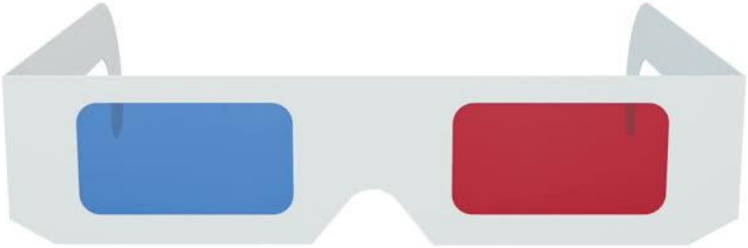
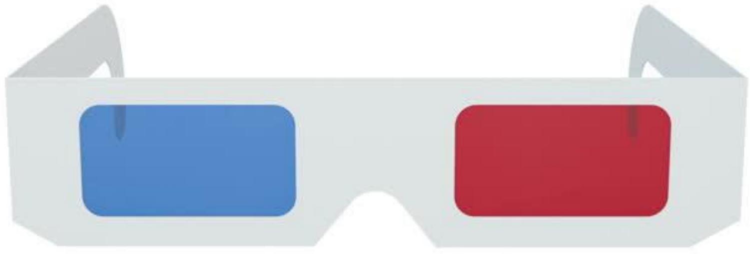
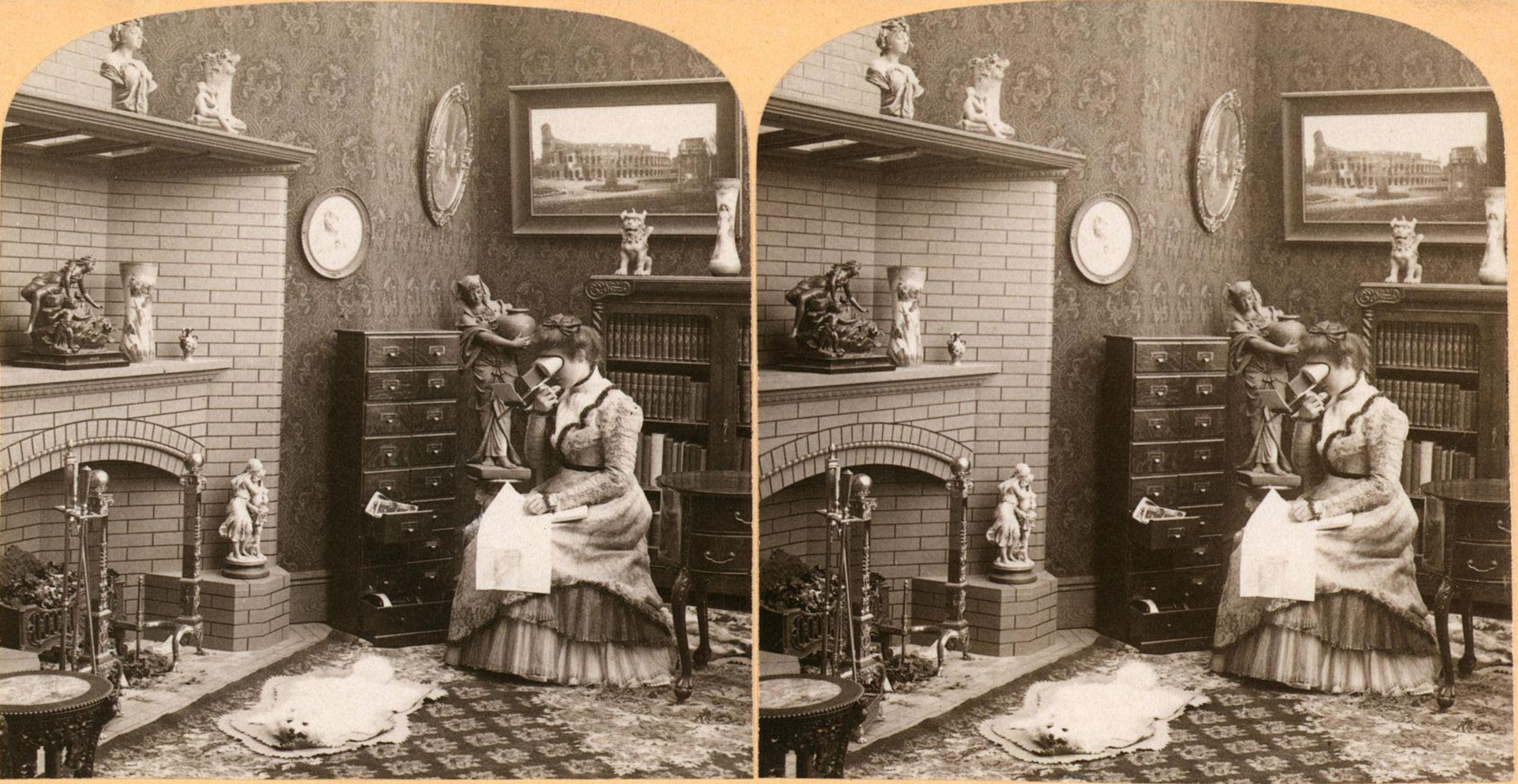
Epipolar Geometry

Hyun Soo Park



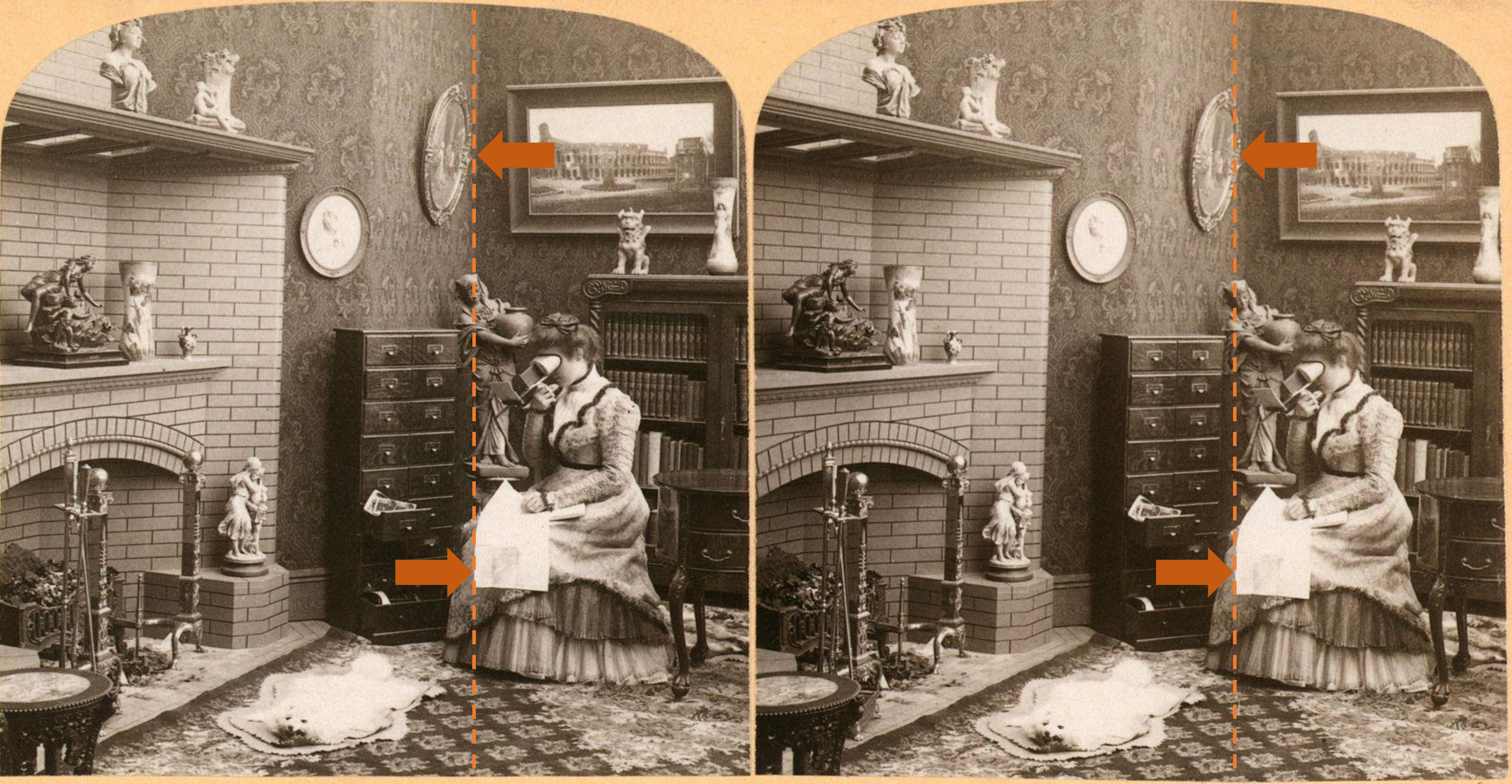


Circa 1900



The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.

The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.



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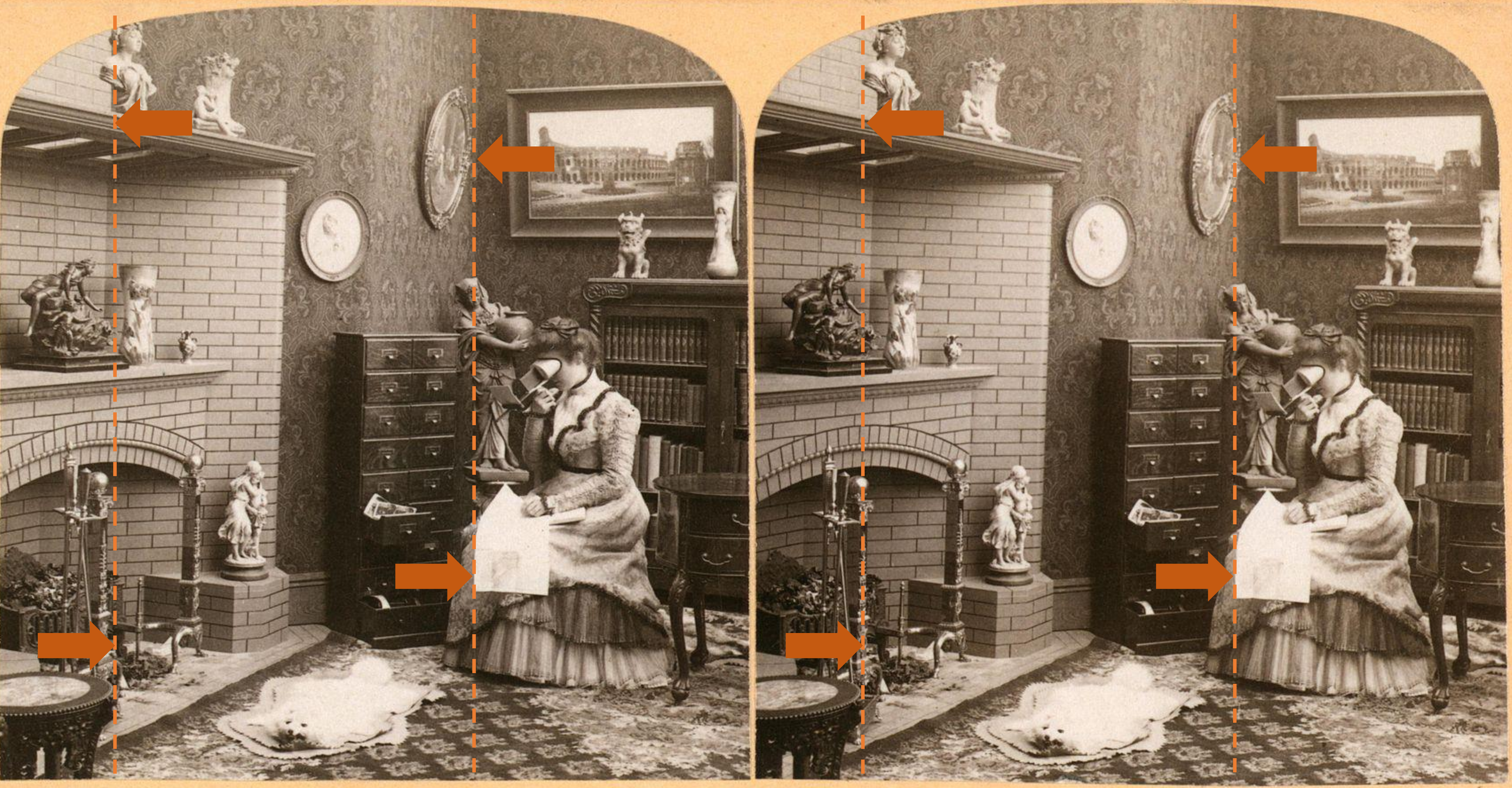

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3DV 2020

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KeystoneDepth

KeystoneDepth: History in 3D

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University of Washington

Yanmeng Kong
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Google Research

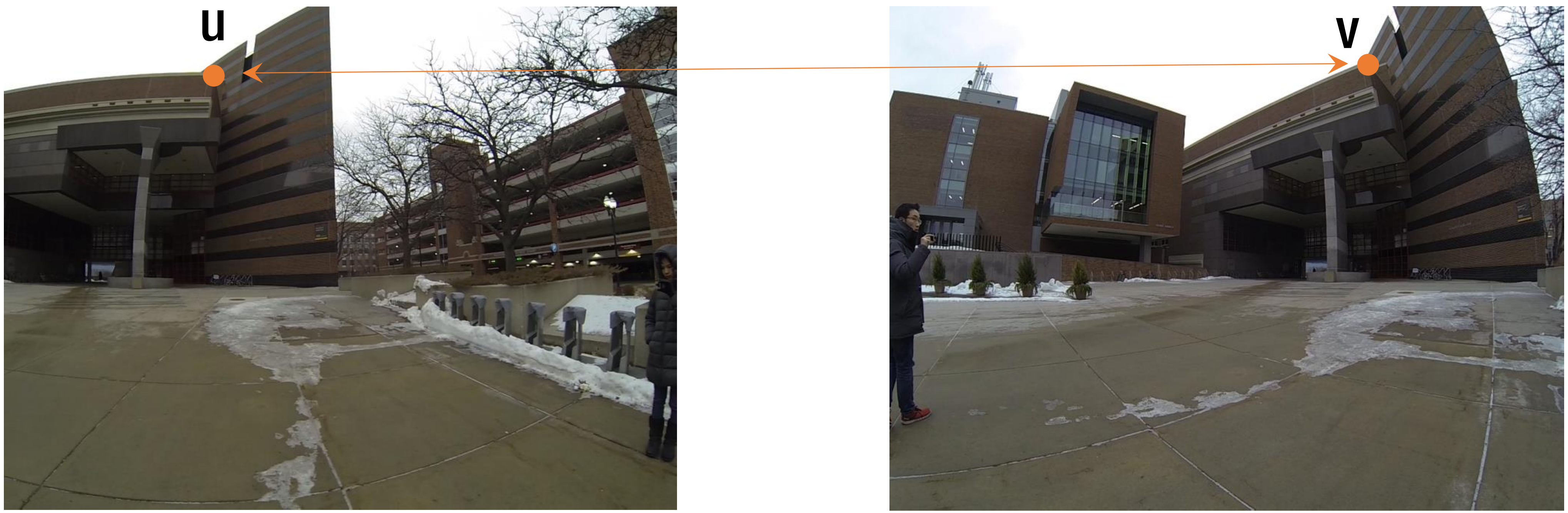
Ricardo Martin-Brualla
Google Research

Steven M. Seitz
University of Washington, Google Research

Left image (Bob)

Right image (Alice)

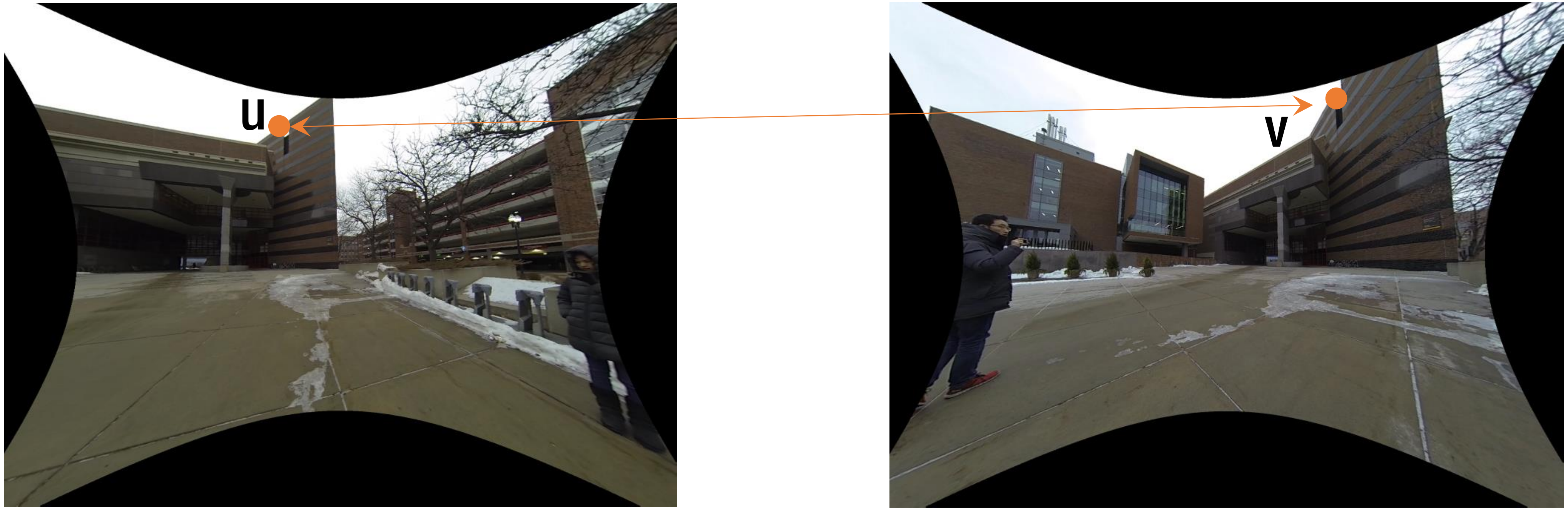
2D Correspondence



Left image (Bob)

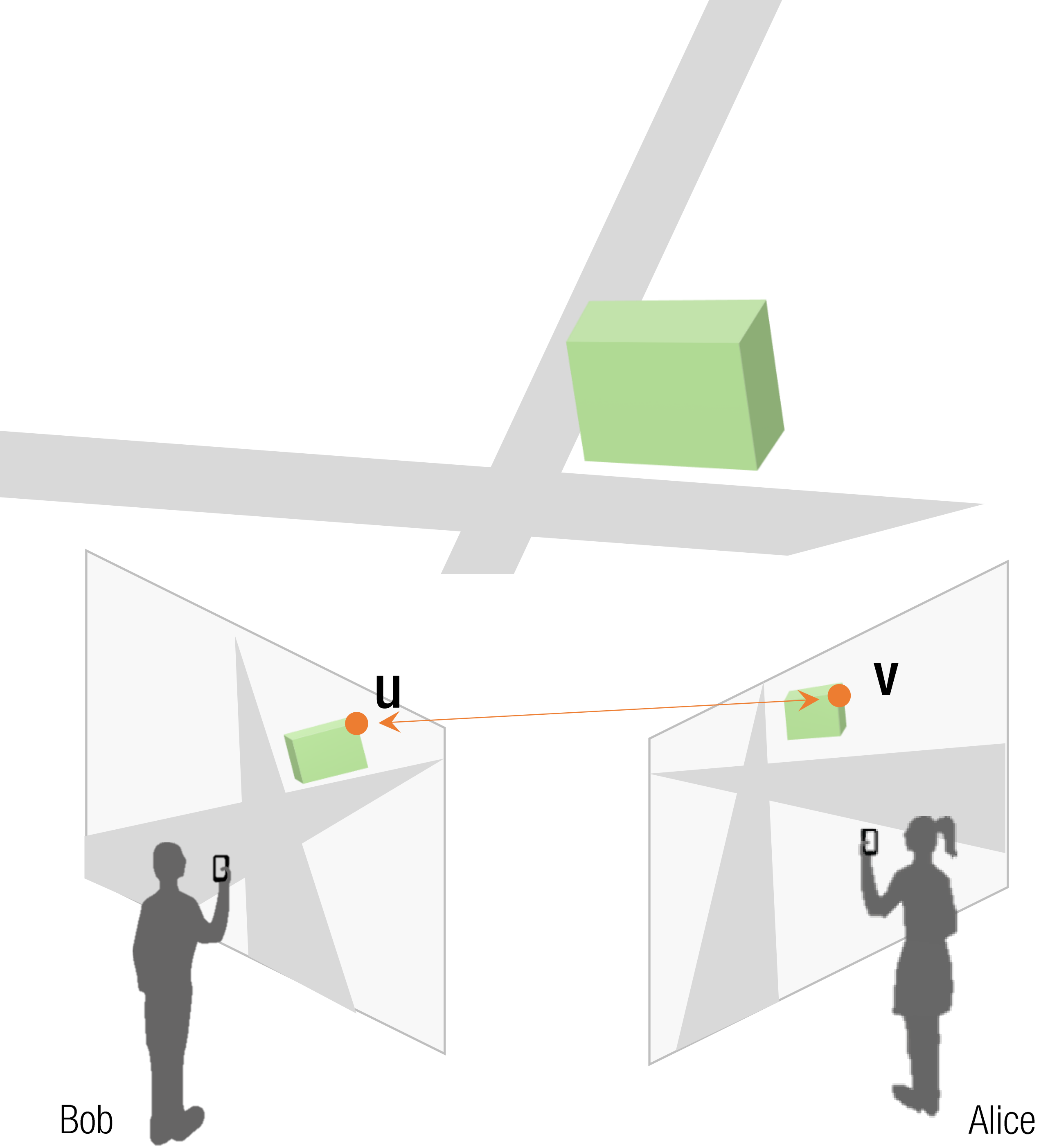
Right image (Alice)

2D Correspondence



Left image (Bob)

Right image (Alice)



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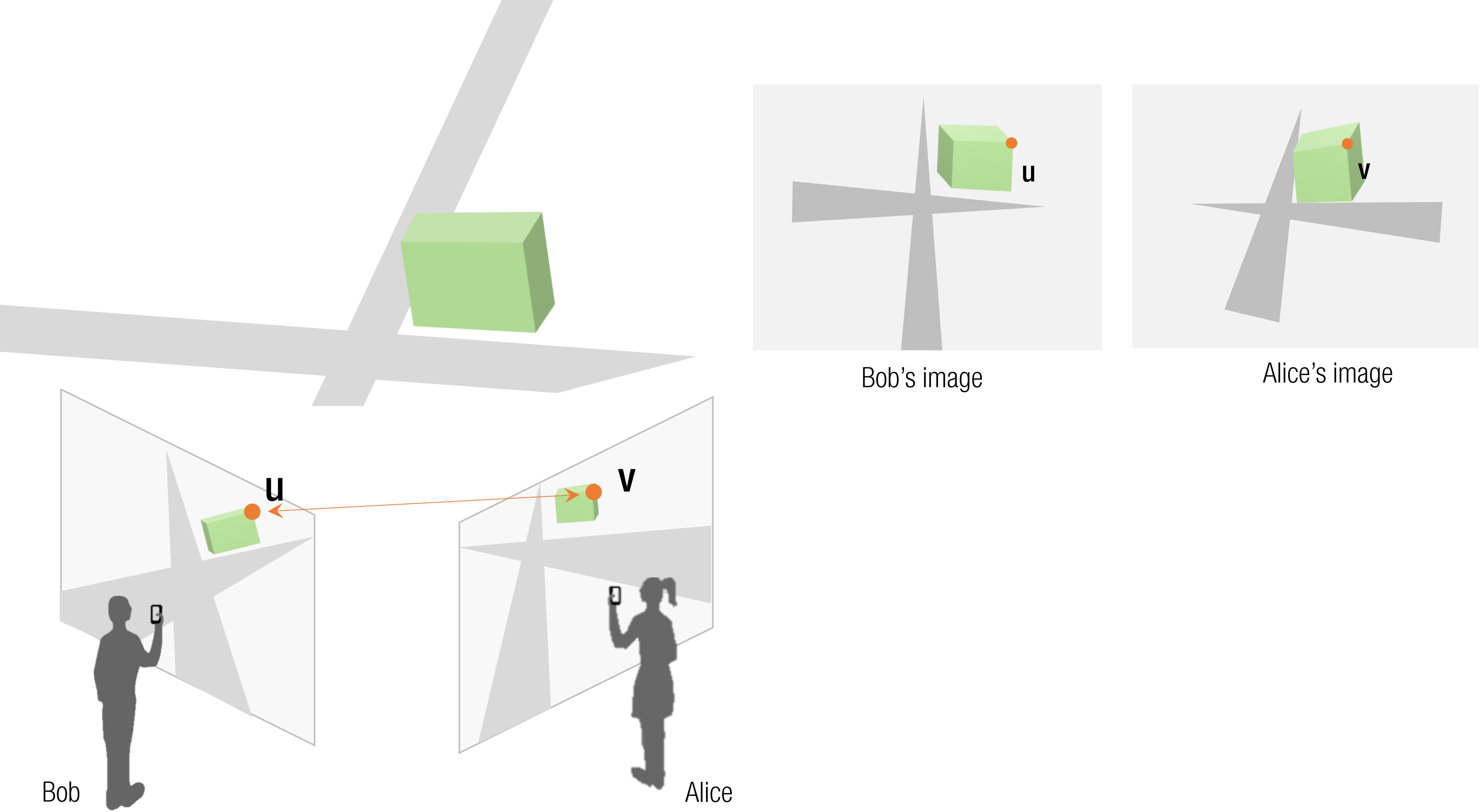
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Bob's image
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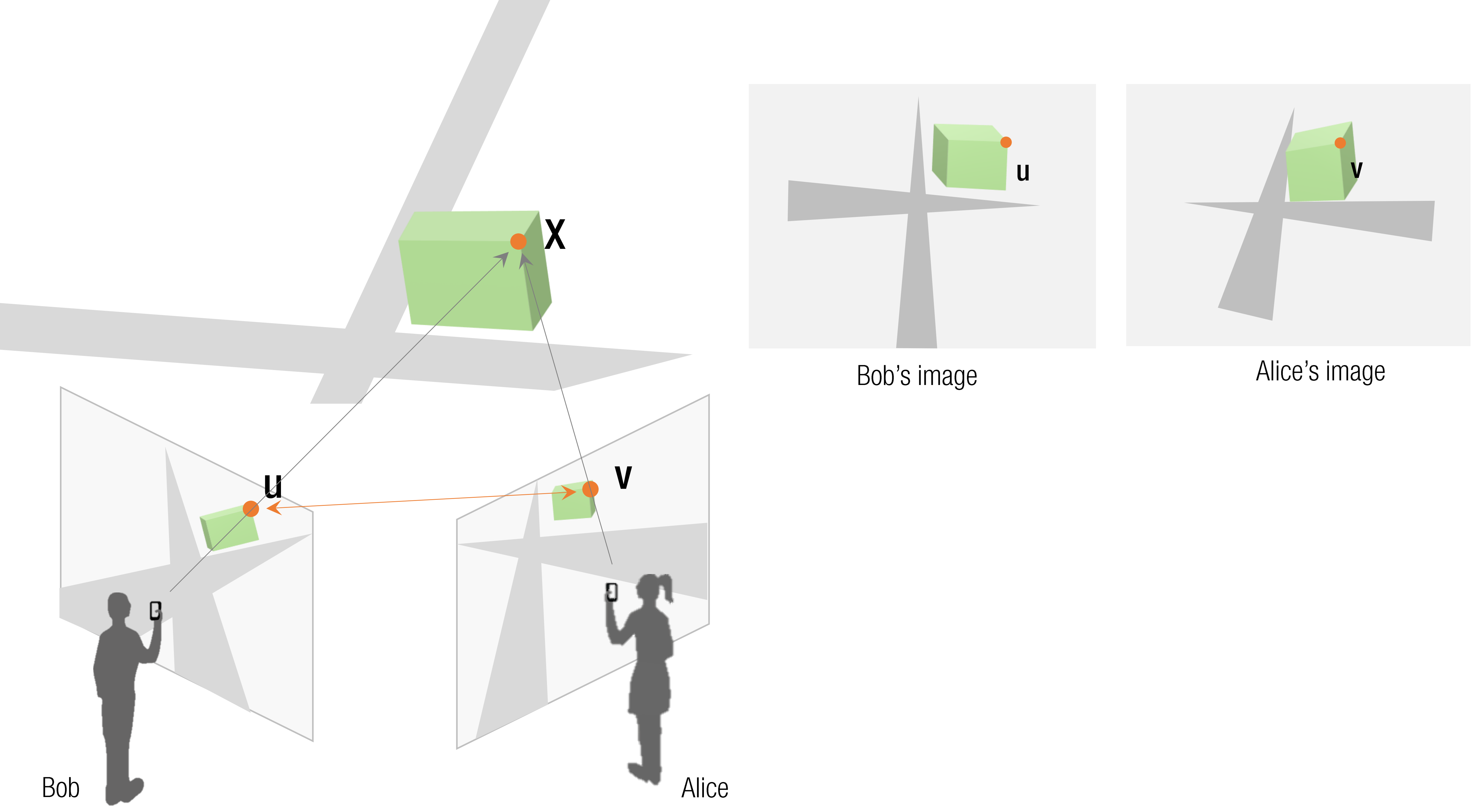

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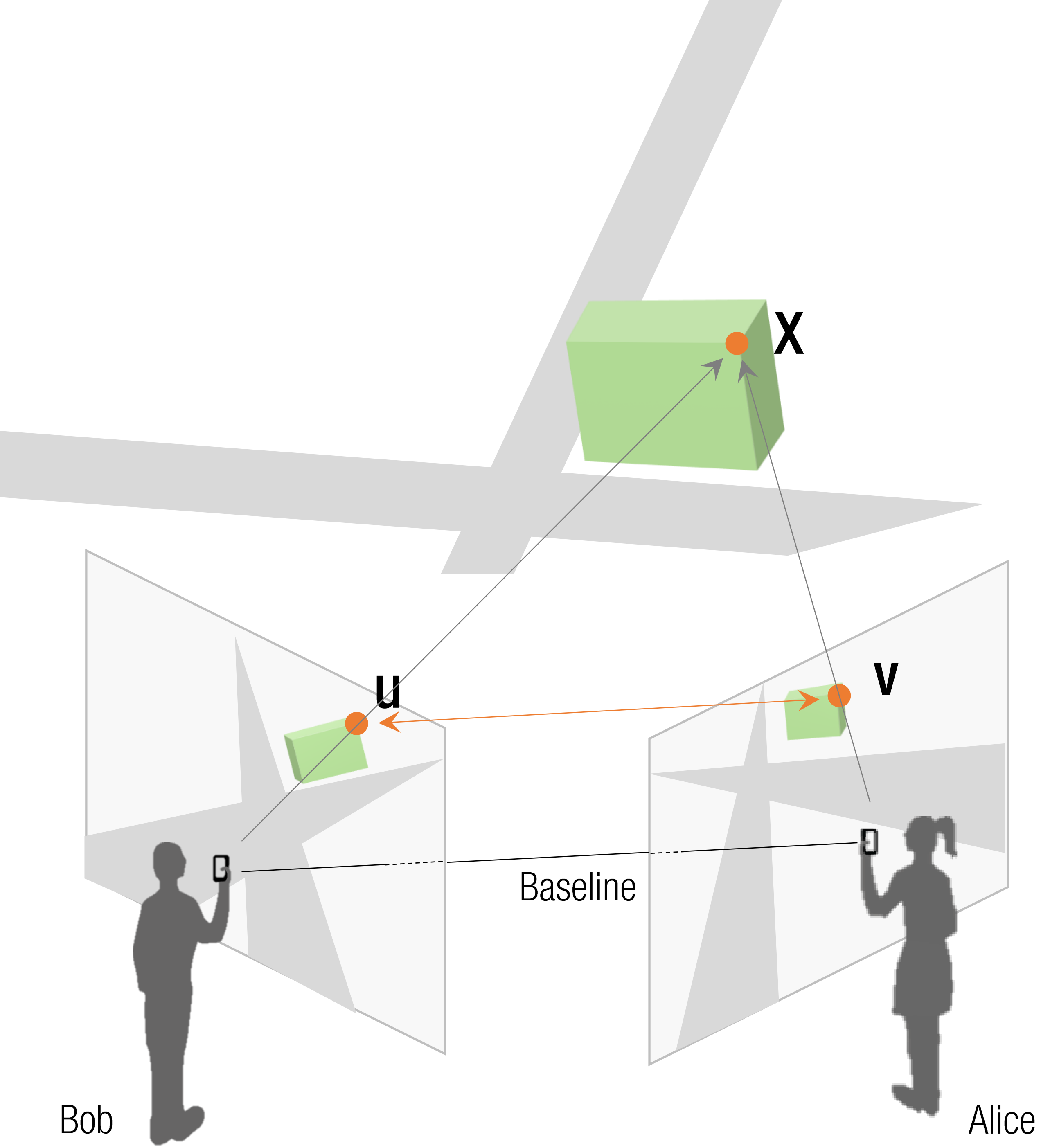
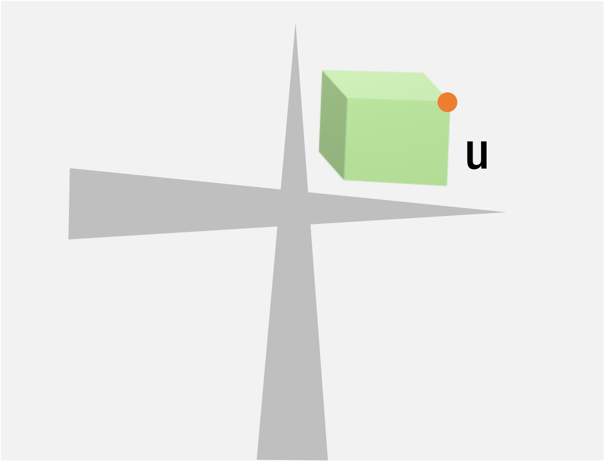
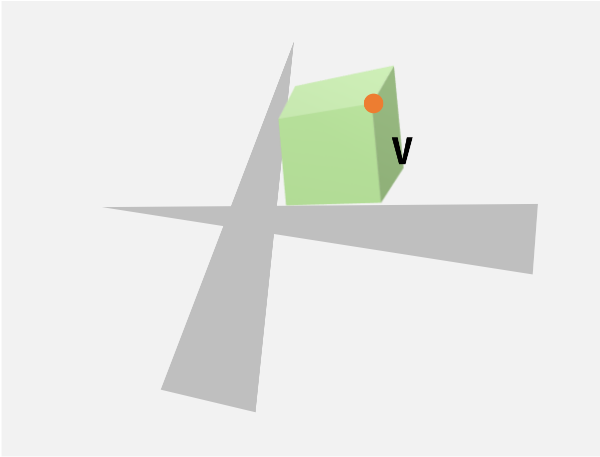
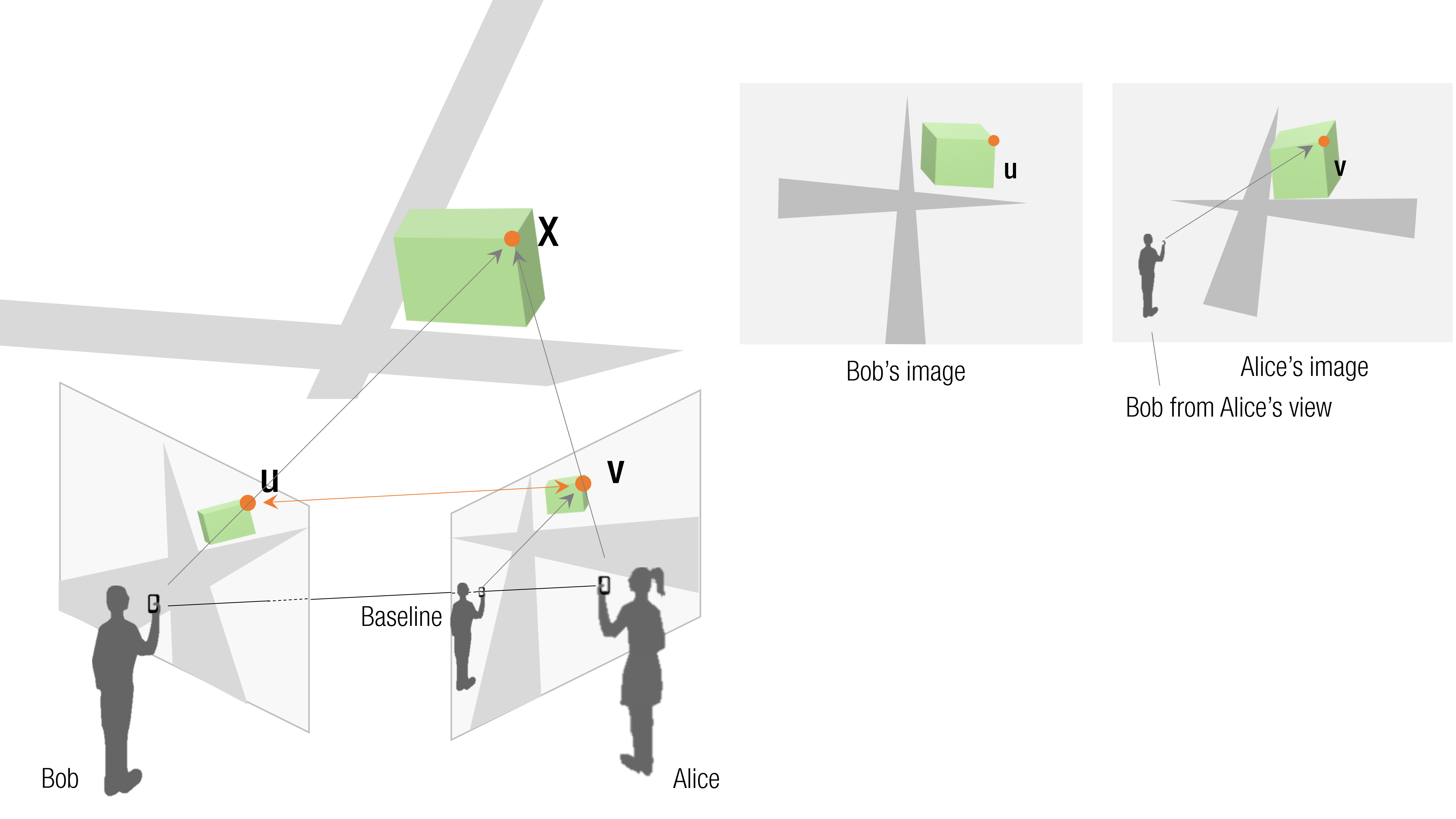




Bob’s image



Alice’s image


X
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Baseline
Bob
Alice
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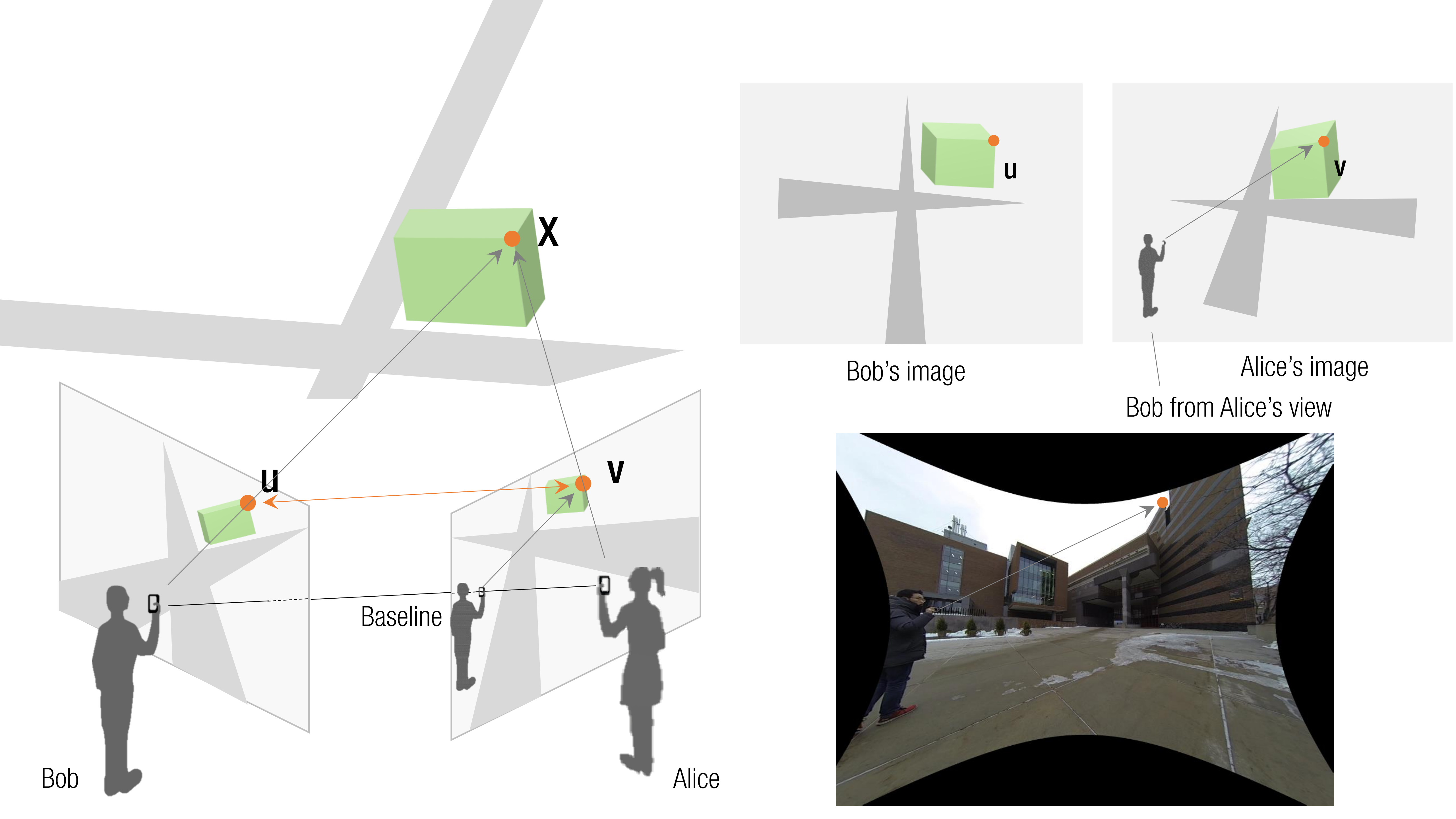
Bob's image
v
Alice's image
Bob from Alice's view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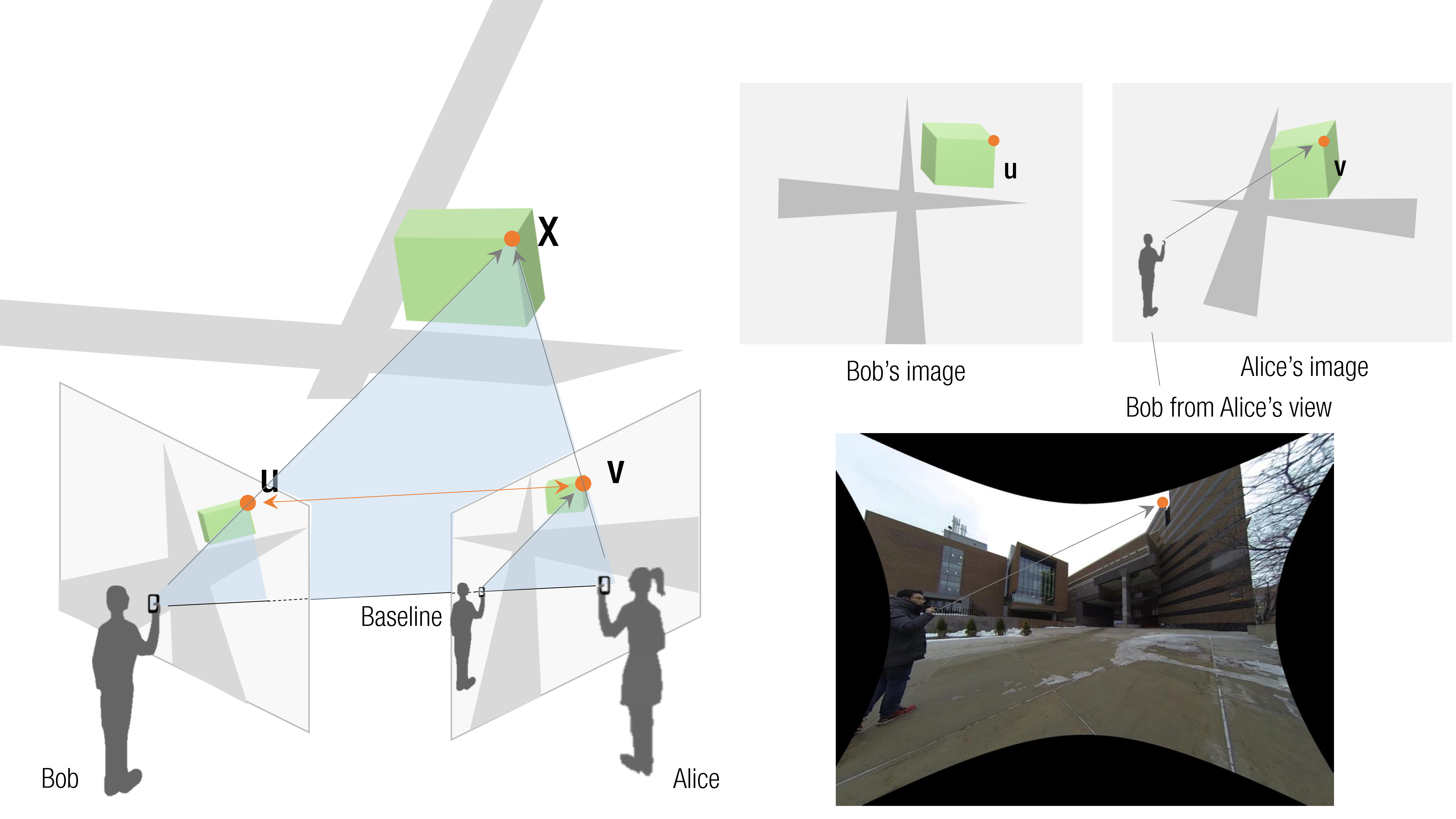
Bob from Alice's view



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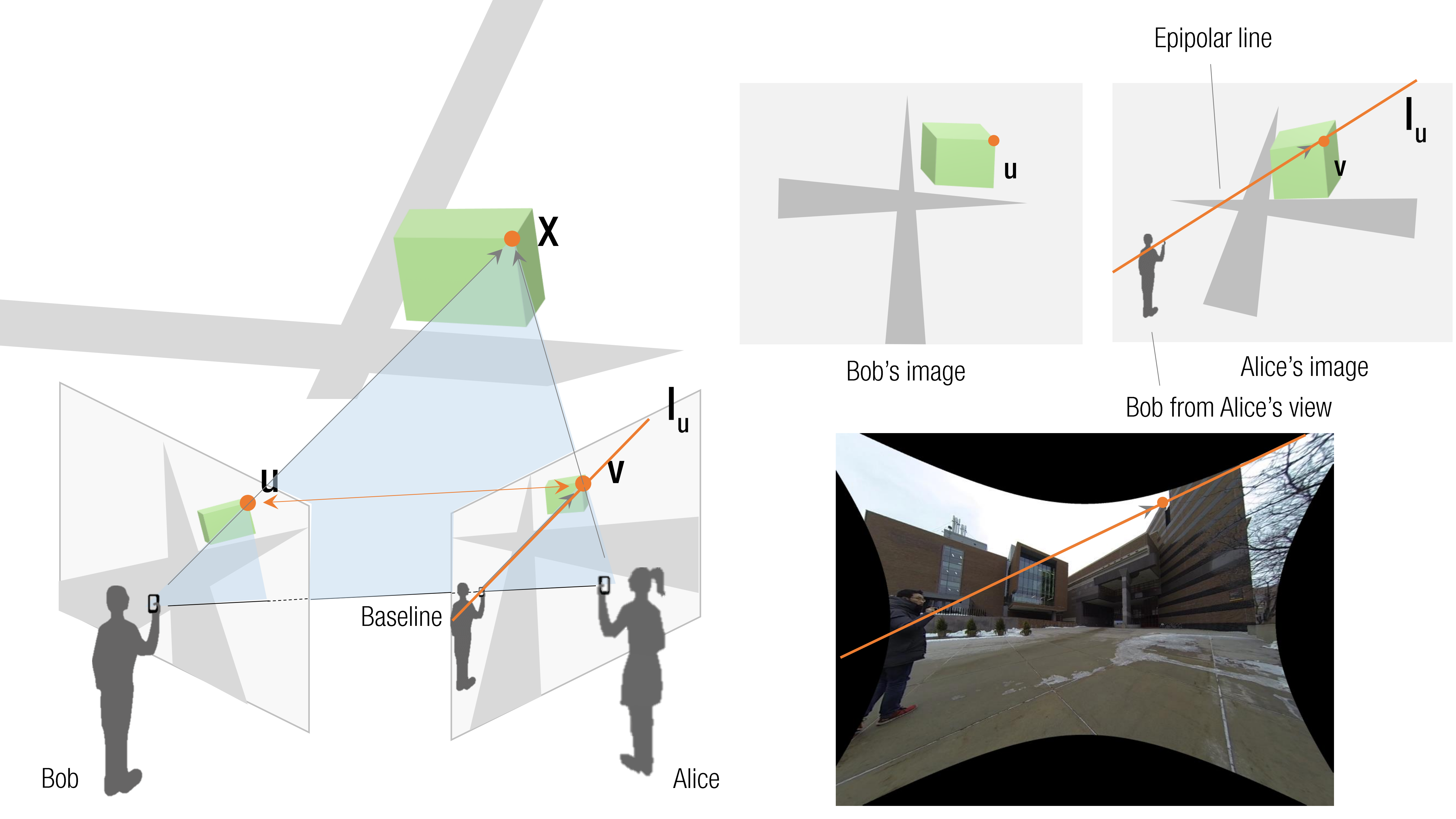
Bob from Alice's view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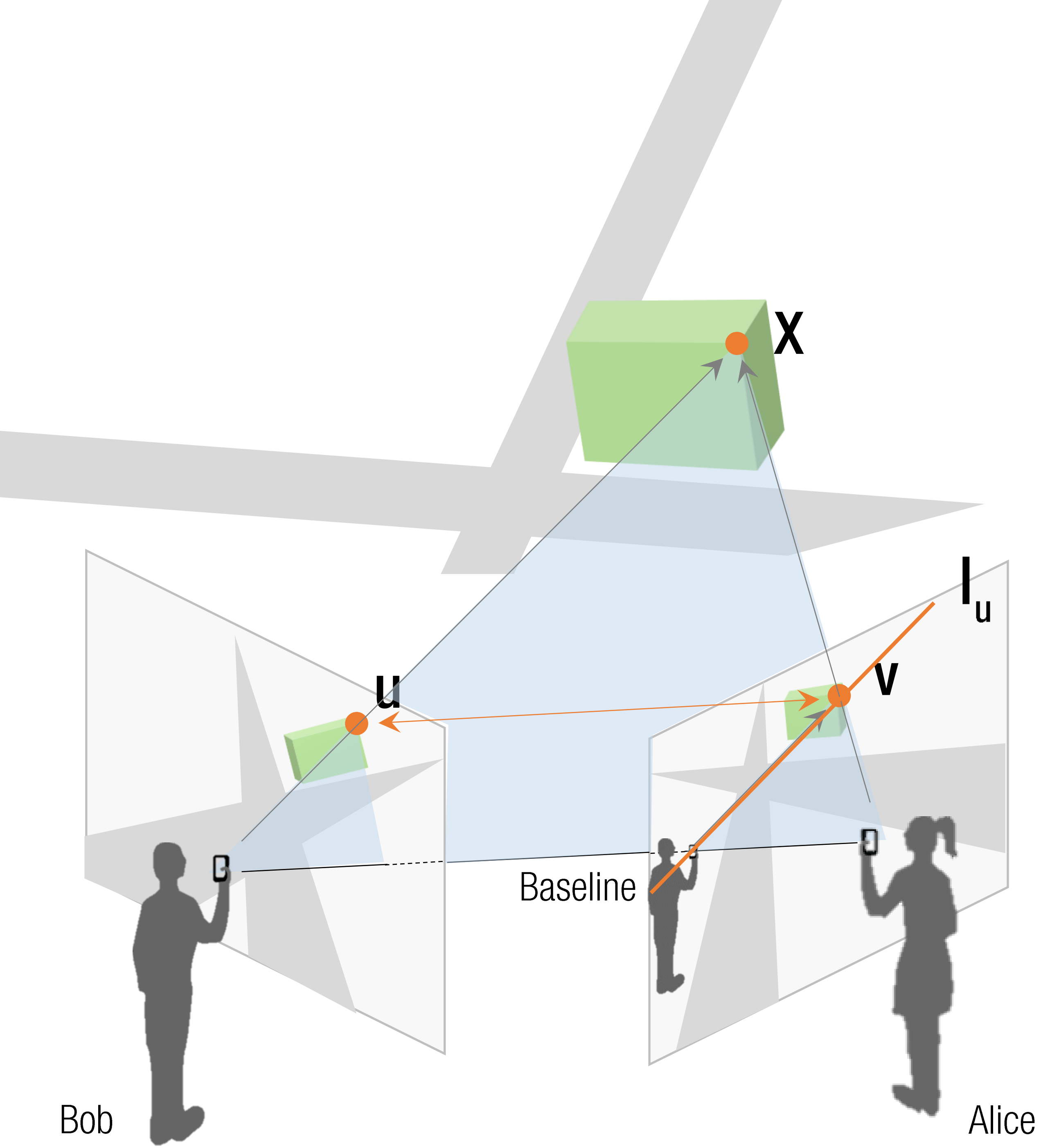
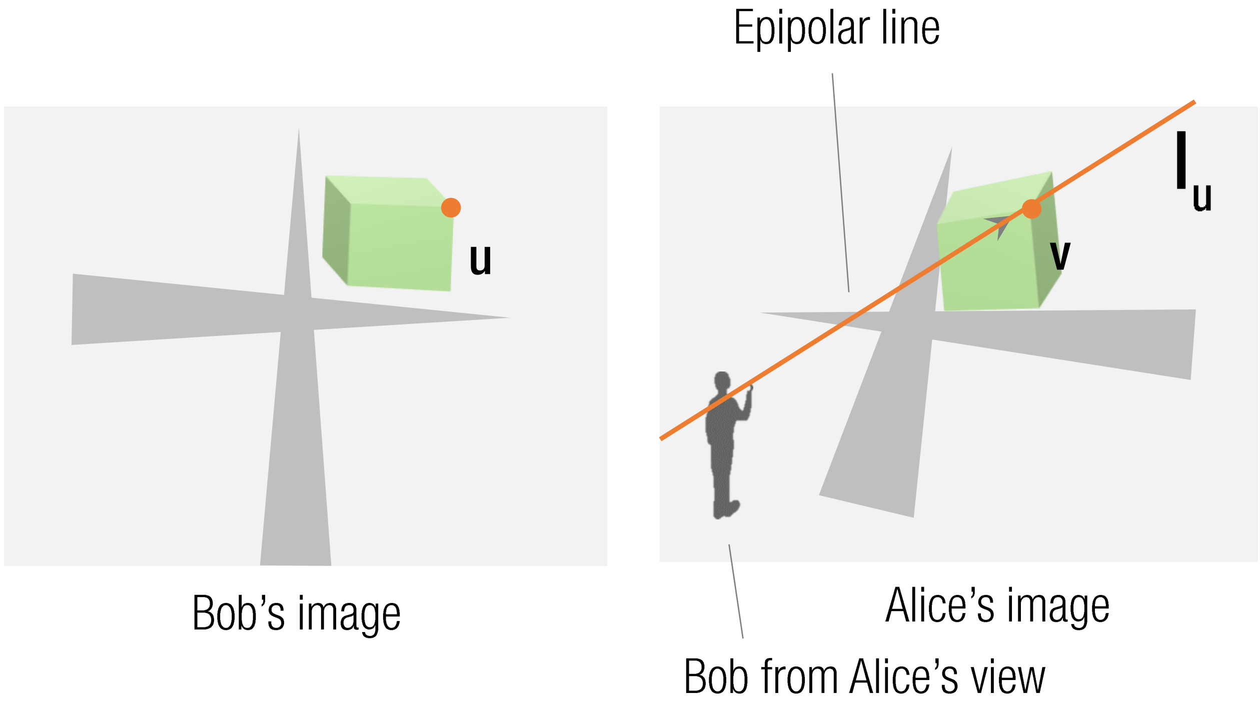
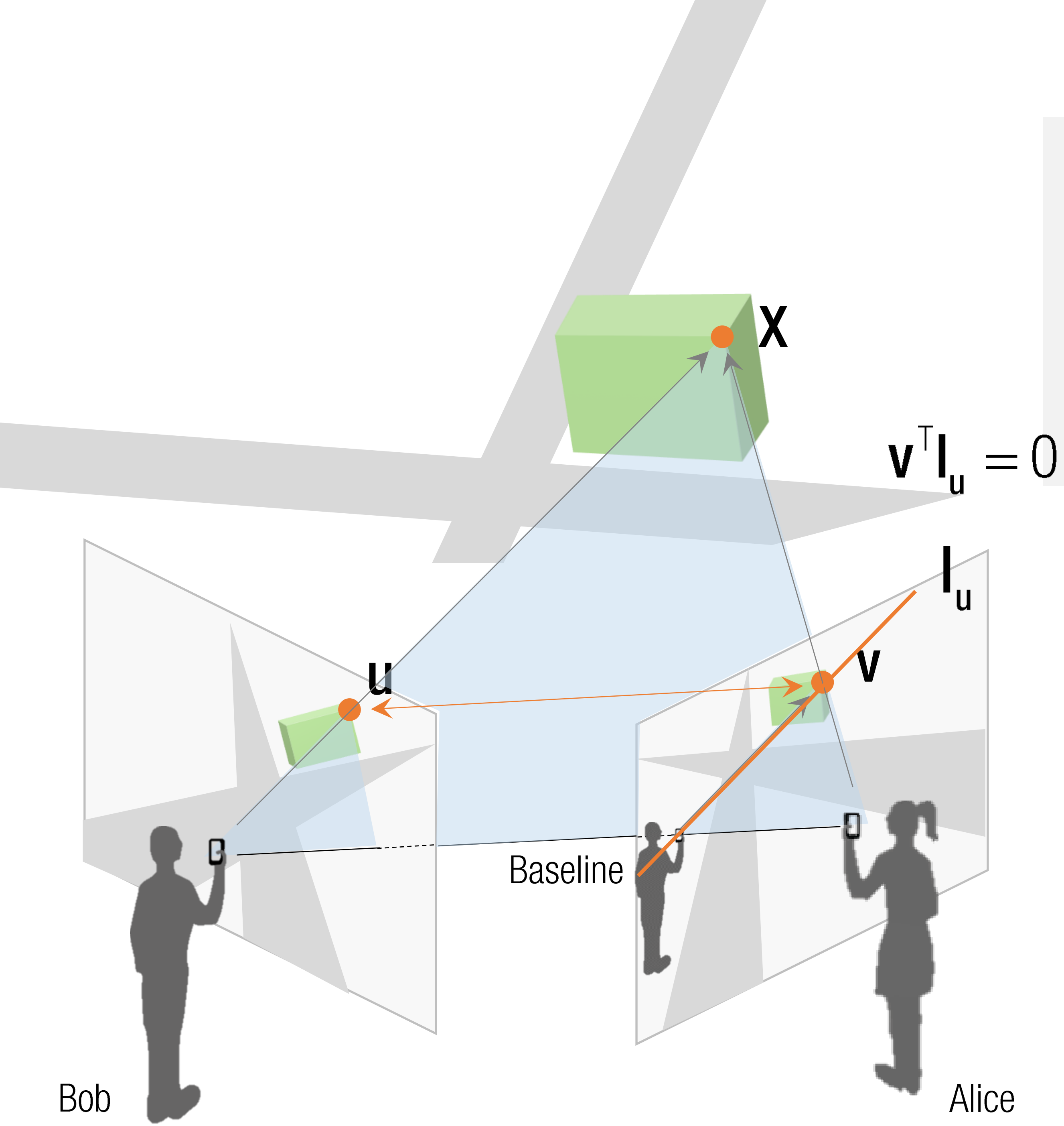
Epipolar line
Bob's image
Alice's image
Bob from Alice's view

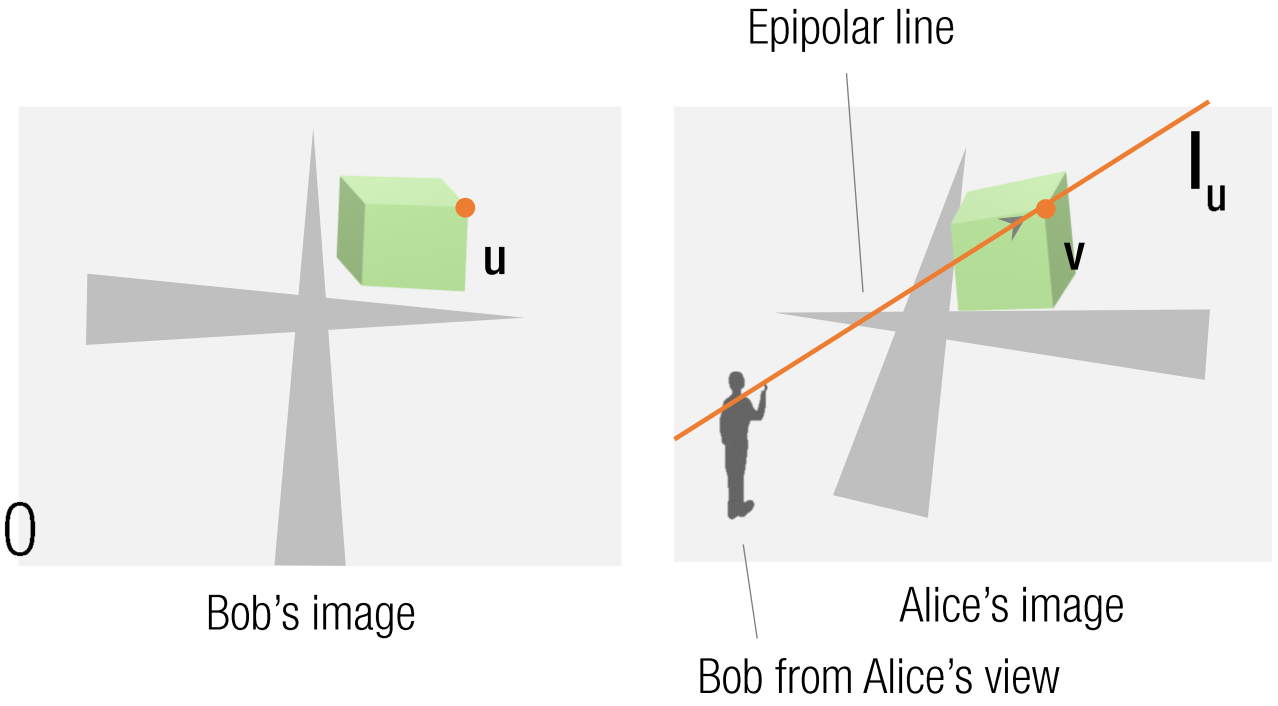




Epipolar constraint between two images:

1. A point, **u**, in Bob’s image corresponds to an epipolar line $\mathbf{l}_u$ in Alice’s image.

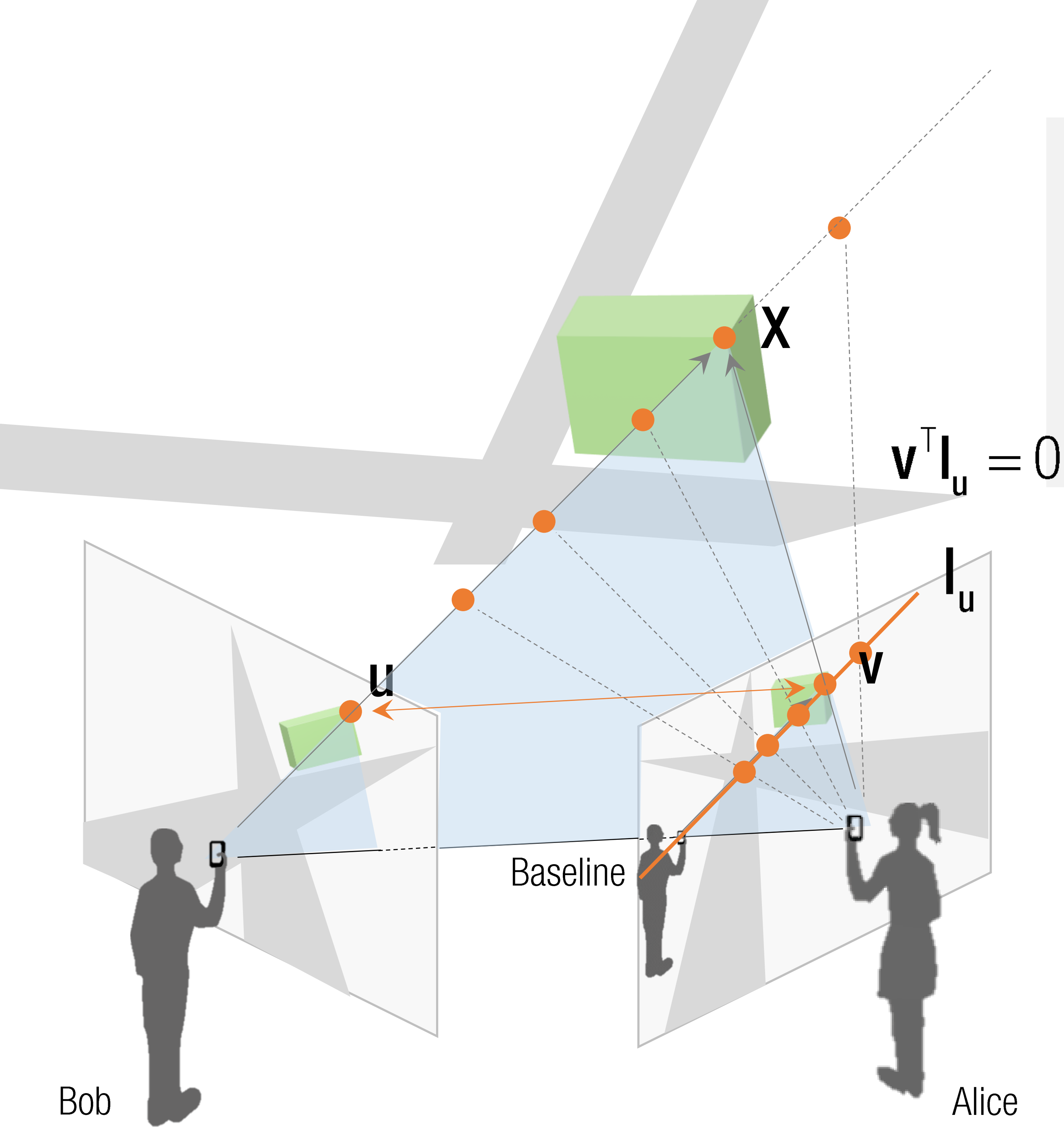


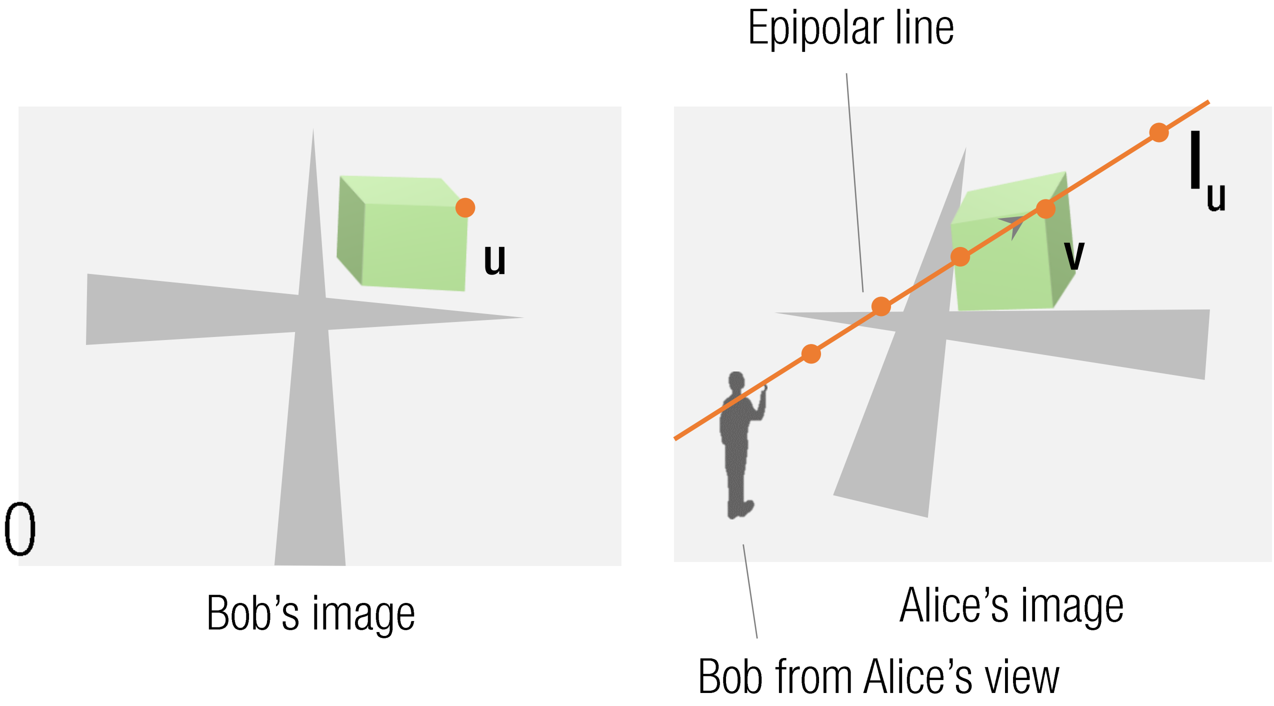


Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.

2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0$



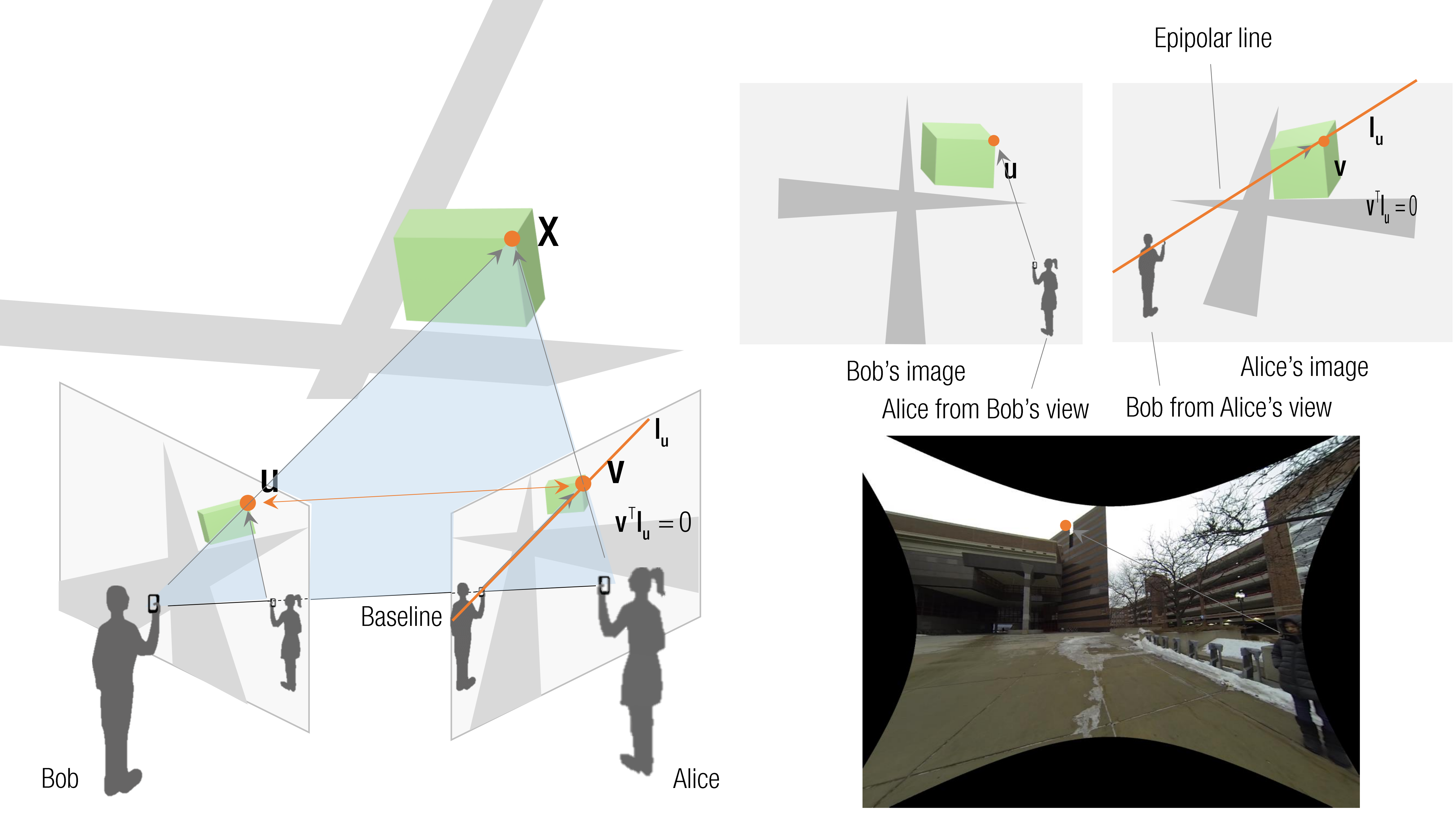


Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.

2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0$

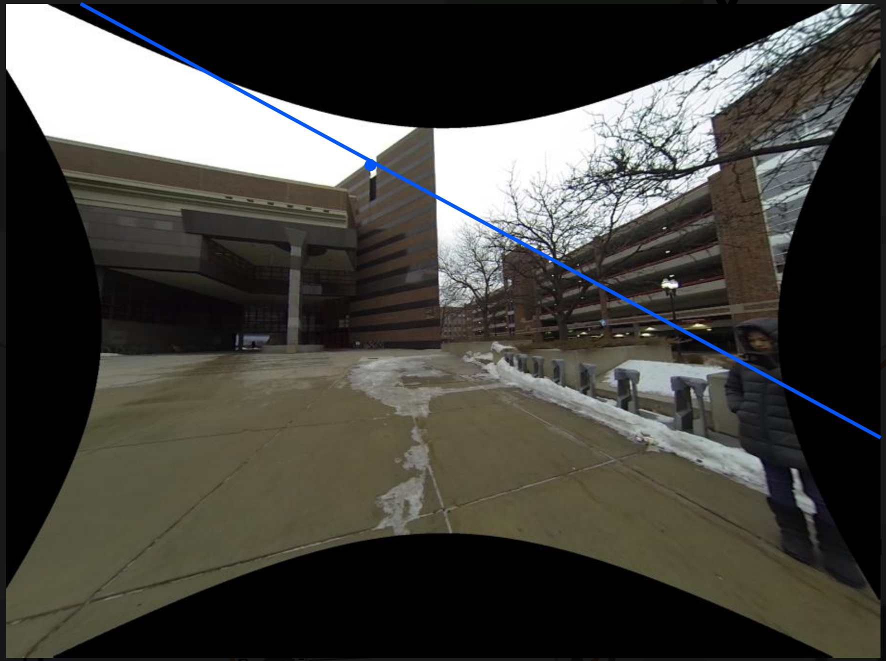
3. All points along the epipolar line are candidates of the correspondence.

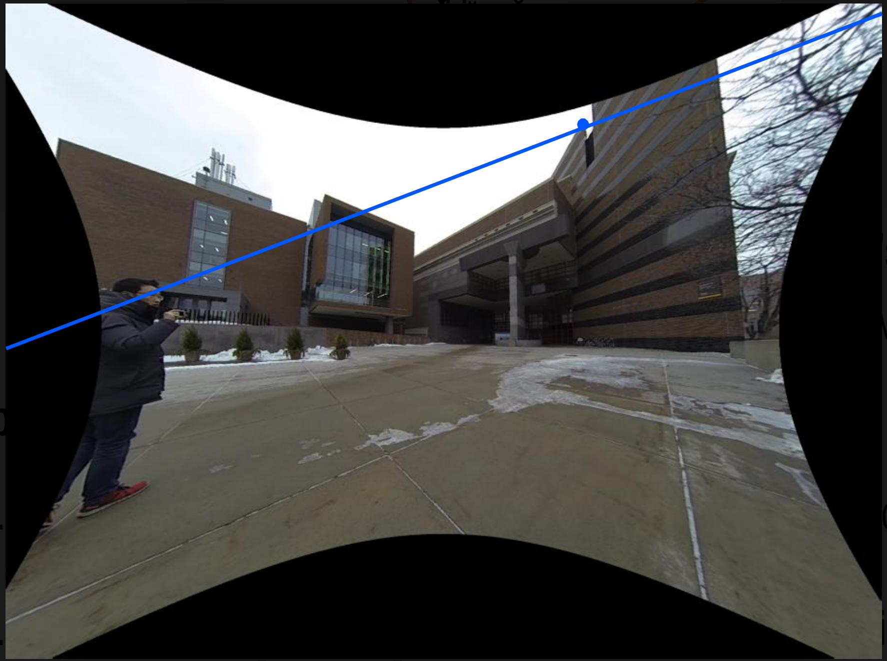

X
u
v
l_u
$v^T l_u = 0$
Baseline
Bob
Alice
Epipolar line
l_u
u
v
$v^T l_u = 0$
Bob's image
Alice from Bob's view
Alice's image
Bob from Alice's view

Epipolar line

$\mathbf{l}_v$ $\mathbf{u}$ $\mathbf{u}^T\mathbf{l}_v = 0$

$\mathbf{l}_u$ $\mathbf{v}$ $\mathbf{v}^T\mathbf{l}_u = 0$





$\mathbf{l}_u$ $= 0$

Bob

Alice

Ep

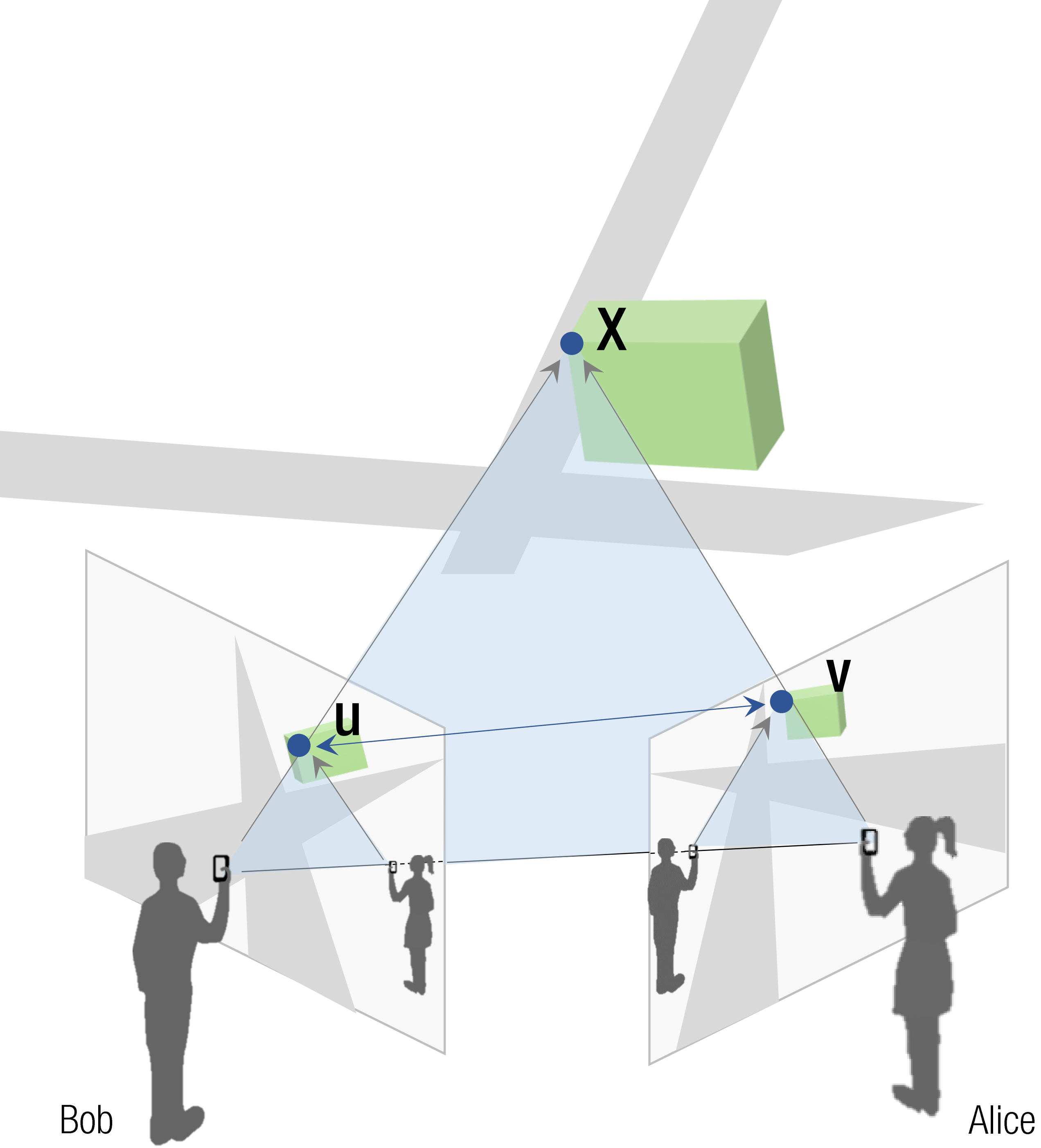
s image

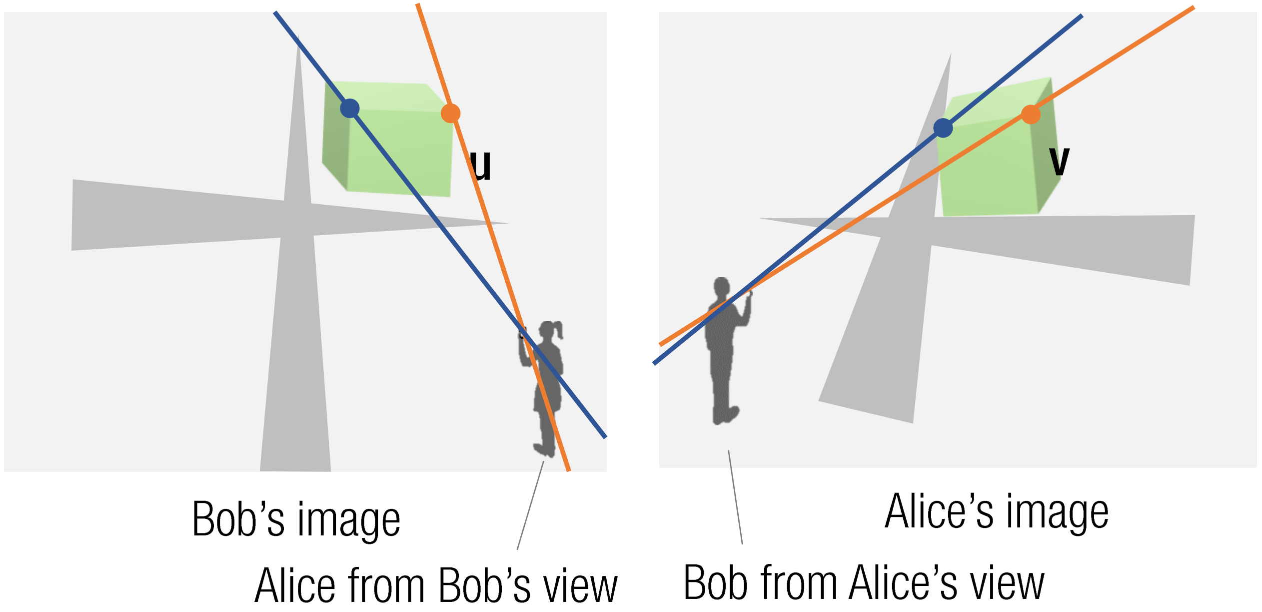
view

1. olar line in

2. in Alice's
image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0$ $\mathbf{u}^T\mathbf{l}_v = 0$

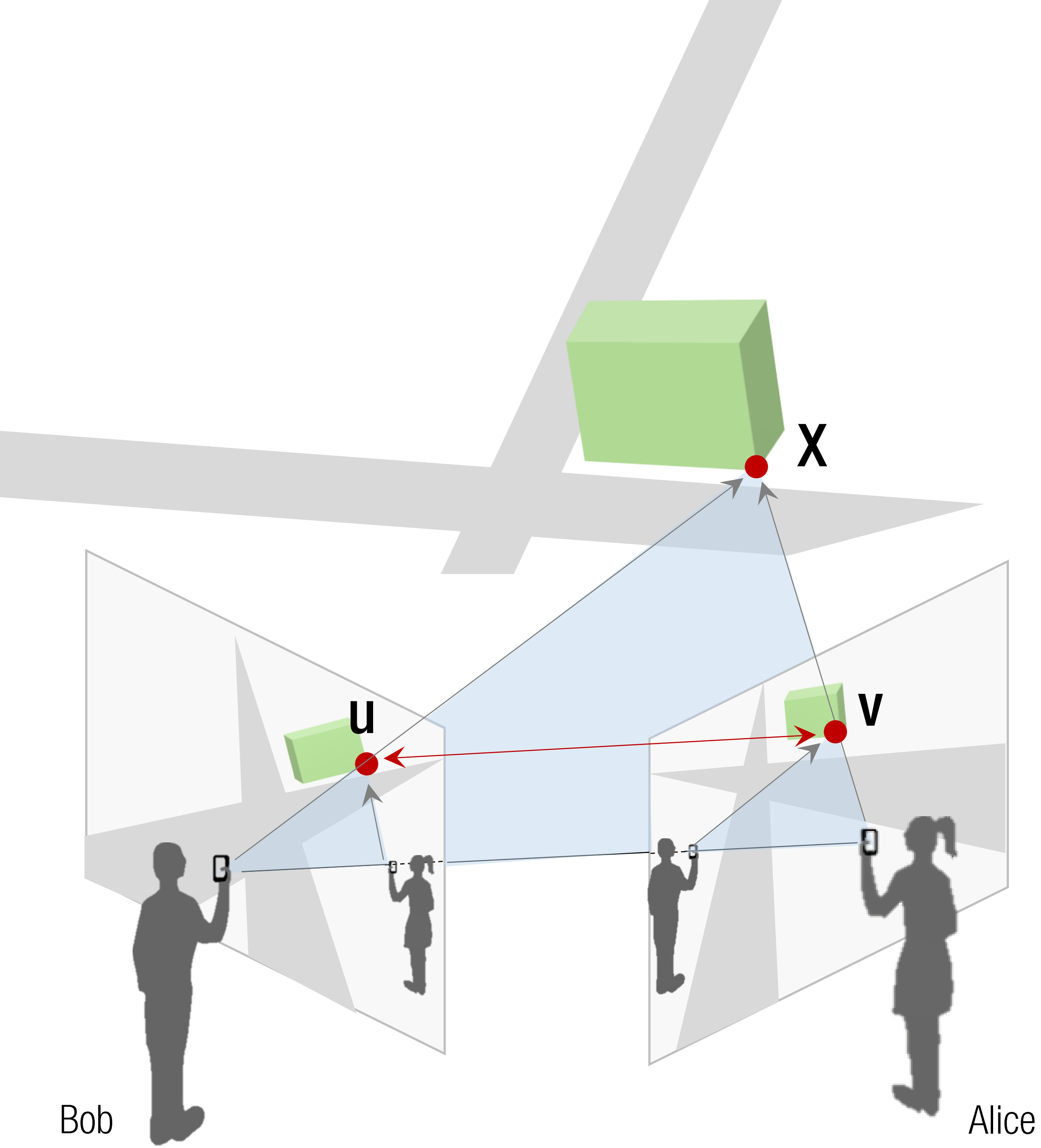
3. Any point along the epipolar line can be a candidate of correspondences.

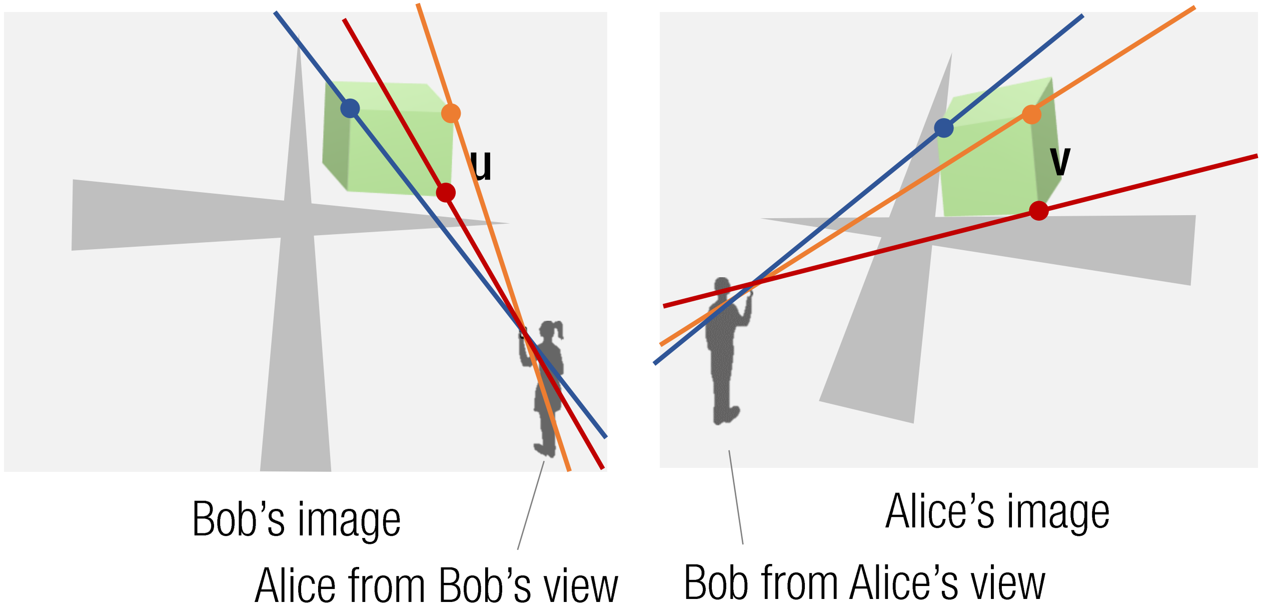




Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^{T}\mathbf{l}_{u} = 0 \quad \mathbf{u}^{T}\mathbf{l}_{v} = 0$
3. Any point along the epipolar line can be a candidate of correspondences.





Epipolar constraint between two images:

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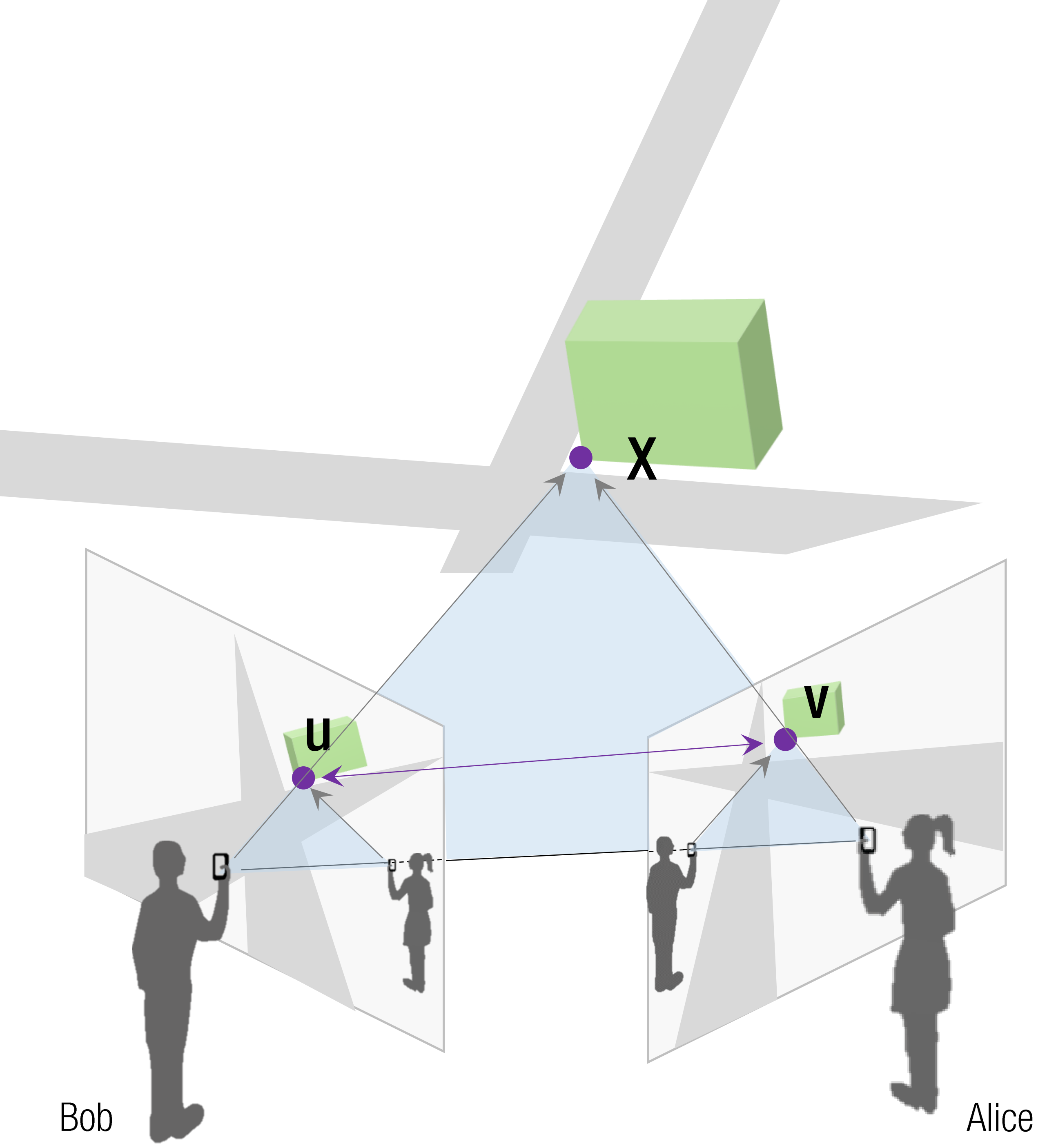

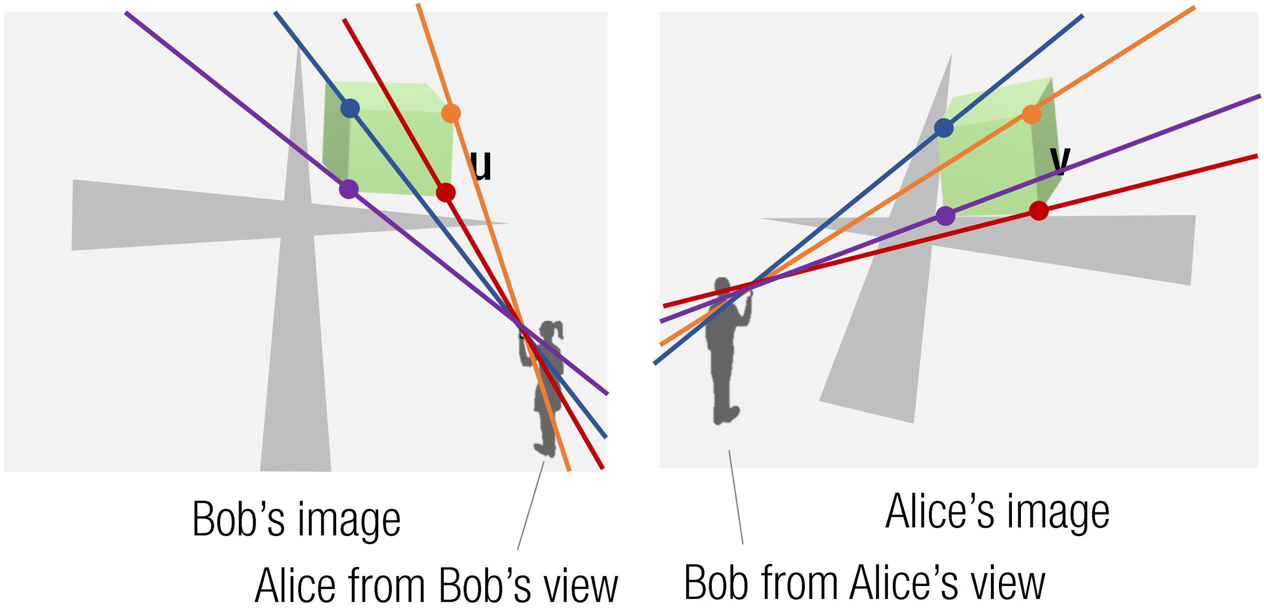

Epipolar constraint between two images:

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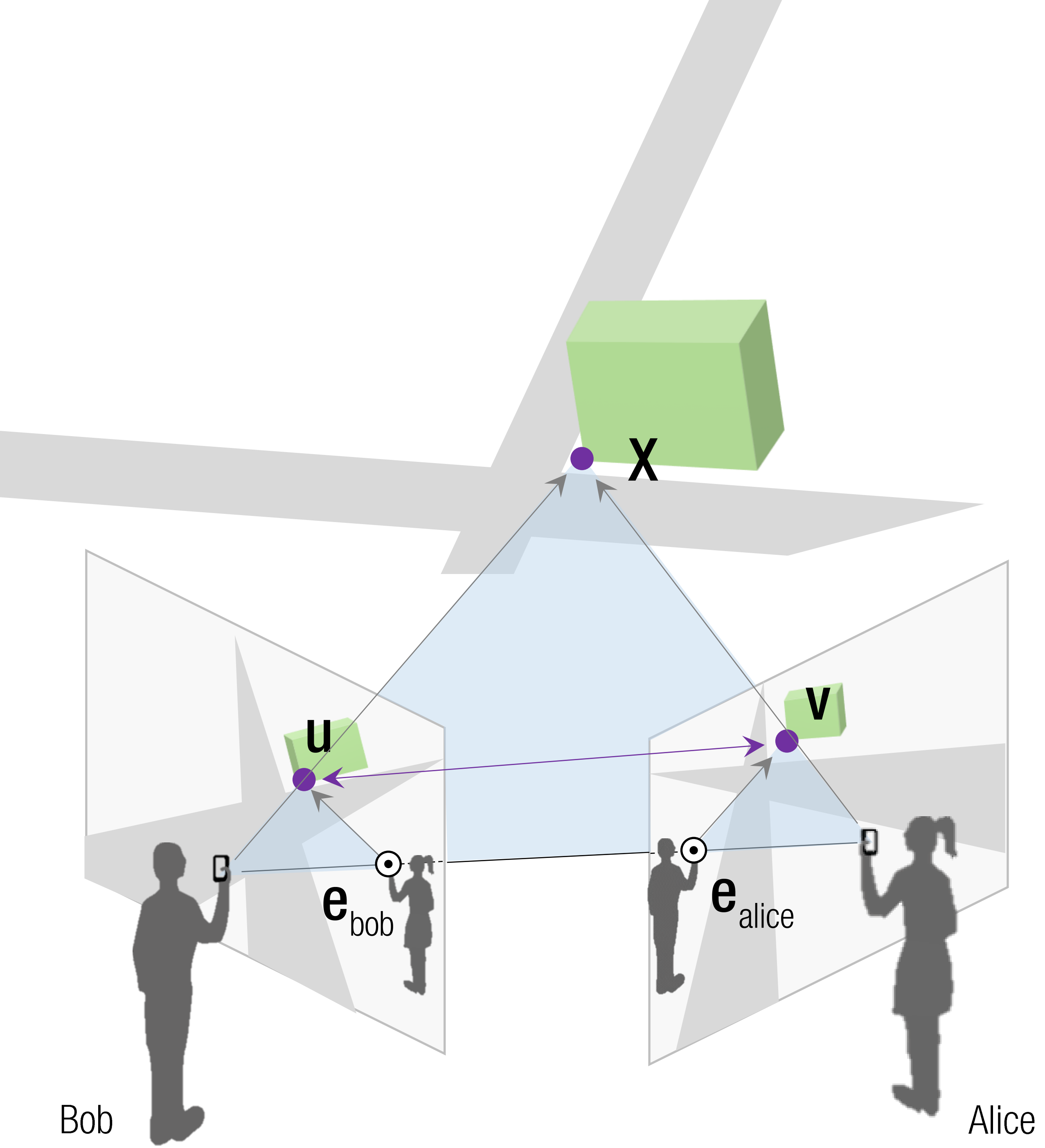
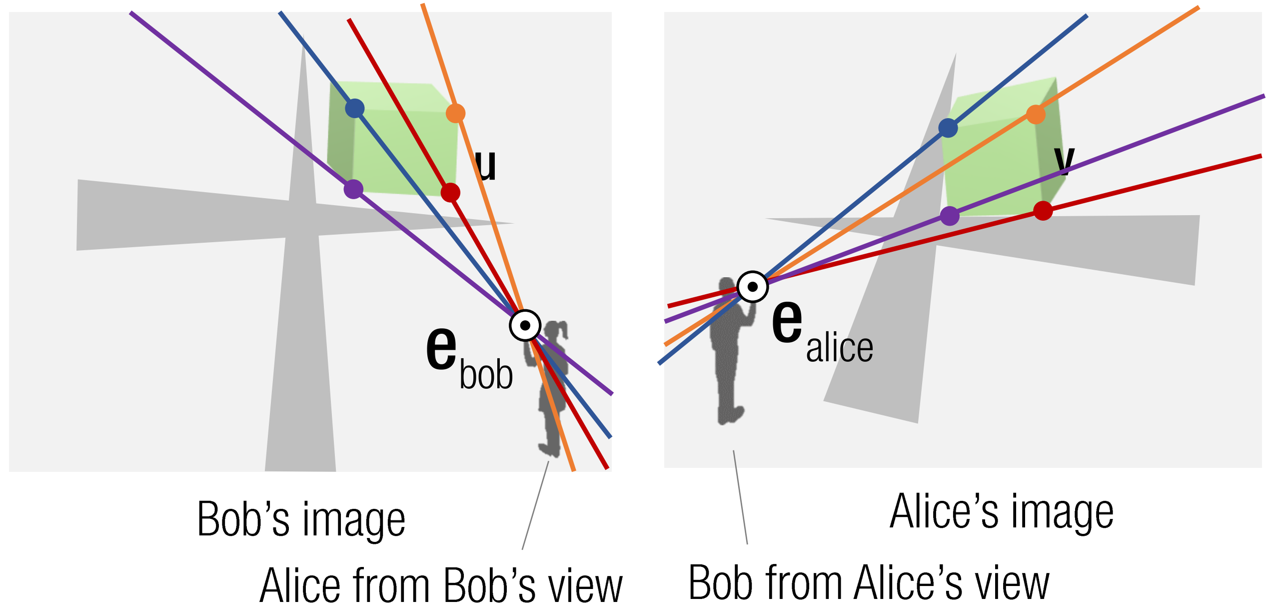
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^{T}\mathbf{l}_{u} = 0 \quad \mathbf{u}^{T}\mathbf{l}_{v} = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole.





Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_\mathbf{u} = 0 \quad \mathbf{u}^T\mathbf{l}_\mathbf{v} = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole: $\mathbf{e}_{bob}^T\mathbf{l}_\mathbf{u} = 0 \quad \mathbf{e}_{alice}^T\mathbf{l}_\mathbf{v} = 0$

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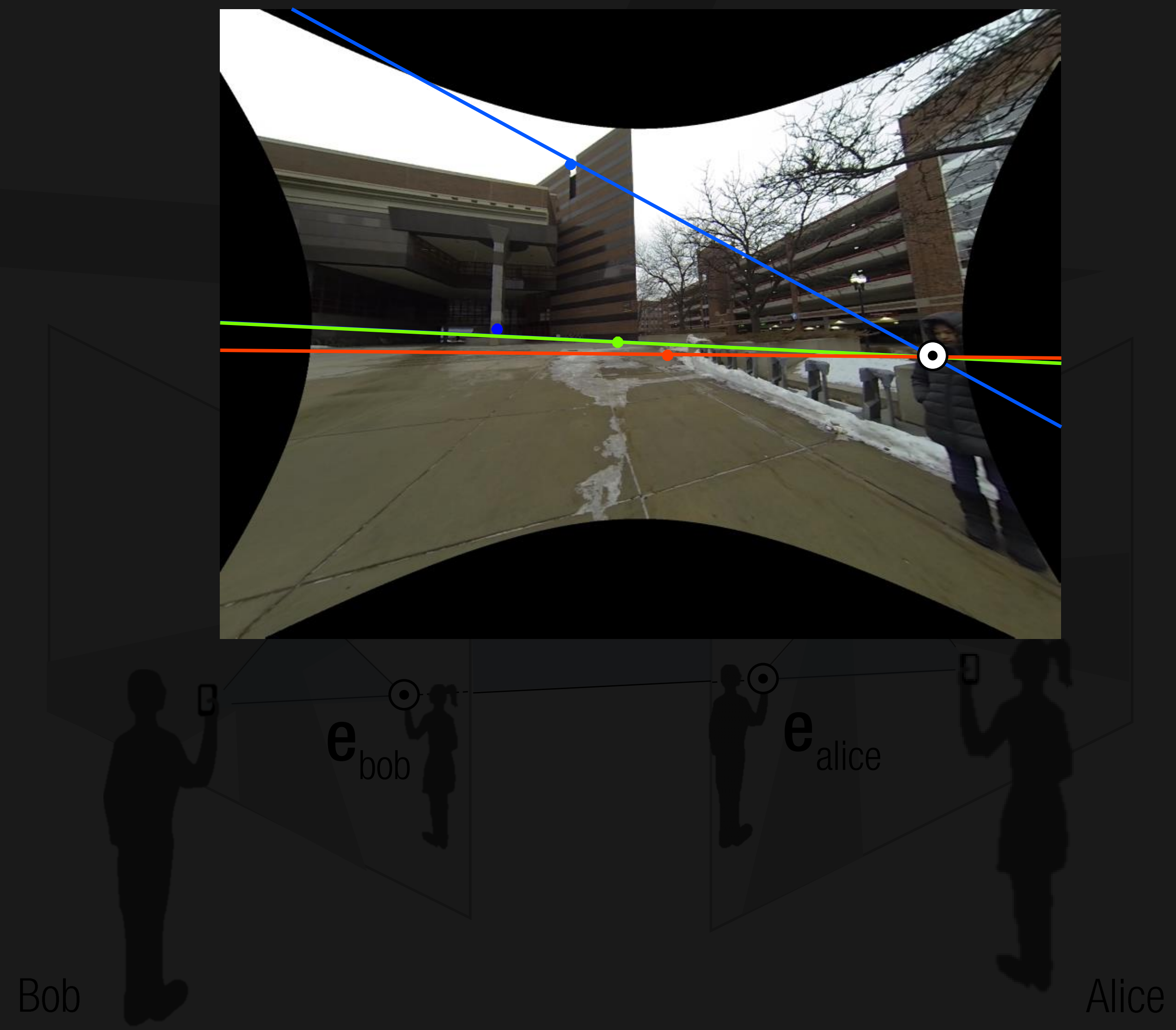

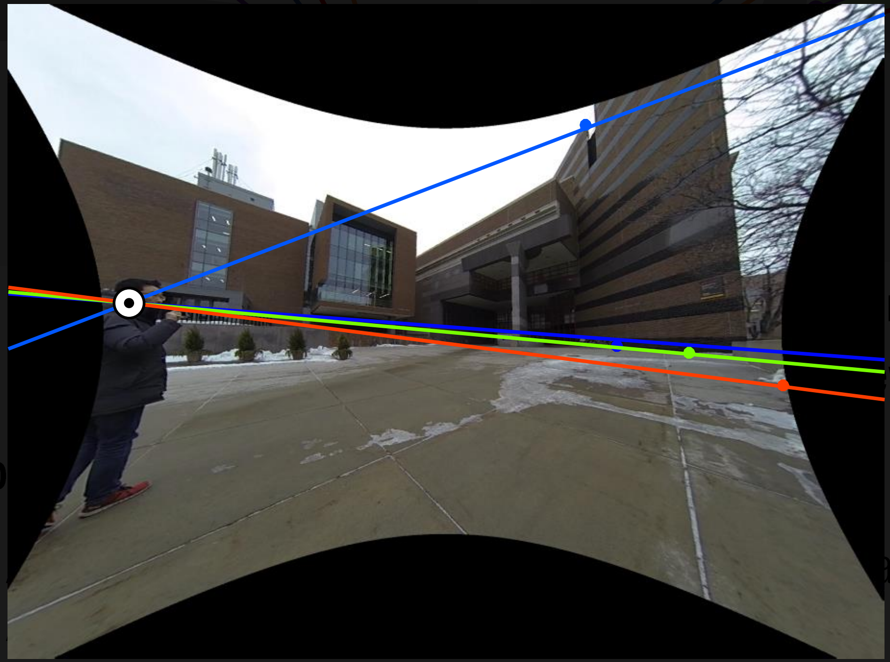

2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T \mathbf{l}_u = 0 \quad \mathbf{u}^T \mathbf{l}_v = 0$
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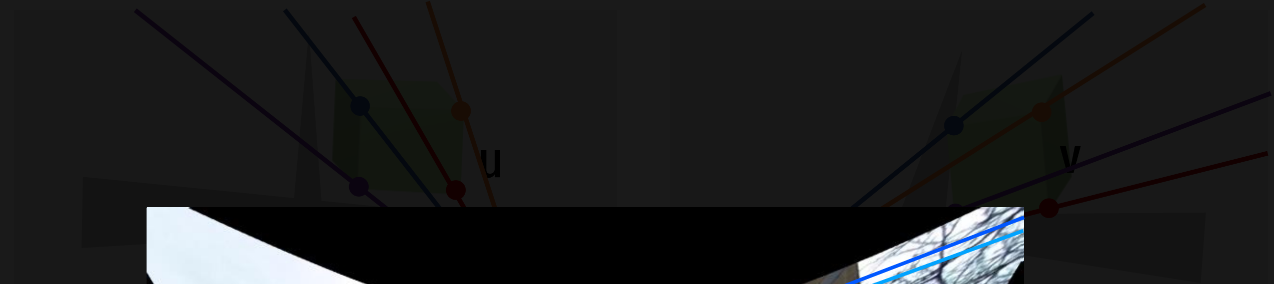

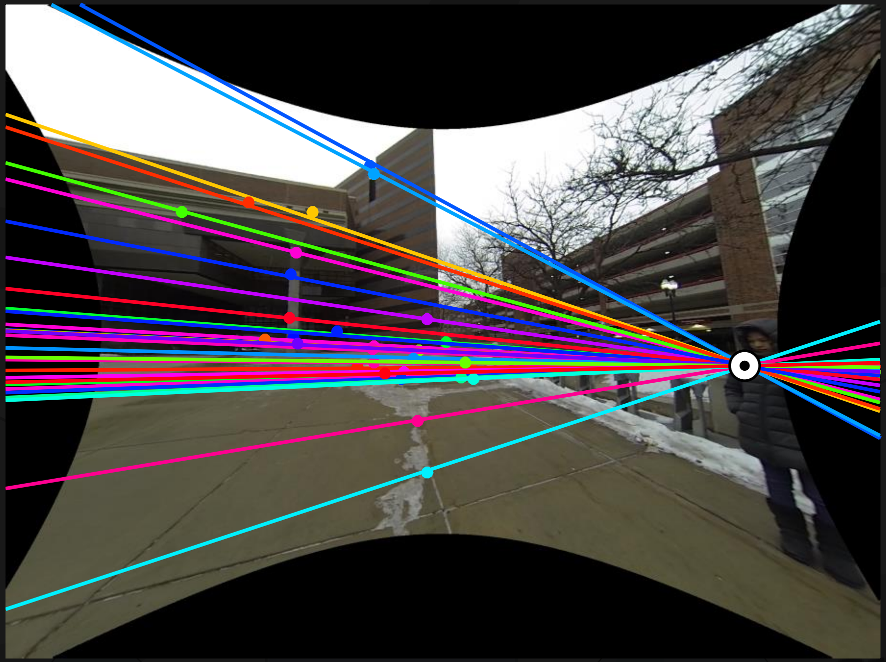

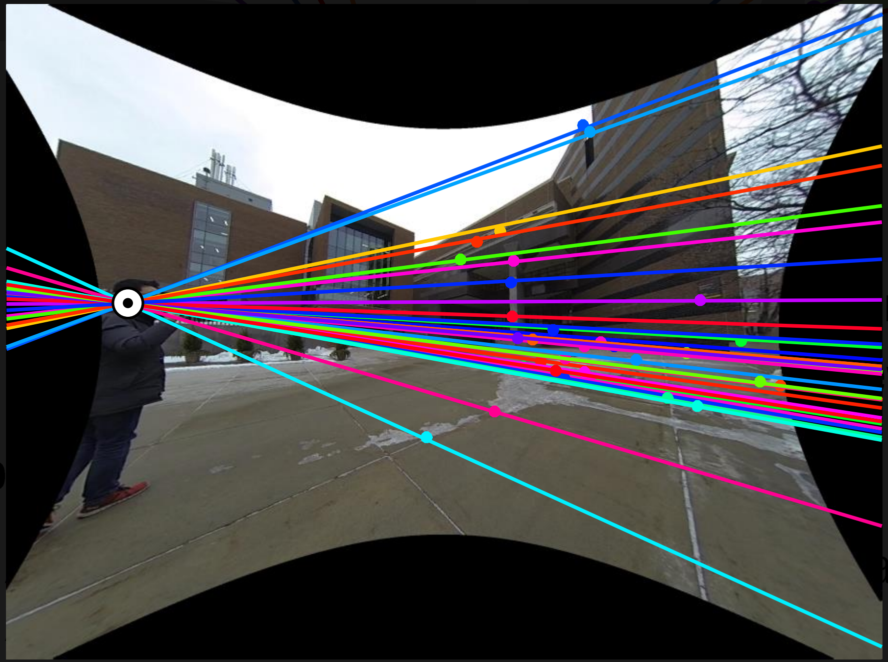

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Epip

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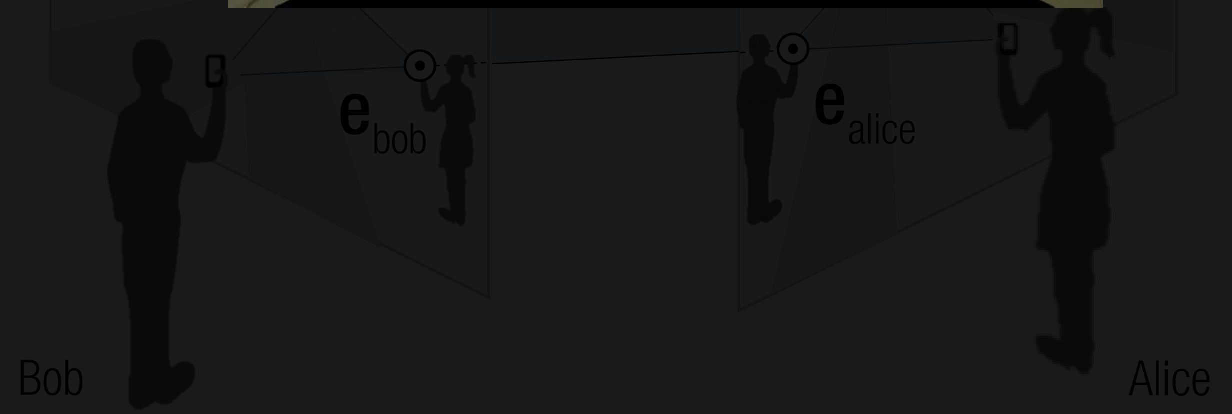
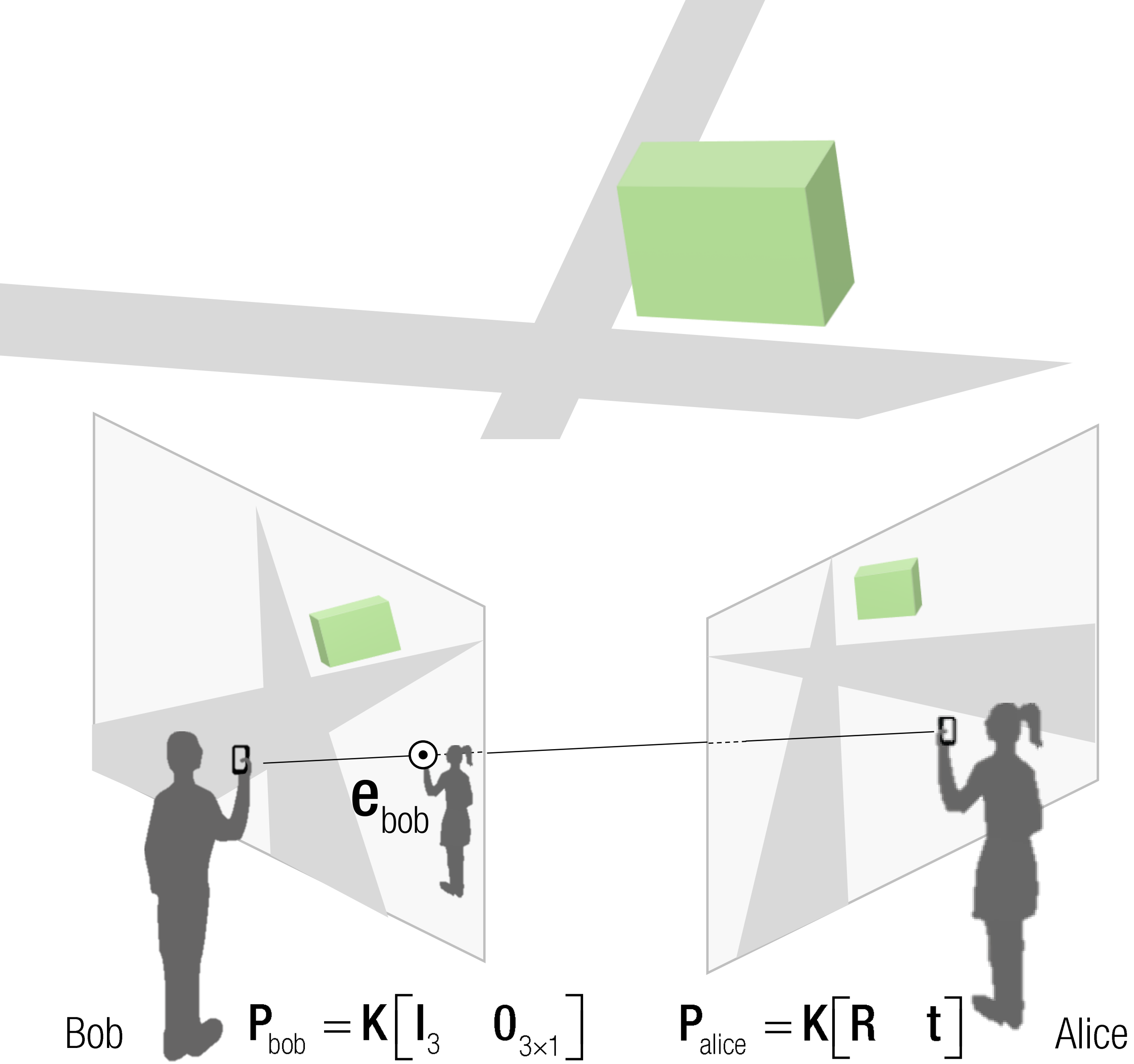
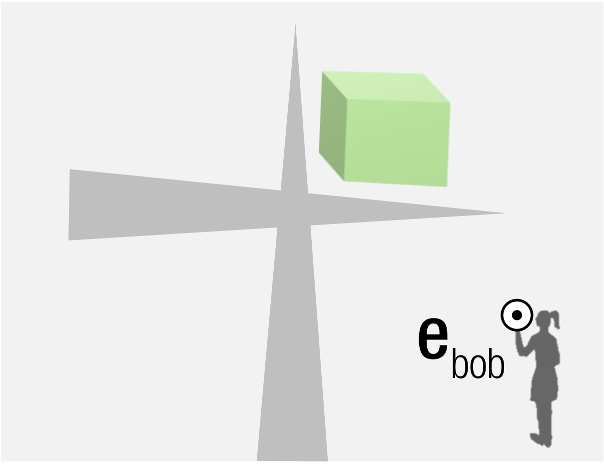
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T \mathbf{l}_u = 0 \quad \mathbf{u}^T \mathbf{l}_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole: $\mathbf{e}_{bob}^T \mathbf{l}_u = 0 \quad \mathbf{e}_{alice}^T \mathbf{l}_v = 0$



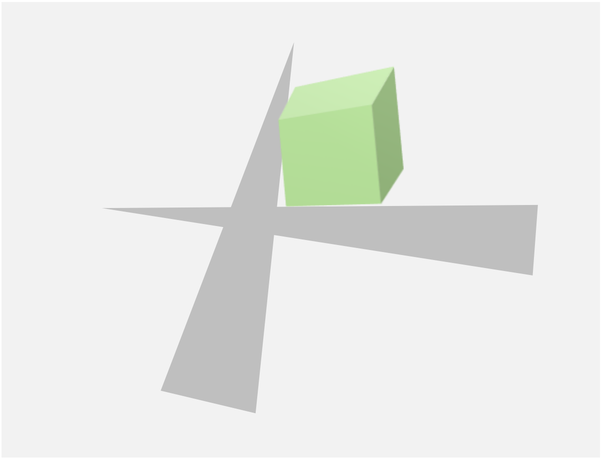
Epipole Computation



$$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix} \qquad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$$



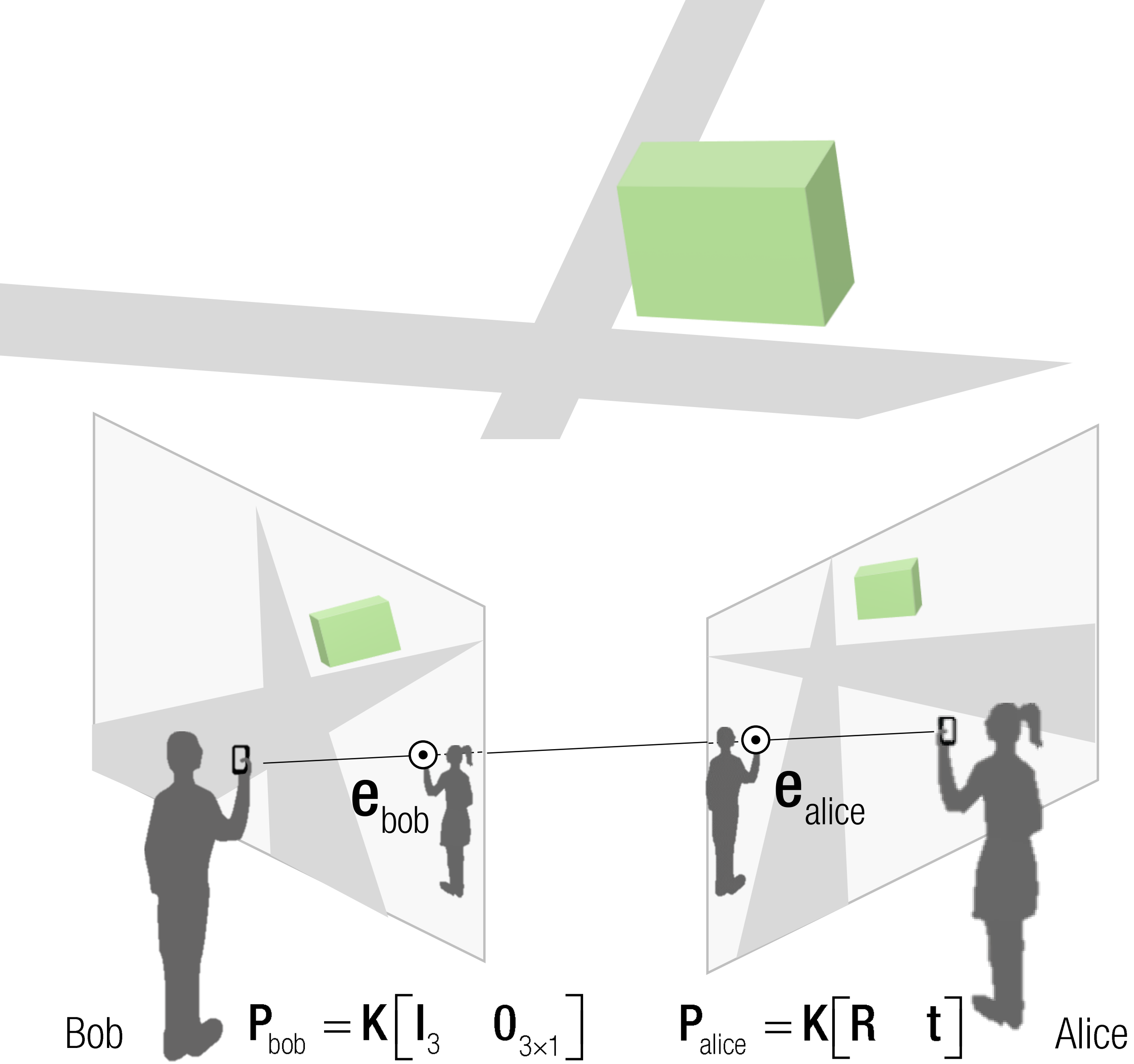
Bob's image

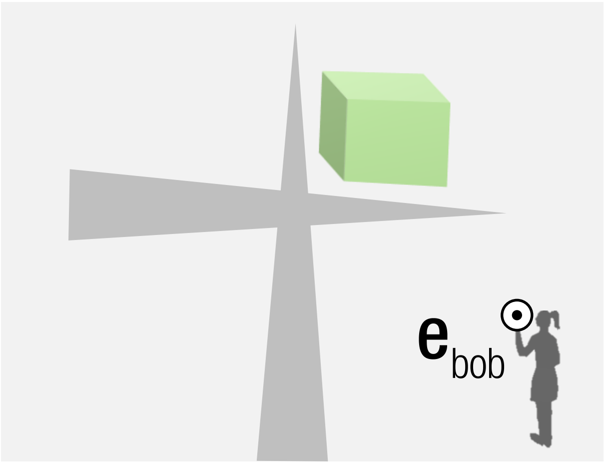


Alice's image

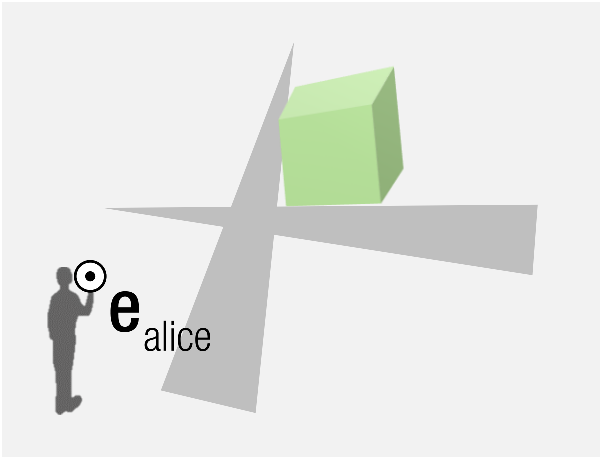
$$\lambda \mathbf{e}_{bob} = \underbrace{K\begin{bmatrix} I_3 & 0 \end{bmatrix}}_{\text{Bob's camera projection matrix}} \begin{bmatrix} -R^{T}t \\ 1 \end{bmatrix} = -KR^{T}t$$

Epipole Computation





Bob's image



Alice's image

$$\lambda \mathbf{e}_{bob} = \underline{\mathbf{K}\begin{bmatrix}\mathbf{I}_3 & \mathbf{0}\end{bmatrix}} \begin{bmatrix} -\mathbf{R}^{\mathsf{T}}\mathbf{t} \\ 1 \end{bmatrix} = -\mathbf{K}\mathbf{R}^{\mathsf{T}}\mathbf{t}$$

Bob's camera projection matrix

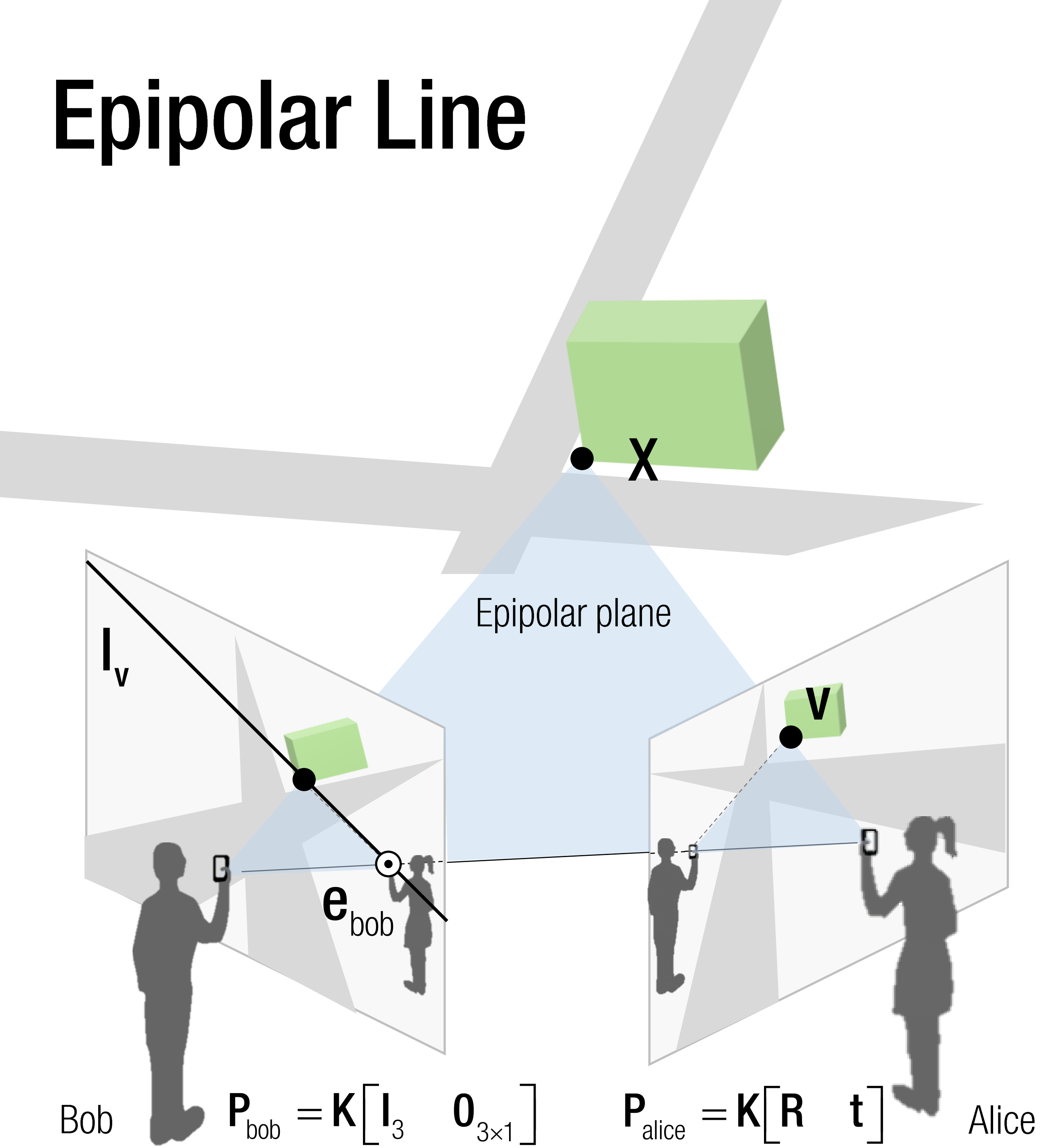
$$\lambda \mathbf{e}_{alice} = \underline{\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}} \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{t}$$

Alice's camera projection matrix

Epipolar Line

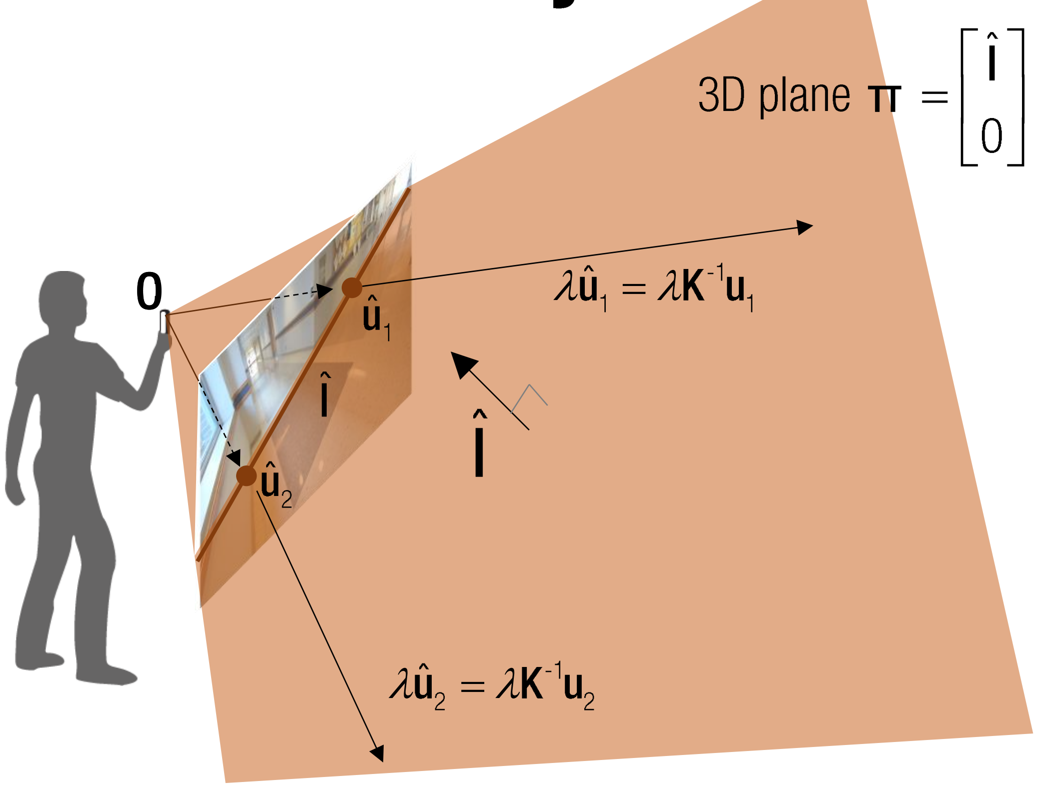
$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^{\top}\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

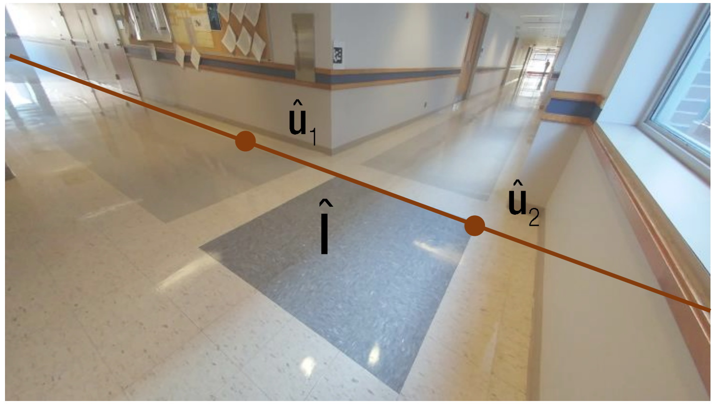
$$\mathbf{l}_{v} =$$



$$\mathbf{P}_{bob} = \mathbf{K}\begin{bmatrix}\mathbf{I}_3 & \mathbf{0}_{3\times1}\end{bmatrix} \qquad \mathbf{P}_{alice} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

Recall: Projective Line vs. Plane





Normalized coordinate:

$$\hat{\mathbf{u}}_1 = \mathbf{K}^{-1}\mathbf{u}_1 \qquad \hat{\mathbf{u}}_2 = \mathbf{K}^{-1}\mathbf{u}_2$$

$$\longrightarrow \quad \hat{\mathbf{l}} = \hat{\mathbf{u}}_1 \times \hat{\mathbf{u}}_2$$

where $\hat{\mathbf{l}} = \left(\mathbf{K}^{-1}\right)^{-\mathsf{T}} \mathbf{l} = \mathbf{K}^{\mathsf{T}}\mathbf{l}$ due to duality

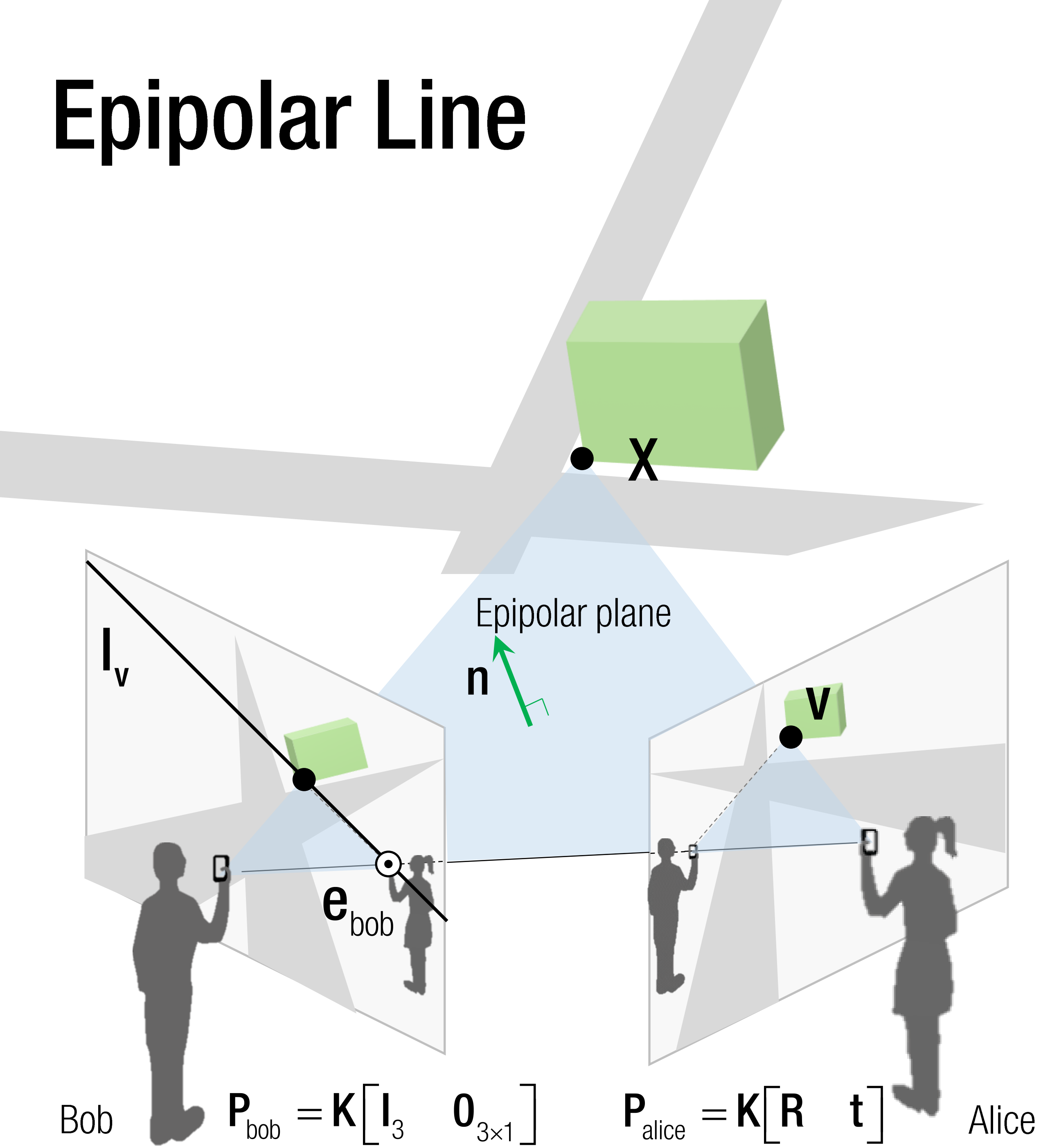
Plane normal: $\left(\lambda_1\hat{\mathbf{u}}_1\right) \times \left(\lambda_2\hat{\mathbf{u}}_2\right) = \lambda\hat{\mathbf{l}}$

$$\therefore \pi = \begin{bmatrix} \hat{\mathbf{l}} \\ 0 \end{bmatrix}$$

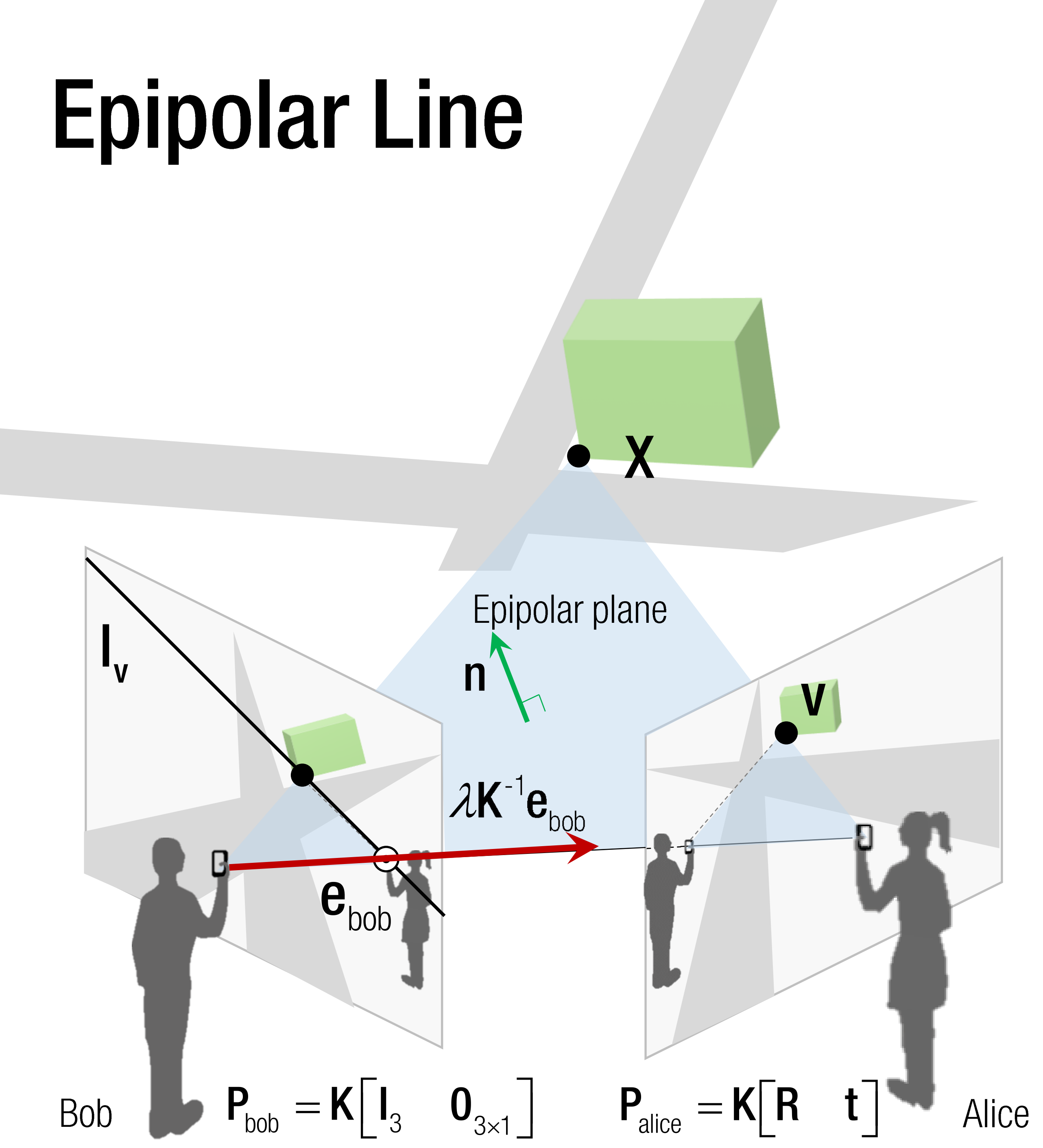
Epipolar Line

$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^{\mathsf{T}}\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_{\mathbf{v}} = \mathbf{K}^{-\mathsf{T}}\mathbf{n}$$



Epipolar Line



$$\lambda e_{bob} = KR^{T}t \qquad \lambda e_{alice} = Kt$$

$$l_v = K^{-T}n$$

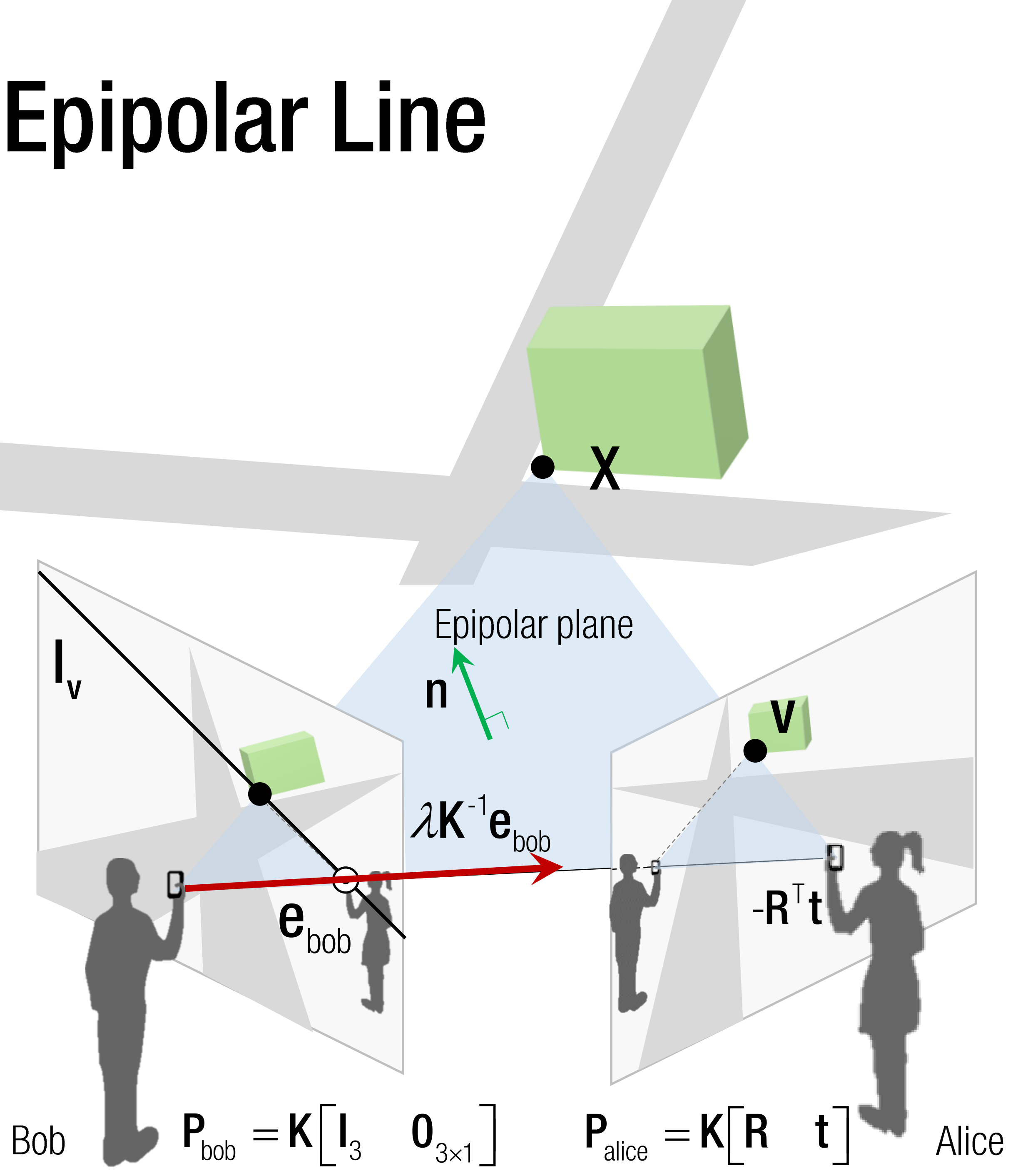
$$\rightarrow \quad \lambda K^{-1} e_{bob} = R^{T}t$$

Epipolar Line

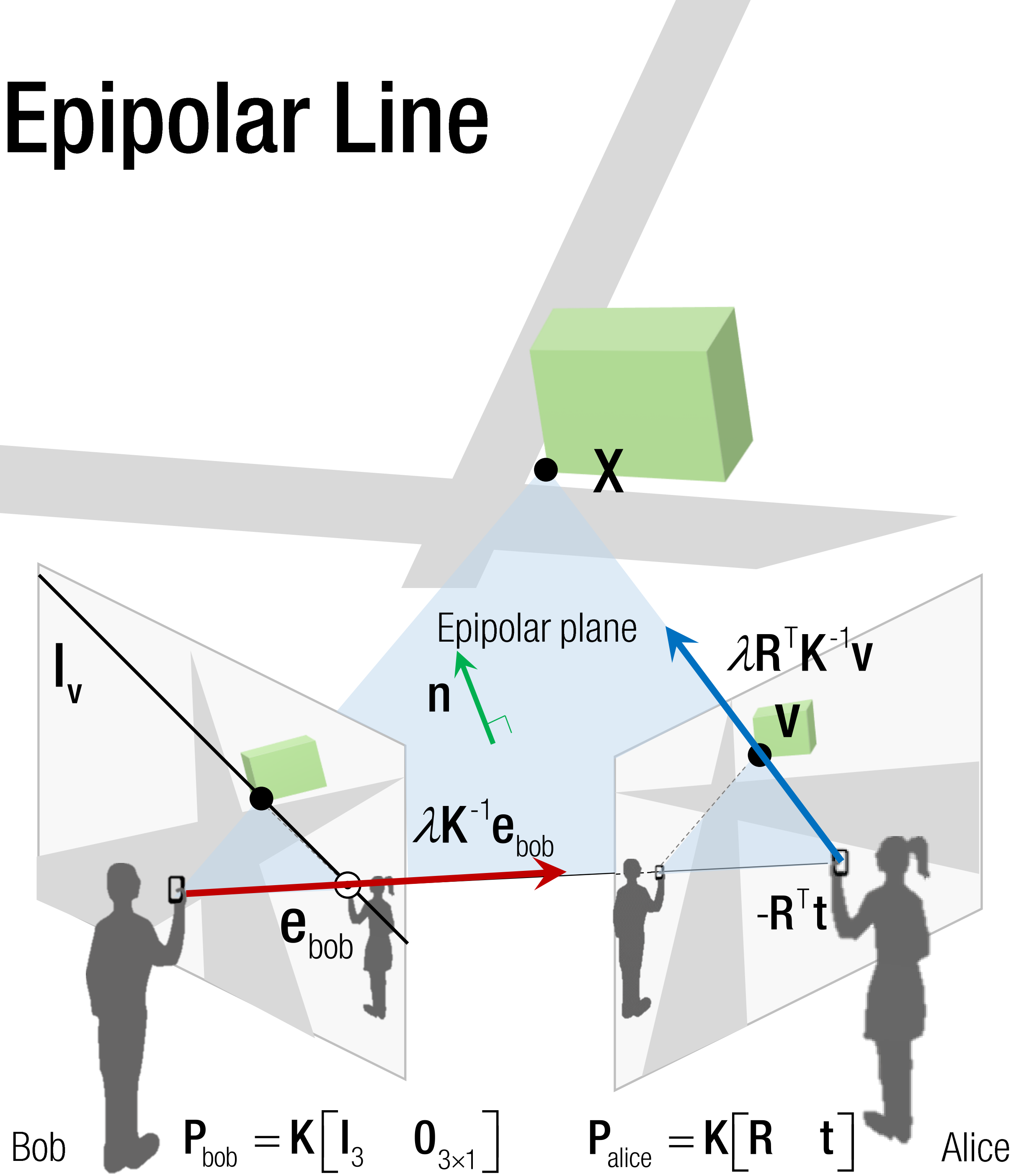
$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^{\mathsf{T}}\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_v = \mathbf{K}^{-\mathsf{T}}\mathbf{n}$$

$$\rightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{e}_{bob} = \mathbf{R}^{\mathsf{T}}\mathbf{t}$$



Epipolar Line



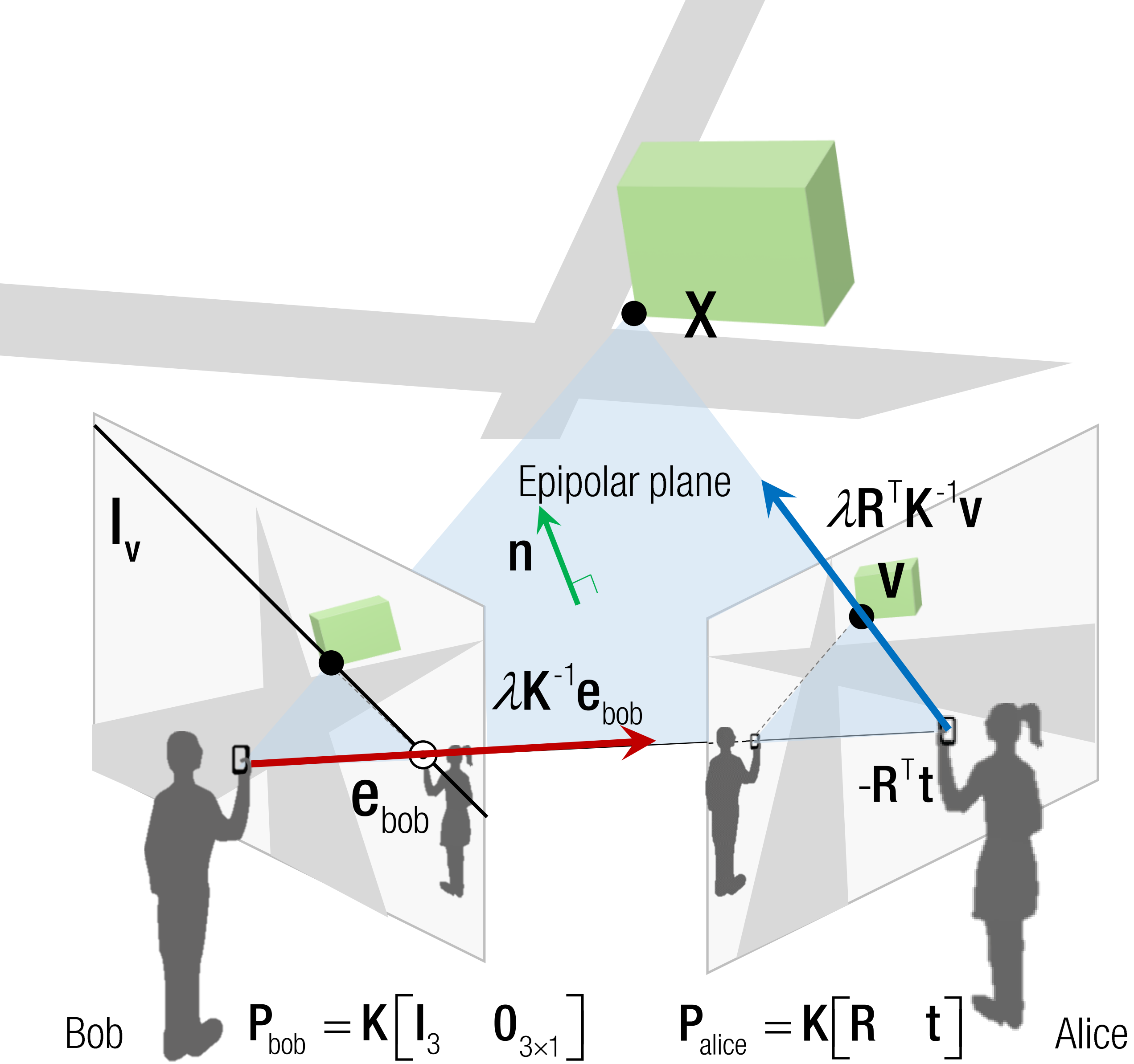
$$\lambda e_{bob} = KR^T t \qquad \lambda e_{alice} = Kt$$

$$l_v = K^{-T} n$$

→ $\lambda K^{-1} e_{bob} = R^T t$

→ $\underbrace{\lambda_1 R^T K^{-1} v}_{\text{Direction}} \underbrace{- R^T t}_{\text{Alice's camer location}}$

Epipolar Line



$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^T\mathbf{t}$ $\lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$

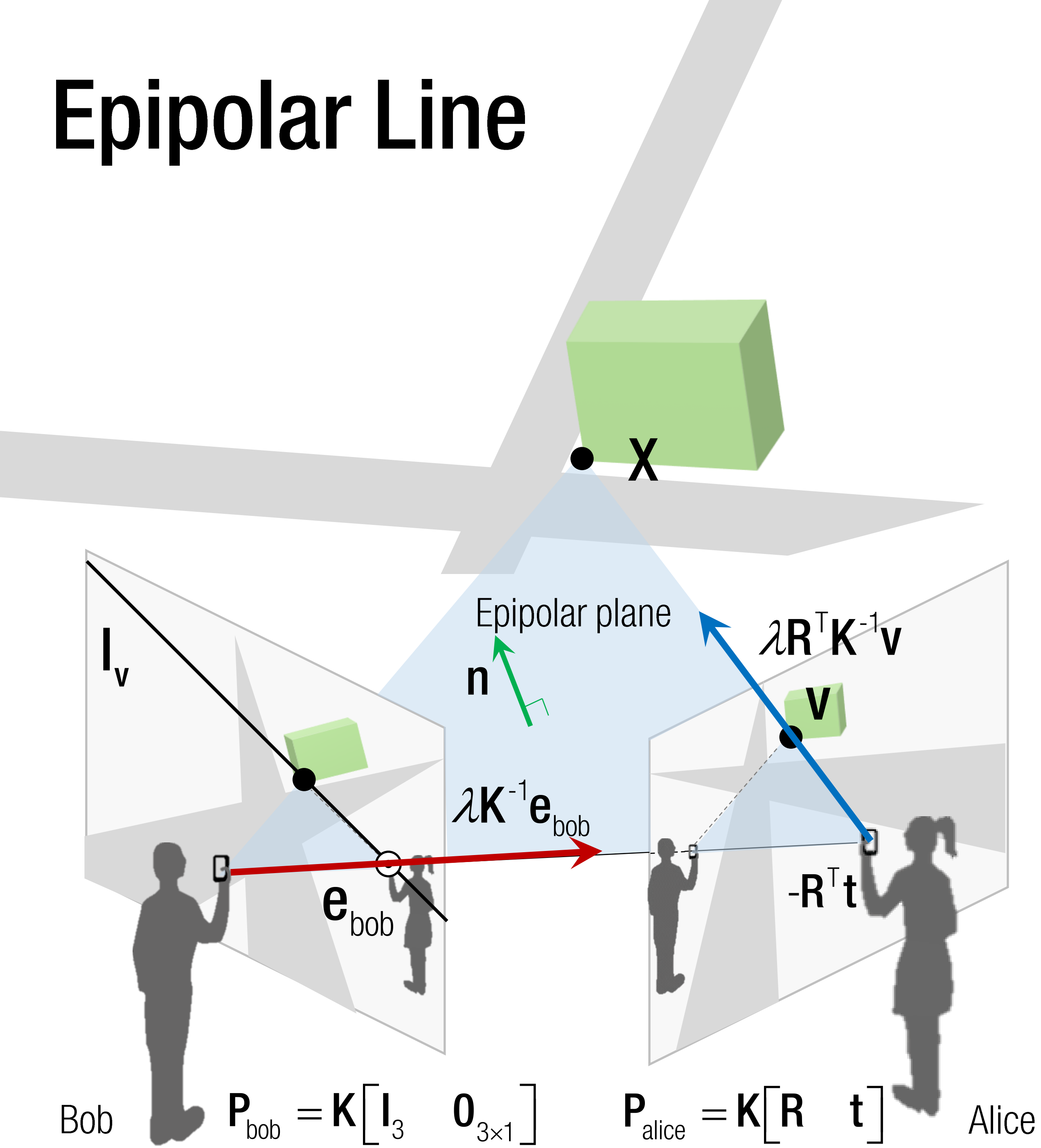
$$\mathbf{l}_v = \mathbf{K}^{-T}\mathbf{n}$$

$$\lambda \mathbf{K}^{-1}\mathbf{e}_{bob} = \mathbf{R}^T\mathbf{t}$$

$$\underbrace{\lambda_1 \mathbf{R}^T\mathbf{K}^{-1}\mathbf{v}}_{\text{Direction}} - \underbrace{\mathbf{R}^T\mathbf{t}}_{\text{Alice's camer location}}$$

$$\begin{aligned}\mathbf{n} &= \mathbf{R}^T\mathbf{t} \times \left(\lambda_1 \mathbf{R}^T\mathbf{K}^{-1}\mathbf{v} - \mathbf{R}^T\mathbf{t}\right)\\ &= \mathbf{R}^T\mathbf{t} \times \mathbf{R}^T\mathbf{K}^{-1}\mathbf{v}\\ &= \mathbf{R}^T\left(\mathbf{t} \times \mathbf{K}^{-1}\mathbf{v}\right) \quad \text{: only works for rotation}\\ &= \mathbf{R}^T\left[\mathbf{t}\right]_\times \mathbf{K}^{-1}\mathbf{v}\end{aligned}$$

Epipolar Line



$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix}$ $P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$

$\lambda e_{bob} = KR^T t$ $\lambda e_{alice} = Kt$

$l_v = K^{-T} n = K^{-T} R^T [t]_\times K^{-1} v$: Epipolar line

→ $\lambda K^{-1} e_{bob} = R^T t$

→ $\underbrace{\lambda_1 R^T K^{-1} v}_{\text{Direction}} - \underbrace{R^T t}_{\text{Alice's camer location}}$

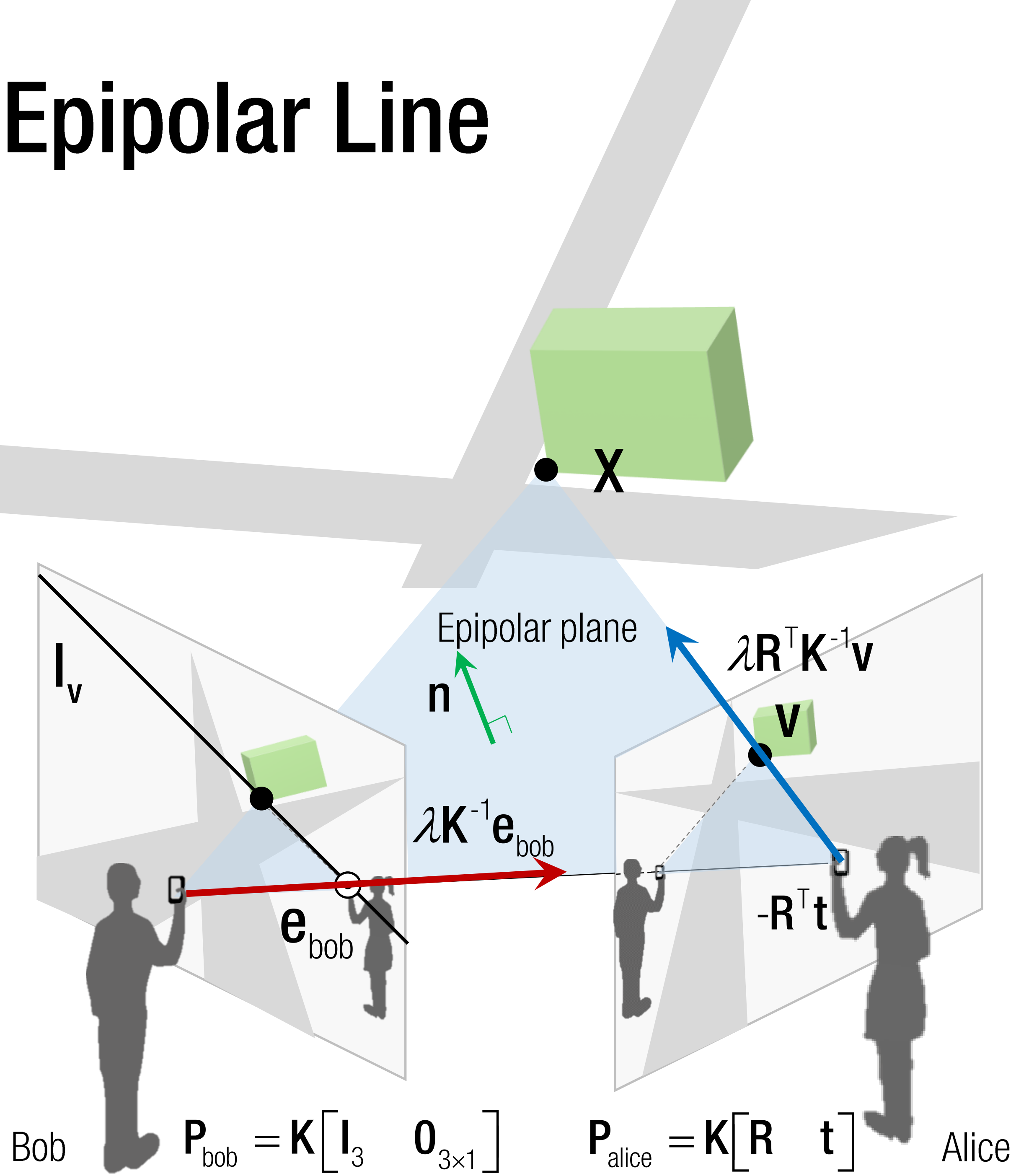
→ $n = R^T t \times \left(\lambda_1 R^T K^{-1} v - R^T t\right)$

$= R^T t \times R^T K^{-1} v$

$= R^T \left(t \times K^{-1} v\right)$: only works for rotation

$= R^T [t]_\times K^{-1} v$

Epipolar Line



$$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix} \qquad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$$

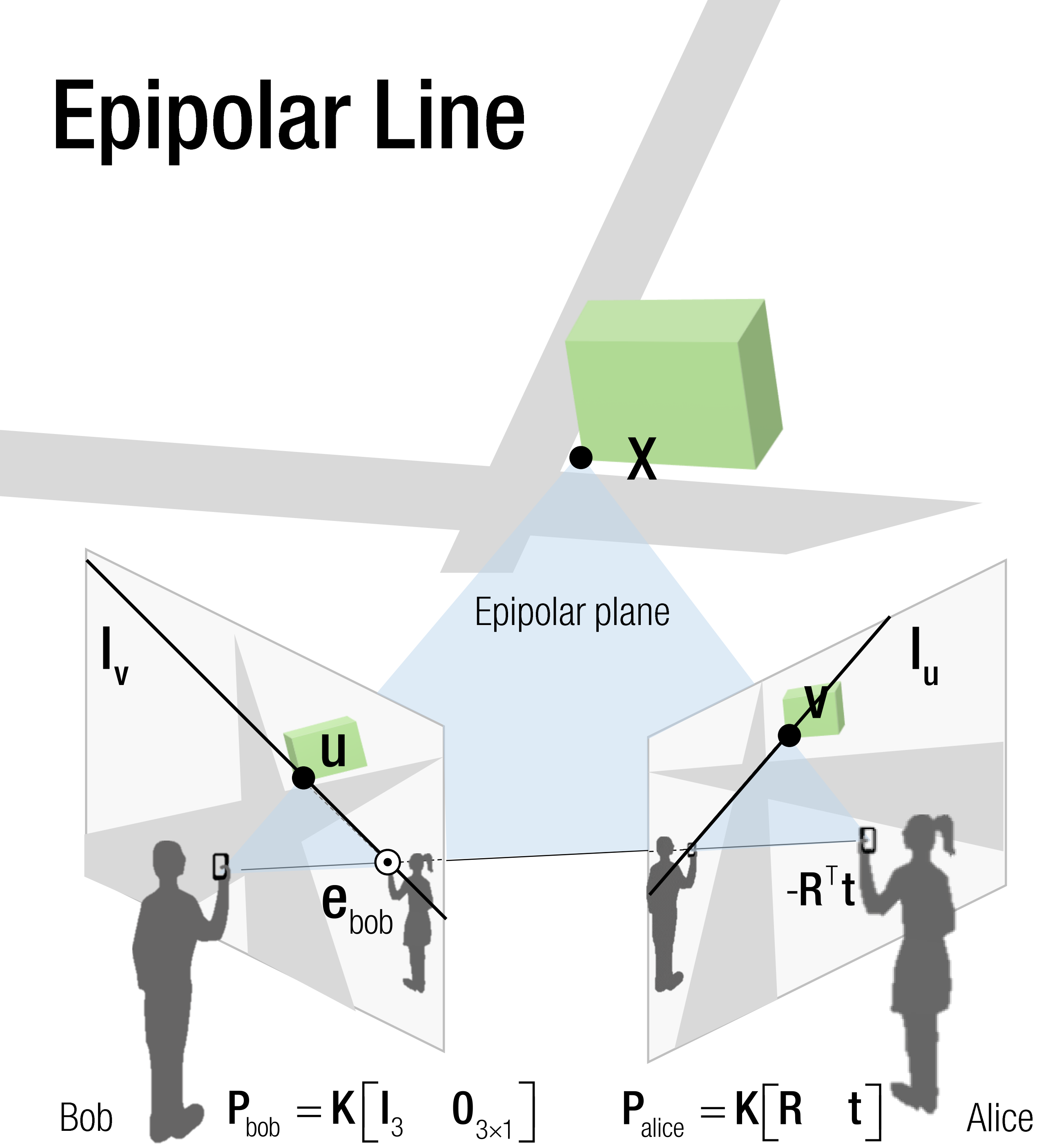
$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^{\mathsf{T}}\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_{\mathbf{v}} = \mathbf{K}^{-\mathsf{T}}\mathbf{n} = \mathbf{K}^{-\mathsf{T}}\mathbf{R}^{\mathsf{T}}\left[\mathbf{t}\right]_{\times}\mathbf{K}^{-1}\mathbf{v}$$: Epipolar line

Epipolar constraint:

$$\mathbf{u}^{\mathsf{T}}\mathbf{l}_{\mathbf{v}} = \mathbf{u}^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{R}^{\mathsf{T}}\left[\mathbf{t}\right]_{\times}\mathbf{K}^{-1}\mathbf{v} = 0$$

Epipolar Line



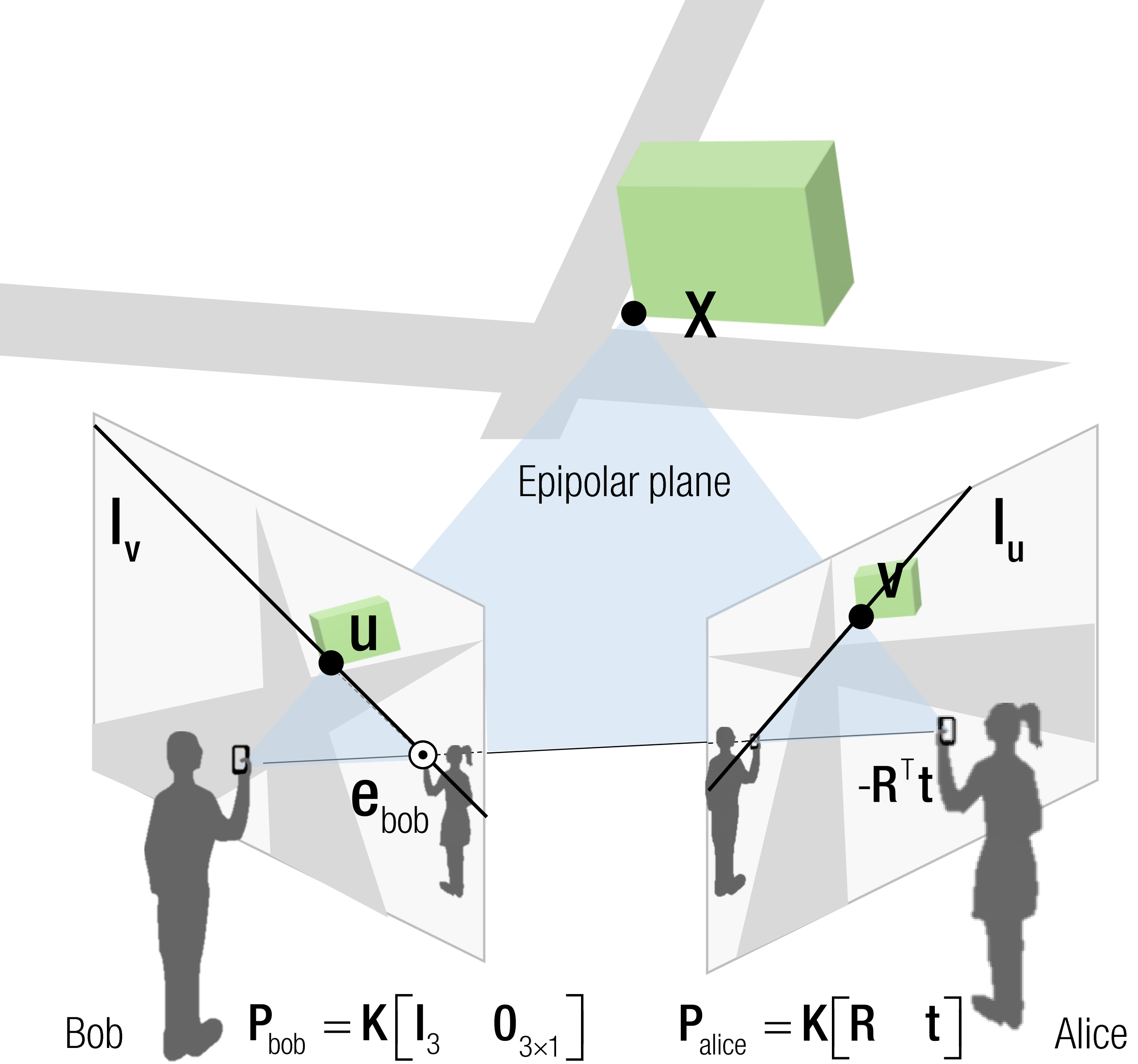
$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^T\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_v = \mathbf{K}^{-T}\mathbf{n} = \mathbf{K}^{-T}\mathbf{R}^T\left[\mathbf{t}\right]_\times \mathbf{K}^{-1}\mathbf{v}$$: Epipolar line

Epipolar constraint:

$$\mathbf{u}^T\mathbf{l}_v = \mathbf{u}^T\mathbf{K}^{-T}\mathbf{R}^T\left[\mathbf{t}\right]_\times \mathbf{K}^{-1}\mathbf{v} = 0$$

Epipolar Line



$$\lambda e_{bob} = KR^T t \qquad \lambda e_{alice} = Kt$$

$$l_v = K^{-T} n = K^{-T} R^T [t]_\times K^{-1} v$$: Epipolar line

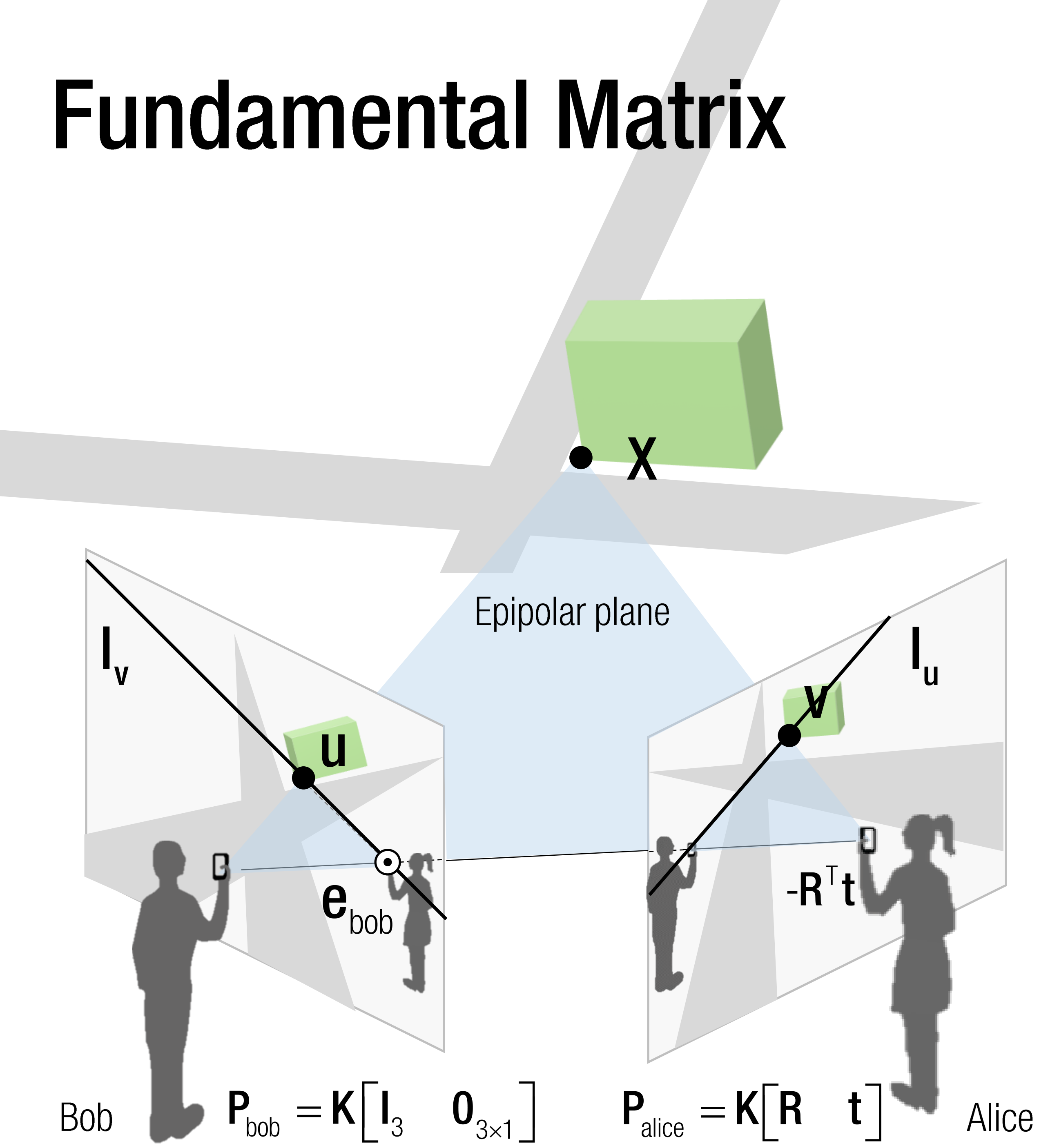
Epipolar constraint:

$$u^T l_v = \underbrace{u^T K^{-T} R^T [t]_\times}_{l_u^T} K^{-1} v = 0$$

$$l_u = -K^{-T} [t]_\times R K^{-1} u \qquad \because [t]_\times^T = -[t]_\times$$

Skew symetric matrix

Fundamental Matrix



$$\lambda e_{bob} = KR^T t \qquad \lambda e_{alice} = Kt$$

$$l_v = K^{-T} n = K^{-T} R^T [t]_\times K^{-1} v$$: Epipolar line

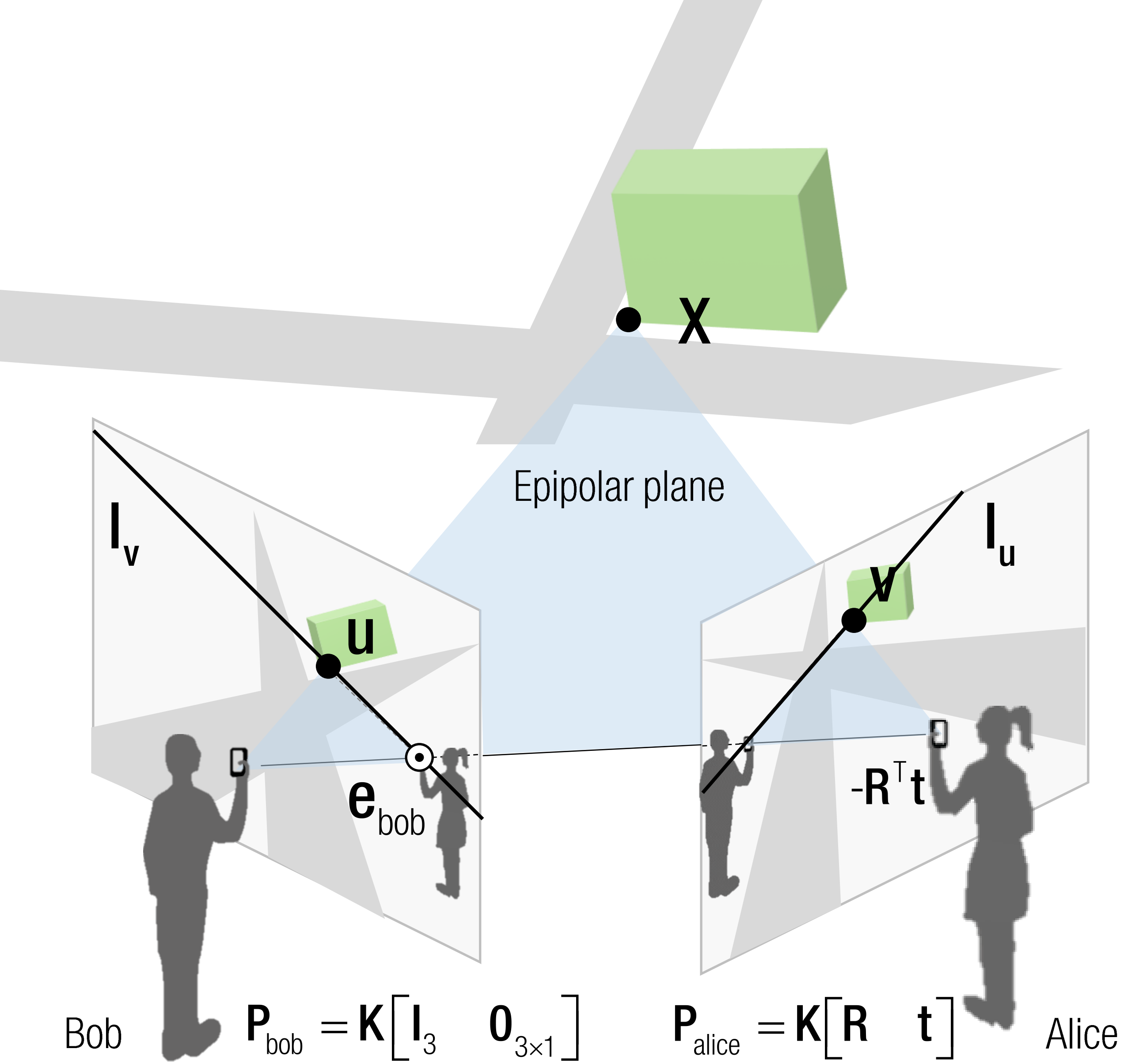
Epipolar constraint:

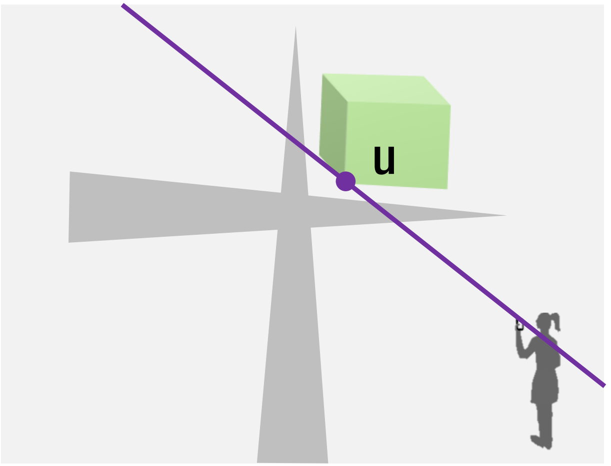
$$u^T l_v = \underbrace{u^T K^{-T} R^T [t]_\times}_{l_u^T} K^{-1} v = 0$$

$$l_u = \underbrace{-K^{-T} [t]_\times R K^{-1}}_{\text{Common for all points}} u \qquad \because [t]_\times^T = -[t]_\times$$

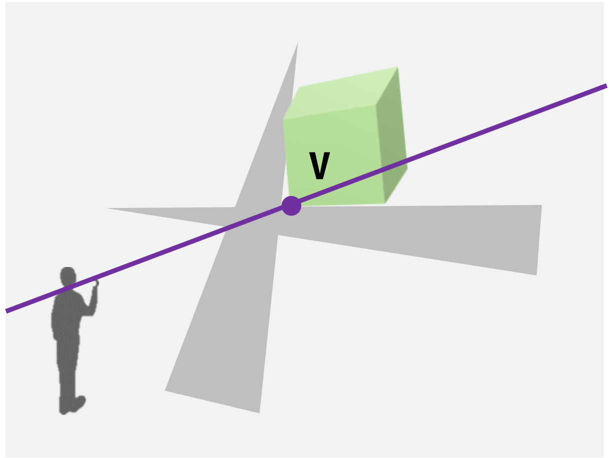
Skew symetric matrix

Fundamental Matrix





Bob's image

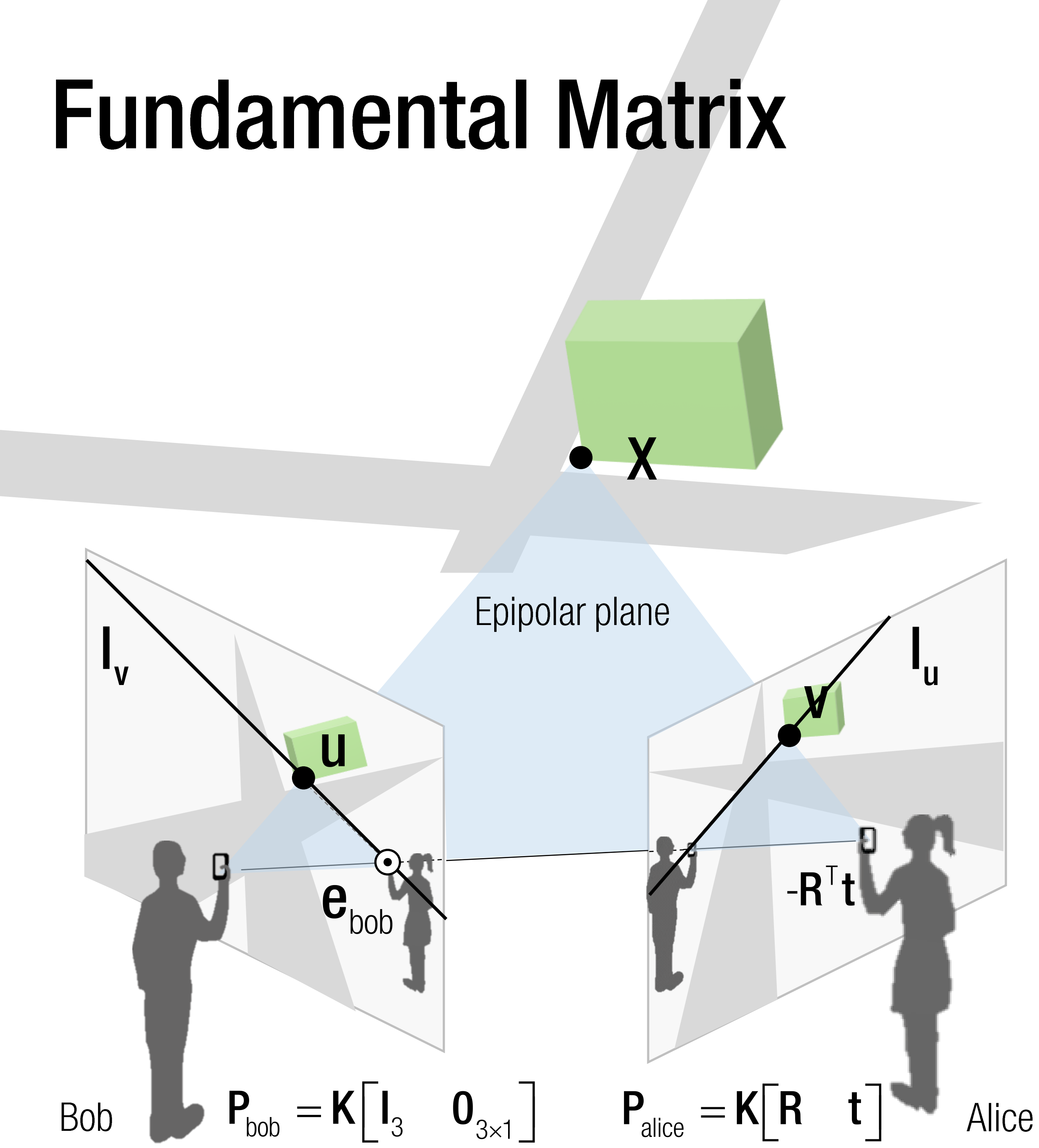


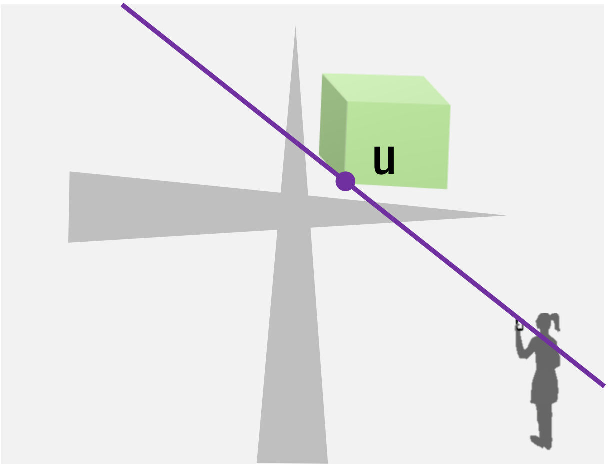
Alice's image

$$\mathbf{v}^{\mathsf{T}}\mathbf{l}_{\mathbf{u}} = \mathbf{v}^{\mathsf{T}}\underline{\mathbf{K}^{-\mathsf{T}}\left[\mathbf{t}\right]_{\times}\mathbf{R}\mathbf{K}^{-1}}\mathbf{u} = 0$$

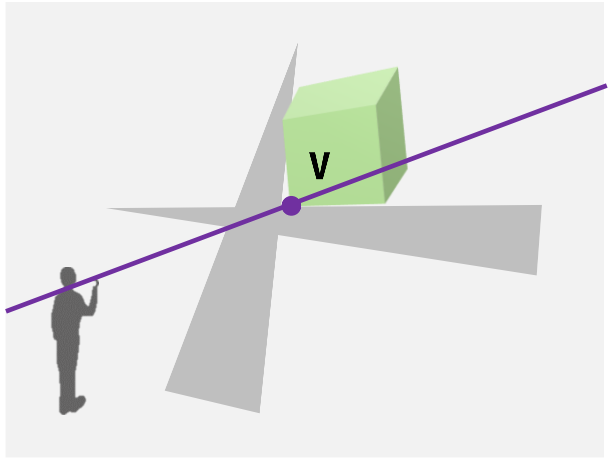
Common for all points

Fundamental Matrix





Bob's image



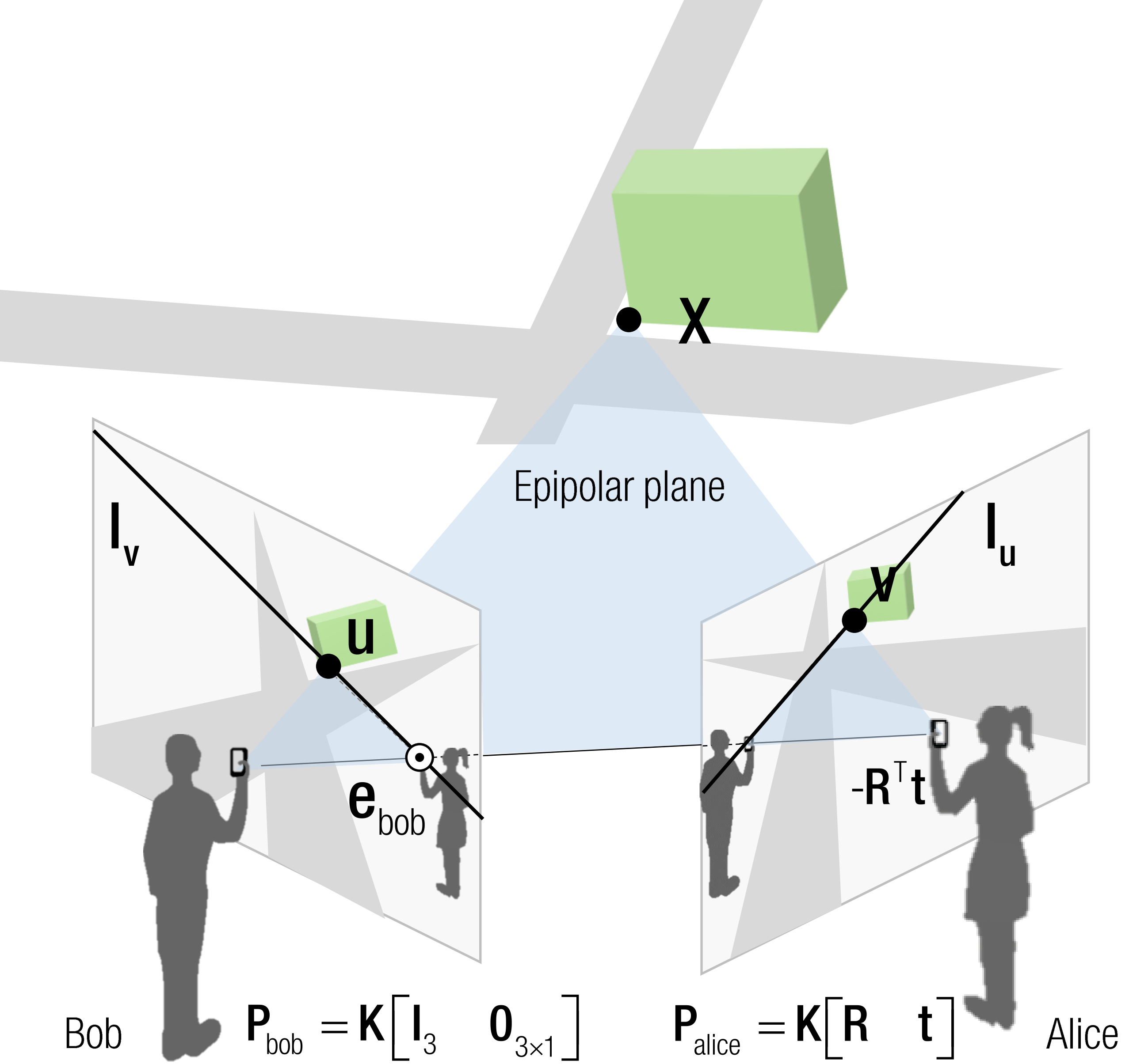
Alice's image

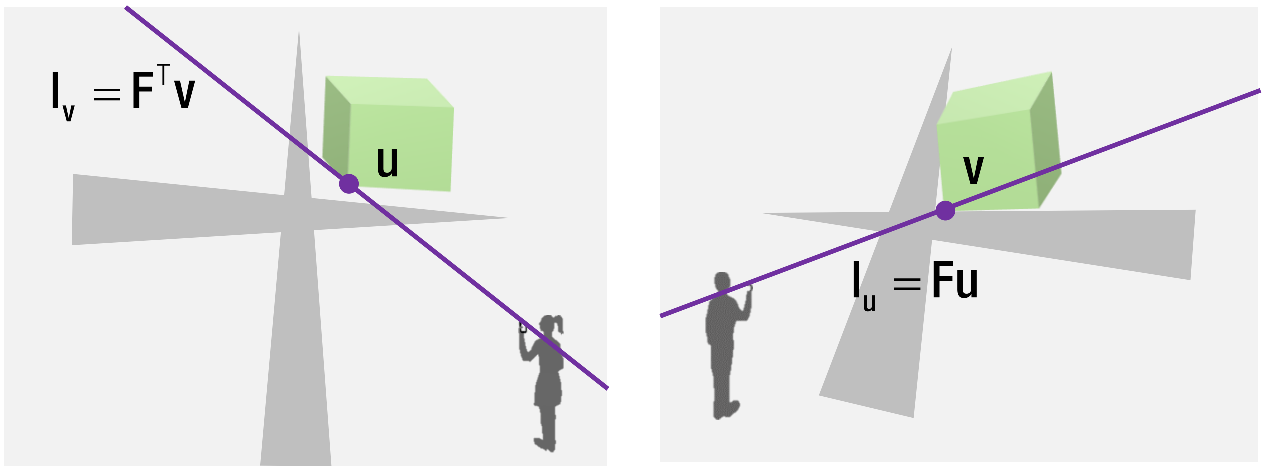
$$v^T l_u = \underbrace{v^T K^{-T}[t]_\times R K^{-1}}_{\text{Common for all points}} u = 0$$

$$= v^T F u = 0$$

where $\underbrace{F = K^{-T}[t]_\times R K^{-1}}_{\text{Fundamental matrix}}$

Fundamental Matrix





Bob's image

Alice's image

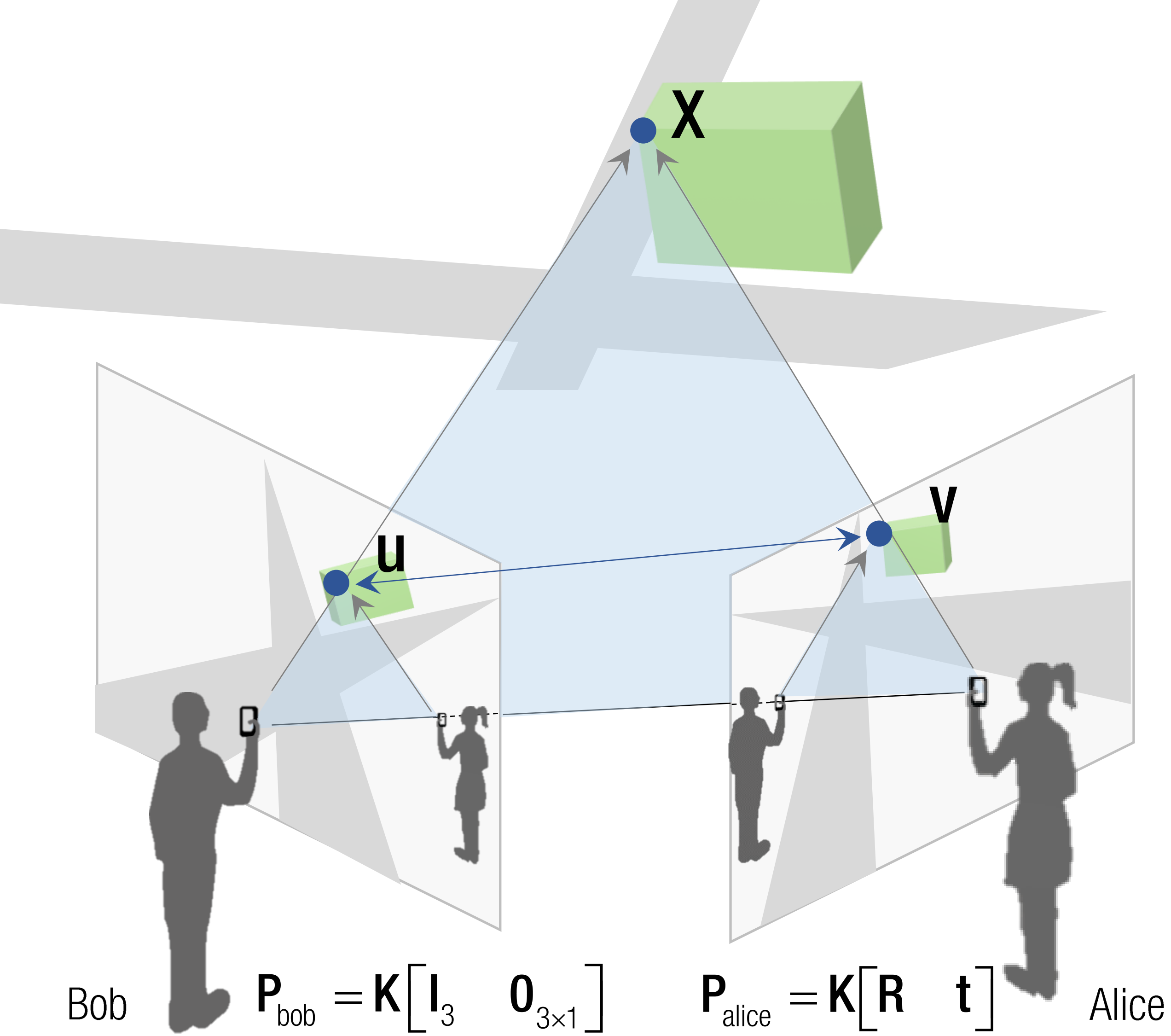
$$v^T l_u = v^T \underline{K^{-T}[t]_\times R K^{-1}} u = 0$$

Common for all points

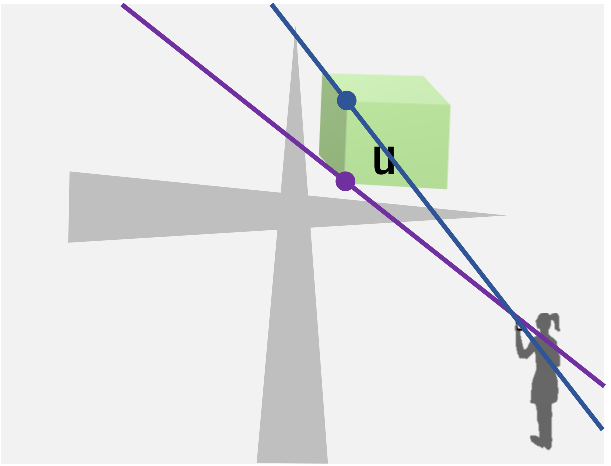
$$= v^T F u = 0$$

$$= v^T (Fu) = u^T (F^T v) = 0$$

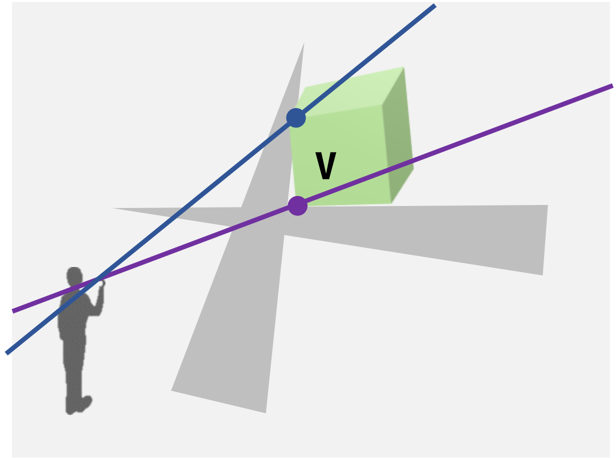
Fundamental Matrix



$$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix} \qquad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$$



Bob's image



Alice's image

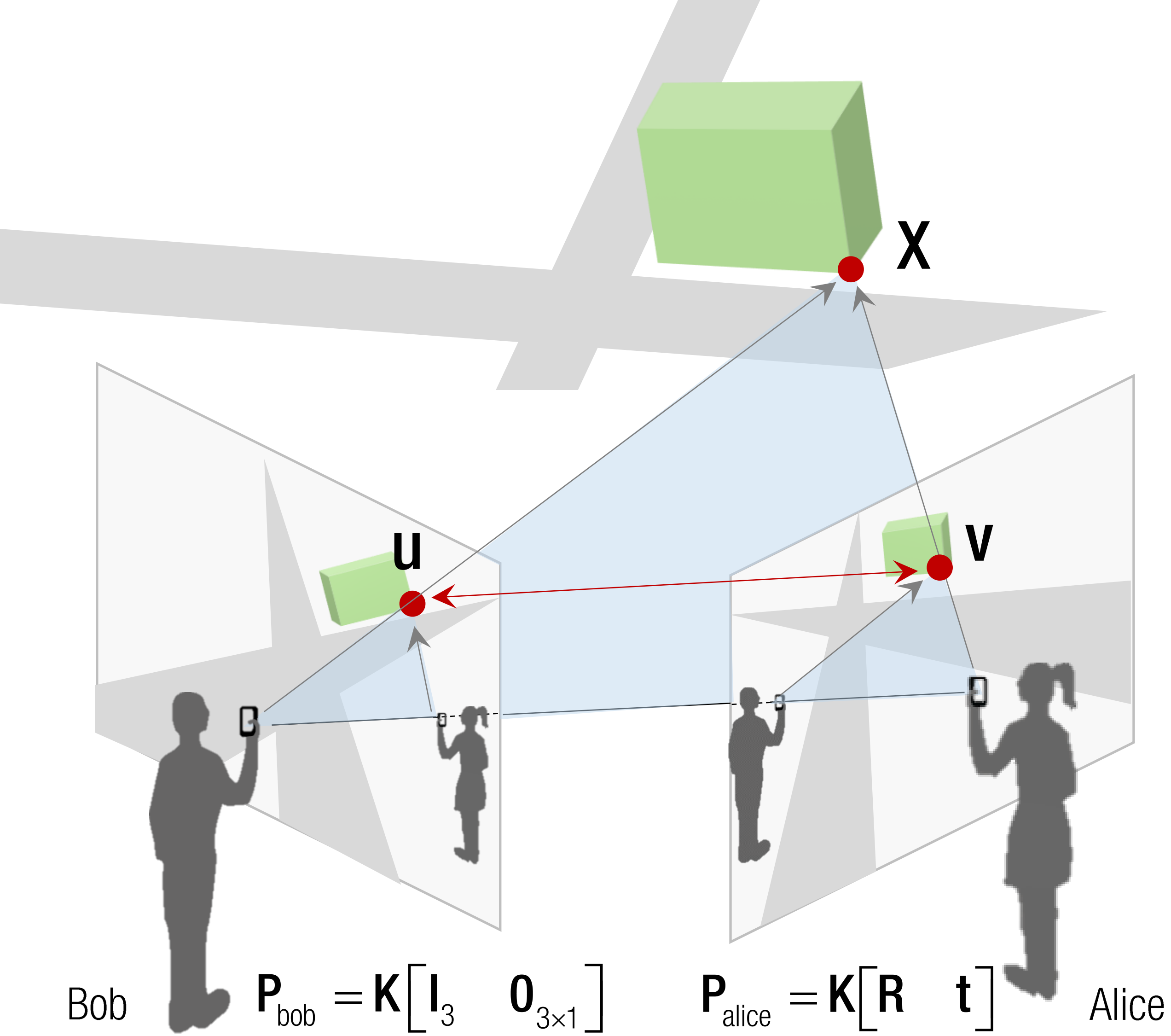
$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \underline{\mathbf{K}^{-T} \left[\mathbf{t}\right]_\times \mathbf{R}\mathbf{K}^{-1}} \mathbf{u} = 0$$

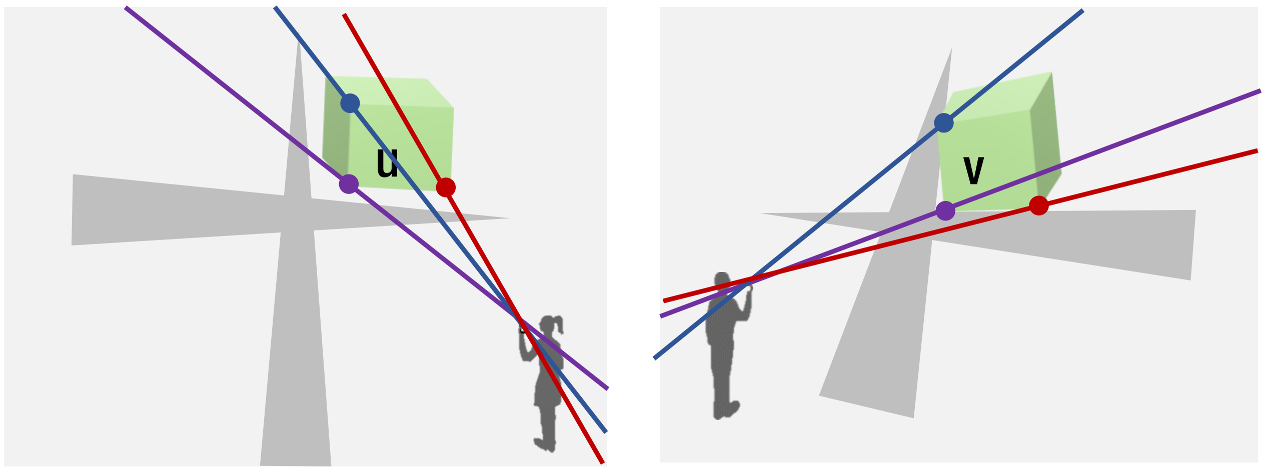
Common for all points

$$= \mathbf{v}^T \mathbf{F}\mathbf{u} = 0$$

$$= \mathbf{v}^T \left(\mathbf{F}\mathbf{u}\right) = \mathbf{u}^T \left(\mathbf{F}^T \mathbf{v}\right) = 0$$

Fundamental Matrix





Bob's image

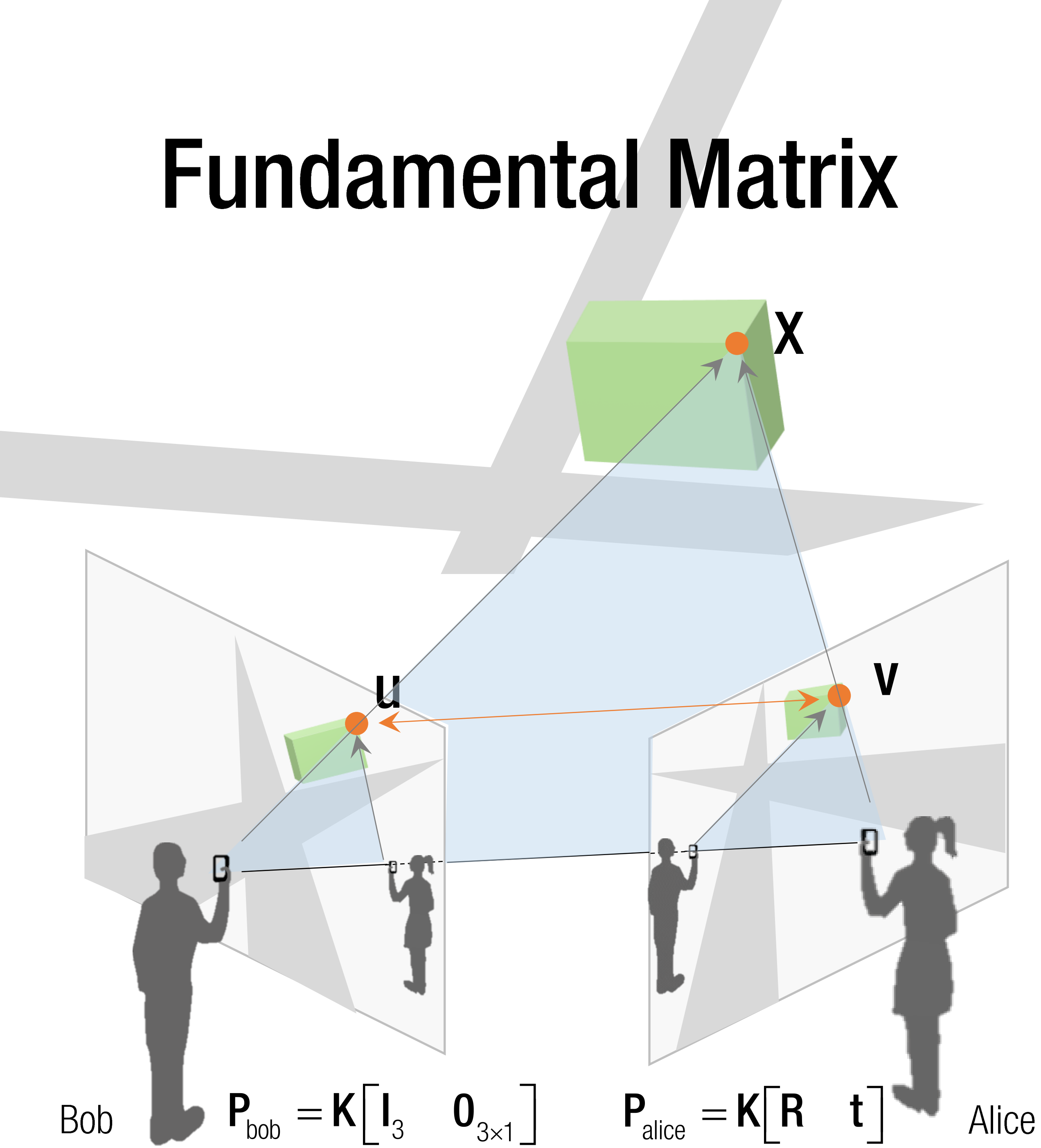
Alice's image

$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \underbrace{\mathbf{K}^{-T} [\mathbf{t}]_\times \mathbf{R} \mathbf{K}^{-1}}_{\text{Common for all points}} \mathbf{u} = 0$$

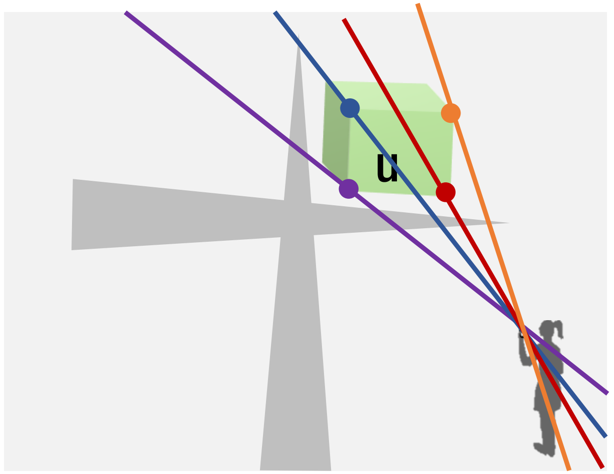
$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

$$= \mathbf{v}^T (\mathbf{F}\mathbf{u}) = \mathbf{u}^T (\mathbf{F}^T \mathbf{v}) = 0$$

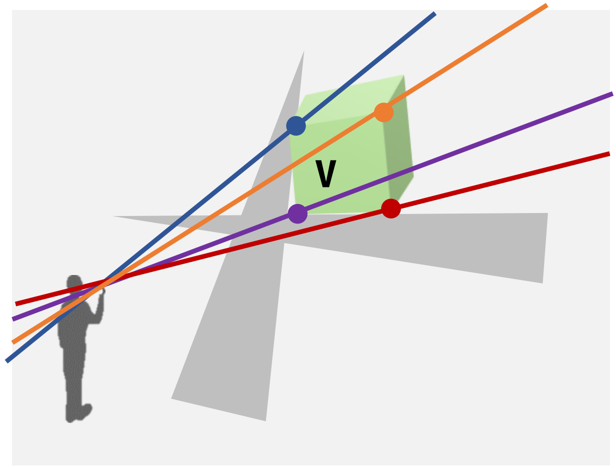
Fundamental Matrix



$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix}$ $P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$



Bob's image



Alice's image

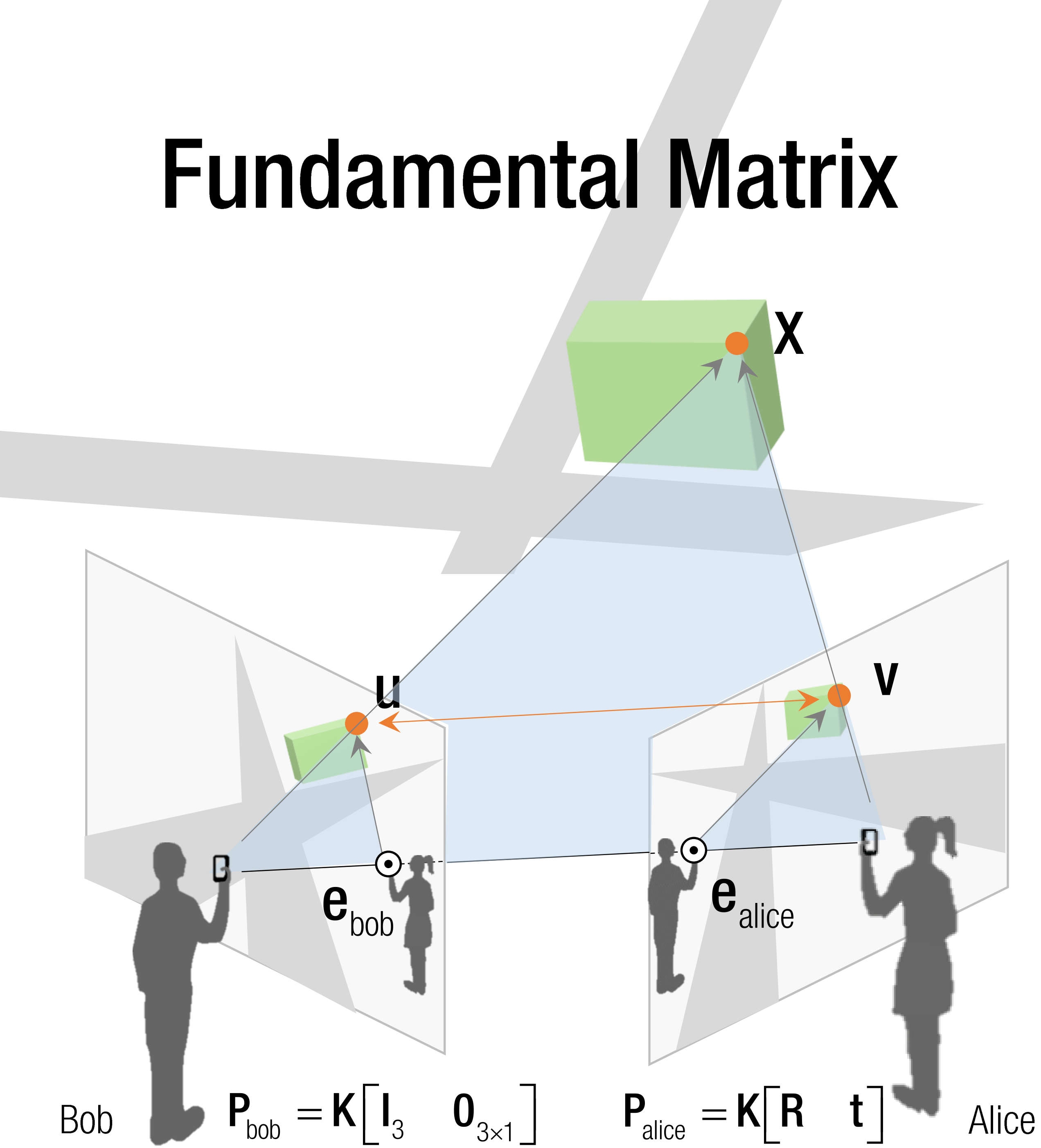
$$v^T l_u = v^T \underline{K^{-T} [t]_\times R K^{-1}} u = 0$$

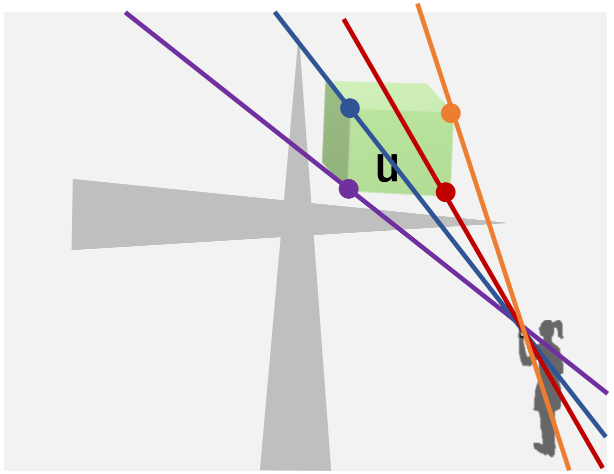
Common for all points

$$= v^T F u = 0$$

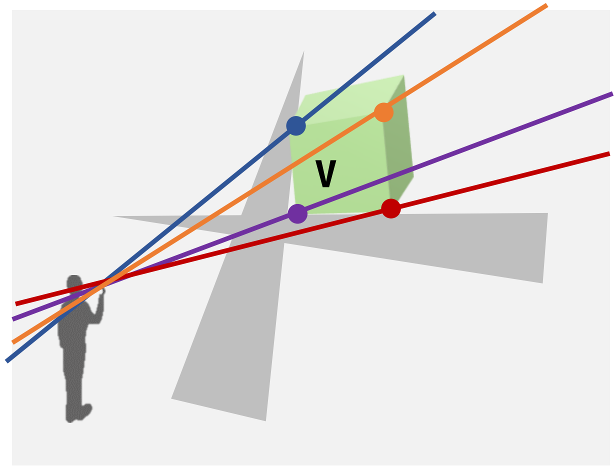
$$= v^T (Fu) = u^T (F^T v) = 0$$

Fundamental Matrix





Bob's image

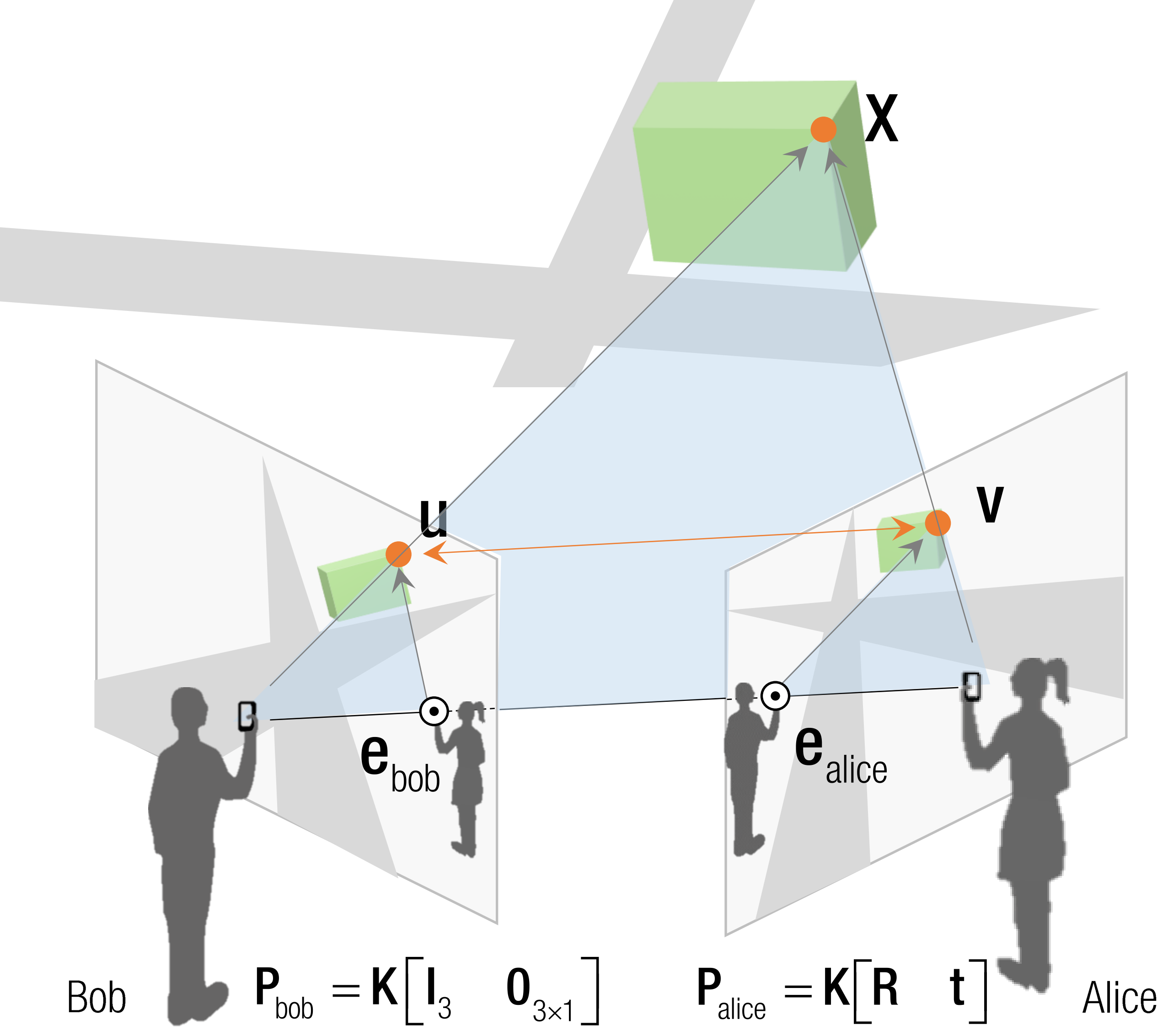


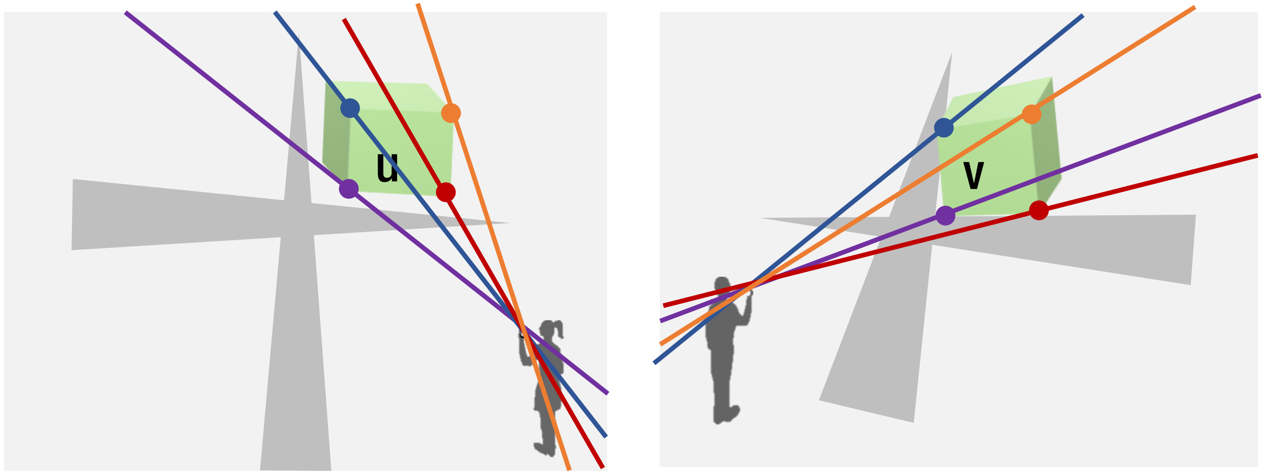
Alice's image

Properties of Fundamental Matrix

- Transpose: if $\mathbf{F}$ is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.

Fundamental Matrix





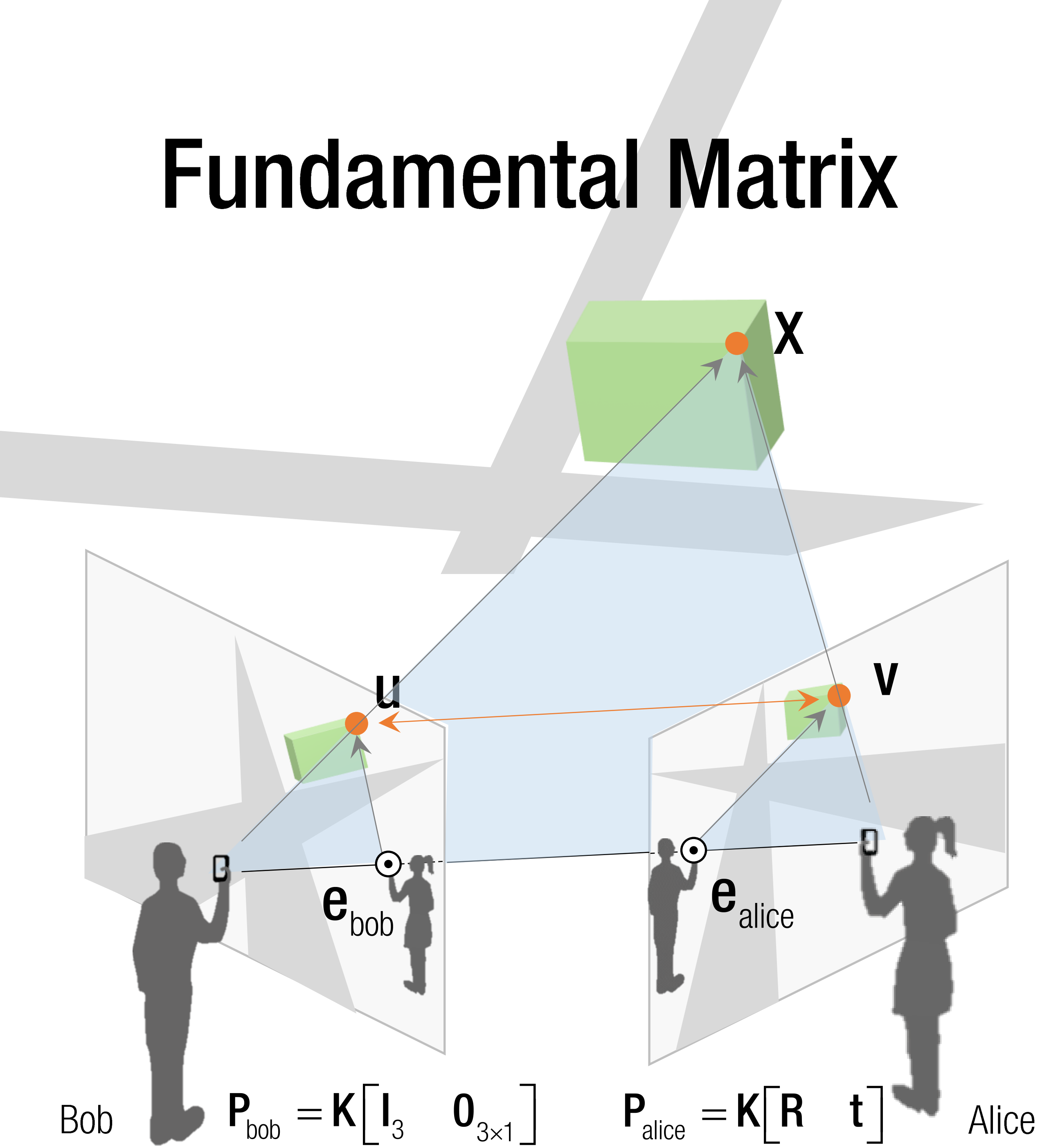
Bob's image

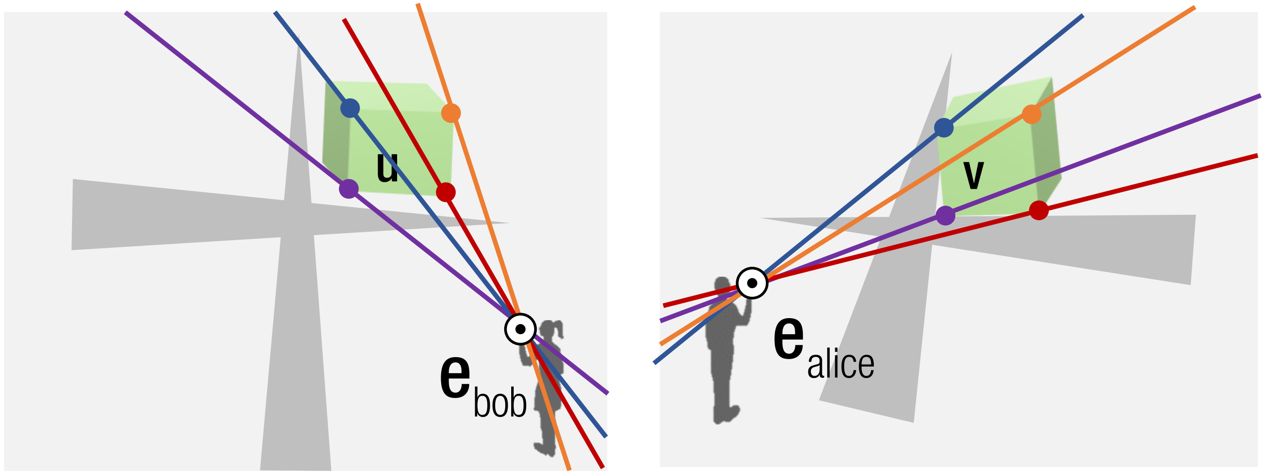
Alice's image

Properties of Fundamental Matrix

- Transpose: if **F** is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^{\mathsf{T}}$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u}$ $\quad \mathbf{l}_v = \mathbf{F}^{\mathsf{T}}\mathbf{v}$

Fundamental Matrix





Bob's image Alice's image

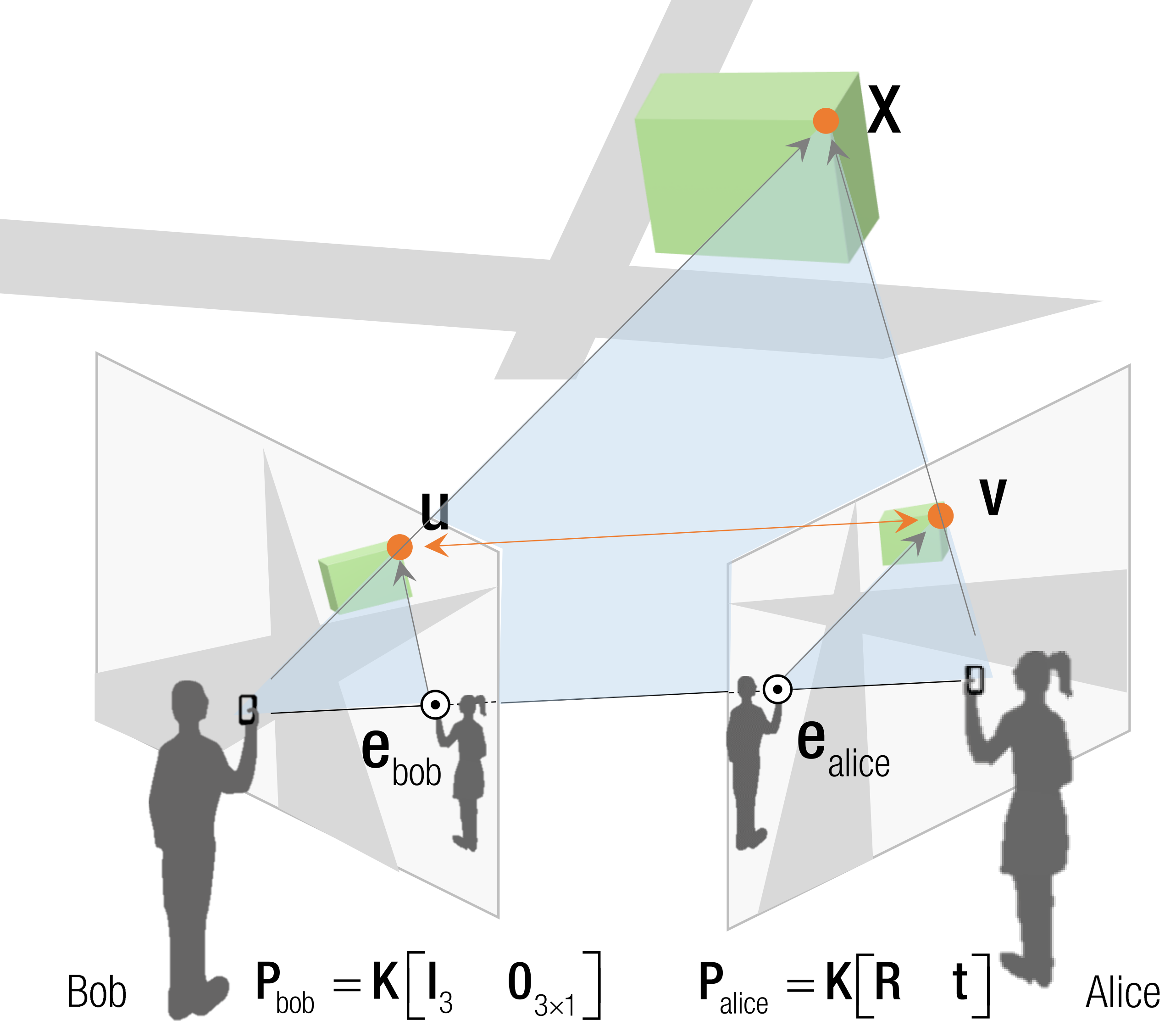
Properties of Fundamental Matrix

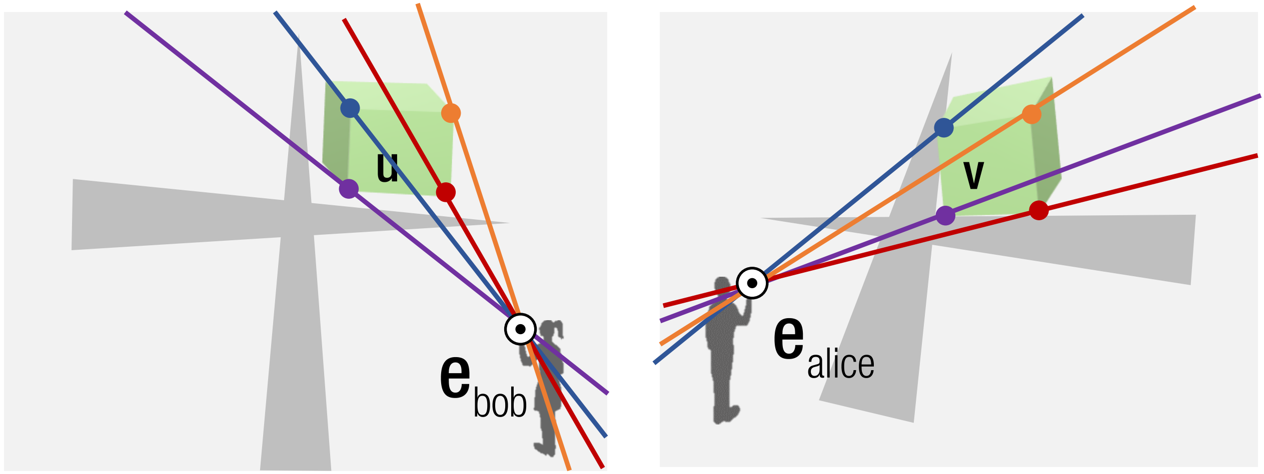
- Transpose: if $\mathbf{F}$ is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u} \quad \mathbf{l}_v = \mathbf{F}^T\mathbf{v}$
- Epipole: $\mathbf{F}\mathbf{e}_{bob} = \mathbf{0} \quad \mathbf{F}^T\mathbf{e}_{alice} = \mathbf{0}$

$$\because \mathbf{v}_i^T\mathbf{F}\mathbf{e}_{bob} = 0, \quad \mathbf{u}_i^T\mathbf{F}^T\mathbf{e}_{alice} = 0, \quad \forall i$$

$$\longrightarrow \mathbf{e}_{bob} = \text{null}(\mathbf{F}), \quad \mathbf{e}_{alice} = \text{null}(\mathbf{F}^T)$$

Fundamental Matrix





Bob's image   Alice's image

Properties of Fundamental Matrix

- Transpose: if **F** is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^{T}$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u}$ $\quad \mathbf{l}_v = \mathbf{F}^{T}\mathbf{v}$
- Epipole: $\mathbf{F}\mathbf{e}_{bob} = \mathbf{0}$ $\quad \mathbf{F}^{T}\mathbf{e}_{alice} = \mathbf{0}$
- rank(**F**)=2: degree of freedom 9 (3x3 matrix)-1 (scale)-1 (rank)=7

$$\mathbf{F} = \mathbf{K}^{-T} \underbrace{\left[\mathbf{t}\right]_{\times}}_{\text{rank 2 matrix}} \mathbf{R}\mathbf{K}^{-1}$$

Camera Motion

Camera Motion

Image 2

Image 1

Forward motion

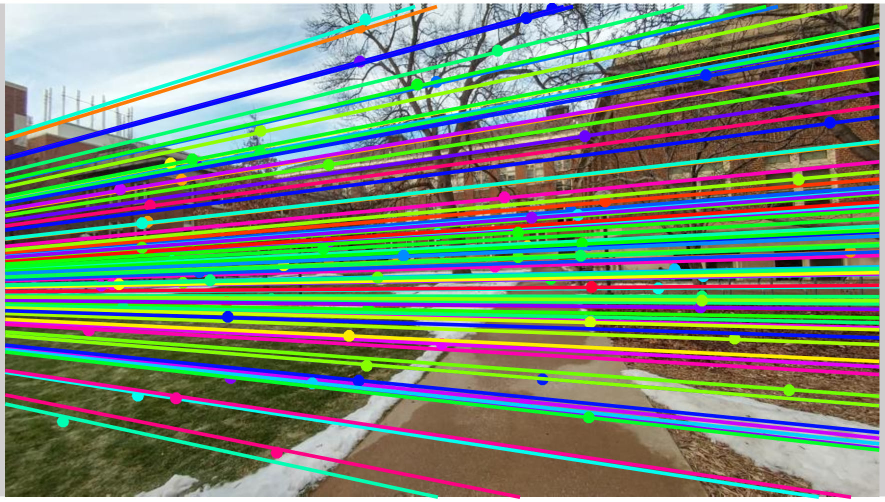


Image 2

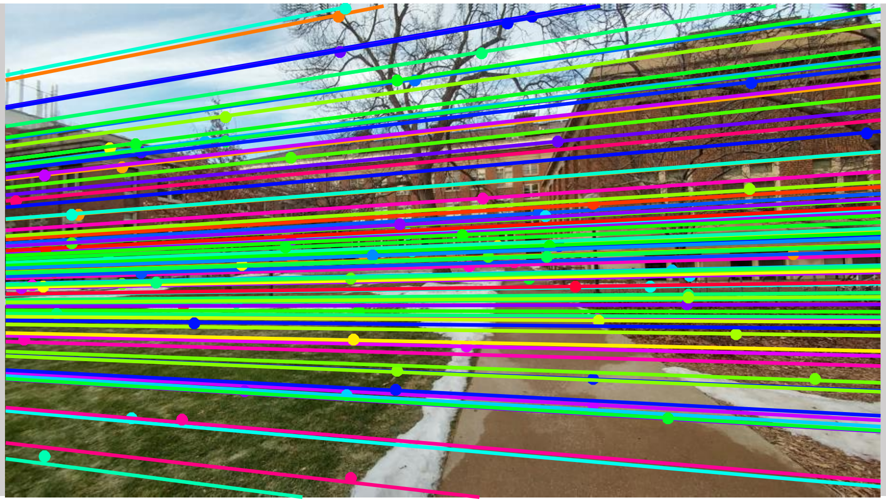
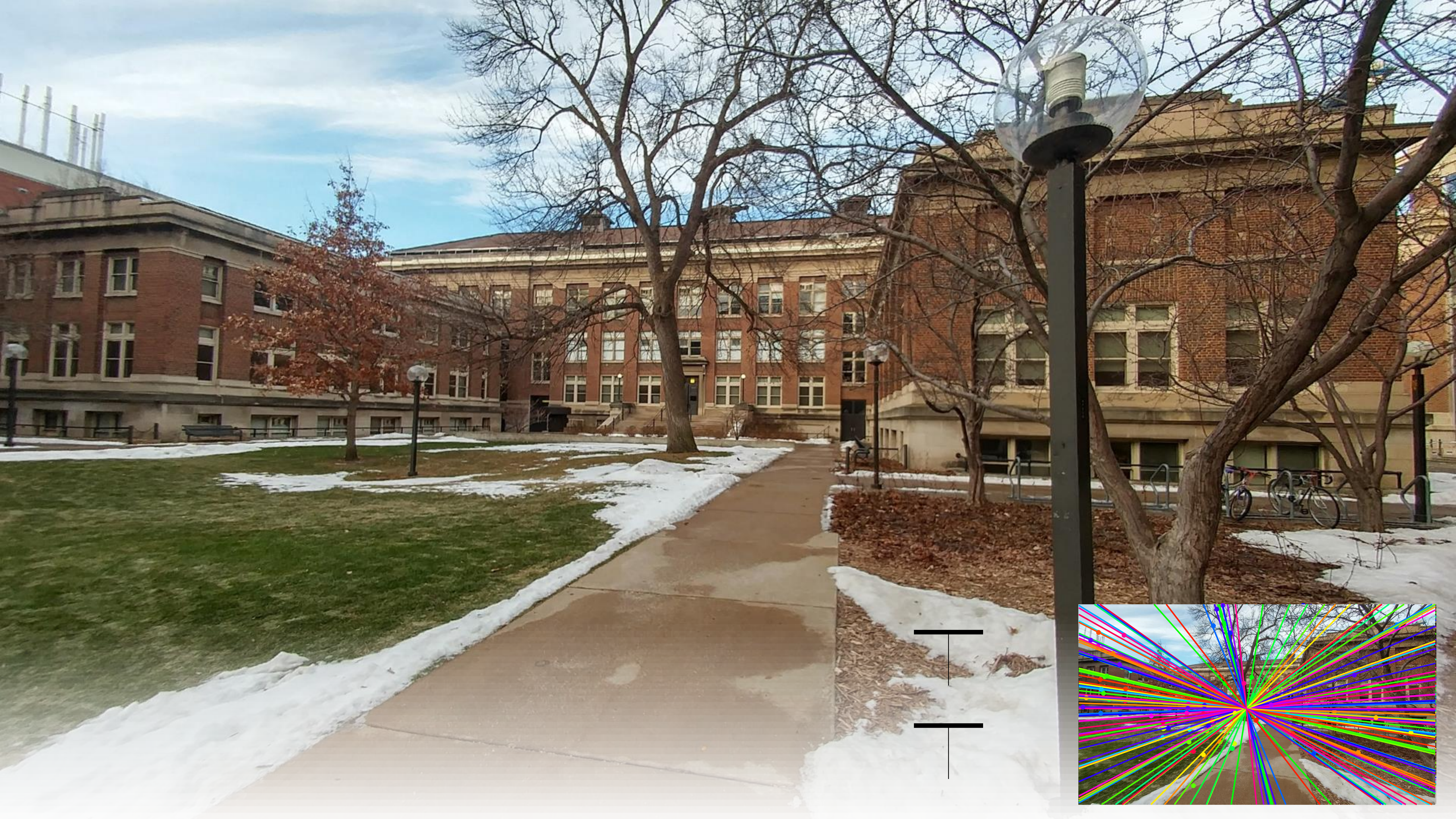
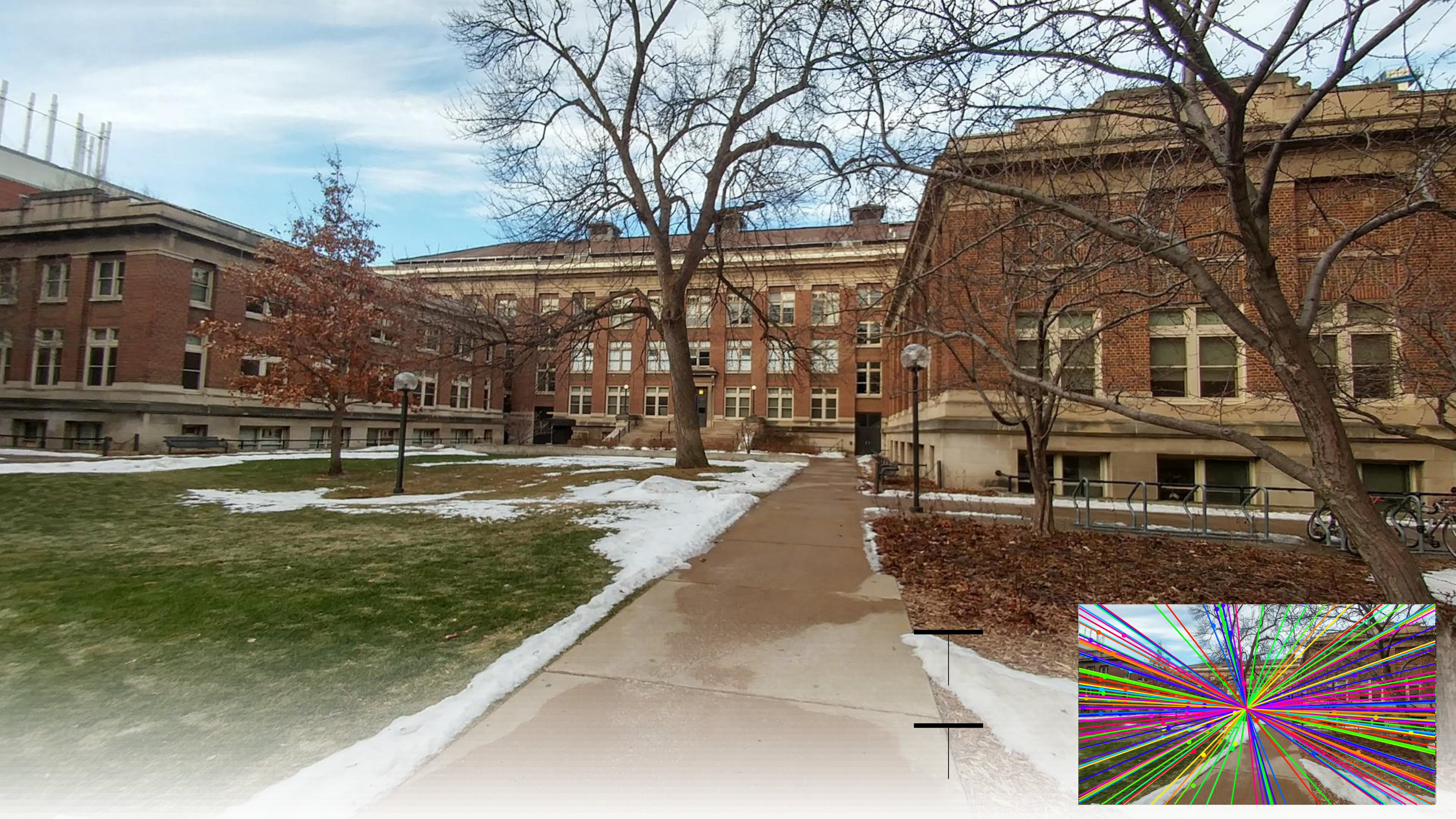


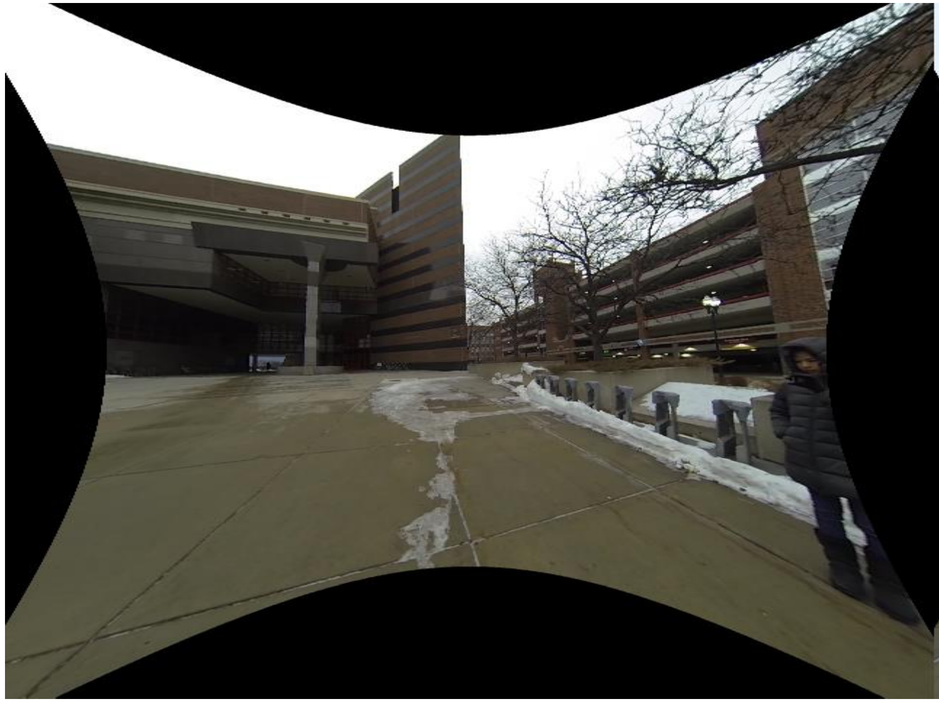
Image 1

Lateral motion

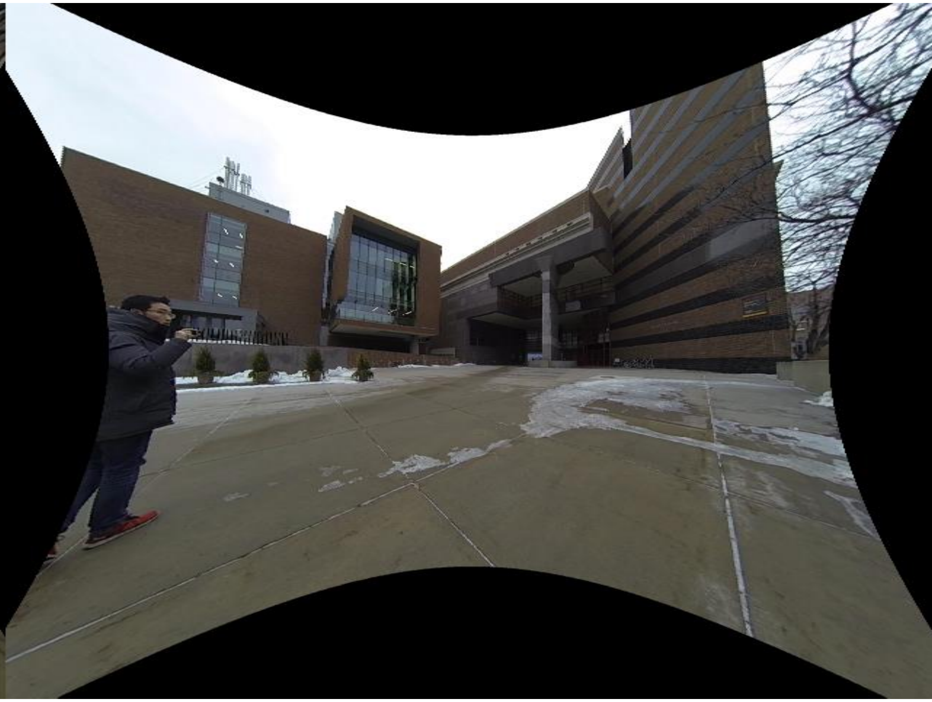




2D Correspondences

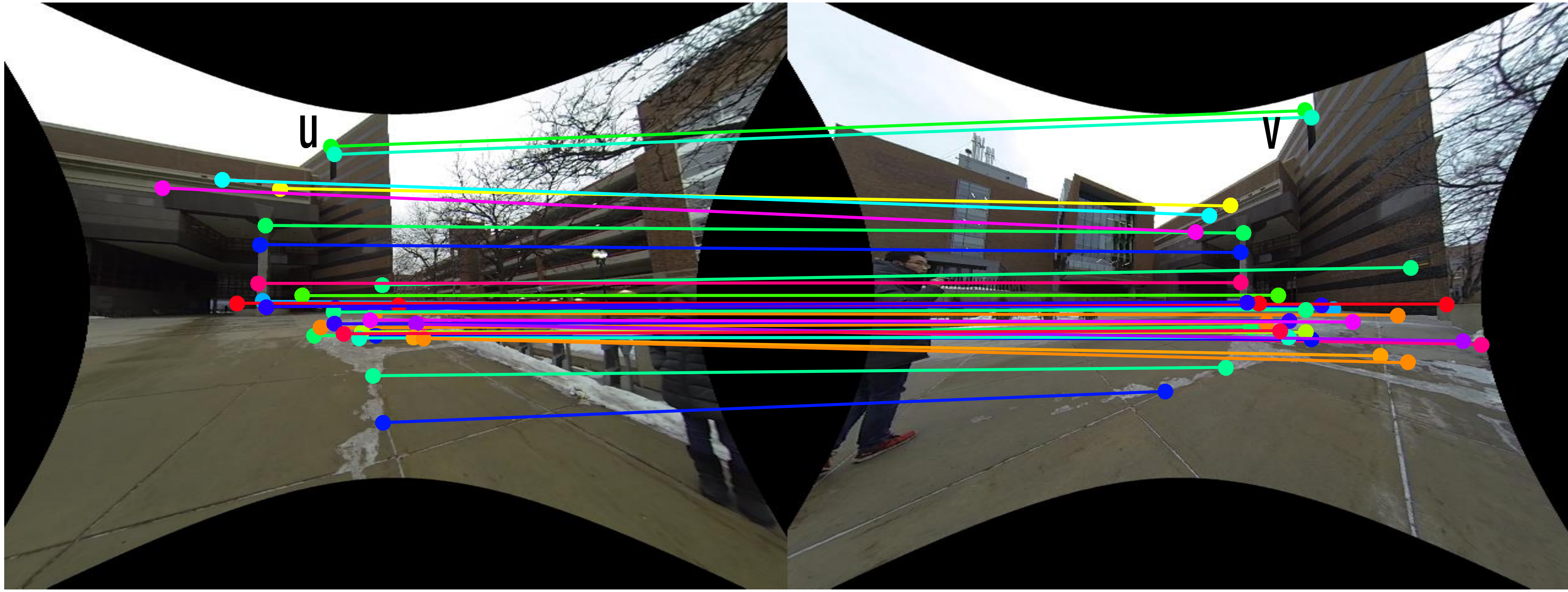


Bob’s image



Alice’s image

2D Correspondences

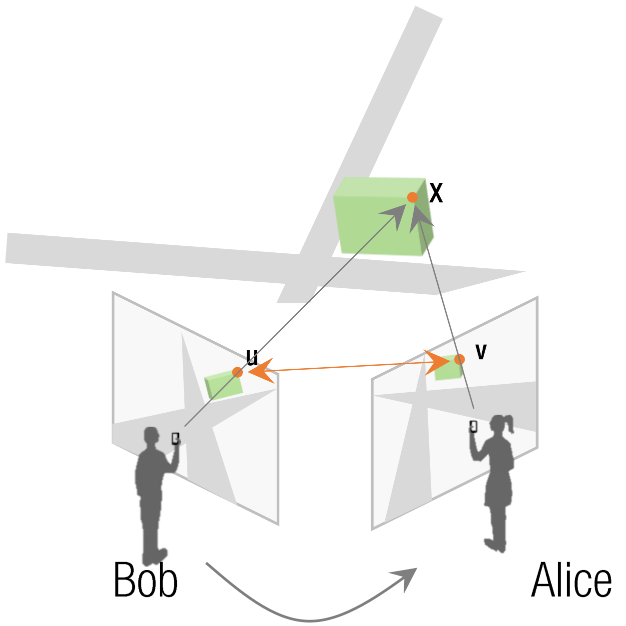


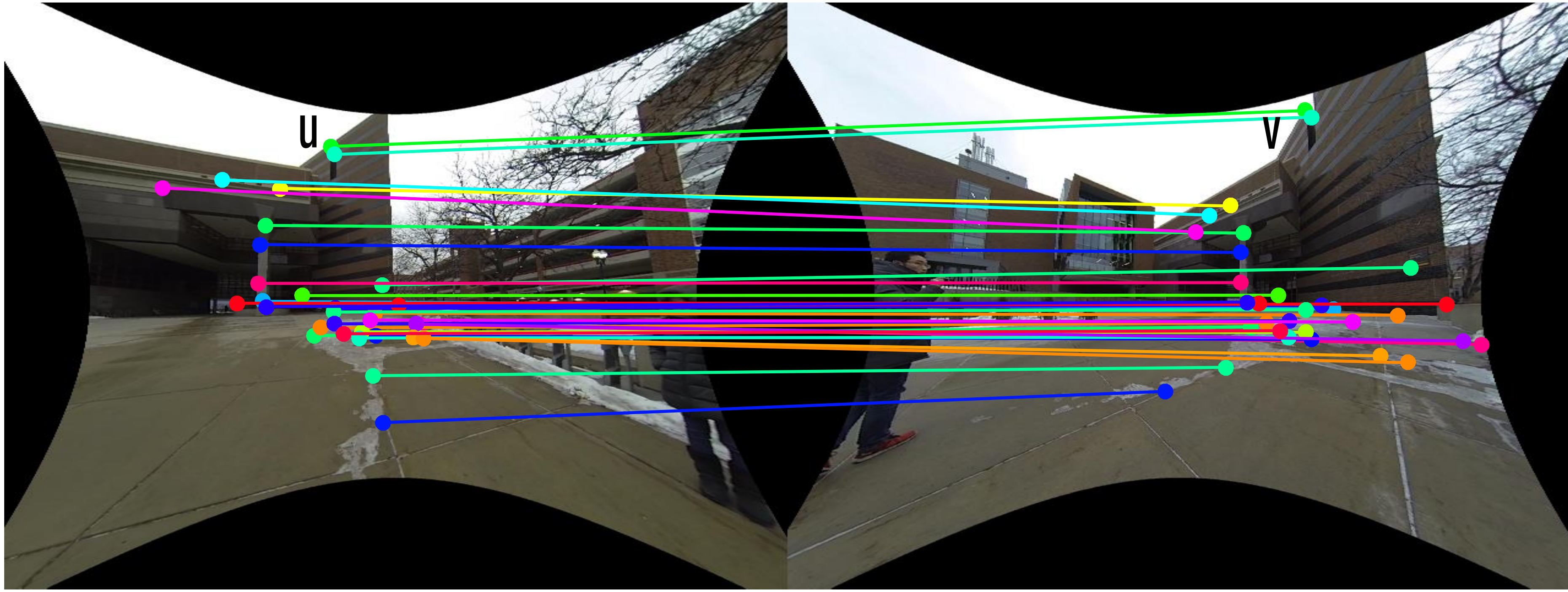
Bob's image

Alice's image

$$\mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

2D Correspondences





Bob's image

Alice's image

$$F = F(R, t)$$

$$= K^{-T}[t]_{\times}RK^{-1}$$

$$v^{T}Fu = 0$$

How to compute fundamental matrix?

8 Point Algorithm (Longuet-Higgins, Nature 1981)

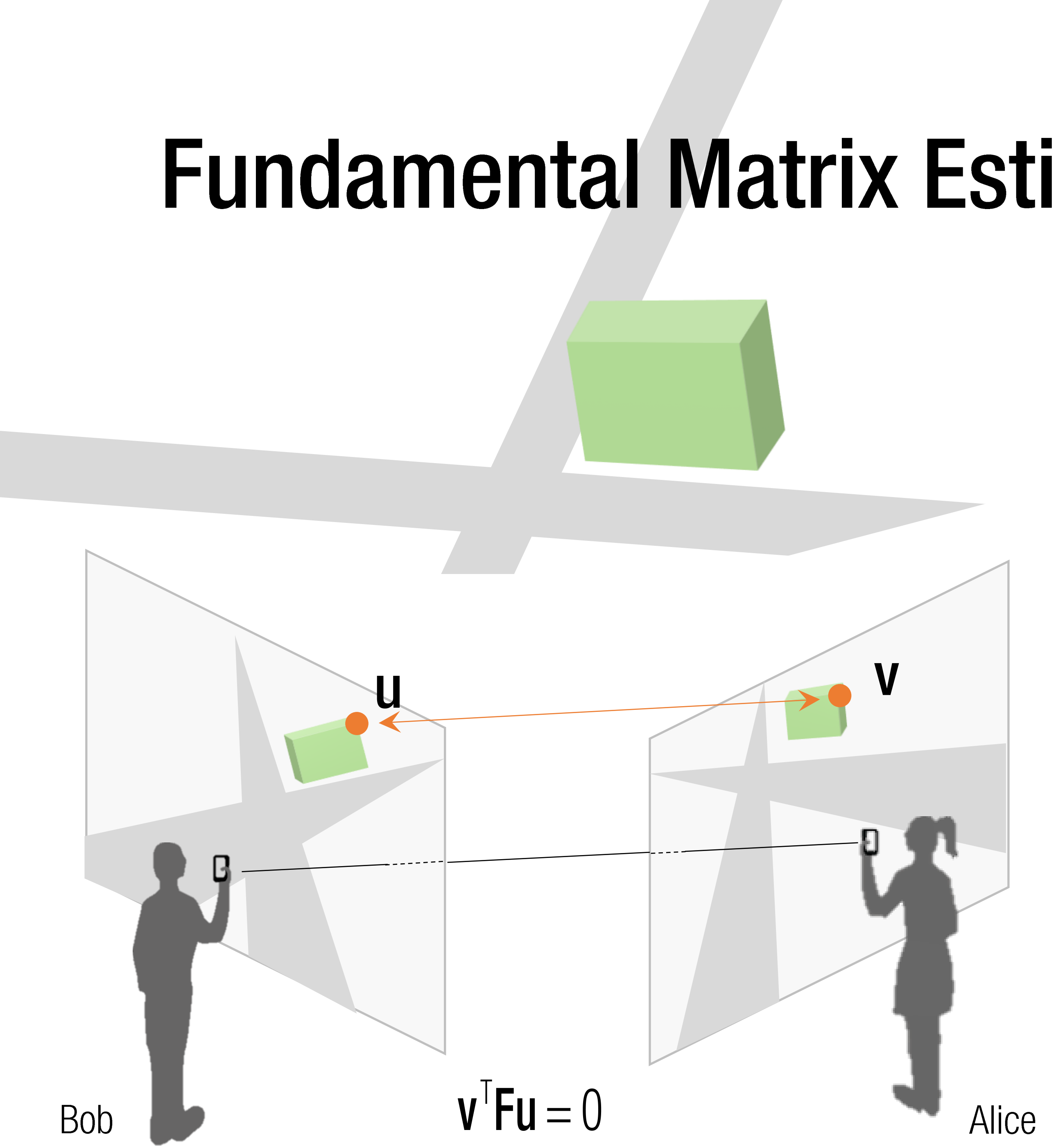
A computer algorithm for reconstructing a scene from two projections

H. C. Longuet-Higgins

Laboratory of Experimental Psychology, University of Sussex,
Brighton BN1 9QG, UK

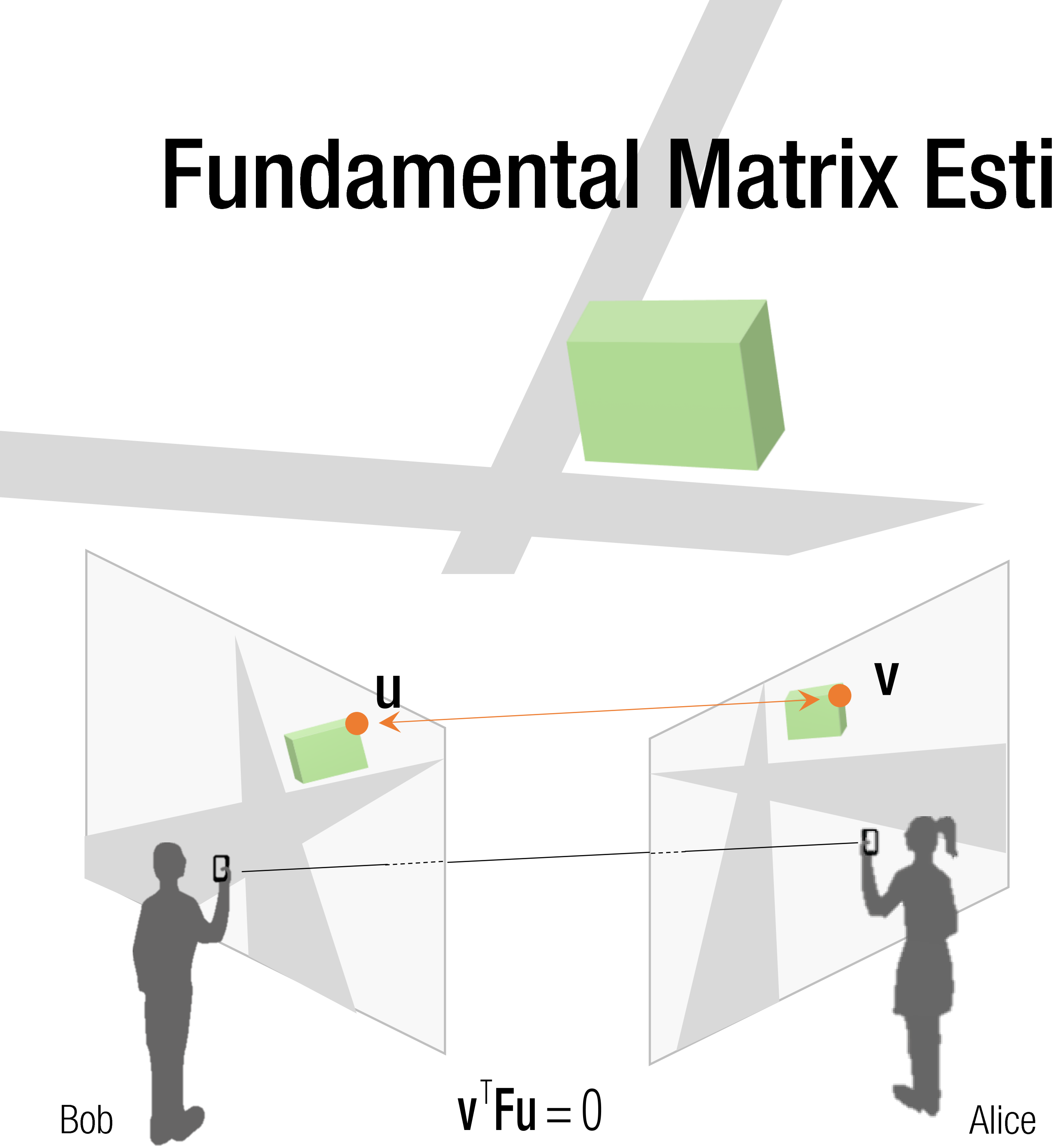
**A simple algorithm for computing the three-dimensiona
ture of a scene from a correlated pair of perspective proj
is described here, when the spatial relationship between t
projections is unknown. This problem is relevant not o
photographic surveying[1] but also to binocular vision[2], wh
non-visual information available to the observer abo**

Fundamental Matrix Estimation



$$v^T F u = 0$$

Fundamental Matrix Estimation

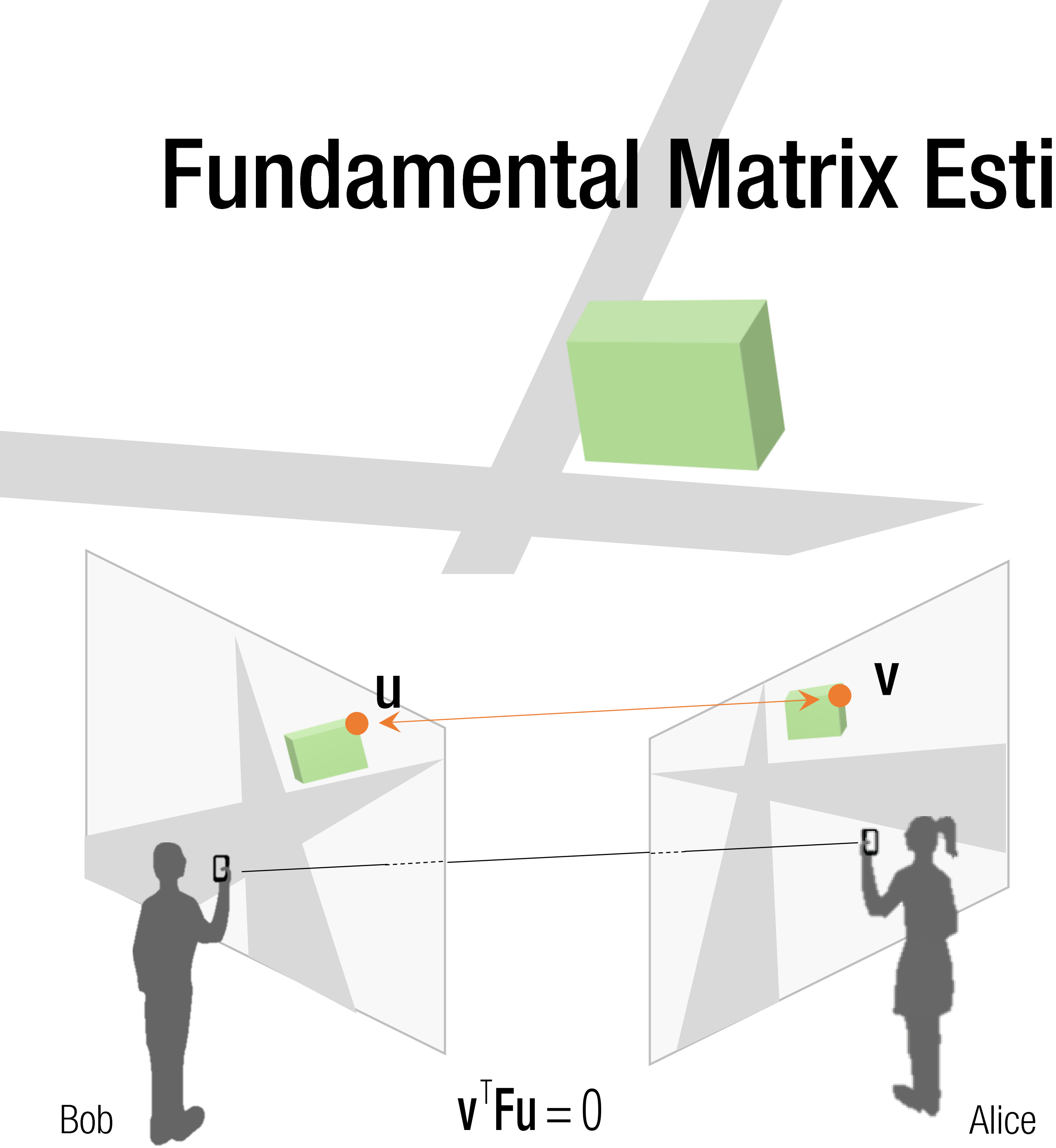


$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = 0$$

$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Degree of freedom of fundamental matrix:

Fundamental Matrix Estimation



$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = 0$$

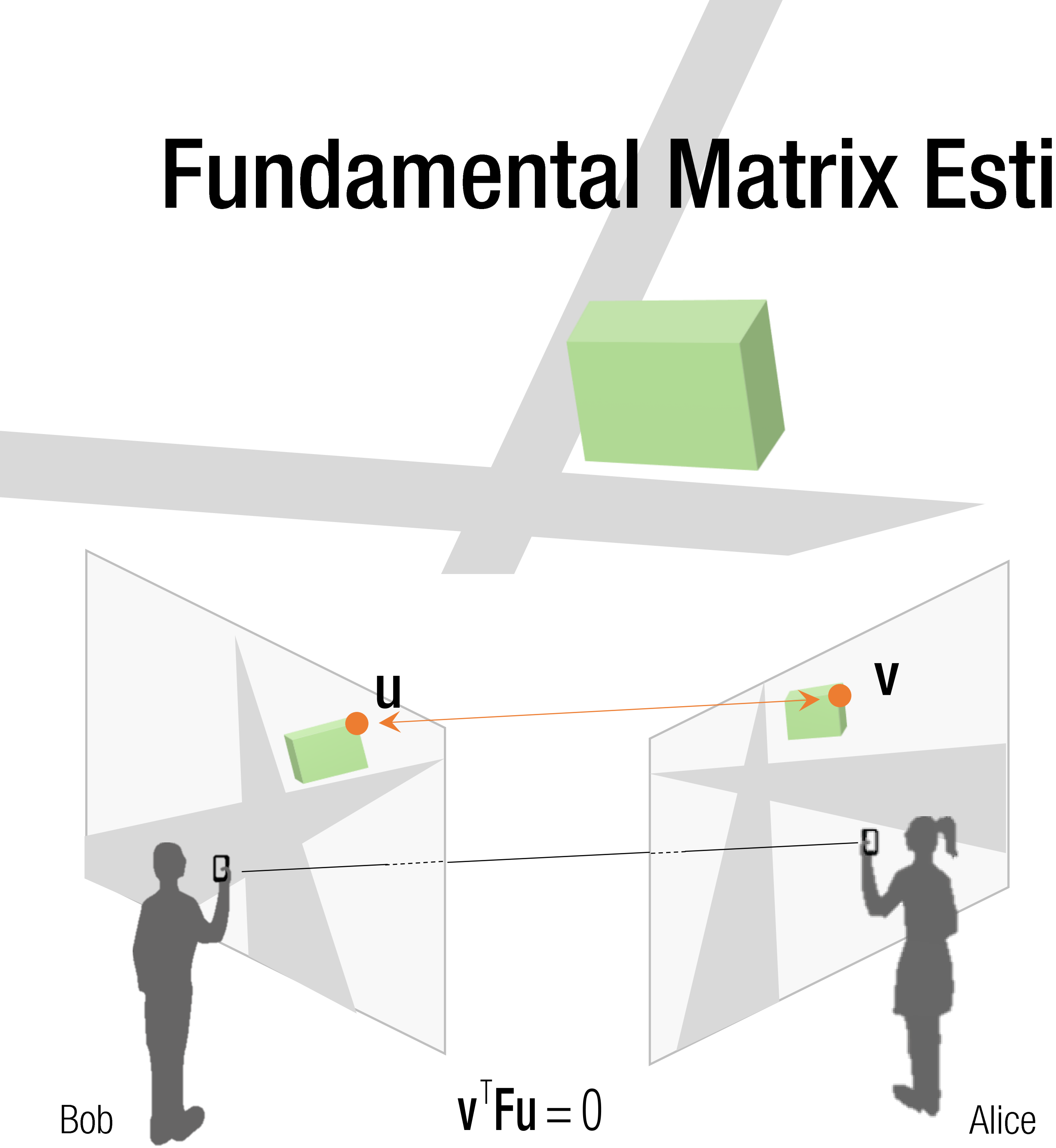
$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Degree of freedom of fundamental matrix:
7 = 9 (3x3 matrix) – 1(scale) – 1 (rank 2)

We will estimate fundamental matrix with 8 parameter by ignoring rank constraint and then project onto rank 2 matrix:

Fundamental Matrix Estimation

$$\mathbf{v}^{\mathrm{T}}\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

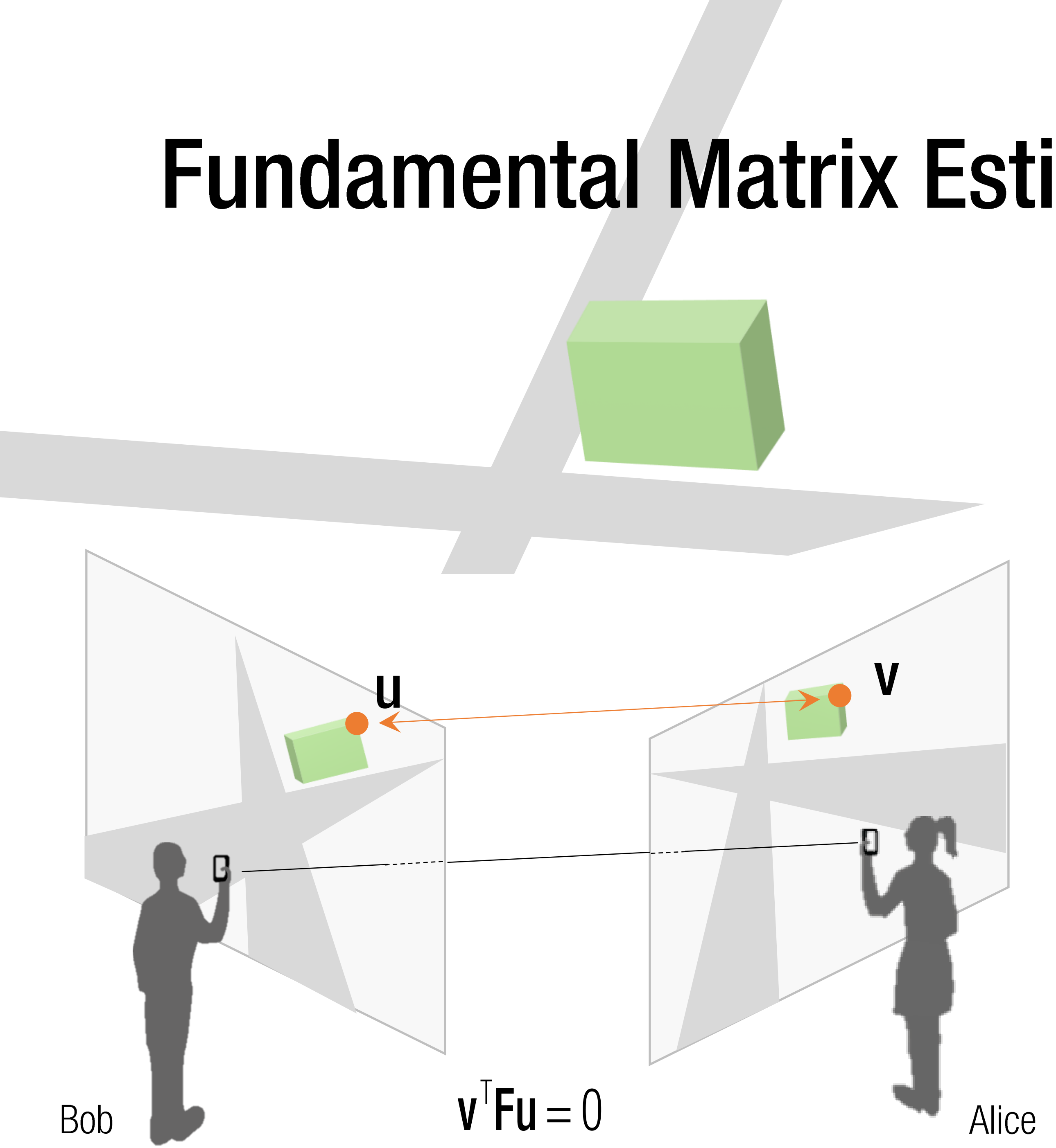


$$\mathbf{v}^{\mathrm{T}}\mathbf{F}\mathbf{u} = 0$$

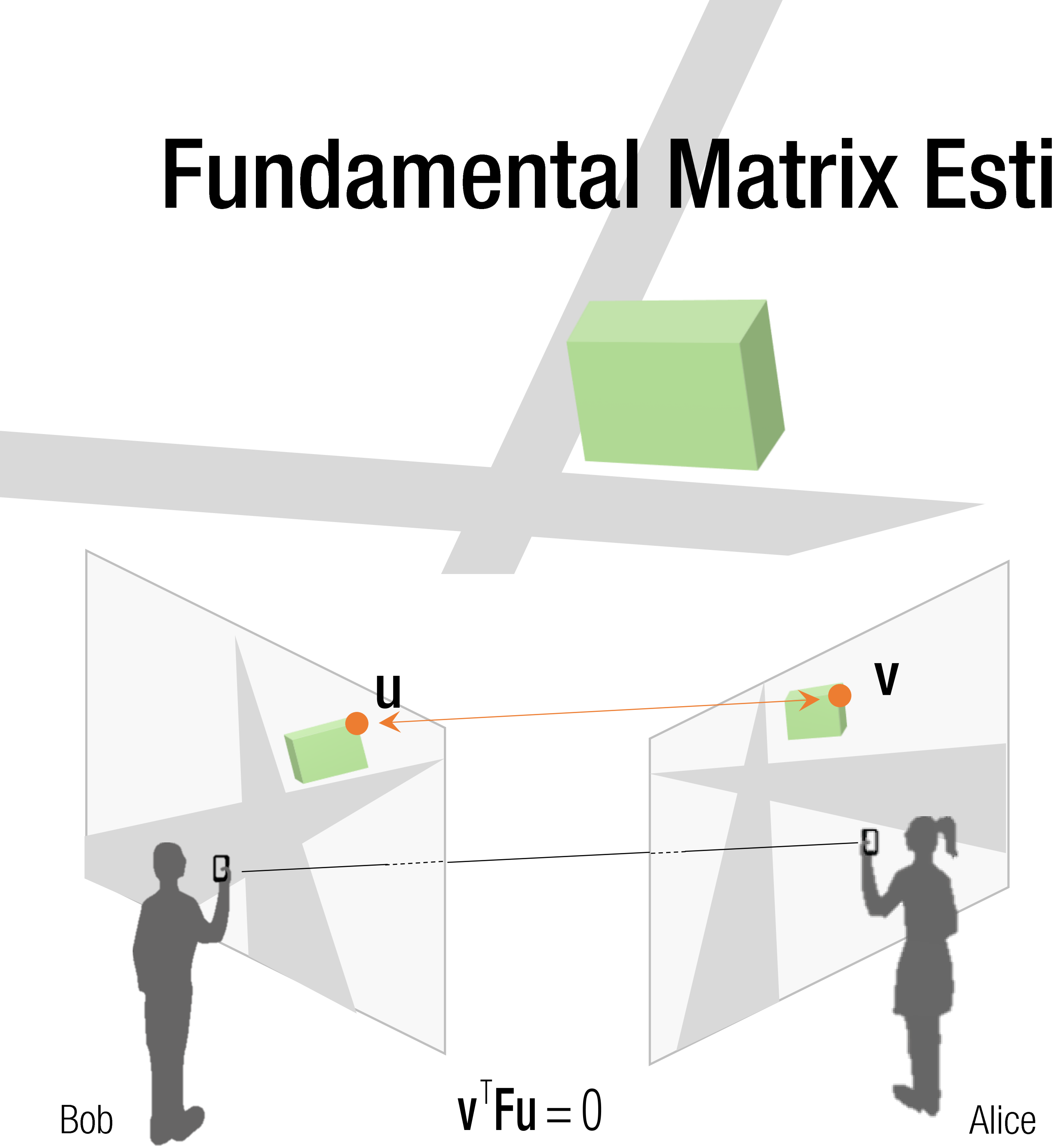
Fundamental Matrix Estimation

$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$



Fundamental Matrix Estimation



$$\mathbf{v}^T\mathbf{F}\mathbf{u} = 0$$

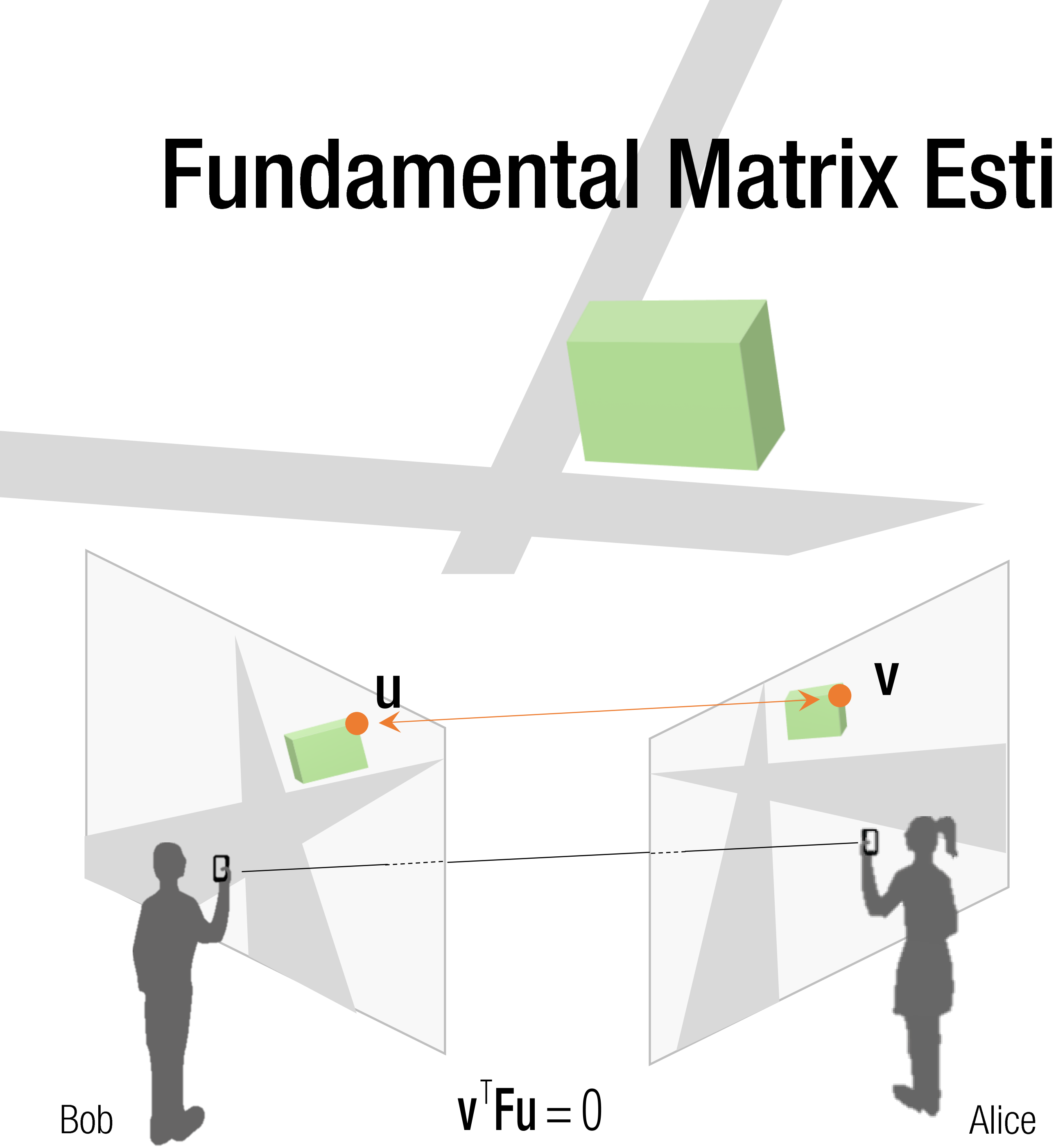
$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

$$= 0$$

Linear in **F**.

Fundamental Matrix Estimation



$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = 0$$

$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

$$= 0$$

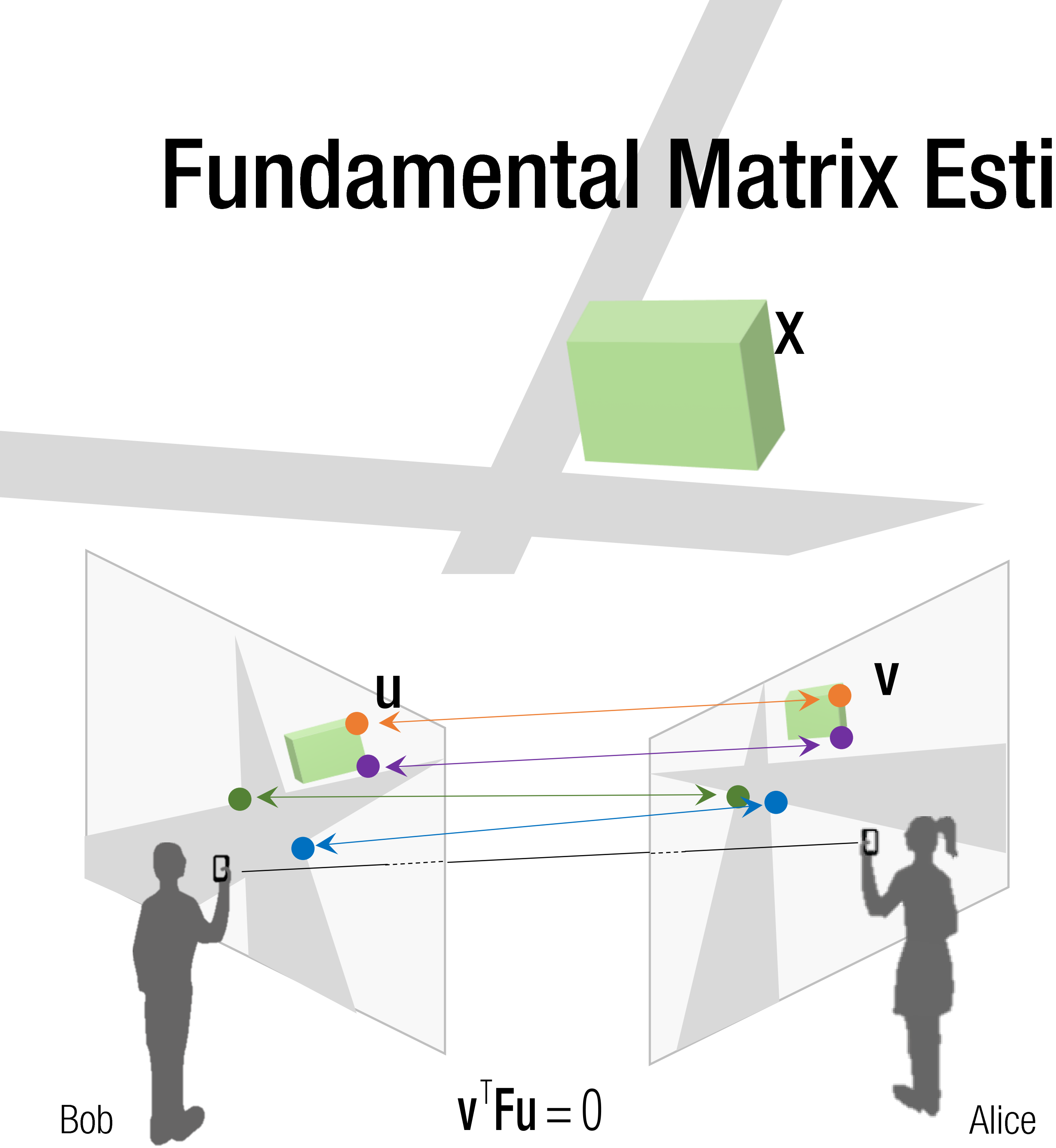
Linear in **F**.

$$\longrightarrow \begin{bmatrix} u^x v^x & u^y v^x & v^x & u^x v^y & u^y v^y & v^y & u^x & u^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

# of unknowns: 9

# of equations per correspondence: 1

Fundamental Matrix Estimation



$$\mathbf{v}^T\mathbf{F}\mathbf{u} = 0$$

$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

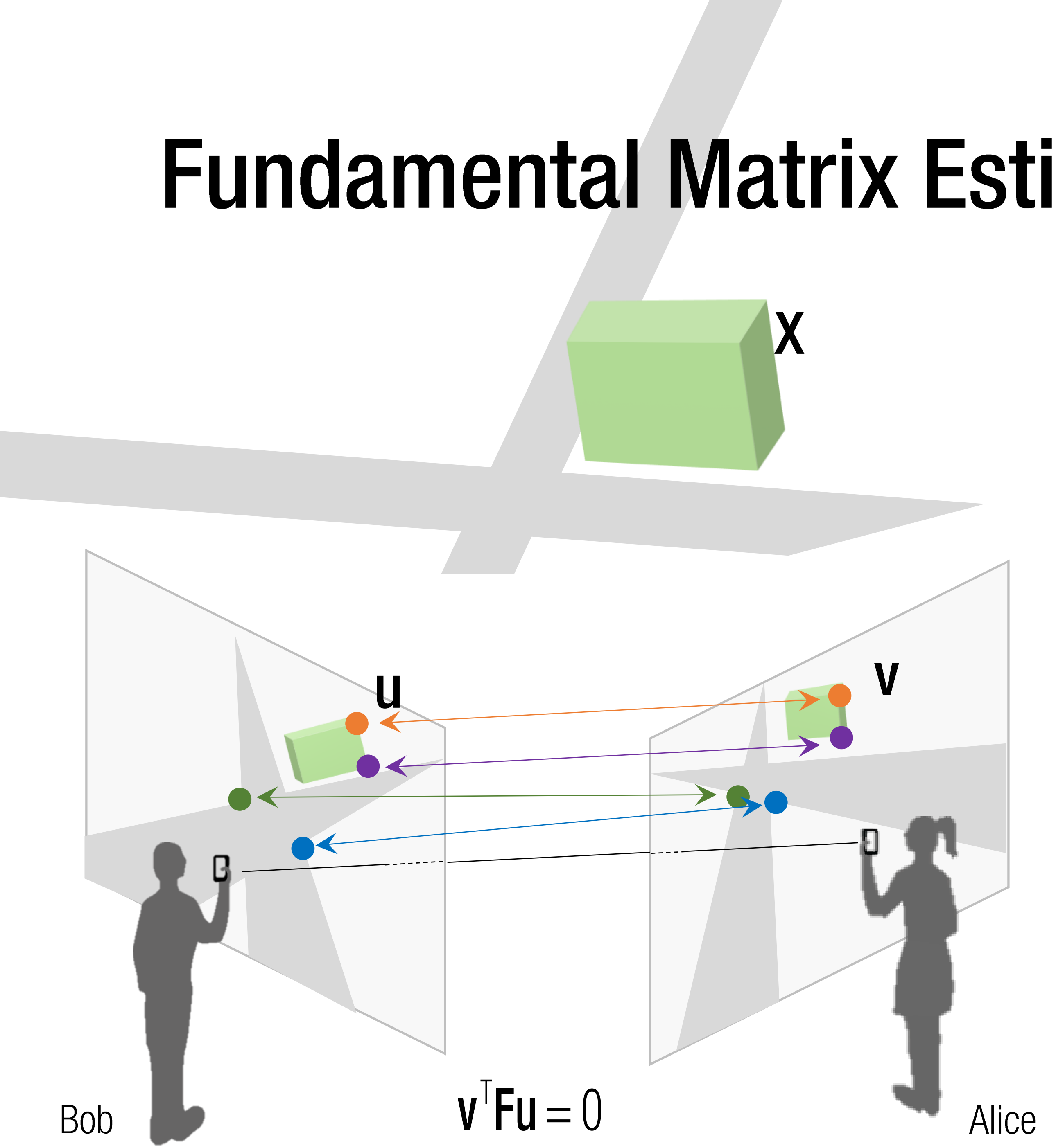
$$= 0$$

Linear in **F**.

$$\begin{bmatrix} u_1^xv_1^x & u_1^yv_1^x & v_1^x & u_1^xv_1^y & u_1^yv_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^xv_m^x & u_m^yv_m^x & v_m^x & u_m^xv_m^y & u_m^yv_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \mathbf{0}_{m\times 1}$$

What is minimum m?

Fundamental Matrix Estimation



$$\mathbf{v}^T\mathbf{F}\mathbf{u} = 0$$

$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^xv^x + f_{12}u^yv^x + f_{13}v^x + f_{21}u^xv^y + f_{22}u^yv^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

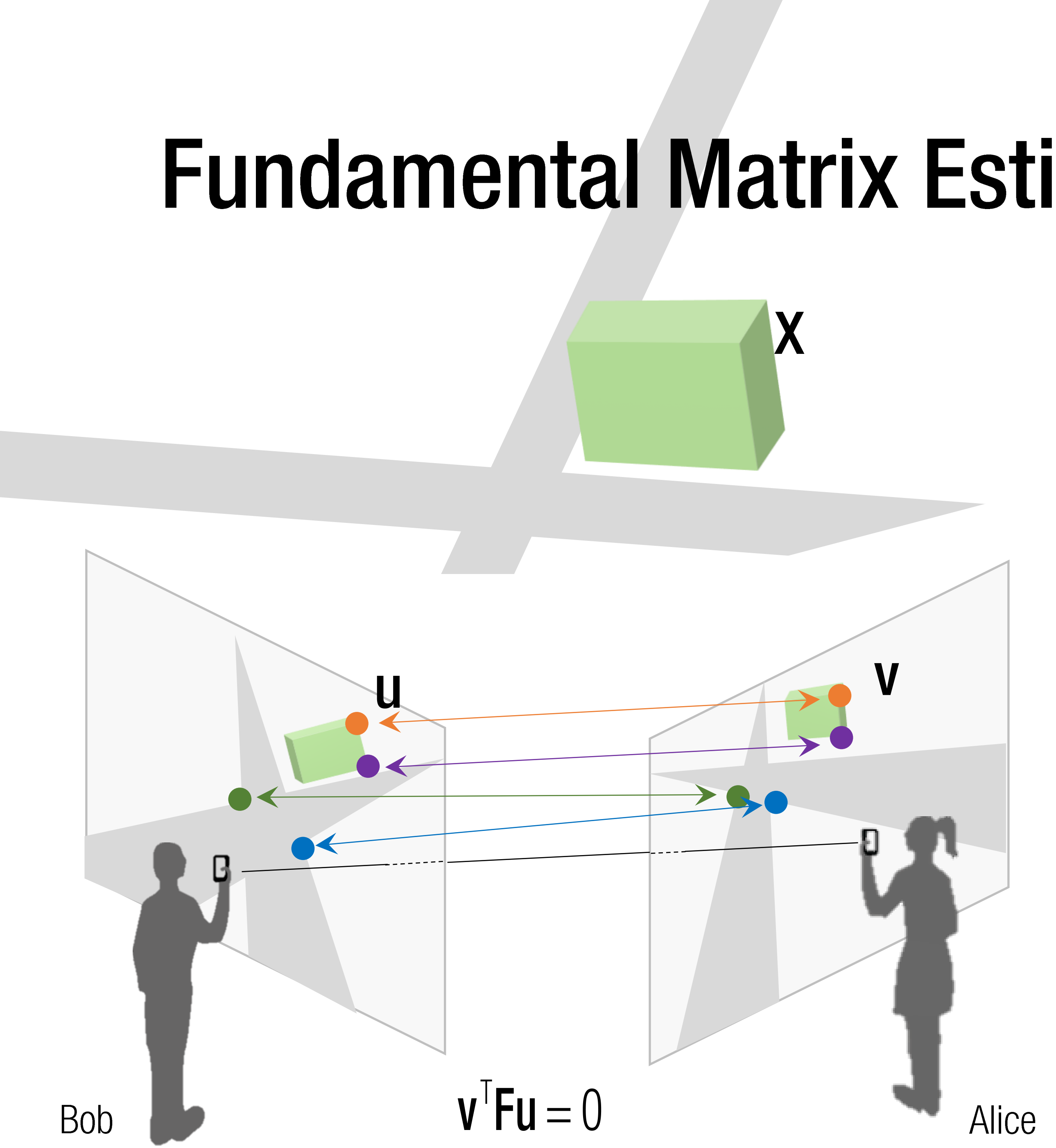
$$= 0$$

Linear in **F**.

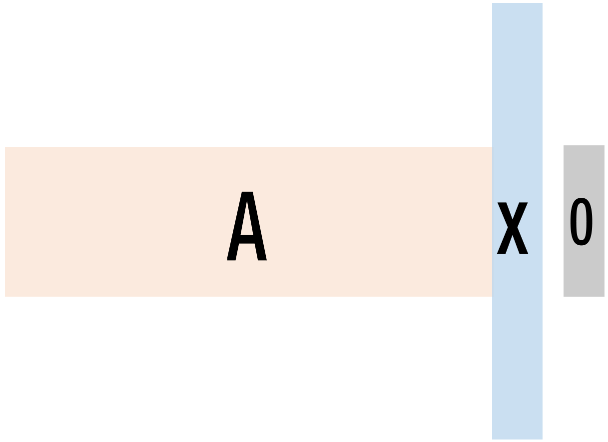
$$\underbrace{\begin{bmatrix} u_1^xv_1^x & u_1^yv_1^x & v_1^x & u_1^xv_1^y & u_1^yv_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^xv_m^x & u_m^yv_m^x & v_m^x & u_m^xv_m^y & u_m^yv_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}}_{\mathbf{X}} = \mathbf{0}_{m\times 1}$$

What is minimum m?

Fundamental Matrix Estimation

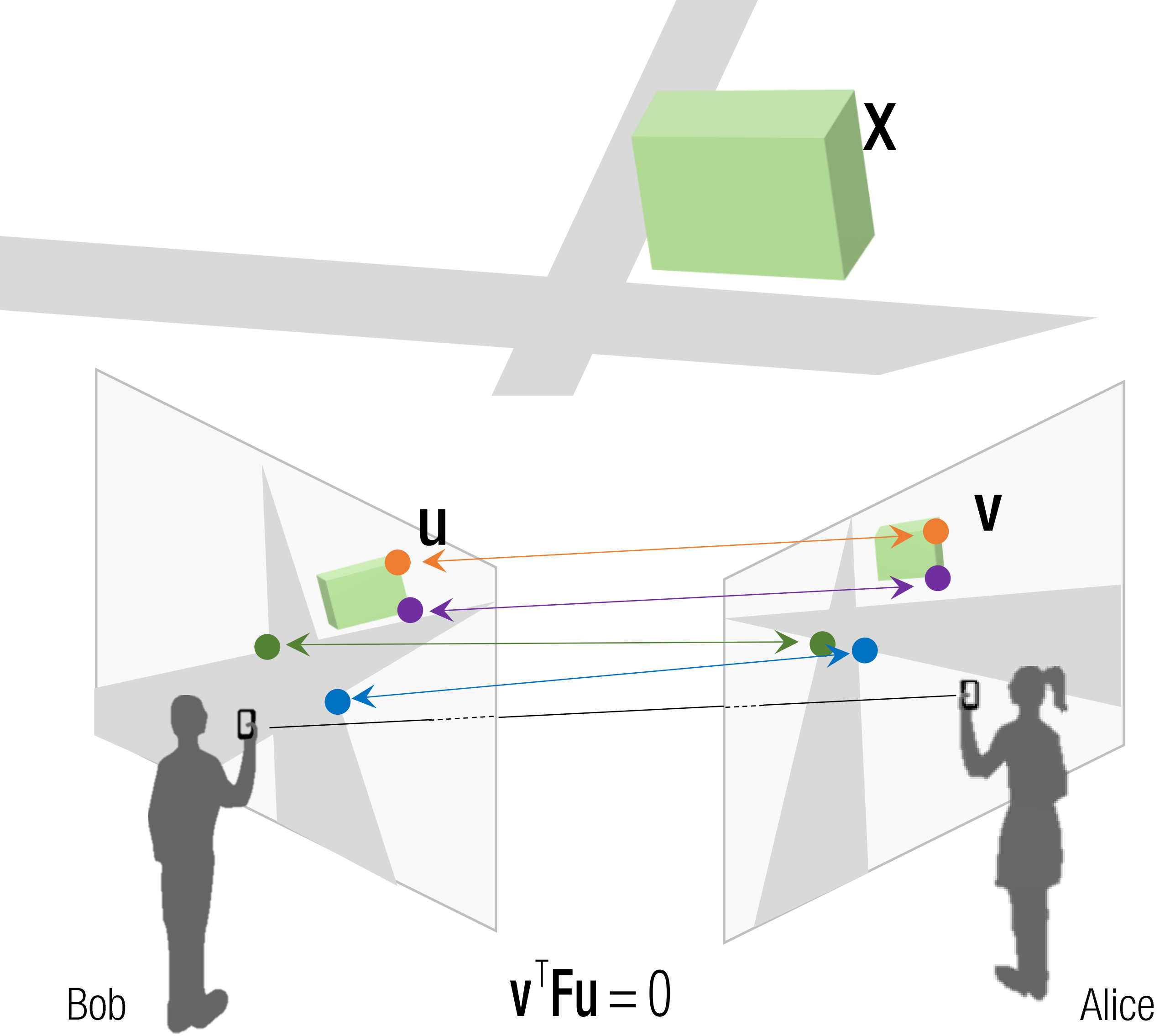


$$v^T F u = 0$$

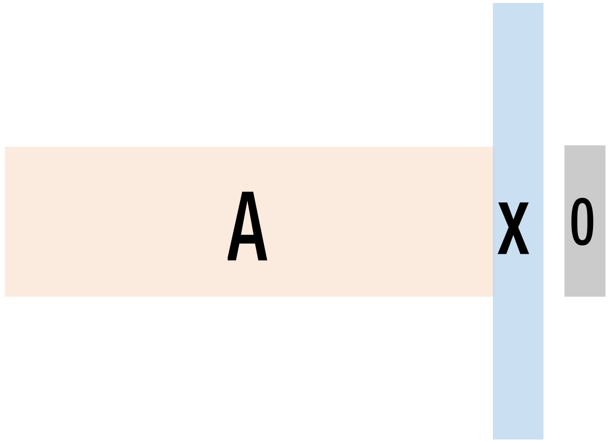


The solution is not necessarily satisfy rank 2 constraint.

Fundamental Matrix Estimation

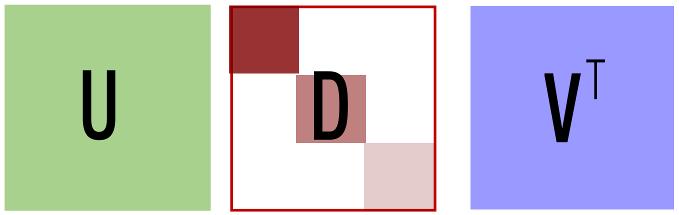


$$v^T F u = 0$$

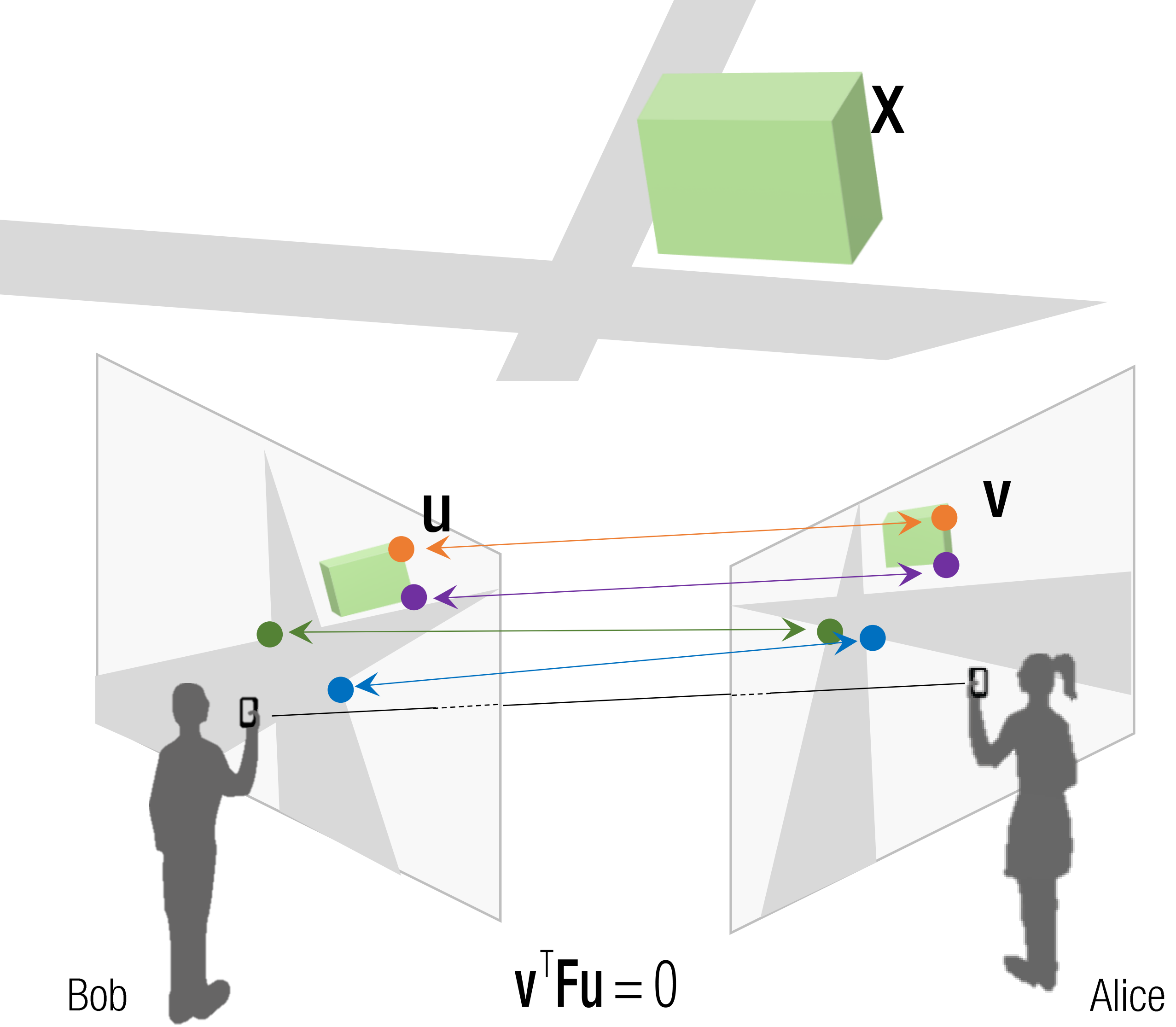


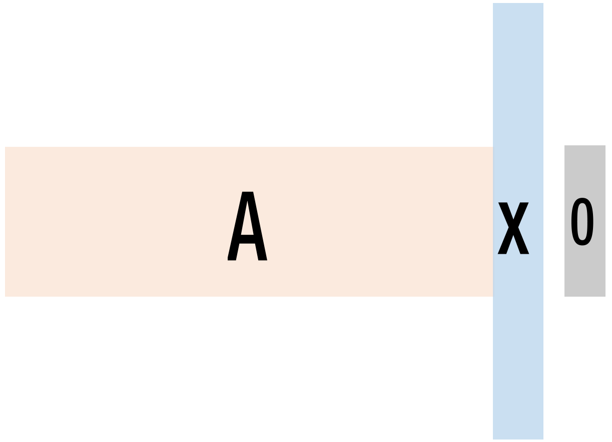
The solution is not necessarily satisfy rank 2 constraint.

$$\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} = U \; D \; V^T$$



Fundamental Matrix Estimation





The solution is not necessarily satisfy rank 2 constraint.