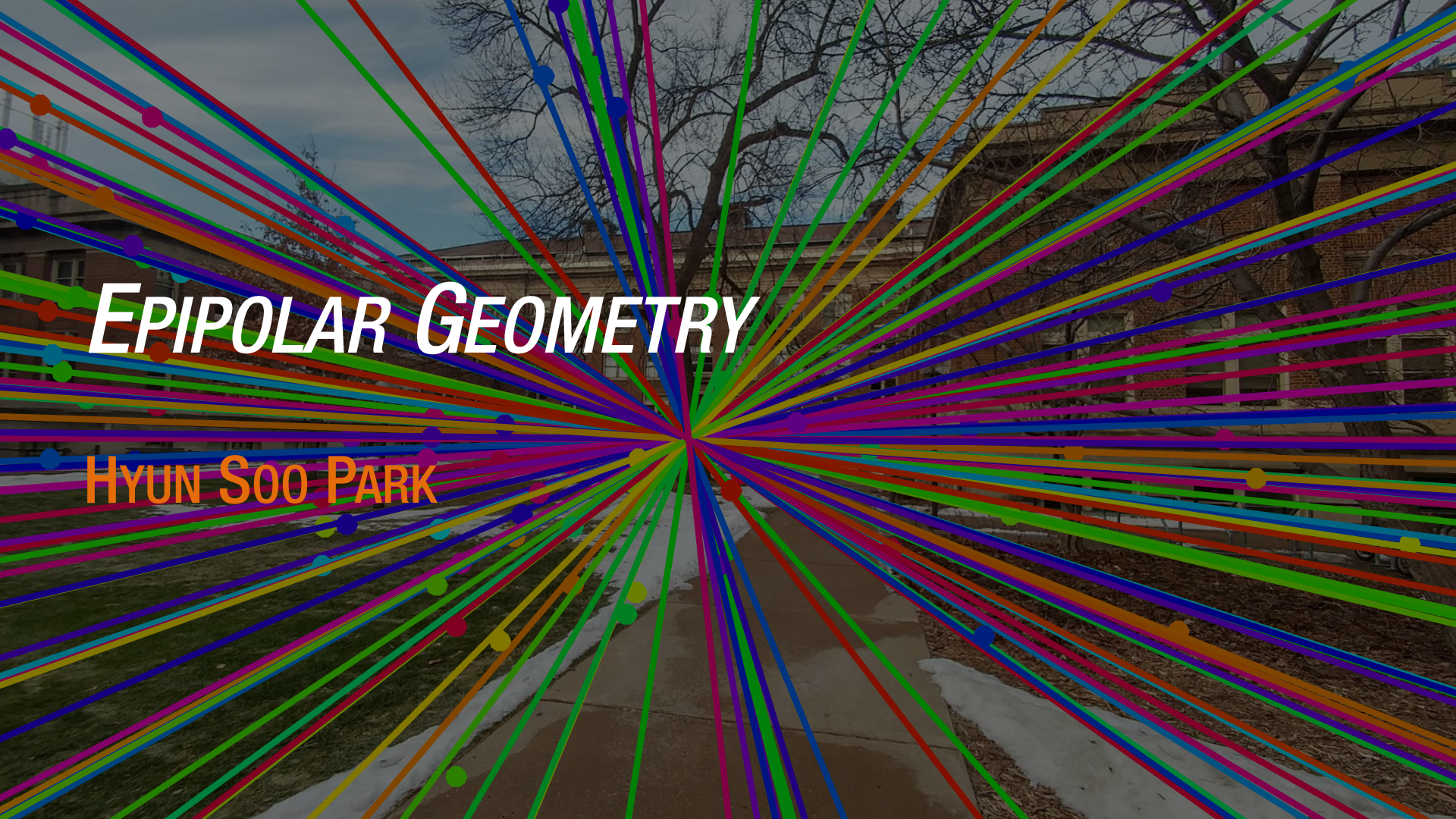
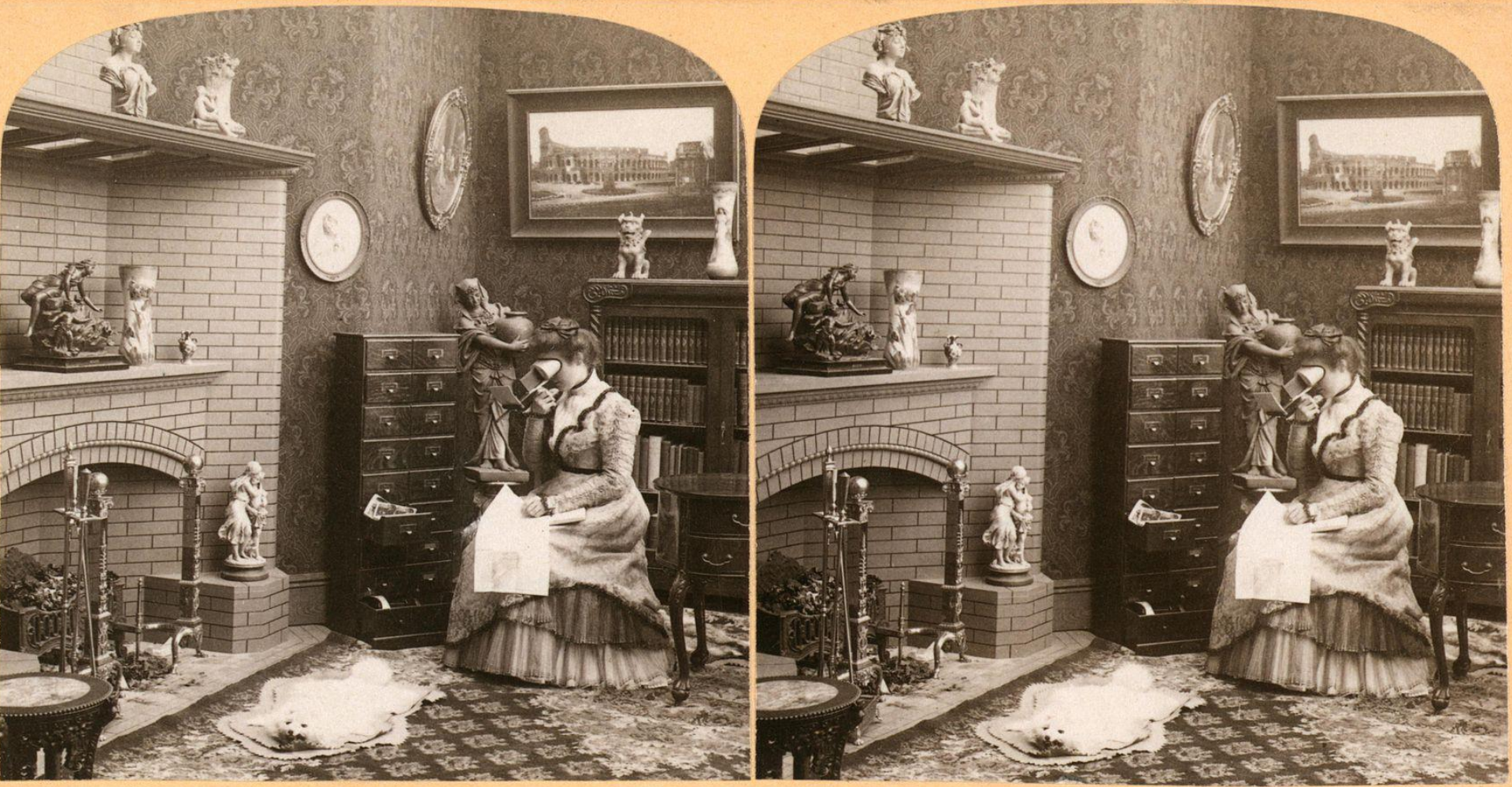


Epipolar Geometry

Hyun Soo Park



The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.

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The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.

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The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.

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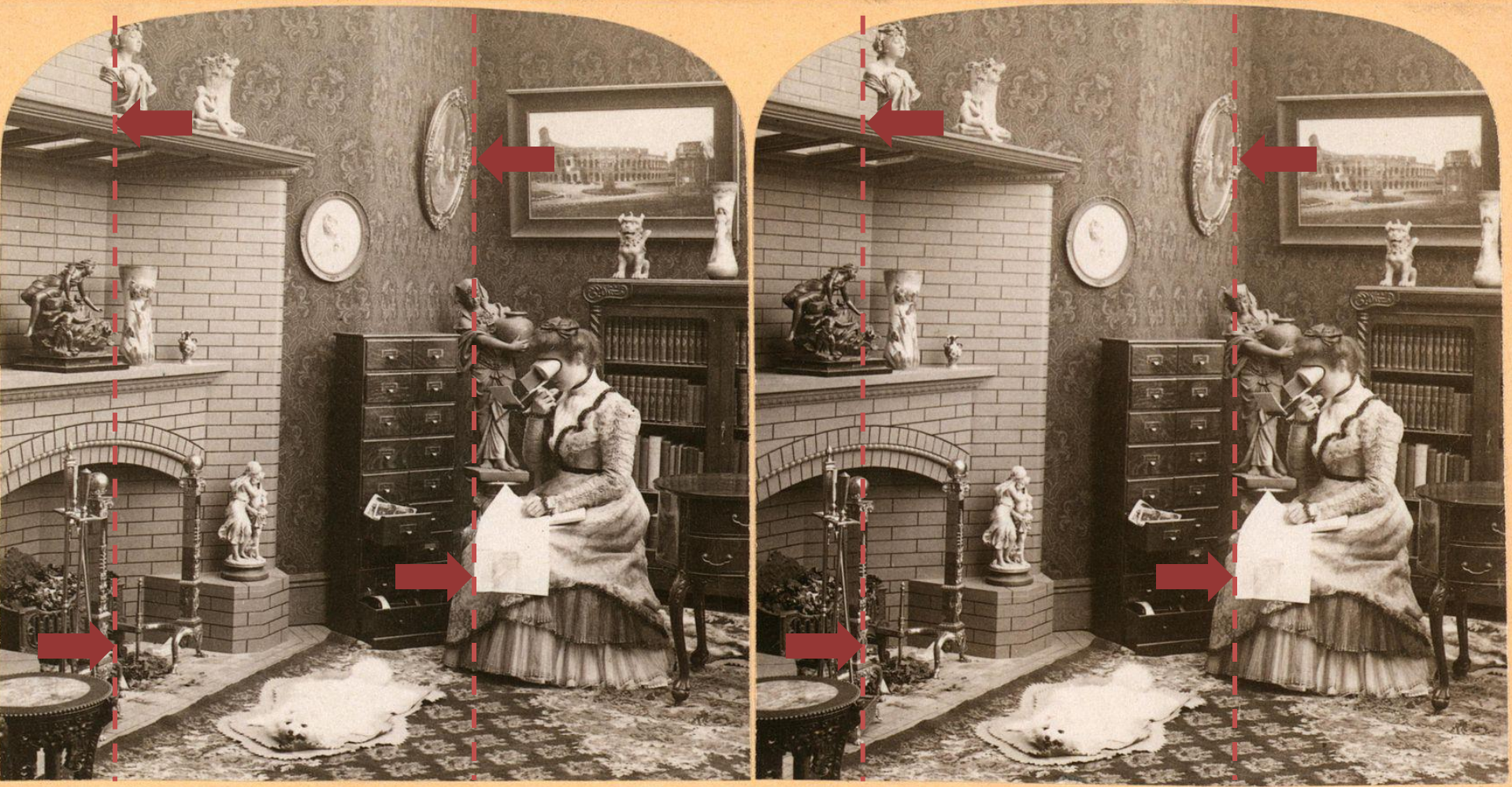

The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.


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Left image (Bob)

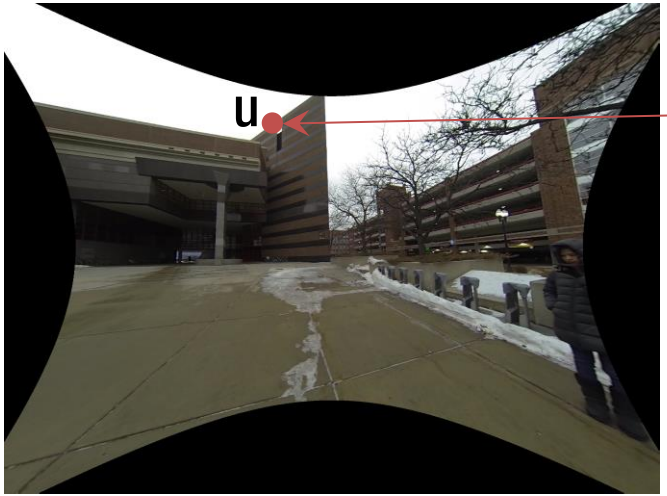
Right image (Alice)

2D Correspondence

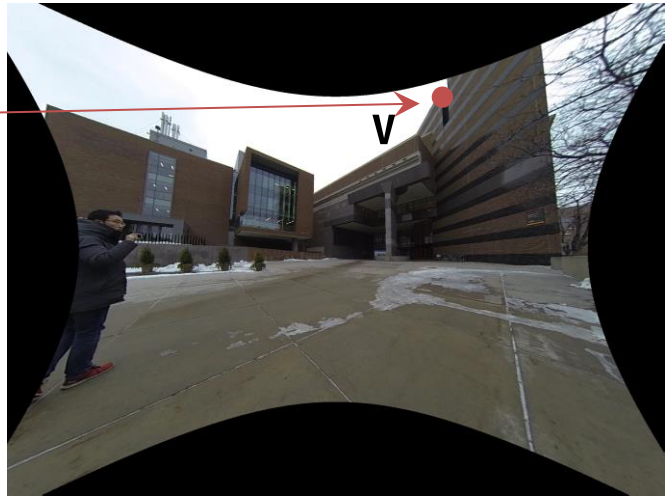
Left image (Bob)

Right image (Alice)

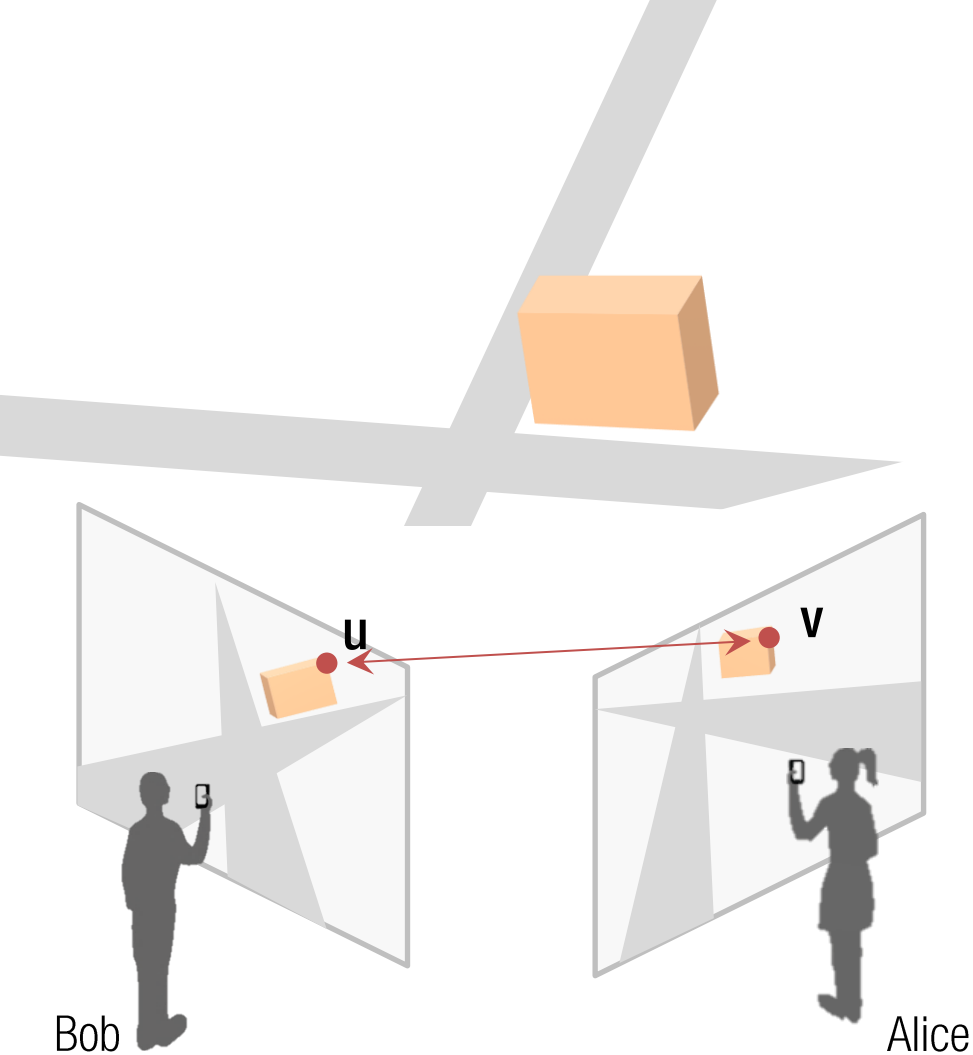
2D Correspondence



Left image (Bob)



Right image (Alice)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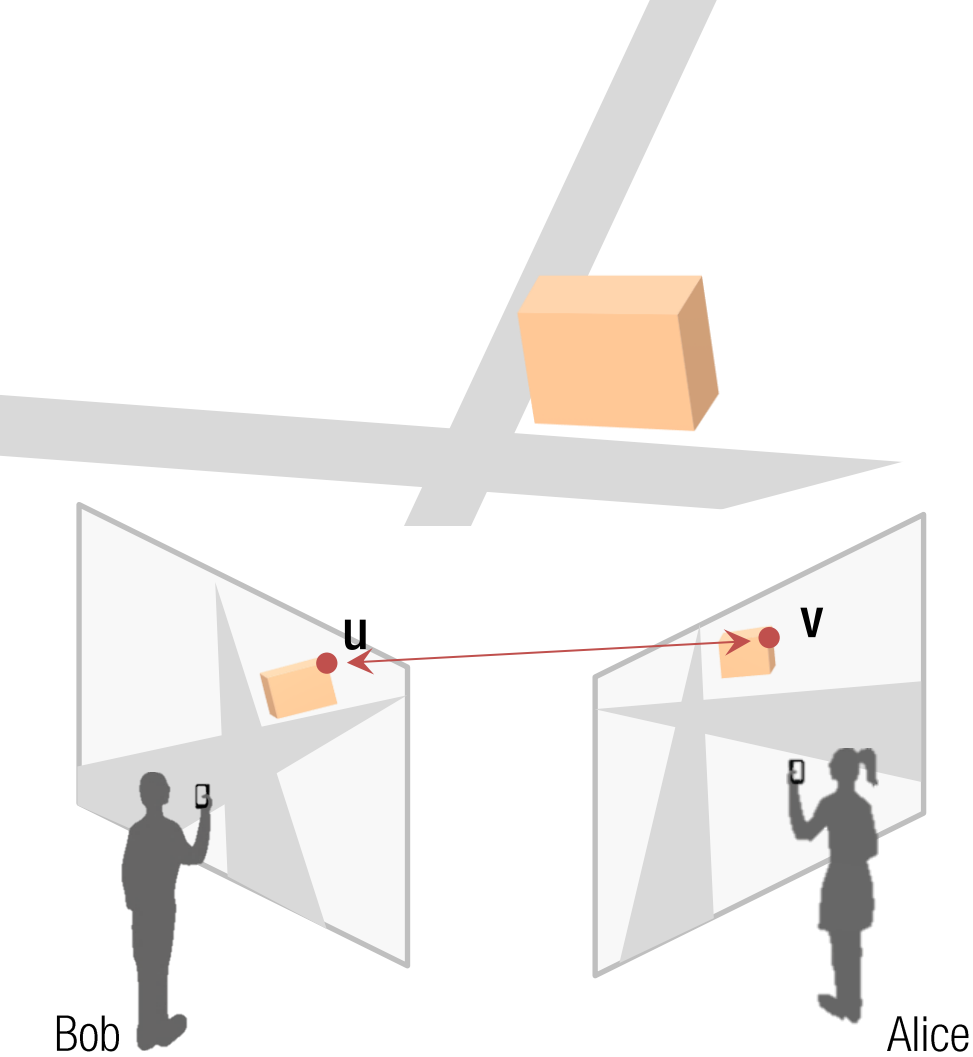
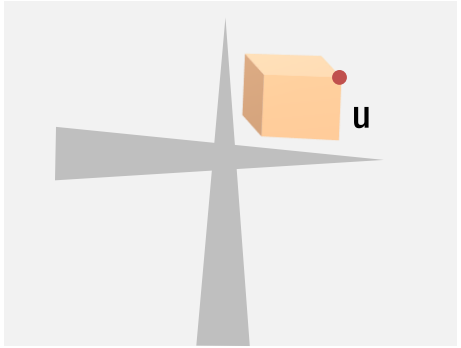
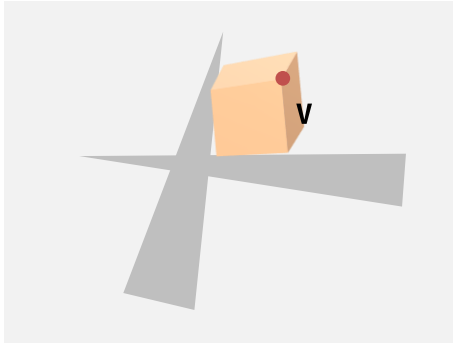
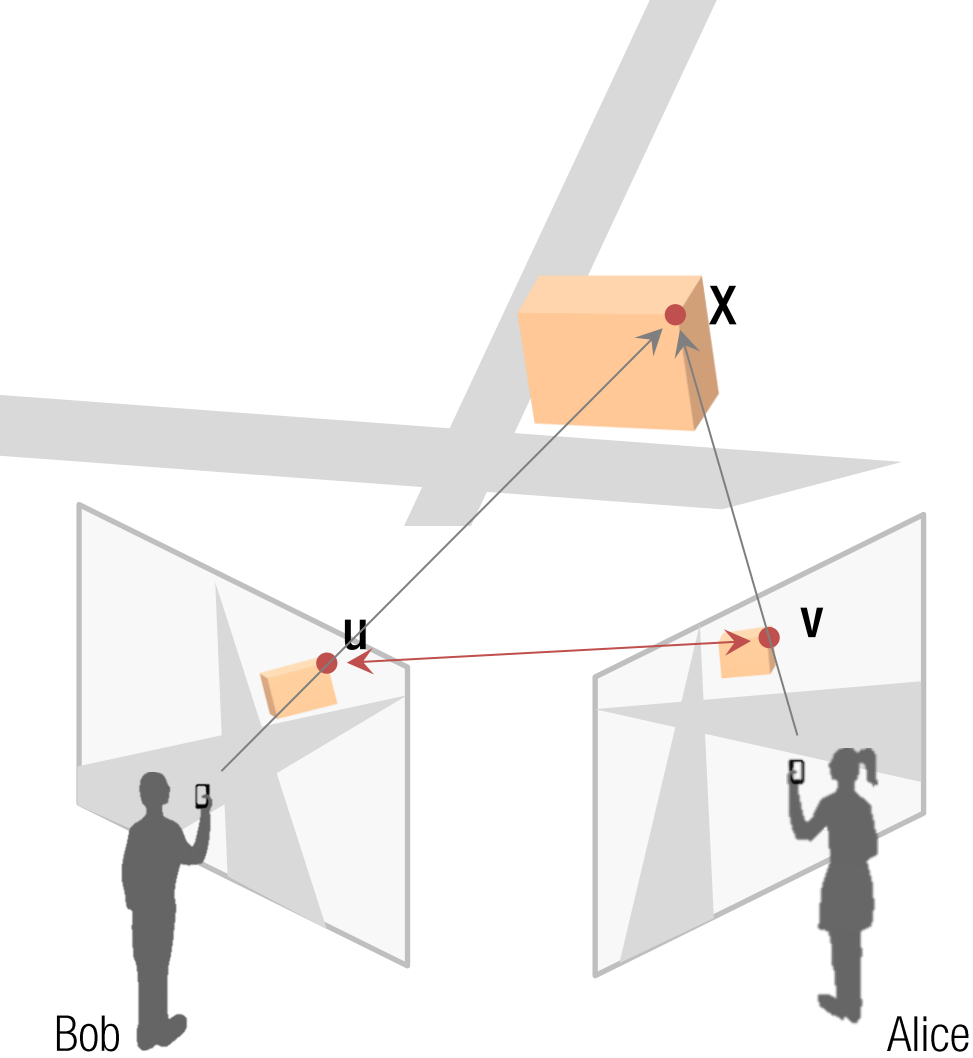


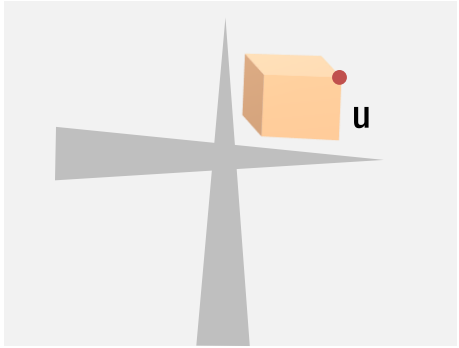


Bob's image

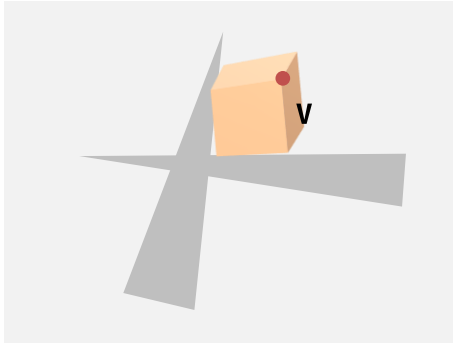


Alice's image

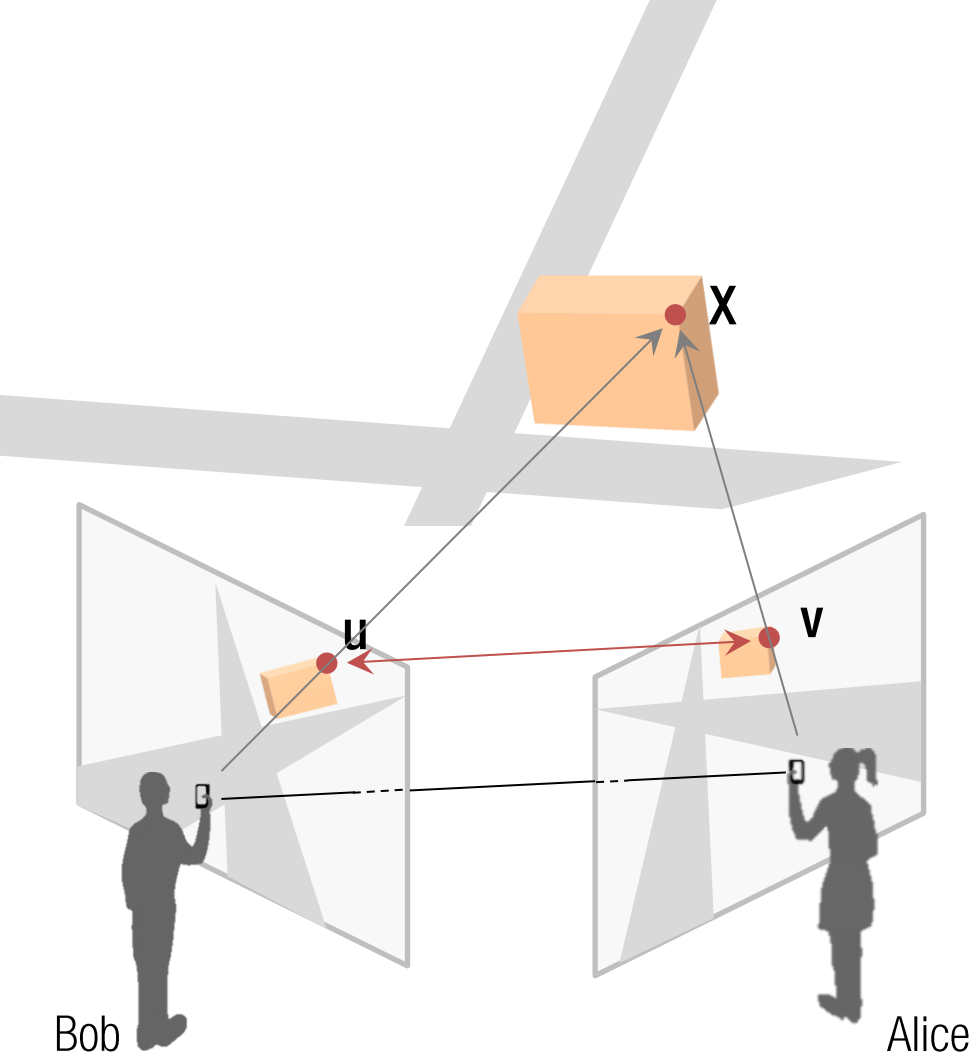


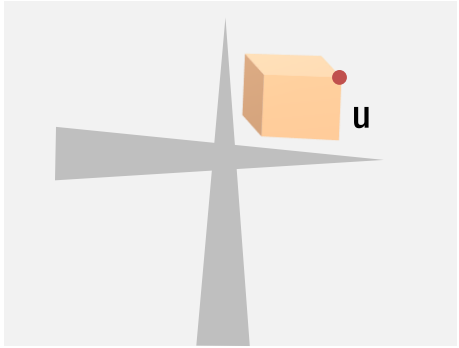


Bob's image



Alice's image





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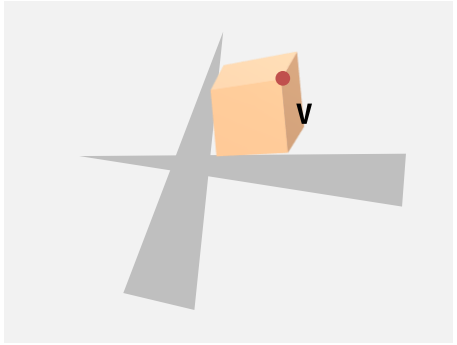

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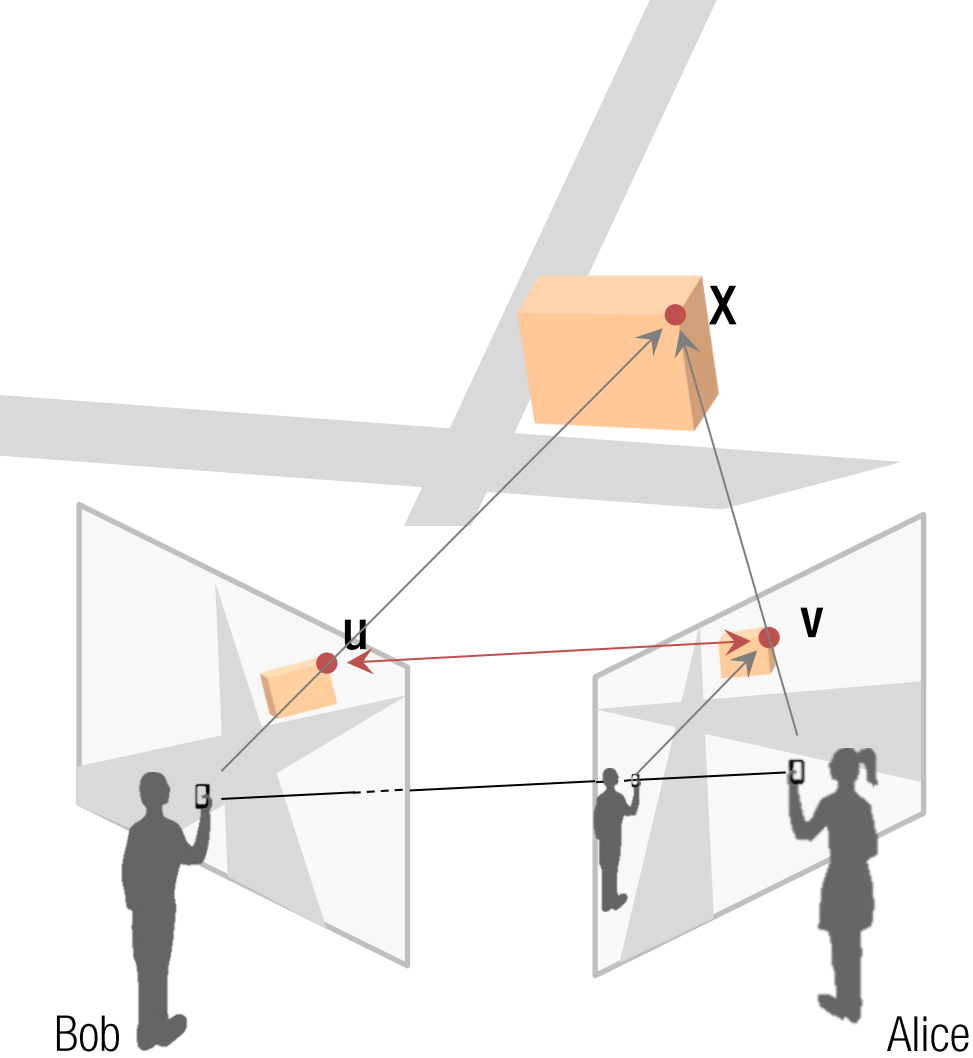
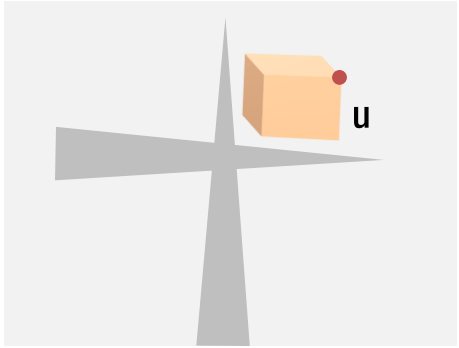
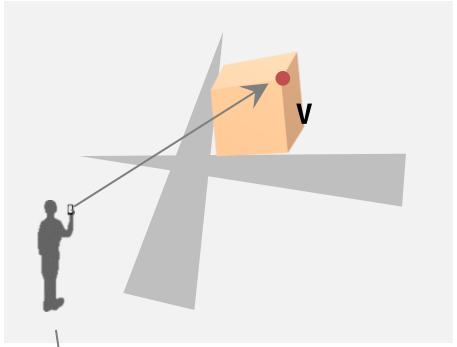


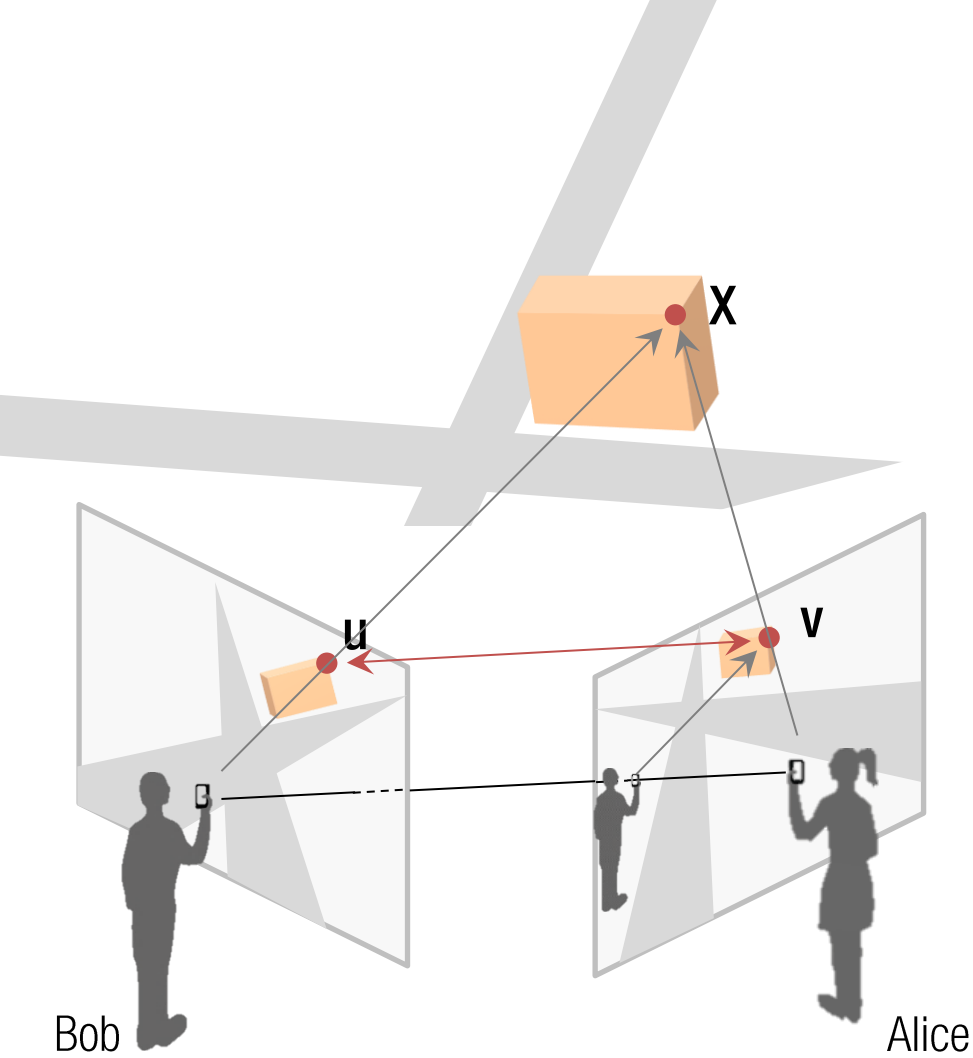


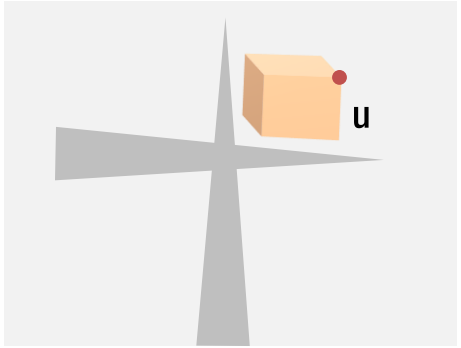
Bob's image



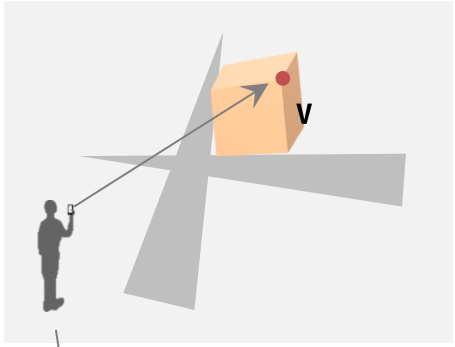
Alice's image

Bob from Alice's view



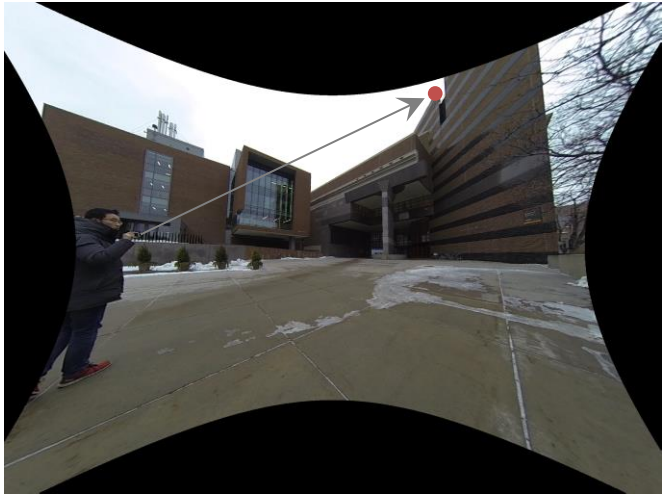


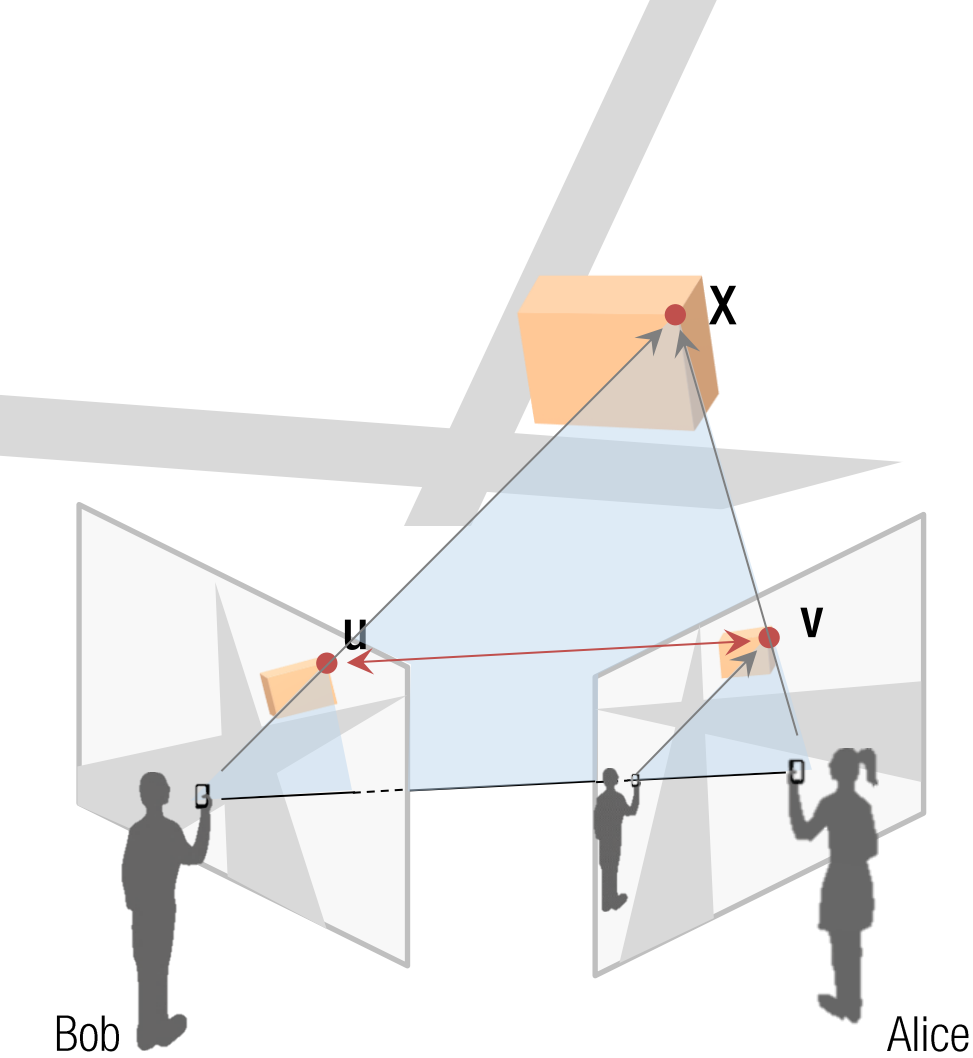
Bob's image

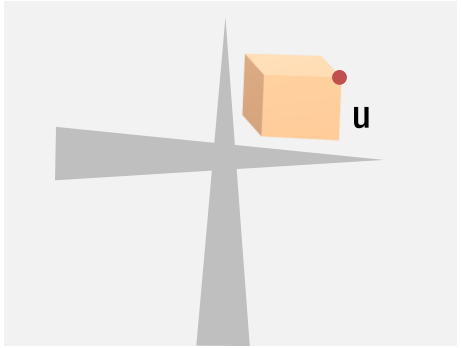


Alice's image

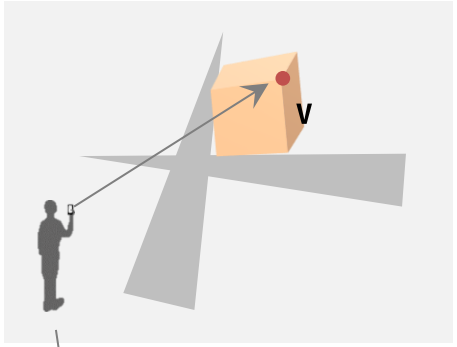
Bob from Alice's view





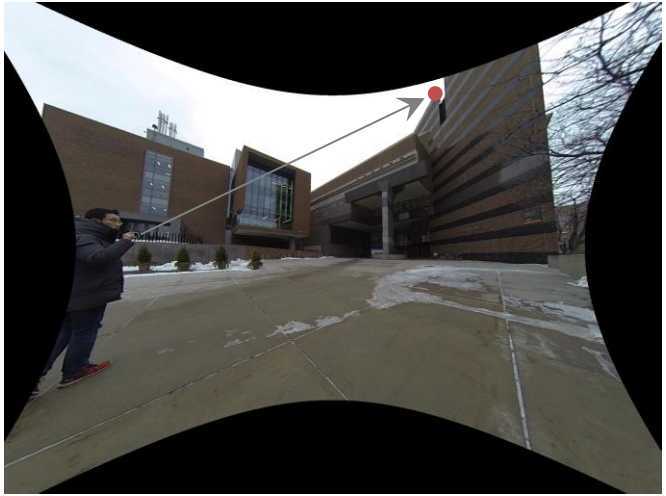


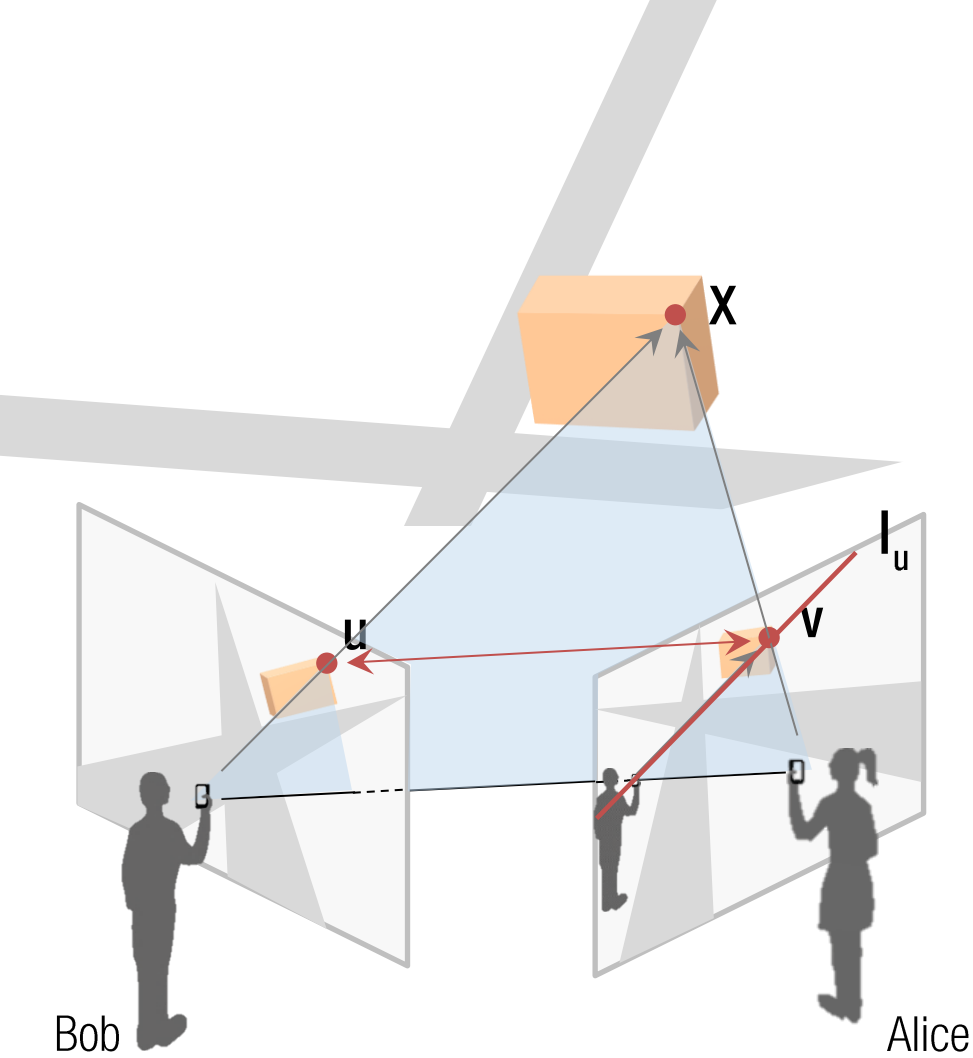
Bob's image

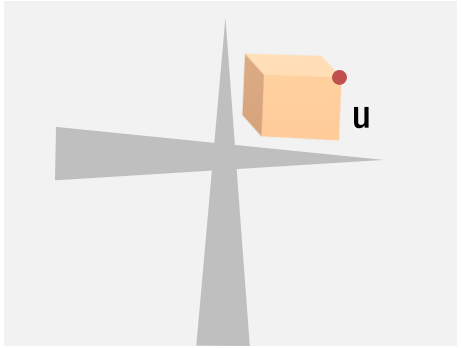


Alice's image

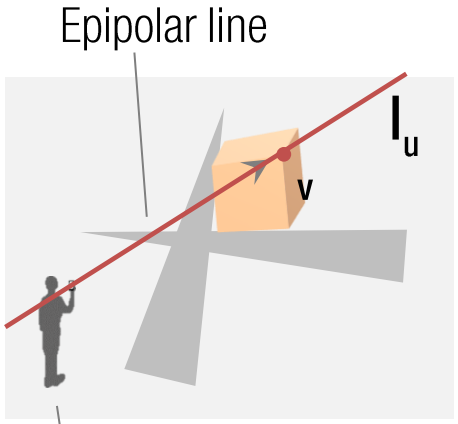
Bob from Alice's view





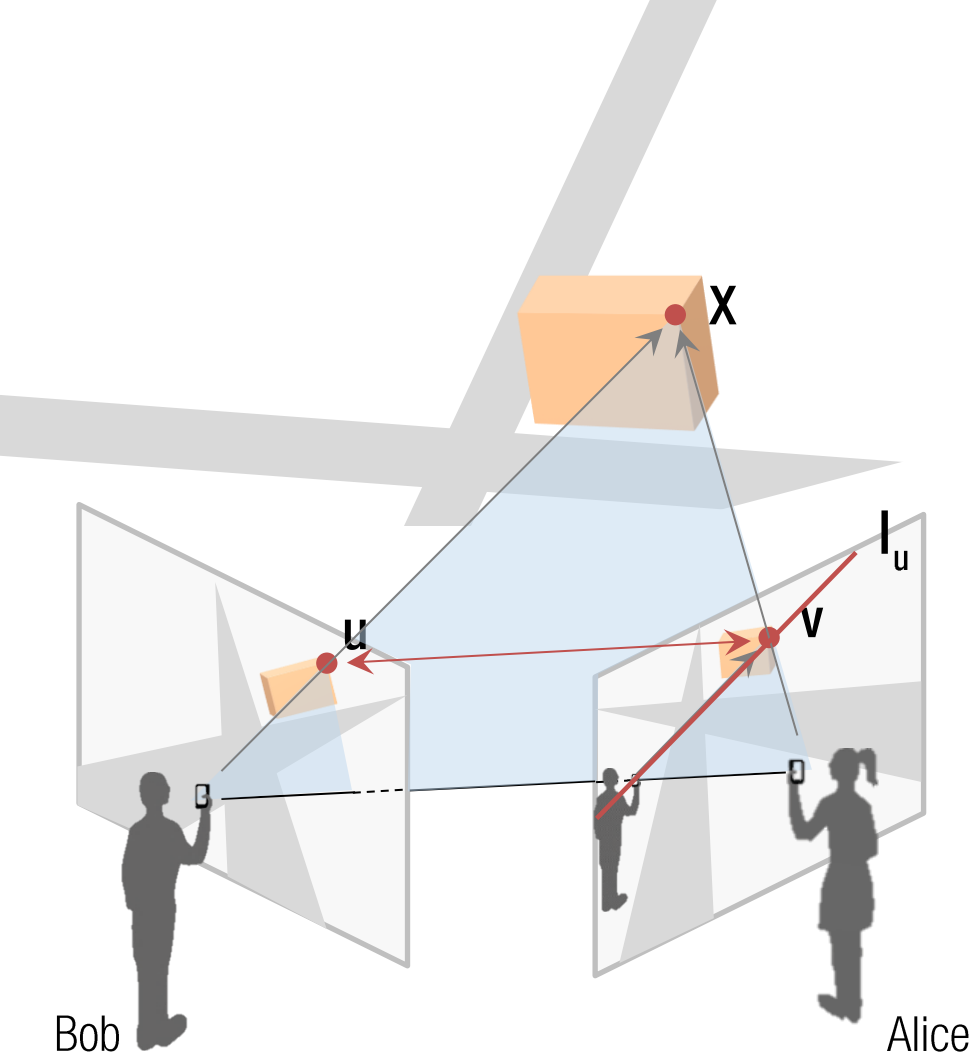


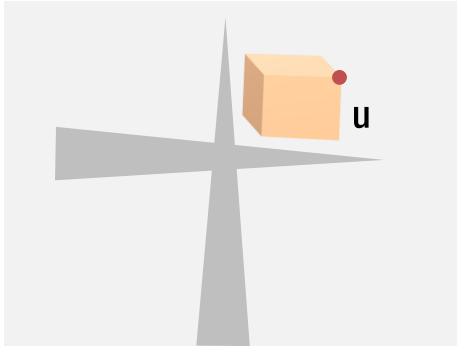
Bob's image



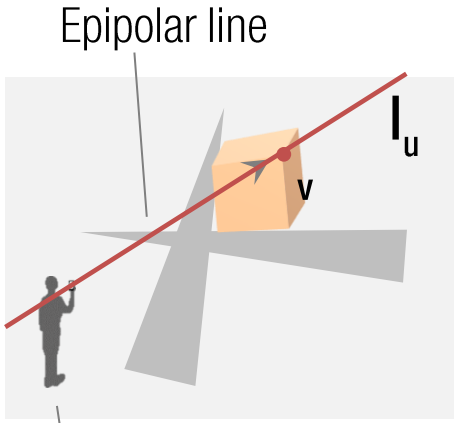
Alice's image

Bob from Alice's view





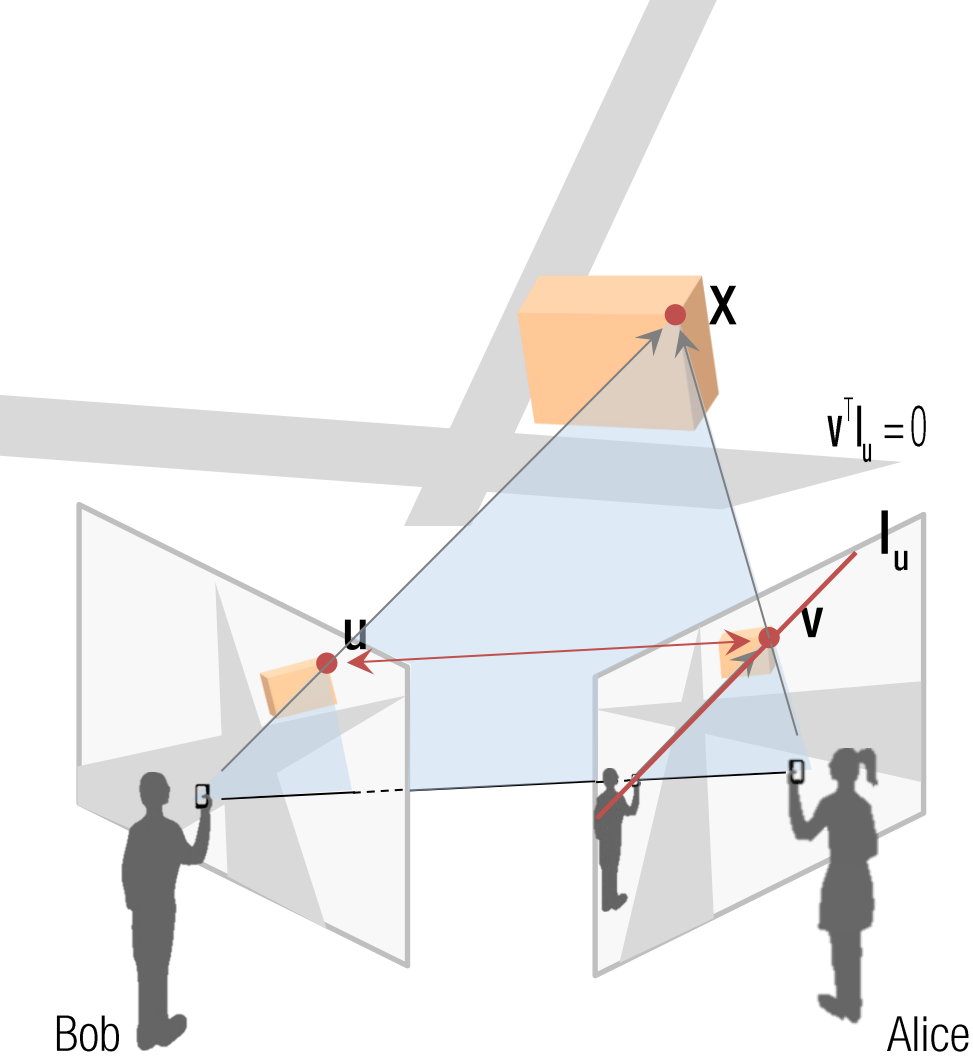
Bob's image

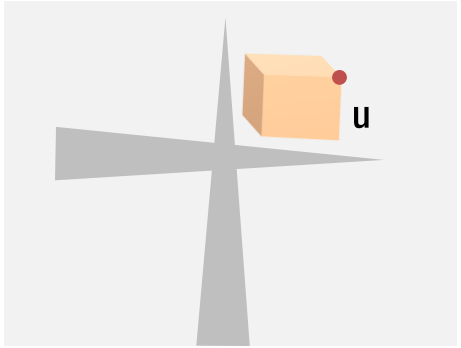


Alice's image
Bob from Alice's view

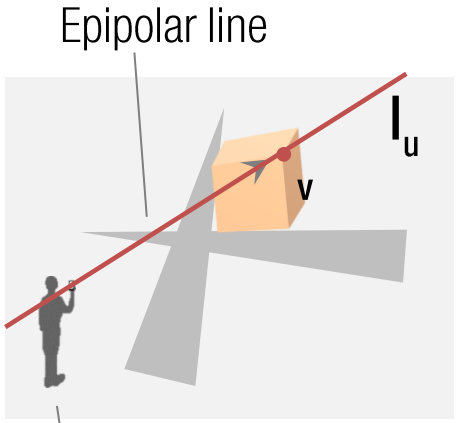
Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line l_u in Alice's image.





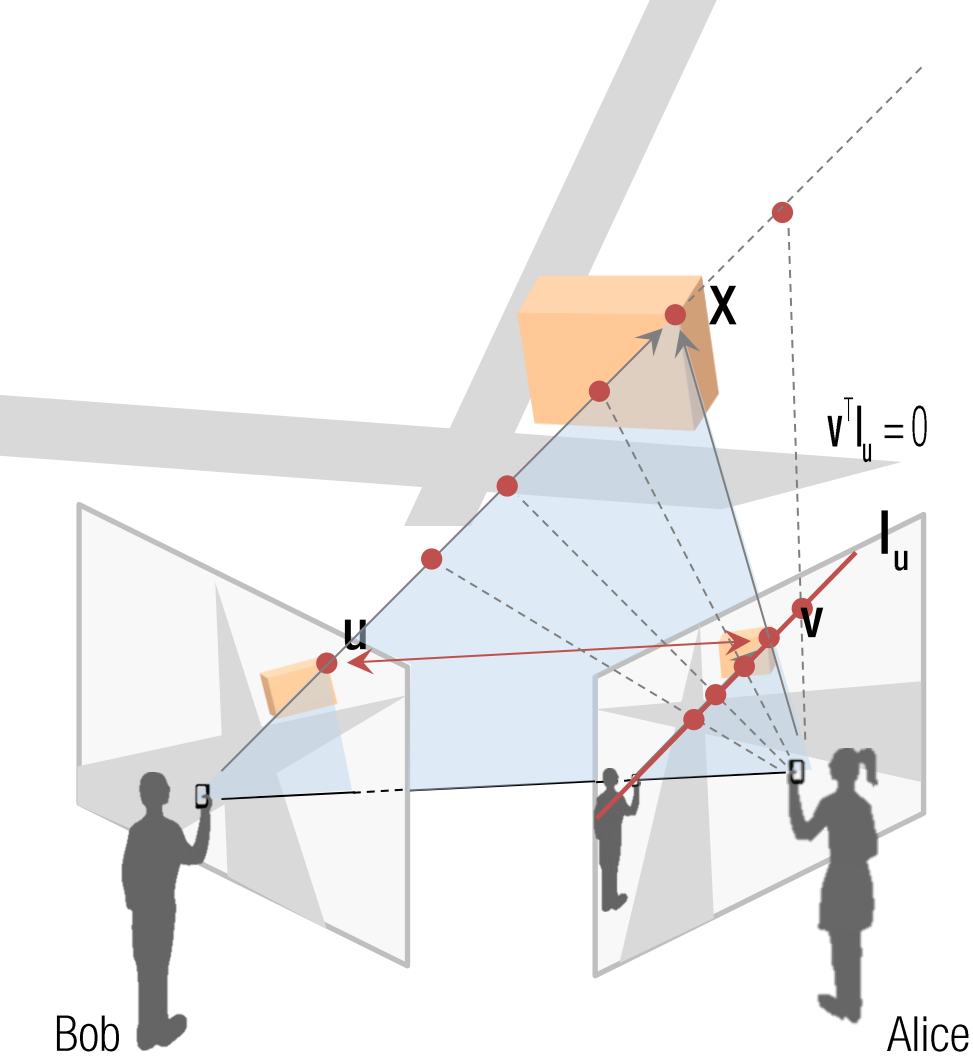
Bob's image

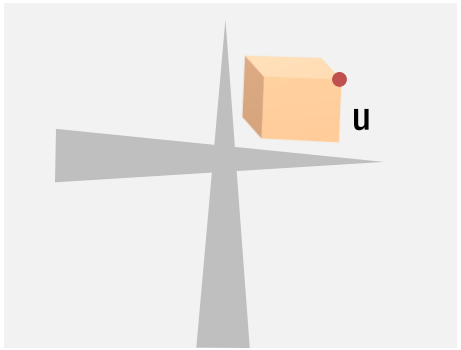


Alice's image
Bob from Alice's view

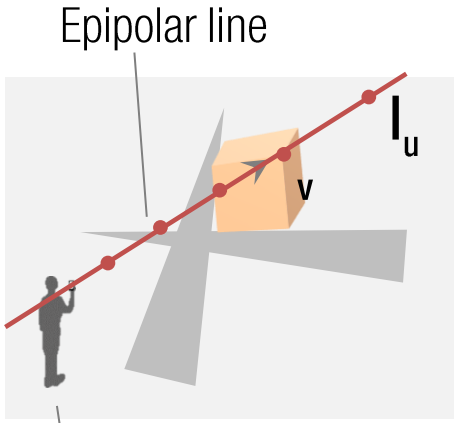
Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line l_u in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$





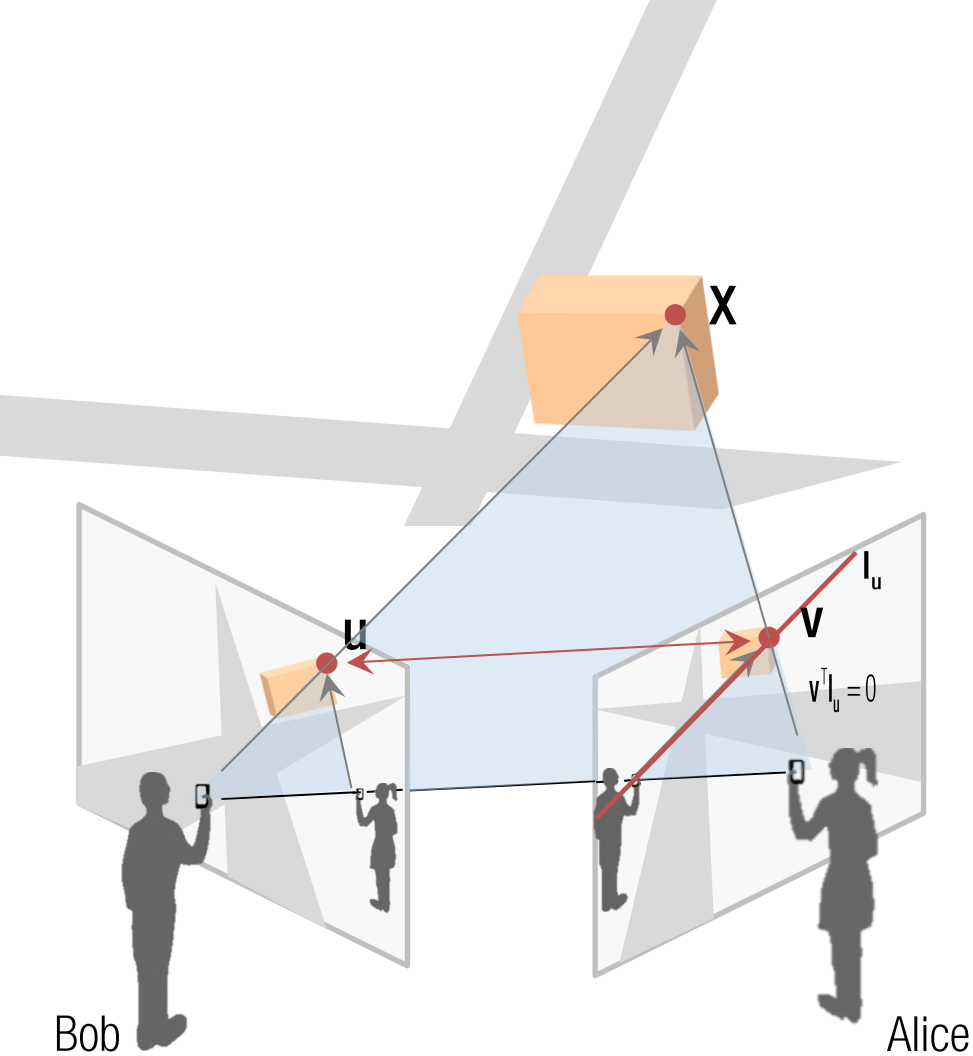
Bob's image



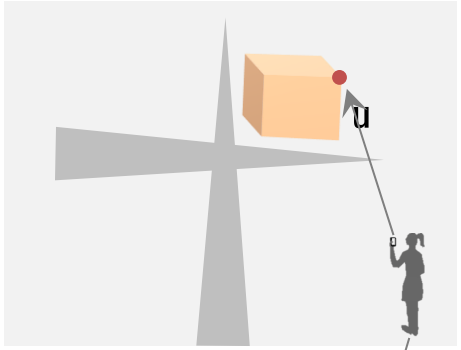
Alice's image
Bob from Alice's view

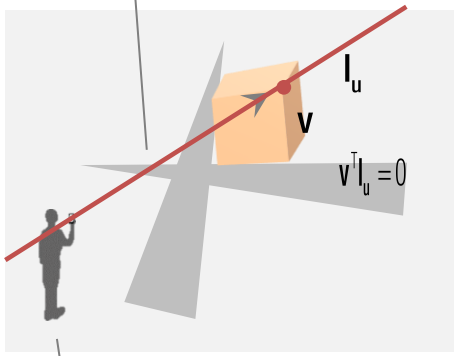
Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line l_u in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.



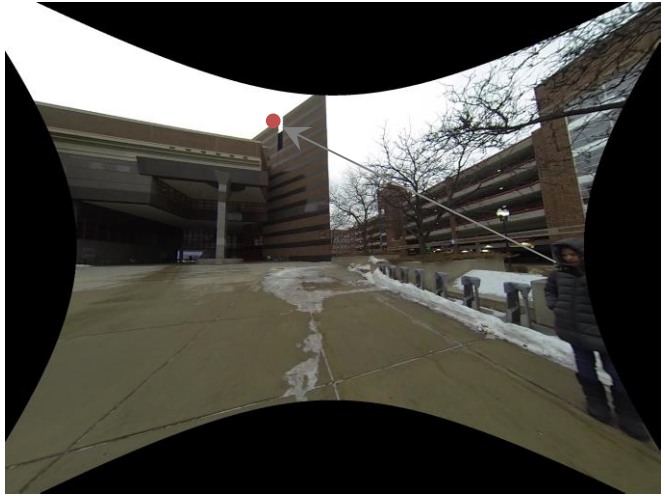
Epipolar line





Bob's image
Alice from Bob's view

Alice's image
Bob from Alice's view



Epipolar line

Bob

Alice

e's image
ce's view

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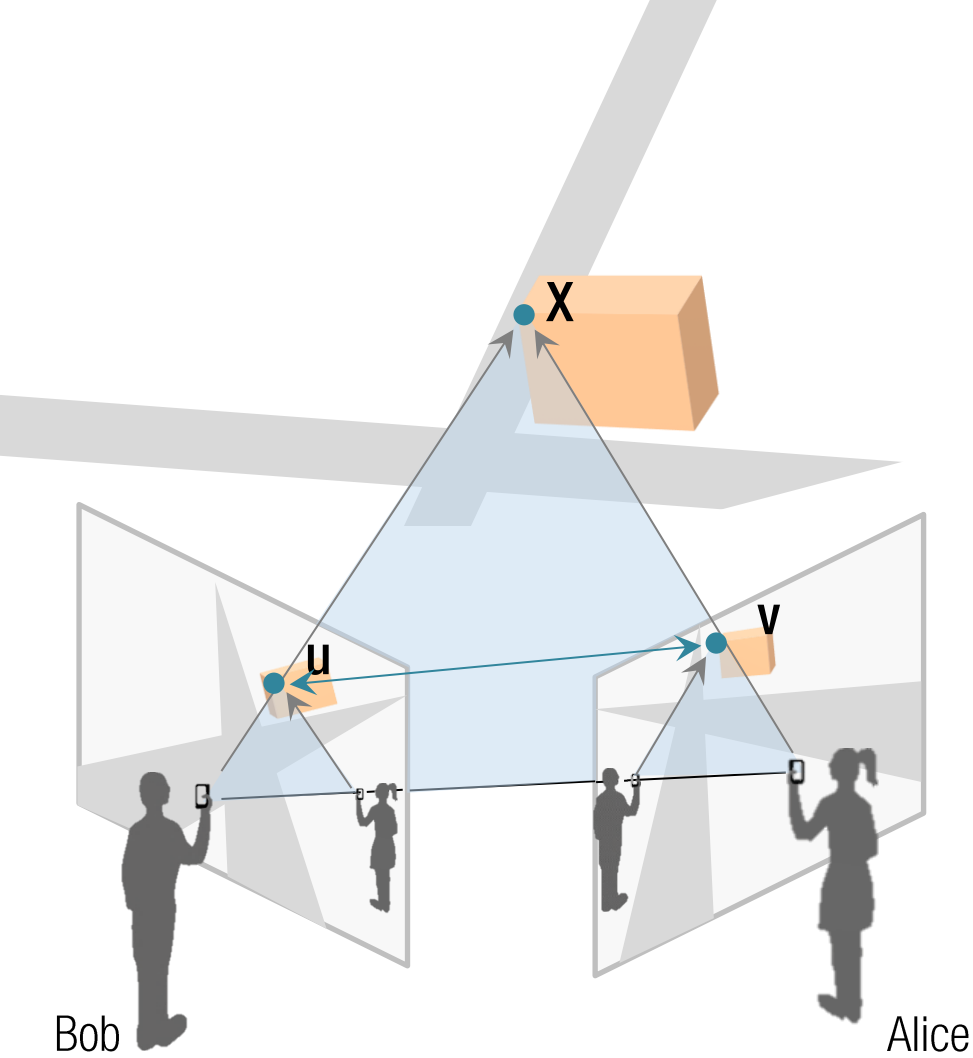
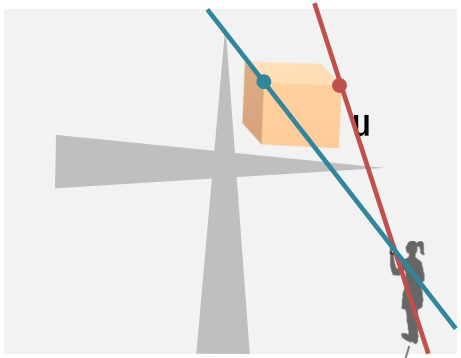
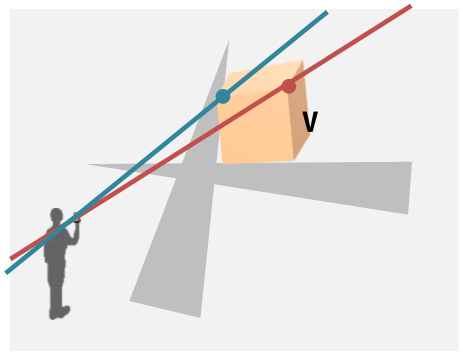
2. The epipolar line passes the corresponding point in Alice's image, **v**:
3. Any point along the epipolar line can be a candidate of correspondences.







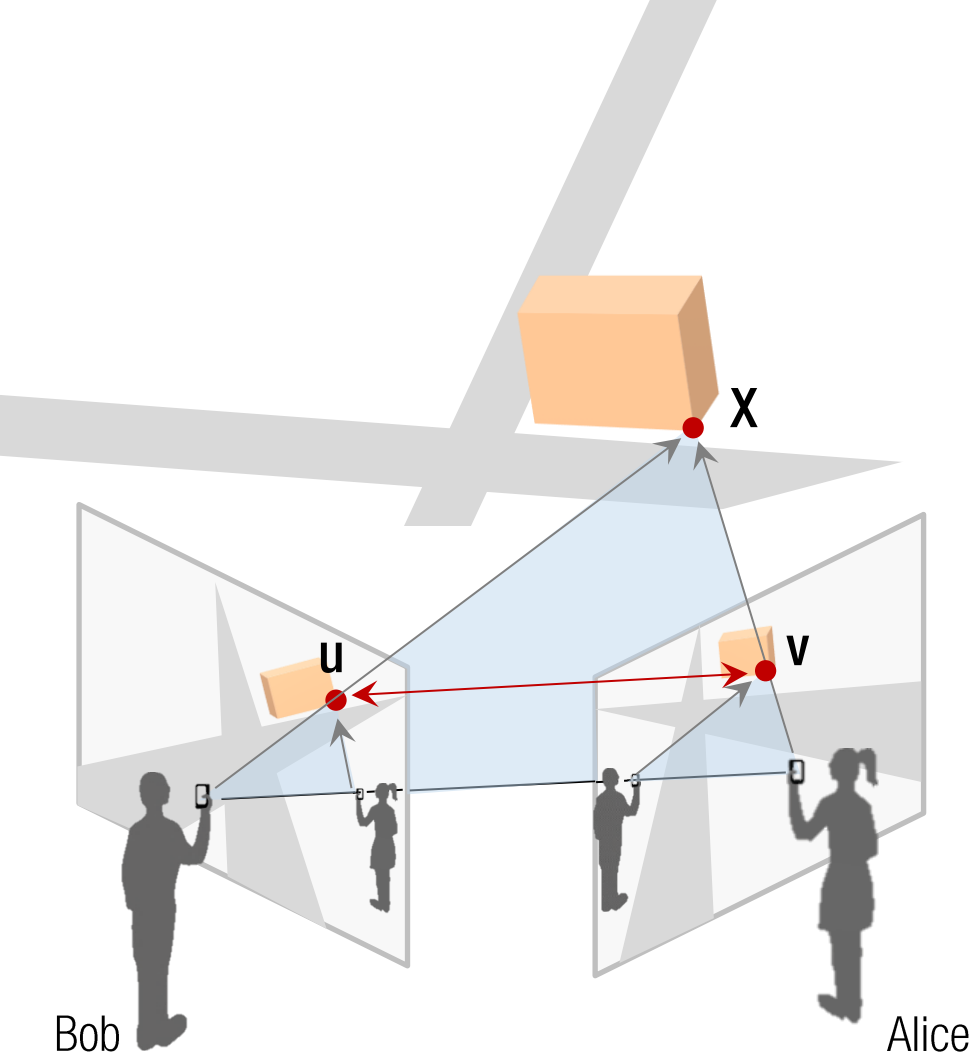
Bob's image

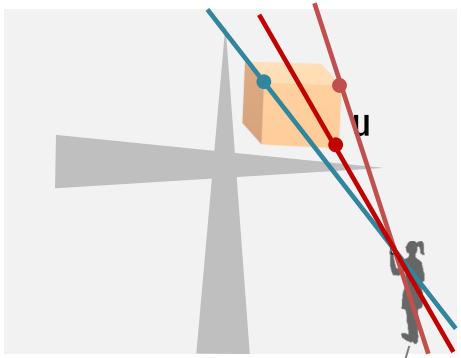
Alice's image

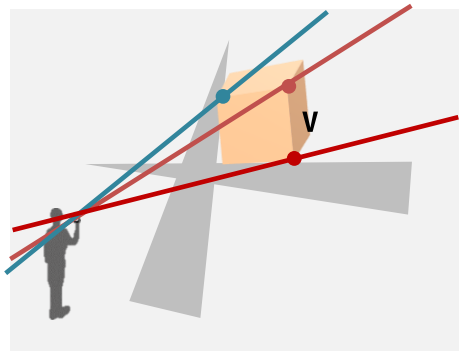
Alice from Bob's view

Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.







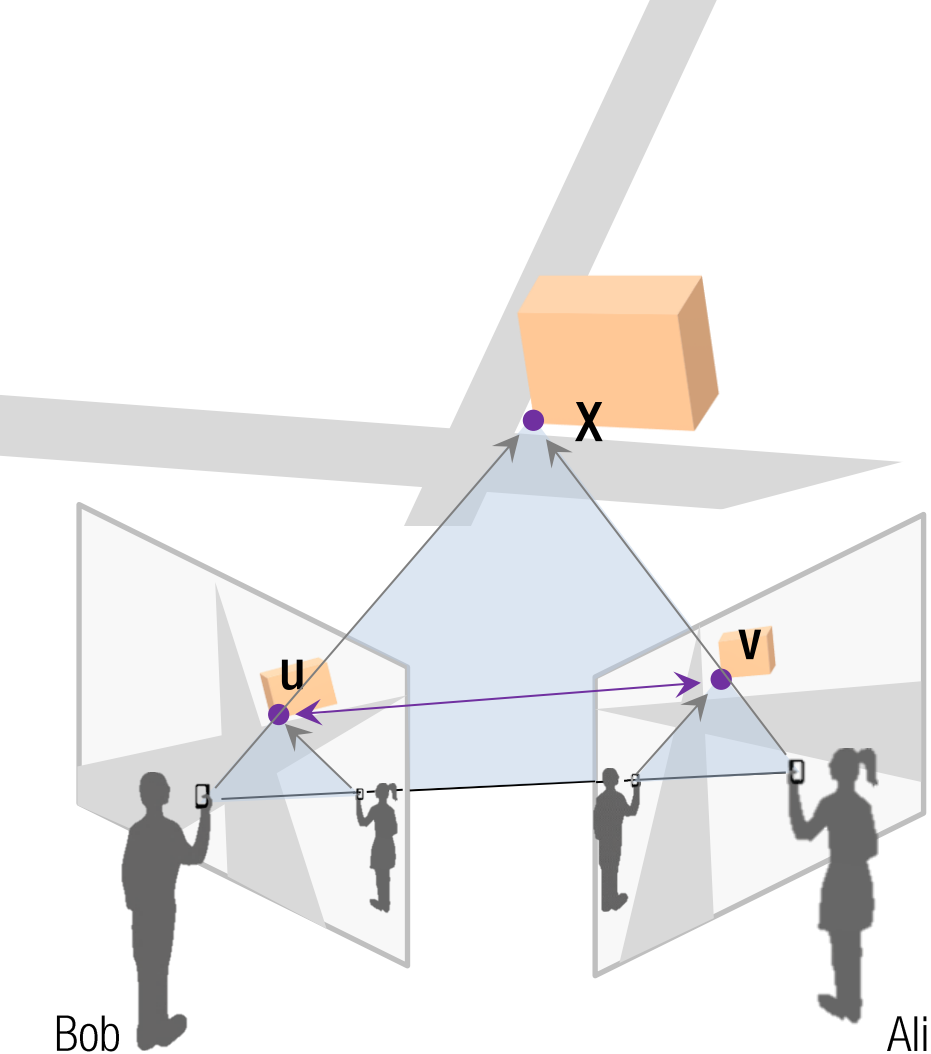
Bob's image

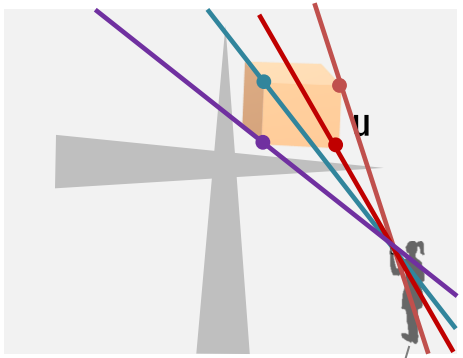
Alice's image

Alice from Bob's view

Epipolar constraint between two images:

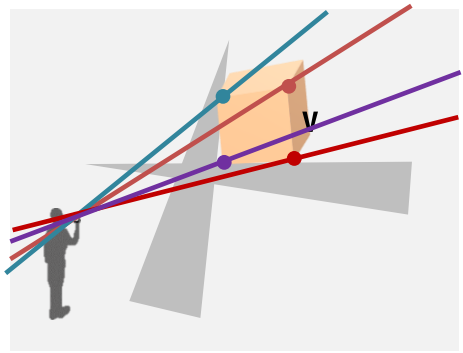
1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.





Bob's image

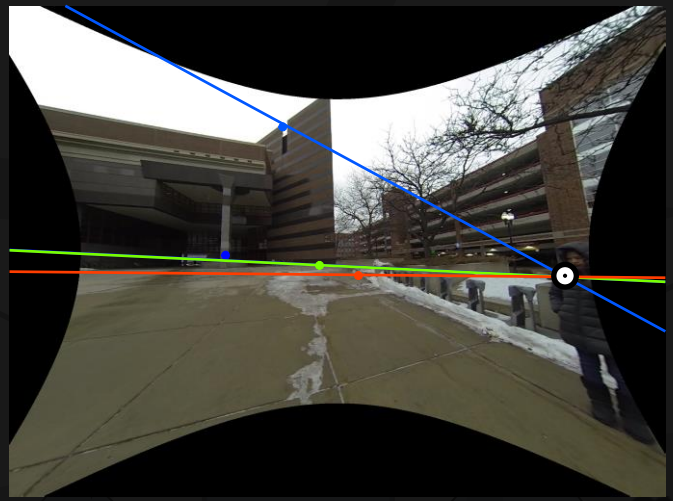
Alice from Bob's view

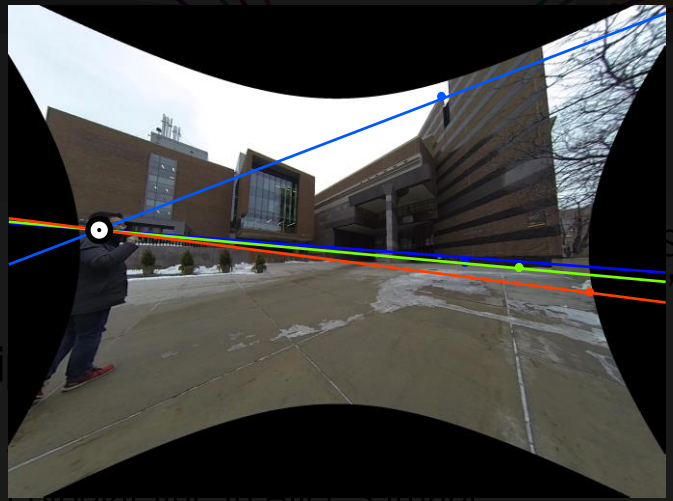


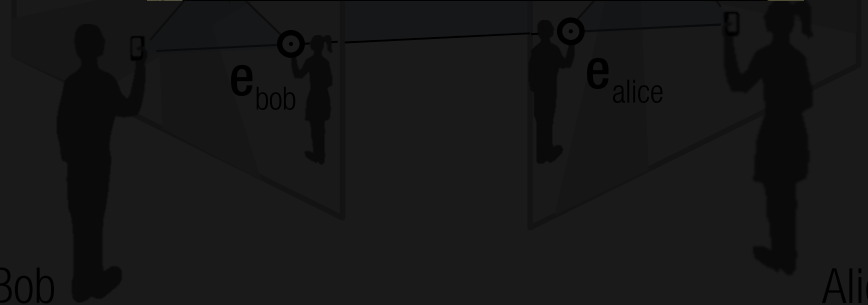
Alice's image

Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole: $\mathbf{e}_{bob}^T\mathbf{l}_u = 0$ $\mathbf{e}_{alice}^T\mathbf{l}_v = 0$







Bob

Alice

u

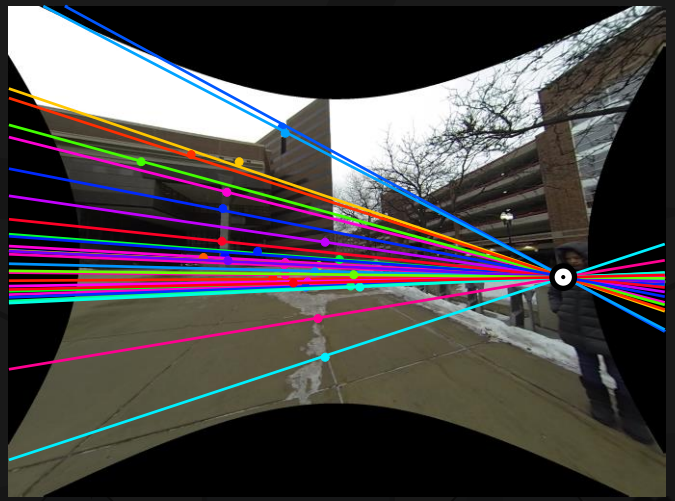
v

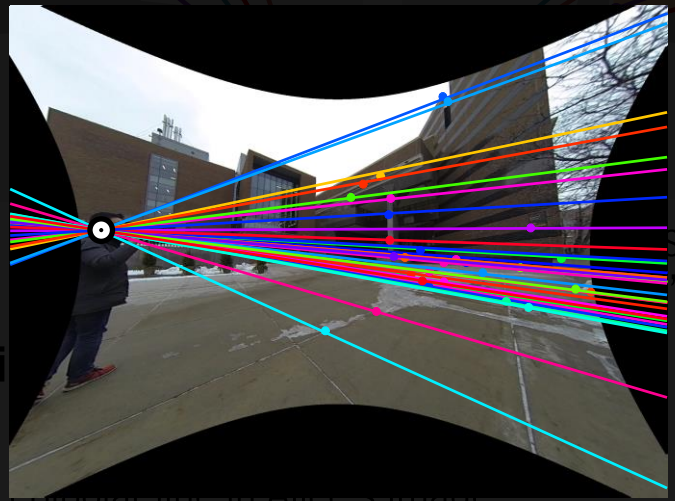
s image

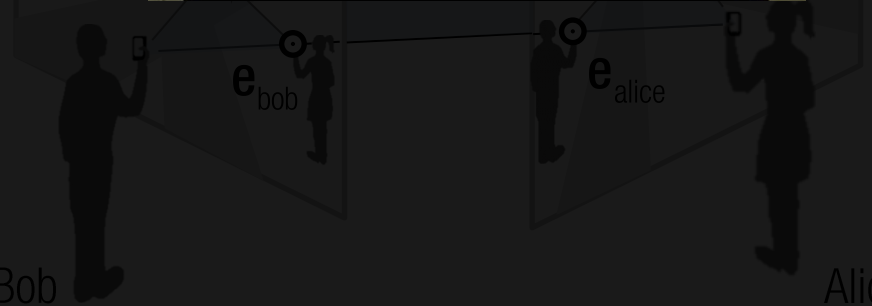
's view

Epi

1. to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**:
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole:







Bob

Alice

u

v

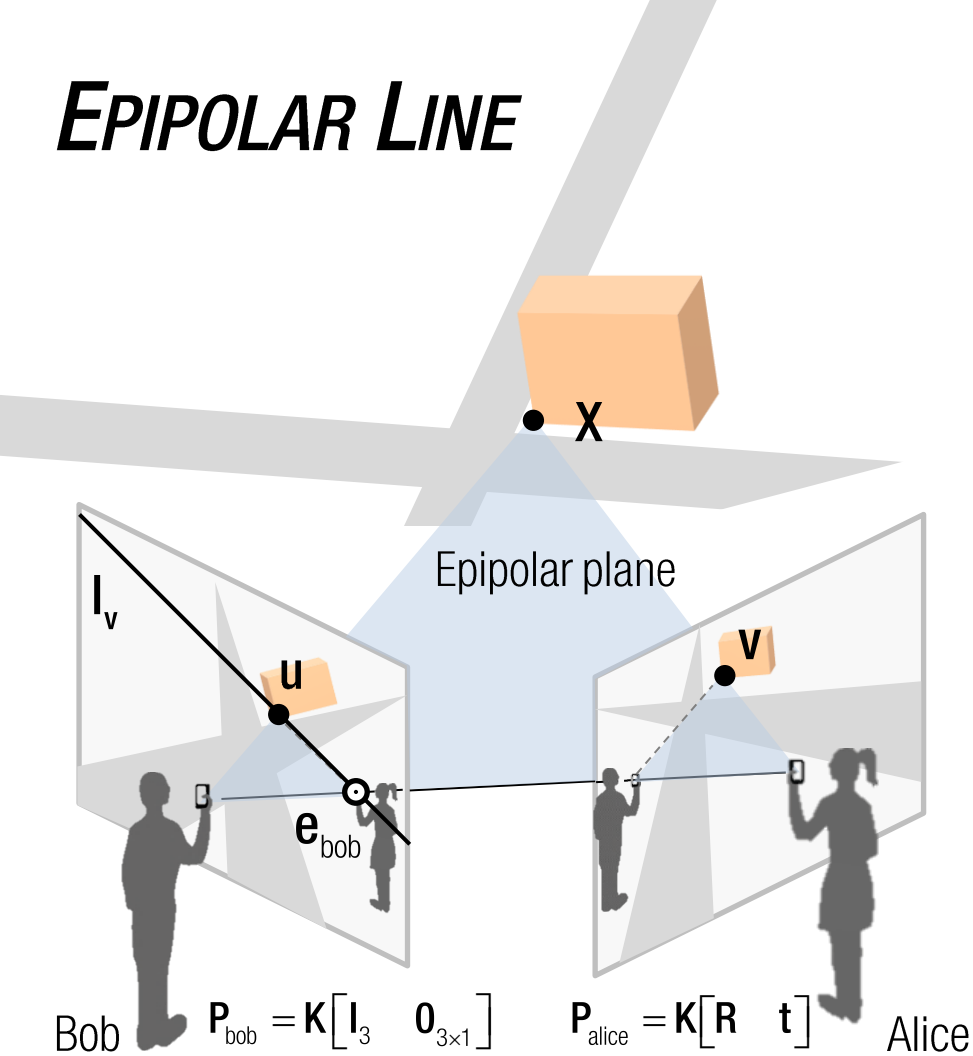
s image

's view

Epi

1. to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**:
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole:

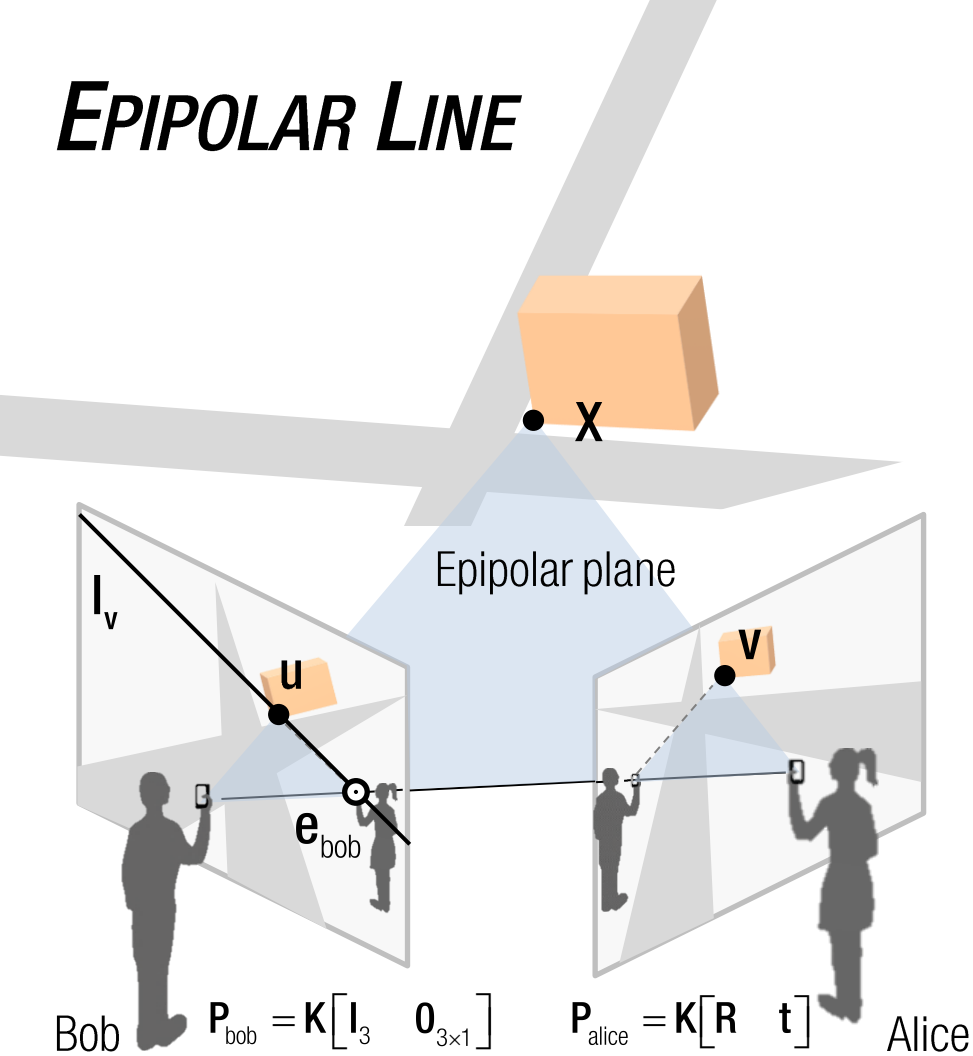
EPIPOLAR LINE



$$l_v = Fv$$

Fundamental matrix

EPIPOLAR LINE



$$l_v = Fv$$

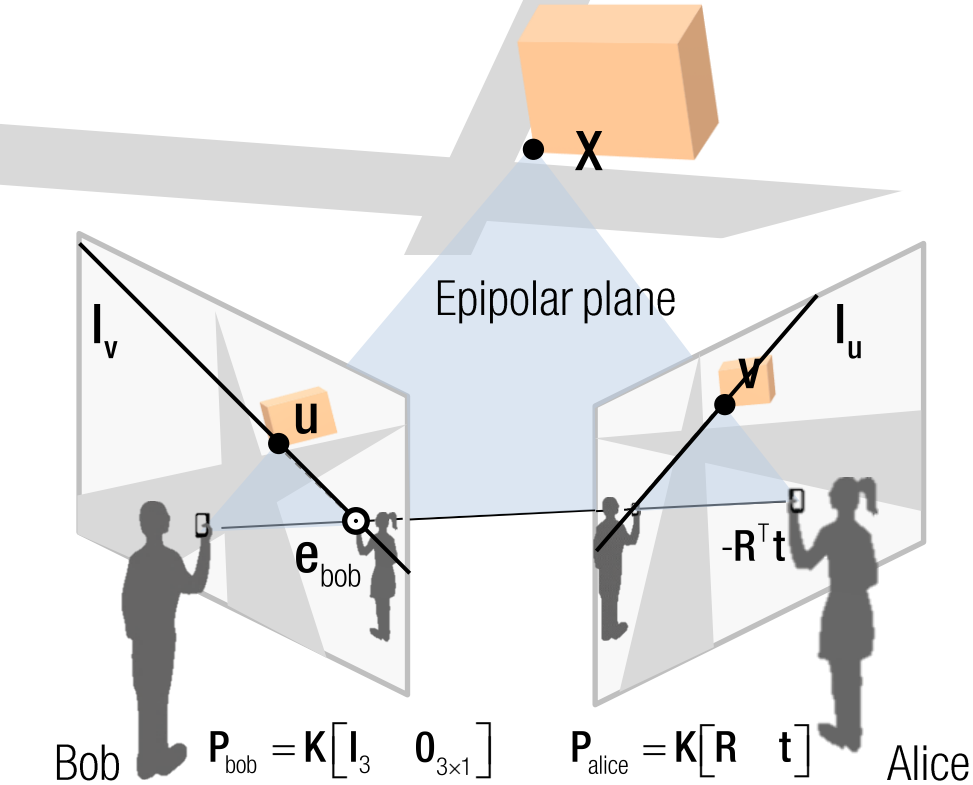
Fundamental matrix

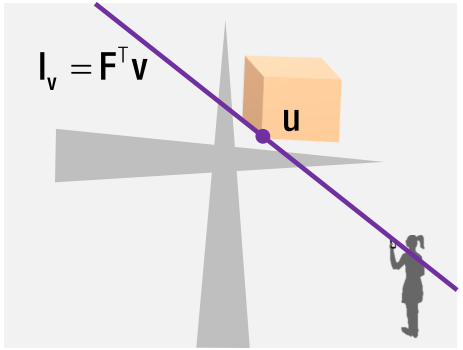
$$u^T Fv = 0$$

where $F = K^{-T}[t]_\times RK^{-1}$

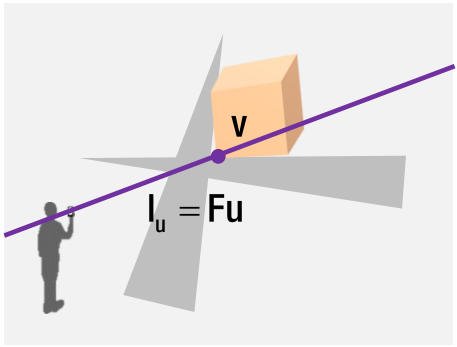
Fundamental matrix

FUNDAMENTAL MATRIX





Bob's image



Alice's image

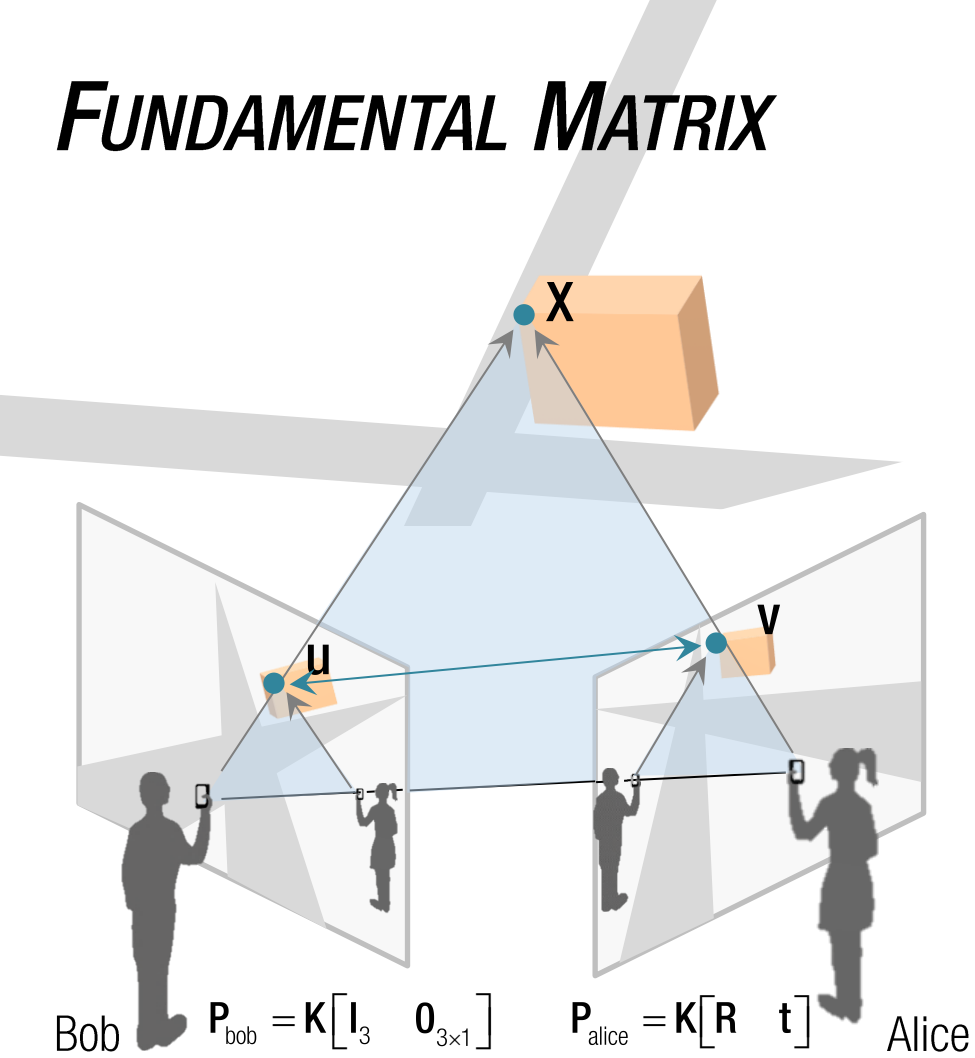
$$v^T l_u = v^T K^{-T} [t]_\times R K^{-1} u = 0$$

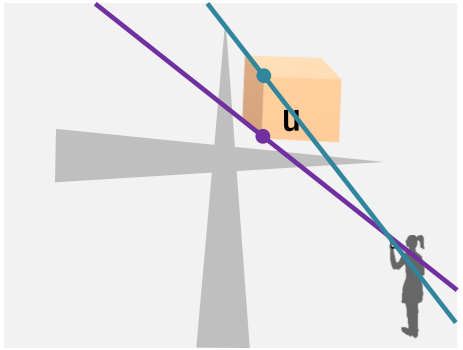
Common for all points

$$= v^T F u = 0$$

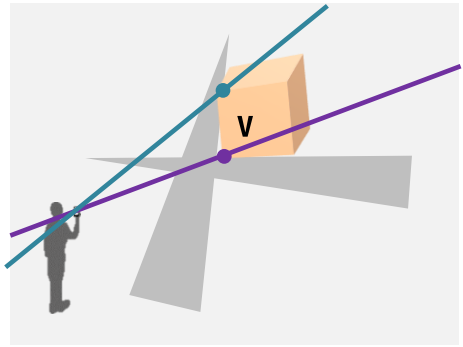
$$= v^T (Fu) = u^T (F^T v) = 0$$

FUNDAMENTAL MATRIX





Bob's image



Alice's image

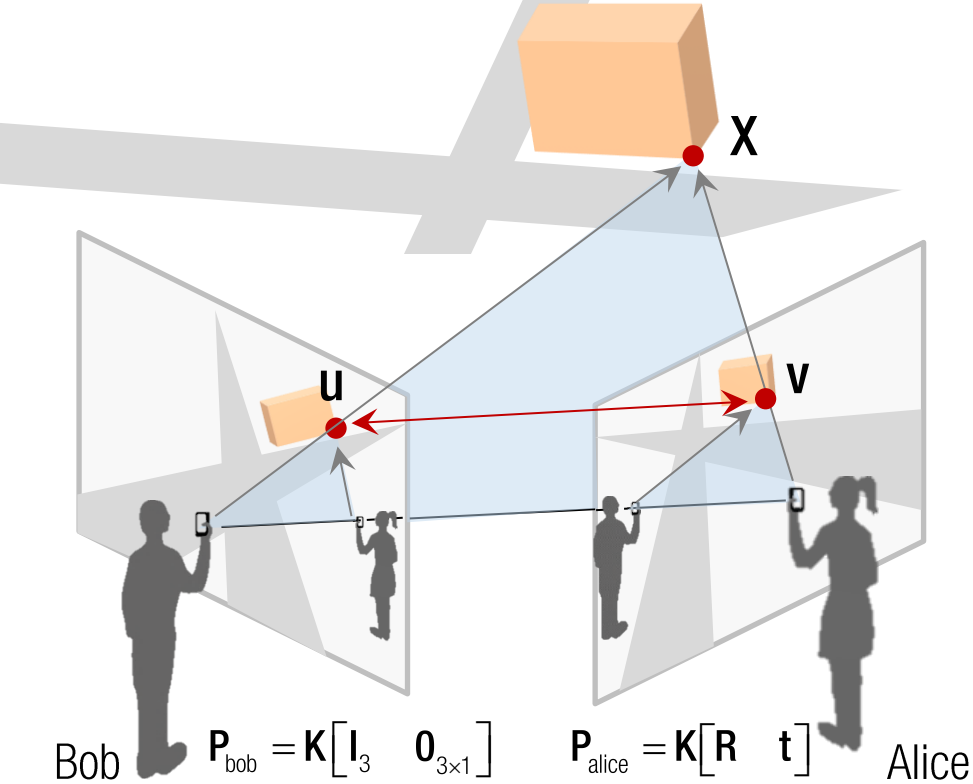
$$\mathbf{v}^T\mathbf{l}_u = \mathbf{v}^T\underbrace{\mathbf{K}^{-T}[\mathbf{t}]_\times \mathbf{R}\mathbf{K}^{-1}}\mathbf{u} = \mathbf{0}$$

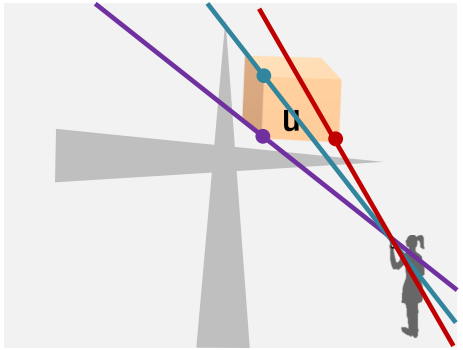
Common for all points

$$= \mathbf{v}^T\mathbf{F}\mathbf{u} = 0$$

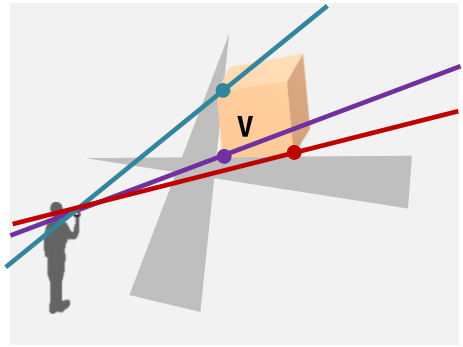
$$= \mathbf{v}^T(\mathbf{F}\mathbf{u}) = \mathbf{u}^T(\mathbf{F}^T\mathbf{v}) = 0$$

FUNDAMENTAL MATRIX





Bob's image



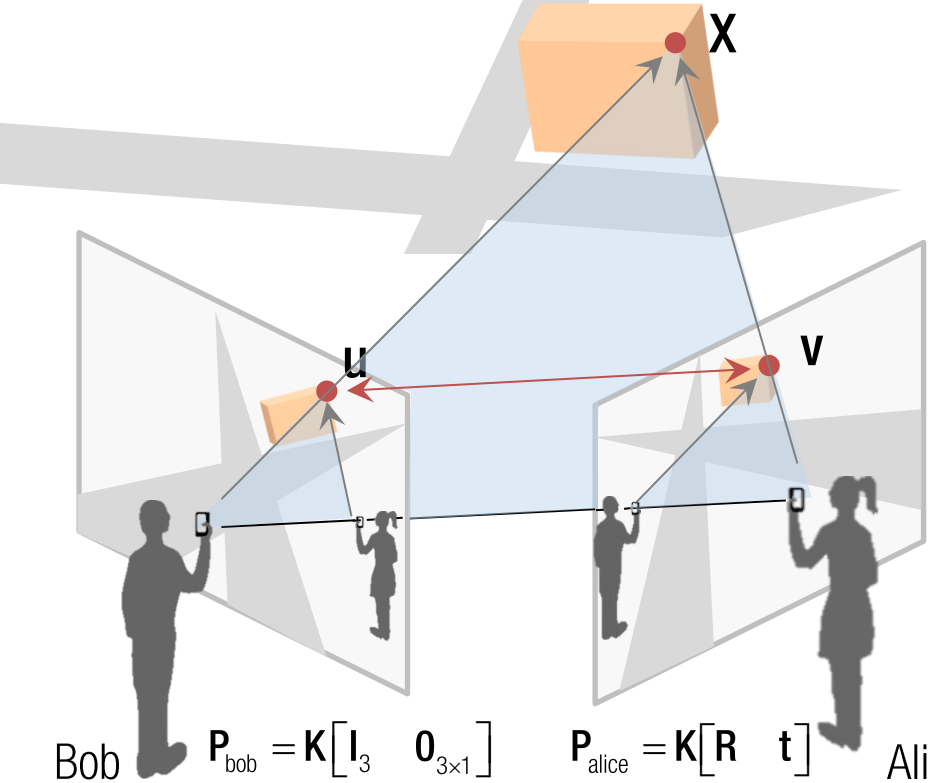
Alice's image

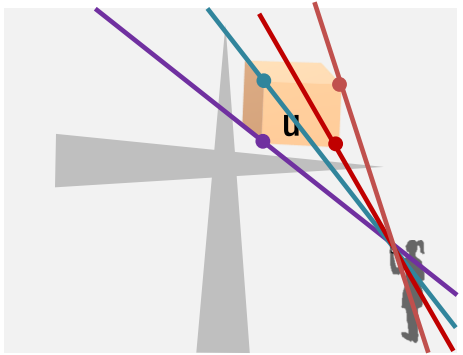
$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \underbrace{\mathbf{K}^{-T} [\mathbf{t}]_\times \mathbf{R} \mathbf{K}^{-1}}_{\text{Common for all points}} \mathbf{u} = \mathbf{0}$$

$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

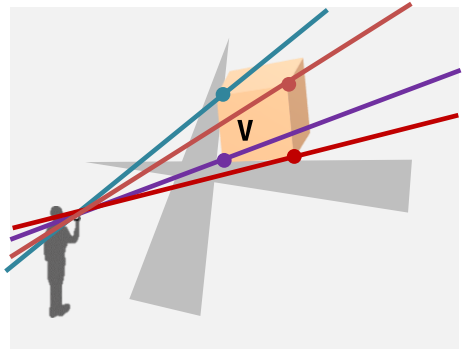
$$= \mathbf{v}^T (\mathbf{F}\mathbf{u}) = \mathbf{u}^T (\mathbf{F}^T \mathbf{v}) = 0$$

FUNDAMENTAL MATRIX





Bob's image



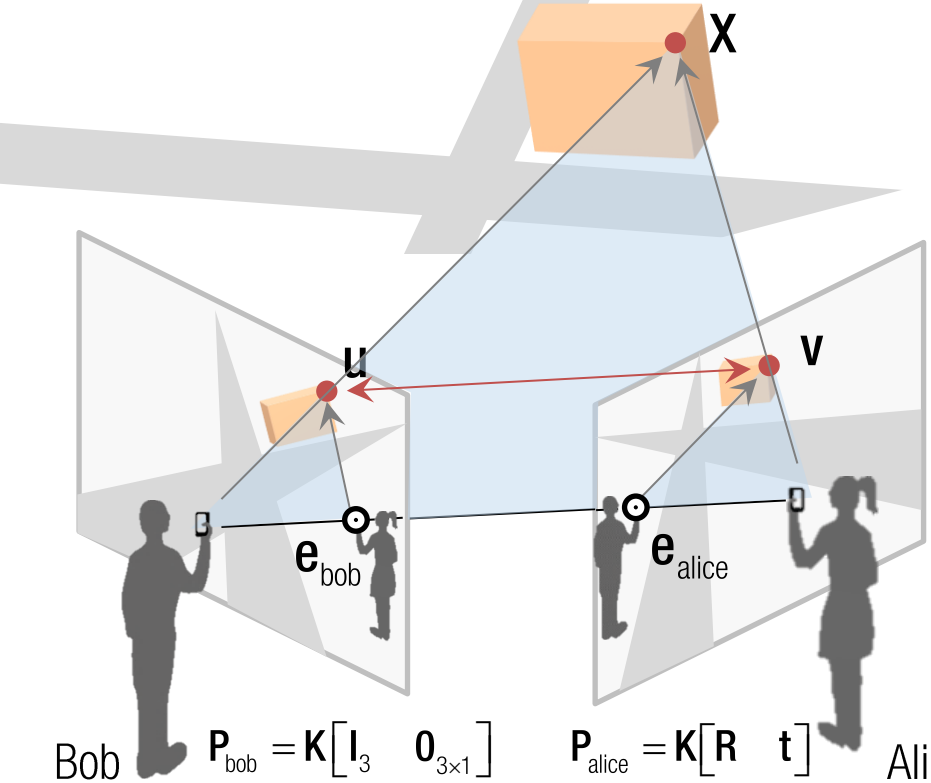
Alice's image

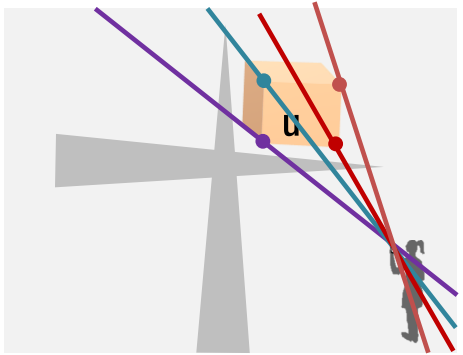
$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \underbrace{\mathbf{K}^{-T} [\mathbf{t}]_\times \mathbf{R} \mathbf{K}^{-1}}_{\text{Common for all points}} \mathbf{u} = \mathbf{0}$$

$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

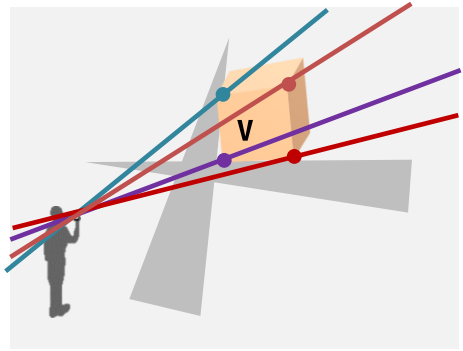
$$= \mathbf{v}^T (\mathbf{F}\mathbf{u}) = \mathbf{u}^T (\mathbf{F}^T \mathbf{v}) = 0$$

FUNDAMENTAL MATRIX





Bob's image

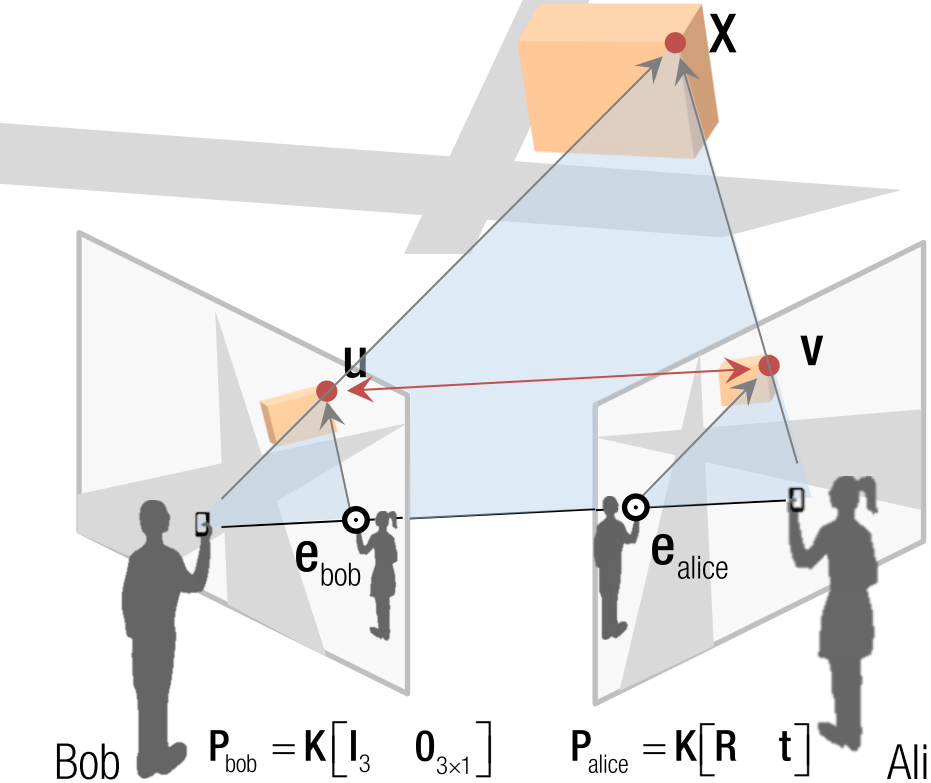


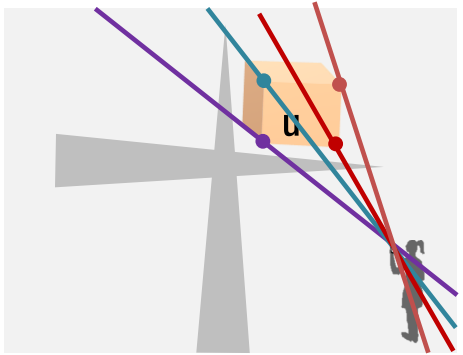
Alice's image

Properties of Fundamental Matrix

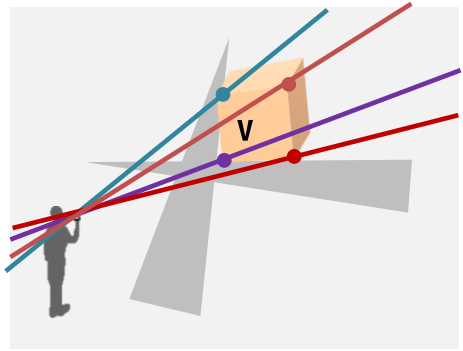
- Transpose: if **F** is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.

FUNDAMENTAL MATRIX





Bob's image

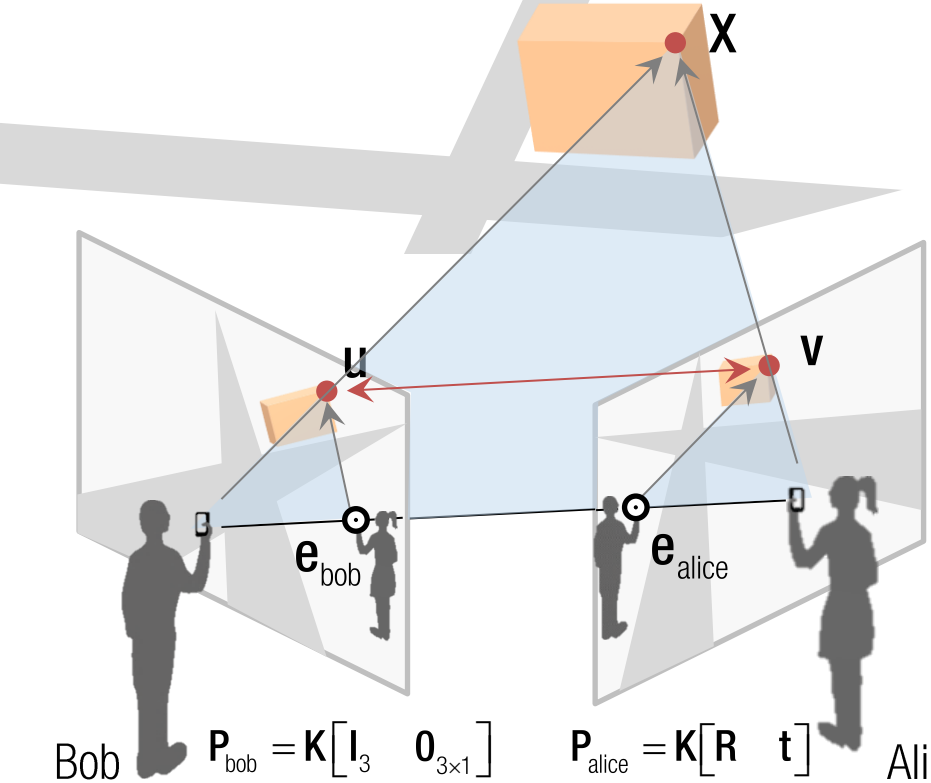


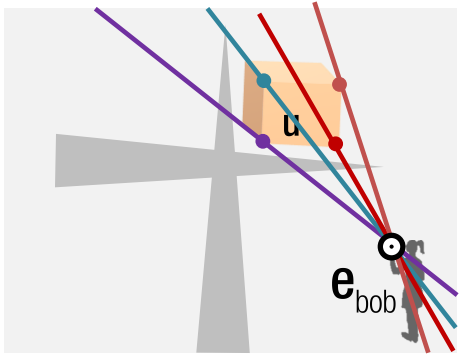
Alice's image

Properties of Fundamental Matrix

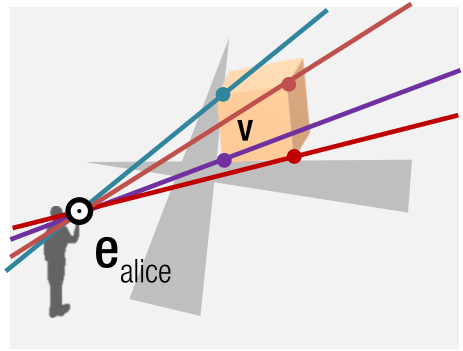
- Transpose: if $\mathbf{F}$ is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $l_u = Fu \quad l_v = F^T v$

FUNDAMENTAL MATRIX





Bob's image



Alice's image

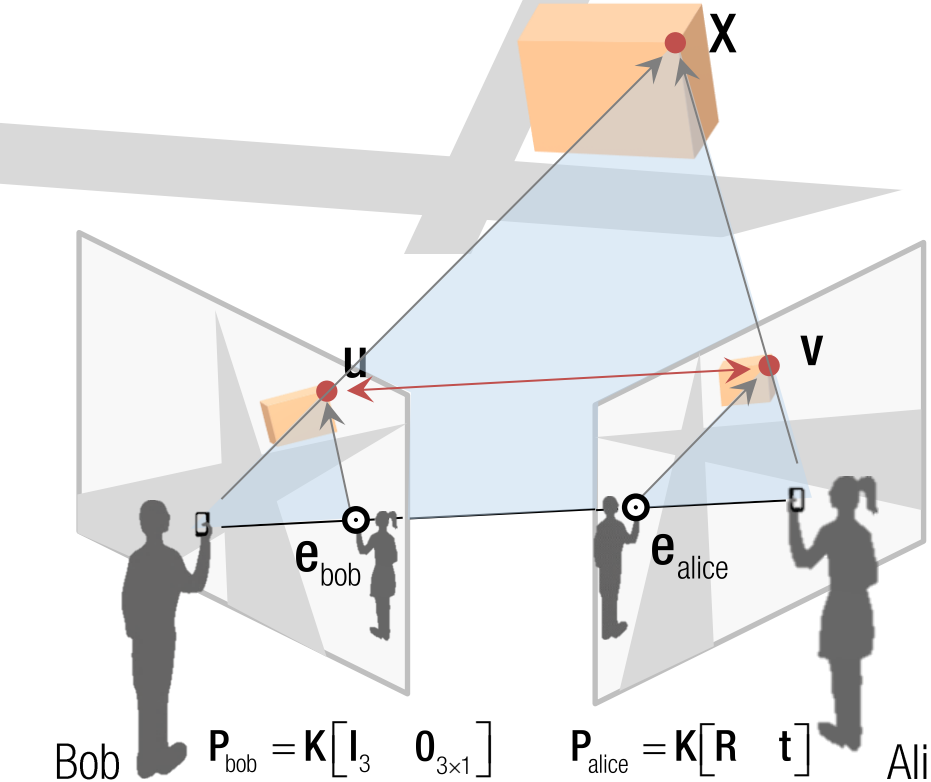
Properties of Fundamental Matrix

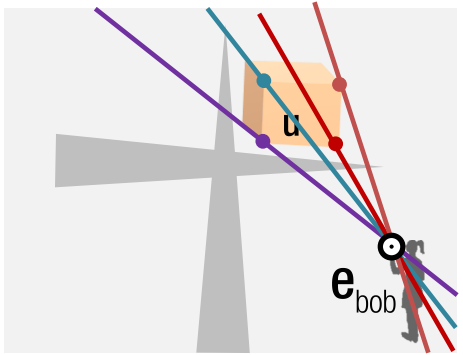
- Transpose: if $\mathbf{F}$ is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u} \quad \mathbf{l}_v = \mathbf{F}^T\mathbf{v}$

  $\mathbf{F}\mathbf{e}_{bob} = \mathbf{0} \quad \mathbf{F}^T\mathbf{e}_{alice} = \mathbf{0}$
- Epipole: $\because \mathbf{v}_i^T\mathbf{F}\mathbf{e}_{bob} = 0, \quad \mathbf{u}_i^T\mathbf{F}^T\mathbf{e}_{alice} = 0, \quad \forall i$

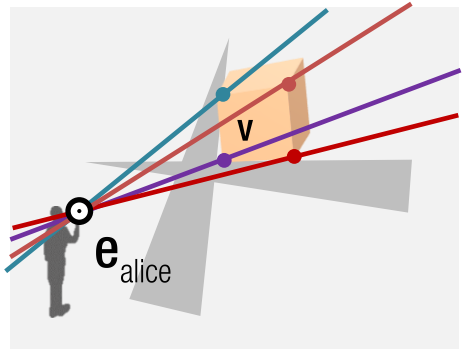
  $\longrightarrow \mathbf{e}_{bob} = \text{null}(\mathbf{F}), \quad \mathbf{e}_{alice} = \text{null}(\mathbf{F}^T)$

FUNDAMENTAL MATRIX





Bob's image



Alice's image

Properties of Fundamental Matrix

- Transpose: if $\mathbf{F}$ is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $l_u = Fu \quad l_v = F^T v$
  $Fe_{bob} = 0 \quad F^T e_{alice} = 0$
- Epipole:
- rank($\mathbf{F}$)=2:
  DoF 9 (3x3 matrix)-1 (scale)-1 (rank)=7

Camera Motion

Camera Motion

Image 2

Image 1

Image

Image

Forward motion

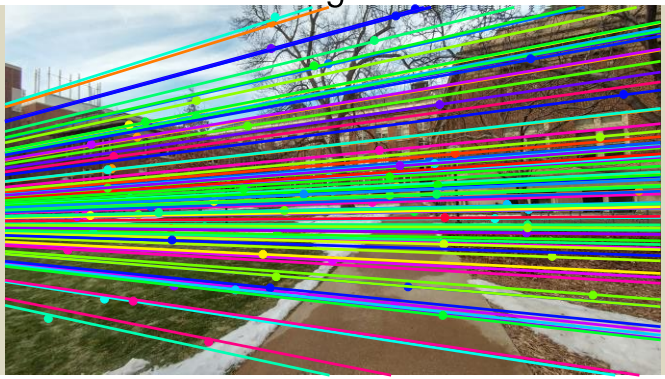


Image 2

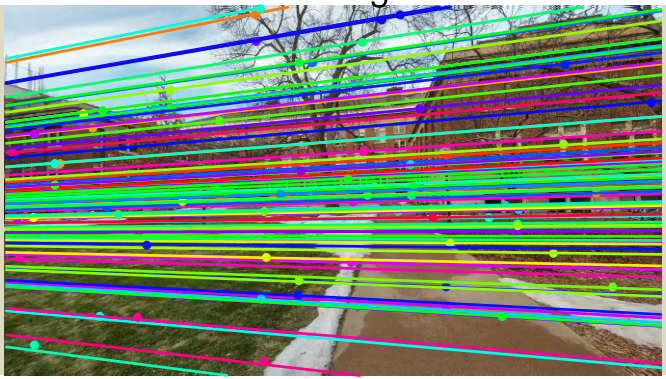
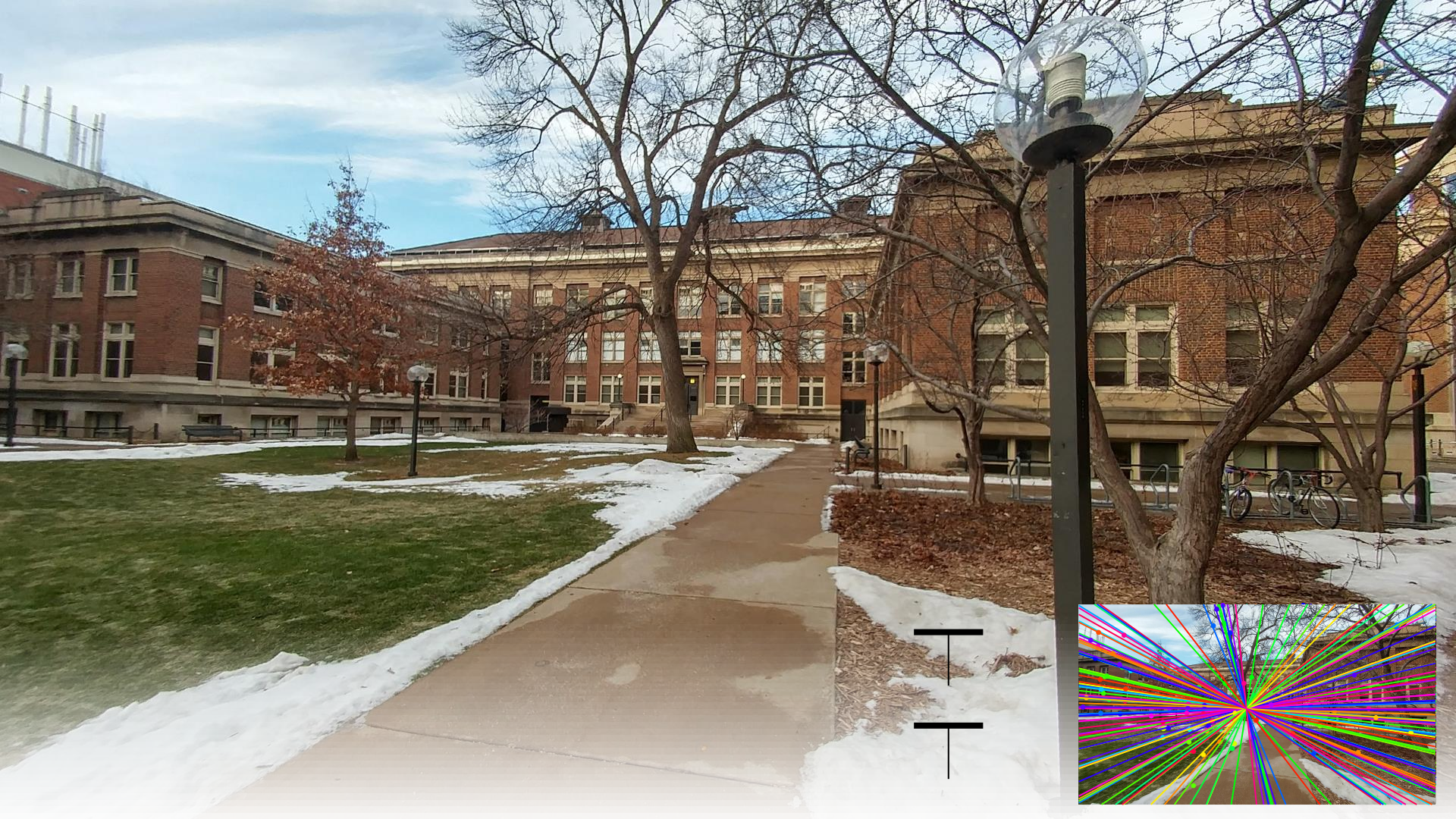
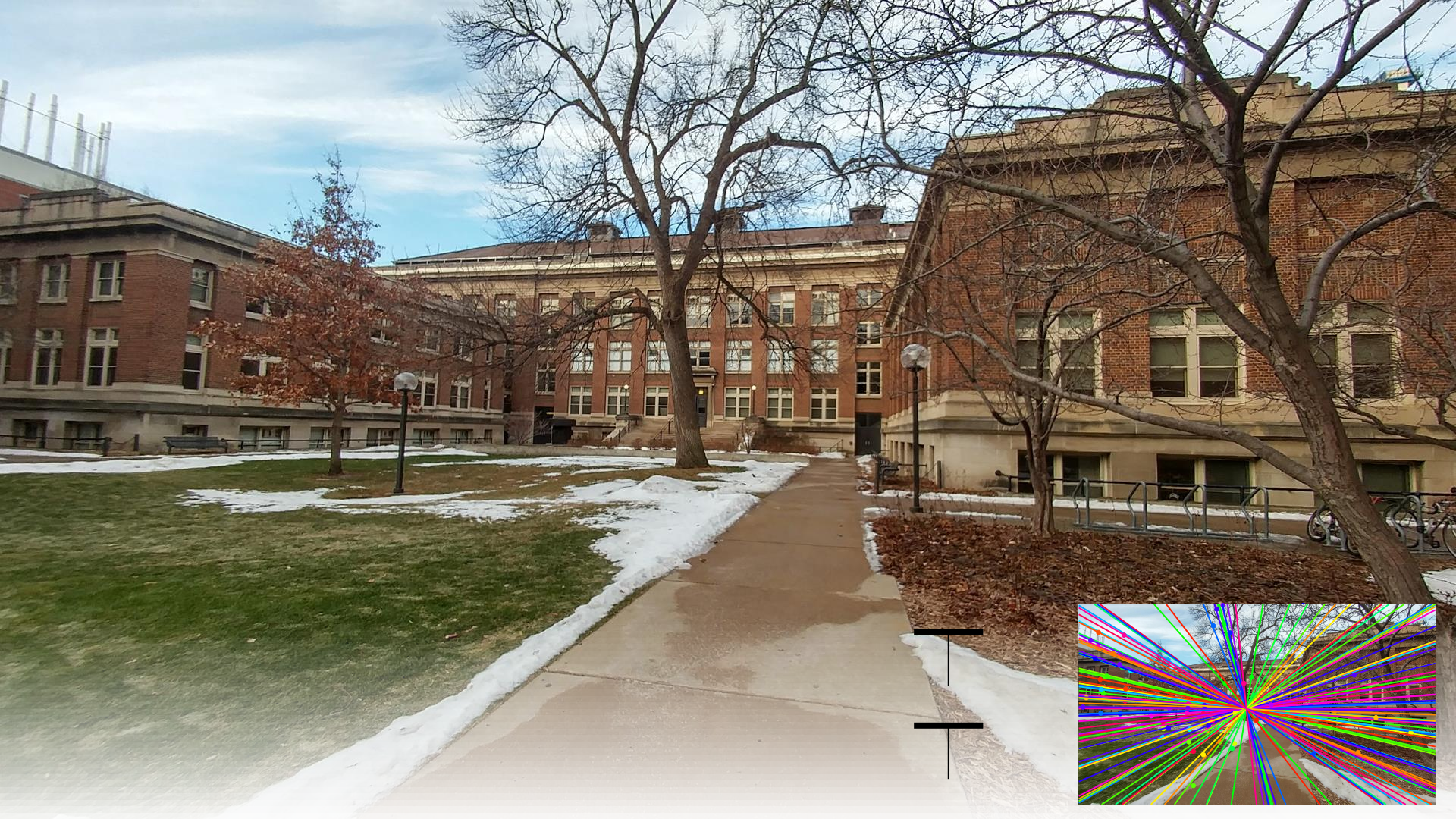


Image 1

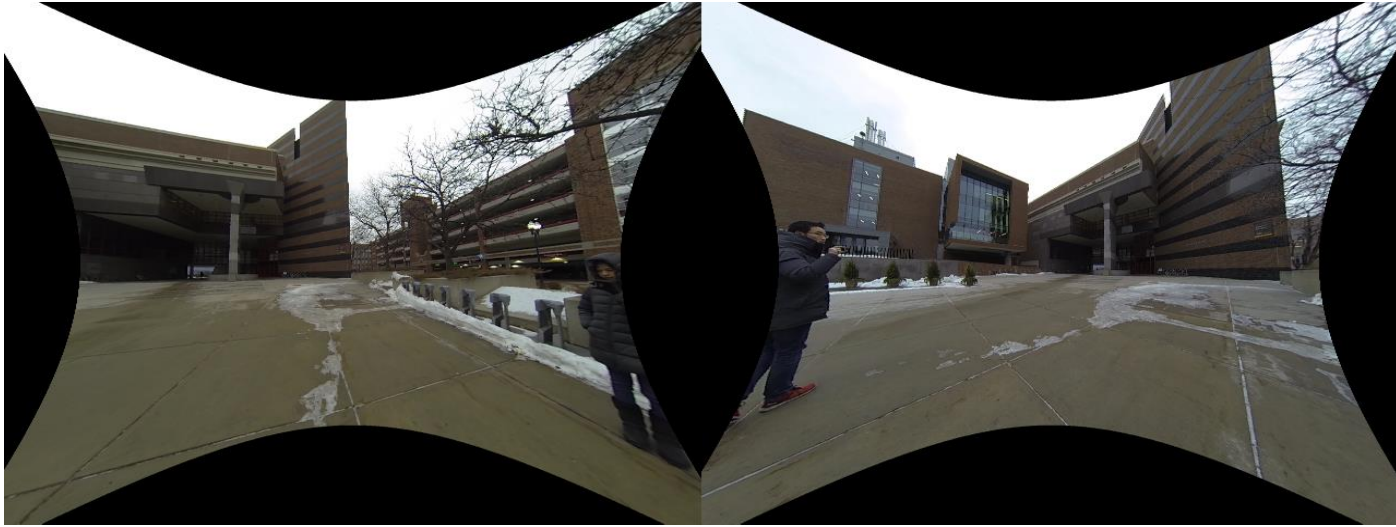
Image 2 Image 1

Lateral motion





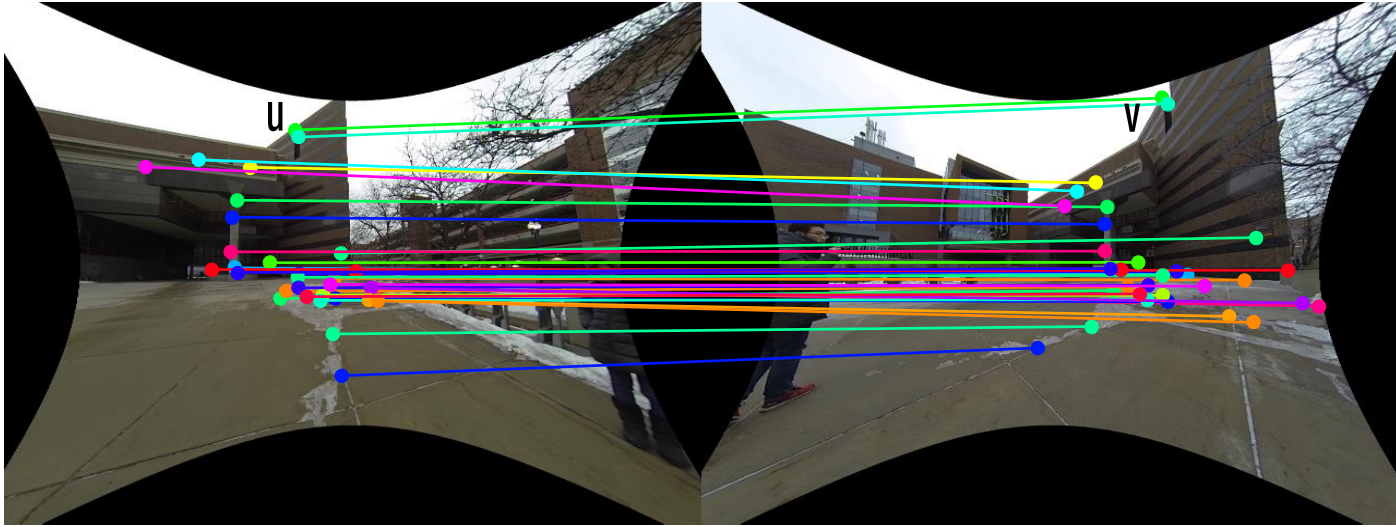
2D Correspondence



Bob's image

Alice's image

2D Correspondence

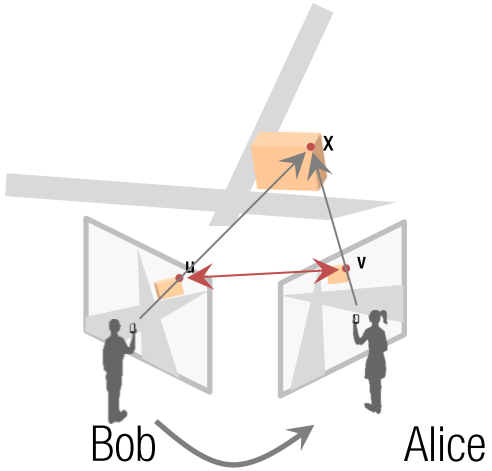


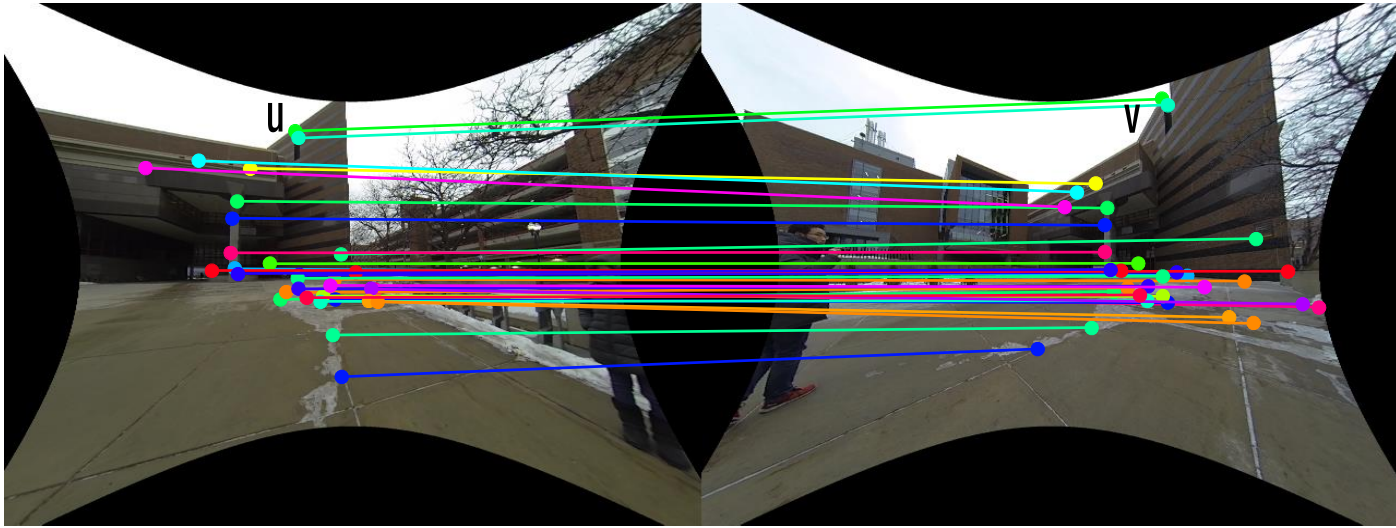
Bob's image

Alice's image

$$\mathbf{v}^{\mathsf{T}}\mathbf{F}\mathbf{u} = 0$$

2D Correspondence





$$\mathbf{F} = \mathbf{F}(\mathbf{R}, \mathbf{t})$$
$$= \mathbf{K}^{-T} [\mathbf{t}]_{\times} \mathbf{R} \mathbf{K}^{-1}$$

Bob's image

Alice's image

$$\mathbf{v}^{T} \mathbf{F} \mathbf{u} = 0$$

How to compute fundamental matrix?

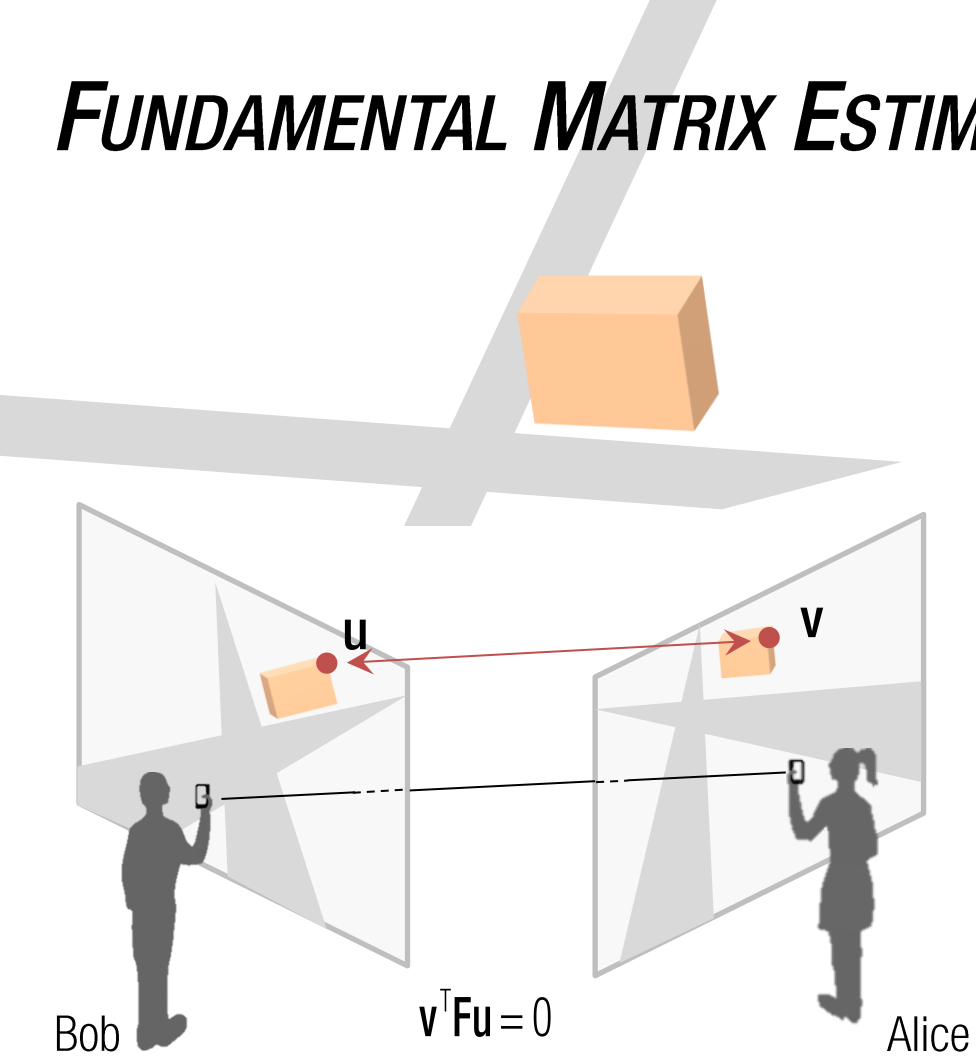
8 Point Algorithm (Longuet-Higgins, Nature 1981)

A computer algorithm for reconstructing a scene from two projections

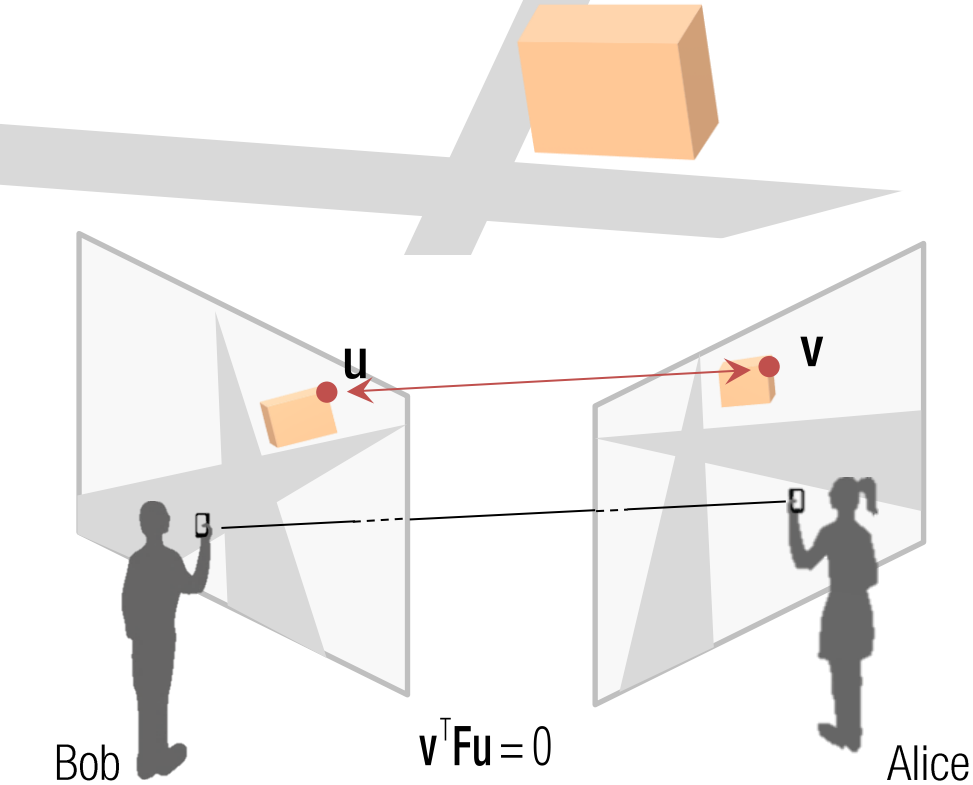
H. C. Longuet-Higgins
Laboratory of Experimental Psychology, University of Sussex, Brighton BN1 9QG, UK

A simple algorithm for computing the three-dimensional structure of a scene from a correlated pair of perspective projections is described here, when the spatial relationship between the projections is unknown. This problem is relevant not only to photographic surveying[1] but also to binocular vision[2], where the non-visual information available to the observer about

Fundamental Matrix Estimation



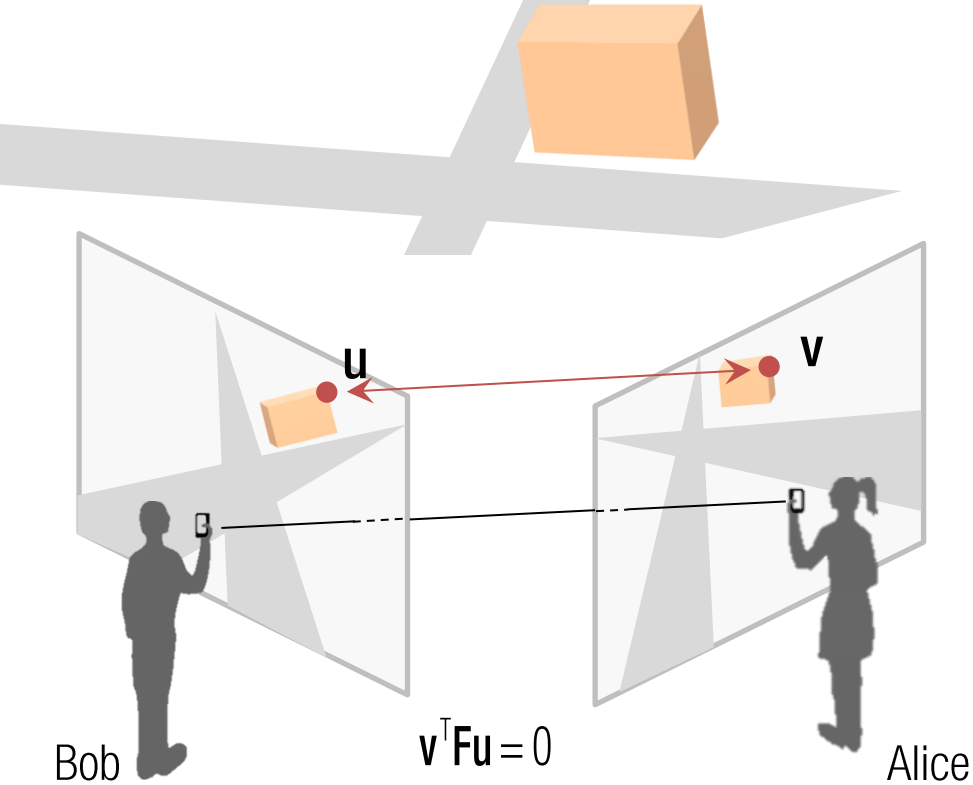
Fundamental Matrix Estimation



$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Degree of freedom of fundamental matrix:

FUNDAMENTAL MATRIX ESTIMATION



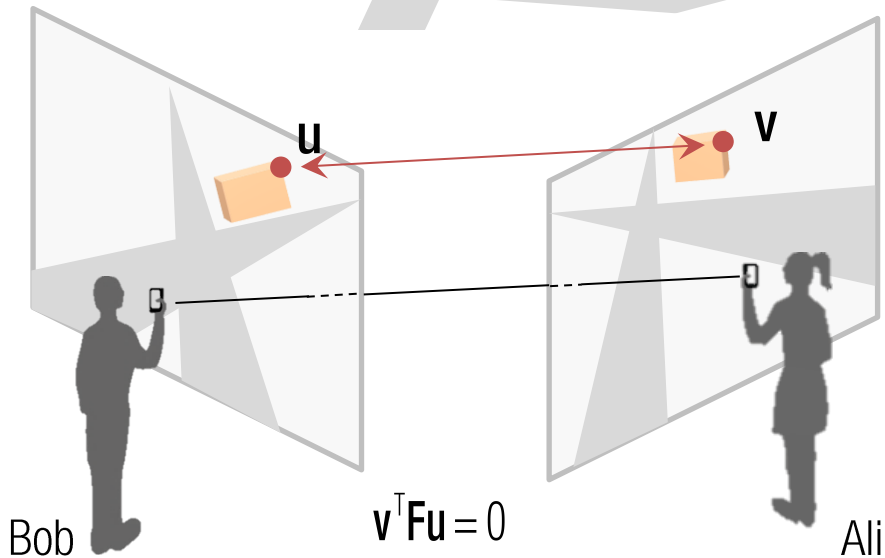
$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Degree of freedom of fundamental matrix:
7 = 9 (3x3 matrix) – 1(scale) – 1 (rank 2)

We will estimate fundamental matrix with 8 parameter by ignoring rank constraint and then project onto rank 2 matrix:

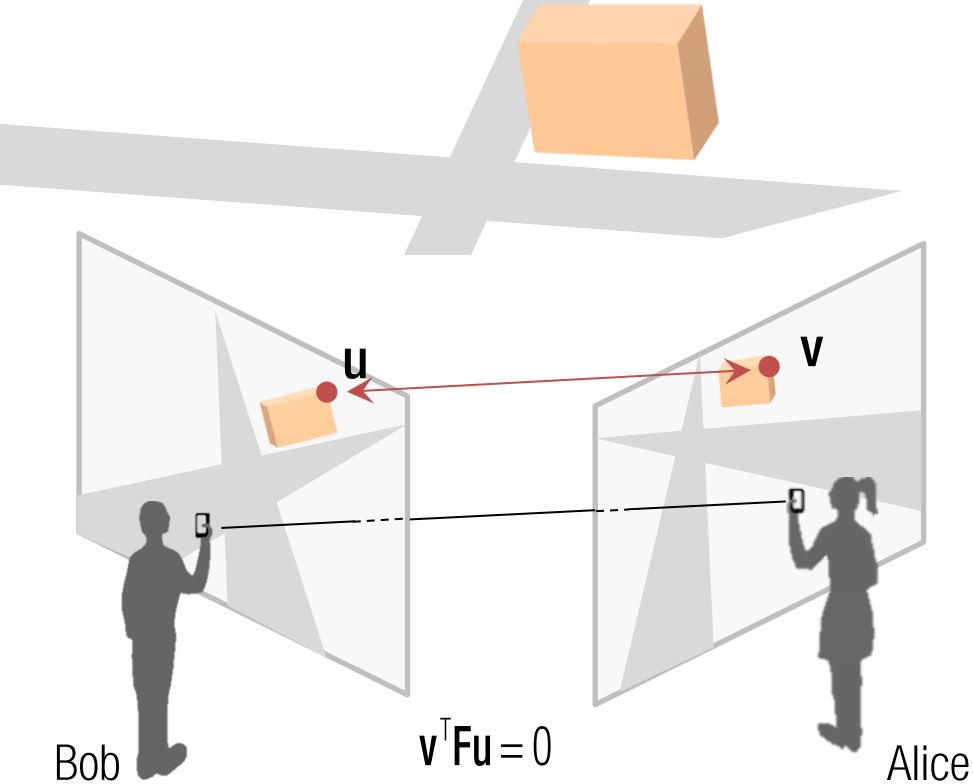
Fundamental Matrix Estimation

$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$



$$\mathbf{v}^T\mathbf{F}\mathbf{u} = 0$$

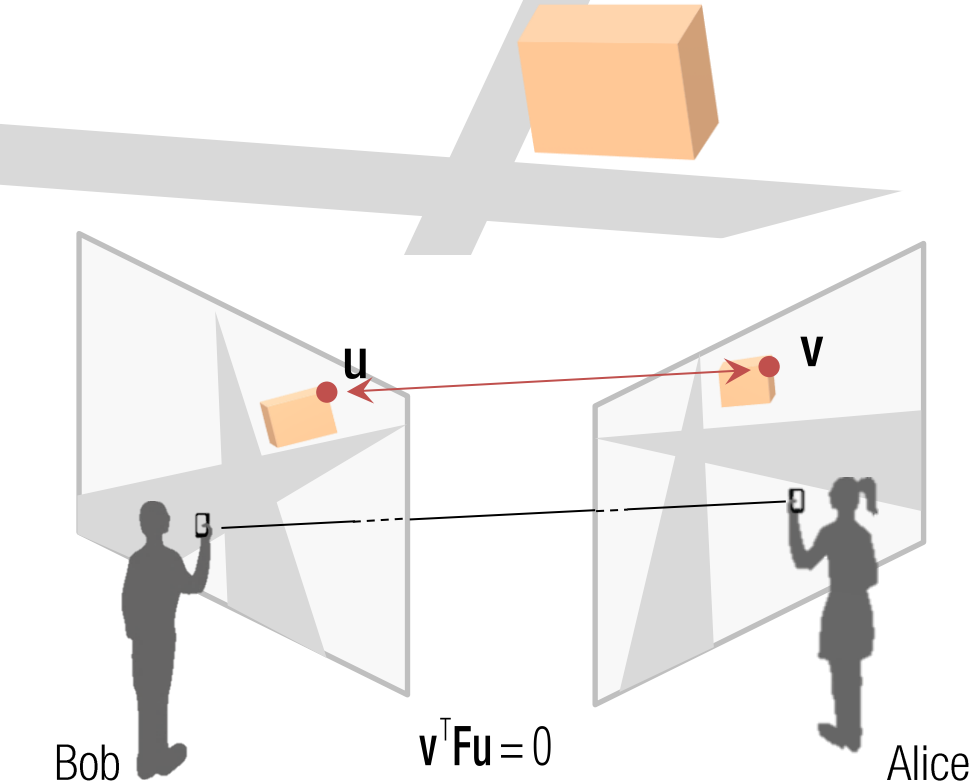
FUNDAMENTAL MATRIX ESTIMATION



$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

FUNDAMENTAL MATRIX ESTIMATION



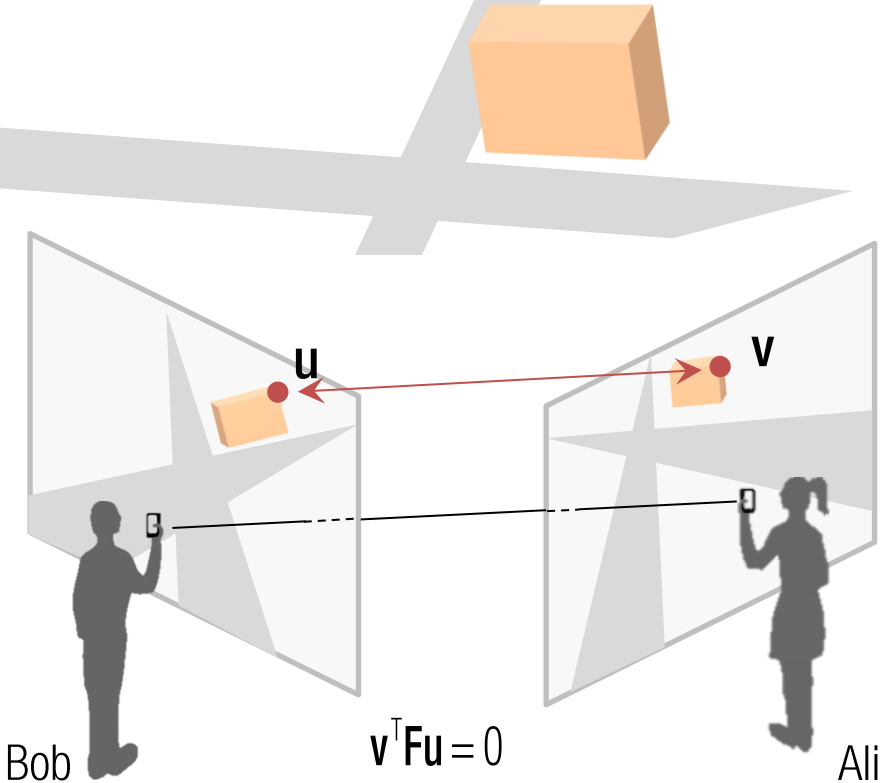
$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

$$= 0$$

linear in $\mathbf{F}$

FUNDAMENTAL MATRIX ESTIMATION



$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

$$= 0$$

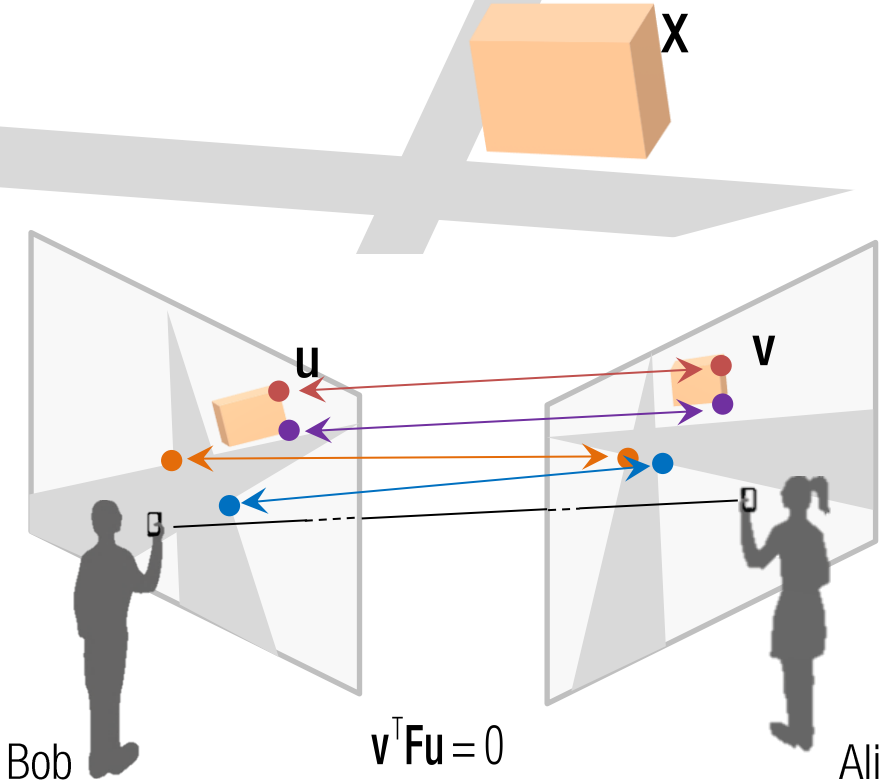
Linear in **F**.

$$\longrightarrow \begin{bmatrix} u^xv^x & u^yv^x & v^x & u^xv^y & u^yv^y & v^y & u^x & u^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

# of unknowns: 9

# of equations per correspondence: 1

Fundamental Matrix Estimation



$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

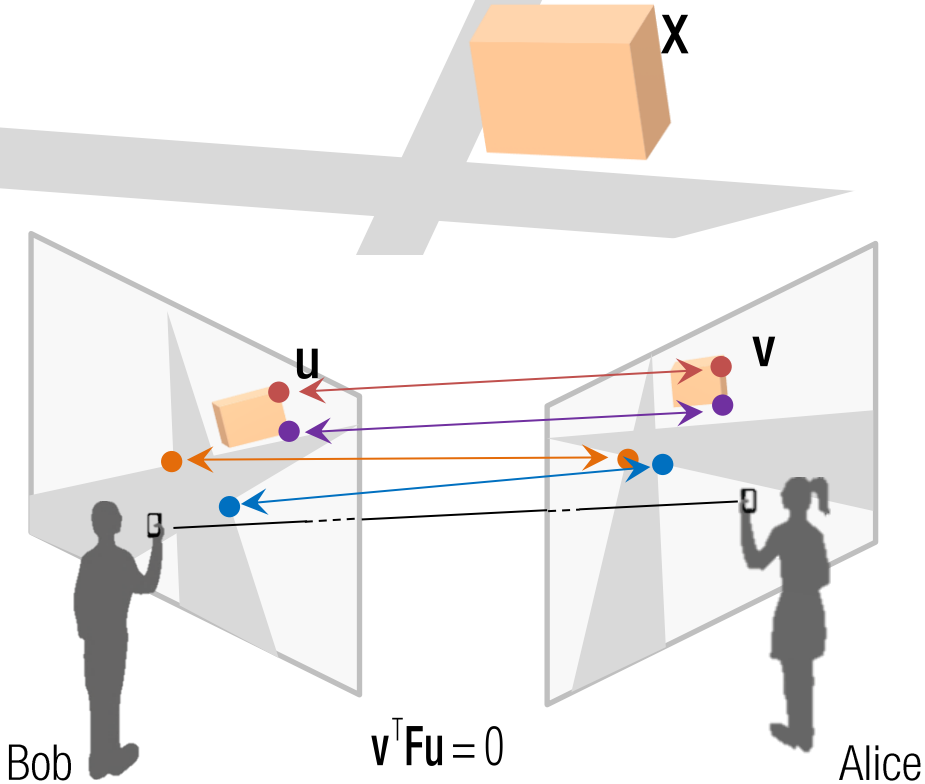
$$= 0$$

Linear in **F**.

$$\rightarrow \begin{bmatrix} u_1^xv_1^x & u_1^yv_1^x & v_1^x & u_1^xv_1^y & u_1^yv_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^xv_m^x & u_m^yv_m^x & v_m^x & u_m^xv_m^y & u_m^yv_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \mathbf{0}_{m\times 1}$$

What is minimum m?

Fundamental Matrix Estimation



$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

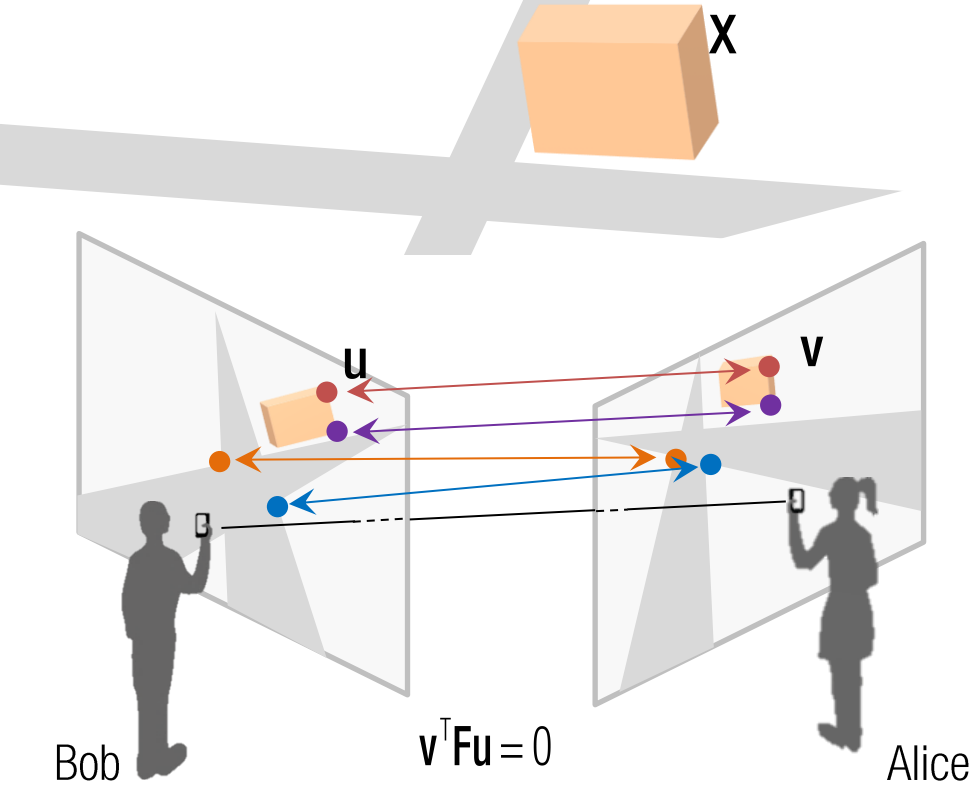
$$= 0$$

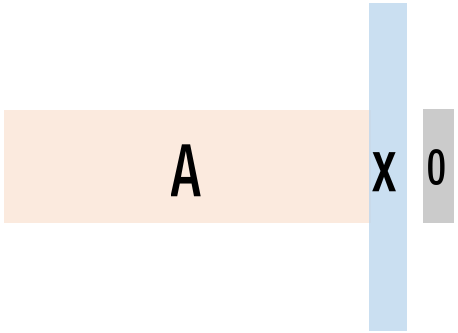
Linear in **F**.

$$\rightarrow \begin{bmatrix} u_1^xv_1^x & u_1^yv_1^x & v_1^x & u_1^xv_1^y & u_1^yv_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^xv_m^x & u_m^yv_m^x & v_m^x & u_m^xv_m^y & u_m^yv_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix} \mathbf{A} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} \mathbf{X} = \mathbf{0}$$

What is minimum m?

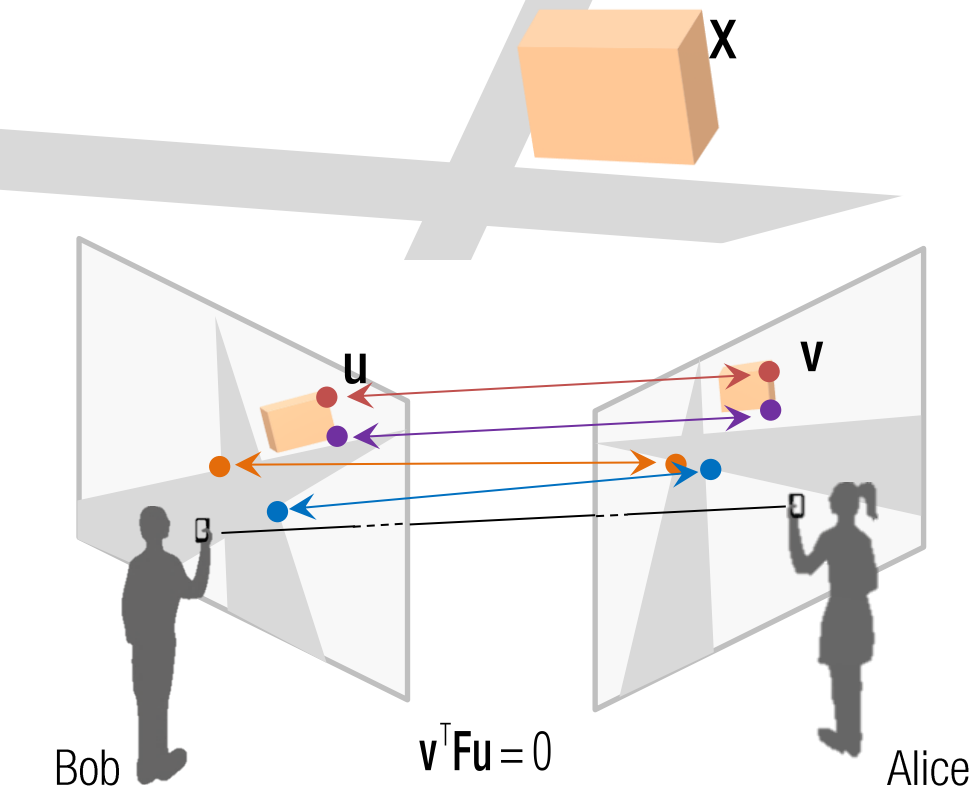
FUNDAMENTAL MATRIX ESTIMATION

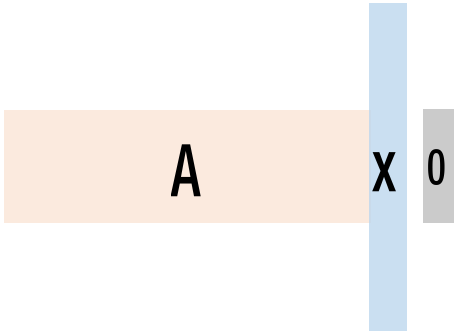




The solution is not necessarily satisfy rank 2 constraint.

FUNDAMENTAL MATRIX ESTIMATION

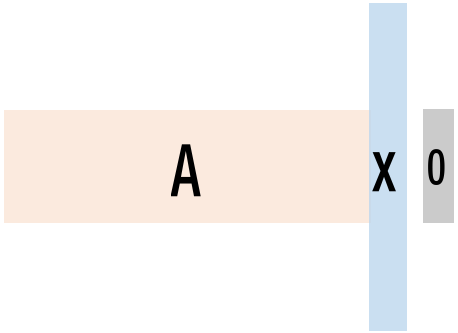


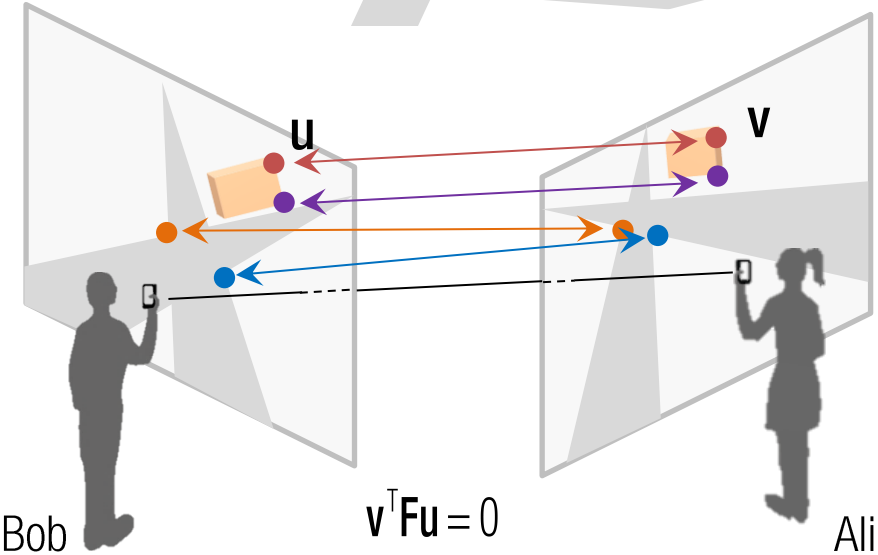


The solution is not necessarily satisfy rank 2.

$$\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} = U \; D \; V^T$$

Fundamental Matrix Estimation





The solution is not necessarily satisfy rank.

$$\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} = U \; D \; V^T$$

$$\approx F_{\text{rank}2} = U \; \tilde{D} \; V^T$$

SVD cleanup

Camera Pose From F