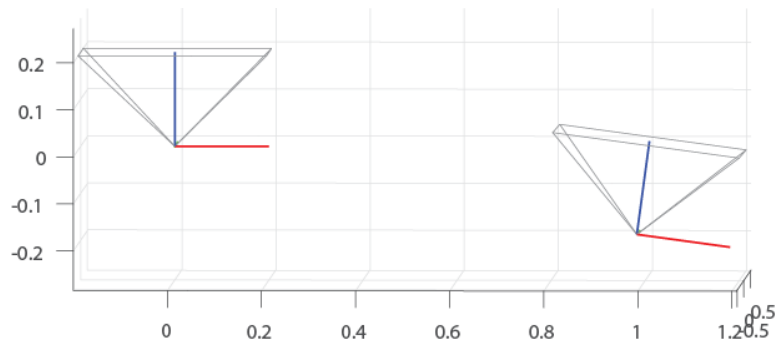
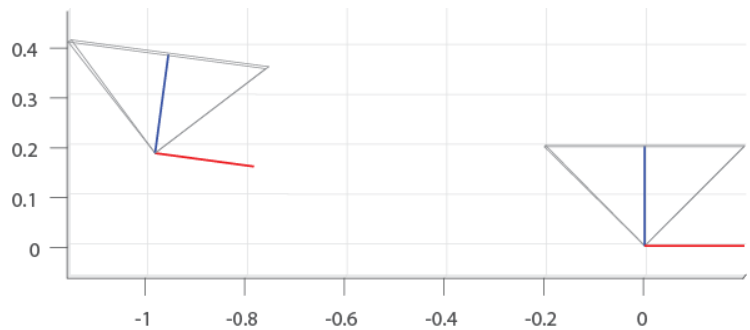
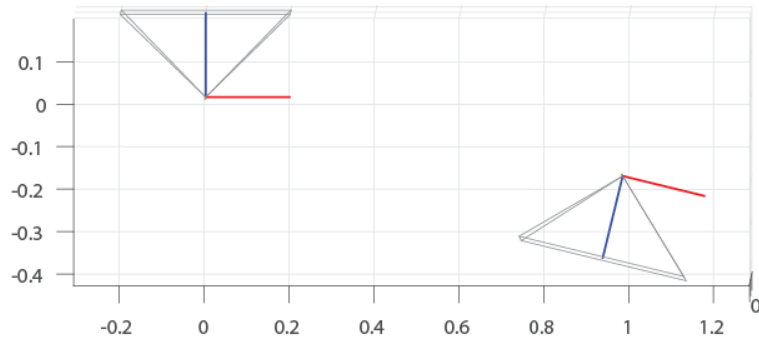
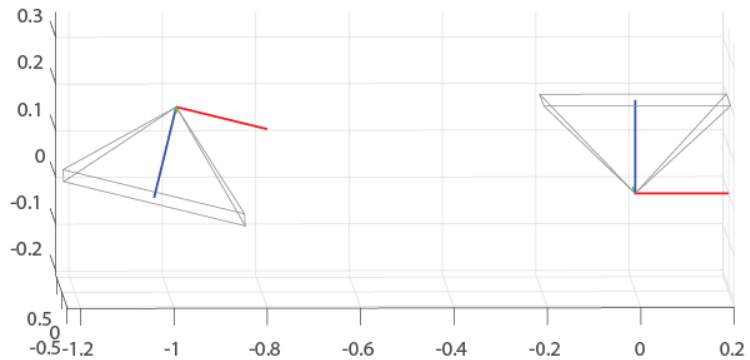


TRIANGULATION

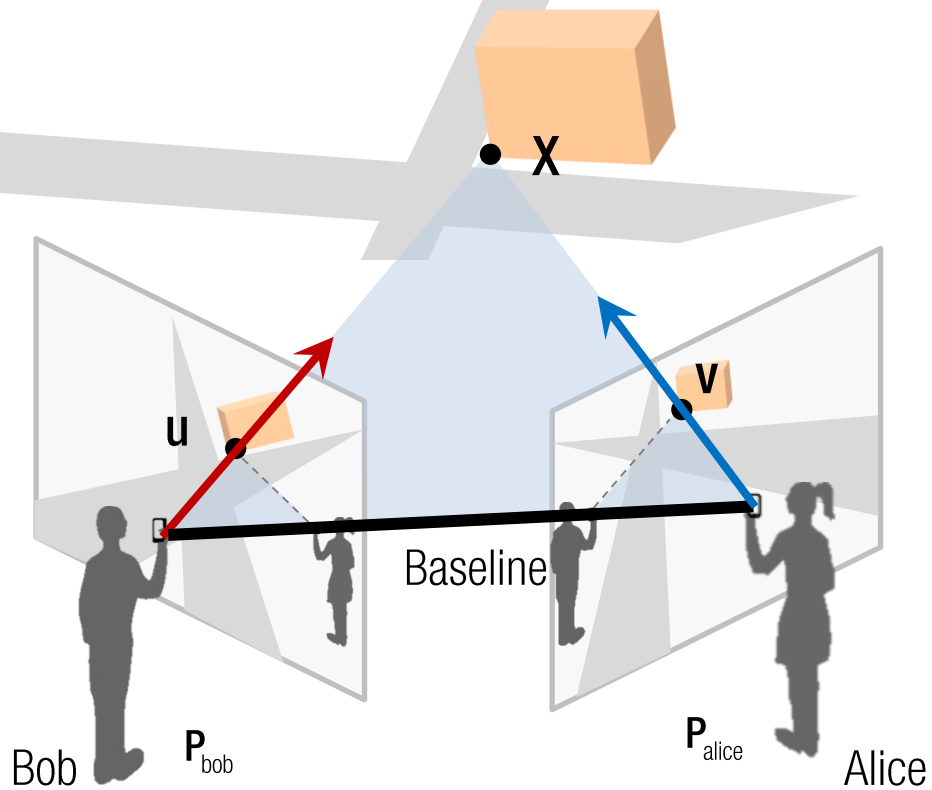
HYUN SOO PARK



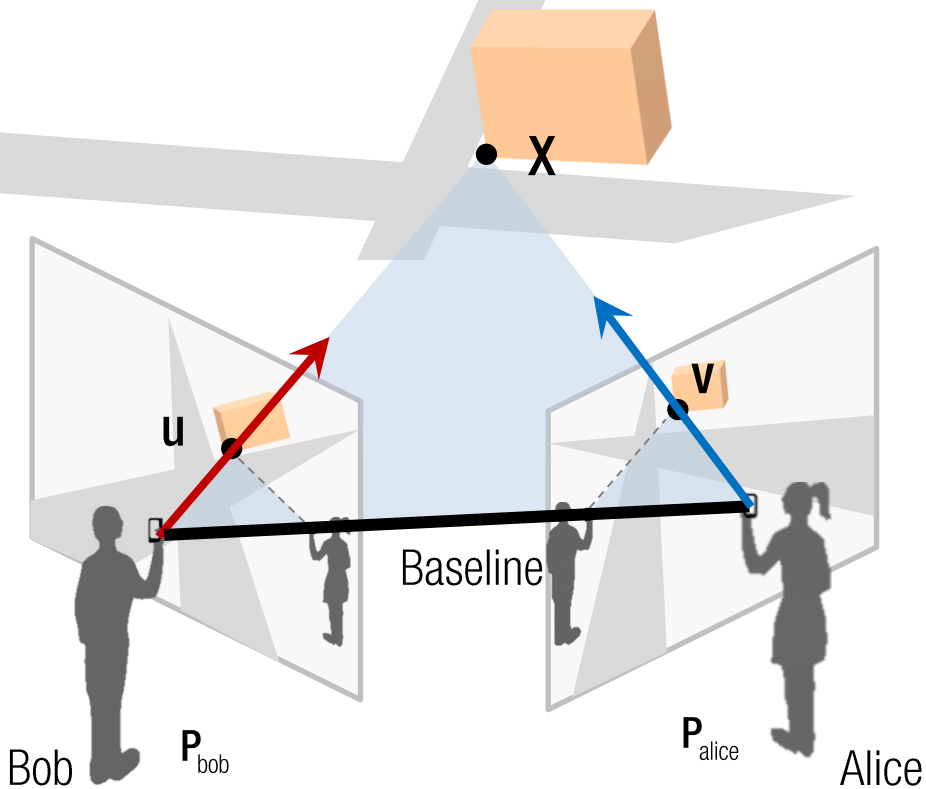
POSE DISAMBIGUATION



TRIANGULATION



TRIANGULATION

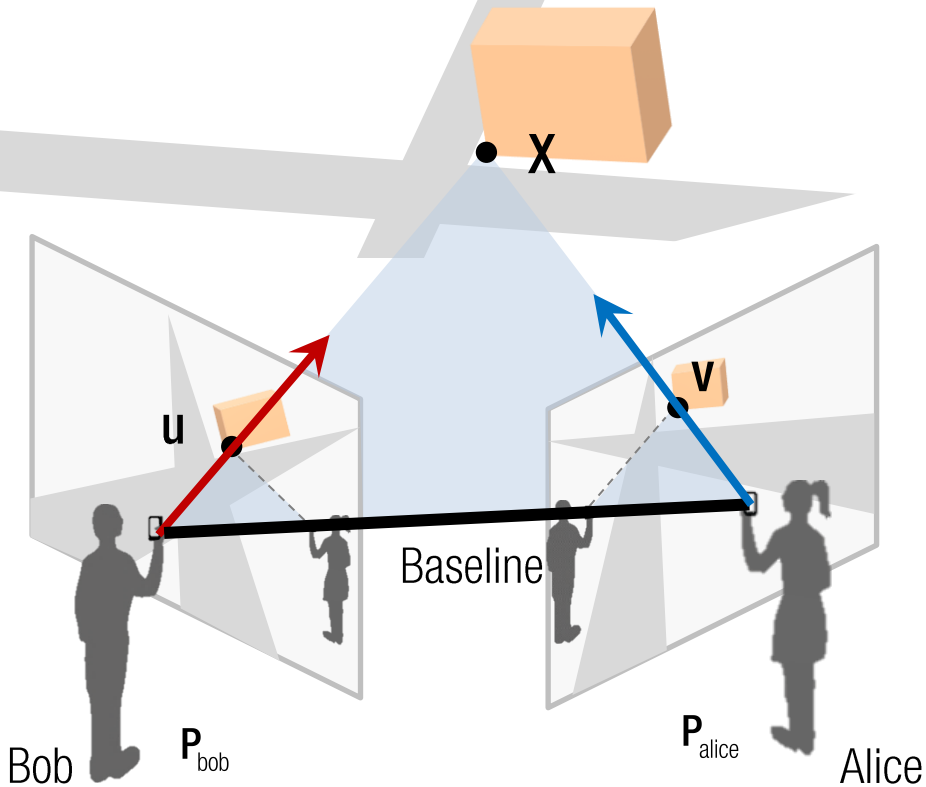


General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

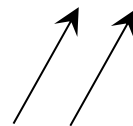
Two 3D vectors are parallel.

TRIANGULATION



General camera pose

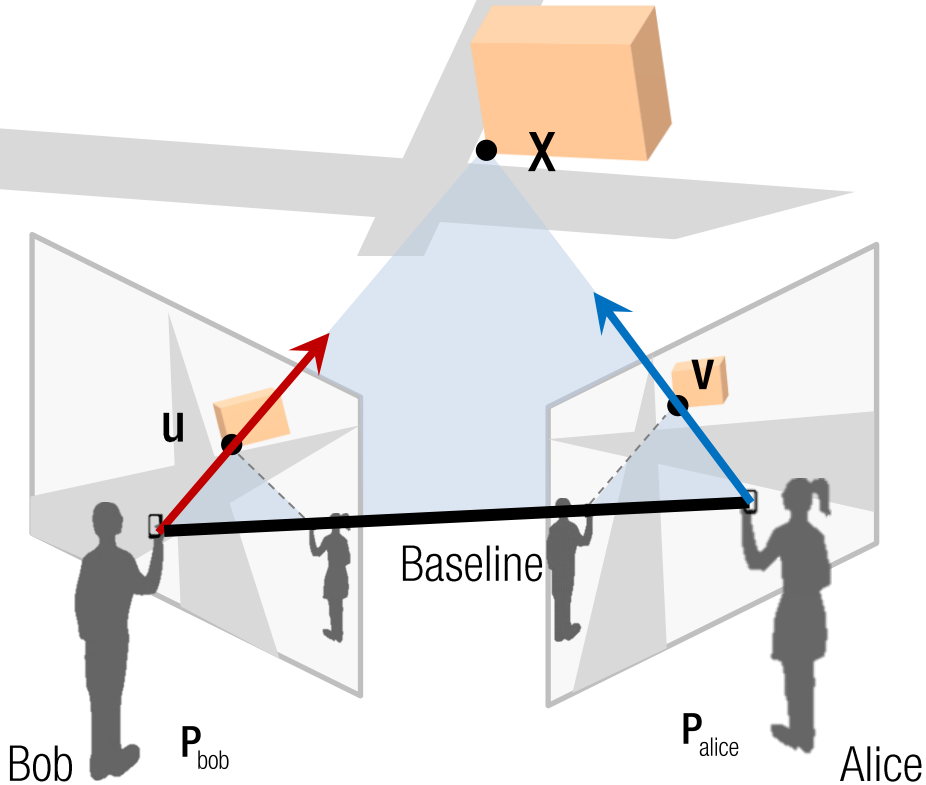
$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

TRIANGULATION



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

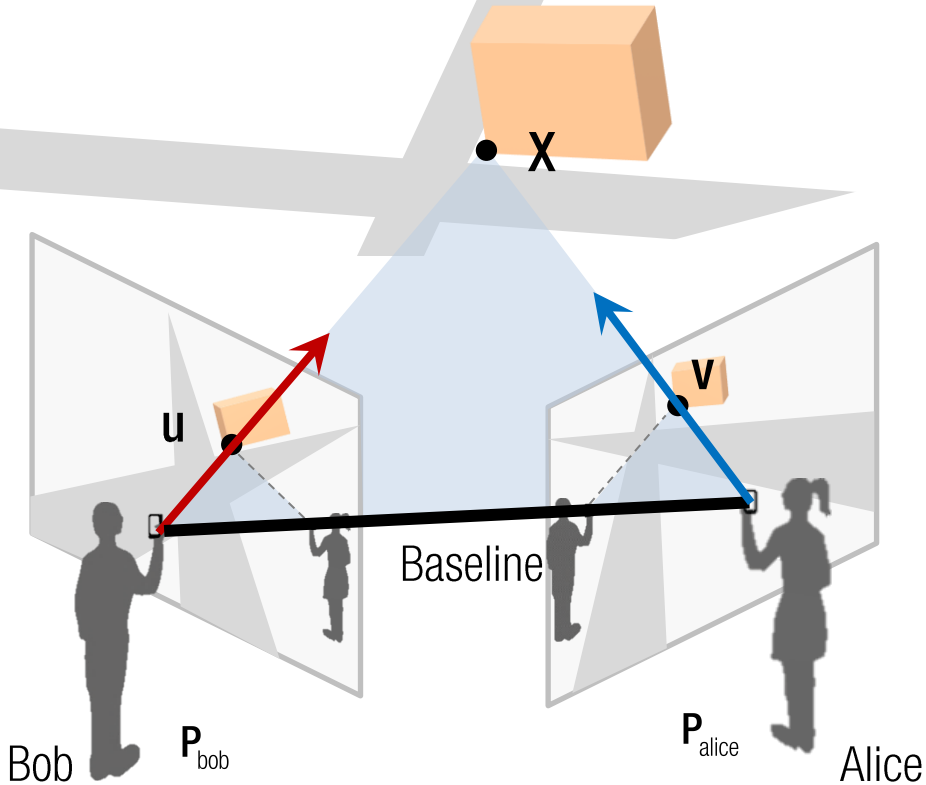
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

: Knowns
 : Unknowns

Skew-symmetric matrix

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \times = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix}$$

TRIANGULATION



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

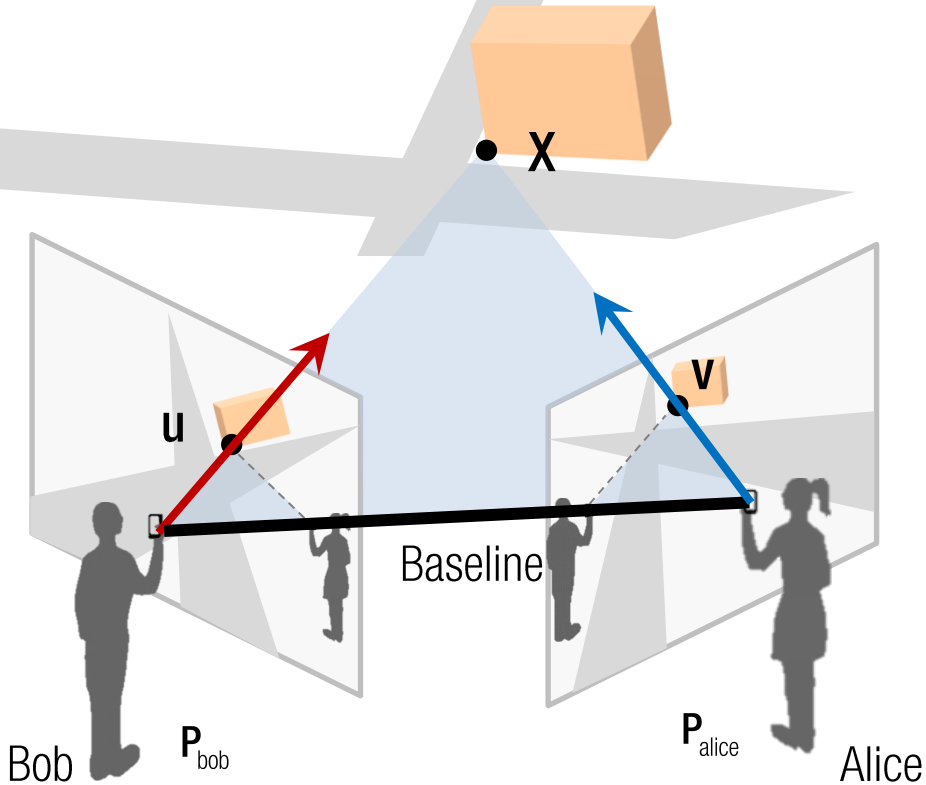
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

3x4

Can we solve for \mathbf{X} ? (single view reconstruction)
Why not?

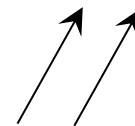
: Knowns
 : Unknowns

TRIANGULATION



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



Two 3D vectors are parallel.

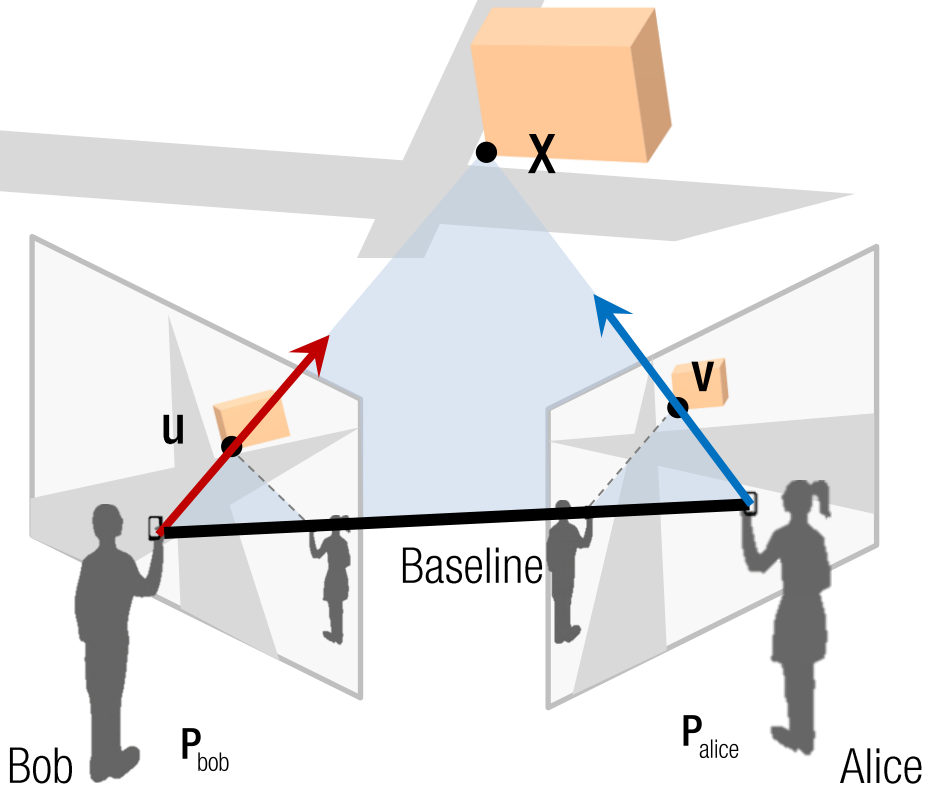
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

2x4

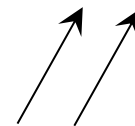
: Knowns
 : Unknowns

TRIANGULATION



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



Two 3D vectors are parallel.

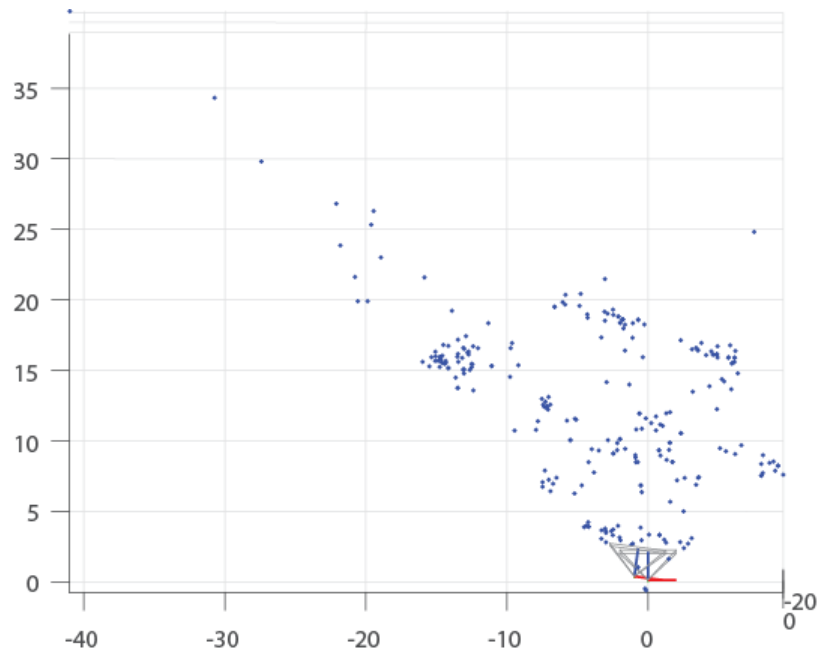
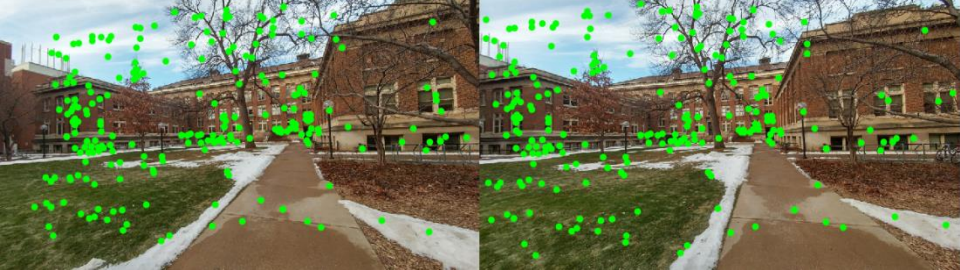
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{alice}}$$

4x4

: Knowns
 : Unknowns



$$\begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} = \mathbf{0}$$
$$\begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{alice}} \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} = \mathbf{0}$$

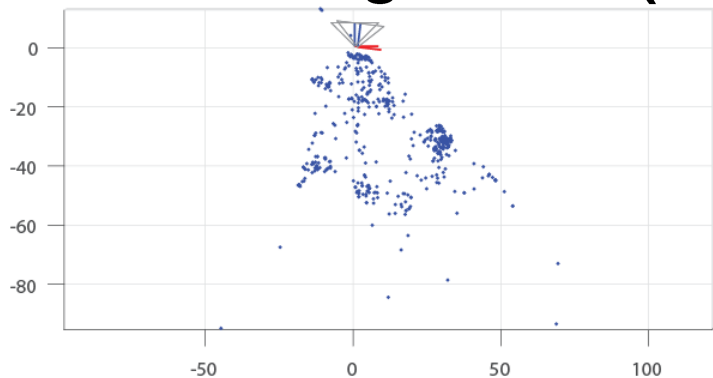
Camera Pose Disambiguation (Cheirality)

Cheirality condition:

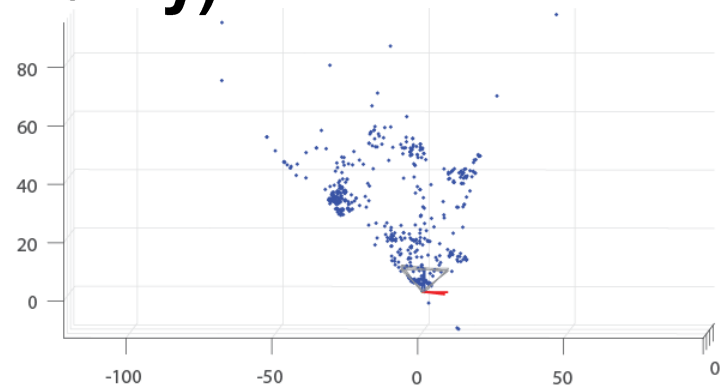
$$\mathbf{r}_3^T (\mathbf{X} - \mathbf{C}) > 0$$

where

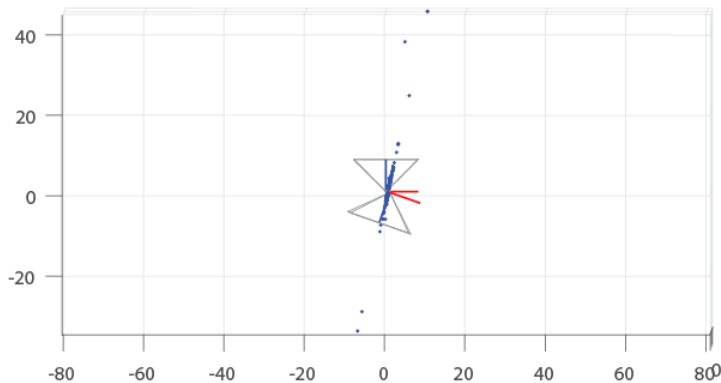
$$\mathbf{R} = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix}$$



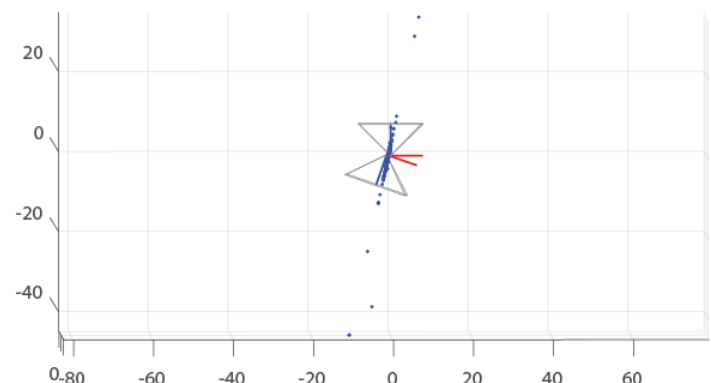
(a) nValid = 10



(b) nValid = 488

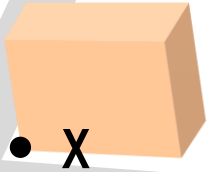


(c) nValid = 0

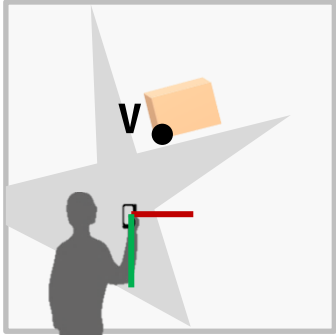
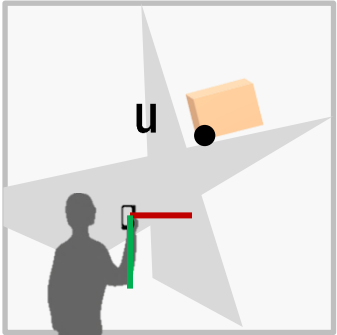


(d) nValid = 0

Special Case: Stereo



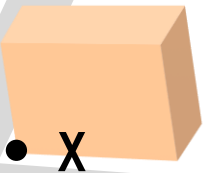
• Same orientation



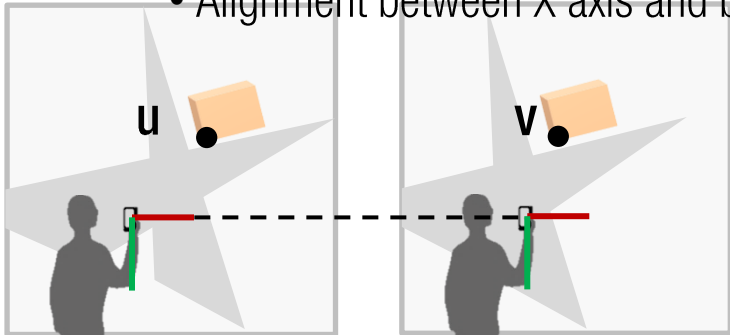
Bob

Mike

Special Case: Stereo



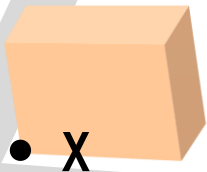
- Same orientation
- Alignment between X axis and baseline



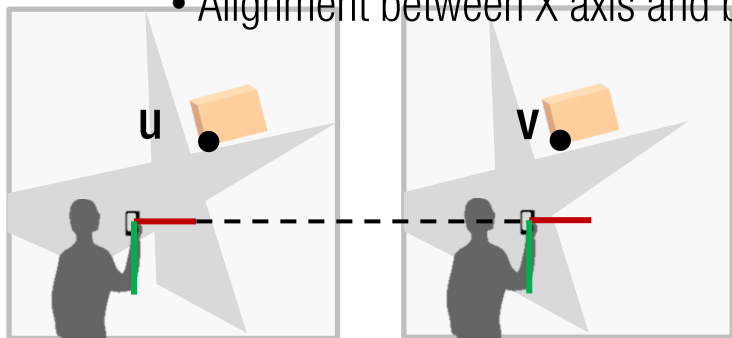
Bob

Mike

Special Case: Stereo



- Same orientation
- Alignment between X axis and baseline

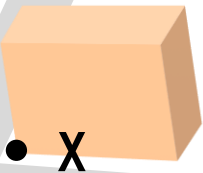


Bob

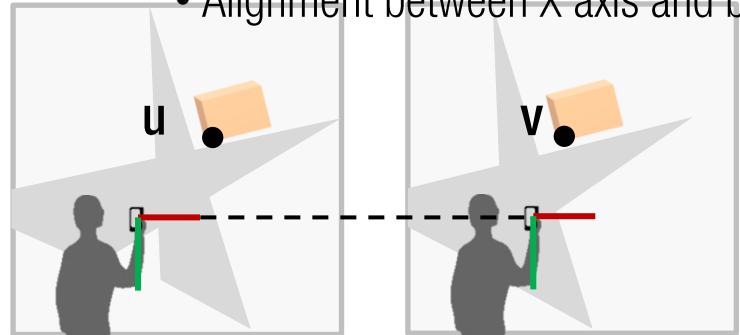
Mike



Special Case: Stereo



- Same orientation
- Alignment between X axis and baseline



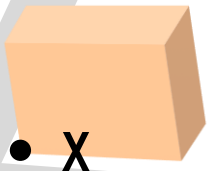
Bob

Mike

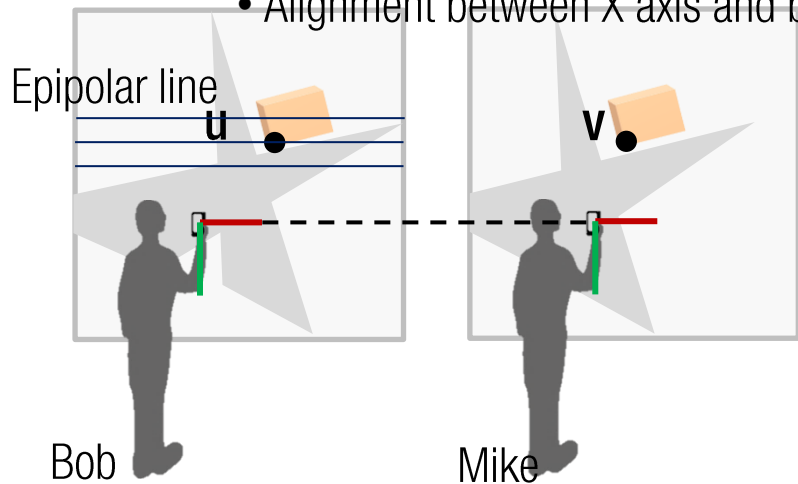


Top view

Special Case: Stereo



- Same orientation
- Alignment between X axis and baseline



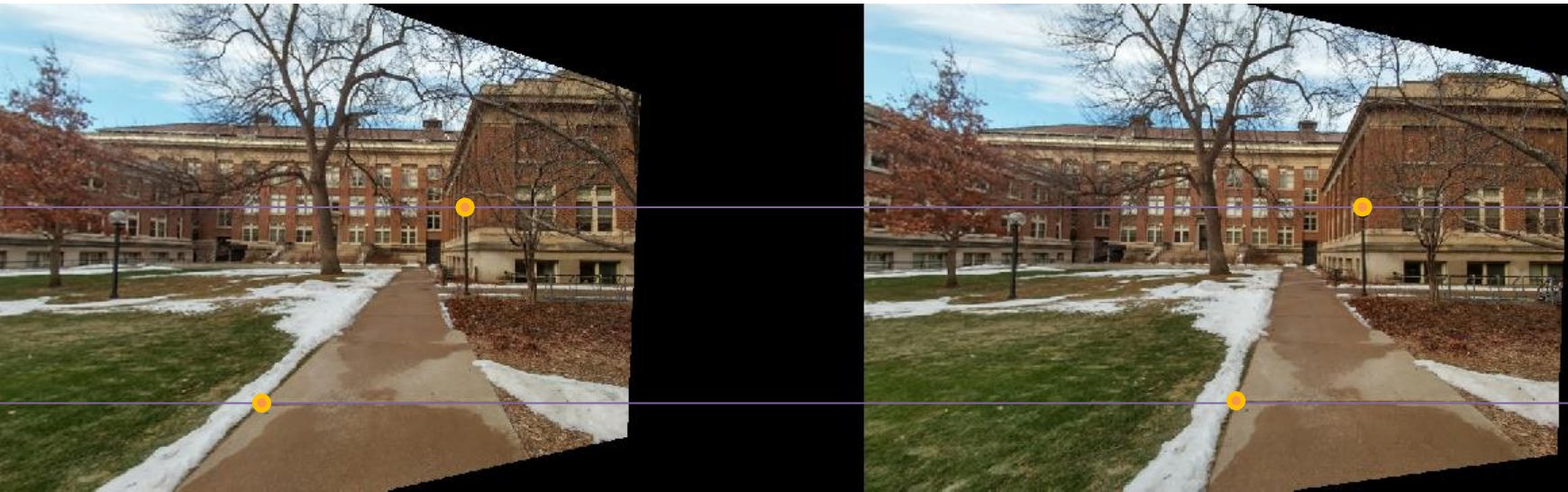
Epipole?
Point at infinity



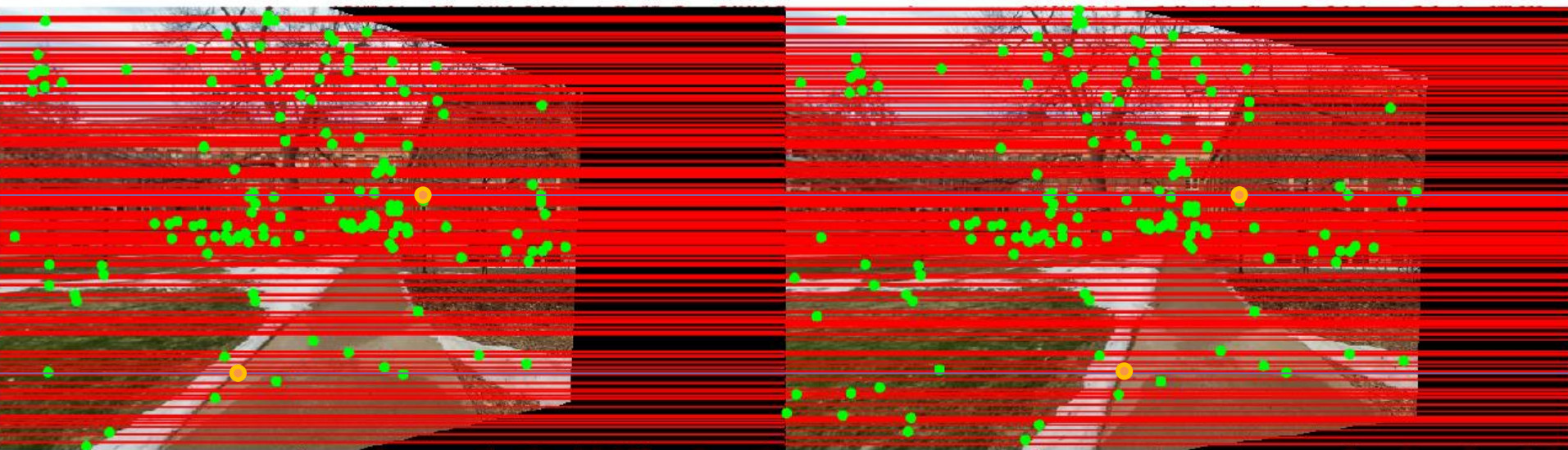
Special Case: Stereo



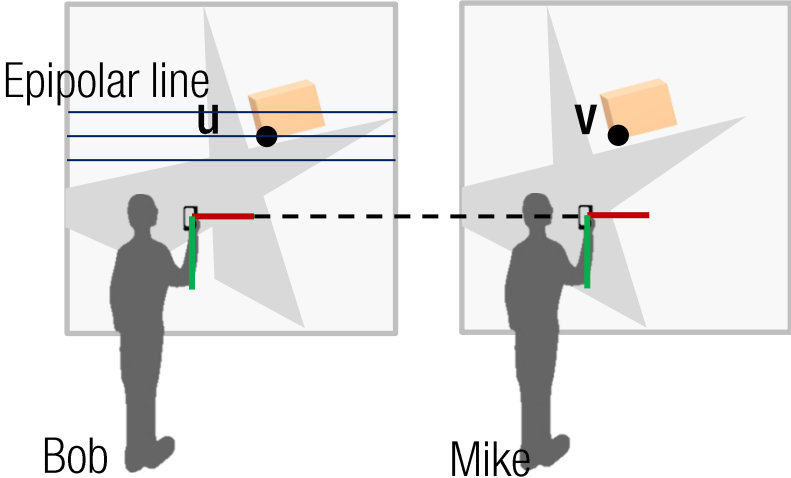
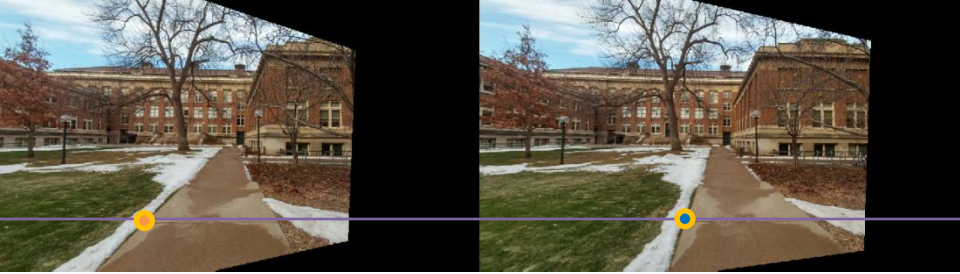
Special Case: Stereo



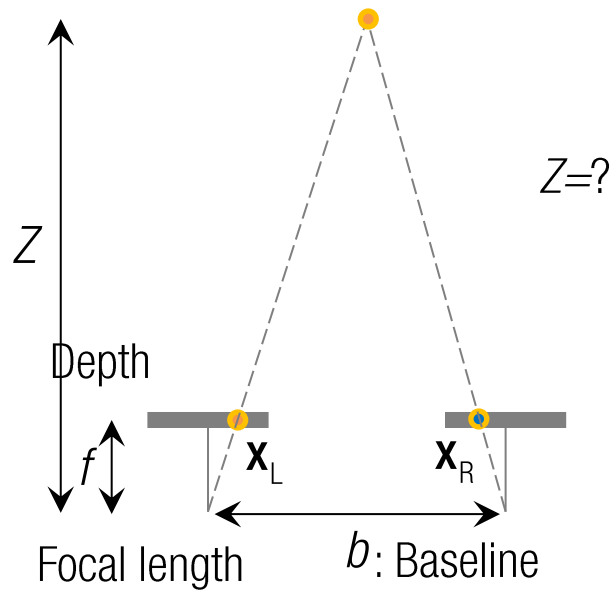
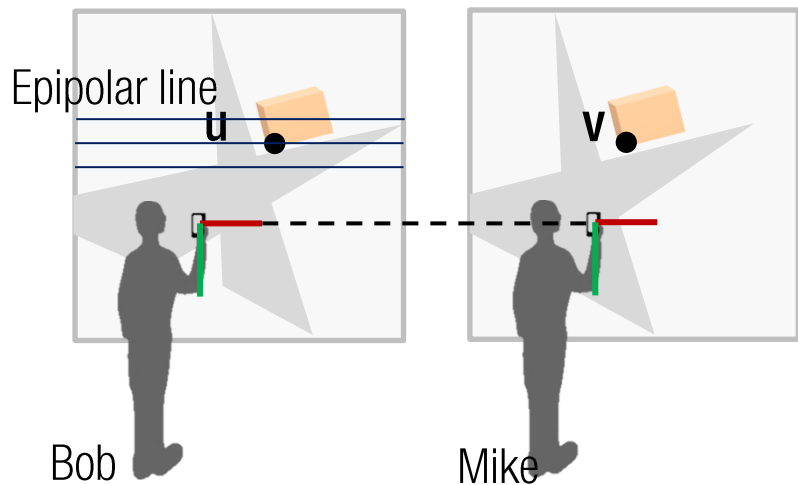
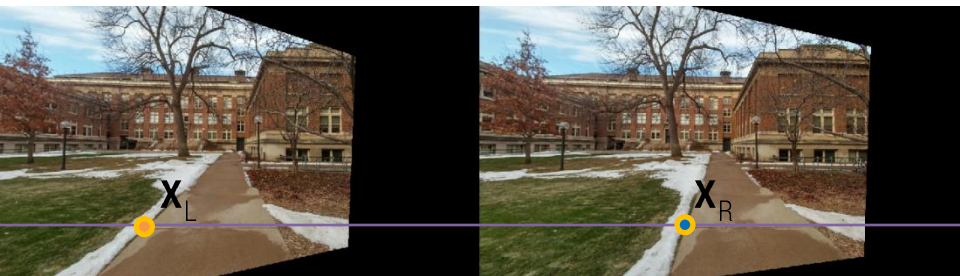
Special Case: Stereo



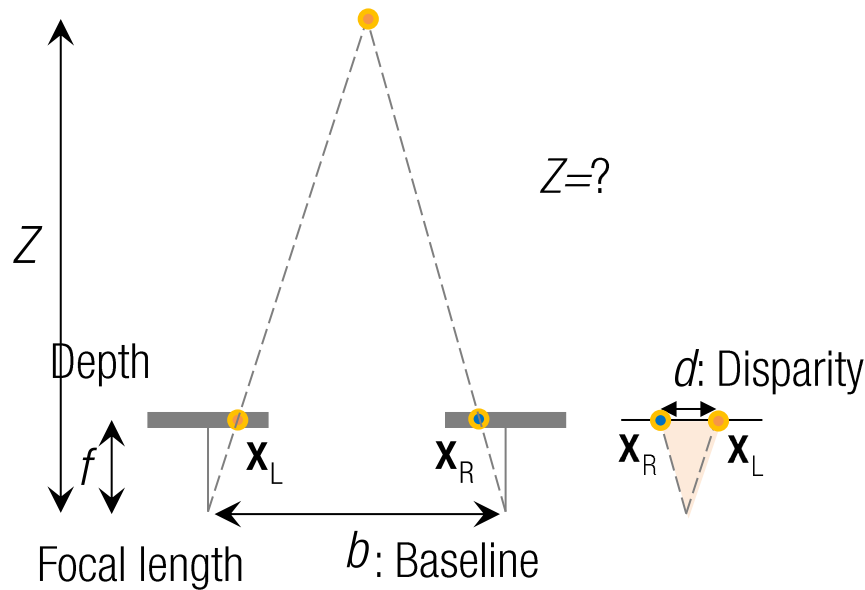
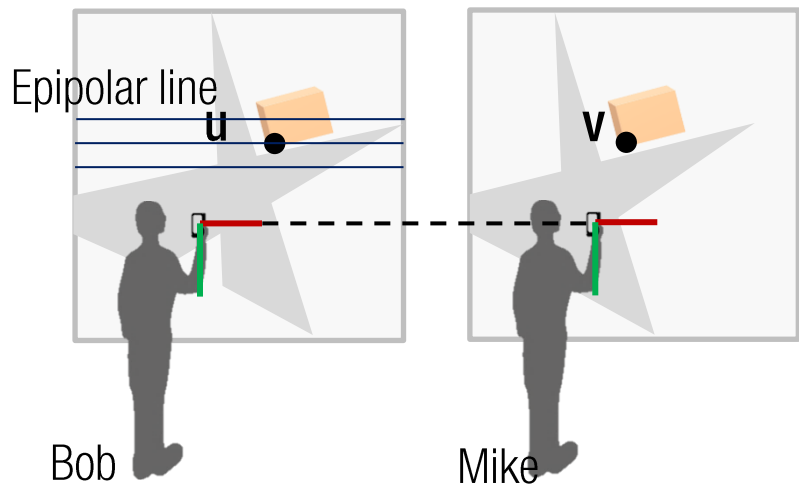
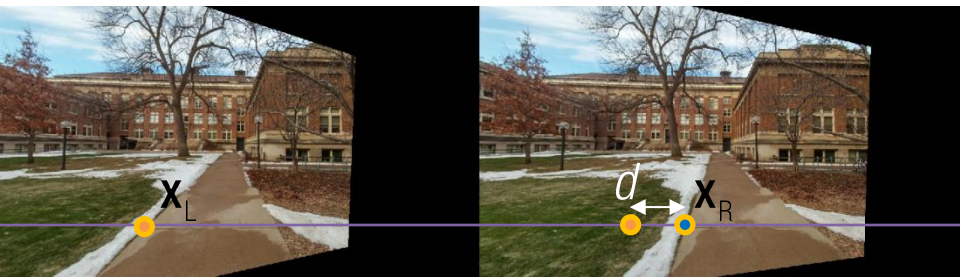
Special Case: Stereo



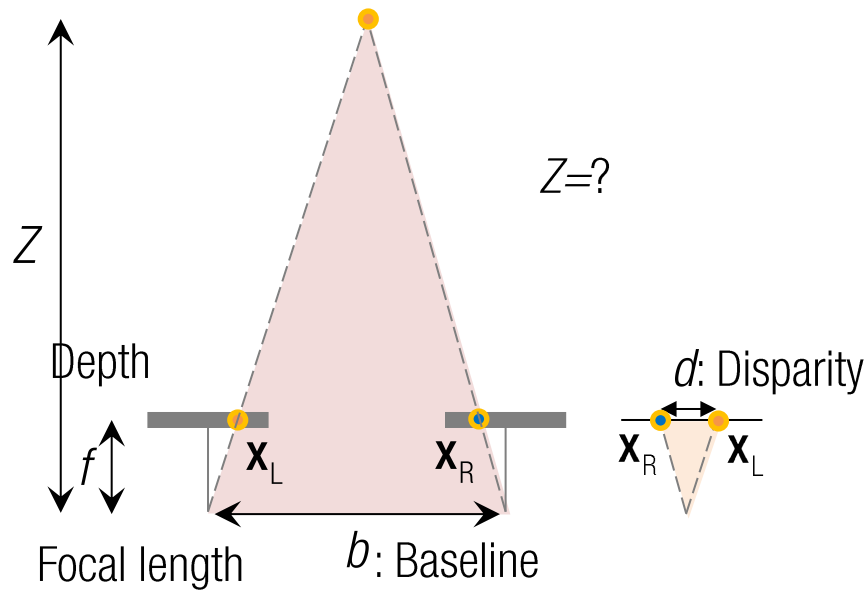
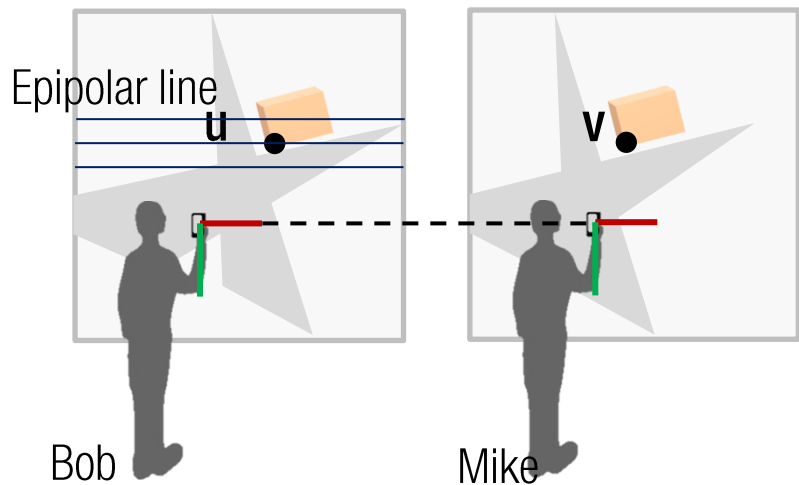
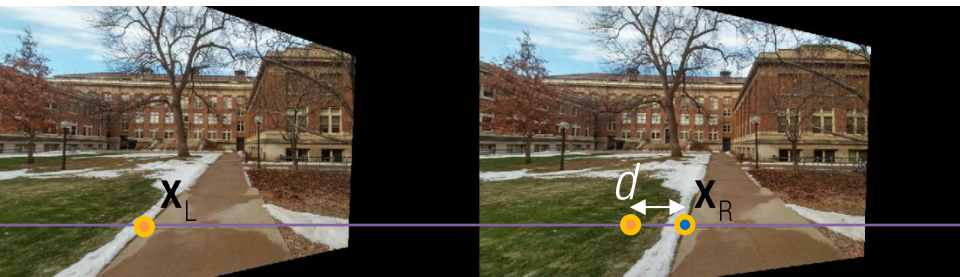
Special Case: Stereo



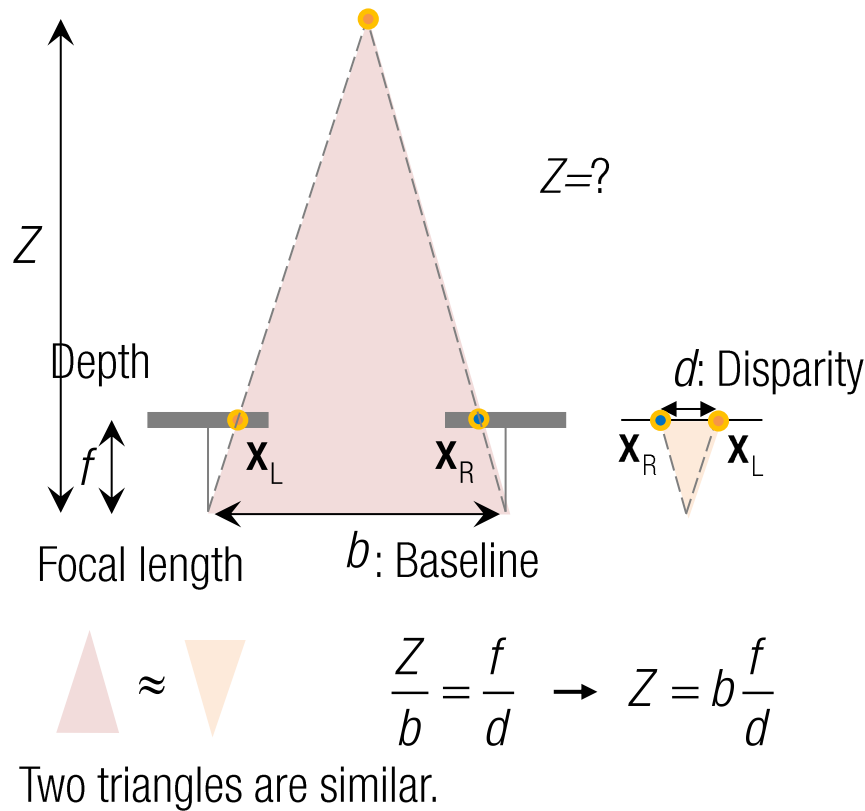
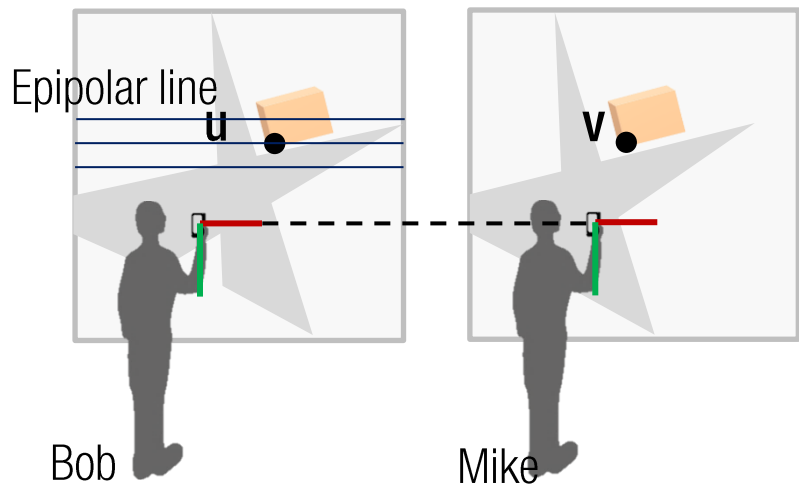
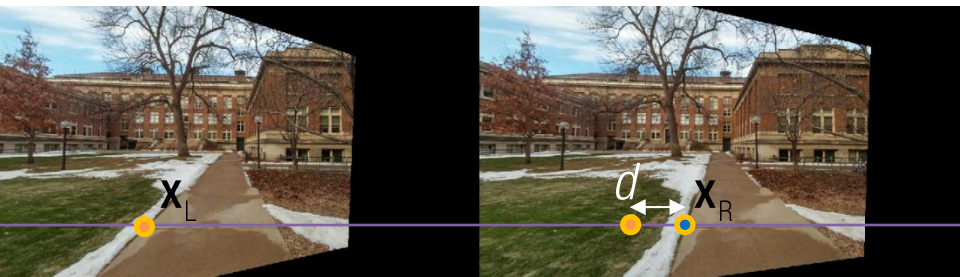
Special Case: Stereo



Special Case: Stereo



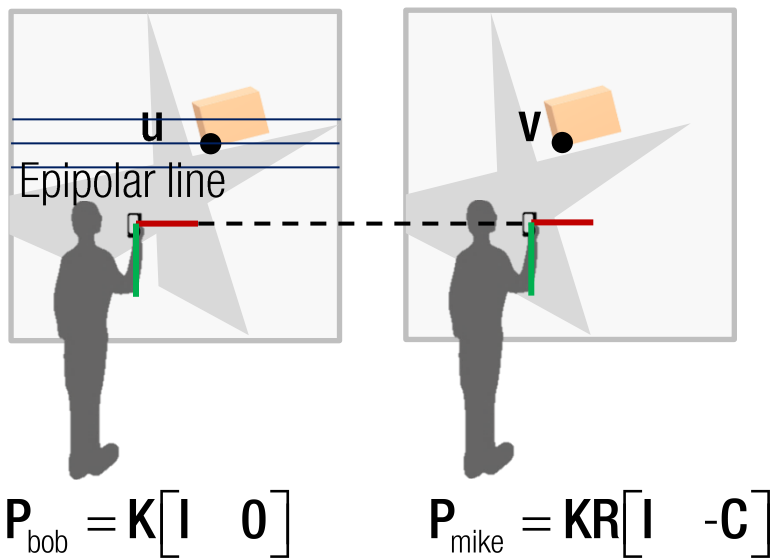
Special Case: Stereo



Special Case: Stereo



Special Case: Stereo



- Same orientation

$$\mathbf{R}_{\text{rect}} = \begin{bmatrix} \mathbf{r}_x^T \\ \mathbf{r}_y^T \\ \mathbf{r}_z^T \end{bmatrix}$$

- Alignment between X axis and baseline

$$\mathbf{r}_x = \frac{\mathbf{C}}{\|\mathbf{C}\|}$$

$$\mathbf{r}_z = \frac{\tilde{\mathbf{r}}_z - (\tilde{\mathbf{r}}_z \cdot \mathbf{r}_x) \mathbf{r}_x}{\|\tilde{\mathbf{r}}_z - (\tilde{\mathbf{r}}_z \cdot \mathbf{r}_x) \mathbf{r}_x\|}$$

: Orthogonal projection

$$\mathbf{r}_y = \mathbf{r}_z \times \mathbf{r}_x$$

$$\text{where } \tilde{\mathbf{r}}_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

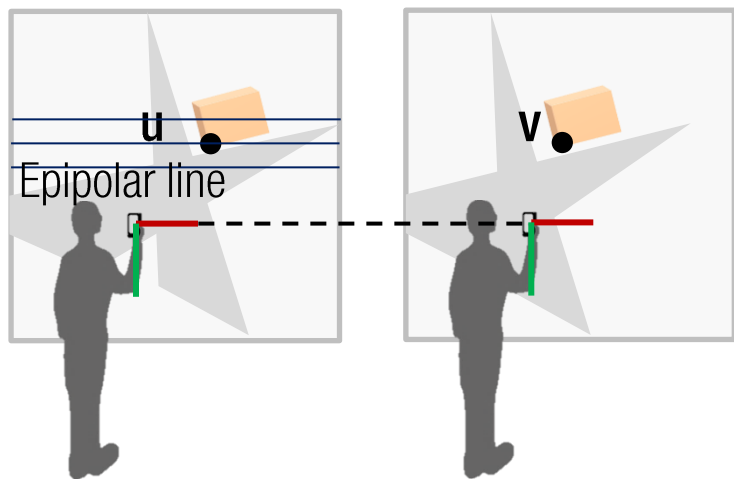
Special Case: Stereo



Homography by pure rotation: R_{rect}

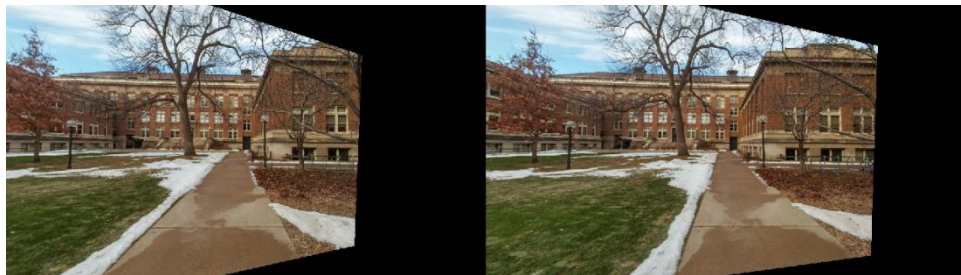
$$H_{\text{bob}} = KR_{\text{rect}}K^{-1}$$

$$H_{\text{mike}} = KR_{\text{rect}}R^TK^{-1}$$

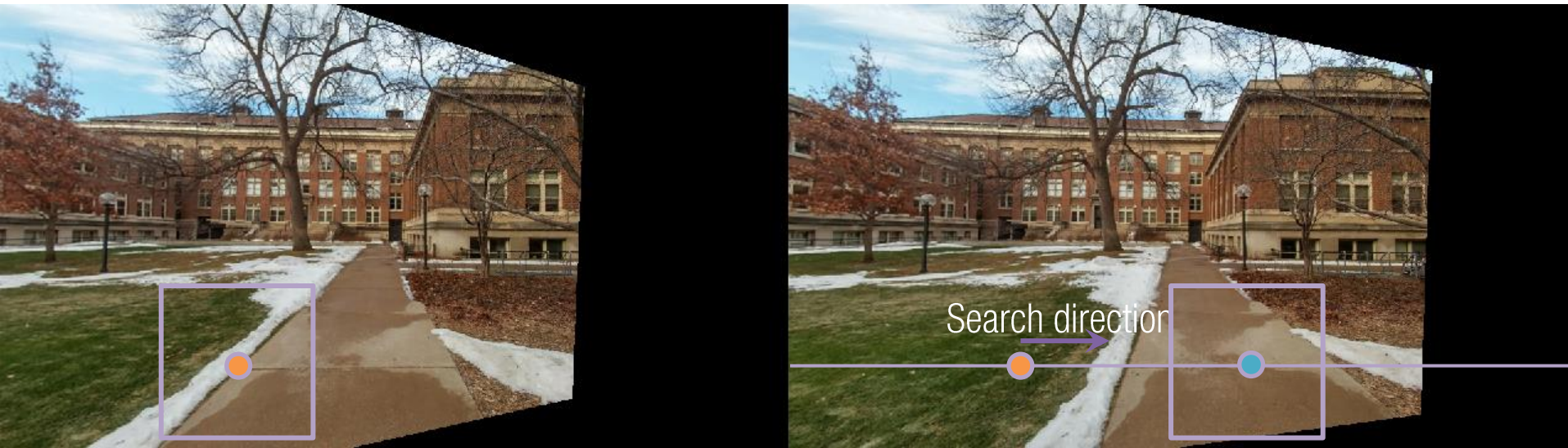


$$P_{\text{bob}} = K \begin{bmatrix} I & 0 \end{bmatrix}$$

$$P_{\text{mike}} = KR \begin{bmatrix} I & -C \end{bmatrix}$$



Dense Feature Matching using SIFT Flow

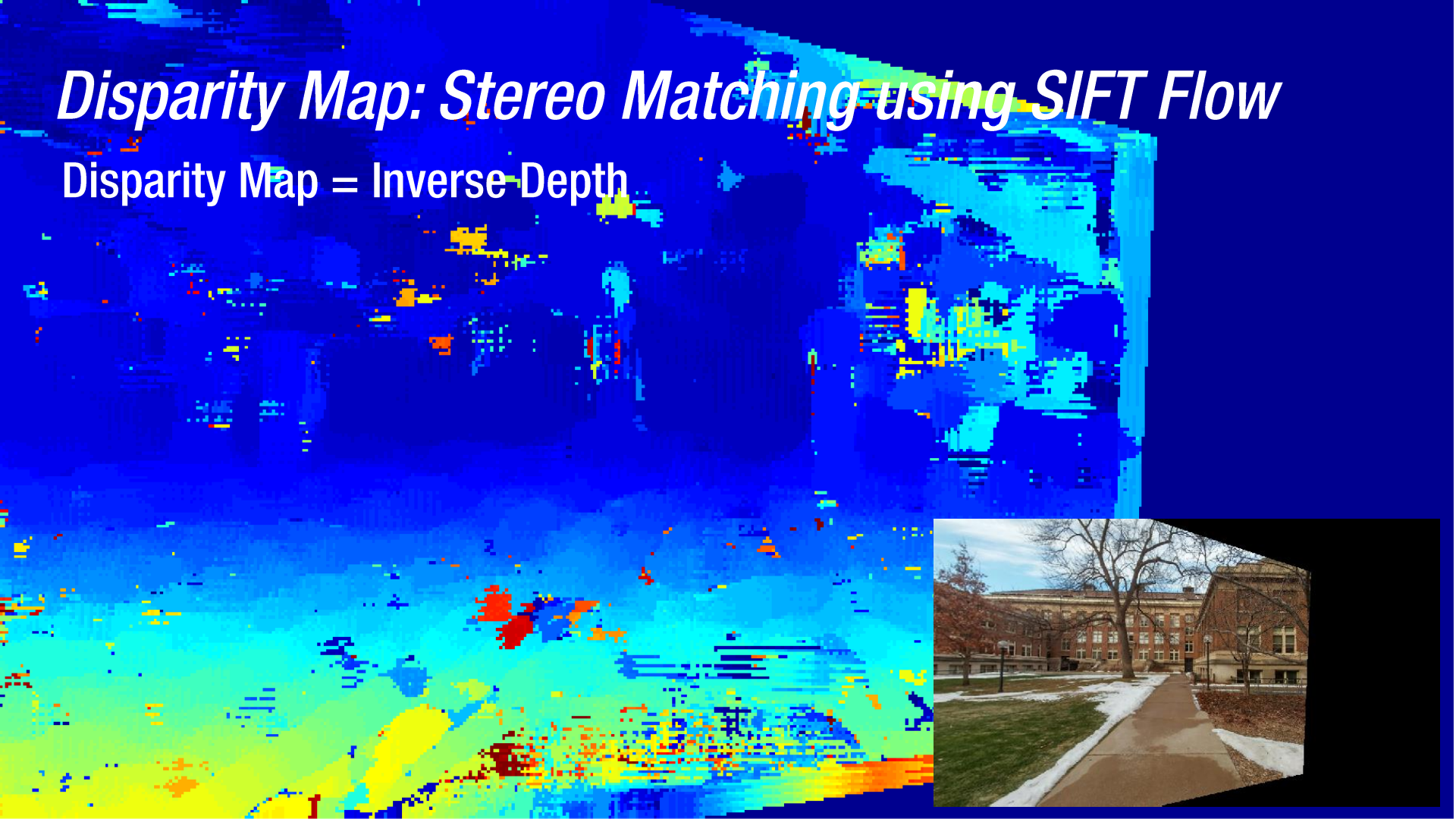


Find a minimum distance over the epipolar line

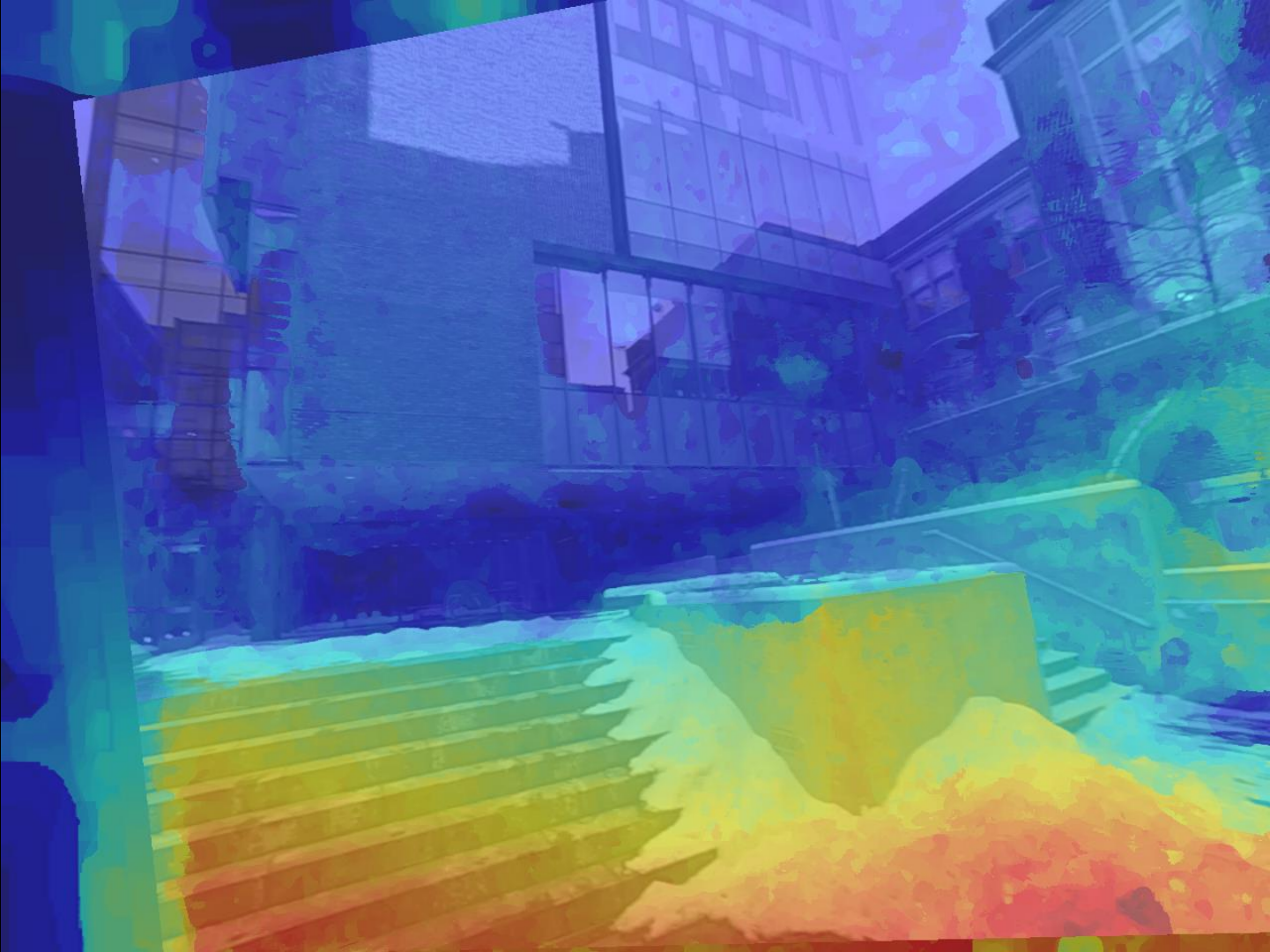


Disparity Map: Stereo Matching using SIFT Flow

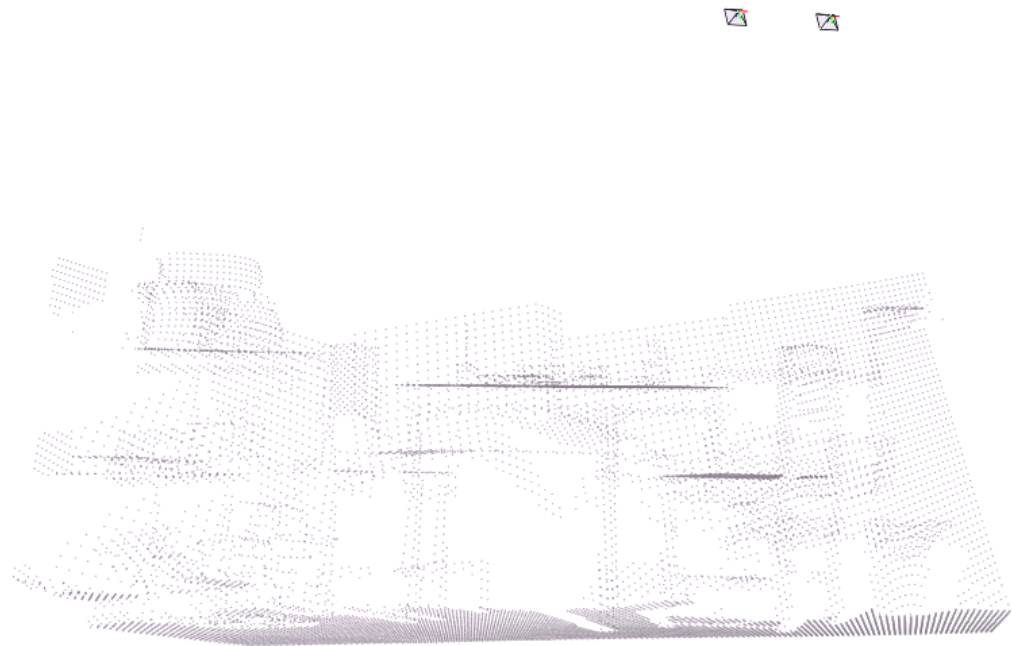
Disparity Map = Inverse Depth











EgoMotion Dataset (outdoor)



Dense Reconstruction using a Monocular Camera



Structure from Motion (SfM)

COLMAP

3.6

Search docs

Installation

Tutorial

Database Format

Camera Models

Output Format

Datasets

Graphical User Interface

Command-line Interface

Frequently Asked Questions

Changelog

Contribution

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Bibliography

Docs » COLMAP

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COLMAP



Sparse model of central Rome using 21K photos produced by COLMAP's SfM pipeline.



Dense models of several landmarks produced by COLMAP's MVS pipeline.

About

<https://www.youtube.com/watch?v=mTBPGuPLI5Y>

https://www.youtube.com/watch?v=PySBQ8Q_R8k

<https://www.insidescience.org/video/motion-capture-inside-out>

<https://www.youtube.com/watch?v=SOpwHaQnRSY>