

A photograph of a brick building and trees with a colorful geometric overlay of lines radiating from a central point. The lines are in various colors (red, green, blue, yellow, purple, orange) and extend across the entire image, creating a starburst effect. The background shows a brick building and trees under a cloudy sky.

EPIPOLAR GEOMETRY

HYUN SOO PARK



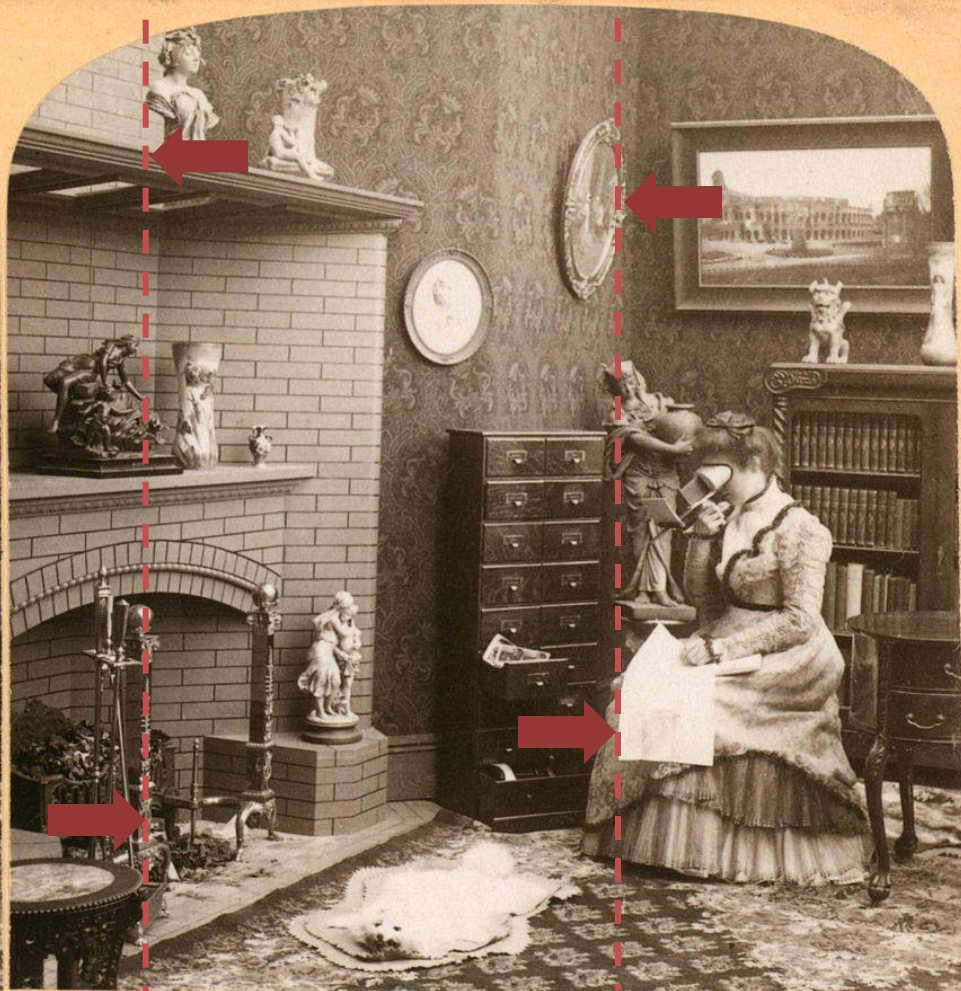
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Left image (Bob)



Right image (Alice)

2D CORRESPONDENCE

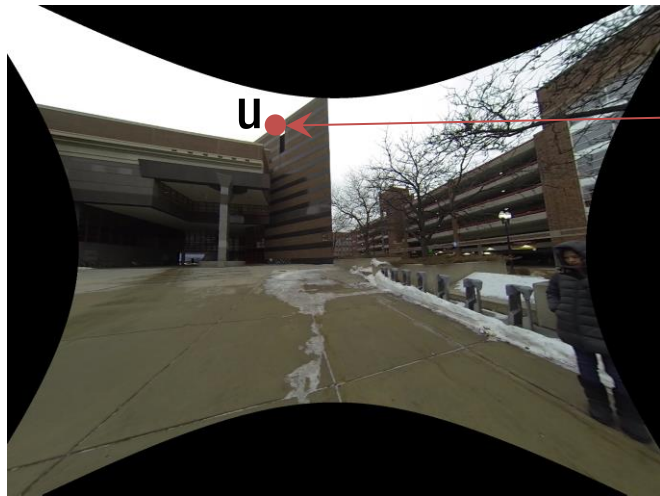


Left image (Bob)



Right image (Alice)

2D CORRESPONDENCE

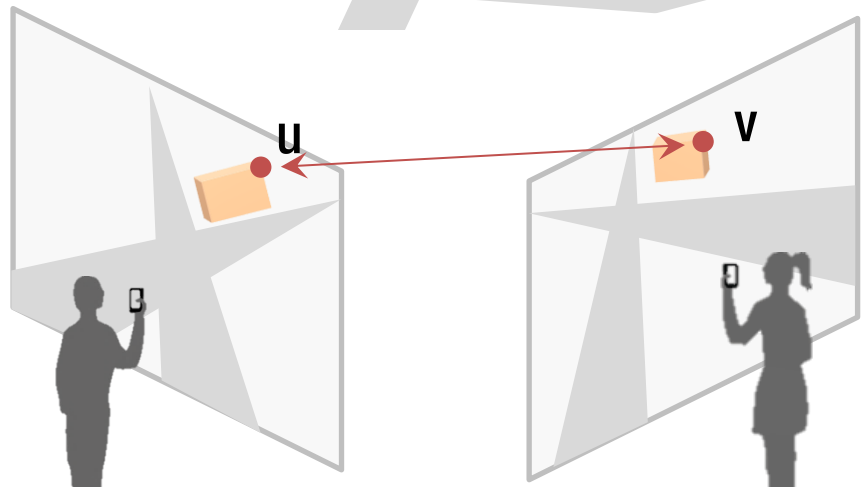
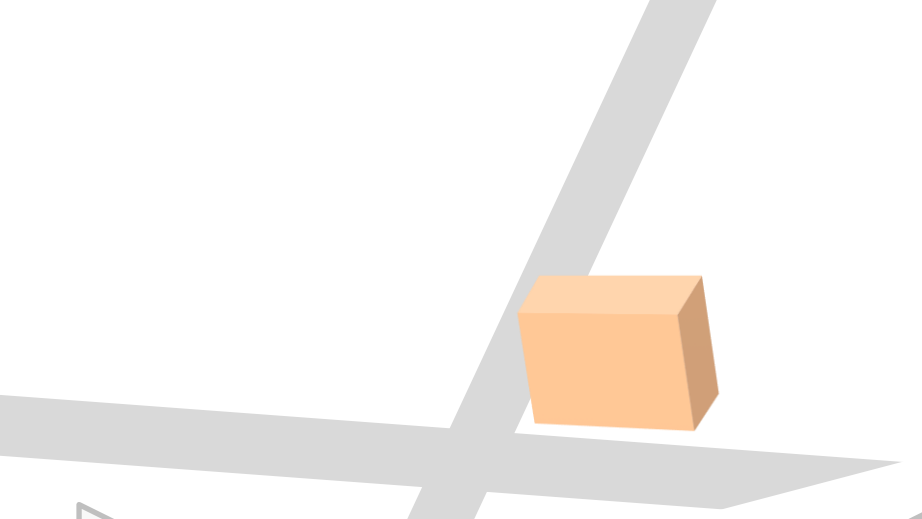


Left image (Bob)



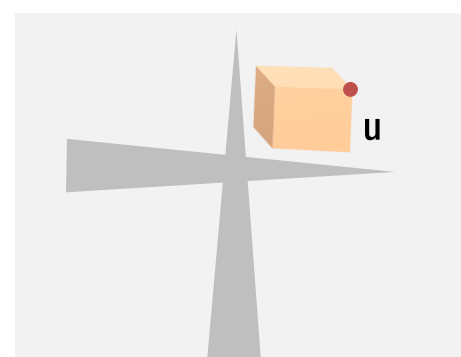
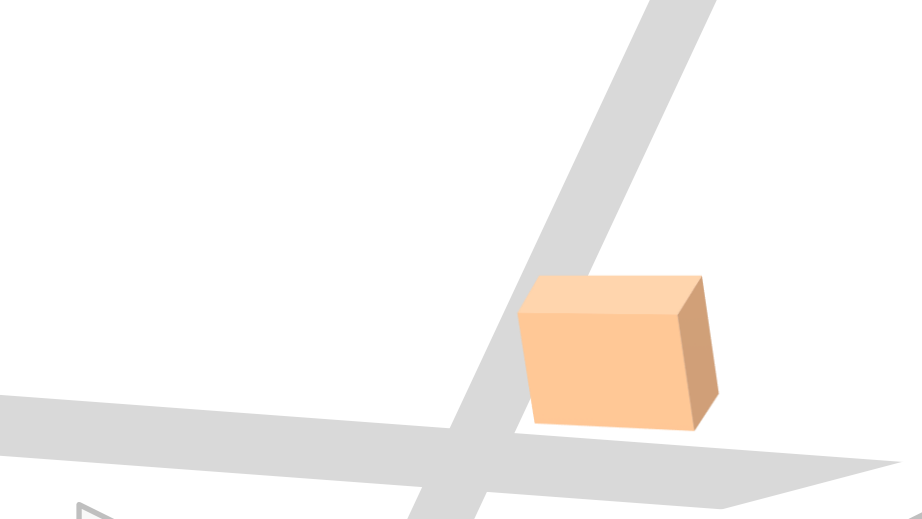
Right image (Alice)



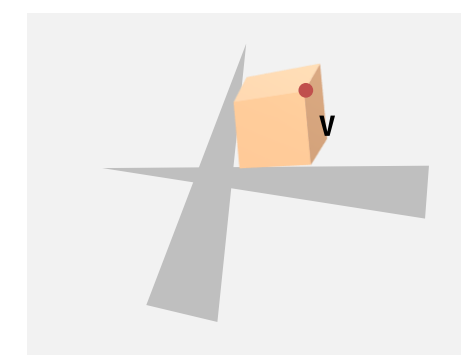


Bob

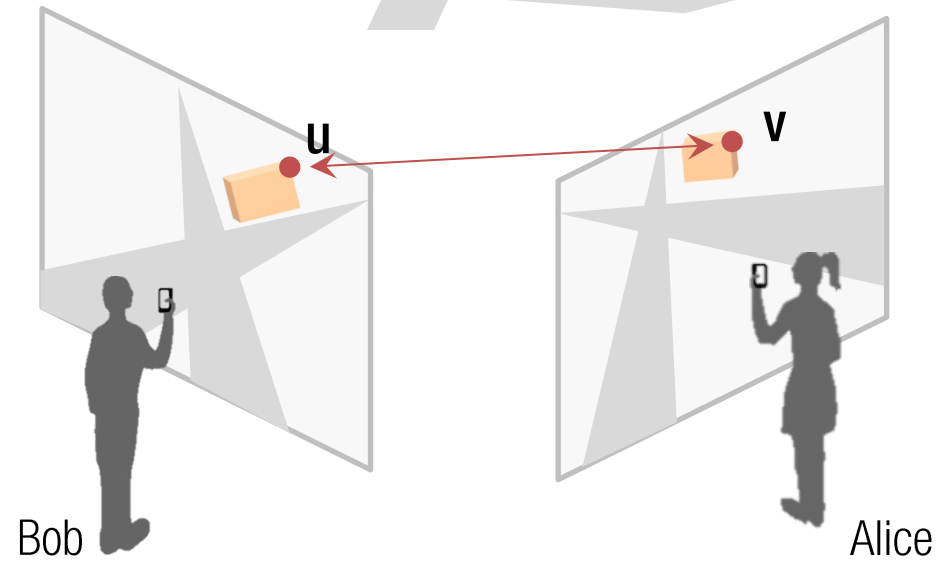
Alice



Bob's image

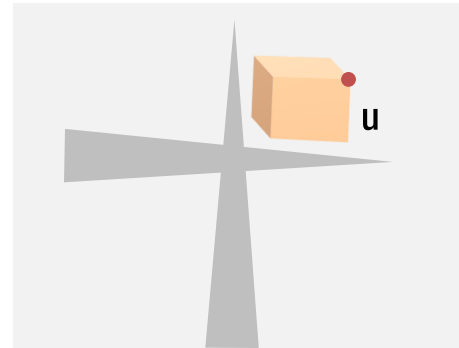
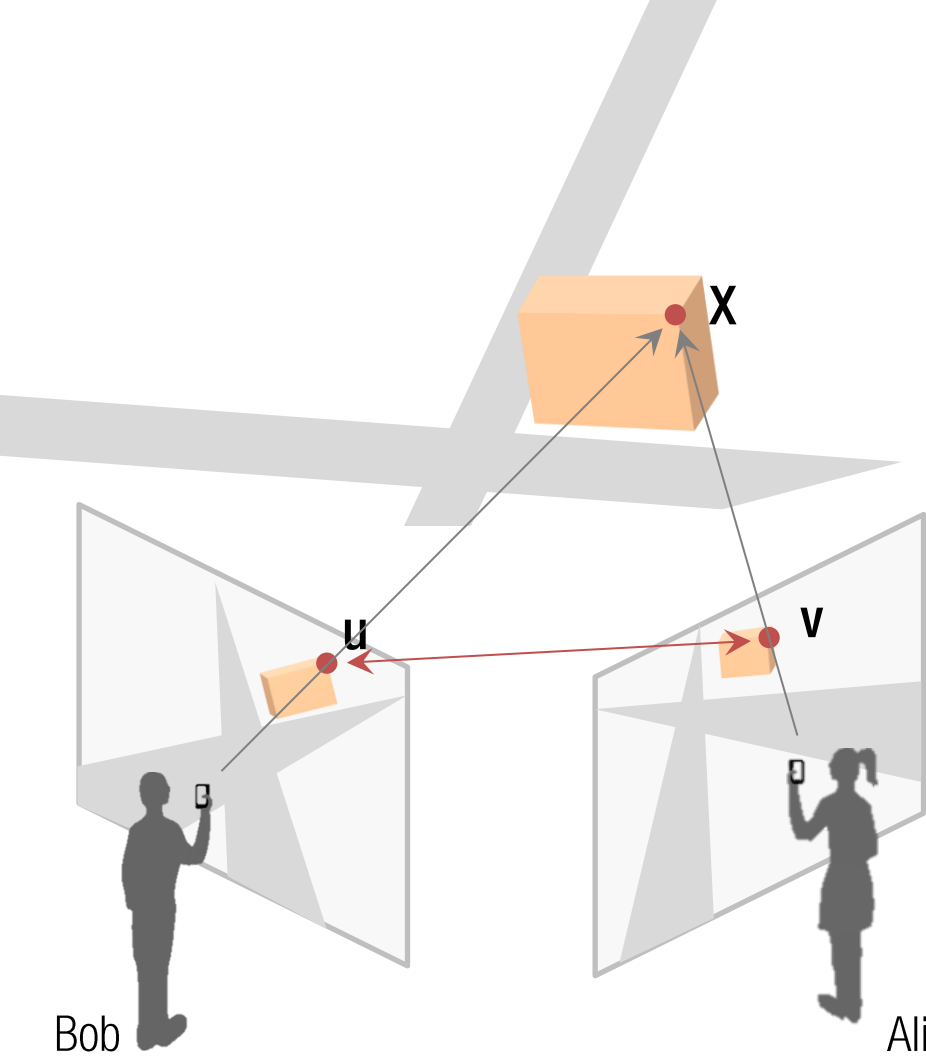


Alice's image

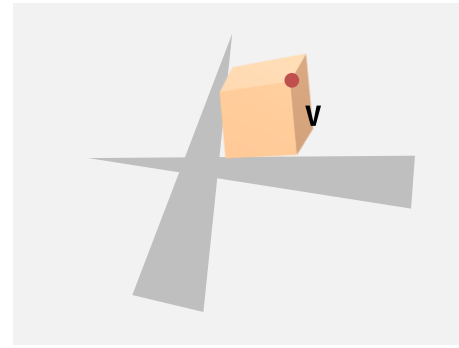


Bob

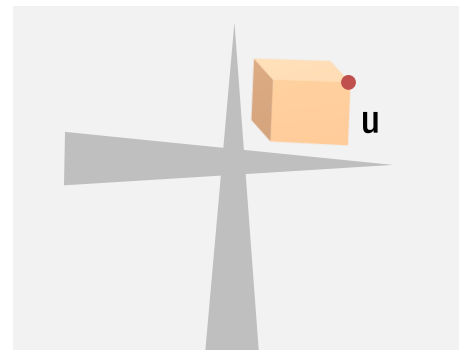
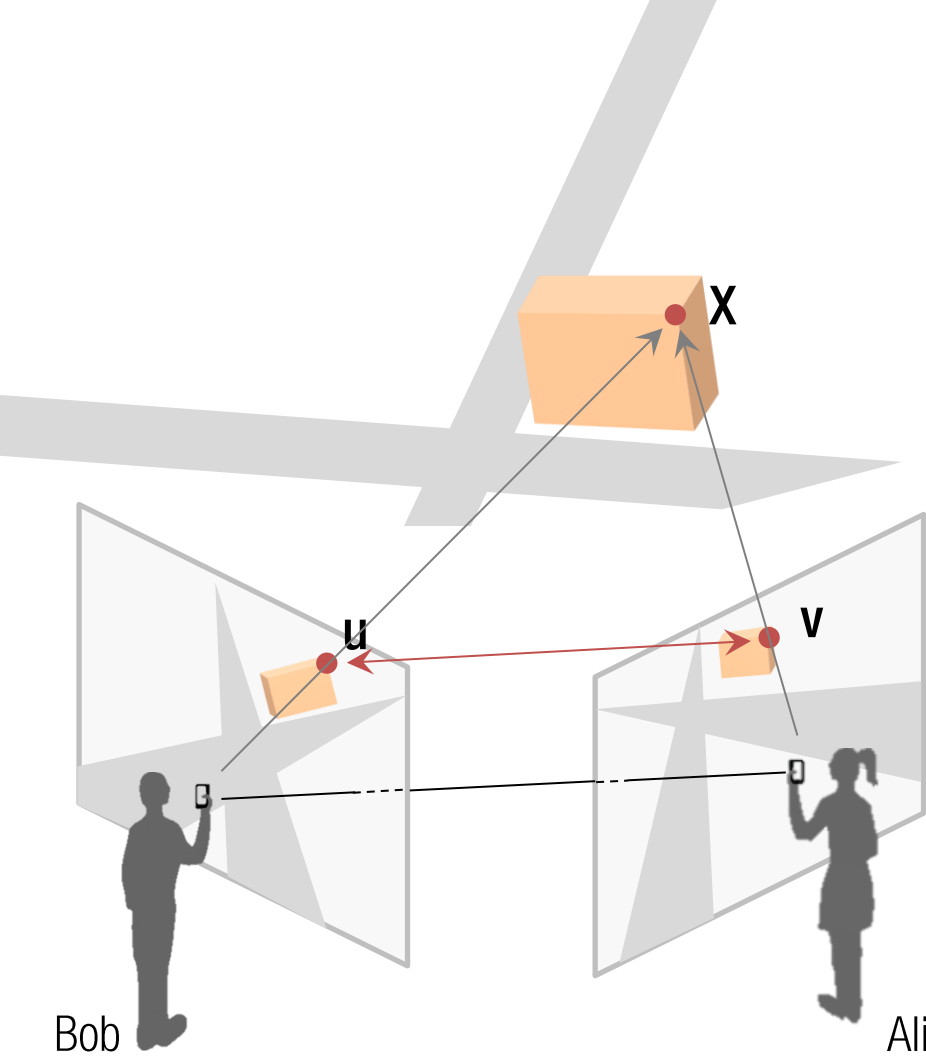
Alice



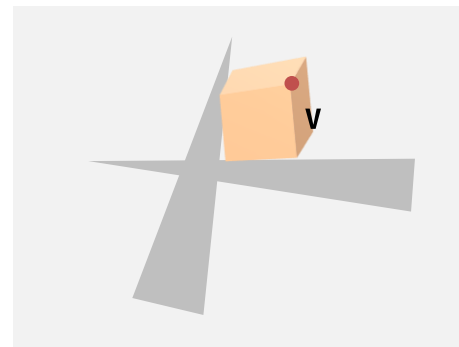
Bob's image



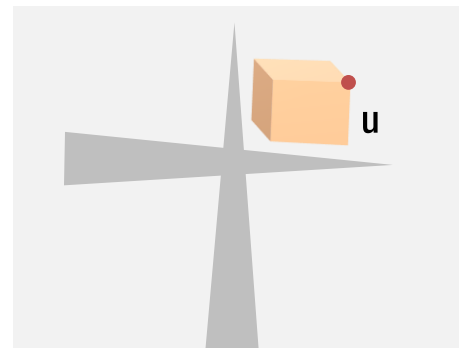
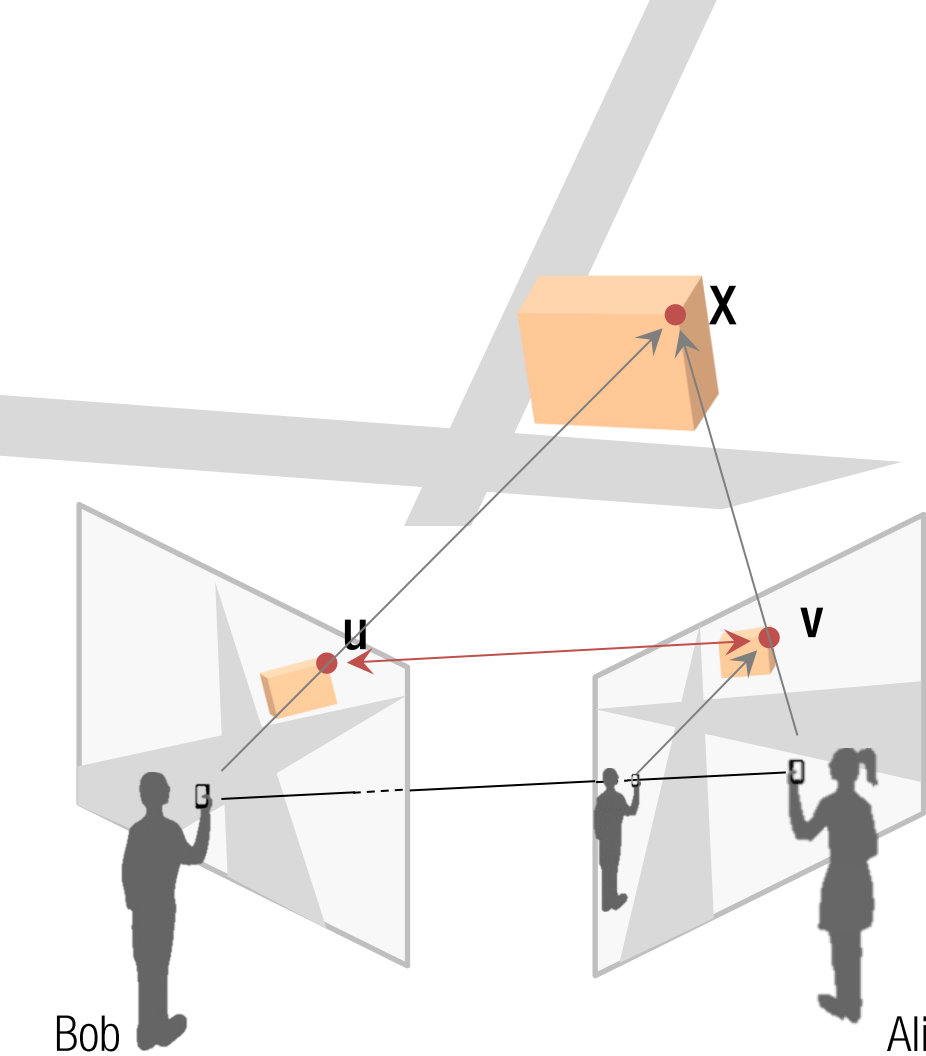
Alice's image



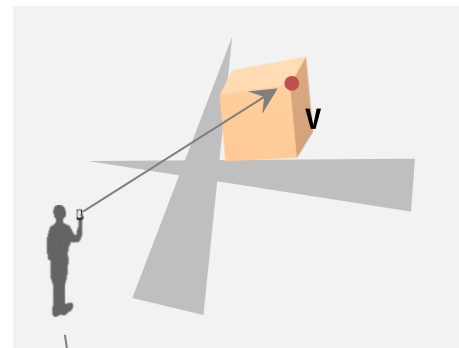
Bob's image



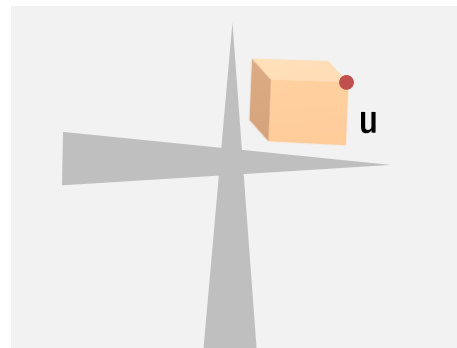
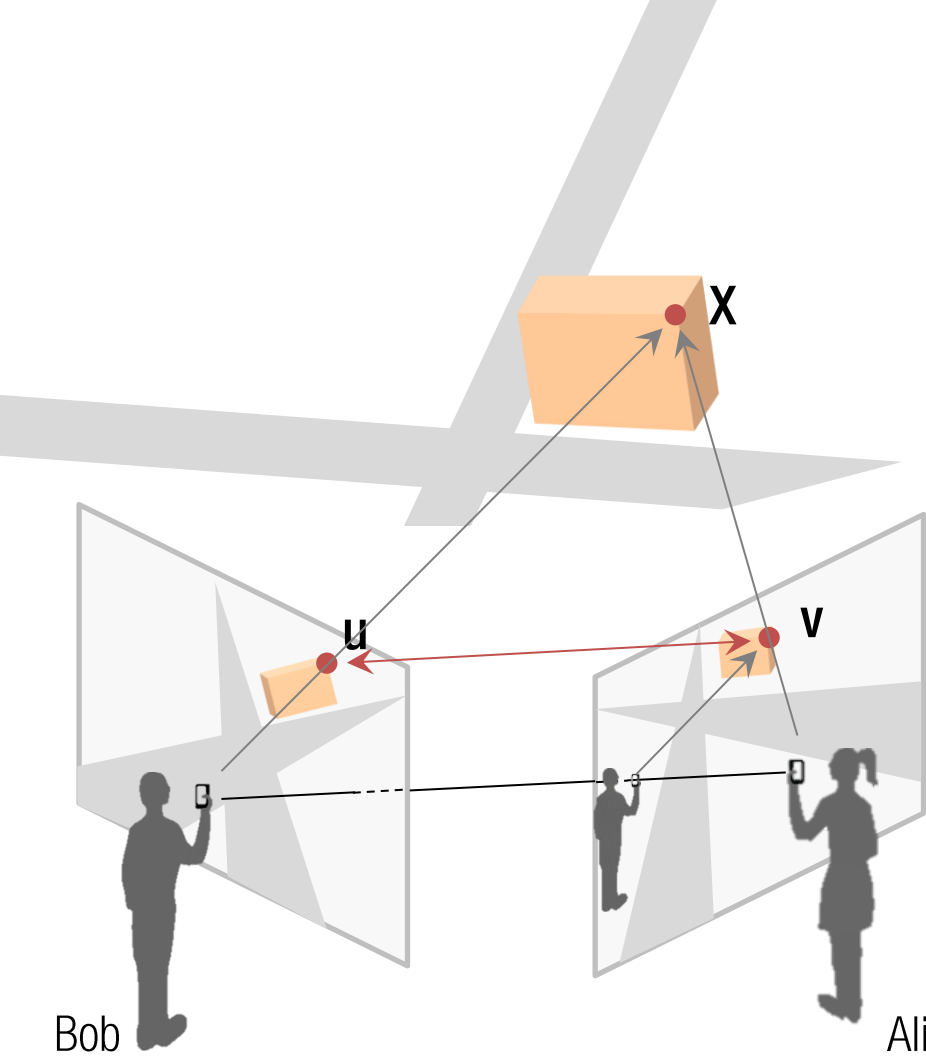
Alice's image



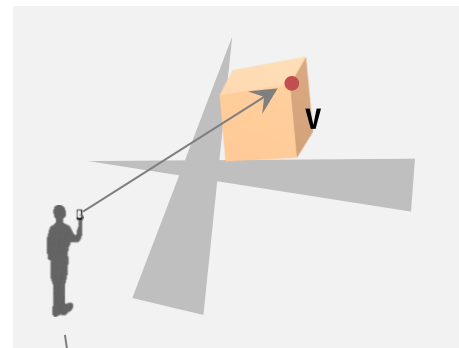
Bob's image



Alice's image
Bob from Alice's view

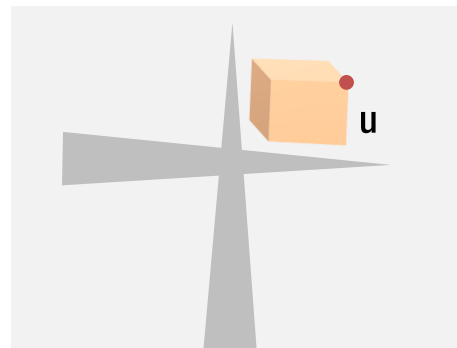
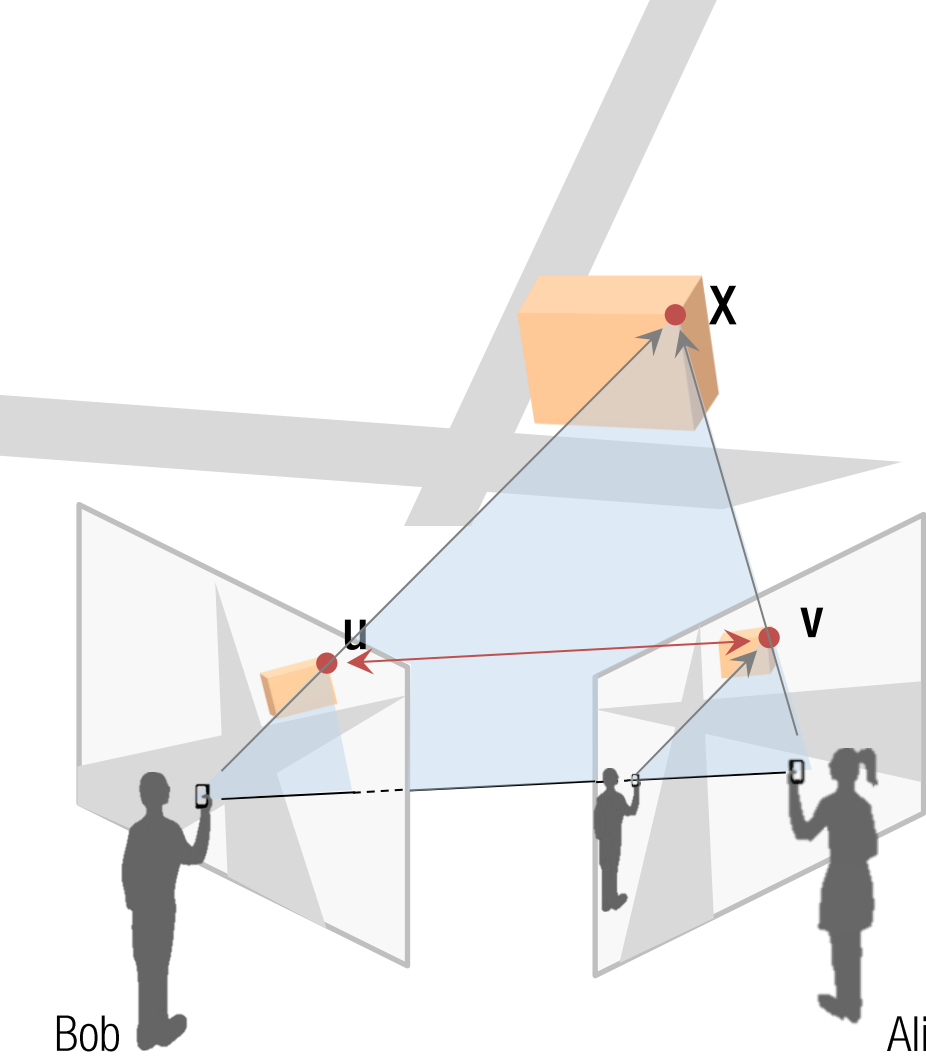


Bob's image

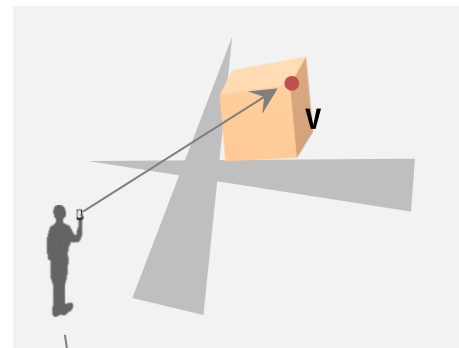


Alice's image
Bob from Alice's view



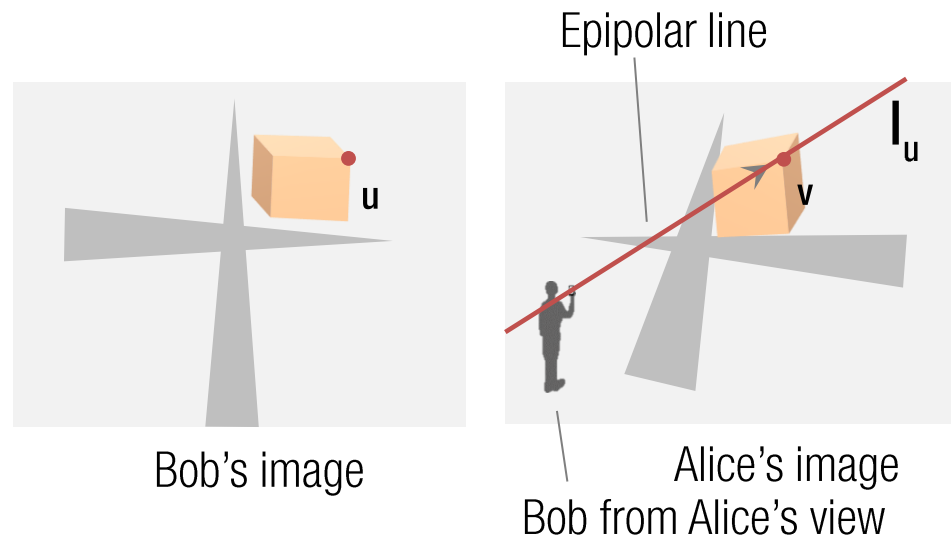
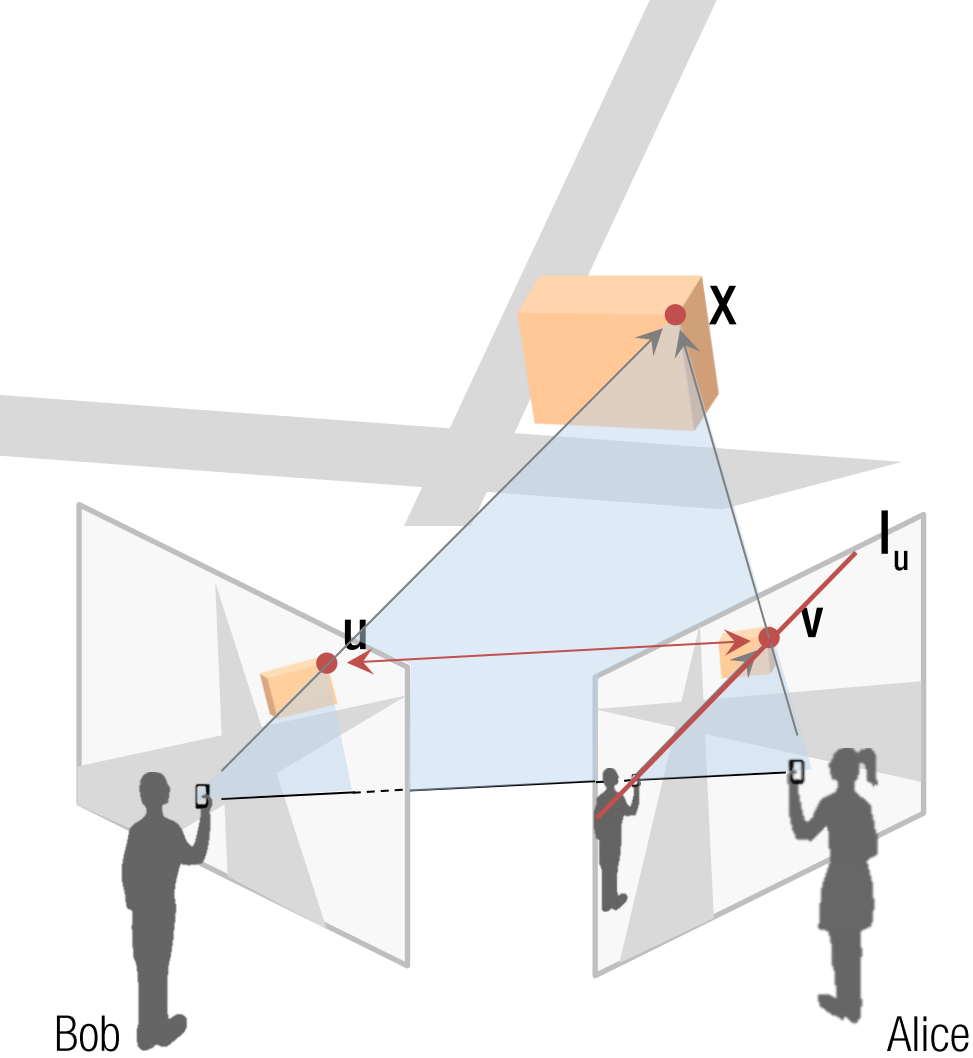


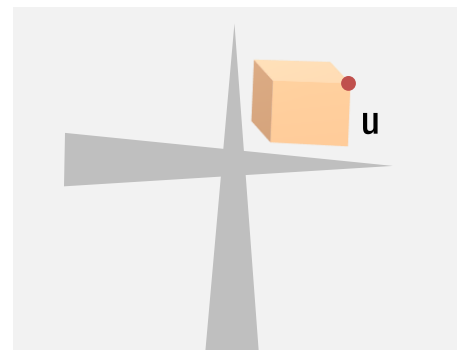
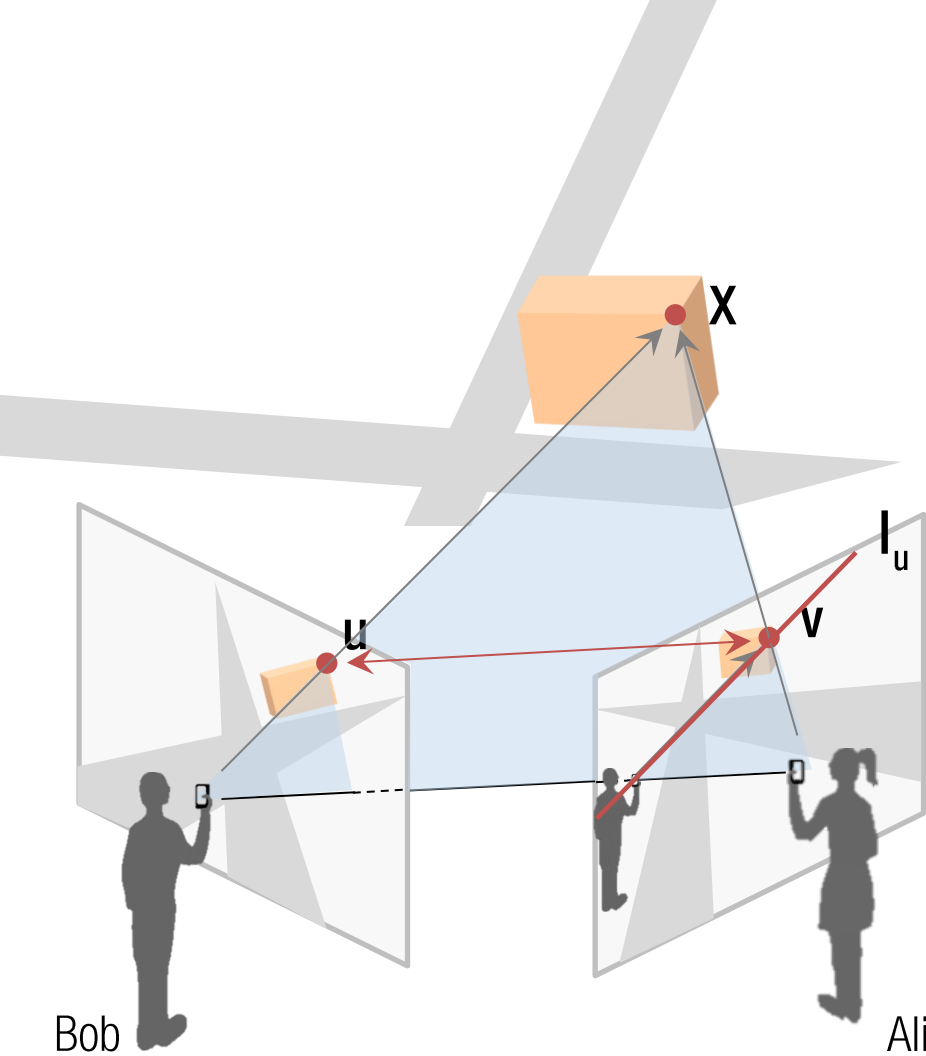
Bob's image



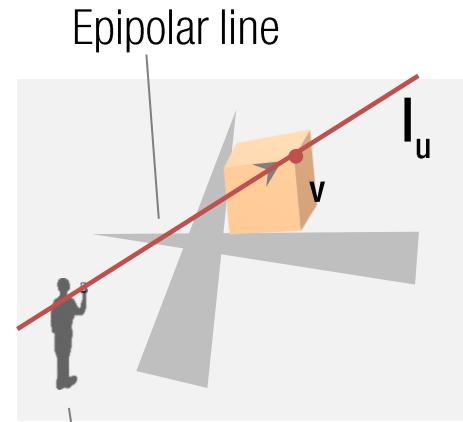
Alice's image
Bob from Alice's view







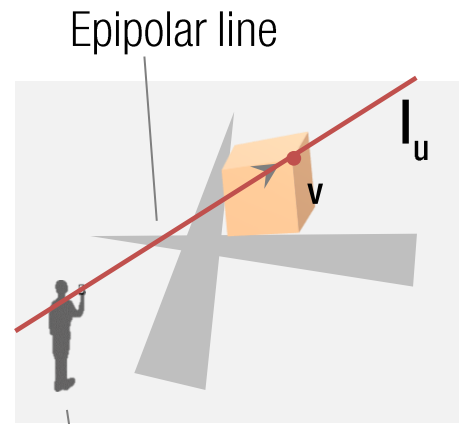
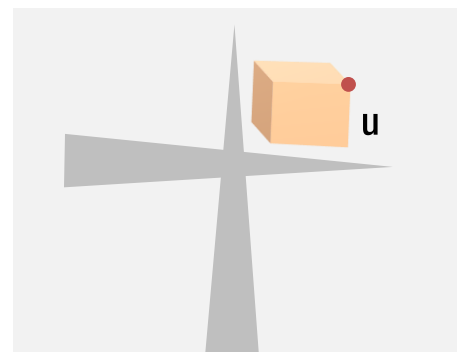
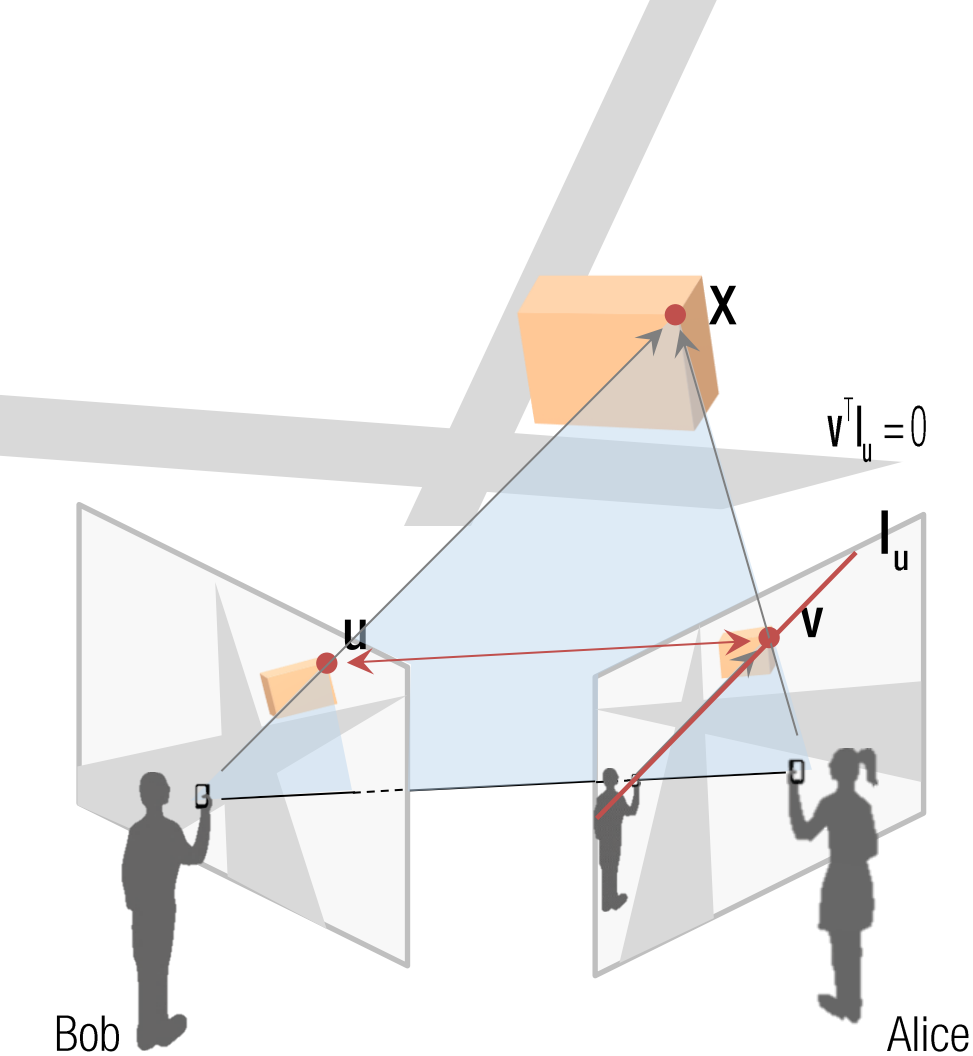
Bob's image



Alice's image
Bob from Alice's view

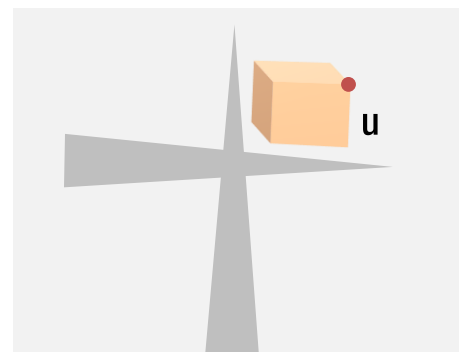
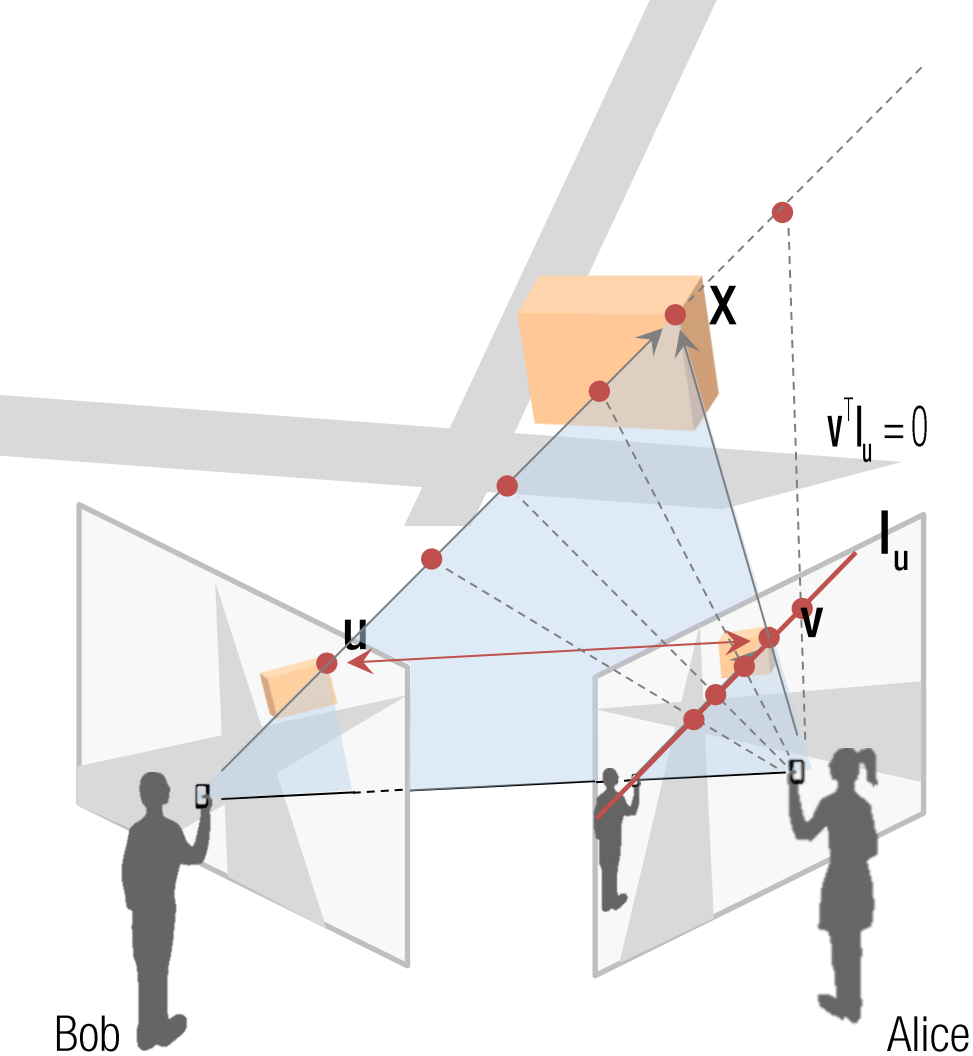
Epipolar constraint between two images:

1. A point, u , in Bob's image corresponds to an epipolar line l_u in Alice's image.

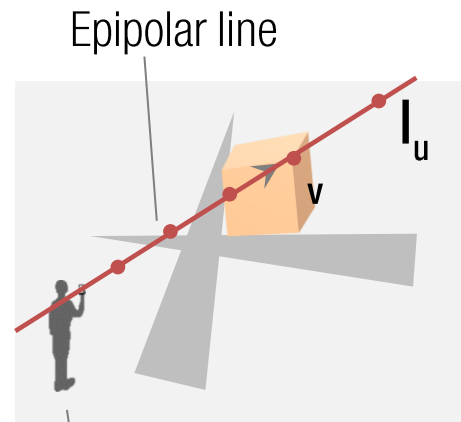


Epipolar constraint between two images:

1. A point, \mathbf{u} , in Bob's image corresponds to an epipolar line \mathbf{l}_u in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, \mathbf{v} : $\mathbf{v}^T \mathbf{l}_u = 0$



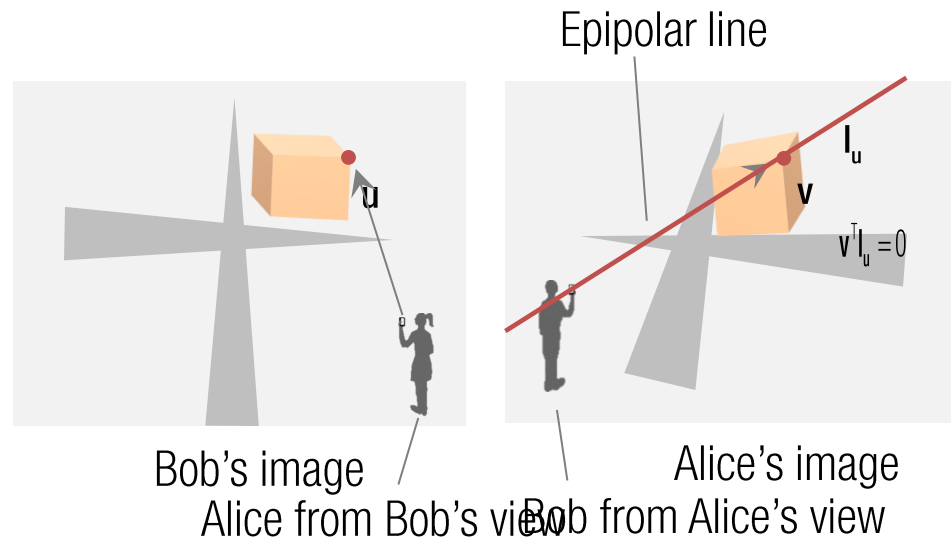
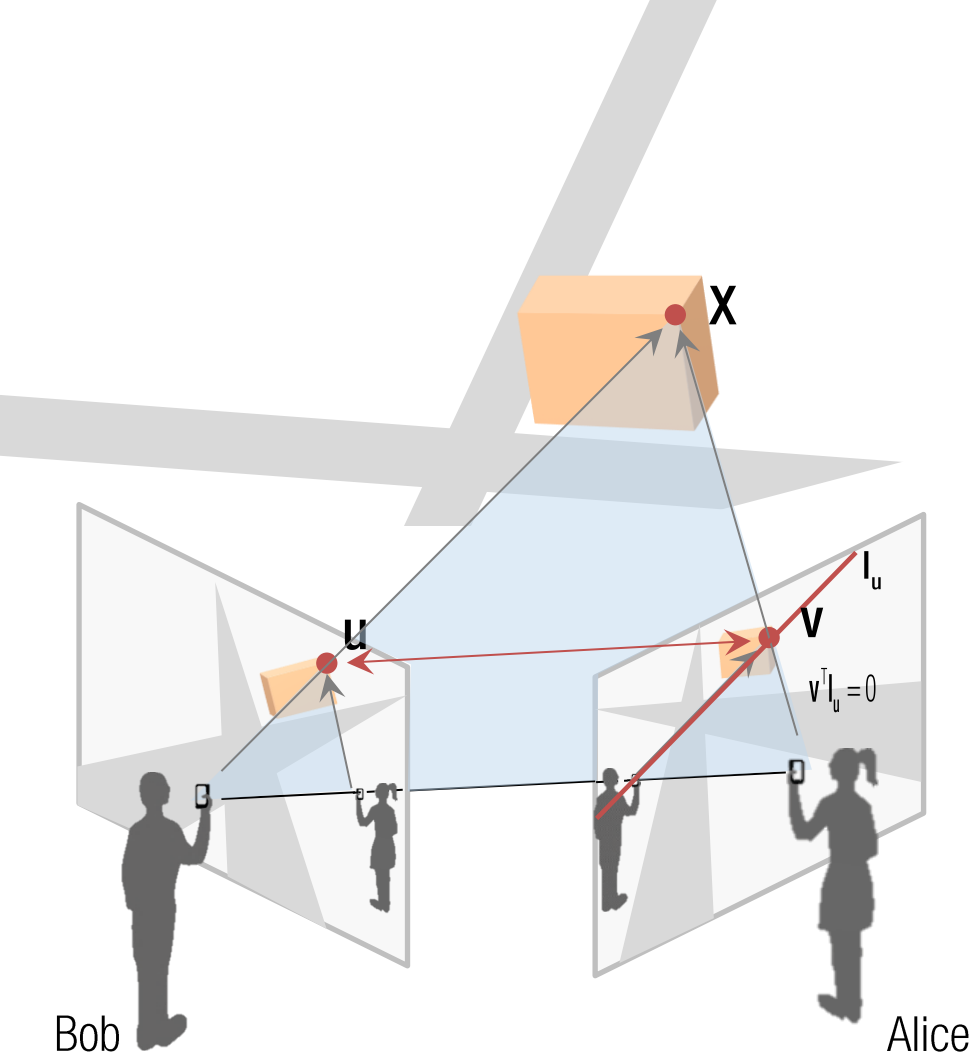
Bob's image



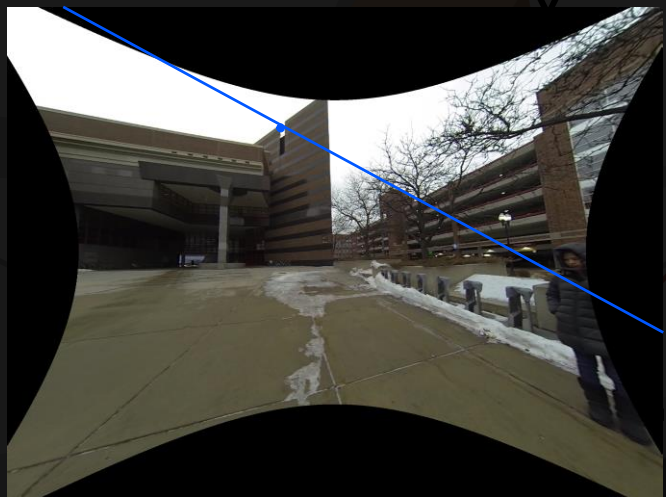
Alice's image
Bob from Alice's view

Epipolar constraint between two images:

1. A point, u , in Bob's image corresponds to an epipolar line l_u in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, v : $v^T l_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.



Epipolar line



l_u
 $= 0$

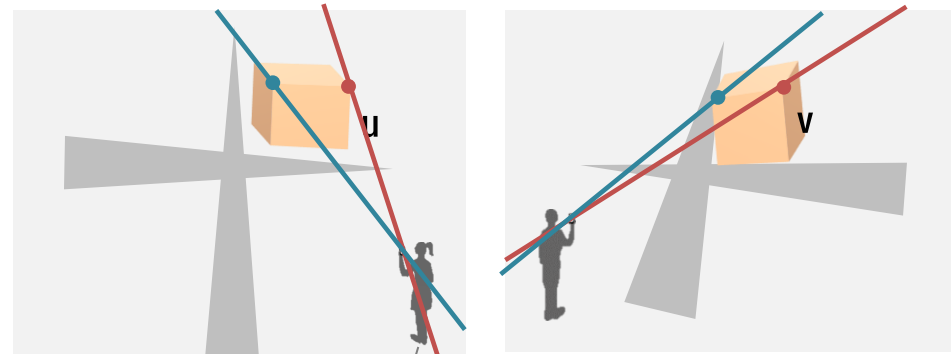
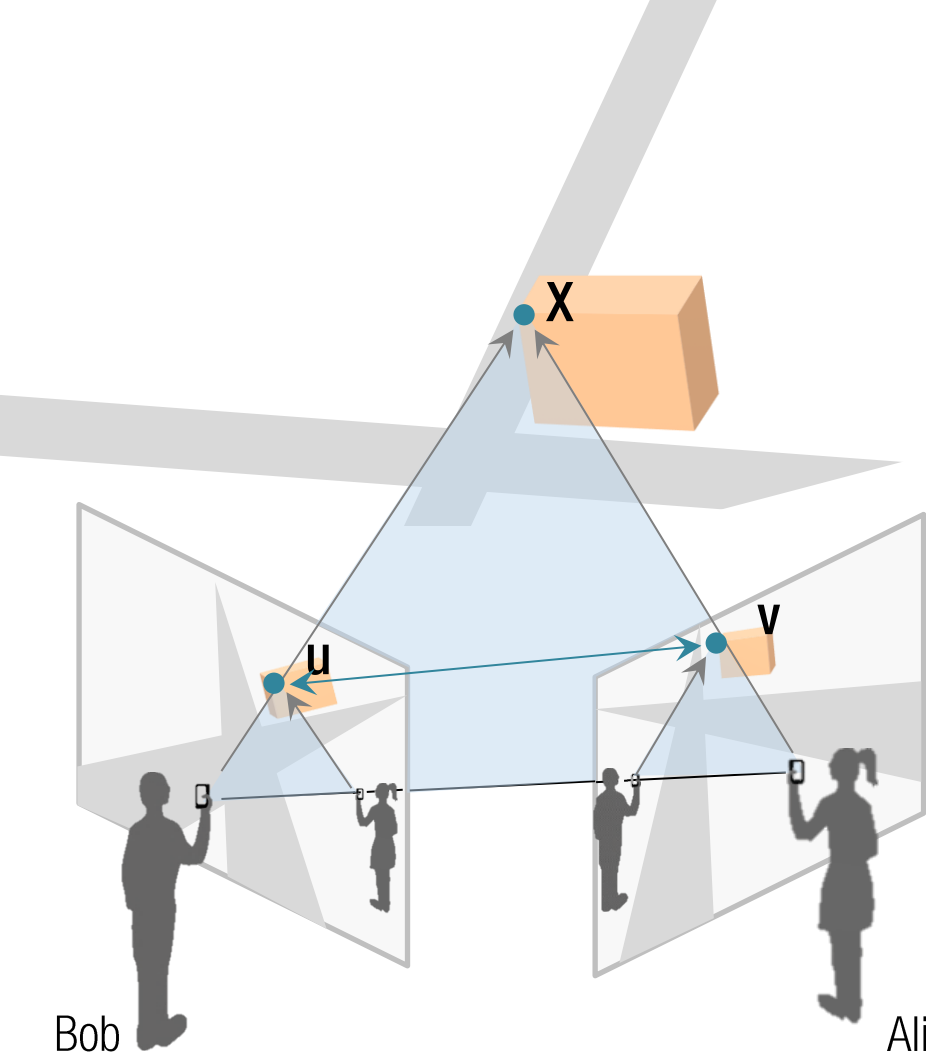


1. Alice's image
2. Alice's view
3. Alice's
4. Alice's
5. Alice's
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100. Alice's

1. The epipolar line passes the corresponding point in Alice's image, v :
2. The epipolar line passes the corresponding point in Alice's image, v :
3. Any point along the epipolar line can be a candidate of correspondences.

Bob

Alice



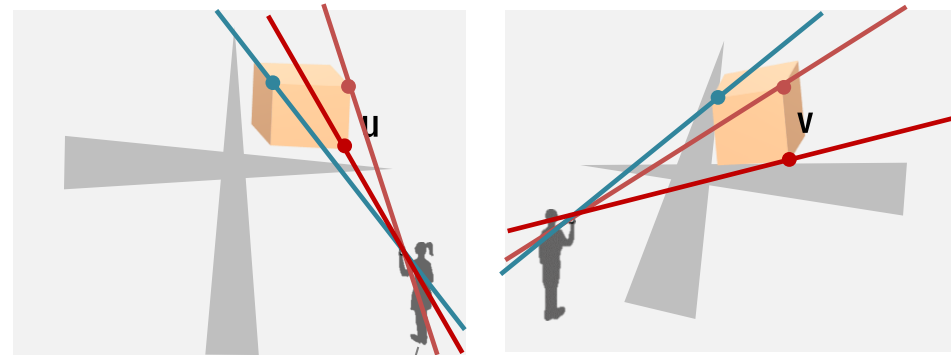
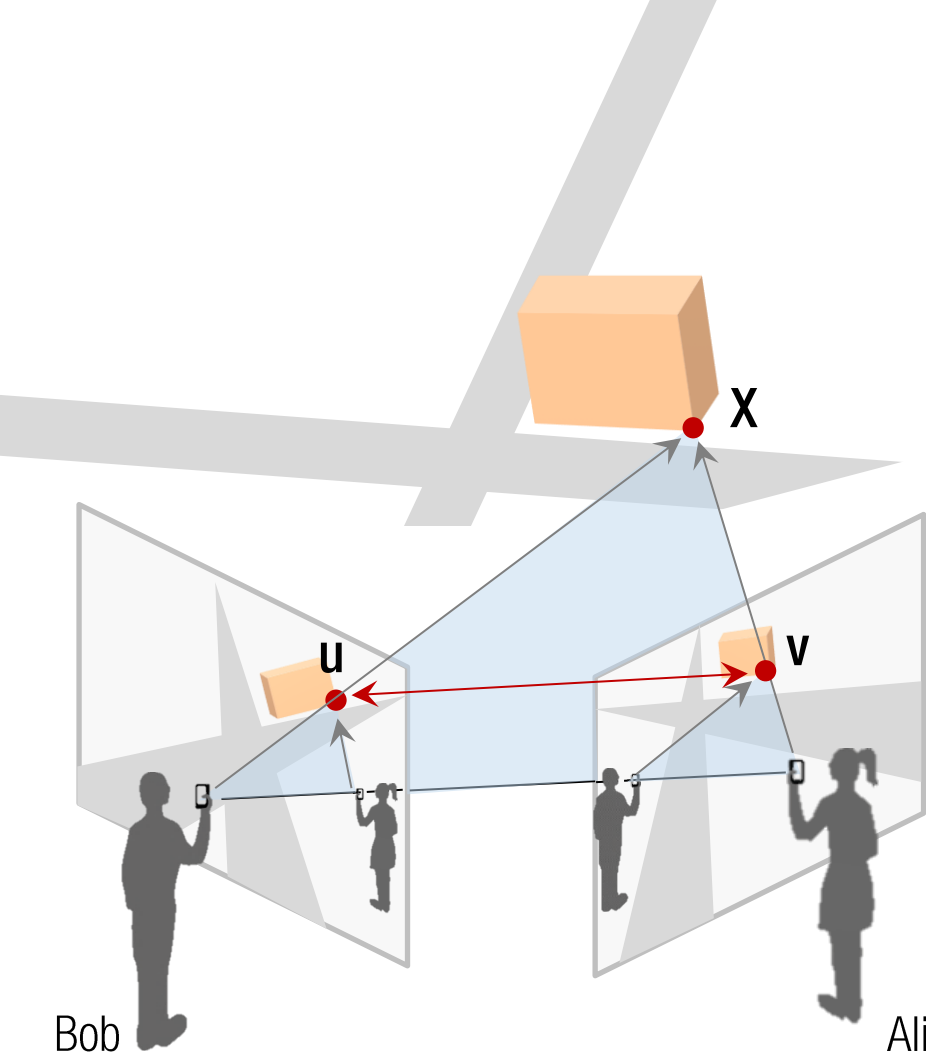
Bob's image

Alice from Bob's view

Alice's image

Epipolar constraint between two images:

1. A point, \mathbf{u} , in Bob's image corresponds to an epipolar line l_u in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, \mathbf{v} : $\mathbf{v}^\top l_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.



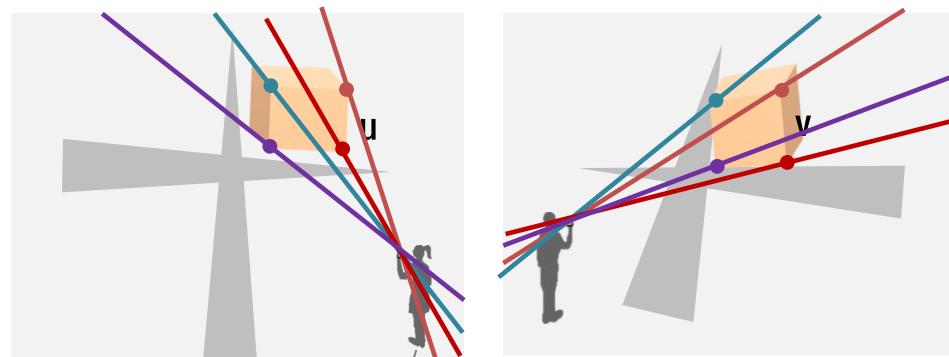
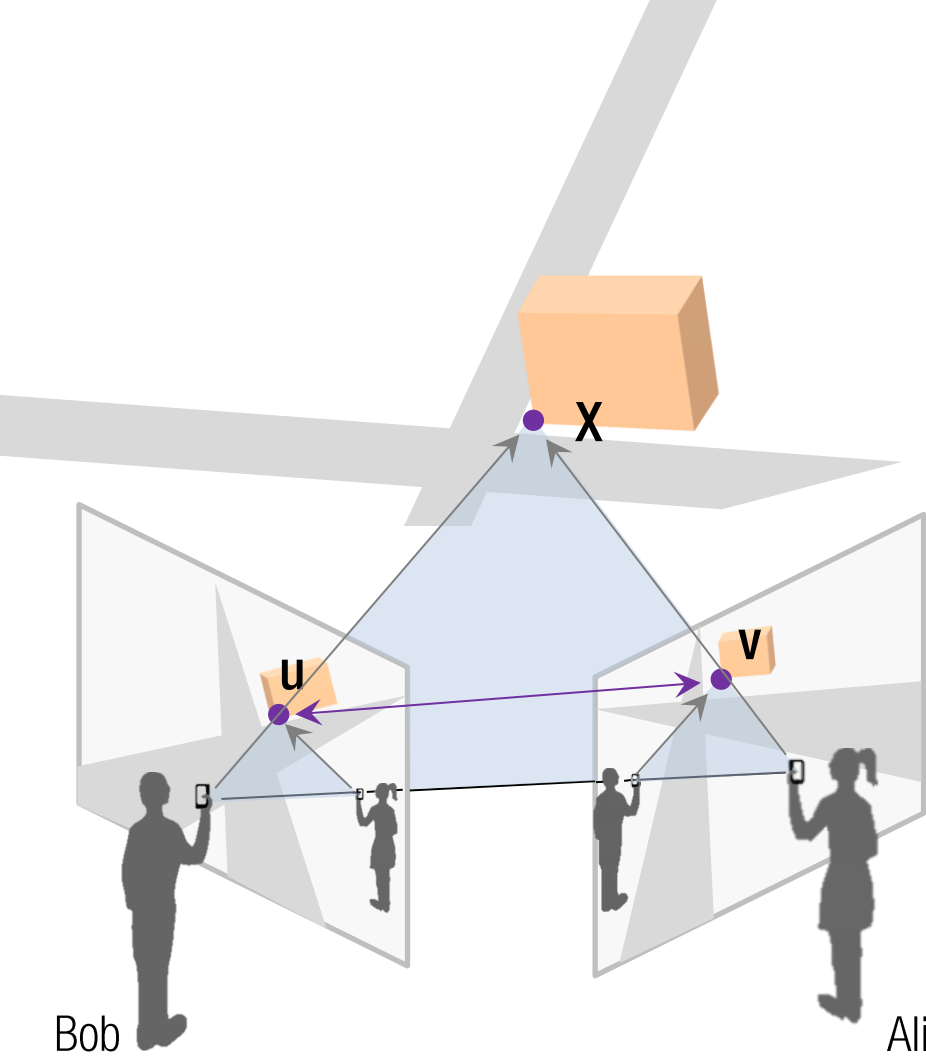
Bob's image

Alice from Bob's view

Alice's image

Epipolar constraint between two images:

1. A point, u , in Bob's image corresponds to an epipolar line l_u in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, v : $v^T l_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.



Bob's image

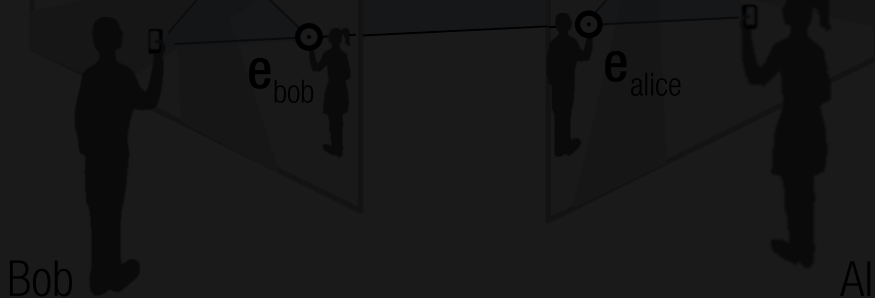
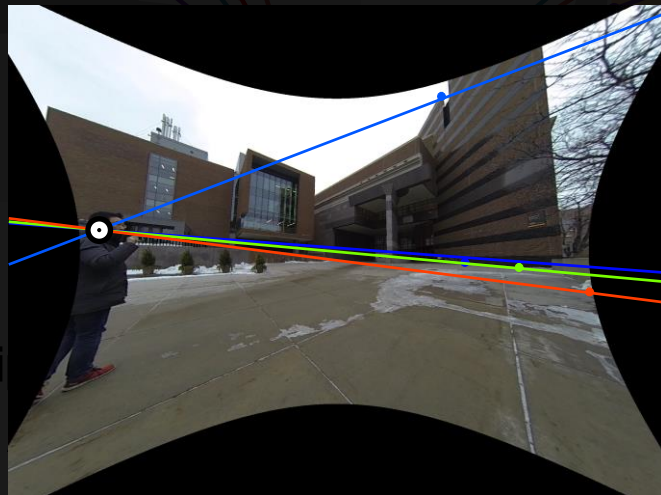
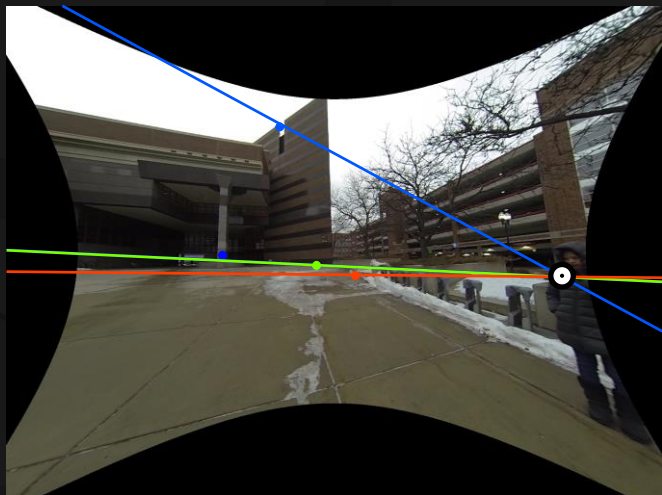
Alice from Bob's view

Alice's image

Epipolar constraint between two images:

1. A point, \mathbf{u} , in Bob's image corresponds to an epipolar line l_u in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, \mathbf{v} : $\mathbf{v}^T l_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.

4. Epipolar lines meet at the epipole: $\mathbf{e}_{\text{bob}}^T l_u = 0$ $\mathbf{e}_{\text{alice}}^T l_v = 0$



Bob

Alice

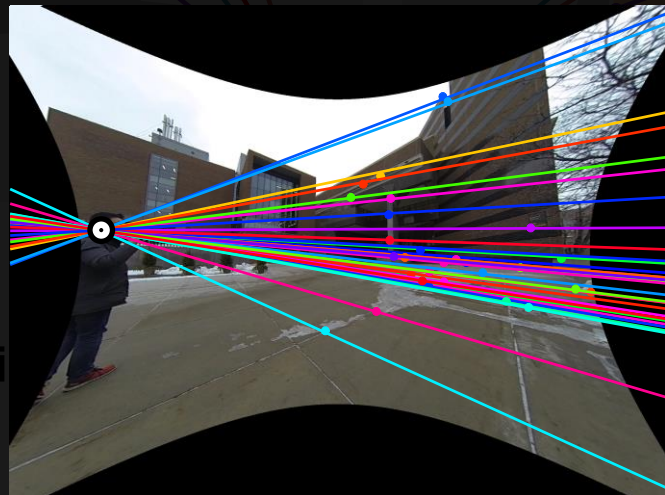
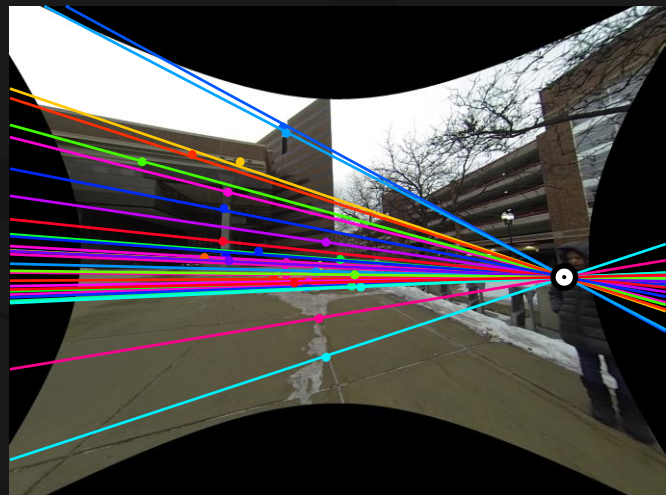
Epipolar geometry

1. The epipolar line in Alice's image, e_{alice} , passes through the epipole e_{bob} and the point u corresponding to the point v in Bob's image.
2. The epipolar line in Bob's image, e_{bob} , passes through the epipole e_{alice} and the point v corresponding to the point u in Alice's image.
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epipolar lines meet at the epipole:

Bob's image
Alice's view

to an

$$e_{bob} \perp u = 0 \quad e_{alice} \perp v = 0$$



Epipolar

1.

2.

3.

4.

epipolar line in Alice's image.

The epipolar line passes the corresponding point in Alice's image, \mathbf{v} :

Any point along the epipolar line can be a candidate of correspondences.

Epipolar lines meet at the epipole:

Bob

Alice

e_{bob}

e_{alice}

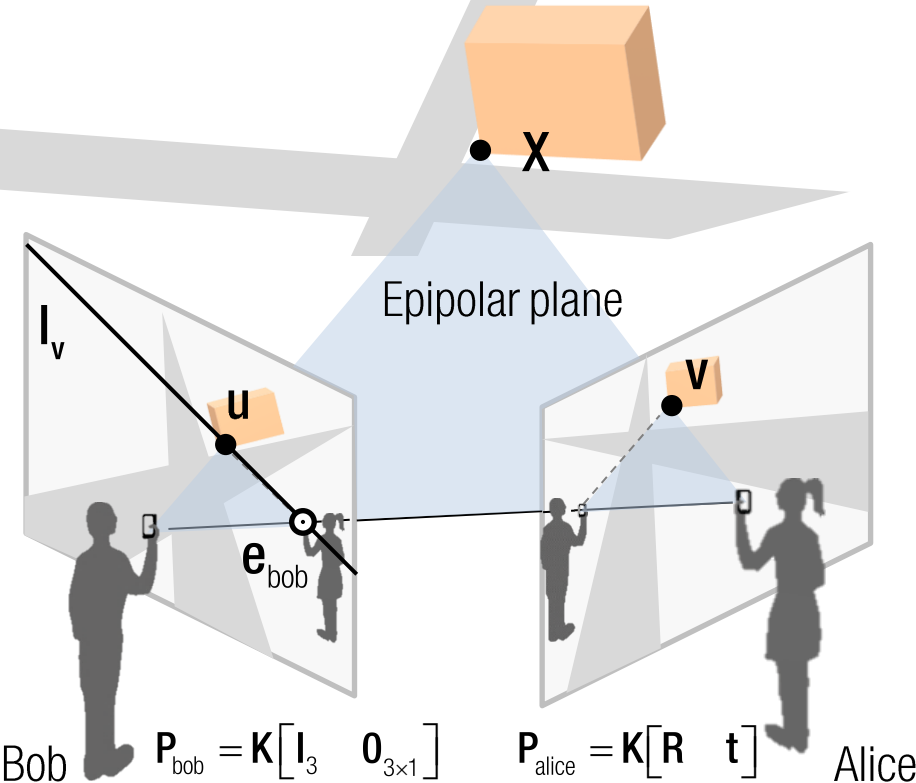
s image
s view

to an

$\mathbf{u}_{\text{bob}} \parallel \mathbf{u}$

$\mathbf{e}_{\text{alice}} \parallel \mathbf{v}$

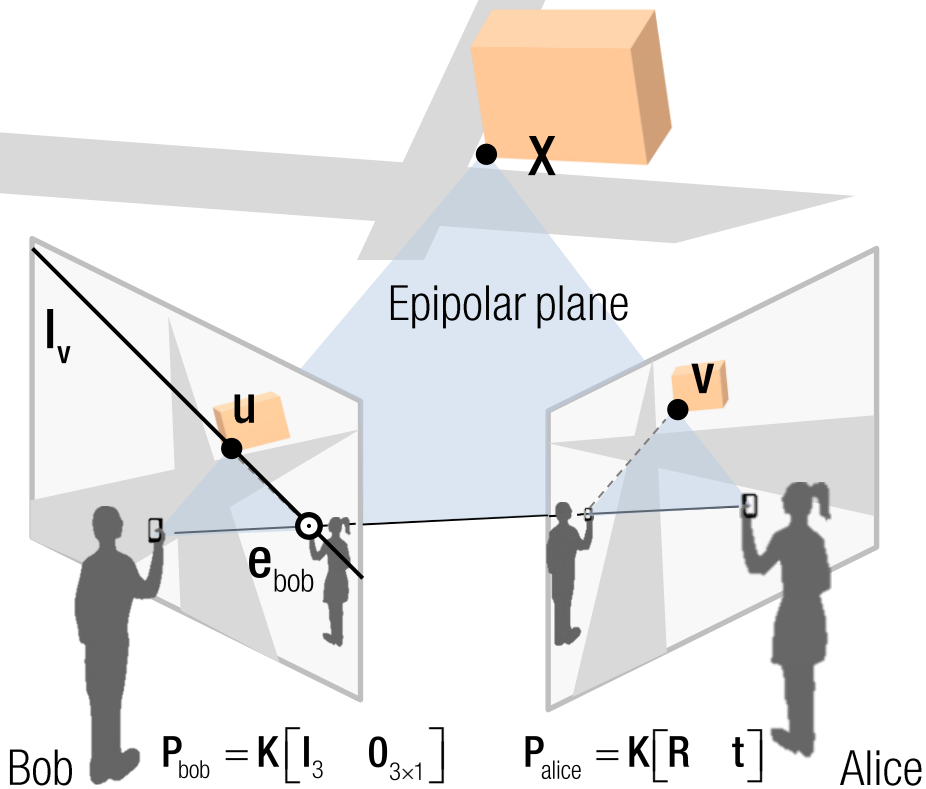
EPIPOLAR LINE



$$l_v = Fv$$

Fundamental matrix

EPIPOLAR LINE



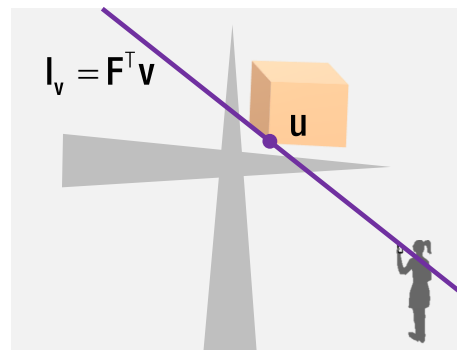
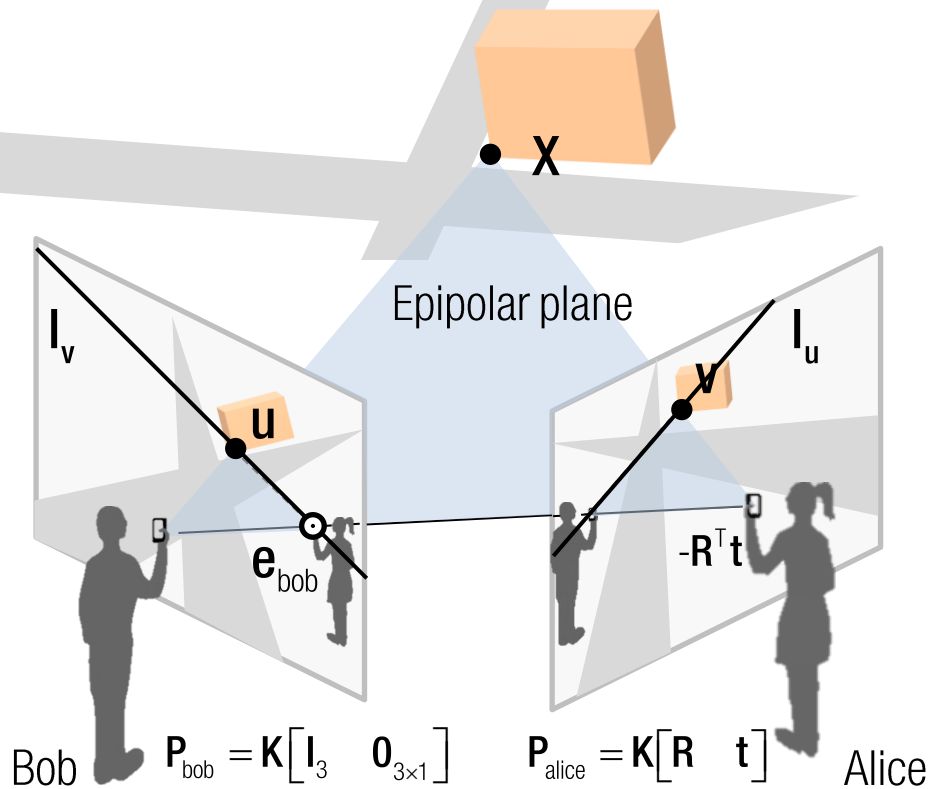
$$I_v = Fv$$

Fundamental matrix

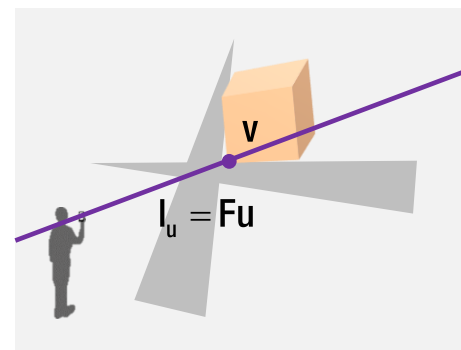
$$u^T Fv = 0$$

where $F = K^{-T} \begin{bmatrix} t \end{bmatrix}_x R K^{-1}$
Fundamental matrix

FUNDAMENTAL MATRIX



Bob's image



Alice's image

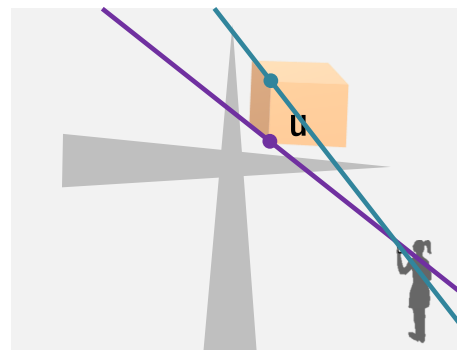
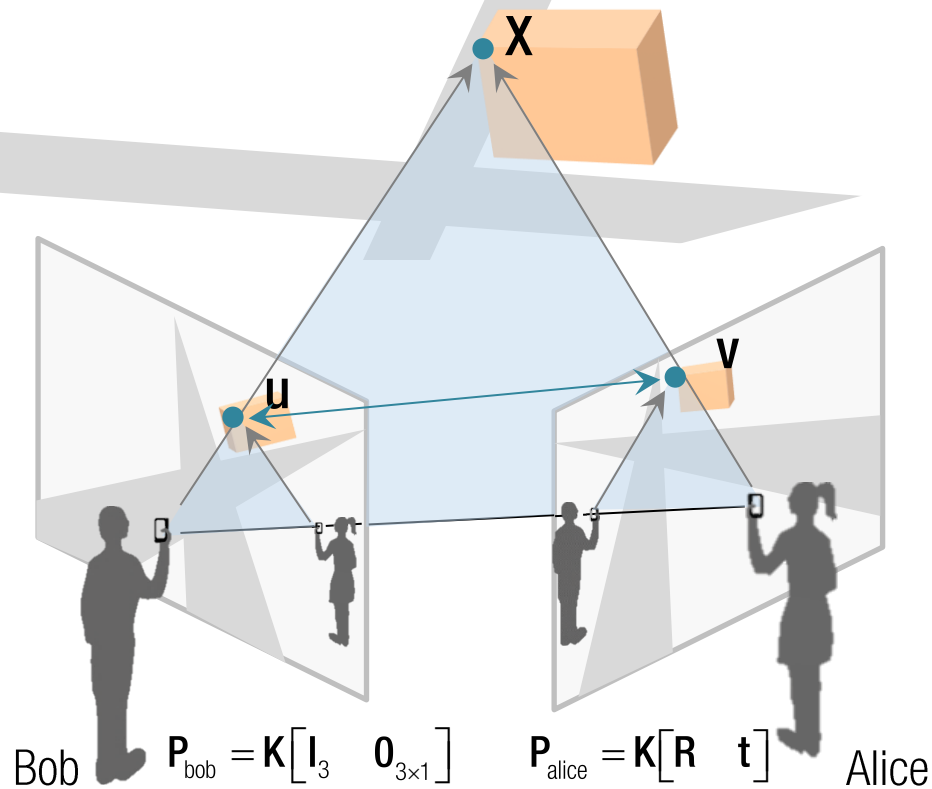
$$\mathbf{v}^T \mathbf{I}_u = \mathbf{v}^T \mathbf{K}^{-T} \begin{bmatrix} \mathbf{t} \\ x \end{bmatrix} \mathbf{R} \mathbf{K}^{-1} \mathbf{u} = 0$$

Common for all points

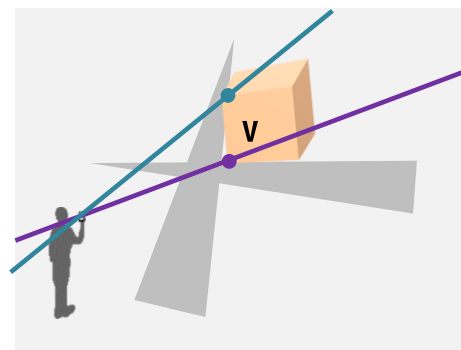
$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

$$= \mathbf{v}^T (\mathbf{F} \mathbf{u}) = \mathbf{u}^T (\mathbf{F}^T \mathbf{v}) = 0$$

FUNDAMENTAL MATRIX



Bob's image



Alice's image

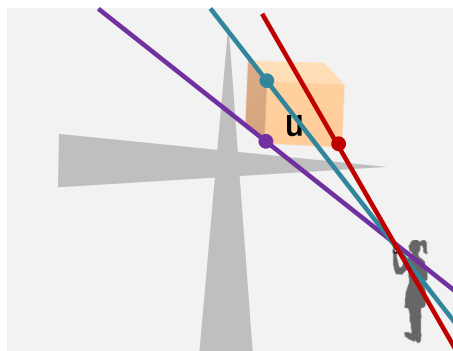
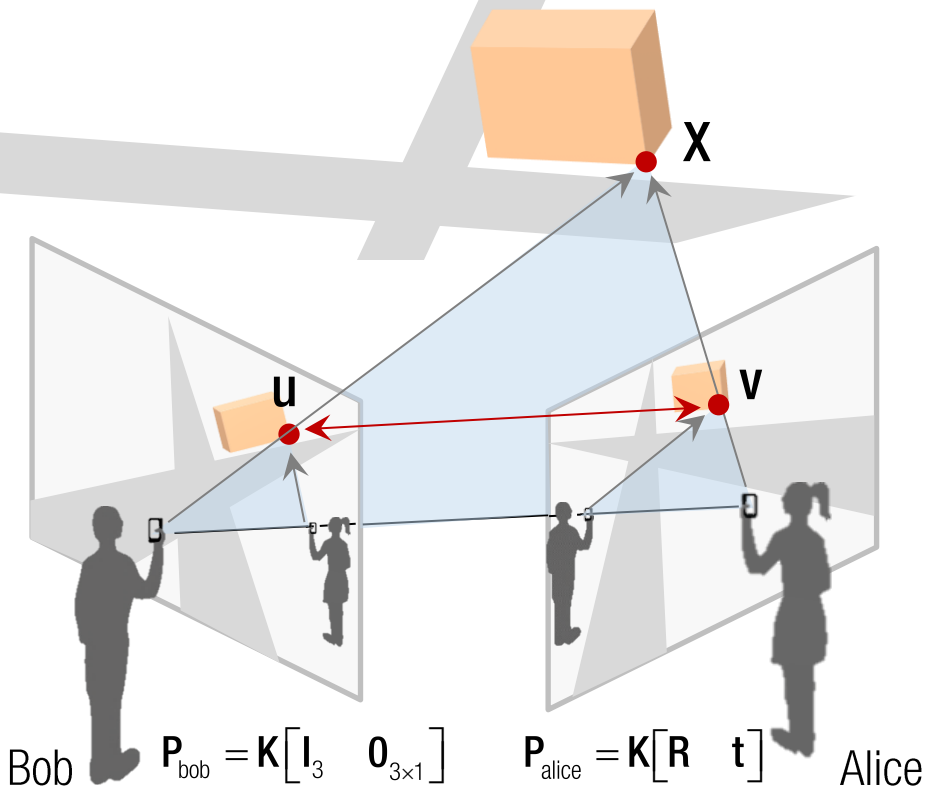
$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \mathbf{K}^{-T} \begin{bmatrix} \mathbf{t} \\ x \end{bmatrix} \mathbf{R} \mathbf{K}^{-1} \mathbf{u} = 0$$

Common for all points

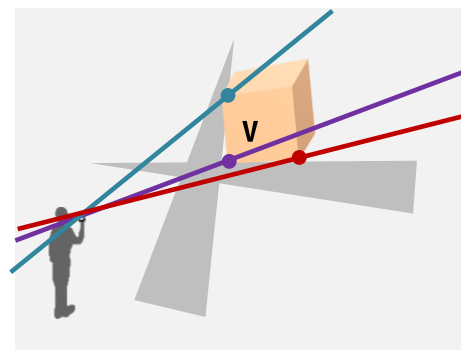
$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

$$= \mathbf{v}^T (\mathbf{F} \mathbf{u}) = \mathbf{u}^T (\mathbf{F}^T \mathbf{v}) = 0$$

FUNDAMENTAL MATRIX



Bob's image



Alice's image

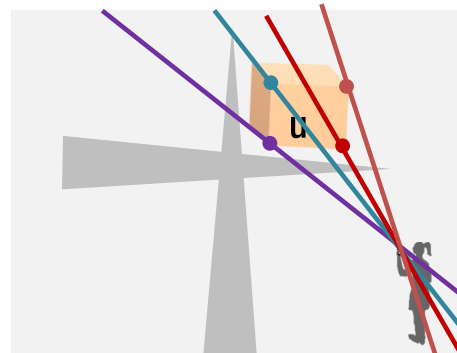
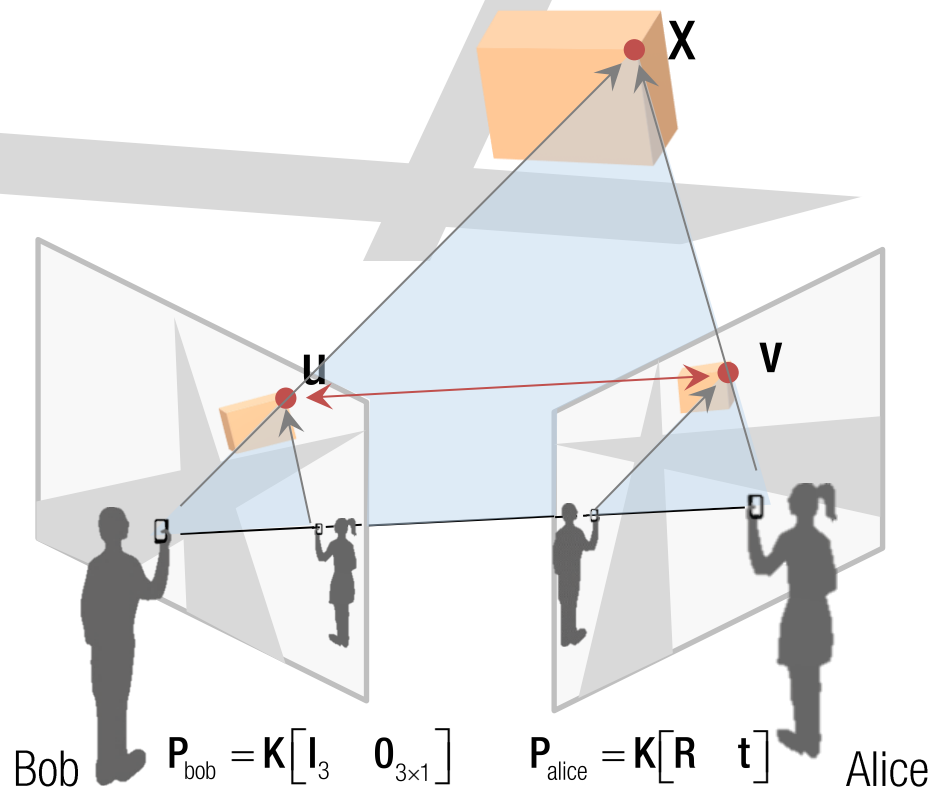
$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \mathbf{K}^{-T} \begin{bmatrix} \mathbf{t} \\ x \end{bmatrix} \mathbf{R} \mathbf{K}^{-1} \mathbf{u} = 0$$

Common for all points

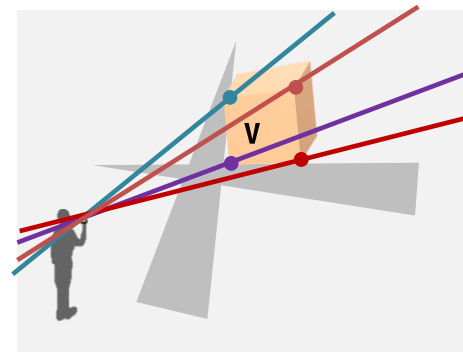
$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

$$= \mathbf{v}^T (\mathbf{F} \mathbf{u}) = \mathbf{u}^T (\mathbf{F}^T \mathbf{v}) = 0$$

FUNDAMENTAL MATRIX



Bob's image



Alice's image

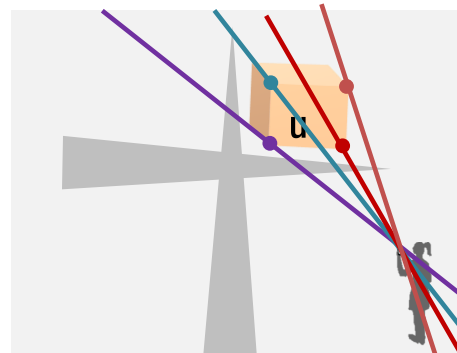
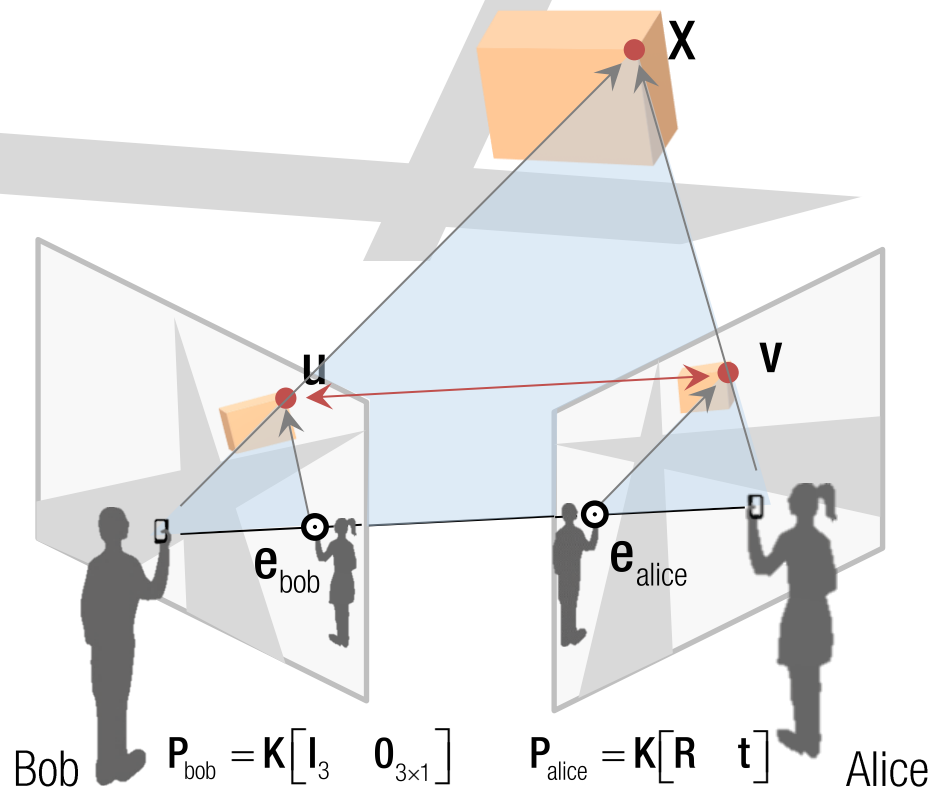
$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \mathbf{K}^{-T} \begin{bmatrix} \mathbf{t} \\ x \end{bmatrix} \mathbf{R} \mathbf{K}^{-1} \mathbf{u} = 0$$

Common for all points

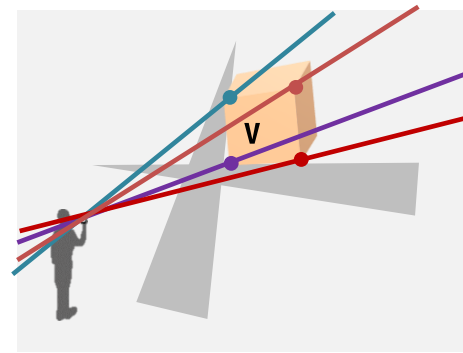
$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

$$= \mathbf{v}^T (\mathbf{F} \mathbf{u}) = \mathbf{u}^T (\mathbf{F}^T \mathbf{v}) = 0$$

FUNDAMENTAL MATRIX



Bob's image

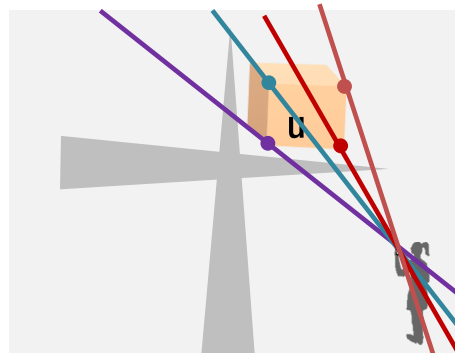
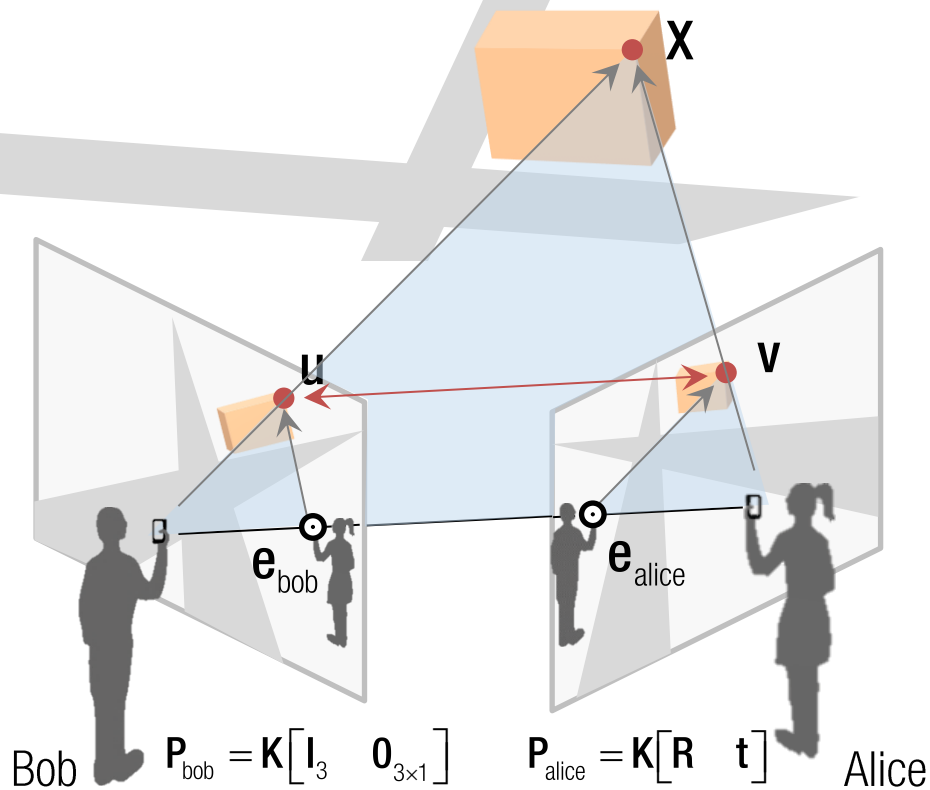


Alice's image

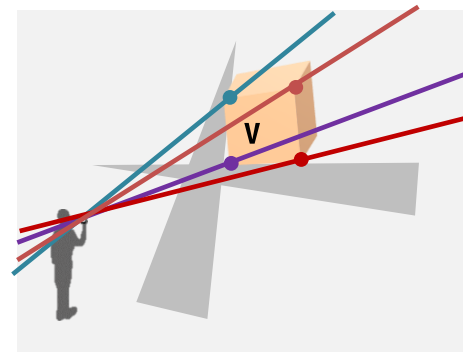
Properties of Fundamental Matrix

- Transpose: if F is for $P_{\text{bob}}, P_{\text{alice}}$, then F^T is for $P_{\text{alice}}, P_{\text{bob}}$.

FUNDAMENTAL MATRIX



Bob's image

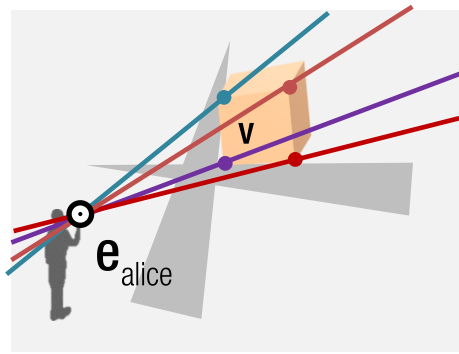
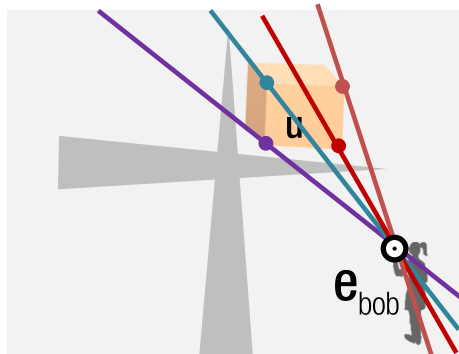
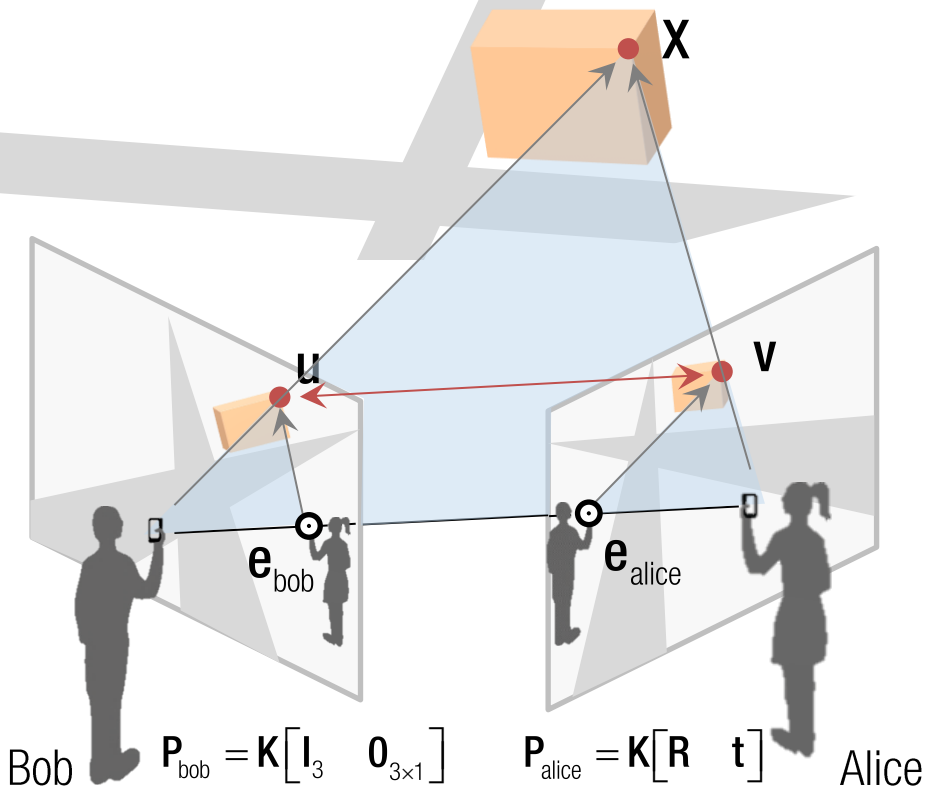


Alice's image

Properties of Fundamental Matrix

- Transpose: if F is for $P_{\text{bob}}, P_{\text{alice}}$, then F^T is for $P_{\text{alice}}, P_{\text{bob}}$.
- Epipolar line: $l_u = Fu \quad l_v = F^T v$

FUNDAMENTAL MATRIX



Properties of Fundamental Matrix

- Transpose: if F is for $P_{\text{bob}}, P_{\text{alice}}$, then F^T is for $P_{\text{alice}}, P_{\text{bob}}$.

$$l_u = Fu \quad l_v = F^T v$$

- Epipolar line:

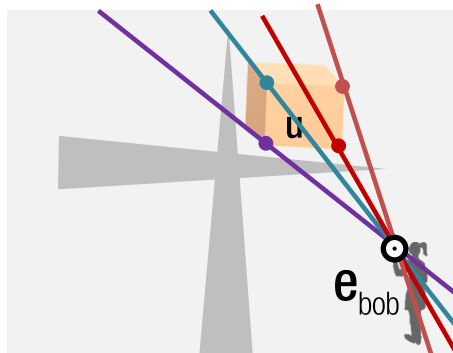
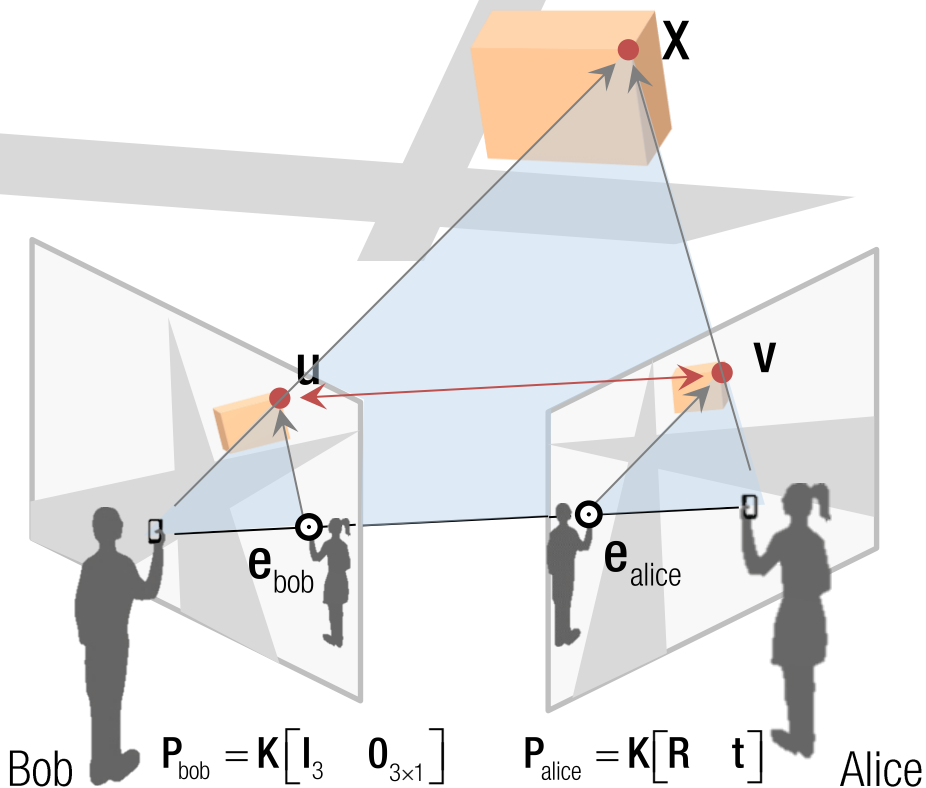
$$F e_{\text{bob}} = 0 \quad F^T e_{\text{alice}} = 0$$

- Epipole:

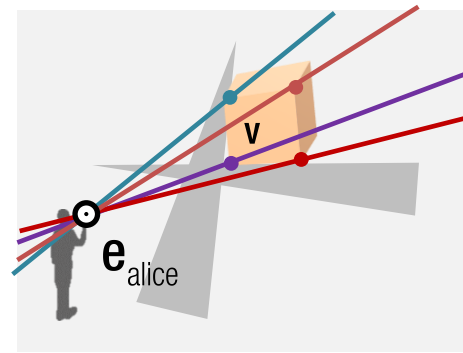
$$\because v_i^T F e_{\text{bob}} = 0, \quad u_i^T F^T e_{\text{alice}} = 0, \quad \forall i$$

$$\rightarrow e_{\text{bob}} = \text{null}(F), \quad e_{\text{alice}} = \text{null}(F^T)$$

FUNDAMENTAL MATRIX



Bob's image



Alice's image

Properties of Fundamental Matrix

- Transpose: if \mathbf{F} is for $\mathbf{P}_{\text{bob}}, \mathbf{P}_{\text{alice}}$, then \mathbf{F}^T is for $\mathbf{P}_{\text{alice}}, \mathbf{P}_{\text{bob}}$.

$$l_u = \mathbf{F}u \quad l_v = \mathbf{F}^T v$$

- Epipolar line:

$$\mathbf{F}e_{\text{bob}} = 0 \quad \mathbf{F}^T e_{\text{alice}} = 0$$

- Epipole:

- $\text{rank}(\mathbf{F})=2$:

DoF 9 (3x3 matrix)-1 (scale)-1 (rank)=7

CAMERA MOTION



CAMERA MOTION



Image 2



Image 1

Image
Image
Forward motion

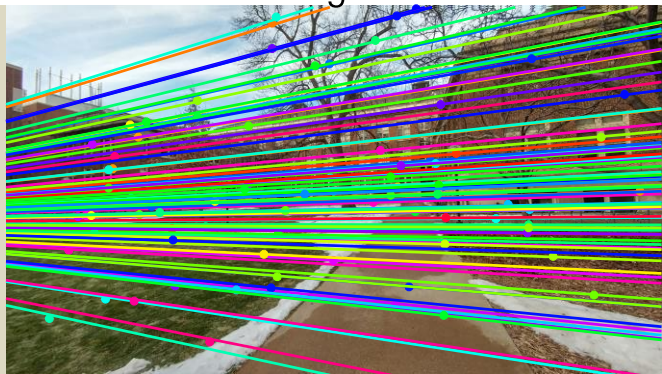


Image 2

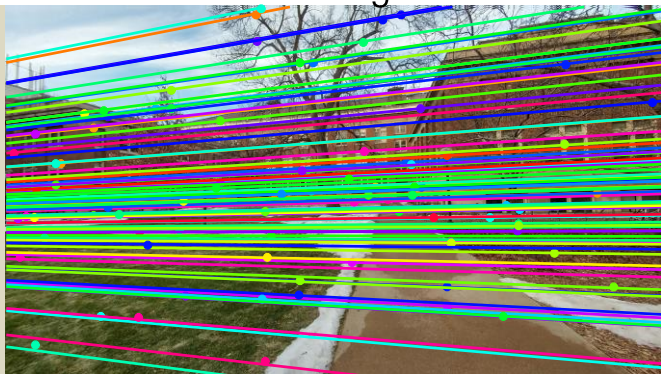
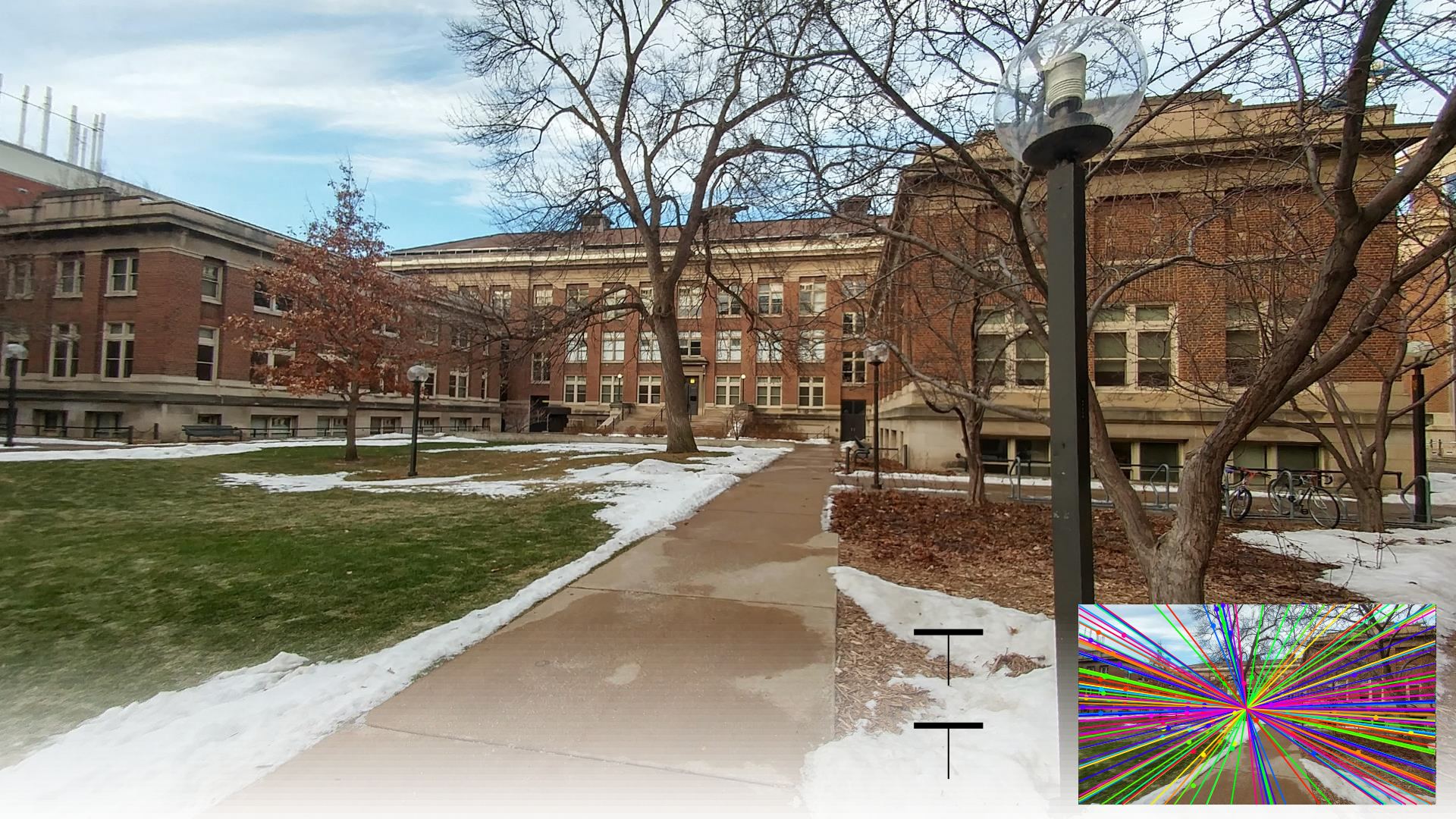
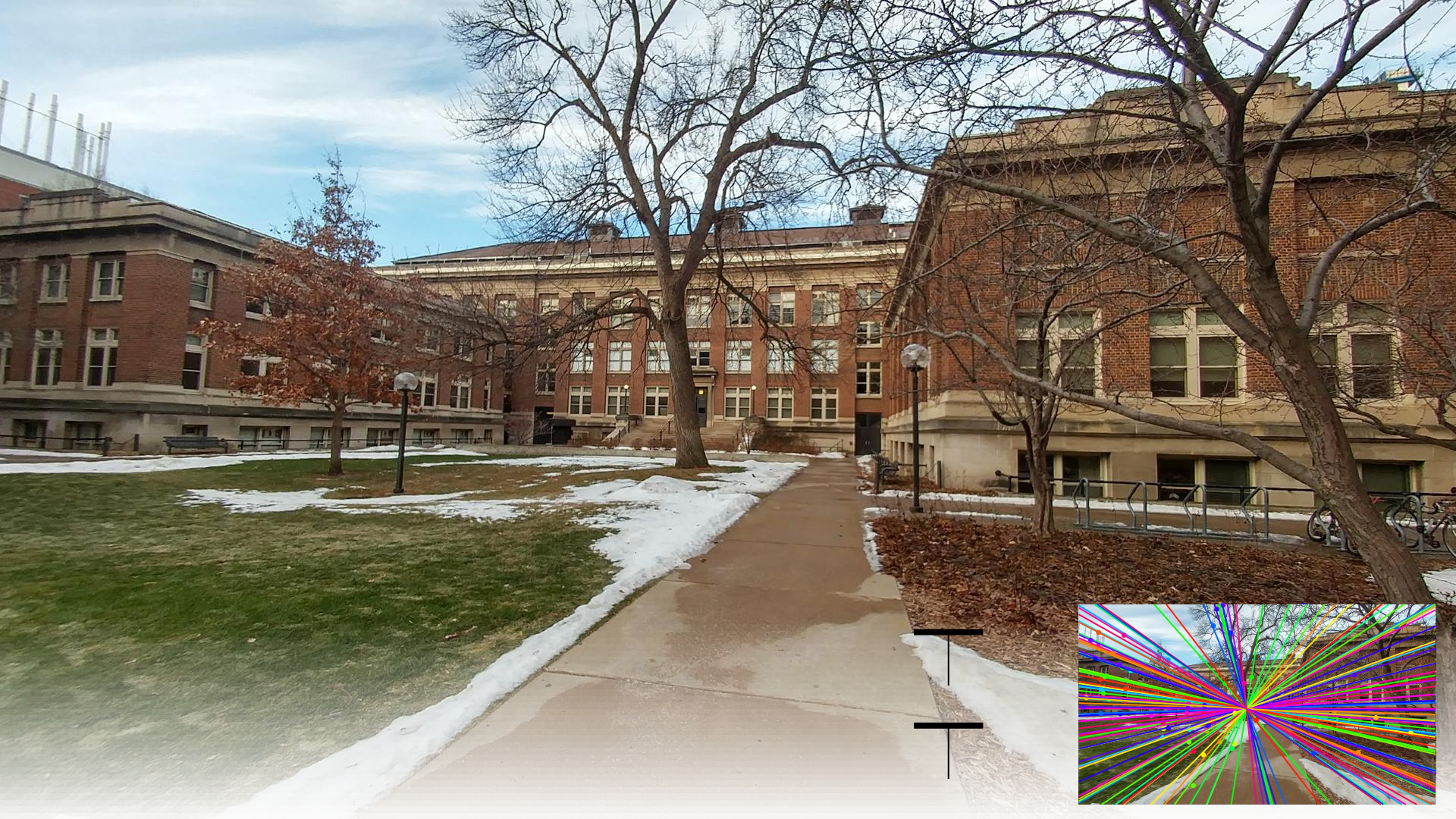
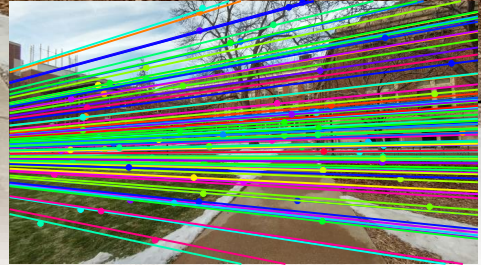


Image 1

Image 2 Image 1
Lateral motion

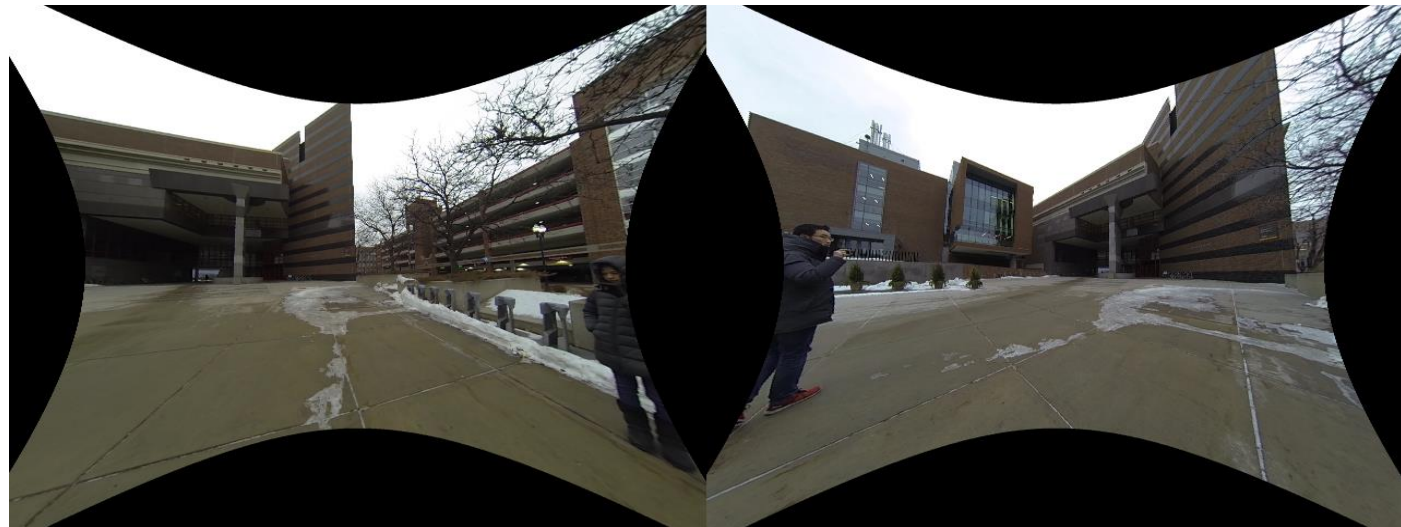








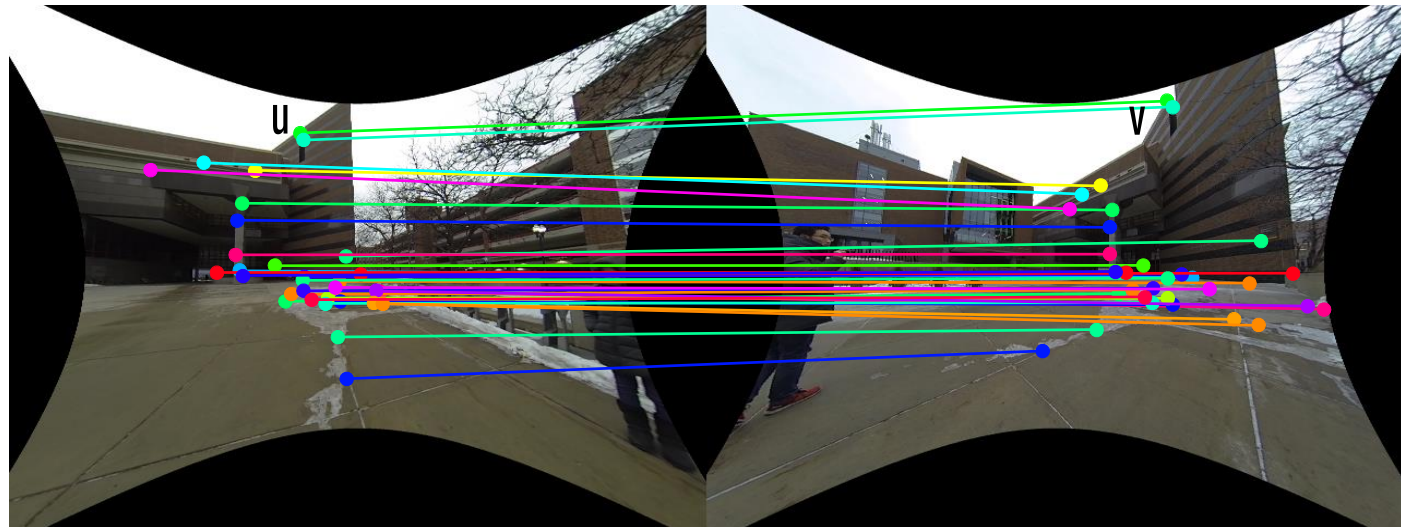
2D CORRESPONDENCE



Bob's image

Alice's image

2D CORRESPONDENCE

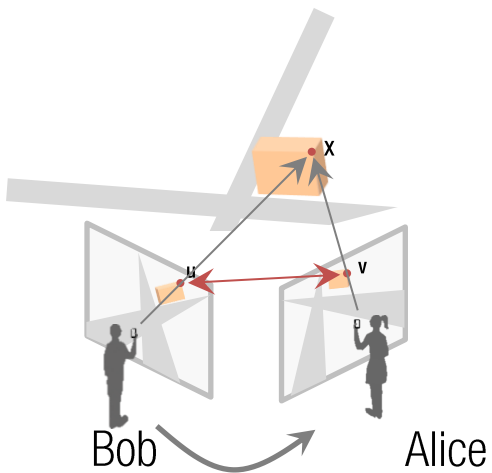


Bob's image

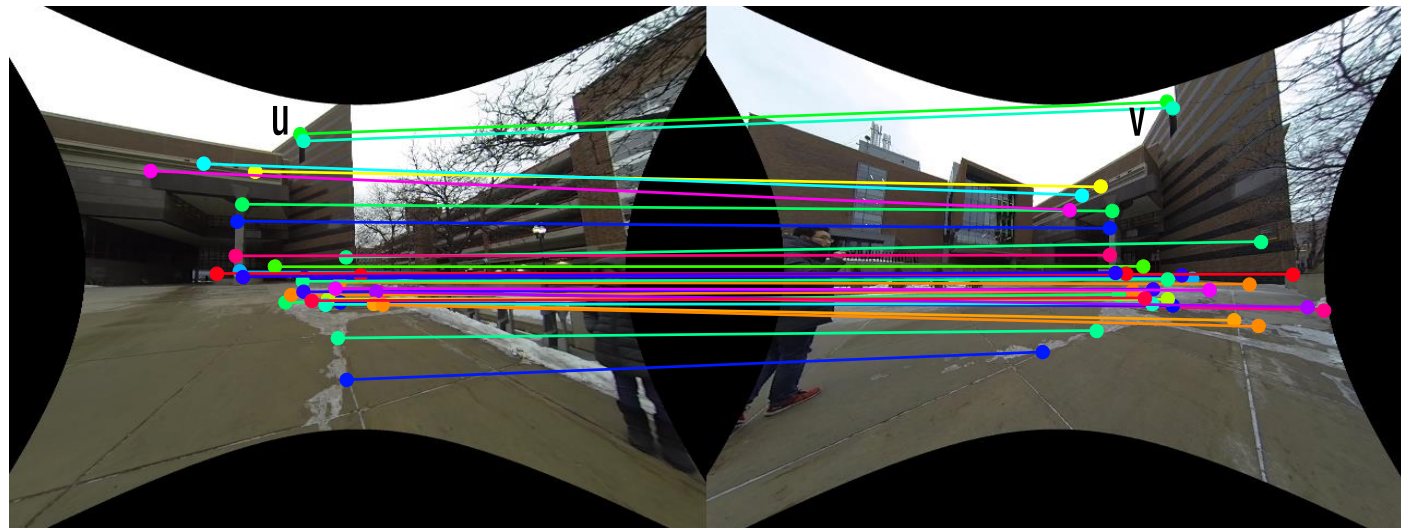
Alice's image

$$\mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

2D CORRESPONDENCE



$$\mathbf{F} = \mathbf{F}(\mathbf{R}, \mathbf{t}) \\ = \mathbf{K}^{-\top} \begin{bmatrix} \mathbf{t} \\ \mathbf{x} \end{bmatrix} \mathbf{R} \mathbf{K}^{-1}$$



Bob's image

Alice's image

$$\mathbf{v}^{\top} \mathbf{F} \mathbf{u} = 0$$

How to compute fundamental matrix?

8 Point Algorithm (Longuet-Higgins, Nature 1981)



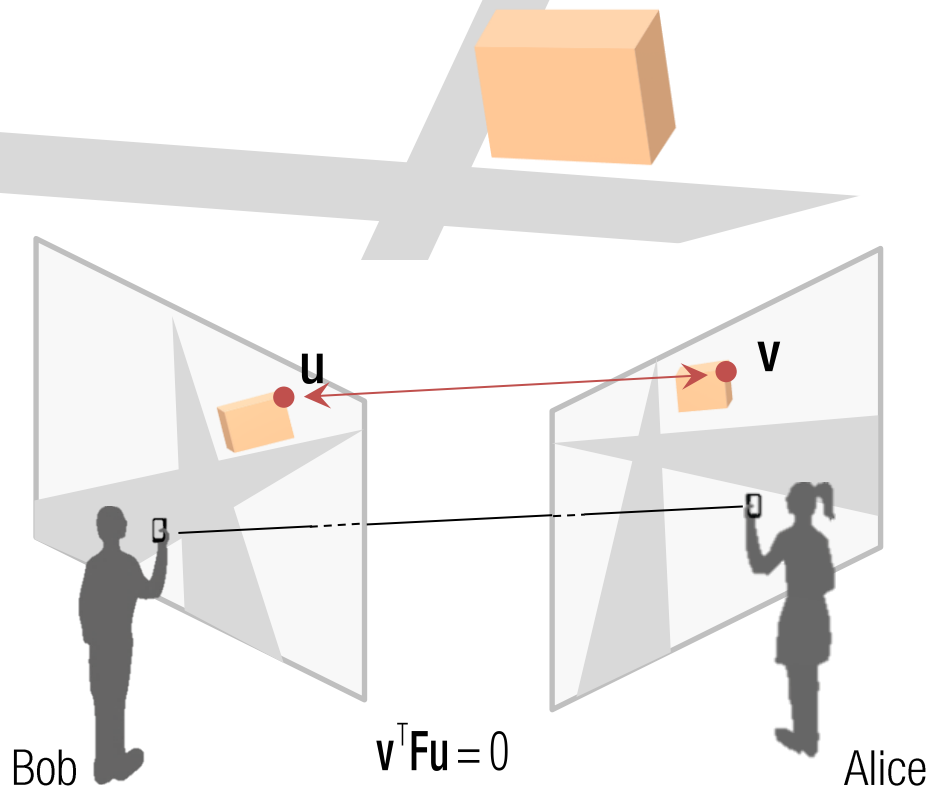
A computer algorithm for reconstructing a scene from two projections

H. C. Longuet-Higgins

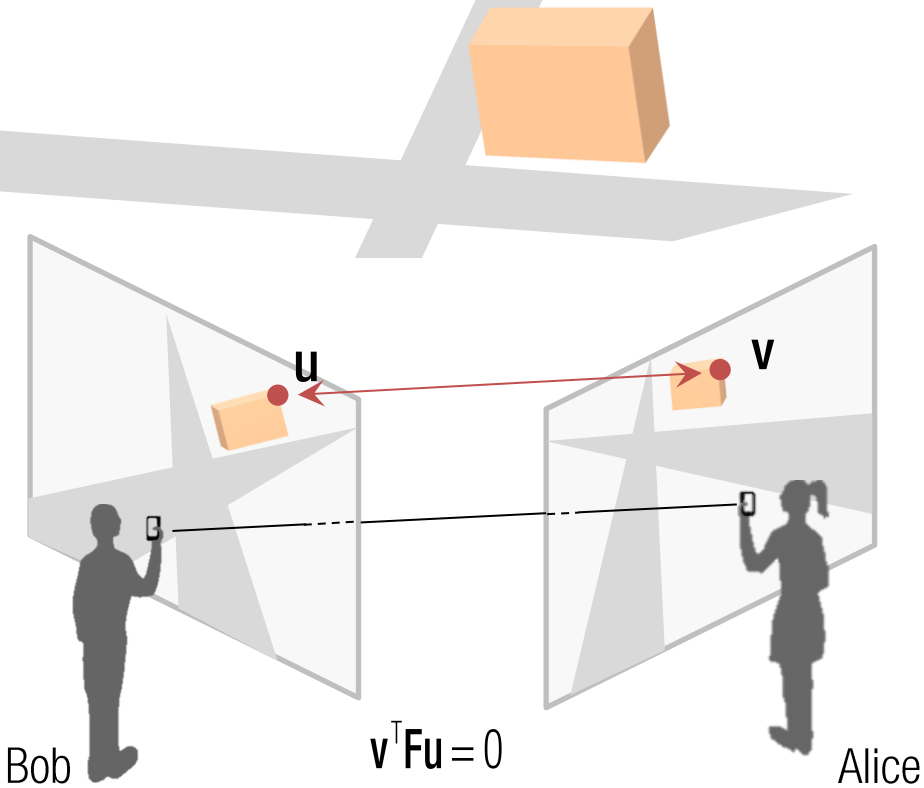
Laboratory of Experimental Psychology, University of Sussex,
Brighton BN1 9QG, UK

A simple algorithm for computing the three-dimensional structure of a scene from a correlated pair of perspective projections is described here, when the spatial relationship between the two projections is unknown. This problem is relevant not only to photographic surveying¹ but also to binocular vision², where the non-visual information available to the observer about the scene is limited to the two retinal images.

FUNDAMENTAL MATRIX ESTIMATION



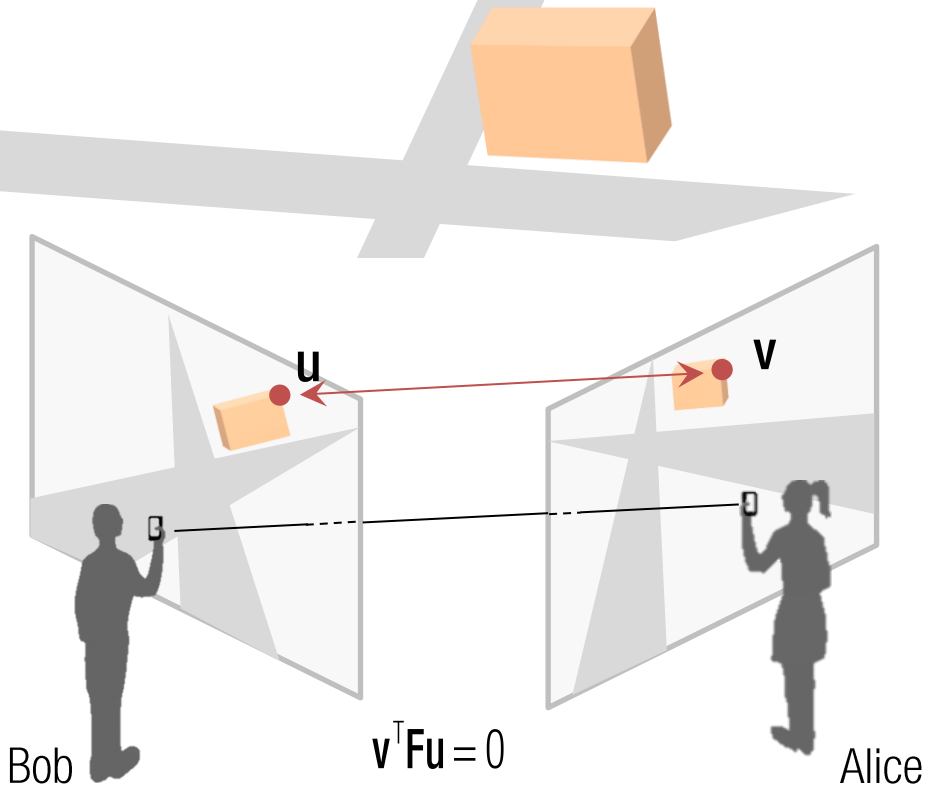
FUNDAMENTAL MATRIX ESTIMATION



$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Degree of freedom of fundamental matrix:

FUNDAMENTAL MATRIX ESTIMATION

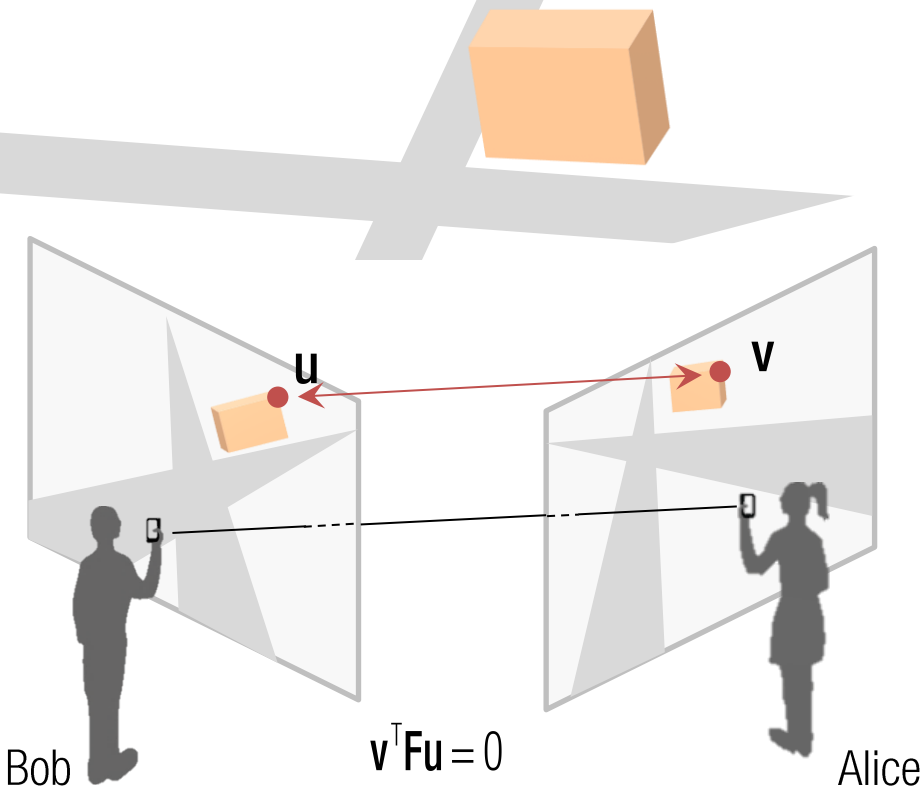


$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Degree of freedom of fundamental matrix:
 $7 = 9$ (3x3 matrix) $- 1$ (scale) $- 1$ (rank 2)

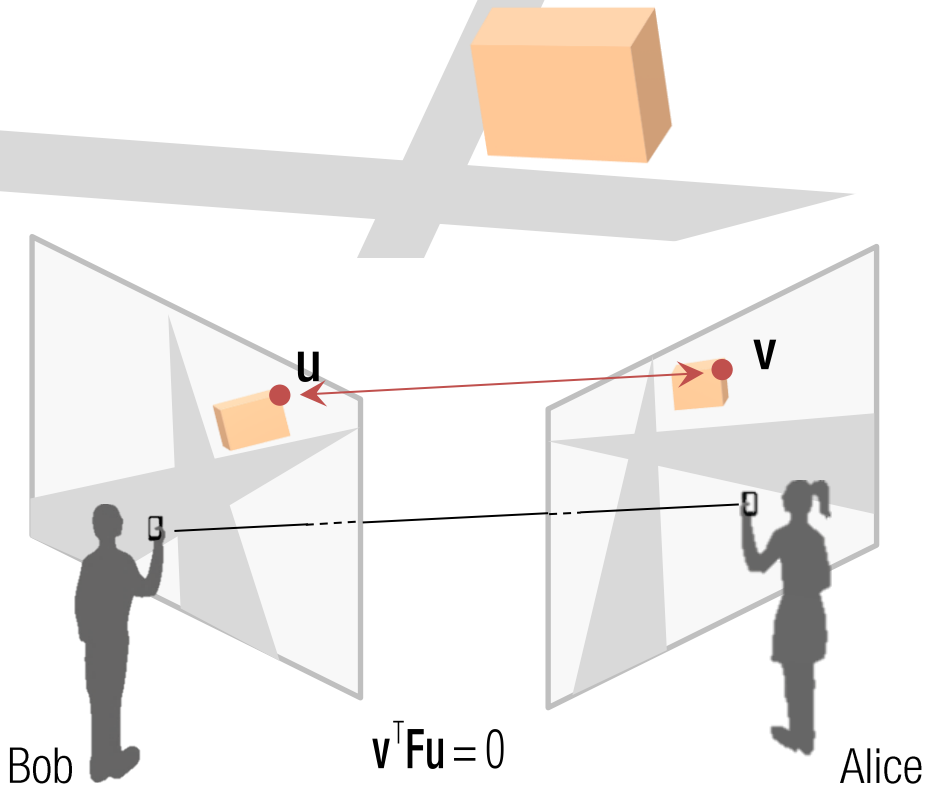
We will estimate fundamental matrix with 8 parameter by ignoring rank constraint and then project onto rank 2 matrix:

FUNDAMENTAL MATRIX ESTIMATION



$$v^T F u = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

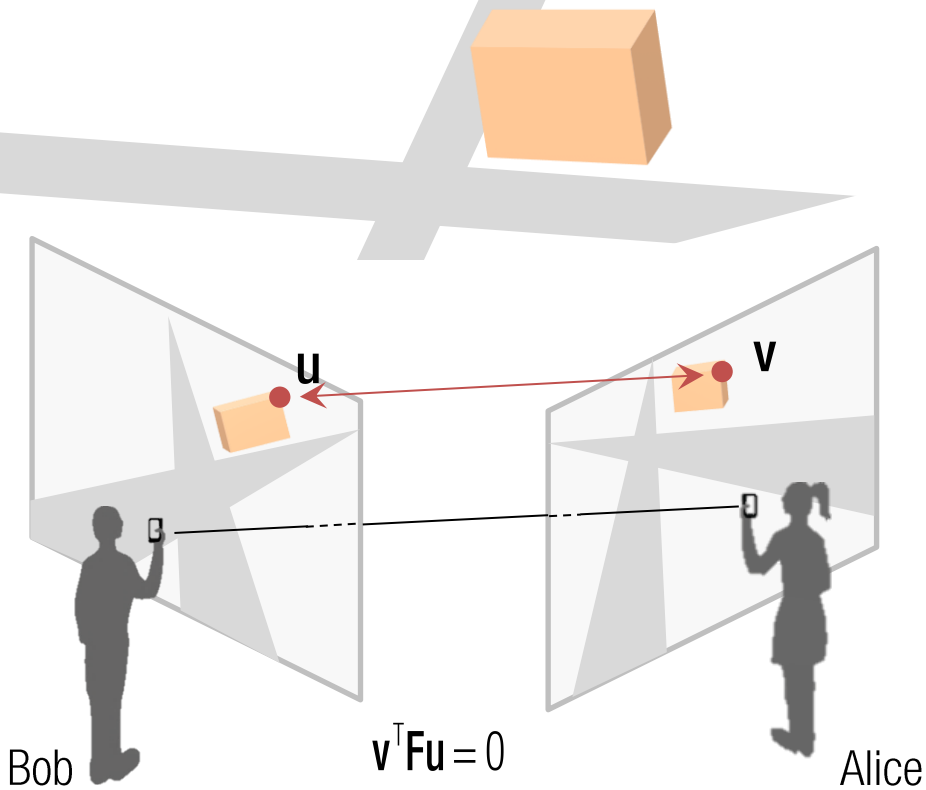
FUNDAMENTAL MATRIX ESTIMATION



$$v^T F u = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

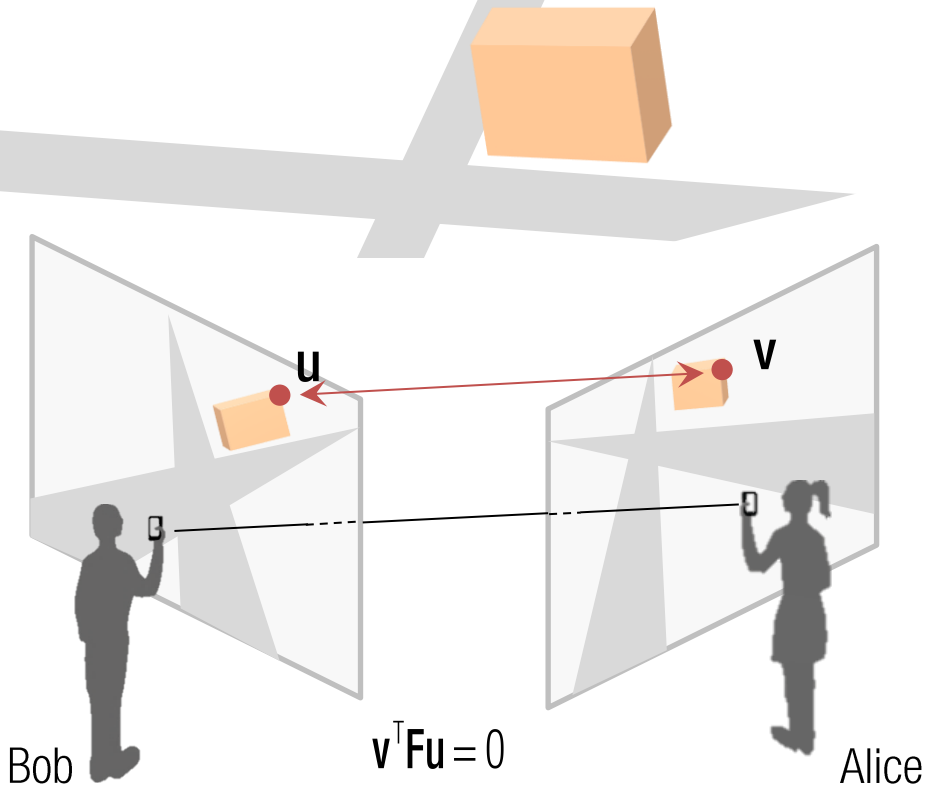
FUNDAMENTAL MATRIX ESTIMATION



$$\mathbf{v}^T \mathbf{F} \mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= \frac{f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}}{= 0} \quad \text{Linear in } \mathbf{F}.$$

FUNDAMENTAL MATRIX ESTIMATION



$$\mathbf{v}^T \mathbf{F} \mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

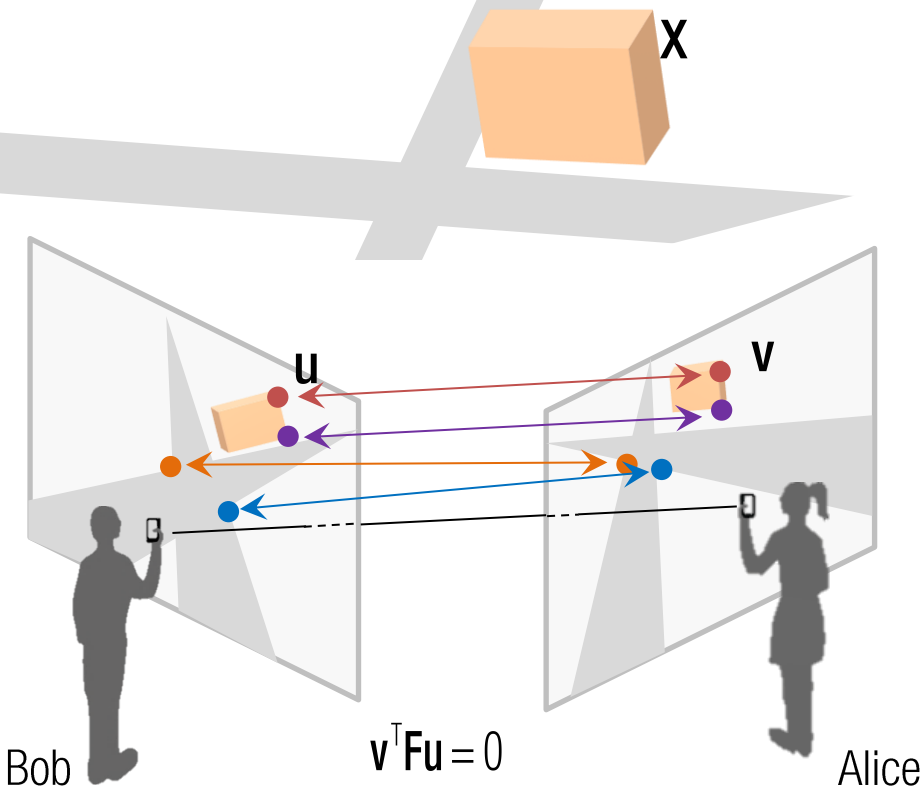
$$= \frac{f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}}{= 0}$$

Linear in \mathbf{F} .

$$\rightarrow \begin{bmatrix} u^xv^x & u^yv^x & v^x & u^xv^y & u^yv^y & v^y & u^x & u^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

of unknowns: 9
of equations per correspondence: 1

FUNDAMENTAL MATRIX ESTIMATION



$$v^T F u = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

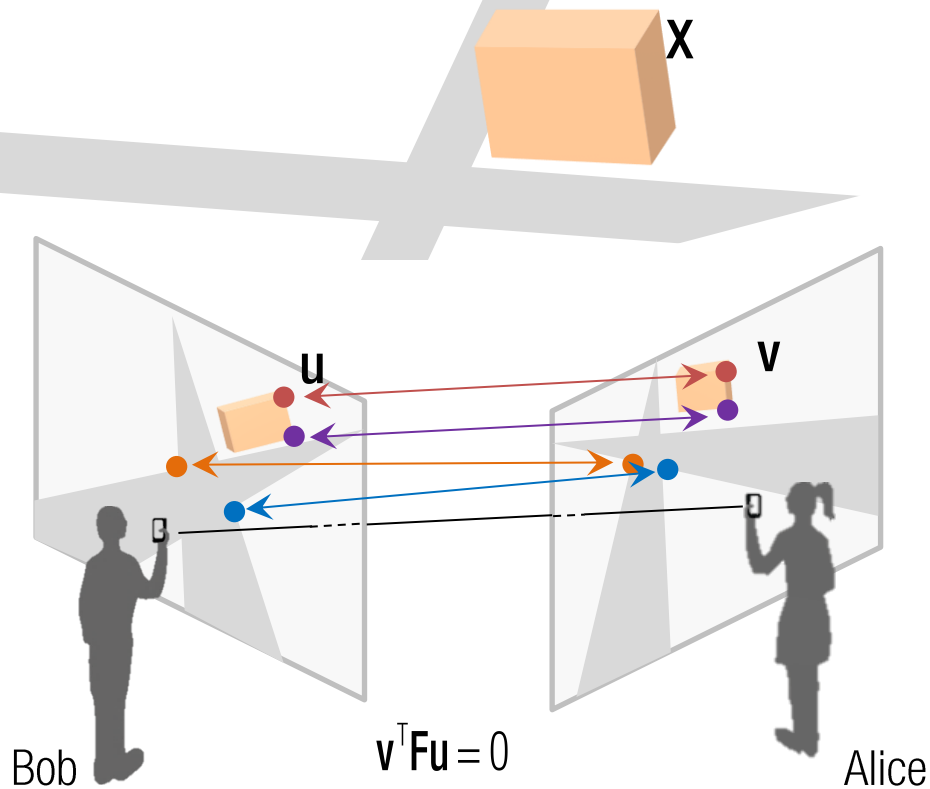
$$= \frac{f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}}{= 0}$$

Linear in F .

$$\begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \mathbf{0}_{m \times 1}$$

What is minimum m ?

FUNDAMENTAL MATRIX ESTIMATION



$$v^T F u = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

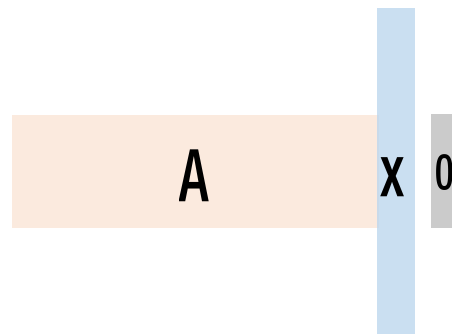
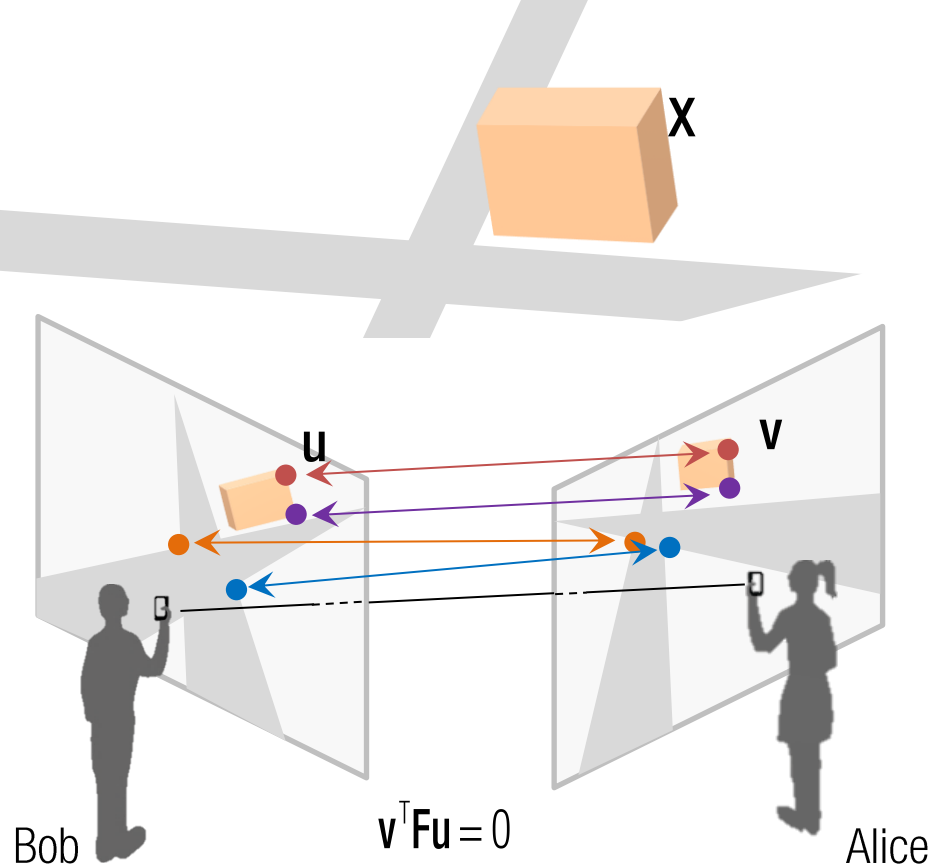
$$= \frac{f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}}{= 0}$$

Linear in F .

$$\begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix} \mathbf{X} = \mathbf{0}$$

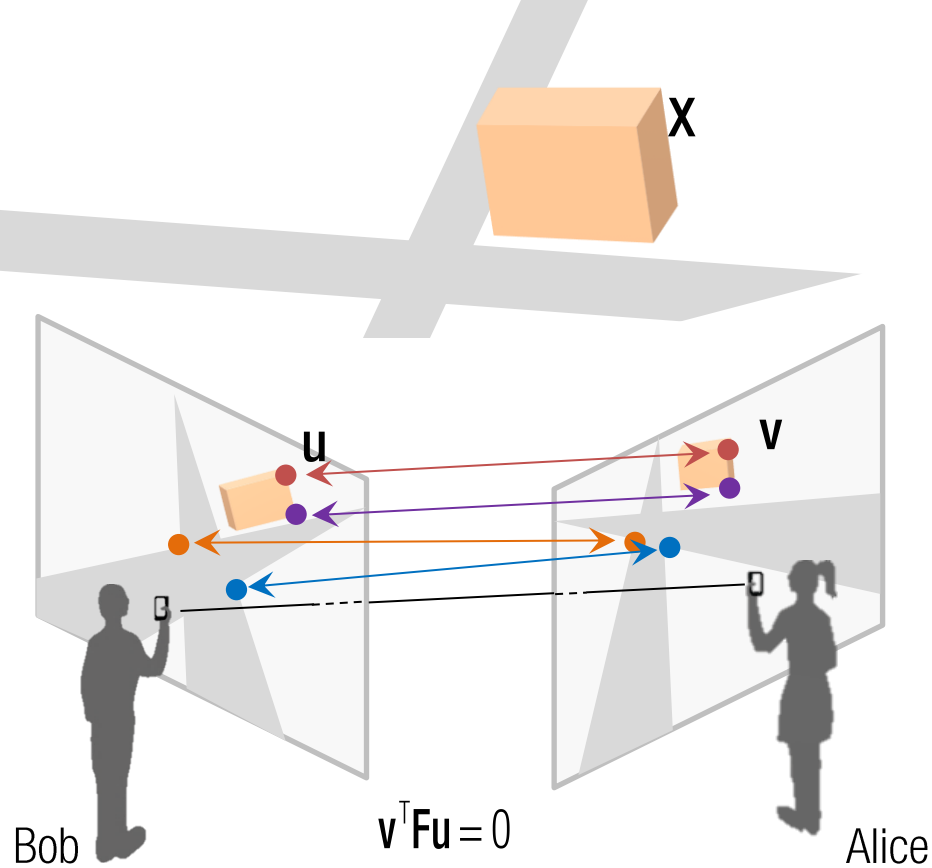
What is minimum m ?

FUNDAMENTAL MATRIX ESTIMATION



The solution is not necessarily satisfy rank 2 constraint.

FUNDAMENTAL MATRIX ESTIMATION

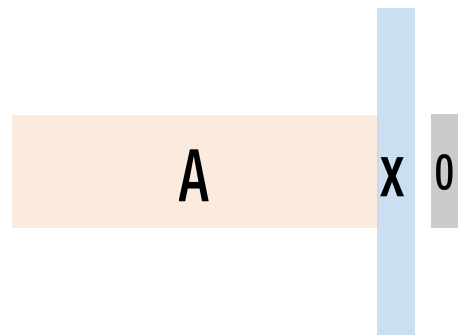
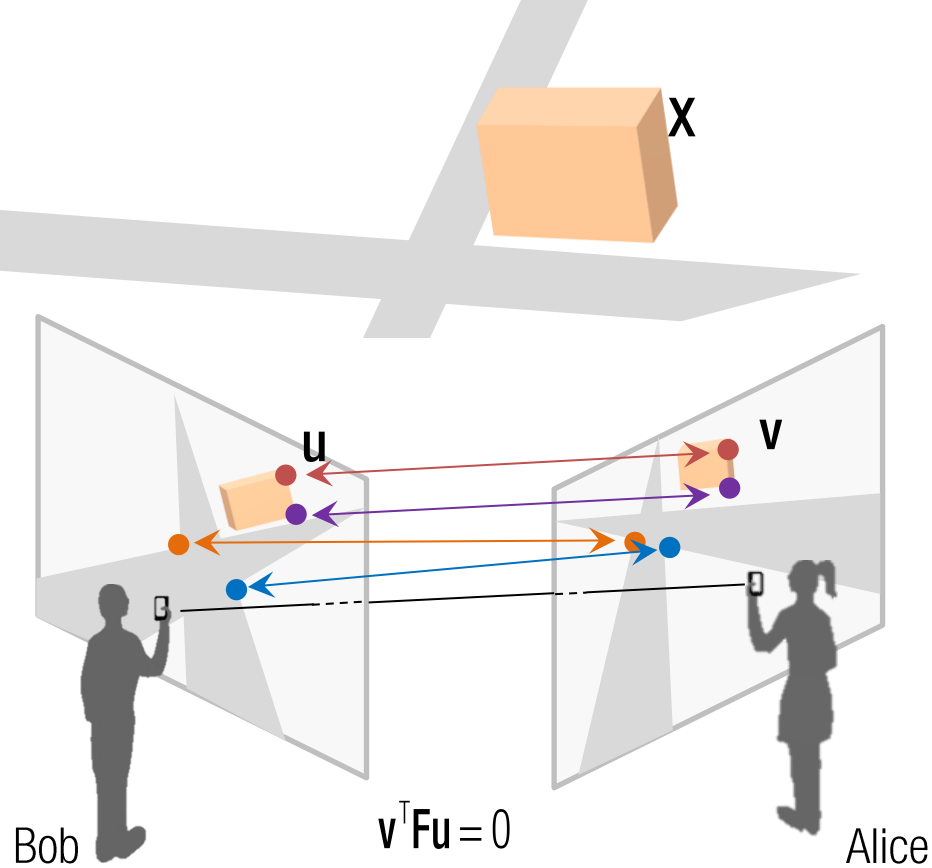


$$A \quad X \quad 0$$

The solution is not necessarily satisfy rank 2.

$$\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} = \begin{bmatrix} \text{U} \end{bmatrix} \begin{bmatrix} \text{D} \end{bmatrix} \begin{bmatrix} \text{V}^T \end{bmatrix}$$

FUNDAMENTAL MATRIX ESTIMATION



The solution is not necessarily satisfy rank.

$$\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} = \begin{bmatrix} \text{U} & \text{D} & \text{V}^T \end{bmatrix}$$

$$\approx_{F_{\text{rank}2}} \begin{bmatrix} \text{U} & \tilde{\text{D}} & \text{V}^T \end{bmatrix}$$

SVD cleanup

CAMERA POSE FROM F

