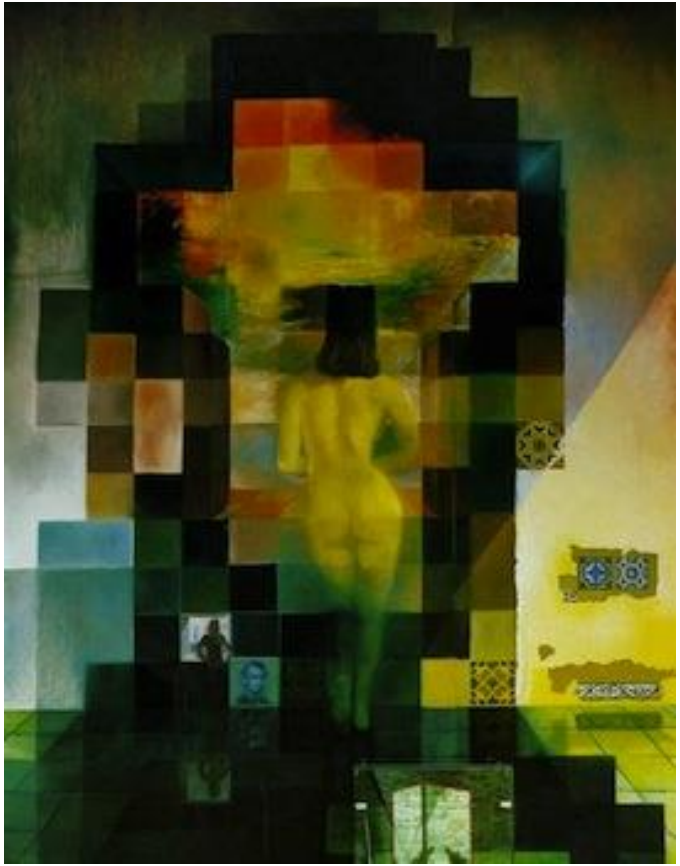
Image Pyramid

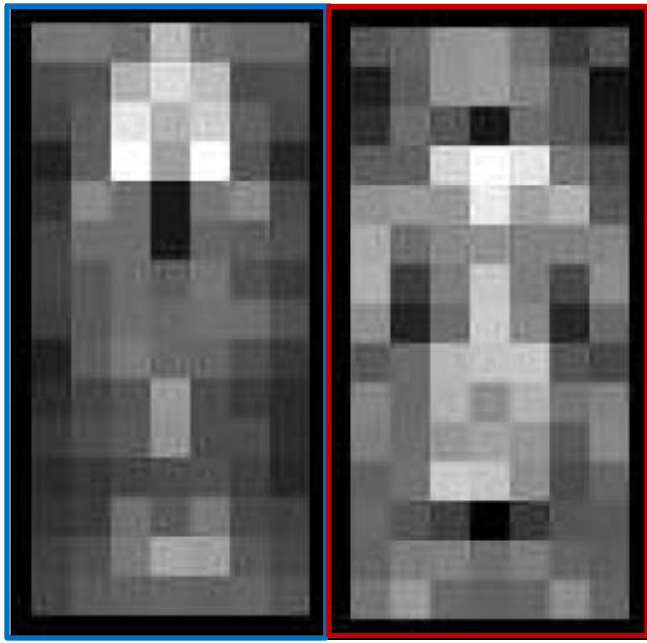
Hyun Soo Park

Salvador Dali, Abraham Lincoln

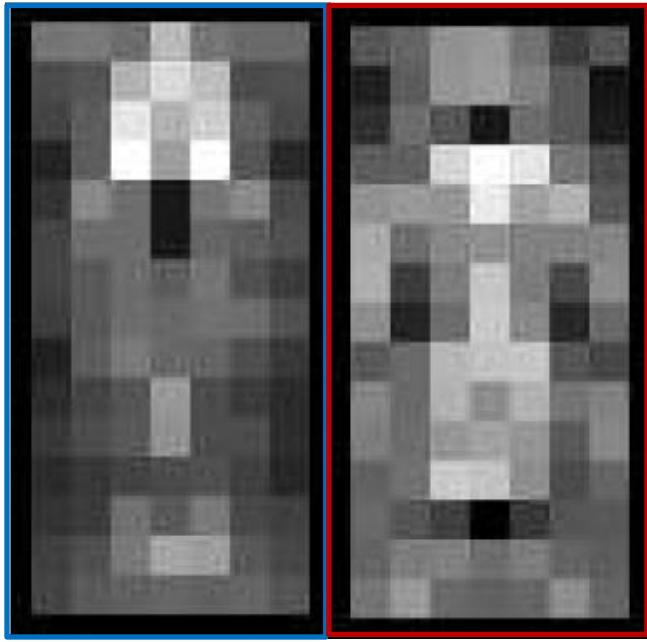


Salvador Dali, Abraham Lincoln

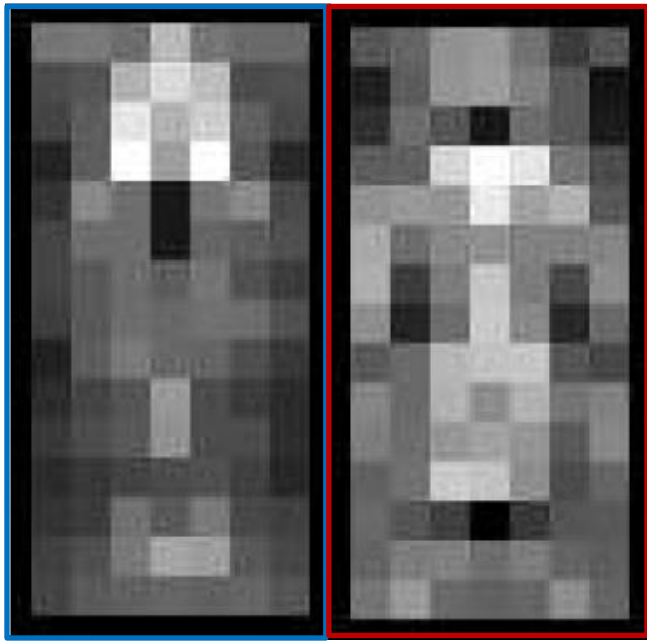
Recall: Object Recognition with HOG



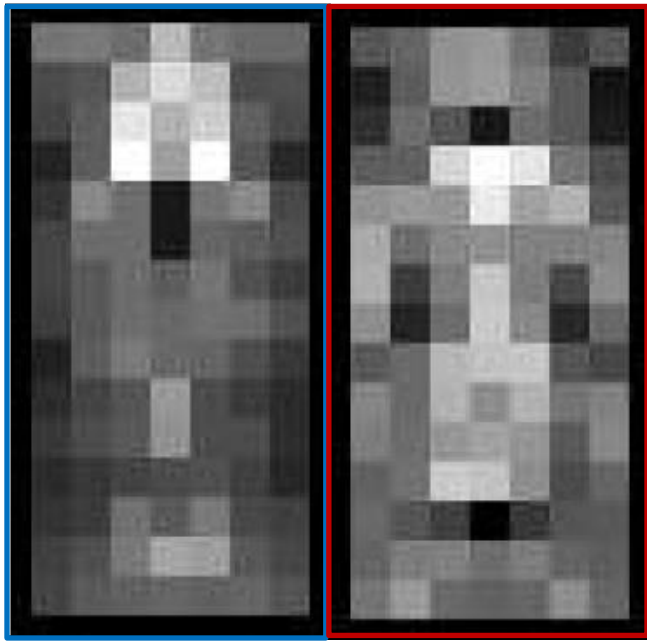
RECALL: OBJECT RECOGNITION WITH HOG



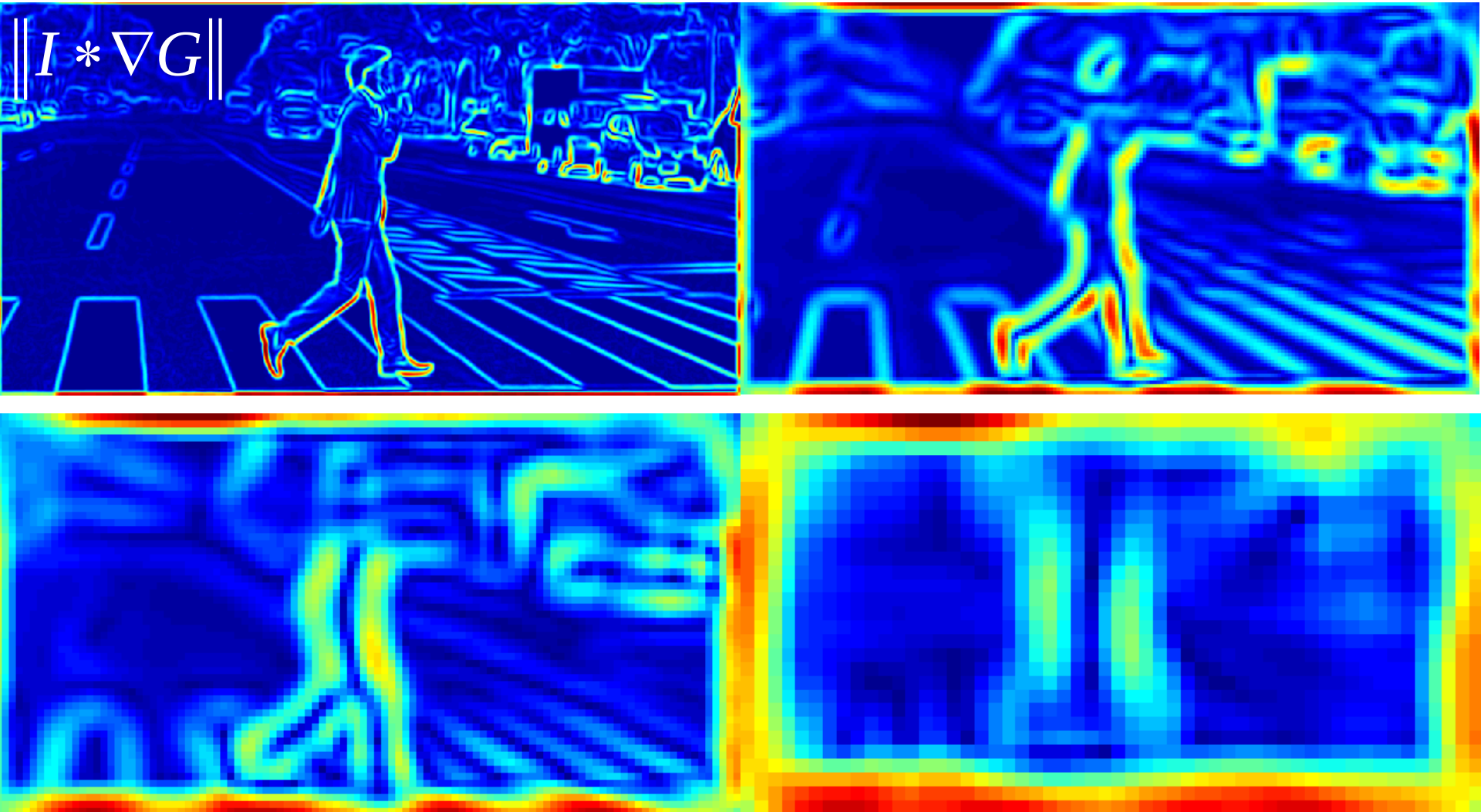
Recall: Object Recognition with HOG



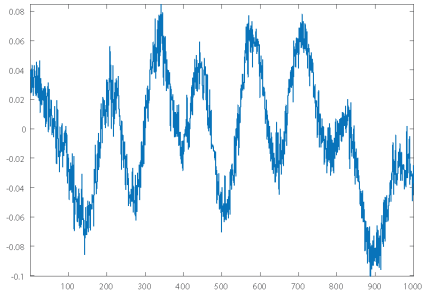
Recall: Object Recognition with HOG







$\|I * \nabla G\|$

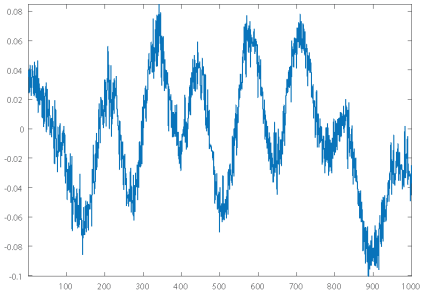
FOURIER TRANSFORM



Time signal

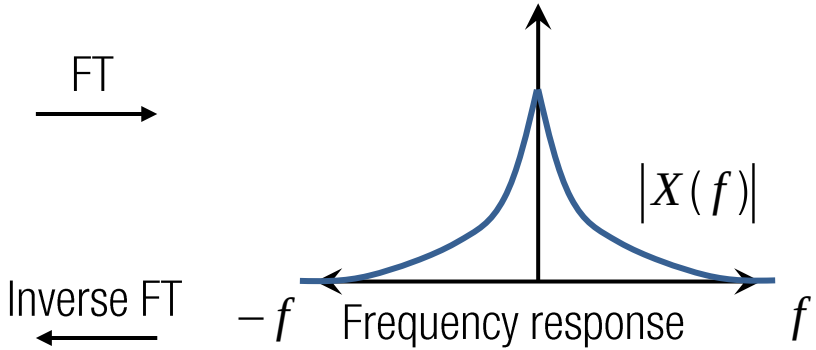
$x(t)$

FOURIER TRANSFORM



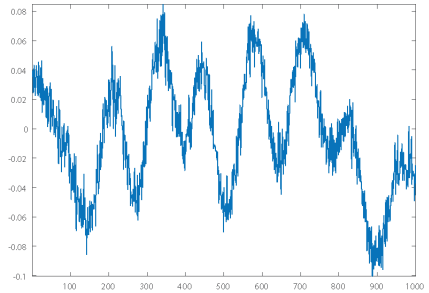
Time signal

$x(t)$



$X(f)$

FOURIER TRANSFORM



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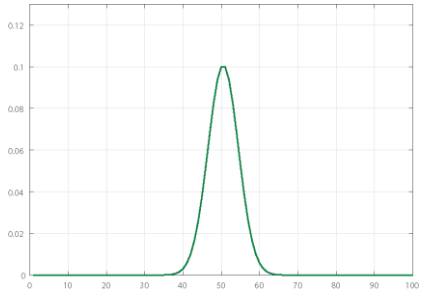
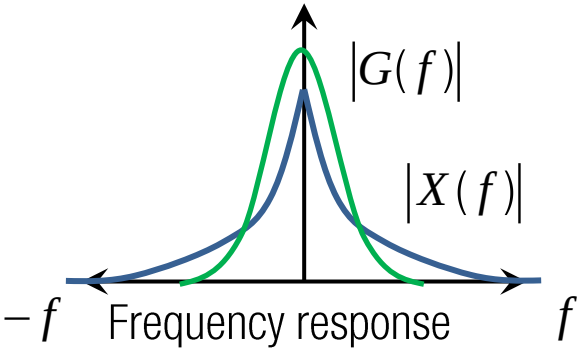


FT

Inverse FT

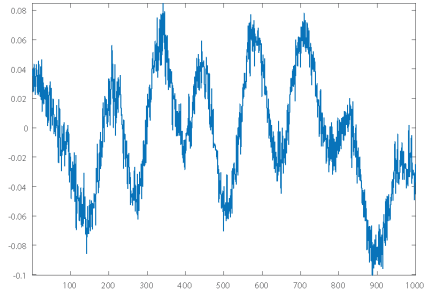


Time signal * Guassian filter

$x(t) * g(t)$

$X(f)\,G(f)$

FOURIER TRANSFORM



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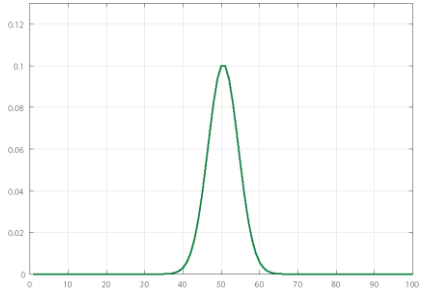
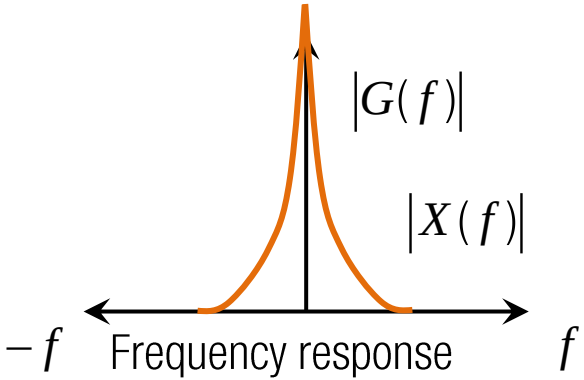


FT

Inverse FT



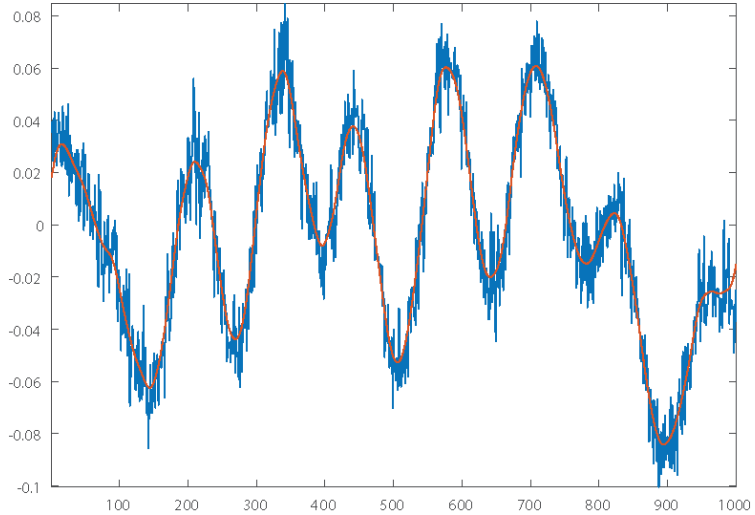
Time signal

Guassian filter

$x(t)$ * $g(t)$

$X(f)\,G(f)$

FOURIER TRANSFORM



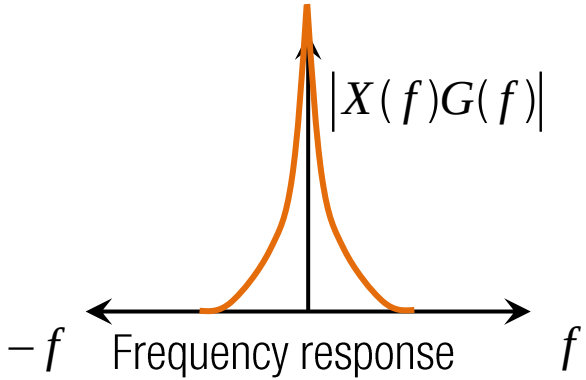
Time signal

Guassian filter

$x(t)$ * $g(t)$

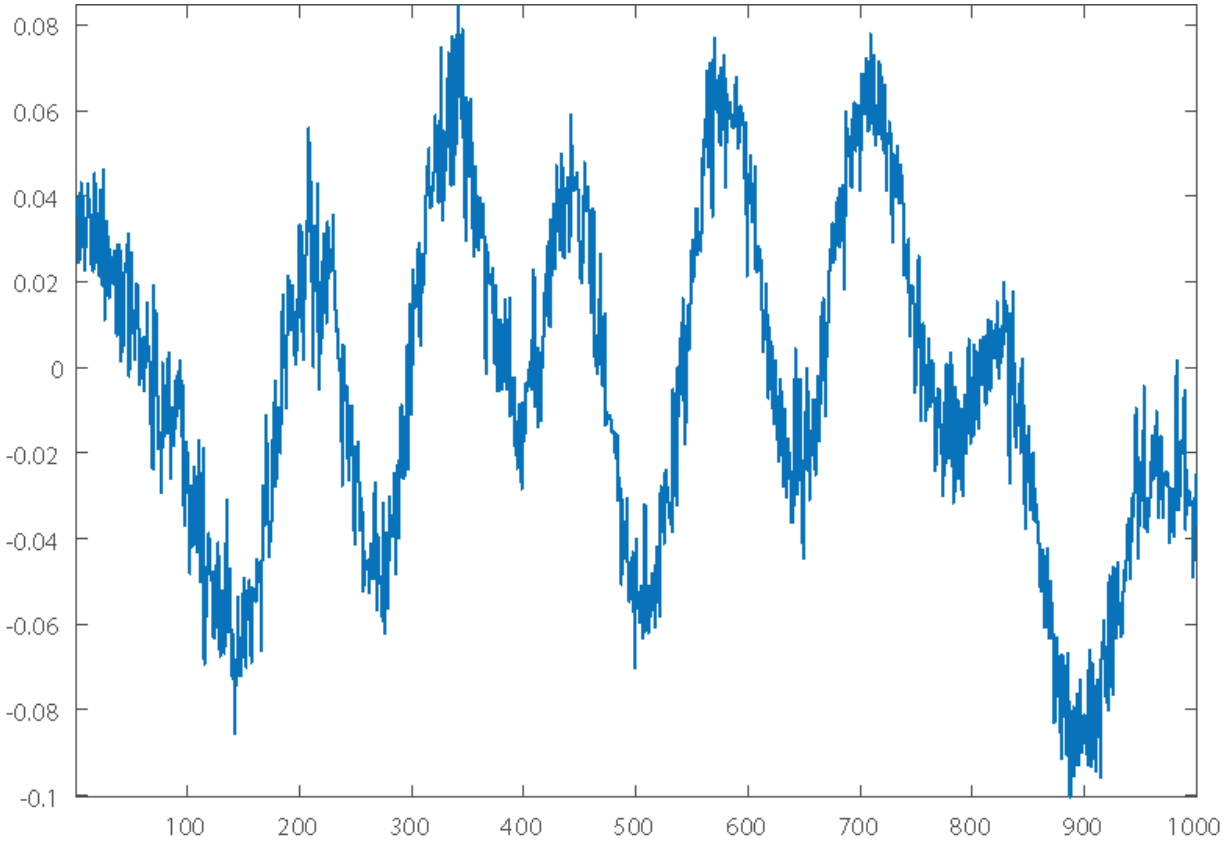
FT

Inverse FT

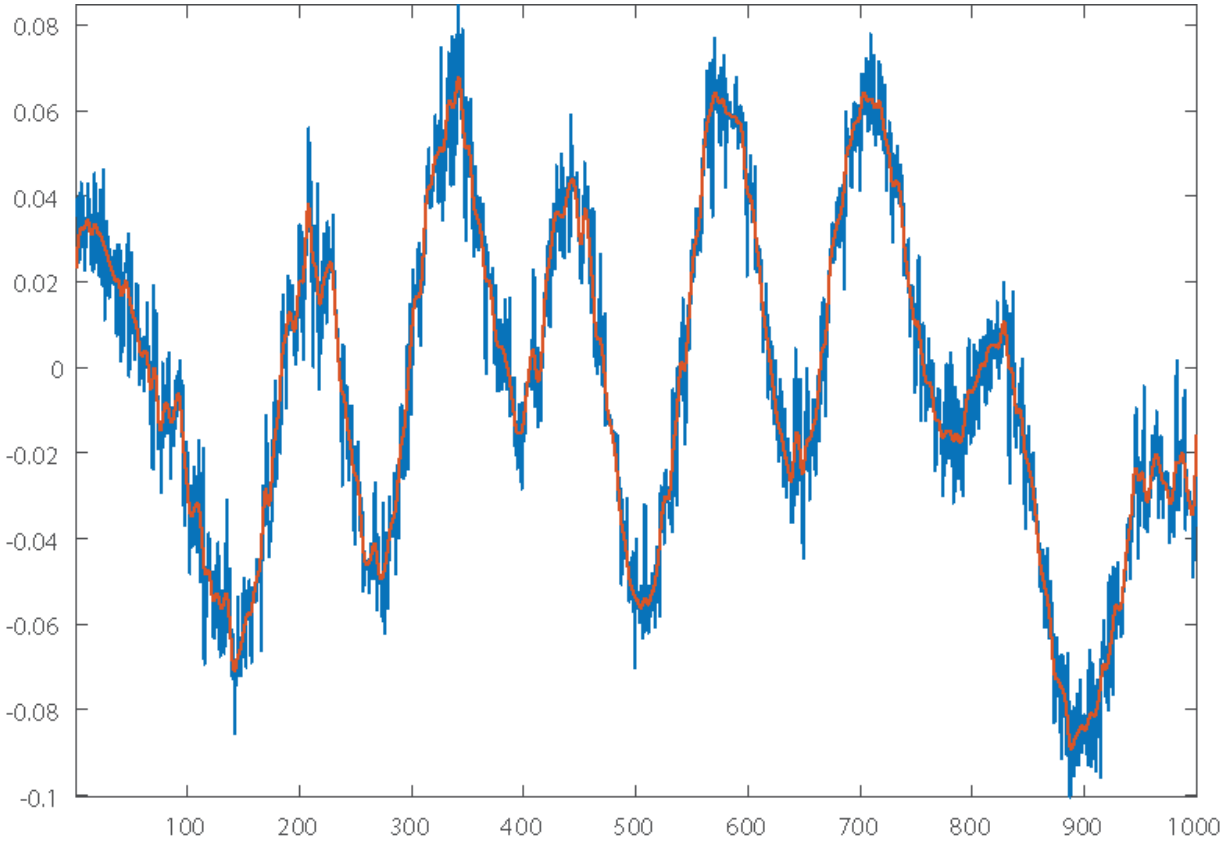


$X(f)\,G(f)$

Gaussian Filtering ~ Low-pass Filtering

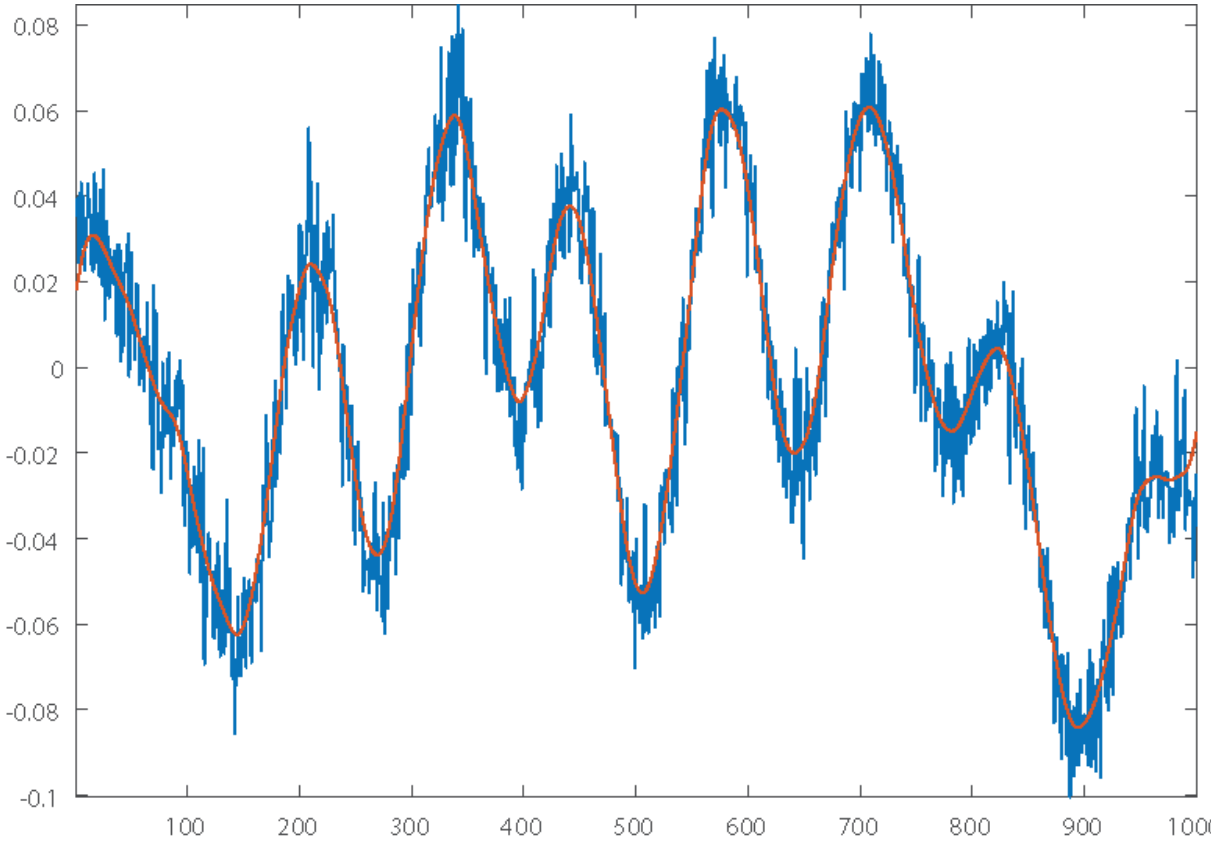


GAUSSIAN FILTERING ~ LOW-PASS FILTERING



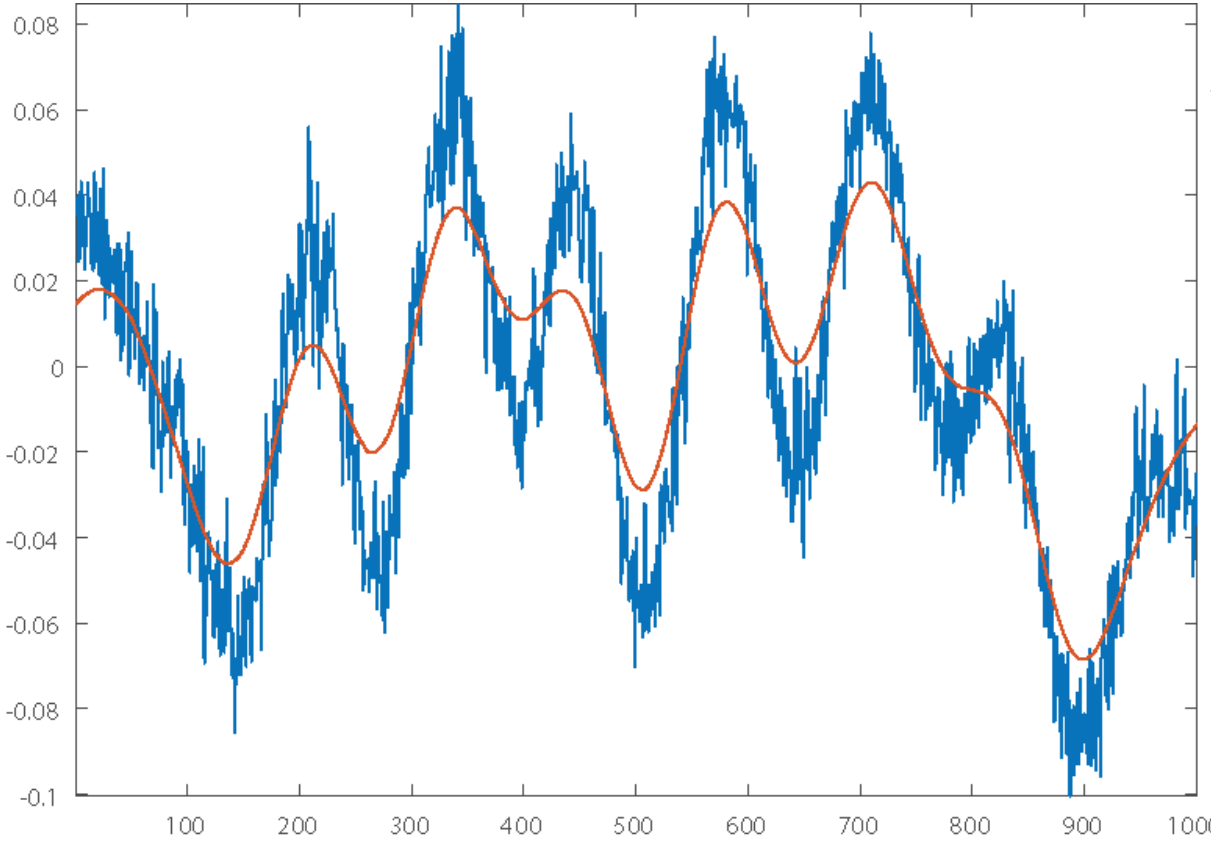
$g(\sigma = 2)$

GAUSSIAN FILTERING ~ LOW-PASS FILTERING



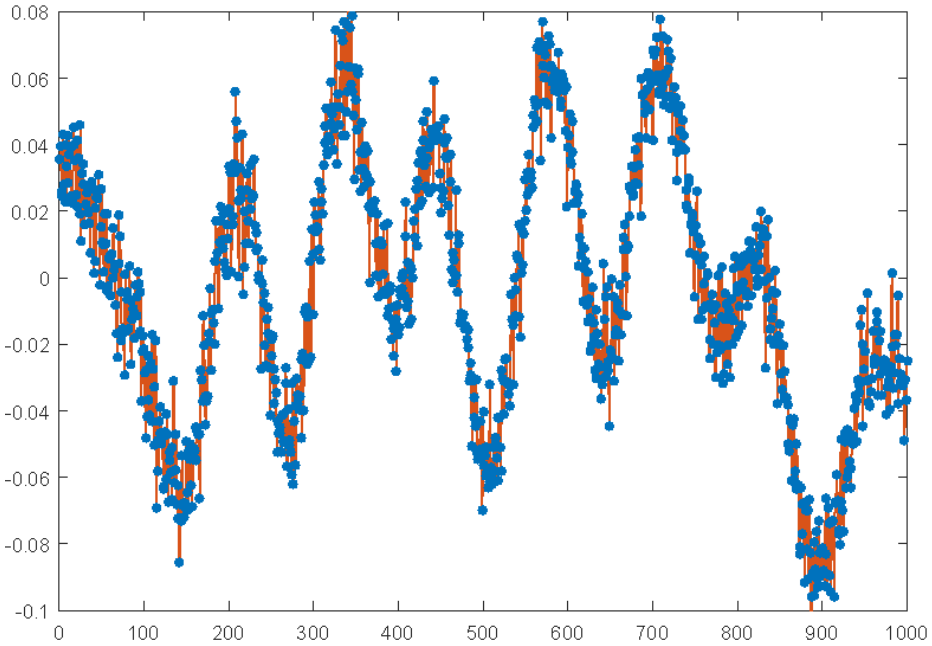
$g(\sigma = 8)$

GAUSSIAN FILTERING ~ LOW-PASS FILTERING



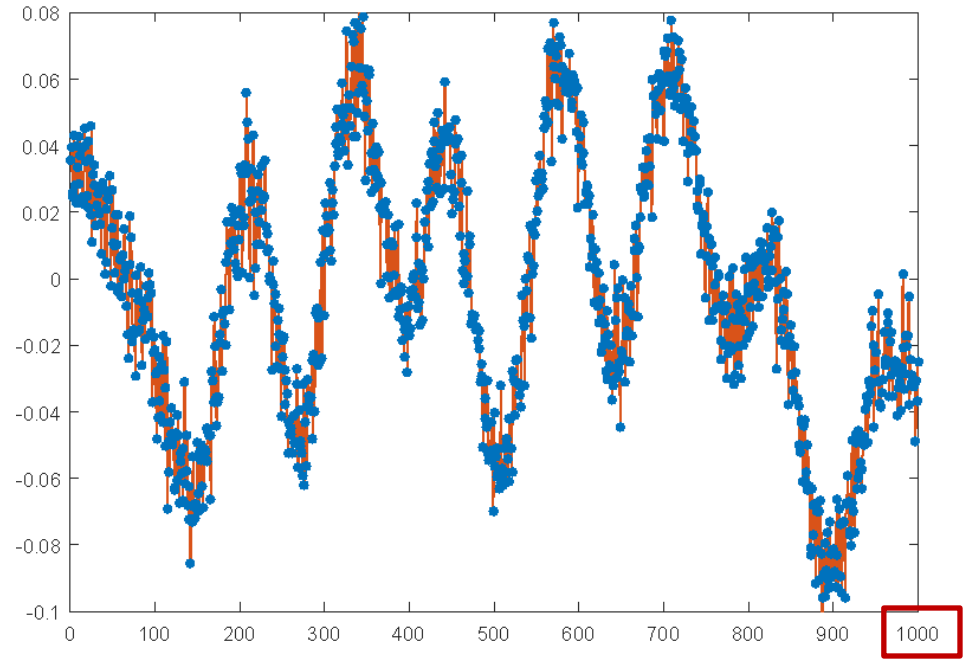
$g(\sigma = 32)$

SUBSAMPLING



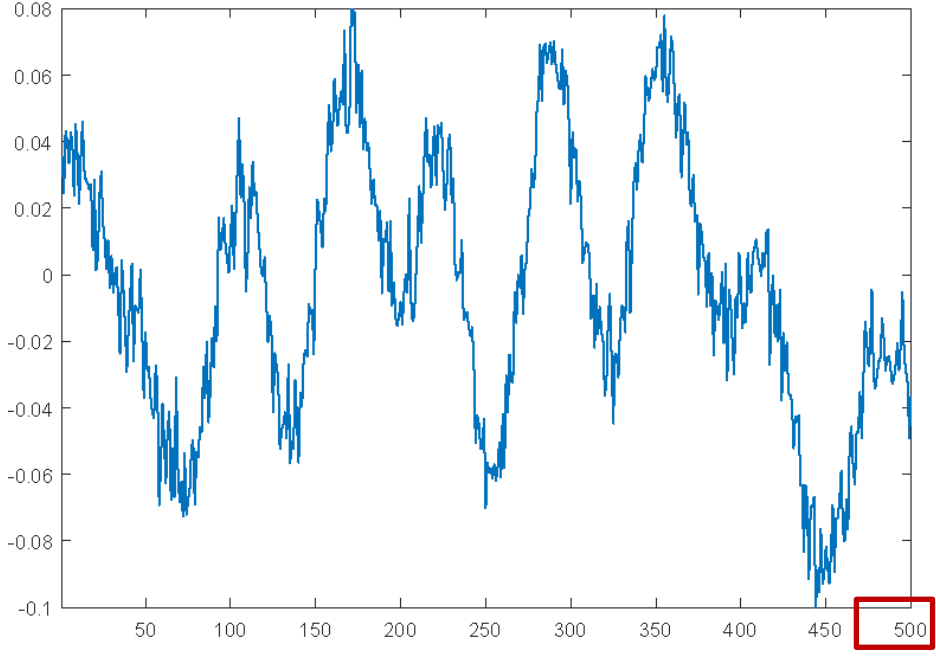
$x(t)$

SUBSAMPLING



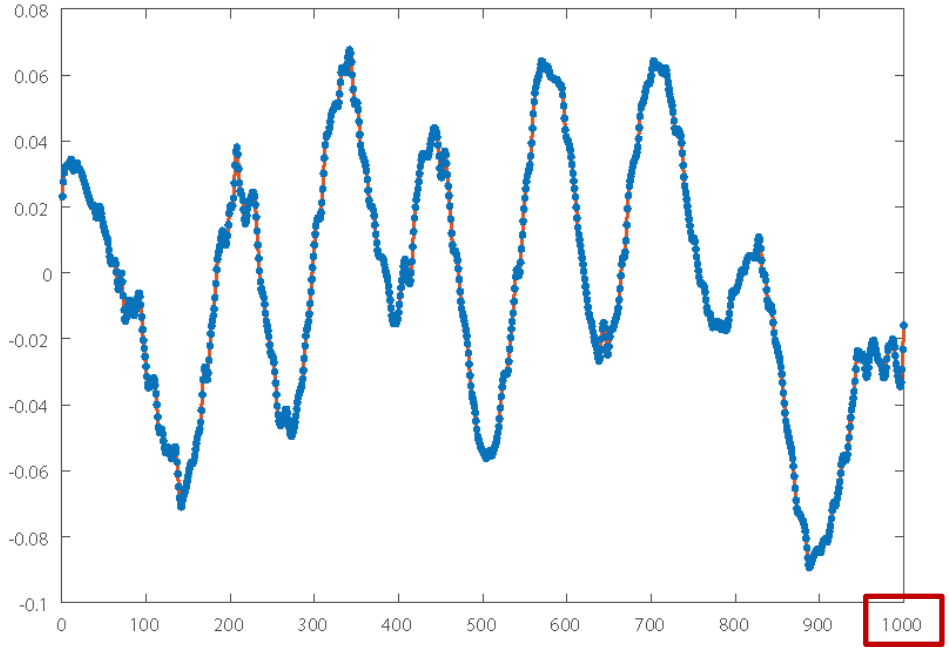
$x(t)$

Subsampling



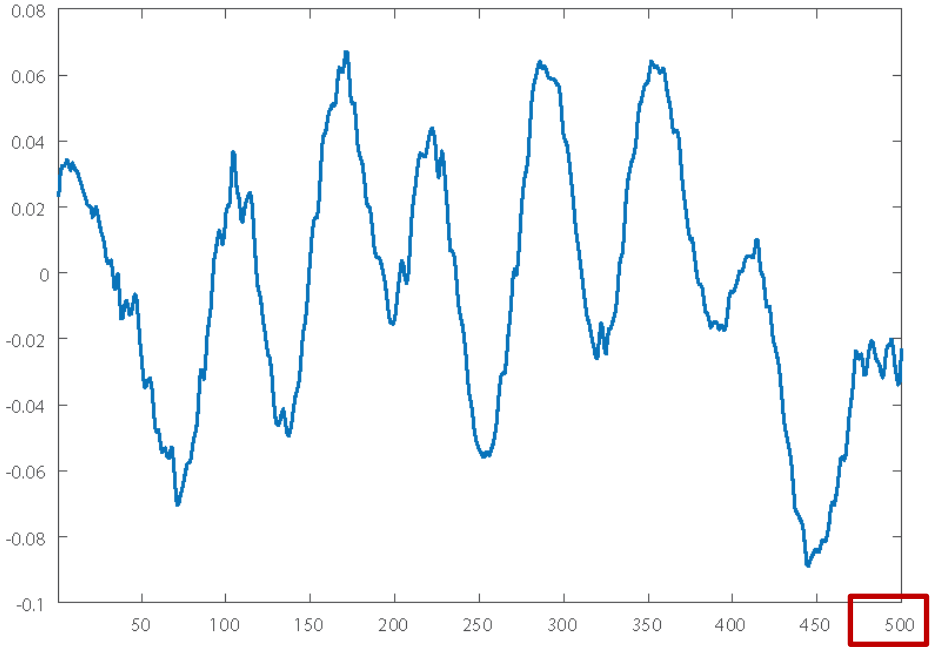
$\downarrow x(t)$

SUBSAMPLING WITH G. FILTERING



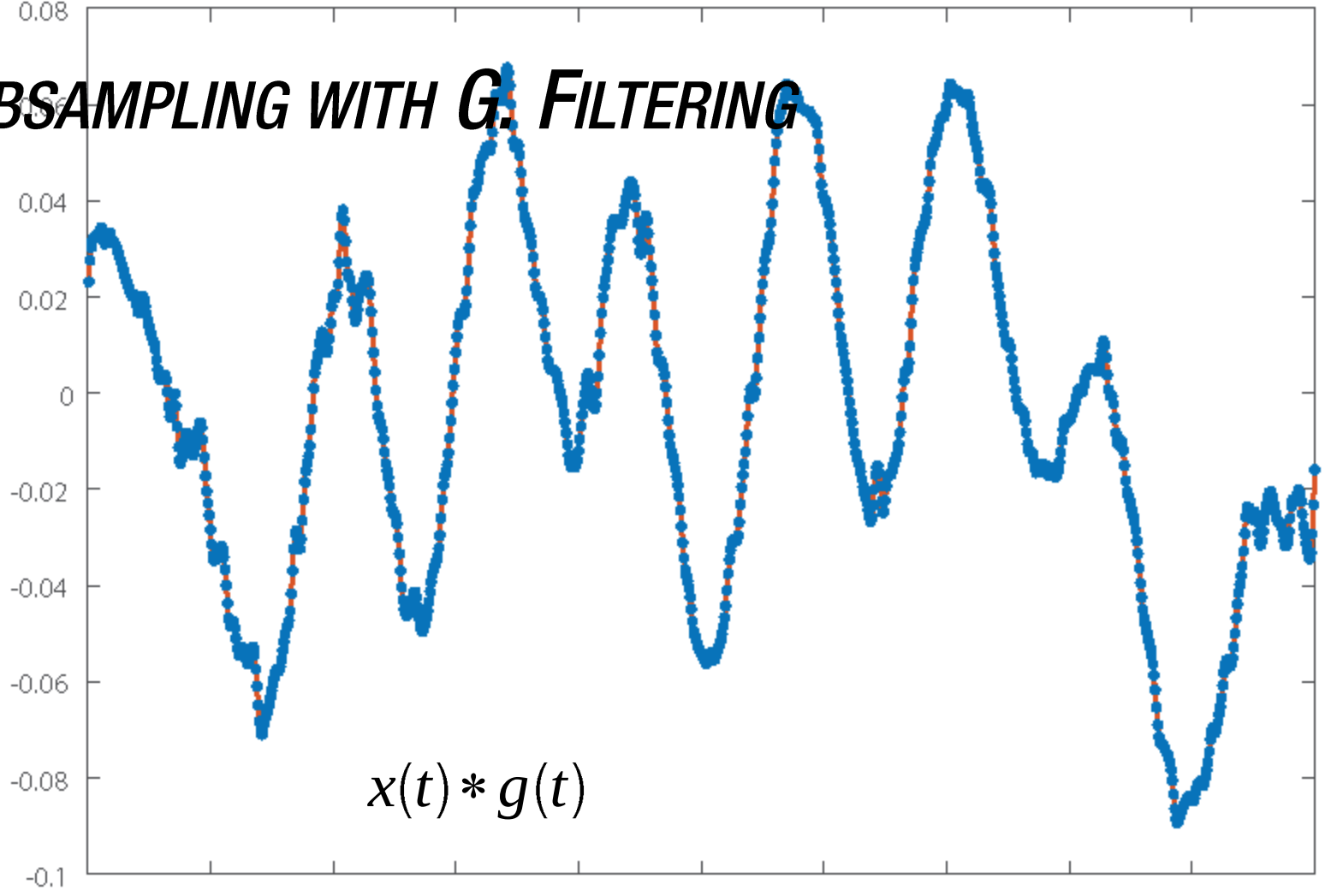
$x(t) * g(t)$

Subsampling

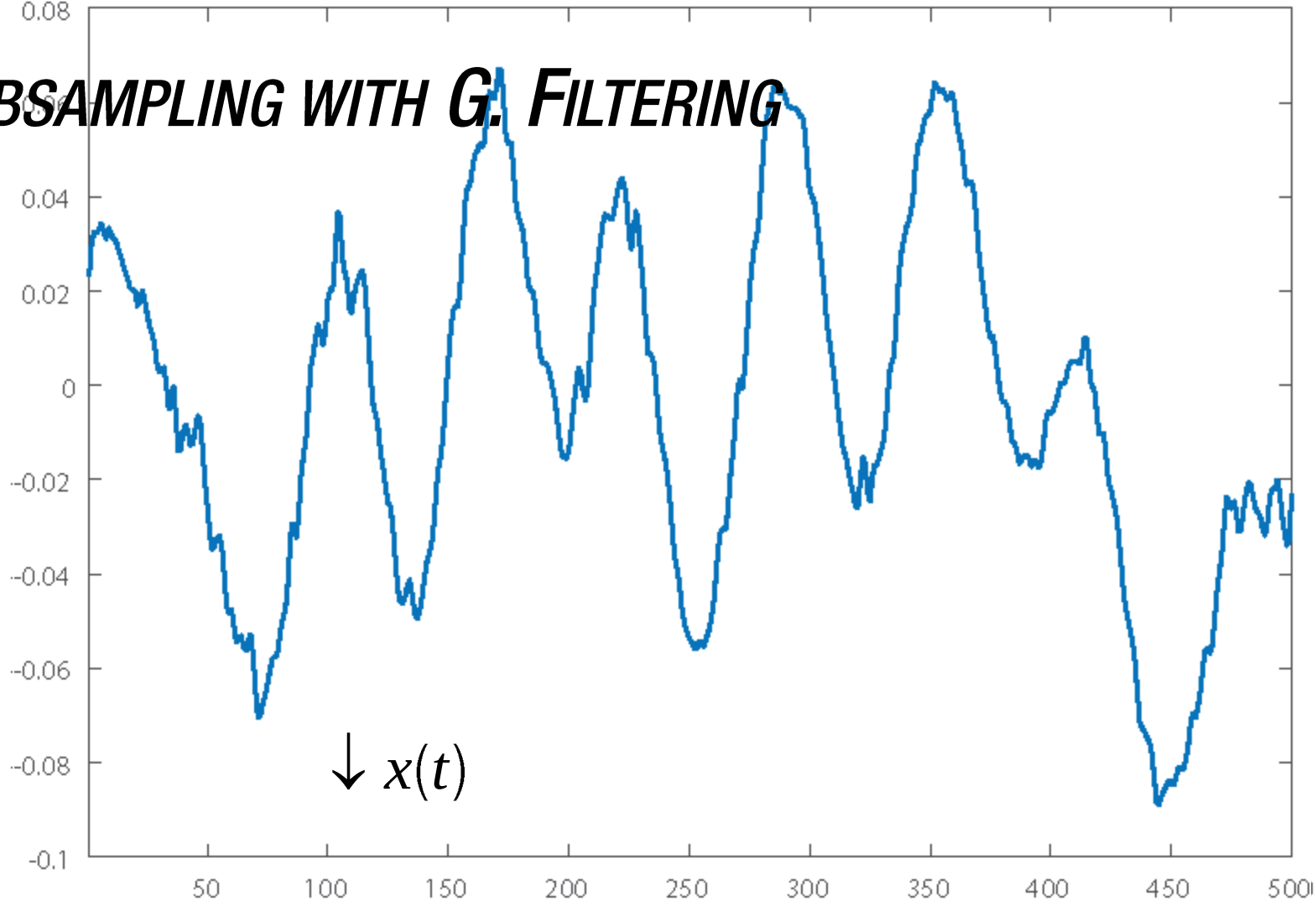


$\downarrow x(t)$

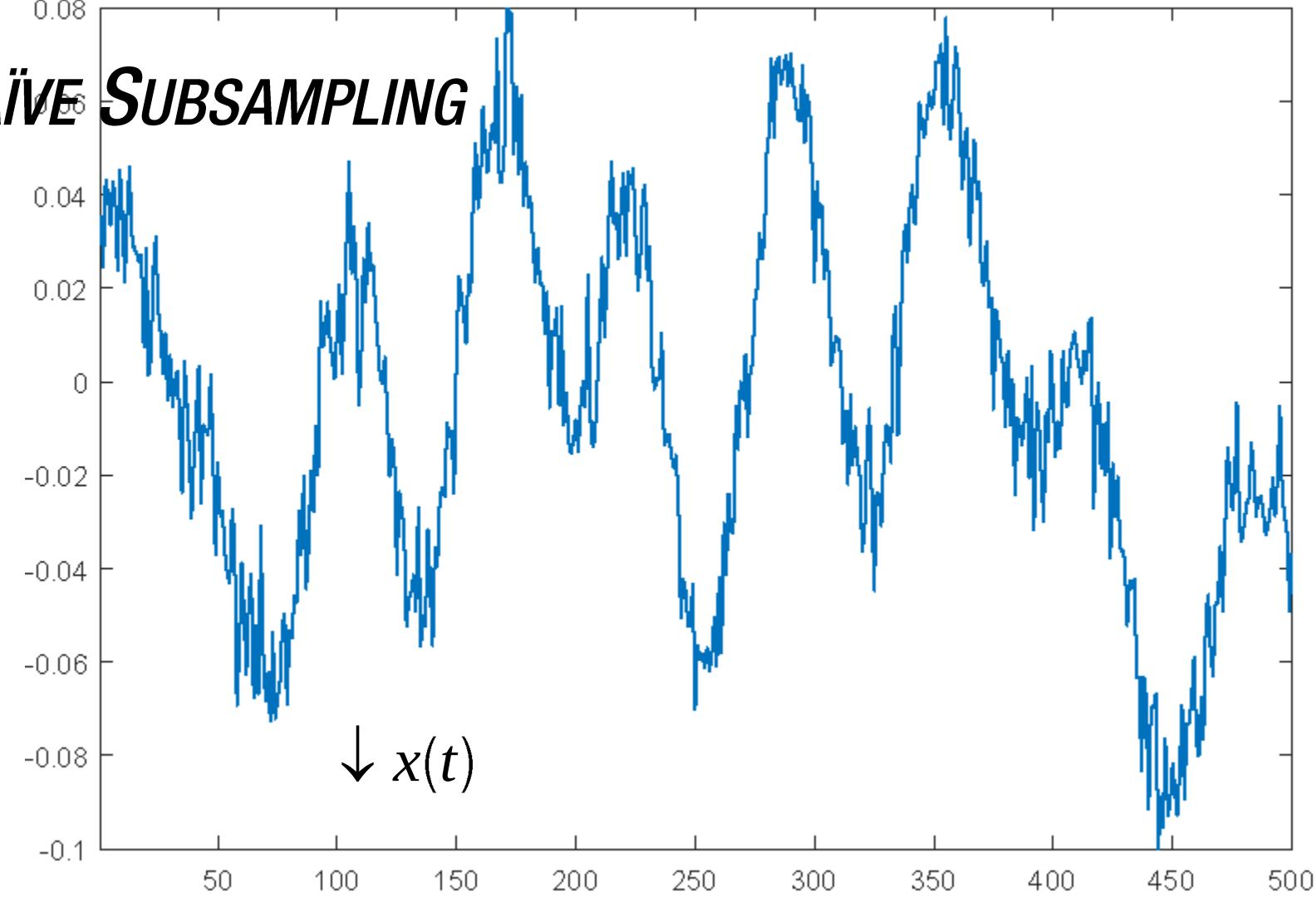
SUBSAMPLING WITH G. FILTERING

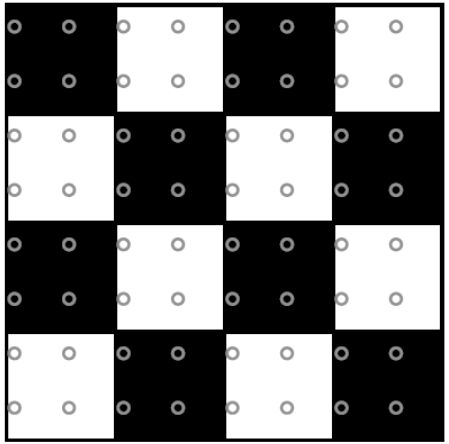


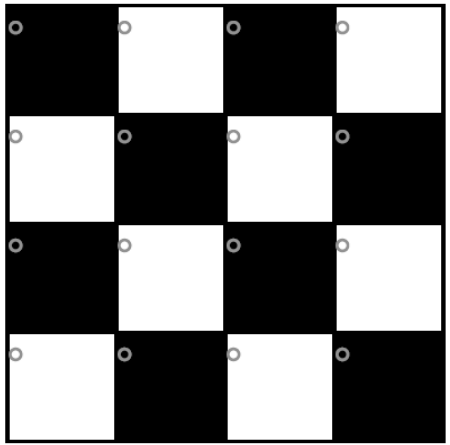
SUBSAMPLING WITH G. FILTERING

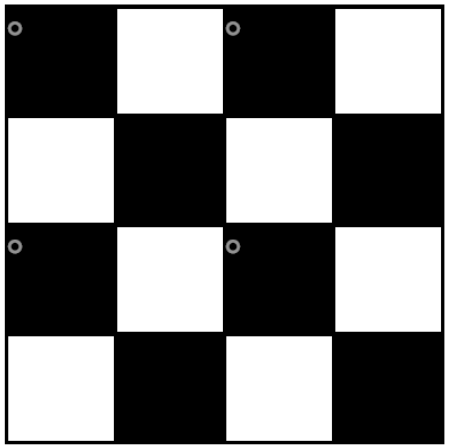


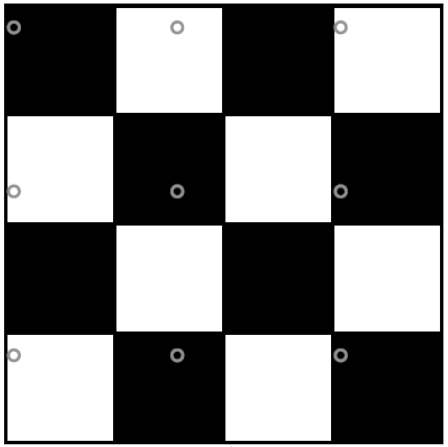
NAÏVE SUBSAMPLING

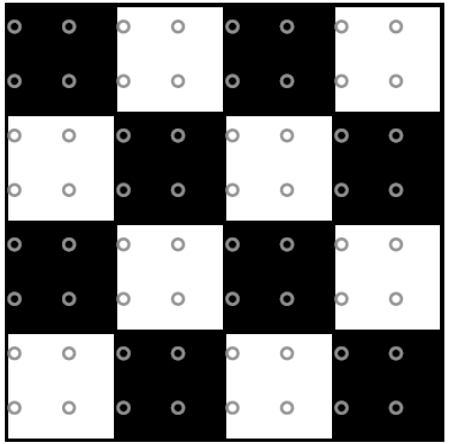


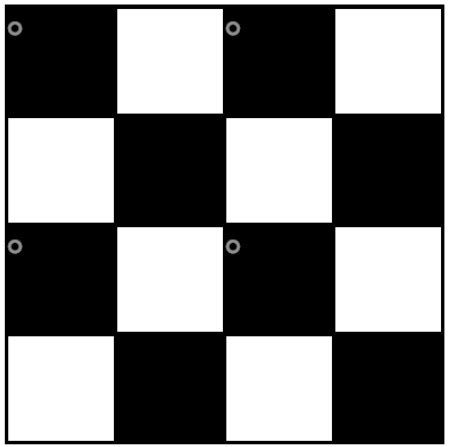








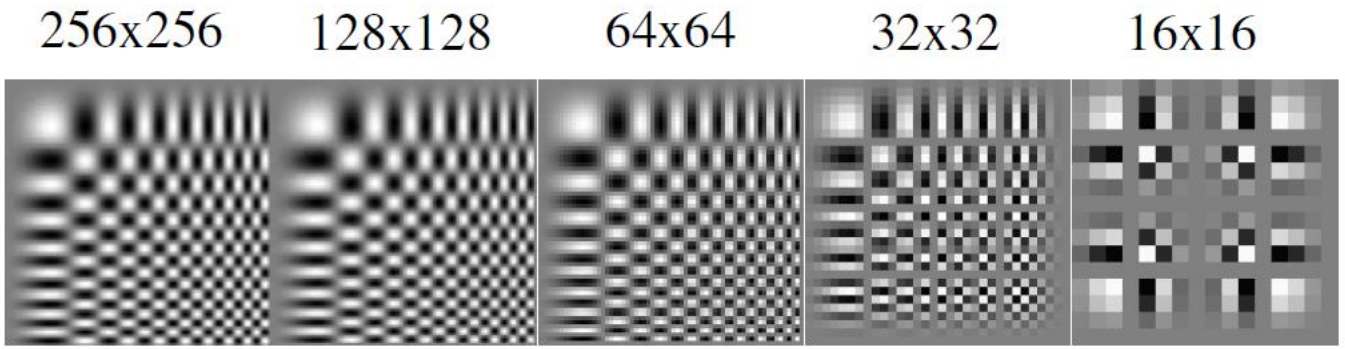




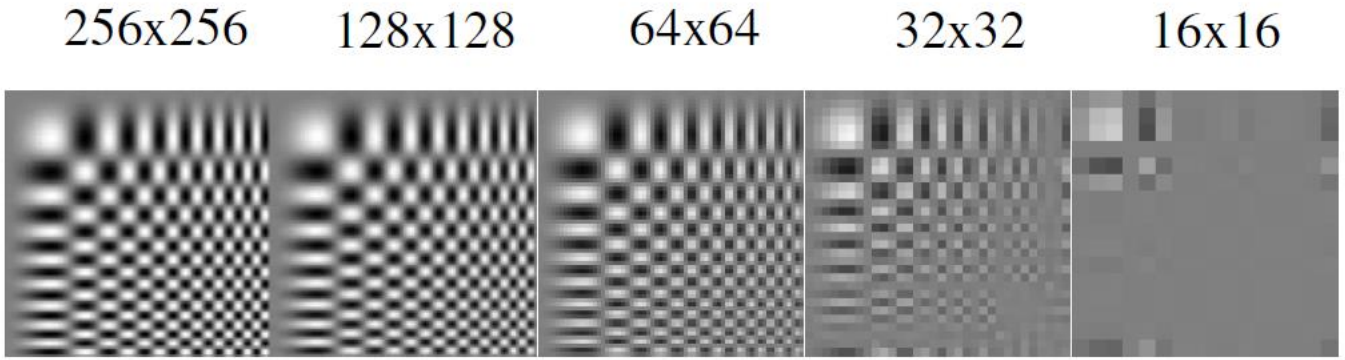
Nyquist's theorem

$$f_{sampling} \geq 2 f_{signal}$$

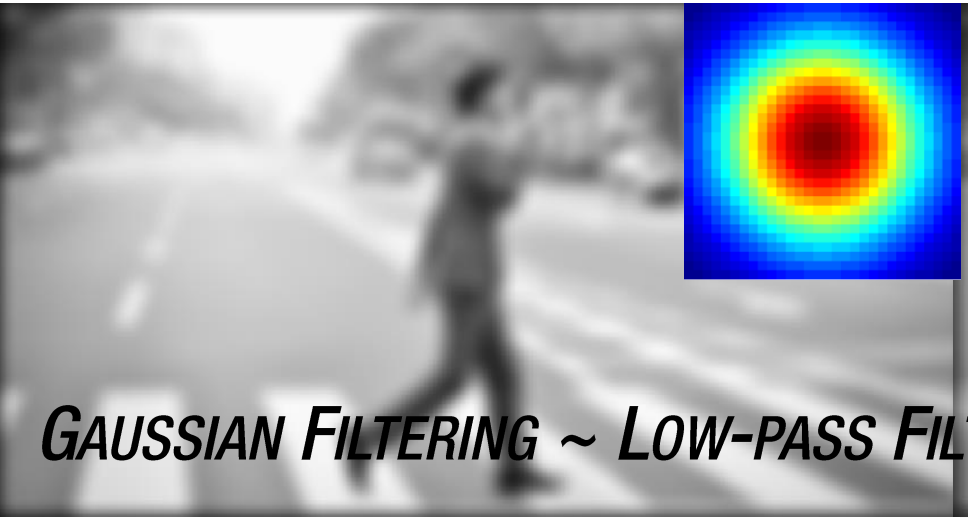
ALIASING



Naïve subsampling

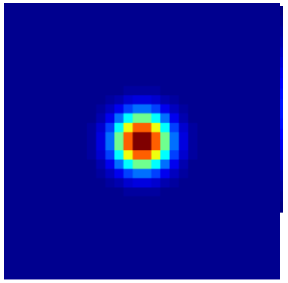


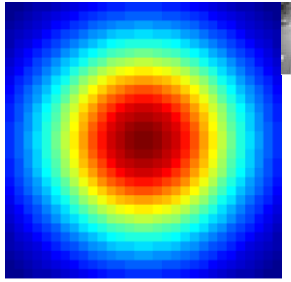
Smoothing and subsampling: eliminating aliasing effects.

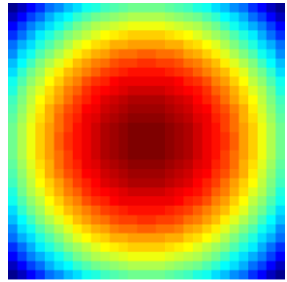


Gaussian Filtering ~ Low-pass Filtering ~ Image Blurring





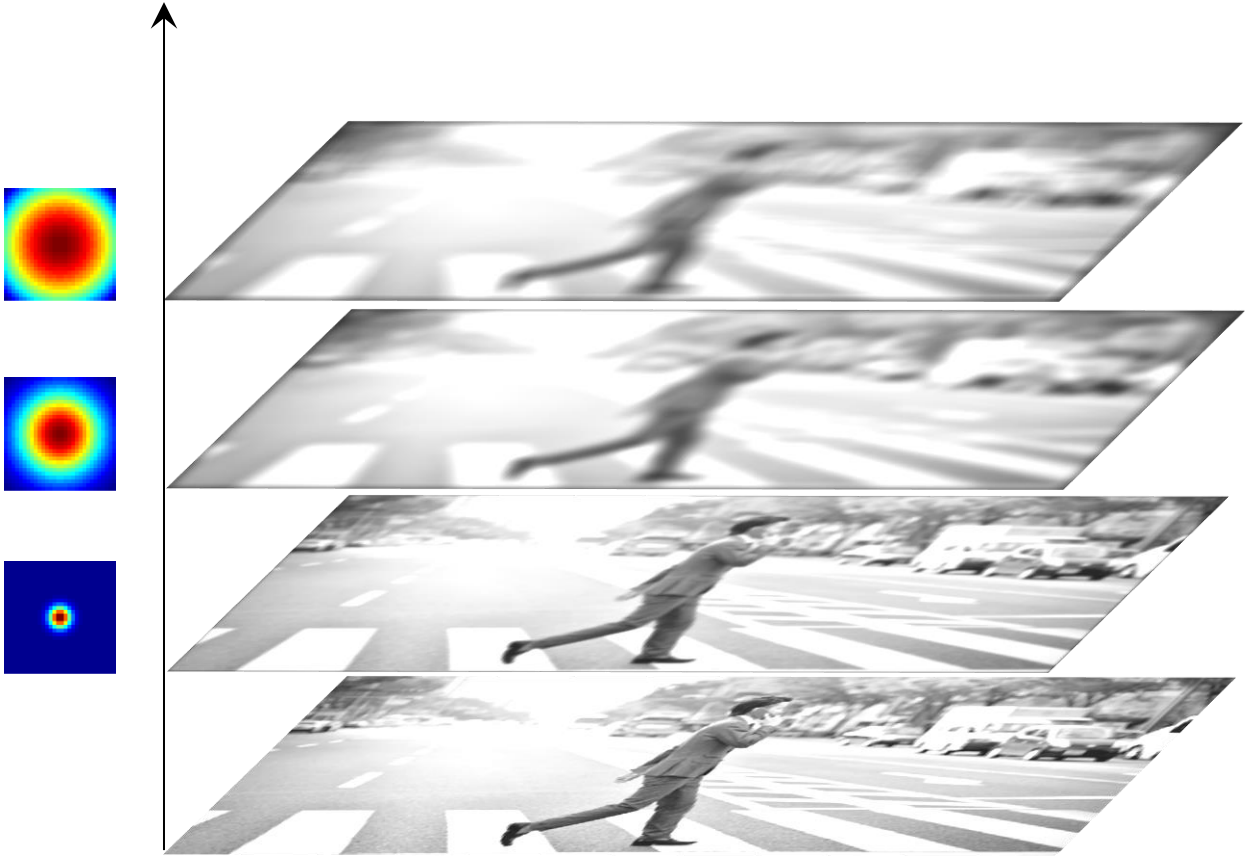




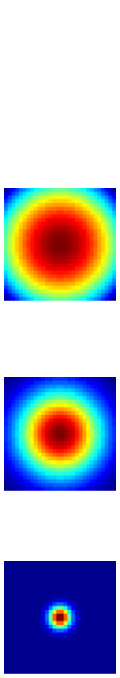
Gaussian Filtering and Then Subsampling

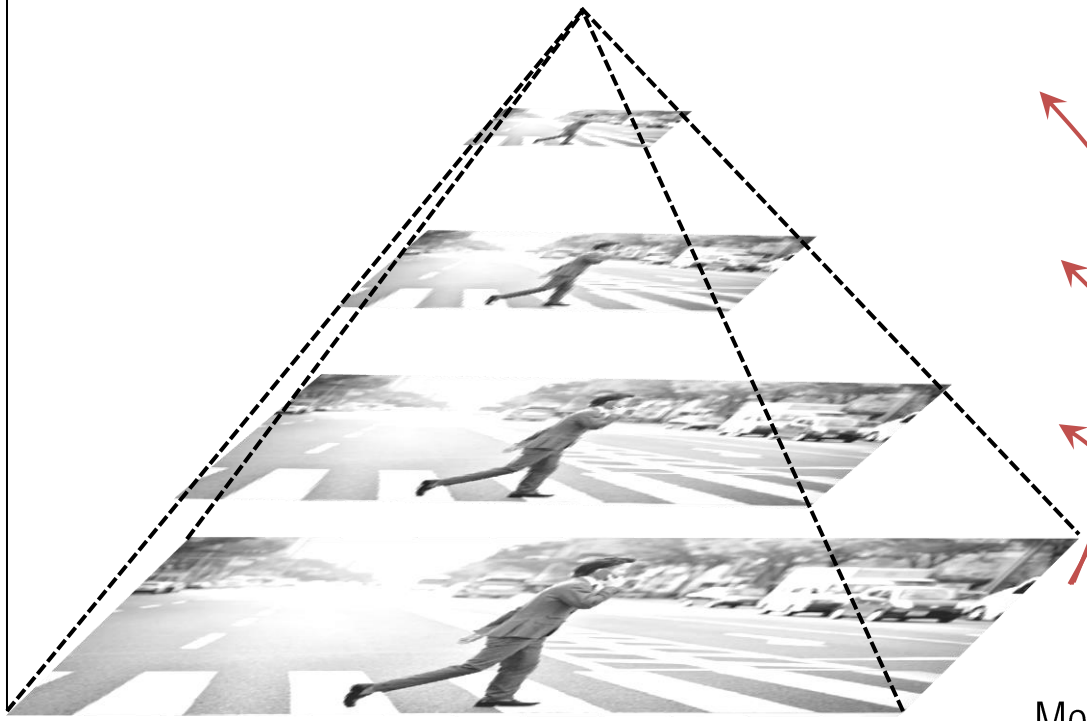
Image Reconstruction: Upsampling and Gaussian Blurring

CF) NAÏVE IMAGE SUBSAMPLING AND UPSAMPLING



MULTI-DIMENSIONAL IMAGE REPRESENTATION





1/16 subsampling

¼ subsampling

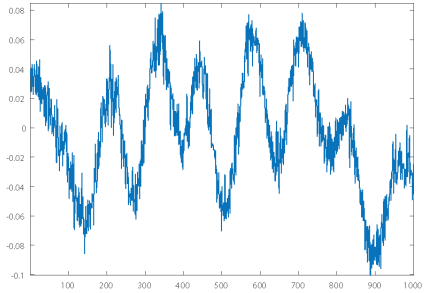
½ subsampling

Memory consumption

$$|I|(1+\frac{1}{4}+\frac{1}{16}+\cdots)=\frac{4}{3}|I|$$

GAUSSIAN IMAGE PYRAMID

Composition of Gaussian Filters



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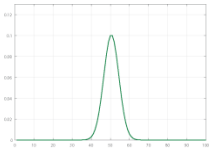

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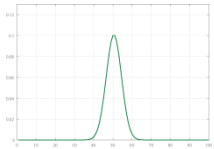
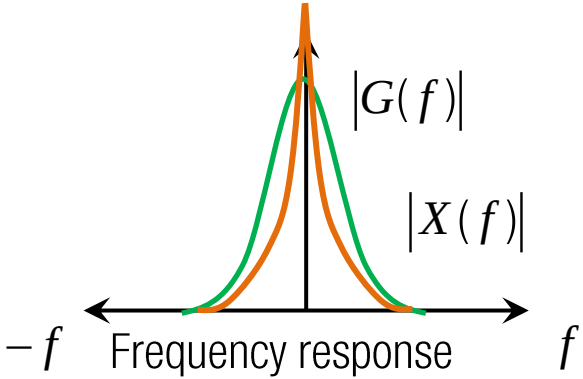


FT

Inverse FT



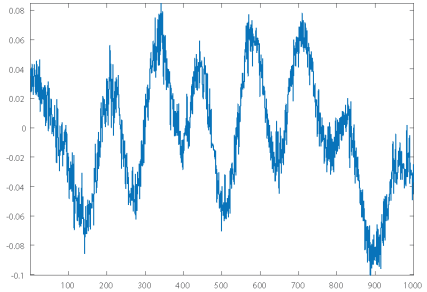
Time signal

Guassian filter

$x(t) \quad * \quad g(t;\sigma_1) \quad * \quad g(t;\sigma_2)$

$X(f)\,G(f)\,G(f)$

Composition of Gaussian Filters



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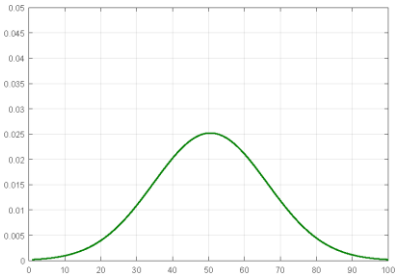
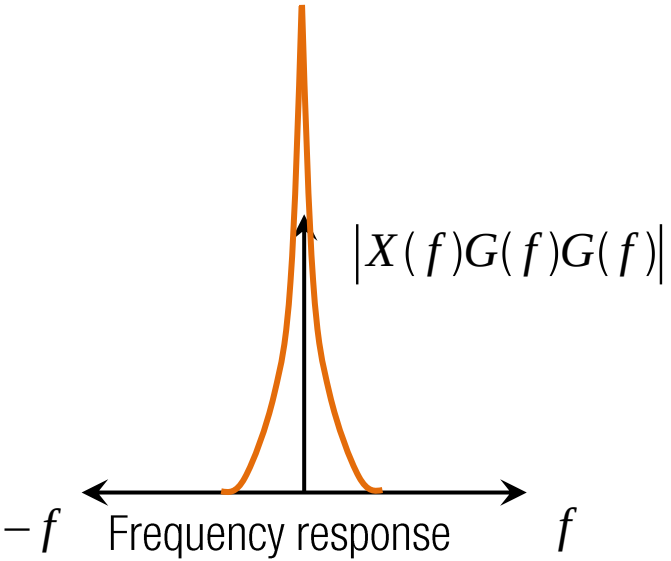


FT

Inverse FT



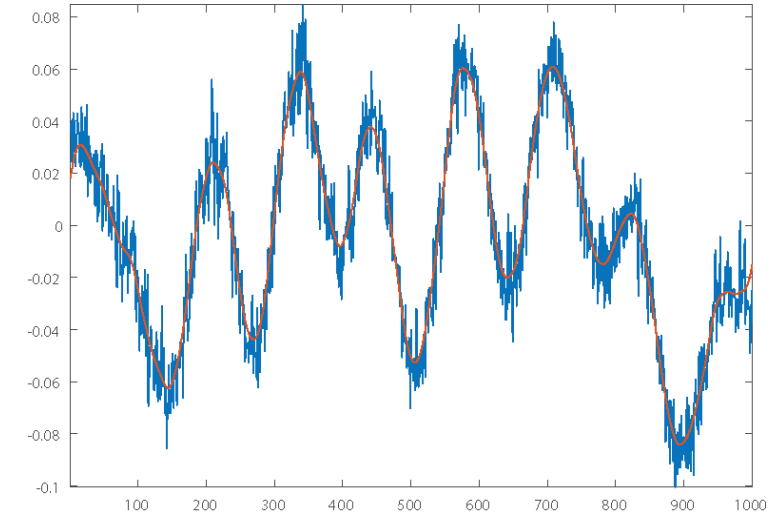
Time signal

Guassian filter

$x(t)$ * $g(t;\sqrt{\sigma_1^2+\sigma_2^2})$

$X(f)\,G(f)\,G(f)$

Composition of Gaussian Filters



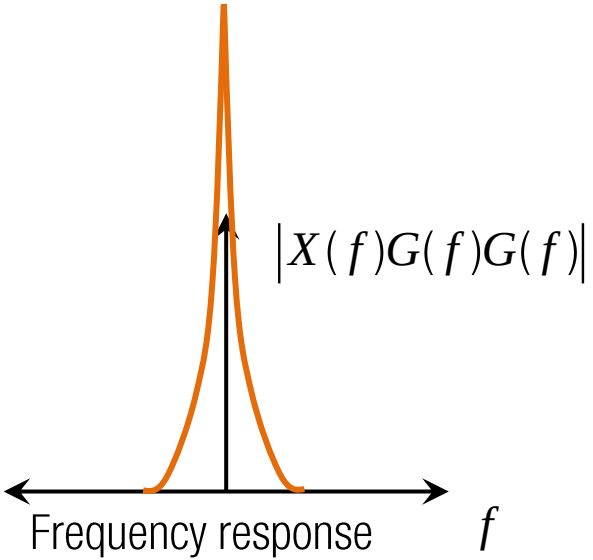
Time signal

Guassian filter

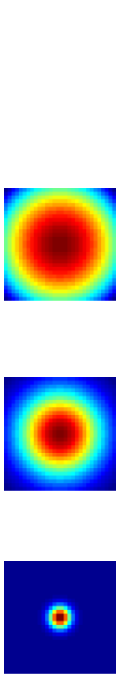
$x(t) \quad * \quad g(t;\sqrt{\sigma_1^2 + \sigma_2^2})$

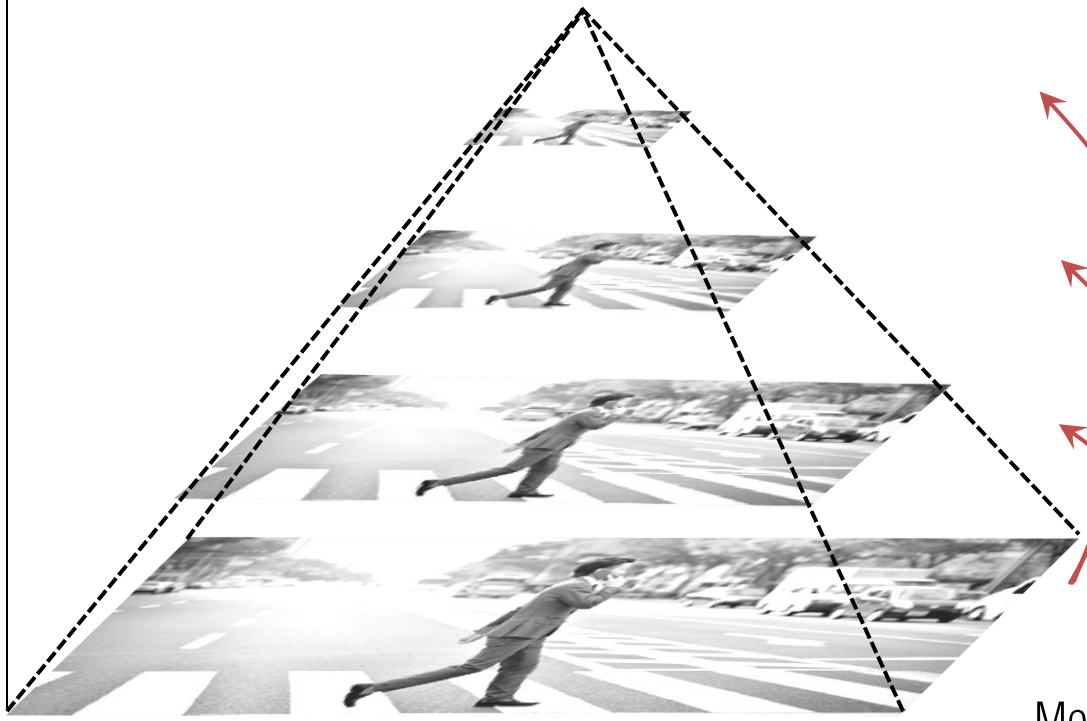
FT

Inverse FT



$X(f)\,G(f)\,G(f)$





1/16 subsampling

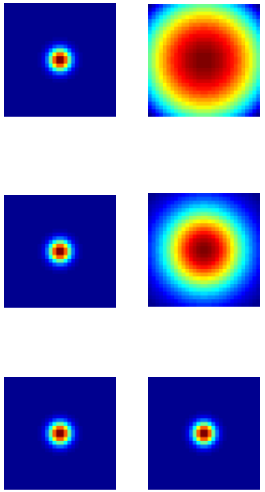
¼ subsampling

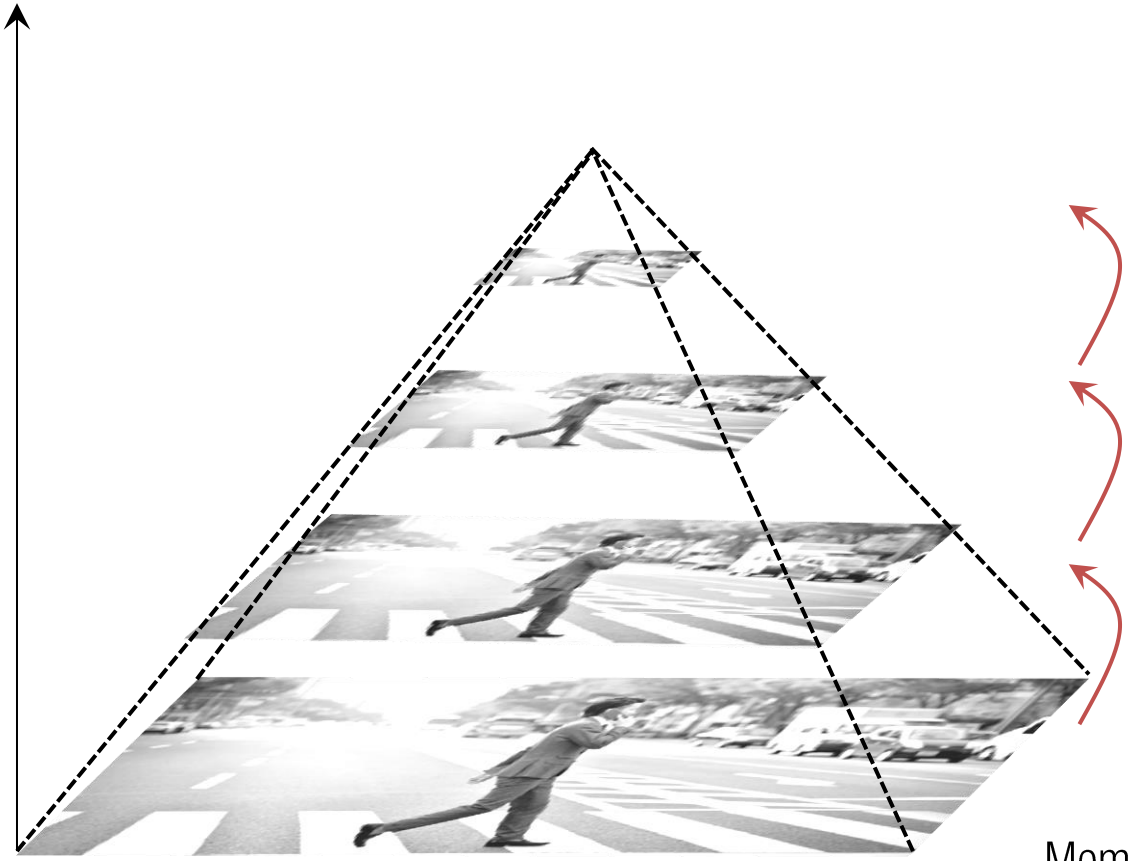
½ subsampling

Memory consumption

$$|I|(1+\frac{1}{4}+\frac{1}{16}+\cdots)=\frac{4}{3}|I|$$

GAUSSIAN IMAGE PYRAMID





½ subsampling

½ subsampling

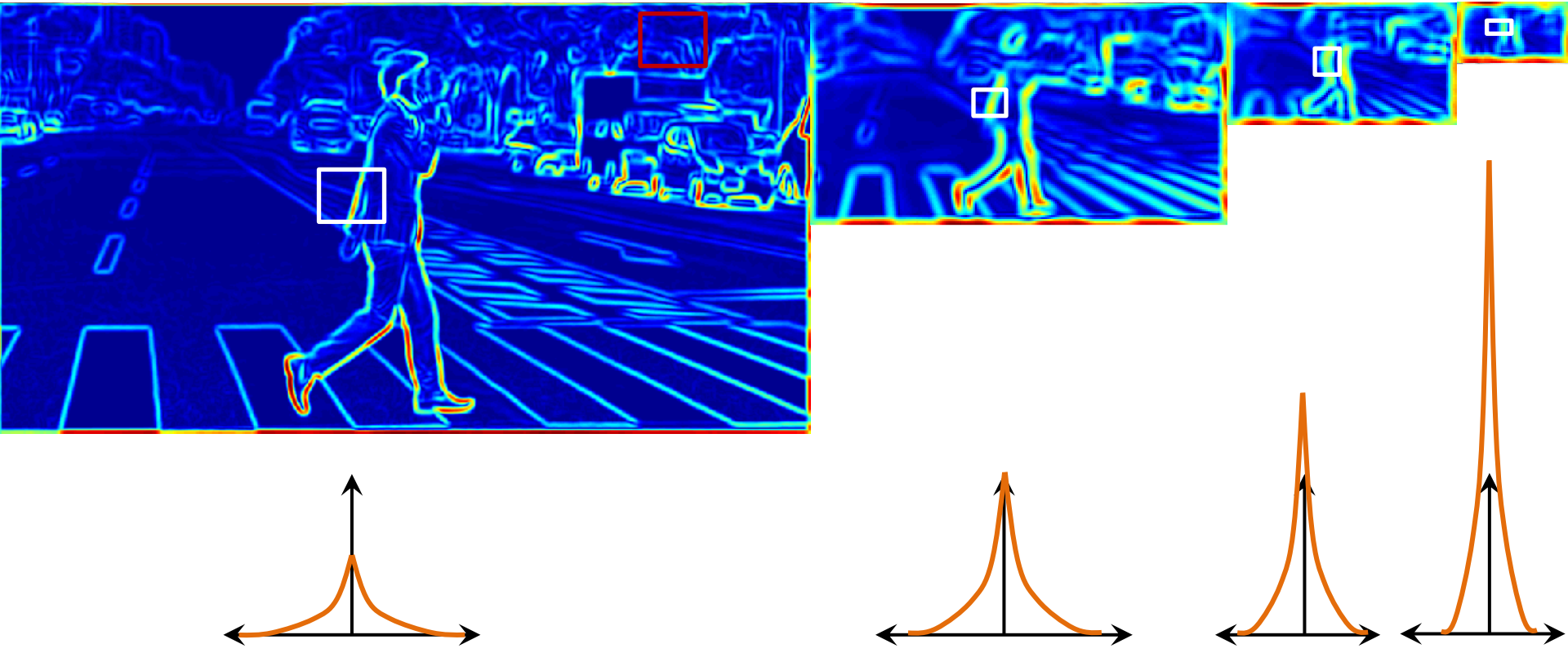
½ subsampling

Memory consumption

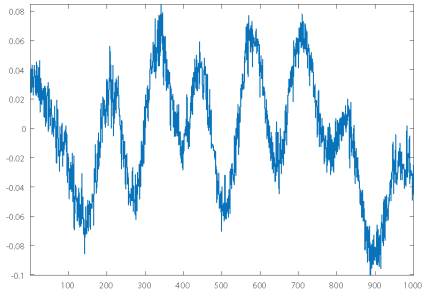
$$|I|(1+\frac{1}{4}+\frac{1}{16}+\cdots)=\frac{4}{3}|I|$$

GAUSSIAN IMAGE PYRAMID

Redundant Representation of Gaussian Pyramid

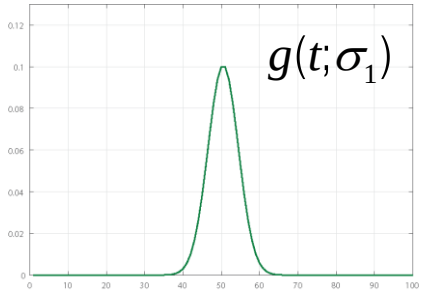


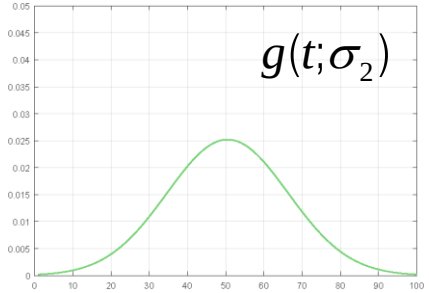
FOURIER TRANSFORM



$x(t)$

$*$



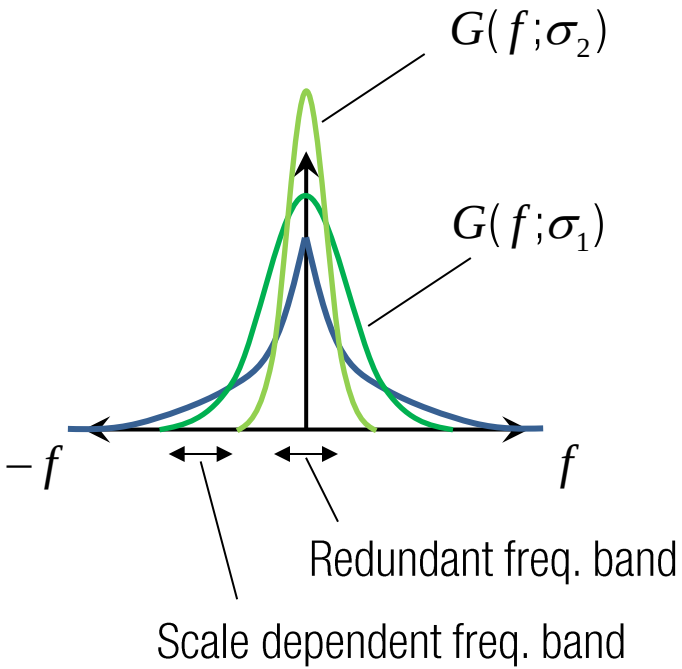


$g(t)$

$\sigma_1 < \sigma_2$

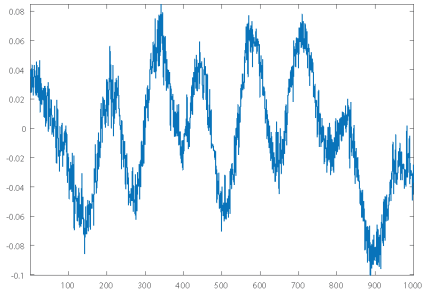
FT

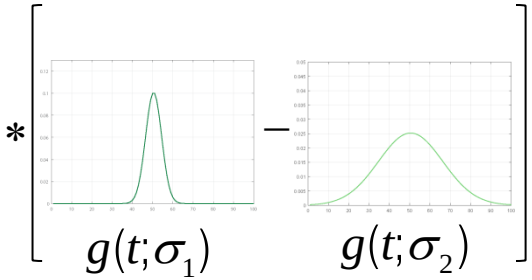
Inverse FT



$X(f)\,G(f)$

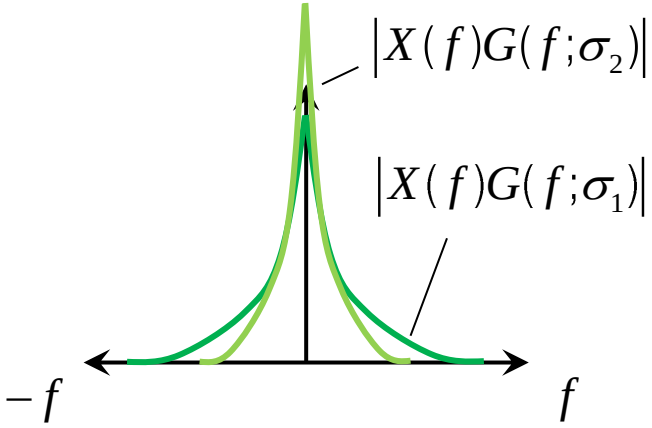
Difference of Gaussian (DoG) ~ Band-pass Filter





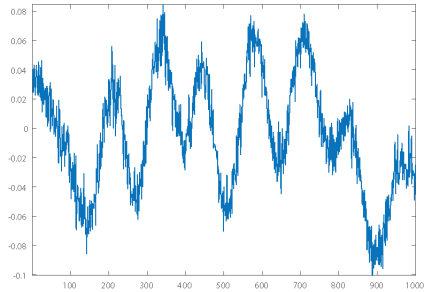
FT

Inverse FT



$x(t)$ $*$ $g(t)$ $X(f)\,G(f)$

Difference of Gaussian (DoG) ~ Band-pass Filter



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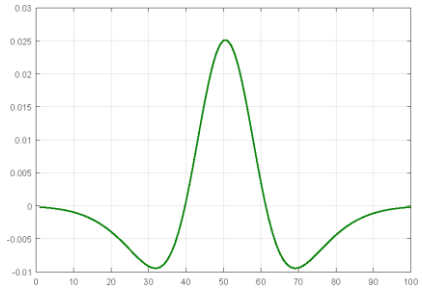
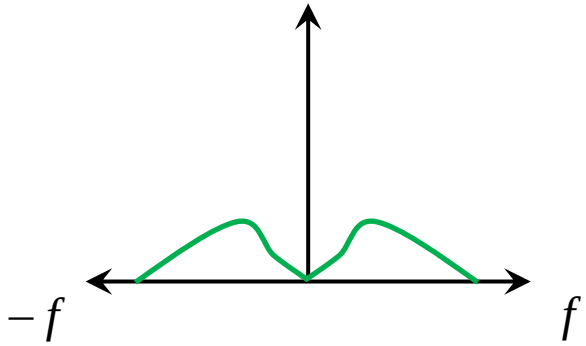


FT

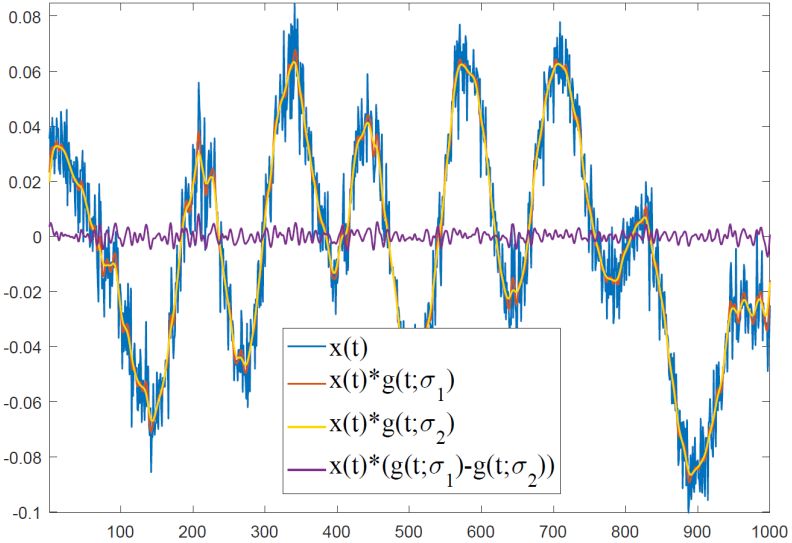
Inverse FT

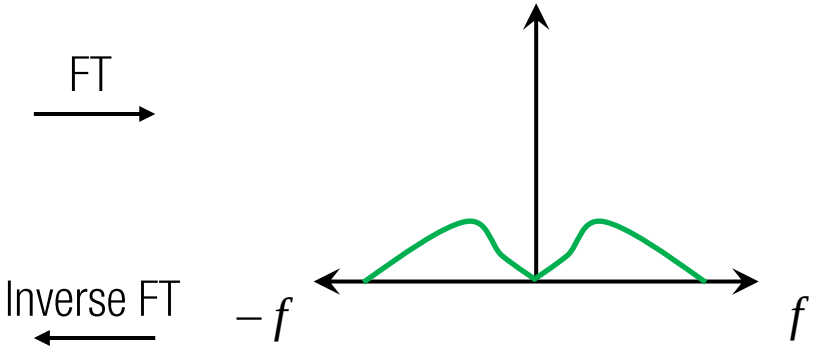


$x(t)$ * $g(t;\sigma_1) - g(t;\sigma_2)$

$X(f)\left(G(f;\sigma_1) - G(f;\sigma_2)\right)$

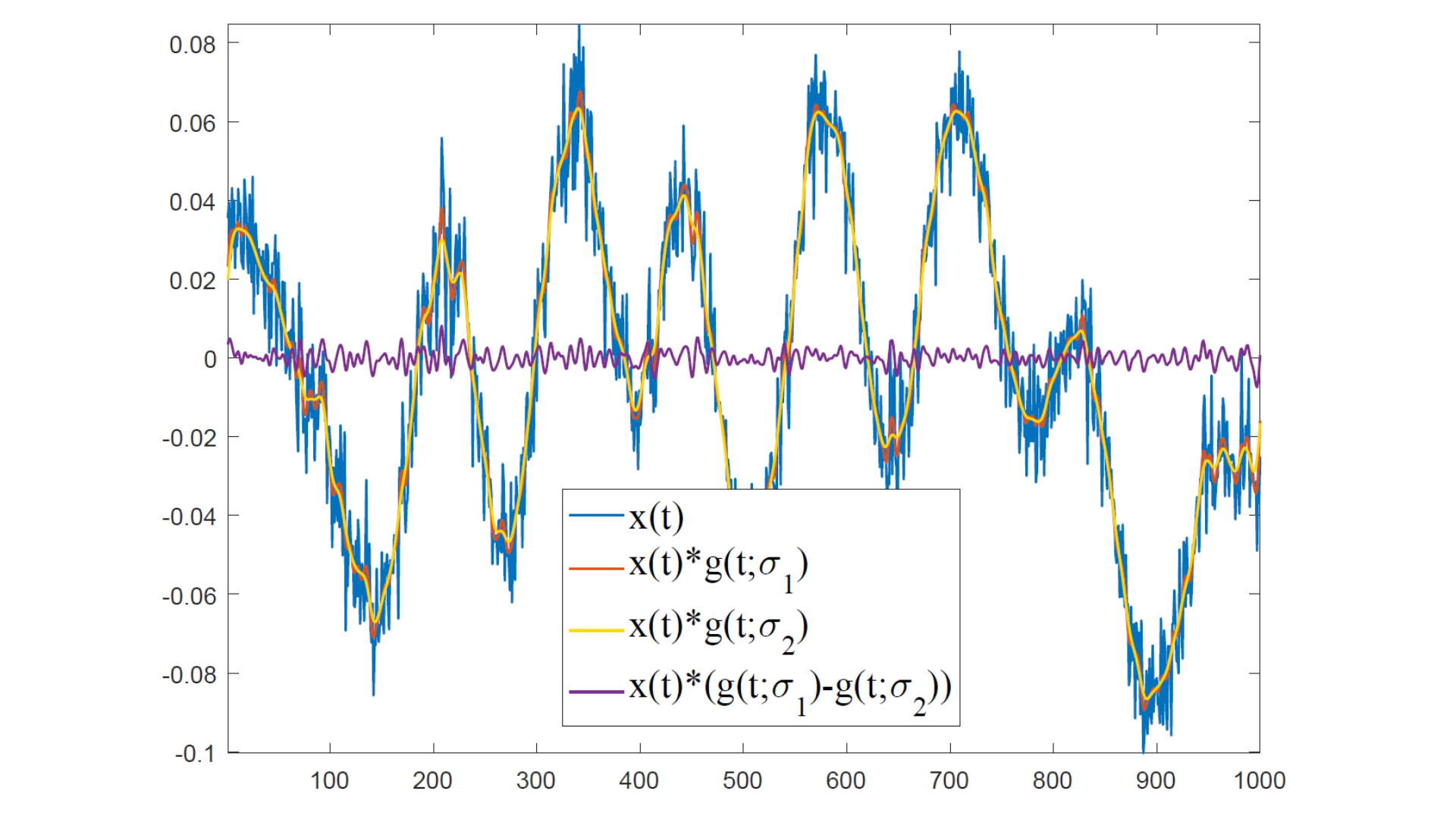
Difference of Gaussian (DoG) ~ Band-pass Filter



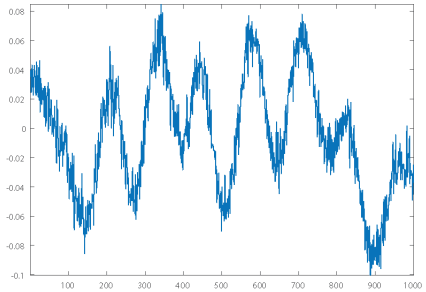


$$x(t) \quad * \quad g(t;\sigma_1) - g(t;\sigma_2)$$

$$X(f)\left(G(f;\sigma_1) - G(f;\sigma_2)\right)$$


0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
100
200
300
400
500
600
700
800
900
1000
x(t)
x(t)*g(t;σ1)
x(t)*g(t;σ2)
x(t)*(g(t;σ1)-g(t;σ2))

Difference of Gaussian (DoG) ~ Band-pass Filter



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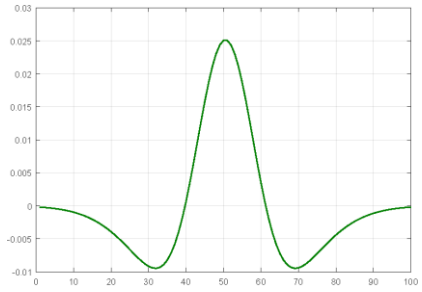
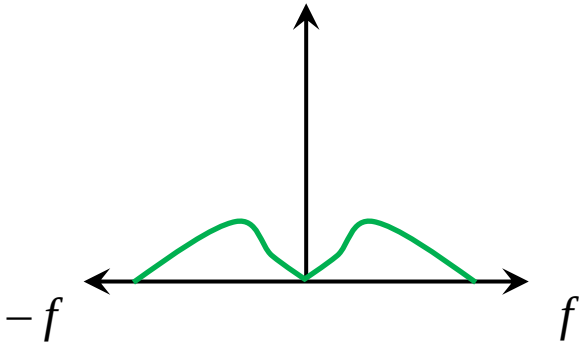


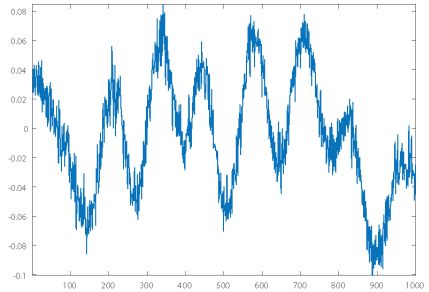
FT

Inverse FT



$$x(t) \quad * \quad g(t;\sigma_1) - g(t;\sigma_2) \qquad X(f)\left(G(f;\sigma_1) - G(f;\sigma_2)\right)$$

LAPLACIAN OF GAUSSIAN (LOG) ~ DOG



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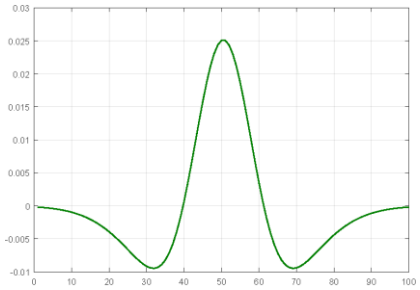
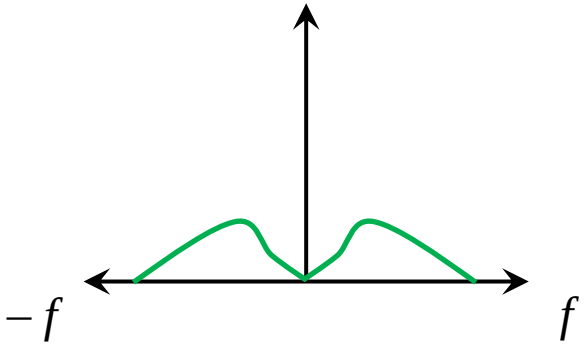


FT

Inverse FT



$x(t)$ * $g(t;\sigma_1) - g(t;\sigma_2)$

$\approx \nabla \cdot \nabla g$

Laplacian of Gaussian

$X(f)\left(G(f;\sigma_1) - G(f;\sigma_2)\right)$

Laplacian of Gaussian (LoG) ~ DoG

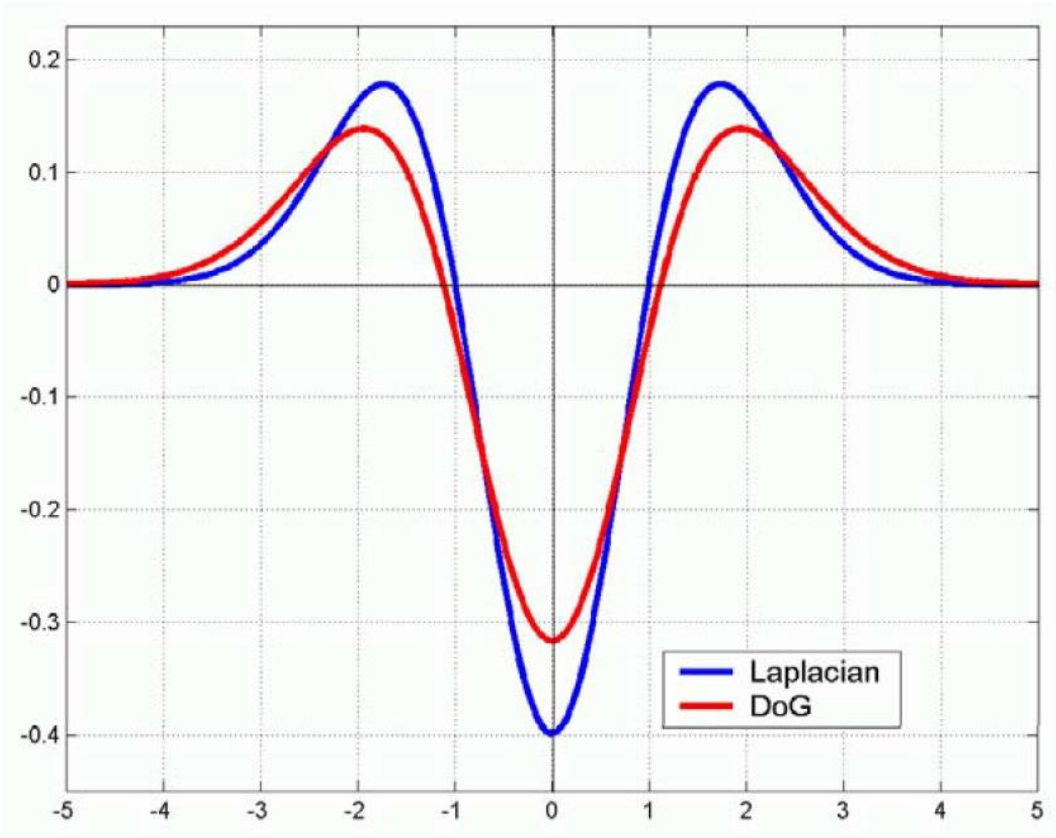
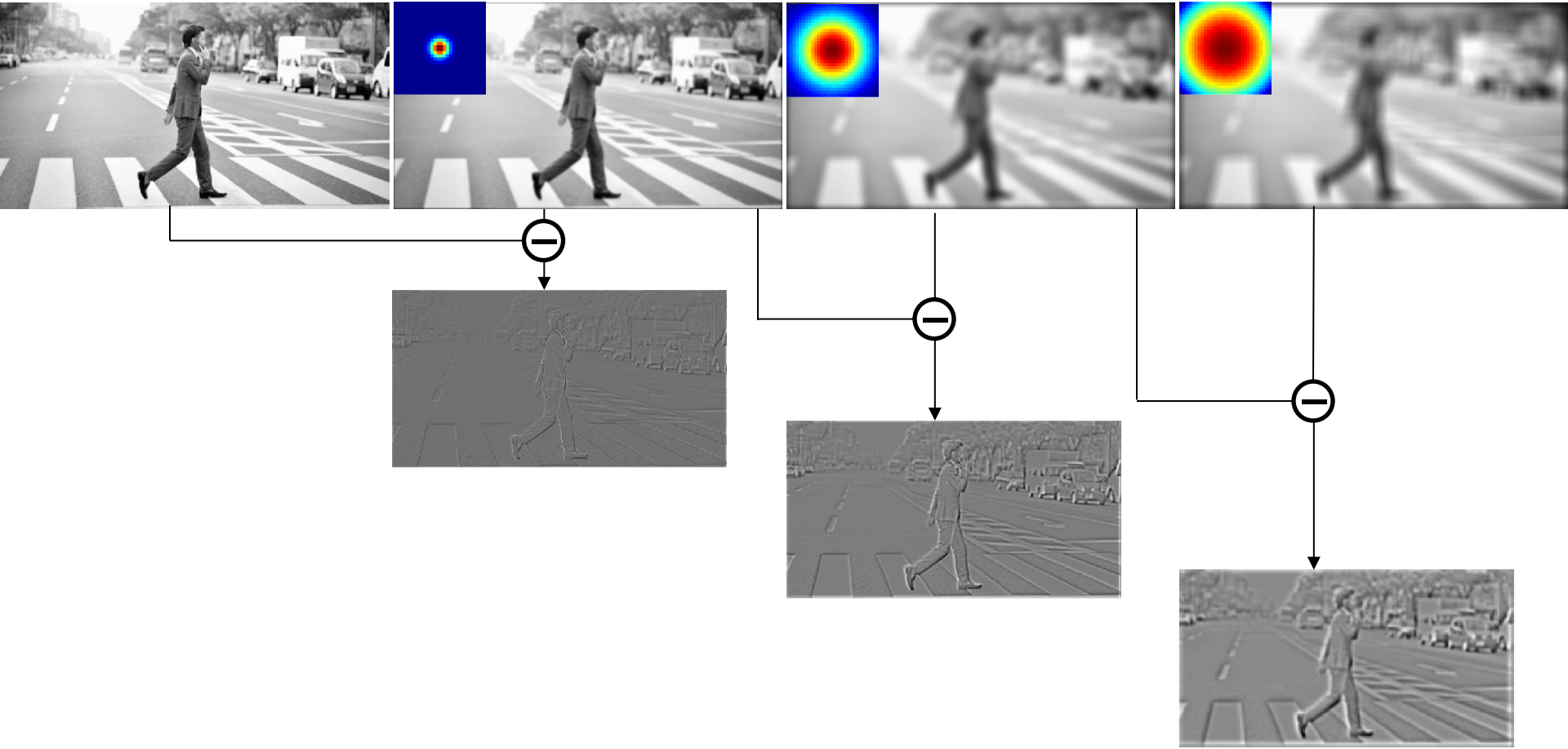
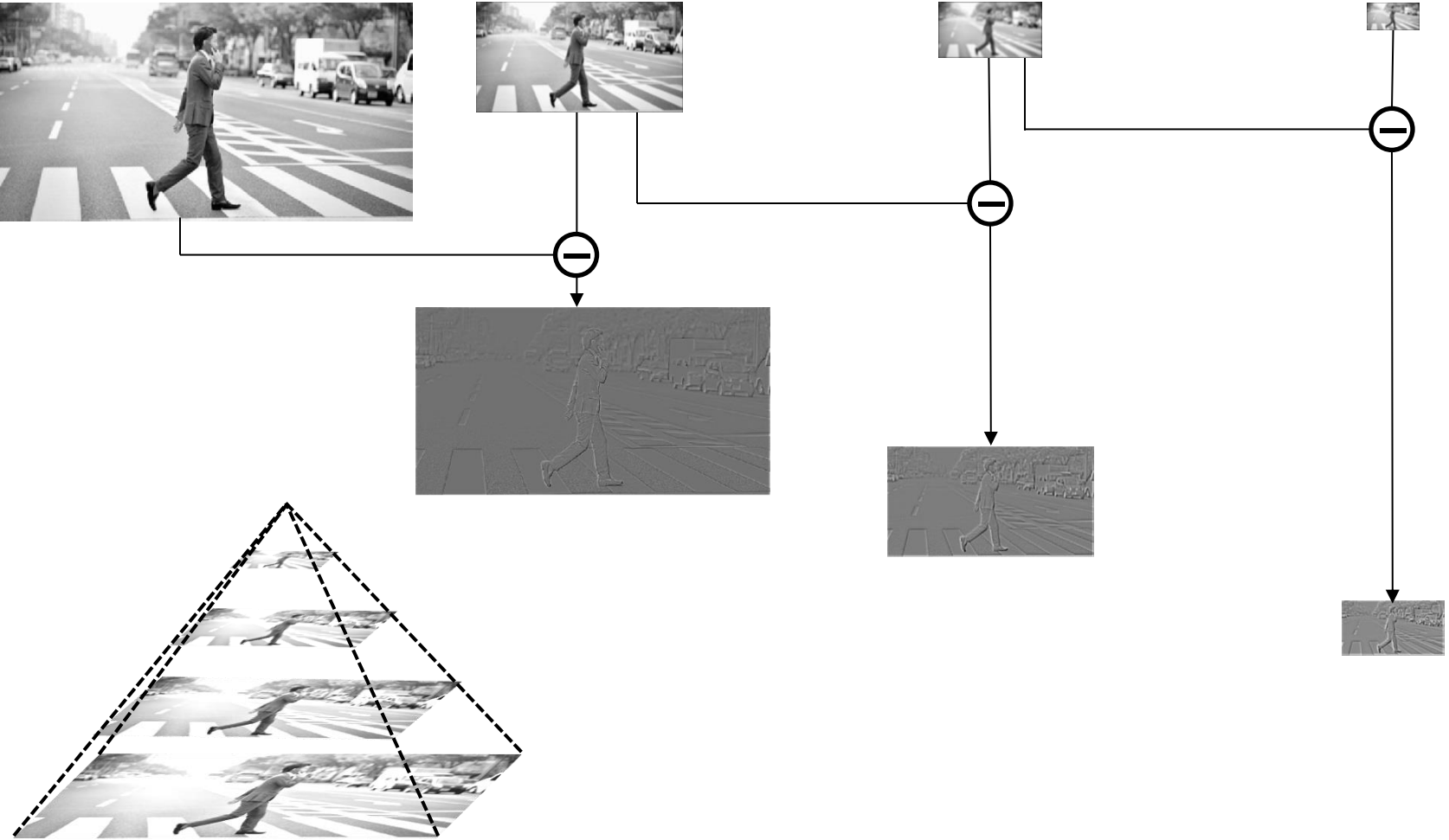
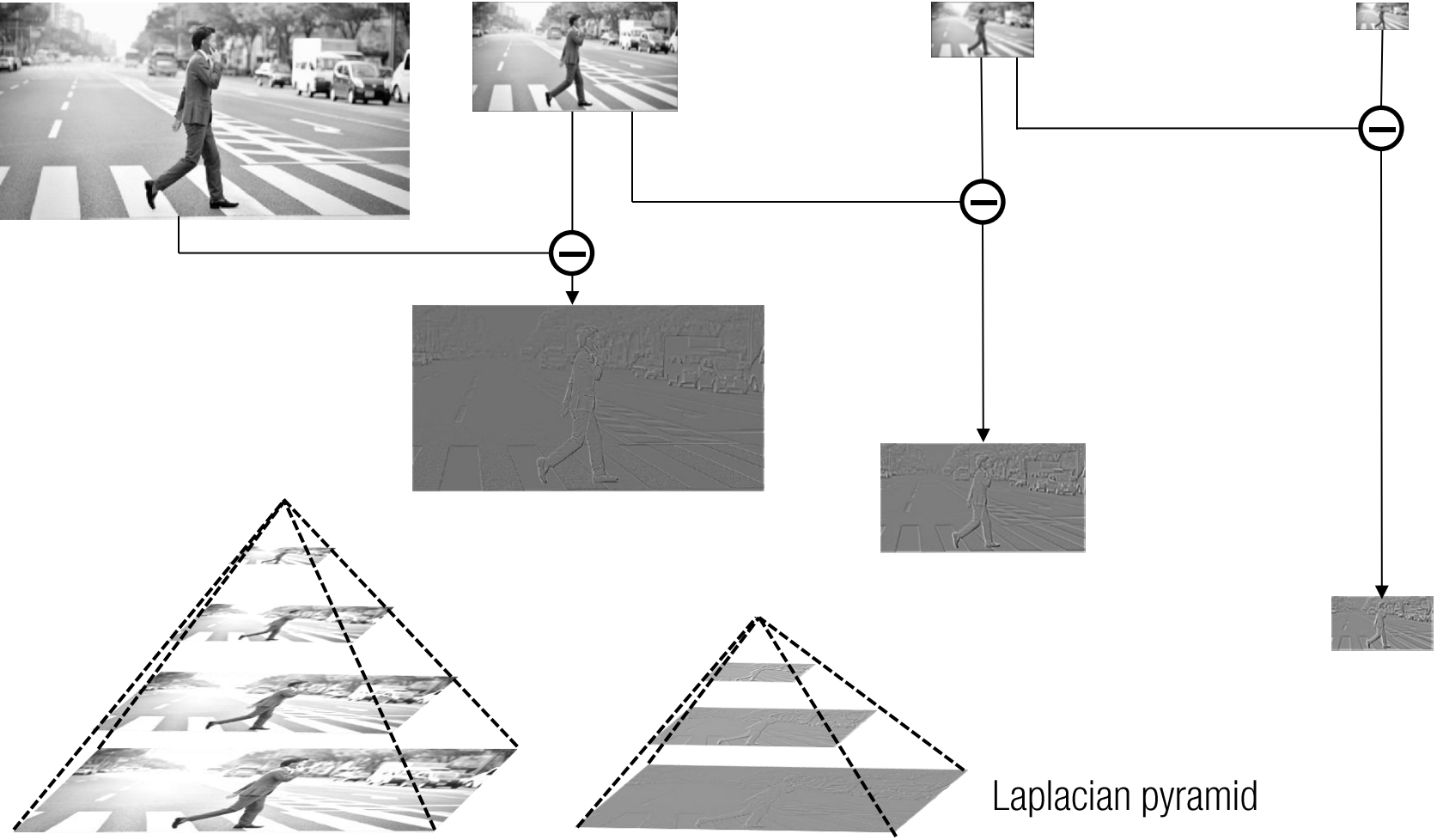


IMAGE LAPLACIAN

IMAGE LAPLACIAN

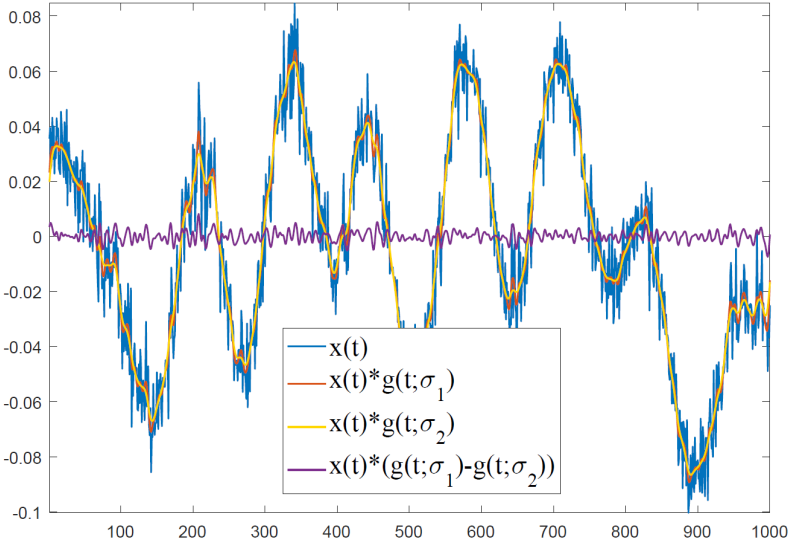


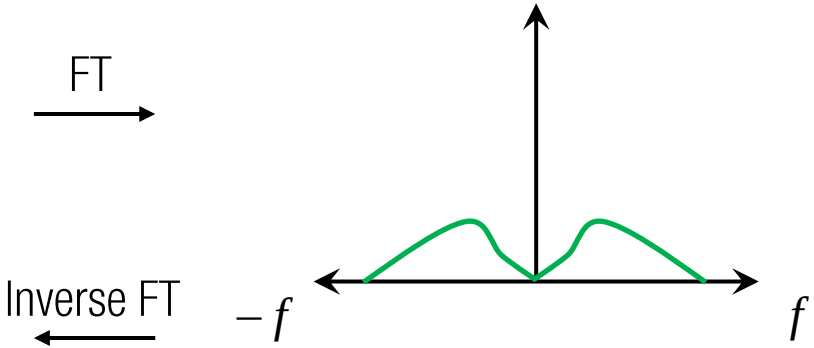




Laplacian pyramid

Difference of Gaussian (DoG) ~ Band-pass Filter

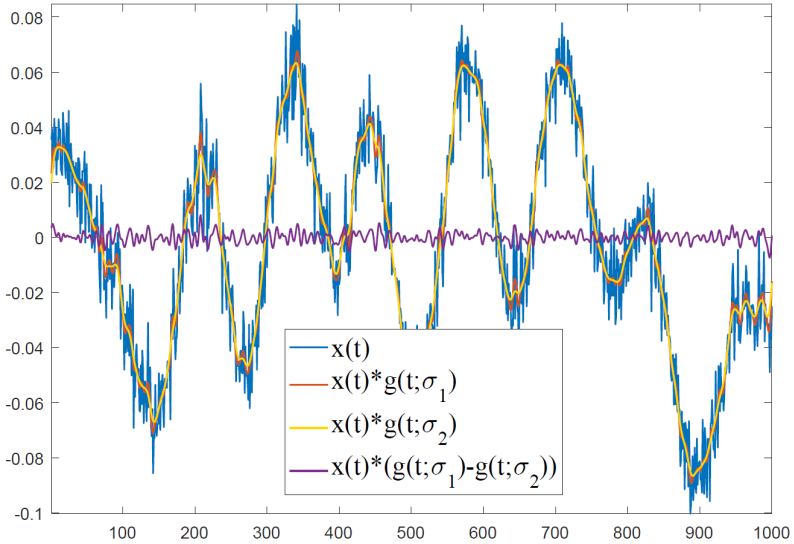


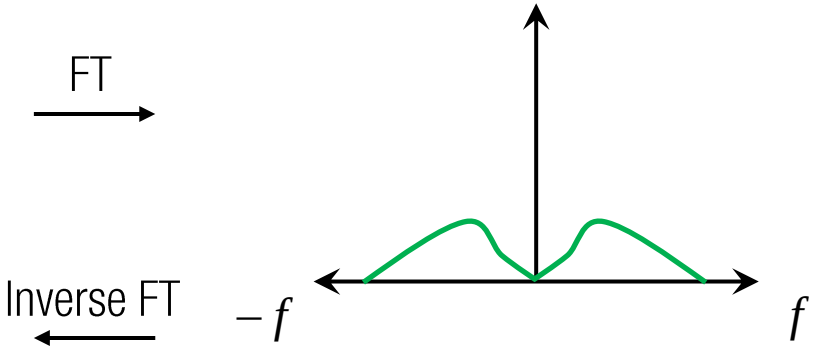


$$x(t) \quad * \quad g(t;\sigma_1) - g(t;\sigma_2)$$

$$X(f)\left(G(f;\sigma_1) - G(f;\sigma_2)\right)$$

Signal Reconstruction ~ LoG + G. Filtering





$$l(t) = x(t) * \left(g(t;\sigma_1) - g(t;\sigma_2) \right)$$

$$x(t) * g(t;\sigma_1) = l(t) + x(t) * g(t;\sigma_2)$$

Signal reconstruction with laplacian

$$L(f) = X(f)\left(G(f;\sigma_1) - G(f;\sigma_2)\right)$$

$$X(f)G(f;\sigma_1) = \underbrace{L(f)}_{\text{LoG}} + \underbrace{X(f)G(f;\sigma_2)}_{\text{Smoother signal}}$$

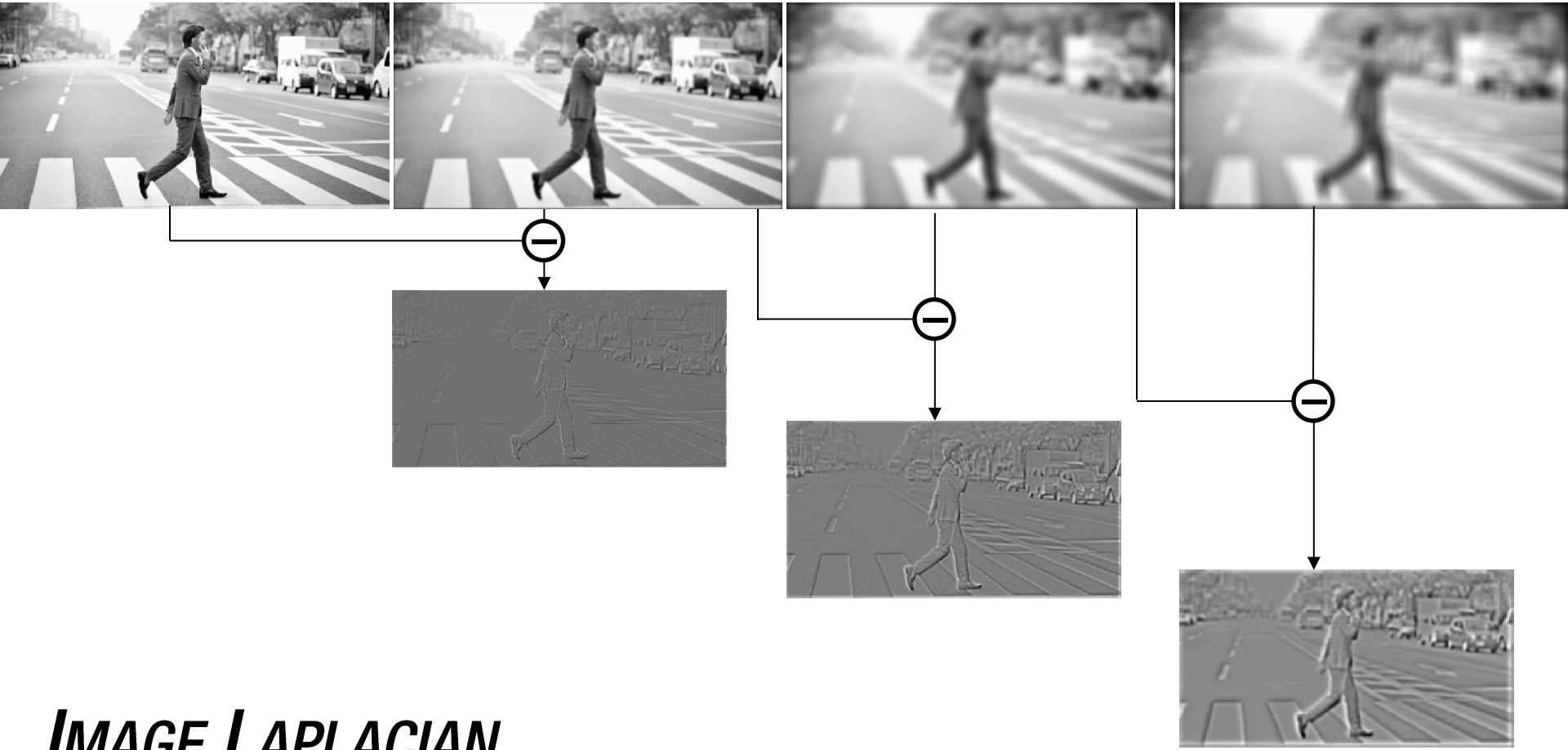


IMAGE LAPLACIAN

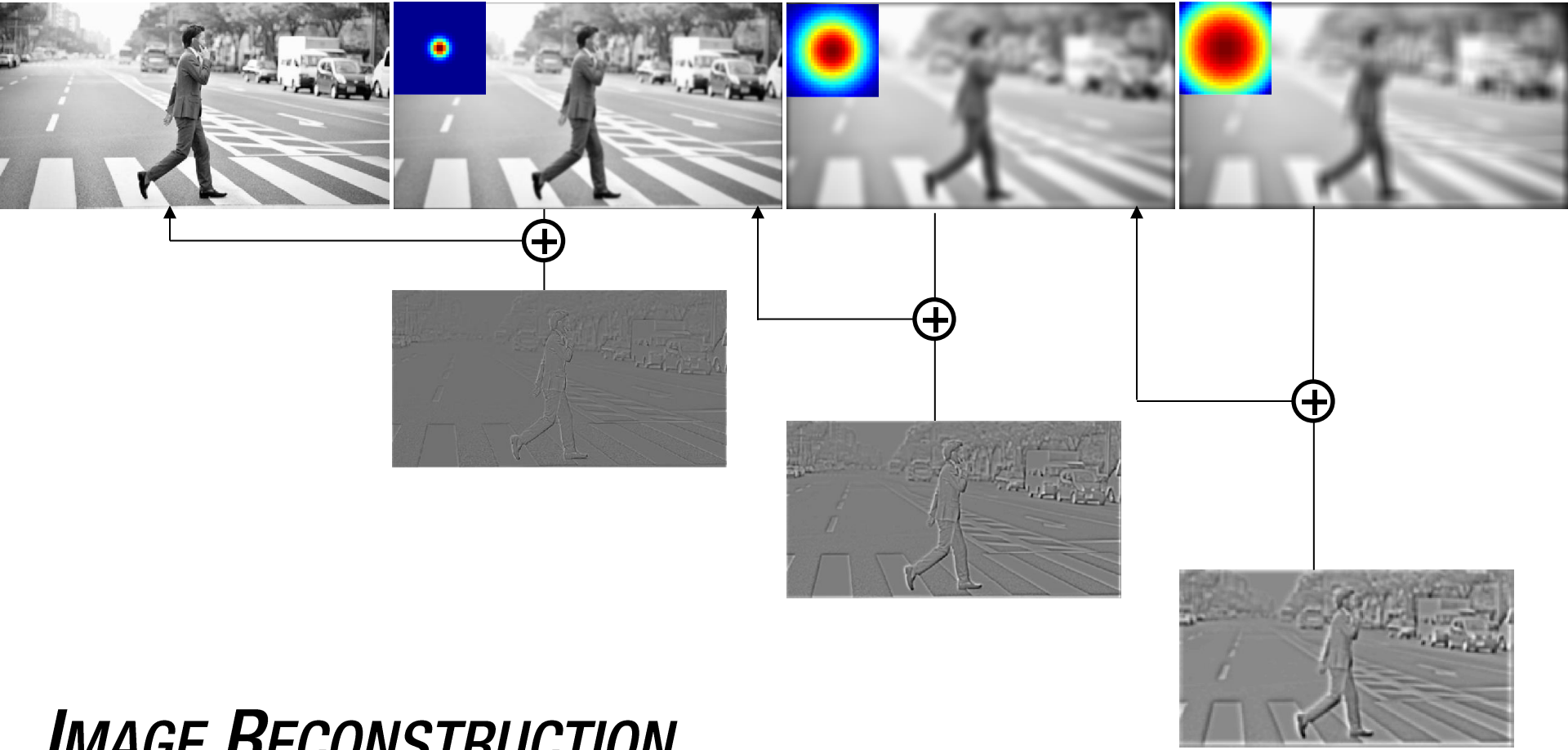
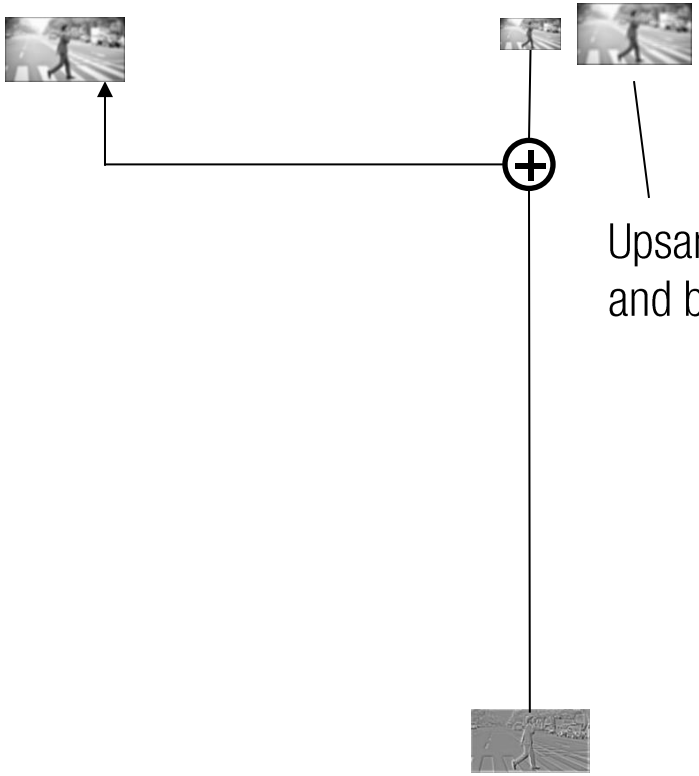
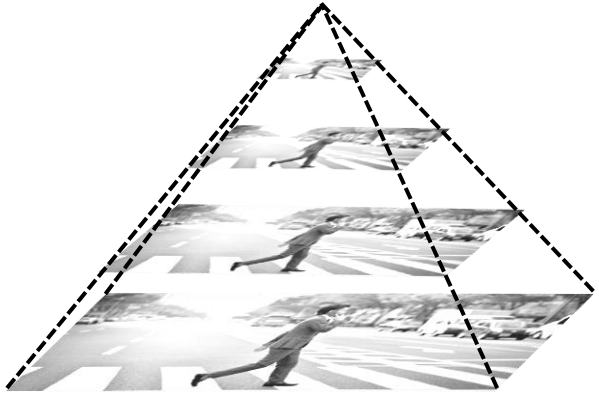
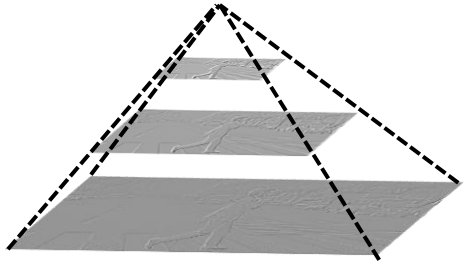


IMAGE RECONSTRUCTION

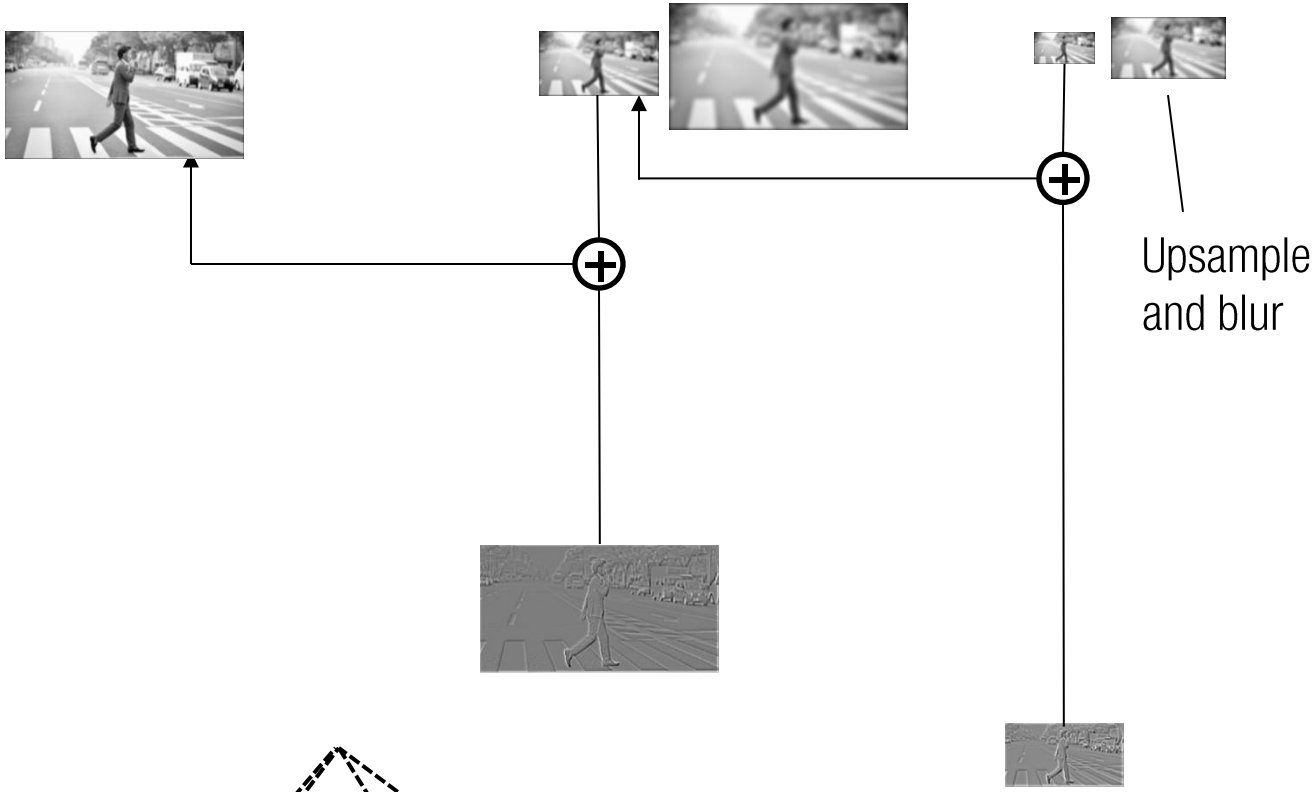


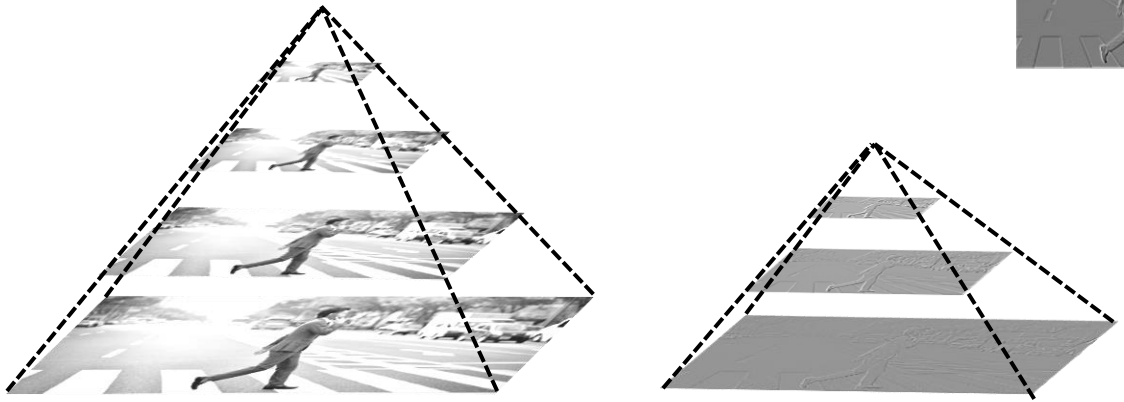
Upsample
and blur





Laplacian pyramid



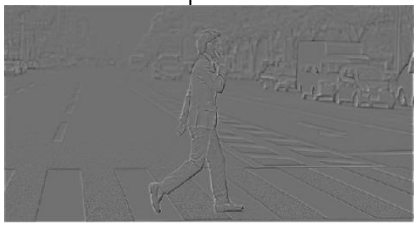


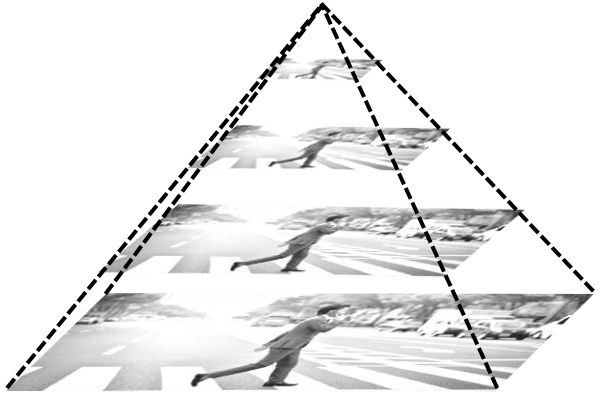
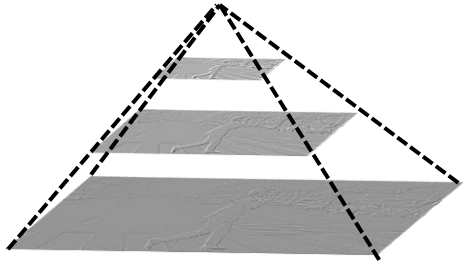
Laplacian pyramid









Upsample and blur

Laplacian pyramid