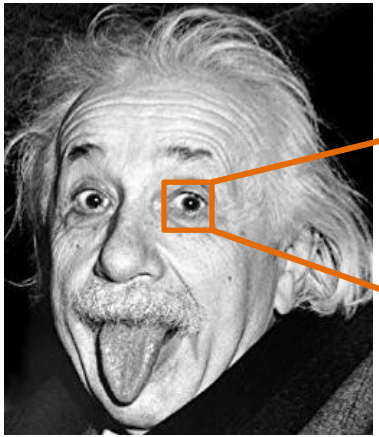
$\otimes \frac{1}{3}$

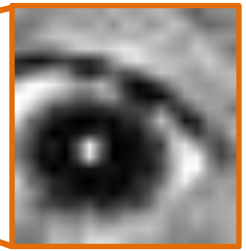
0	0	0
1	1	1
0	0	0

Horizontal blur

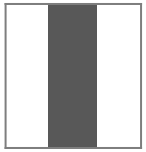
DIRECTIONAL BLUR

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$

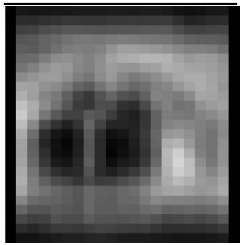




$\otimes$



$=$



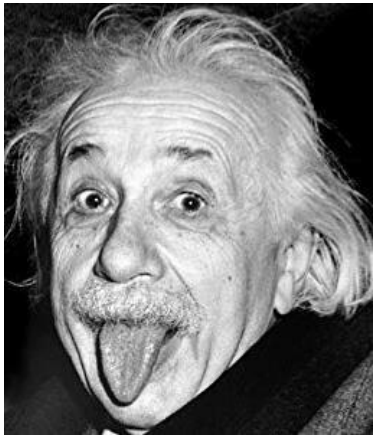
2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$ $\frac{1}{3}$

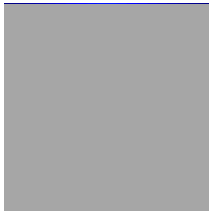
0	1	0
0	1	0
0	1	0

Vertical blur

Box Filter (Moving Averaging)



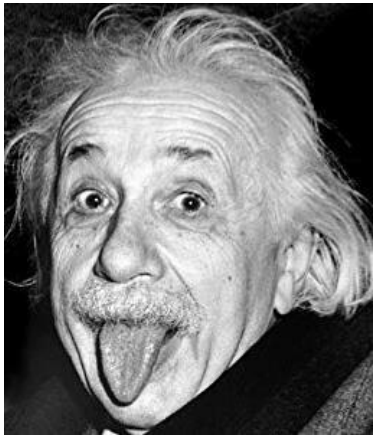
$\otimes$



$=$

Pixel averaging

GAUSSIAN BLURRING (MOVING WEIGHTED AVERAGE)



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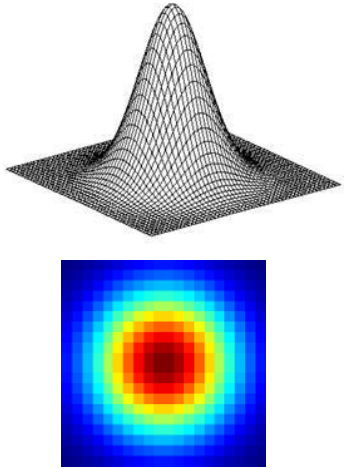
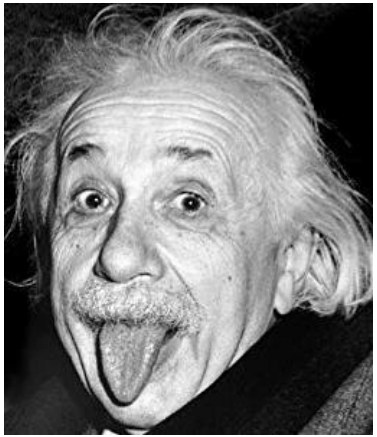


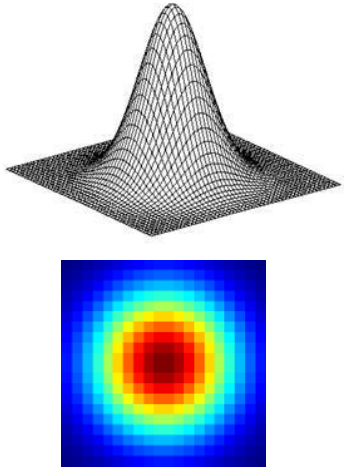
$=$

$$z(k,l) = \frac{1}{2\pi\sigma^2} e^{-\frac{k^2+l^2}{2\sigma^2}}$$

Gaussian Blurring (Moving Weighted Average)



$\otimes$

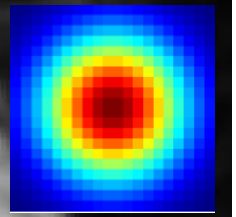


$=$

$$z(k,l) = \frac{1}{2\pi\sigma^2} e^{-\frac{k^2+l^2}{2\sigma^2}}$$

Gaussian blur

Gaussian Blurring~Denoising

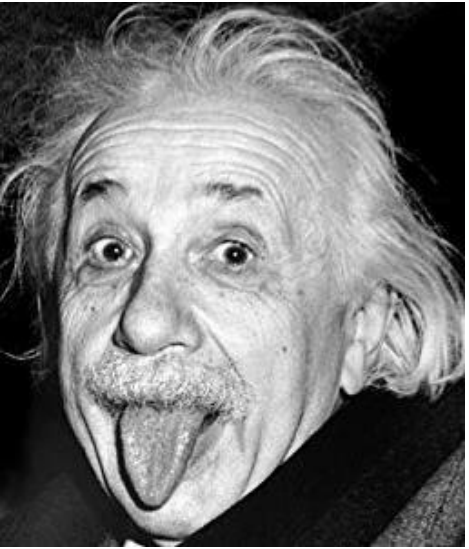


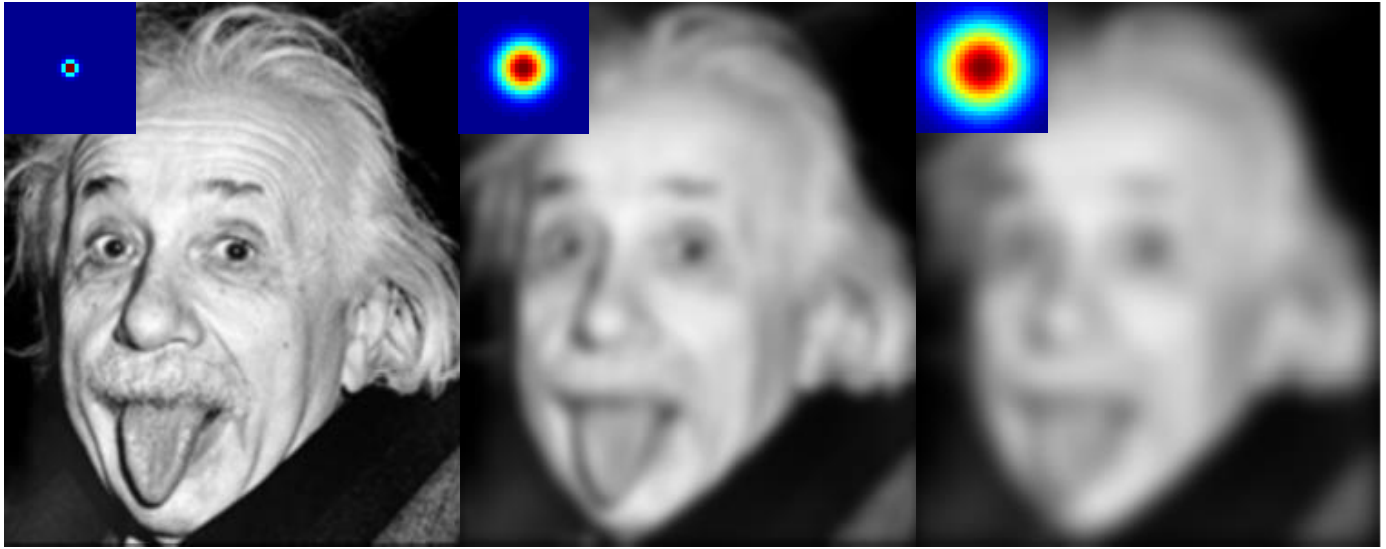
Noisy image

Denoised image

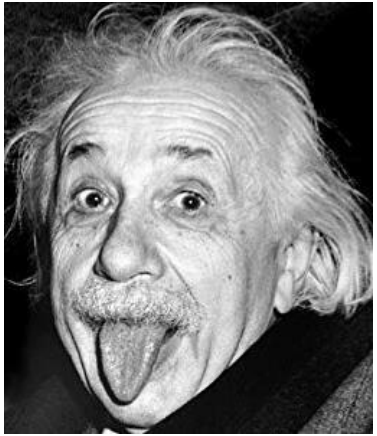
GAUSSIAN BLURRING

$$z(k,l) = \frac{1}{2\pi\sigma^2} e^{-\frac{k^2+l^2}{2\sigma^2}}$$

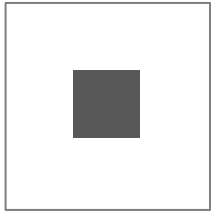




$\sigma = 1$ $\sigma = 4$ $\sigma = 7$



$\otimes$



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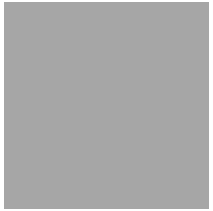
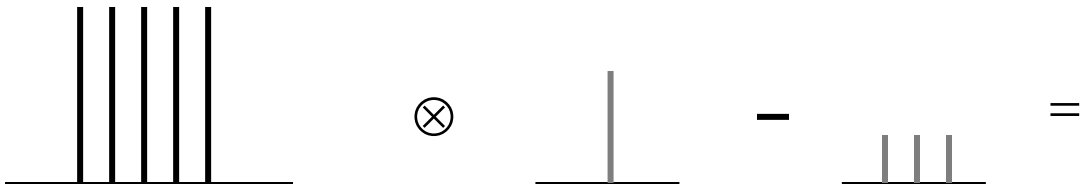
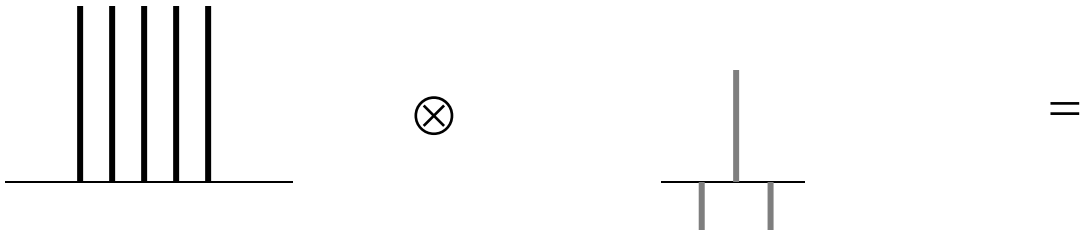
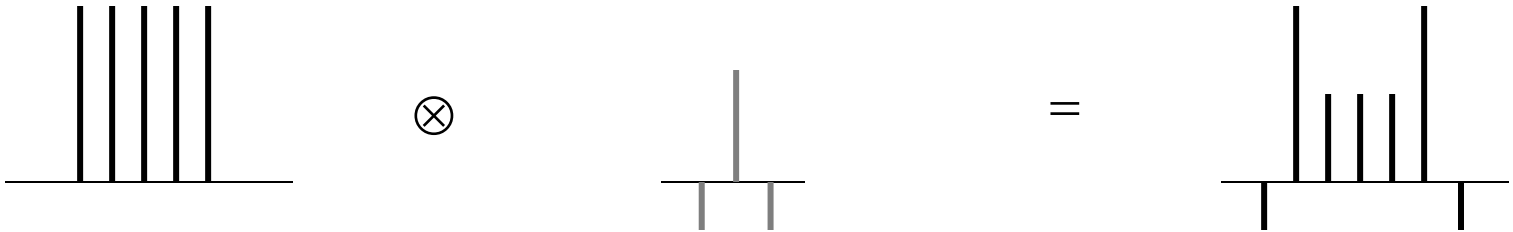


$=$







High response at edge

IMAGE SHARPENING

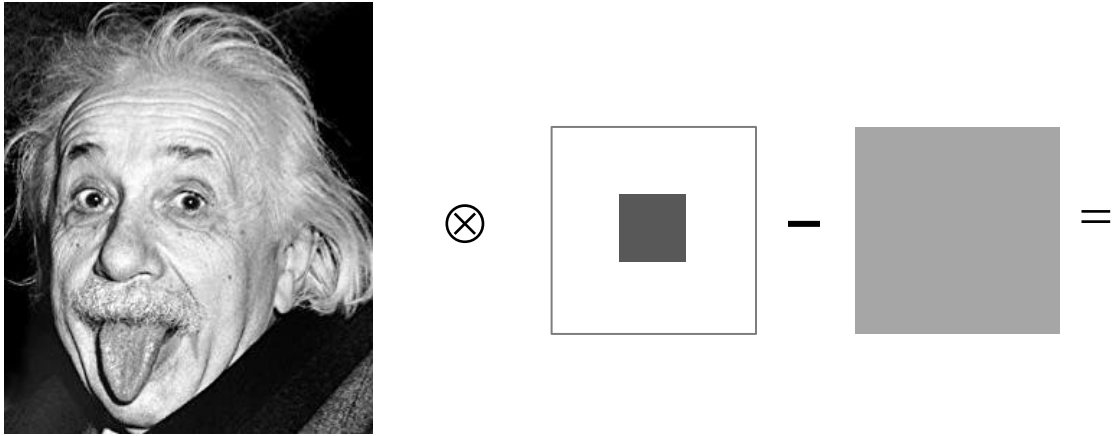
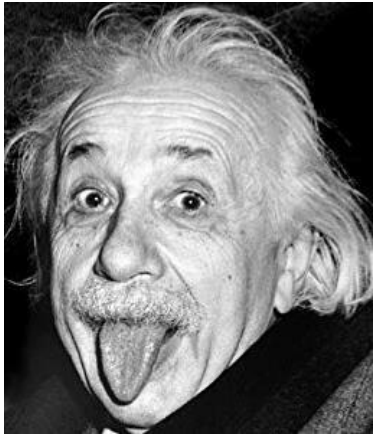
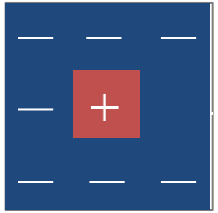


IMAGE SHARPENING



⊗



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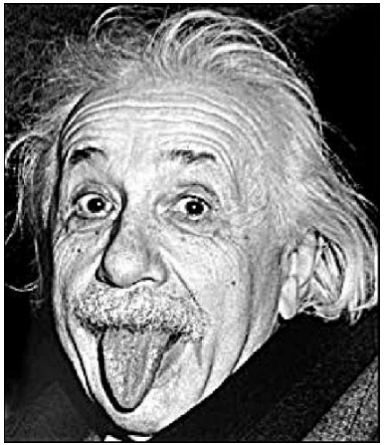
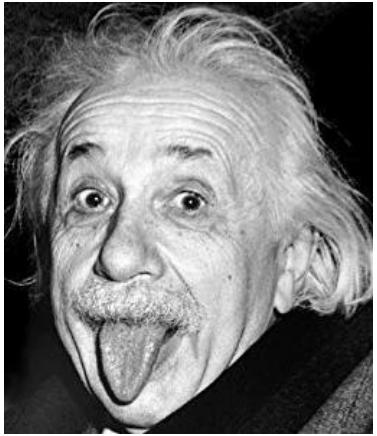
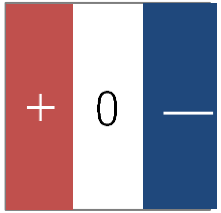


Sharpening



$\otimes$



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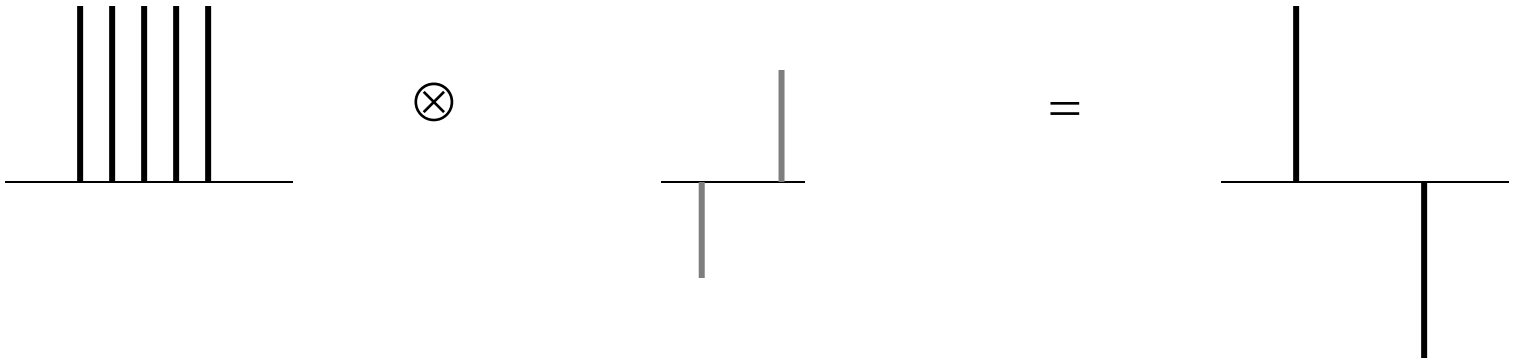
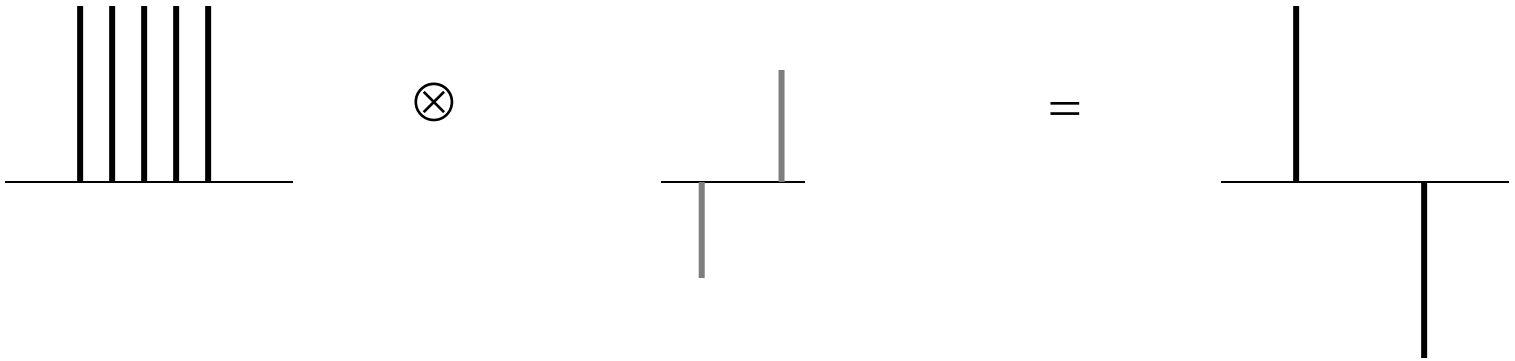
DIFFERENTIATION

DIFFERENTIATION

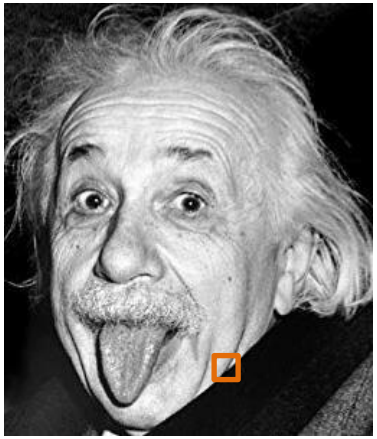


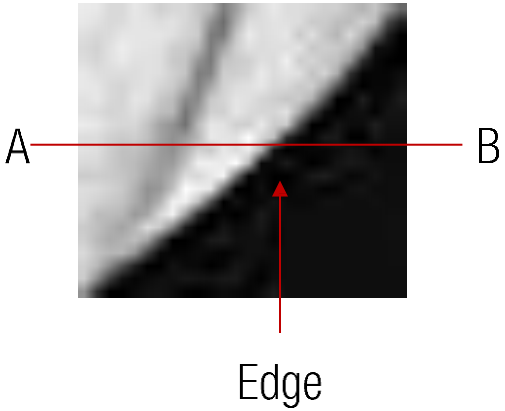
DIFFERENTIATION

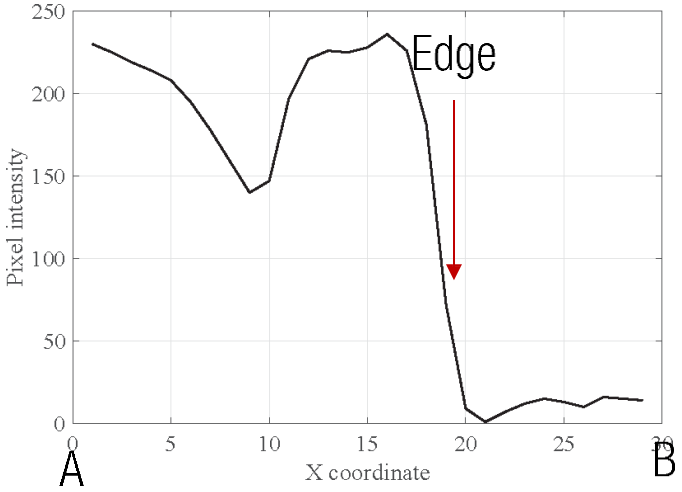


$$\frac{df}{du} = \lim_{h \to 0} \frac{f(u+h) - f(u-h)}{2h} \longrightarrow I \otimes z = \frac{\partial I}{\partial u}$$

DIFFERENTIATION







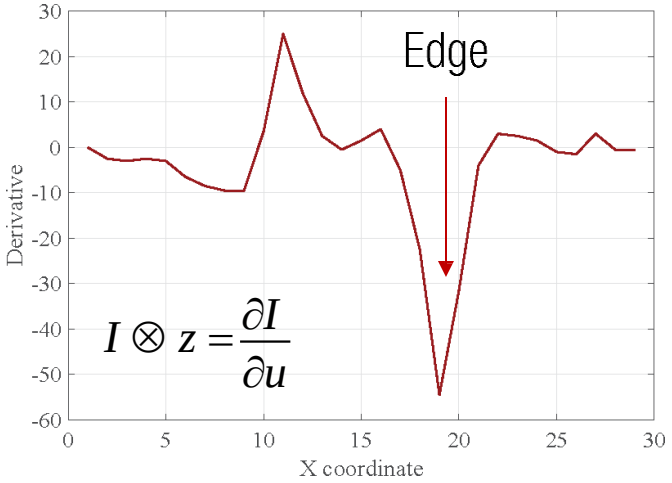
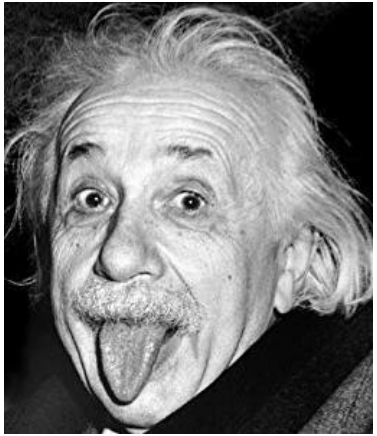
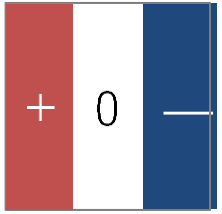


IMAGE DIFFERENTIATION

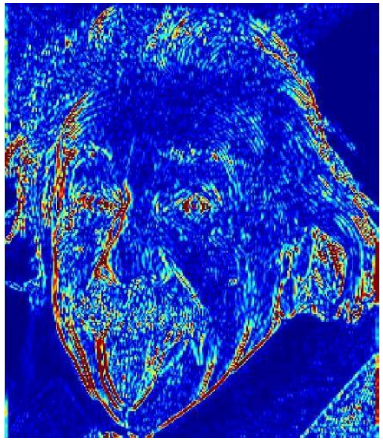


$\otimes$



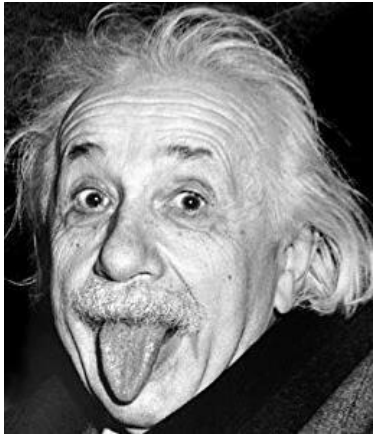
$$I \otimes z = \frac{\partial I}{\partial u}$$

=

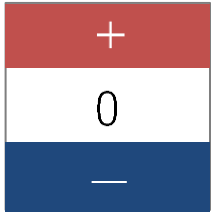


Differentiation

IMAGE DIFFERENTIATION

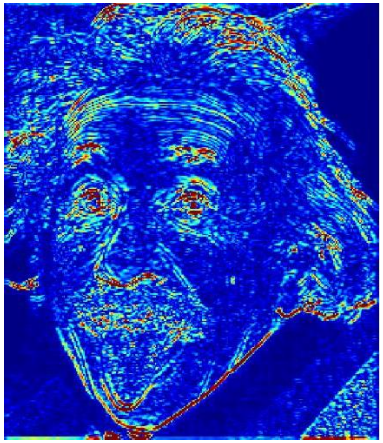


$\otimes$



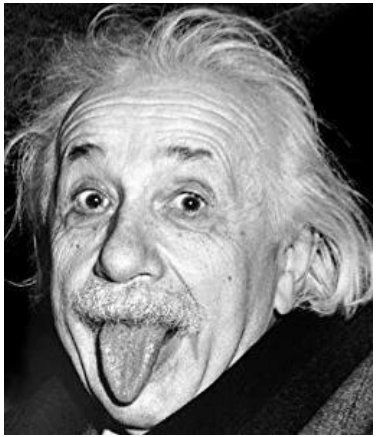
$$I \otimes z = \frac{\partial I}{\partial v}$$

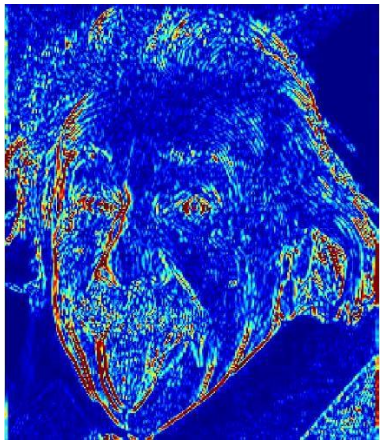
=

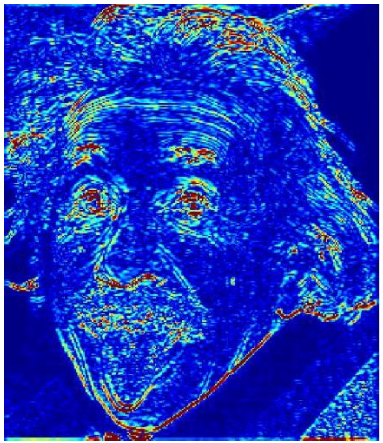


Differentiation

Edge Response







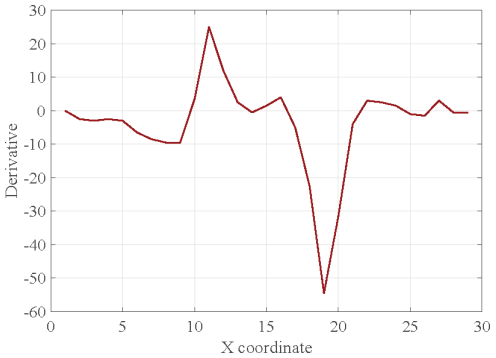


Image differentiation can be used for edge detection but it is VERY noisy.

Nonlinear Filter