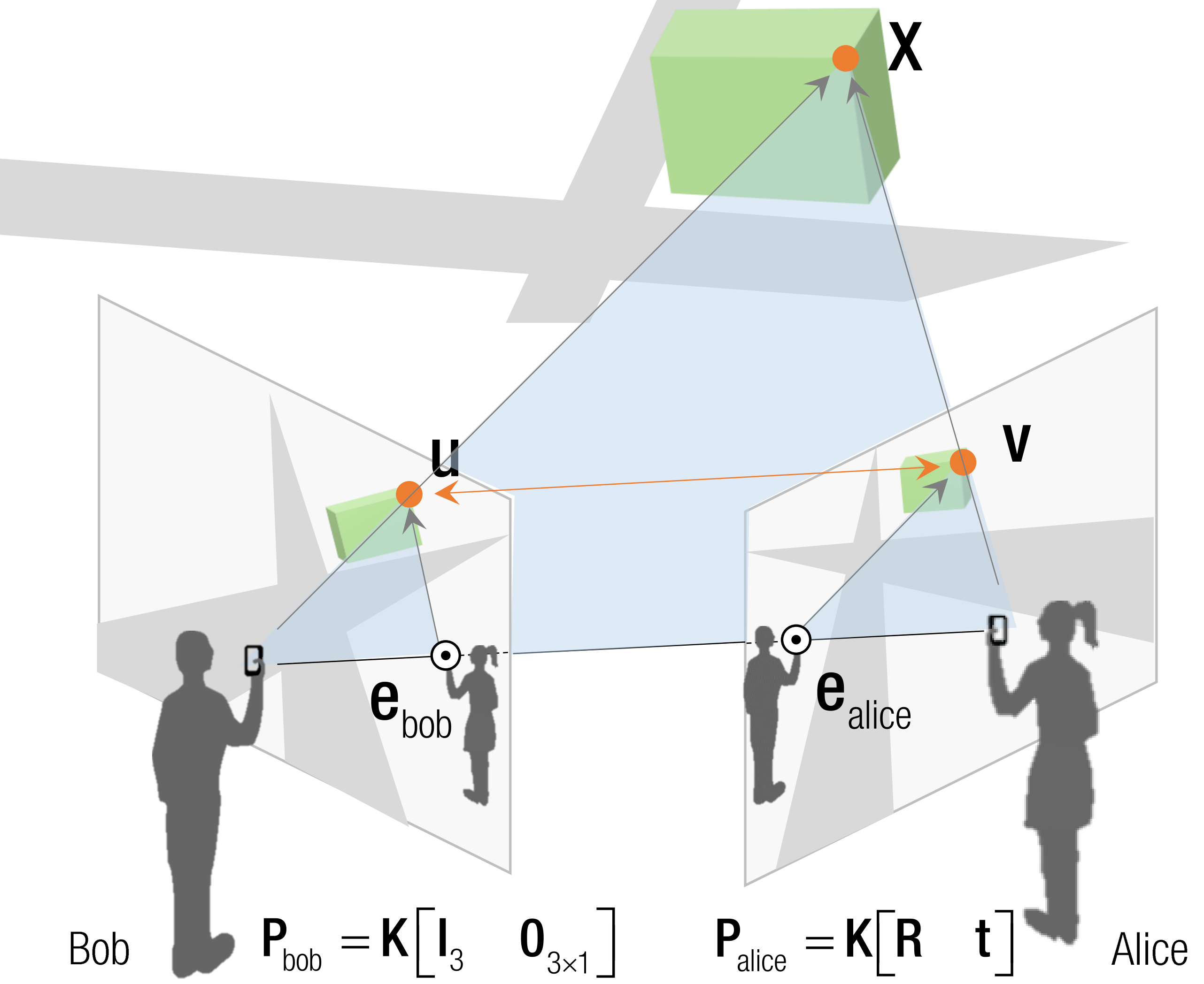
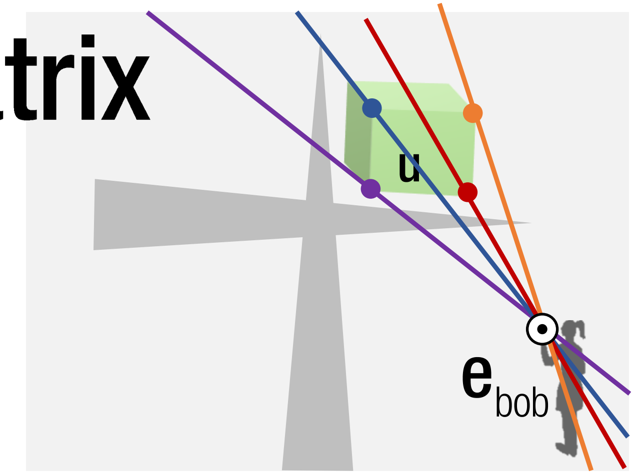
Camera Pose Estimation

Hyun Soo Park

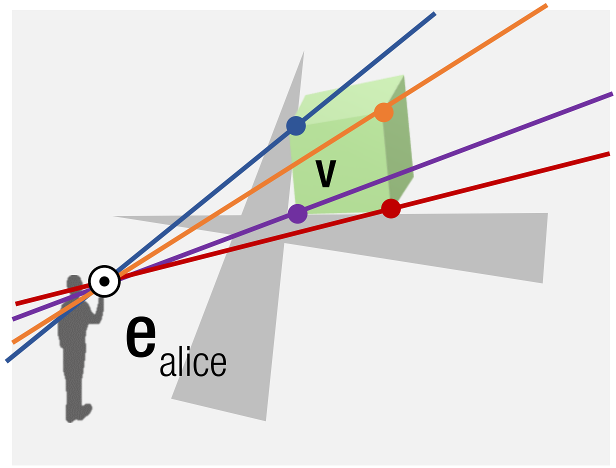
Recall: Fundamental Matrix



$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times 1} \end{bmatrix}$ $\quad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$



Bob's image



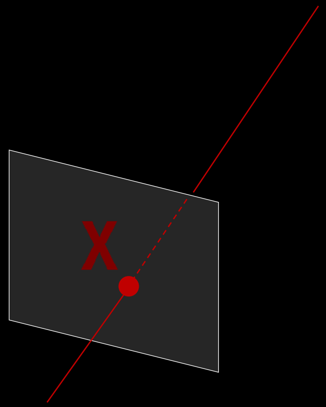
Alice's image

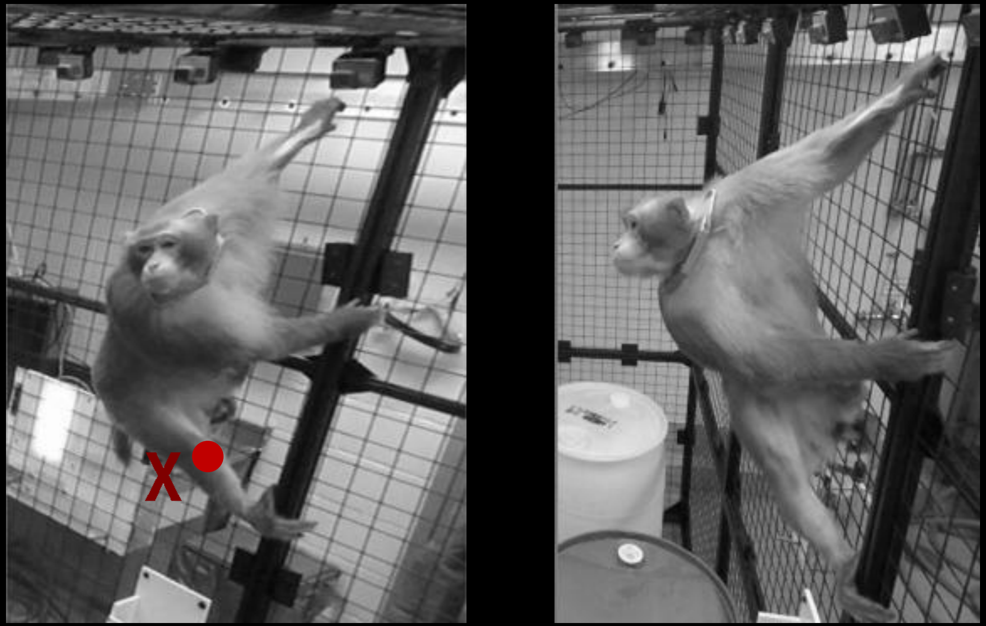
Properties of Fundamental Matrix

- Transpose: if $\mathbf{F}$ is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u} \quad \mathbf{l}_v = \mathbf{F}^T\mathbf{v}$
- Epipole: $\mathbf{F}\mathbf{e}_{bob} = \mathbf{0} \quad \mathbf{F}^T\mathbf{e}_{alice} = \mathbf{0}$
- rank($\mathbf{F}$)=2: degree of freedom 9 (3x3 matrix)-1 (scale)-1 (rank)=7

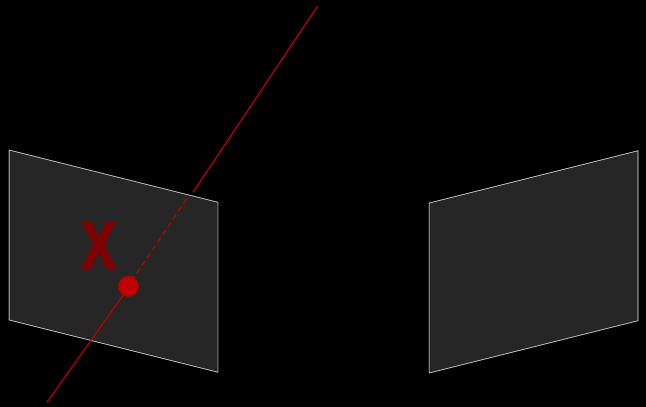
$$\mathbf{F} = \mathbf{K}^{-T}\underbrace{\left[\mathbf{t}\right]_\times}_{\text{rank 2 matrix}}\mathbf{R}\mathbf{K}^{-1}$$

Knee label

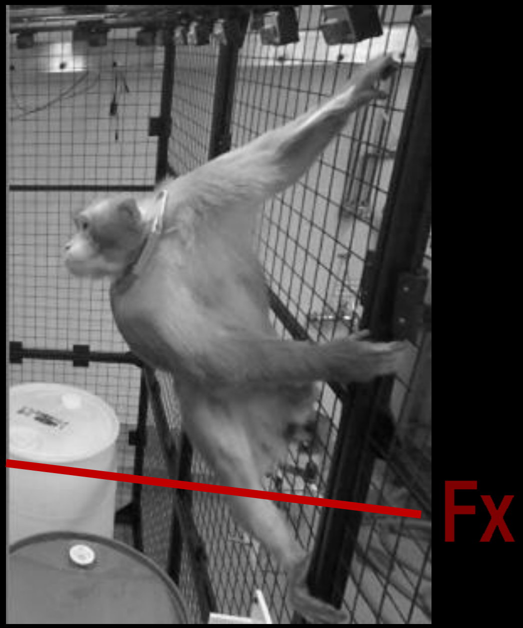




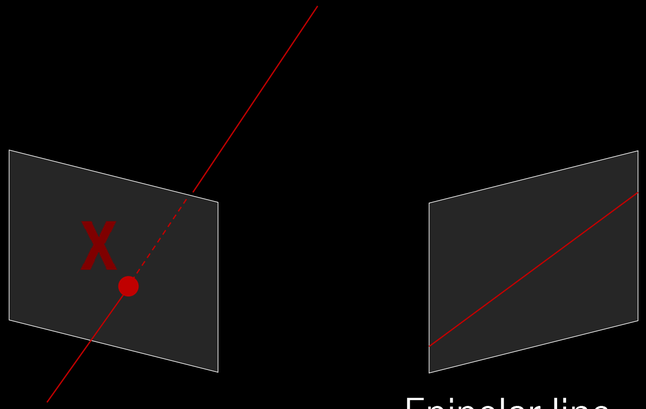
Knee label



Knee label



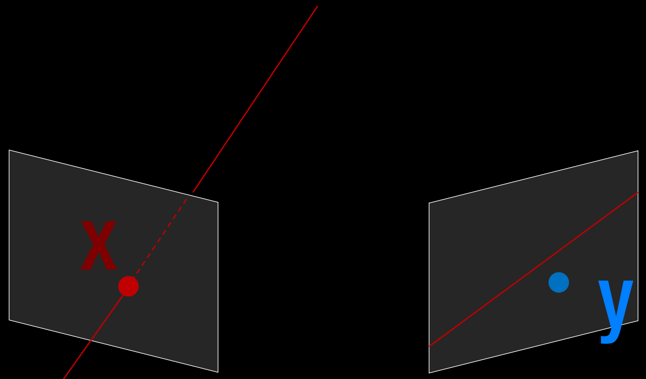
Transferred knee label



Epipolar line

Knee label

Transferred knee label

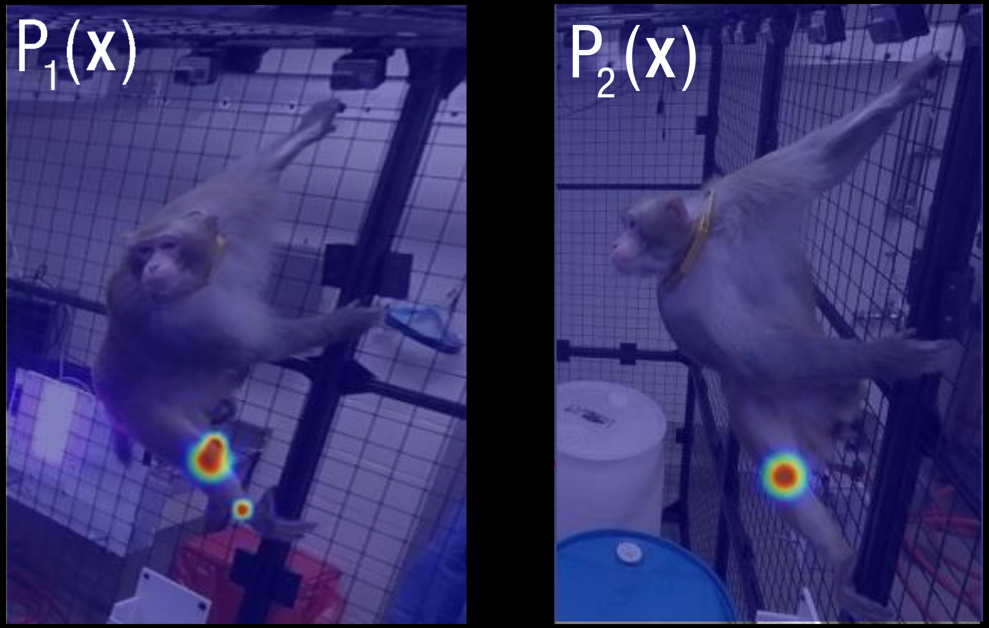


Epipolar line

Epipolar constraint

$$y^{T}Fx = 0$$

Knee inference



Epipolar constraint

$$y^T F x = 0$$

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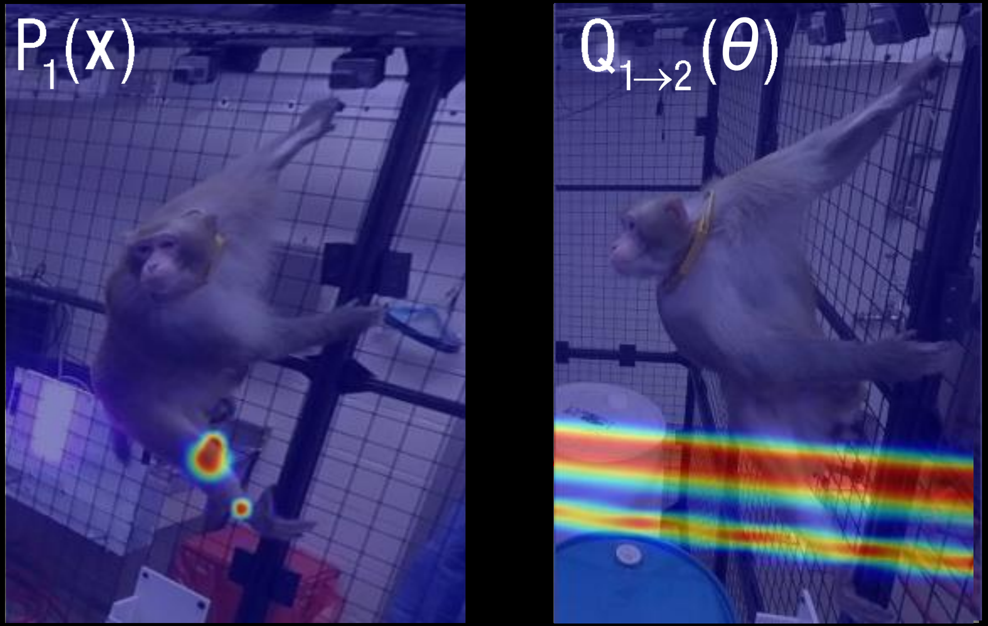
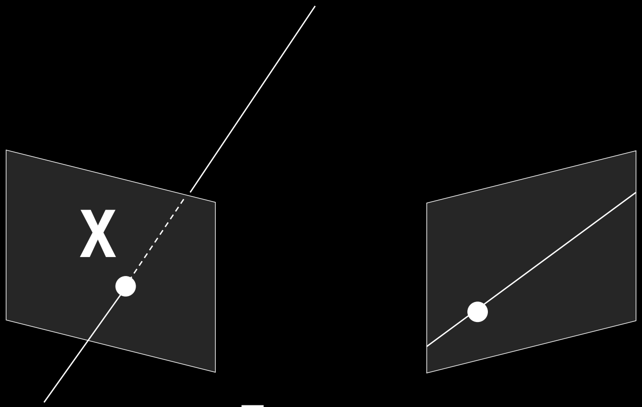
Knee inference



Epipolar constraint

$$y^T F x = 0$$

$$Q_{1\to 2}(\theta) = EP\big(P_1(x)\big) = \sup_{x \in \theta} P_1(x)$$



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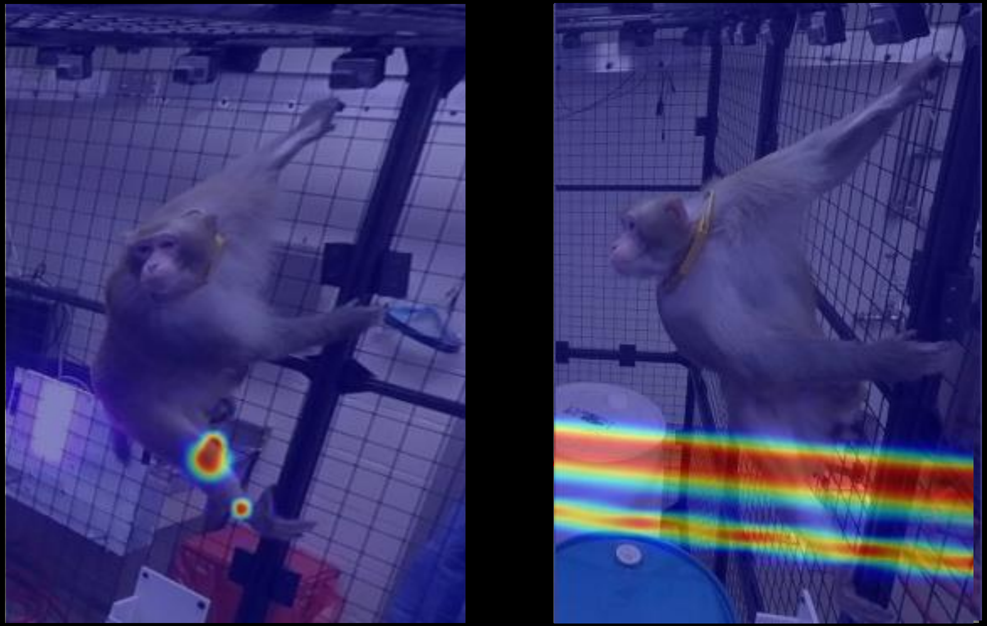
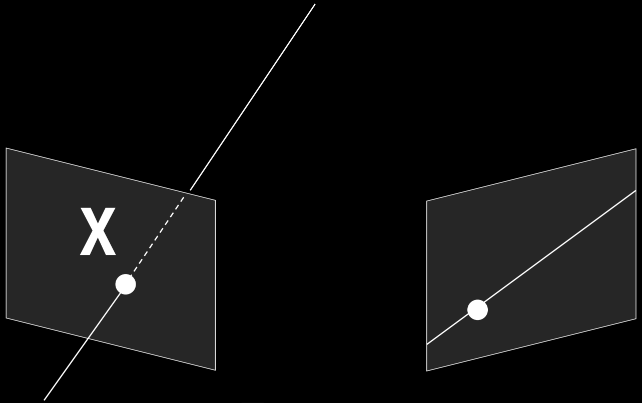
Knee inference



Knee inference

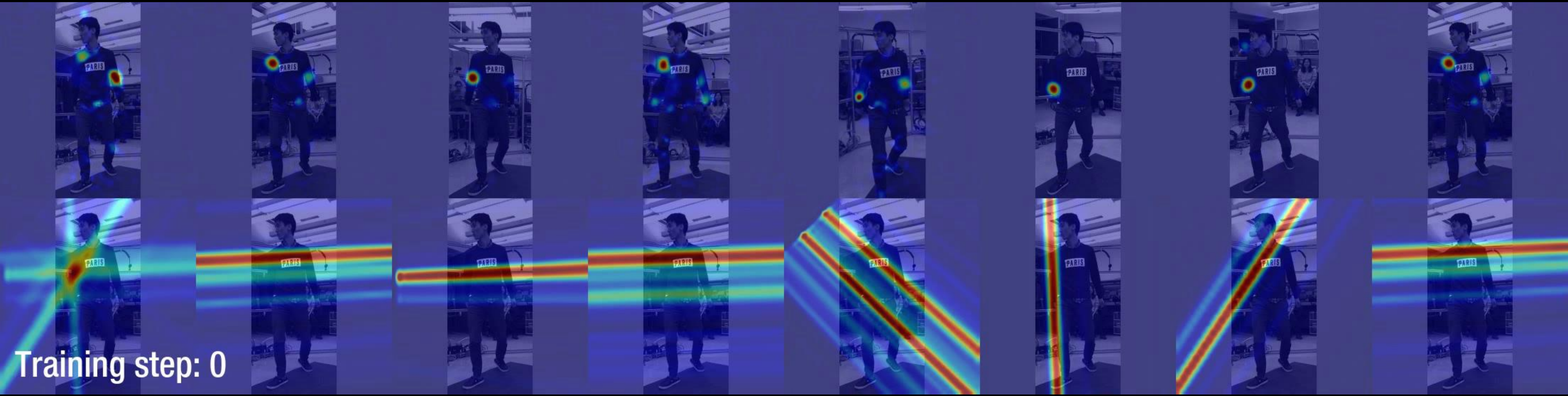
$$Q_{1\to 2}(\theta) = EP\left(P_1(\mathbf{x})\right) = \sup_{\mathbf{x}\in\theta} P_1(\mathbf{x})$$



$$y^{T}Fx = 0$$

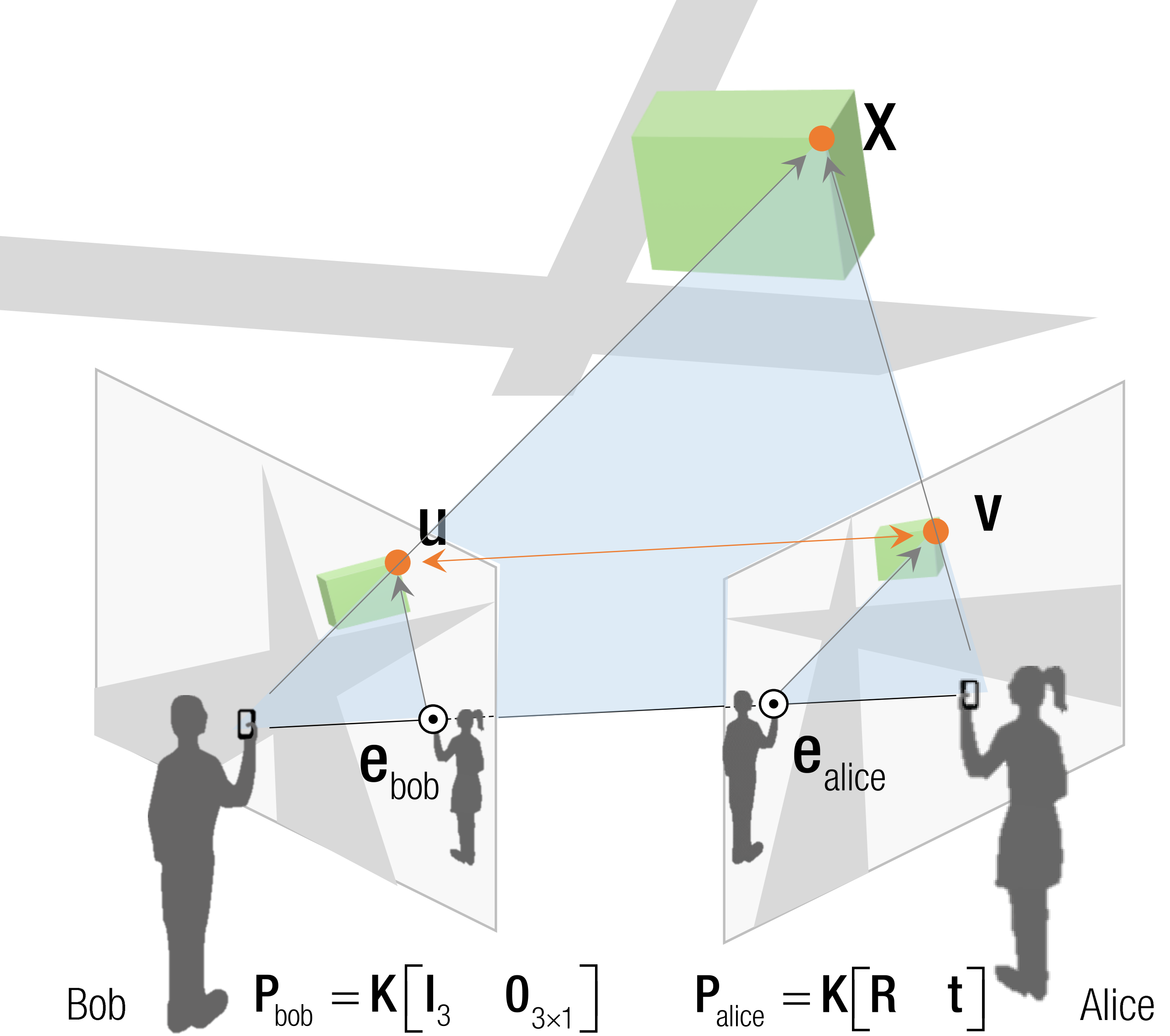
$$\mathcal{L}_{\text{multiview}} = D_{KL}\left(Q_{1\to 2}(\theta)\,||\,Q_2(\theta)\right)$$

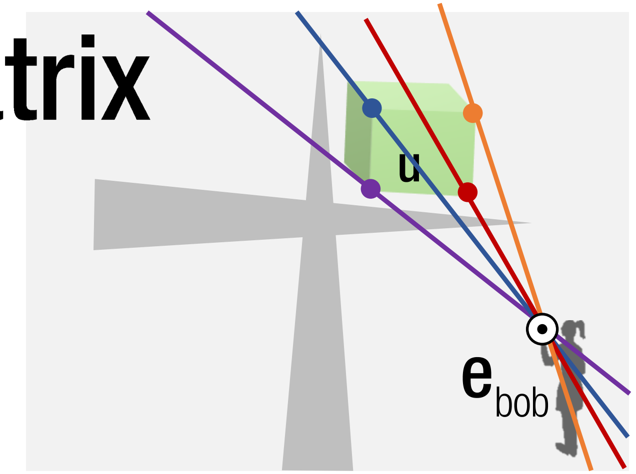
Left hand recognition



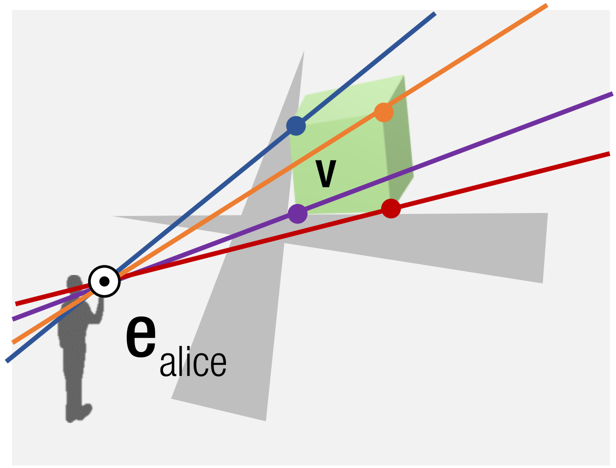
Left elbow recognition

Recall: Fundamental Matrix





Bob's image



Alice's image

Properties of Fundamental Matrix

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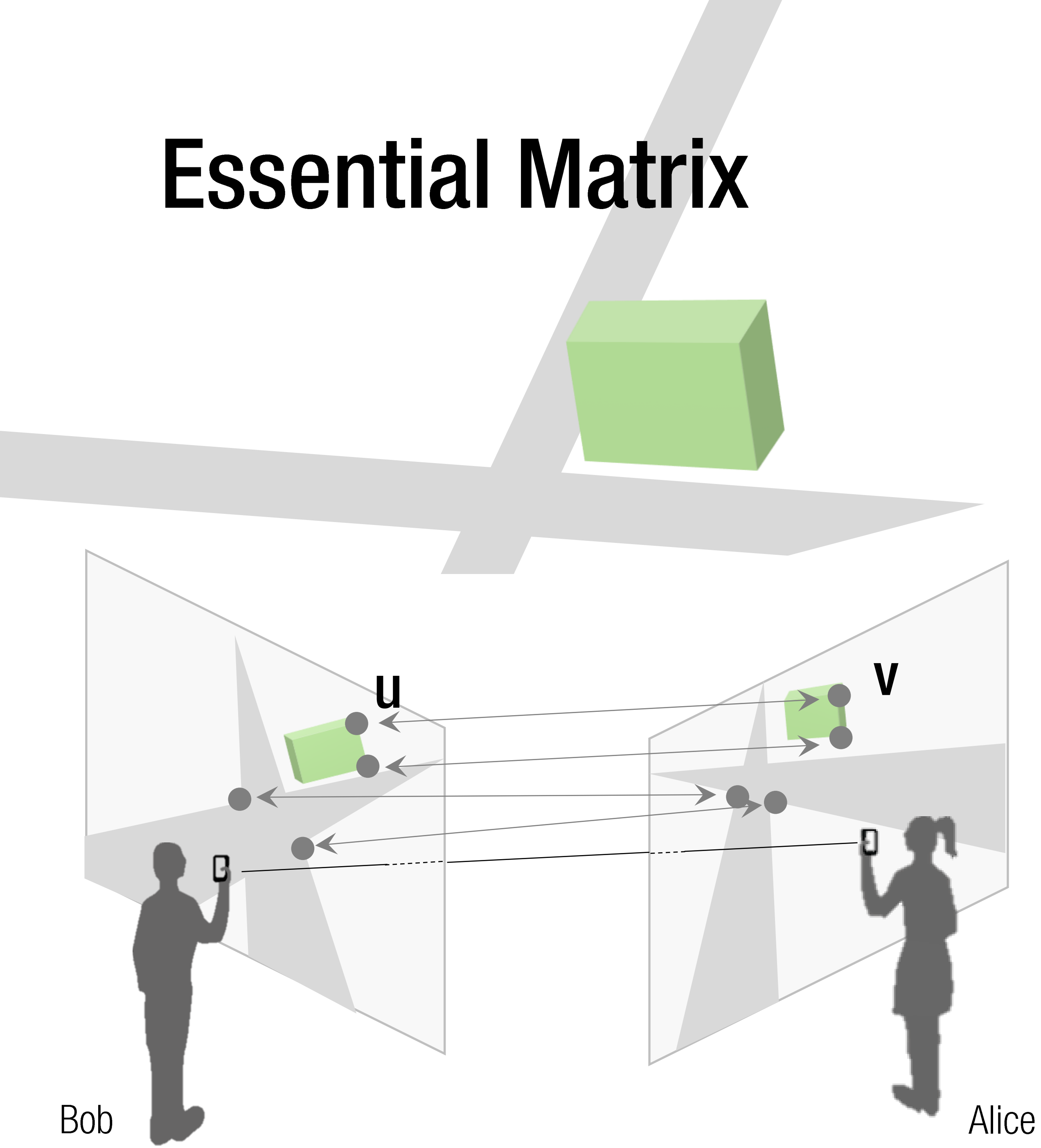
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u} \quad \mathbf{l}_v = \mathbf{F}^T\mathbf{v}$
- Epipole: $\mathbf{F}\mathbf{e}_{bob} = \mathbf{0} \quad \mathbf{F}^T\mathbf{e}_{alice} = \mathbf{0}$
- rank(**F**)=2: degree of freedom 9 (3x3 matrix)-1 (scale)-1 (rank)=7

$$\mathbf{F} = \mathbf{K}^{-T}\underbrace{\left[\mathbf{t}\right]_\times \mathbf{R}}_{\text{rank 2 matrix}}\mathbf{K}^{-1}$$

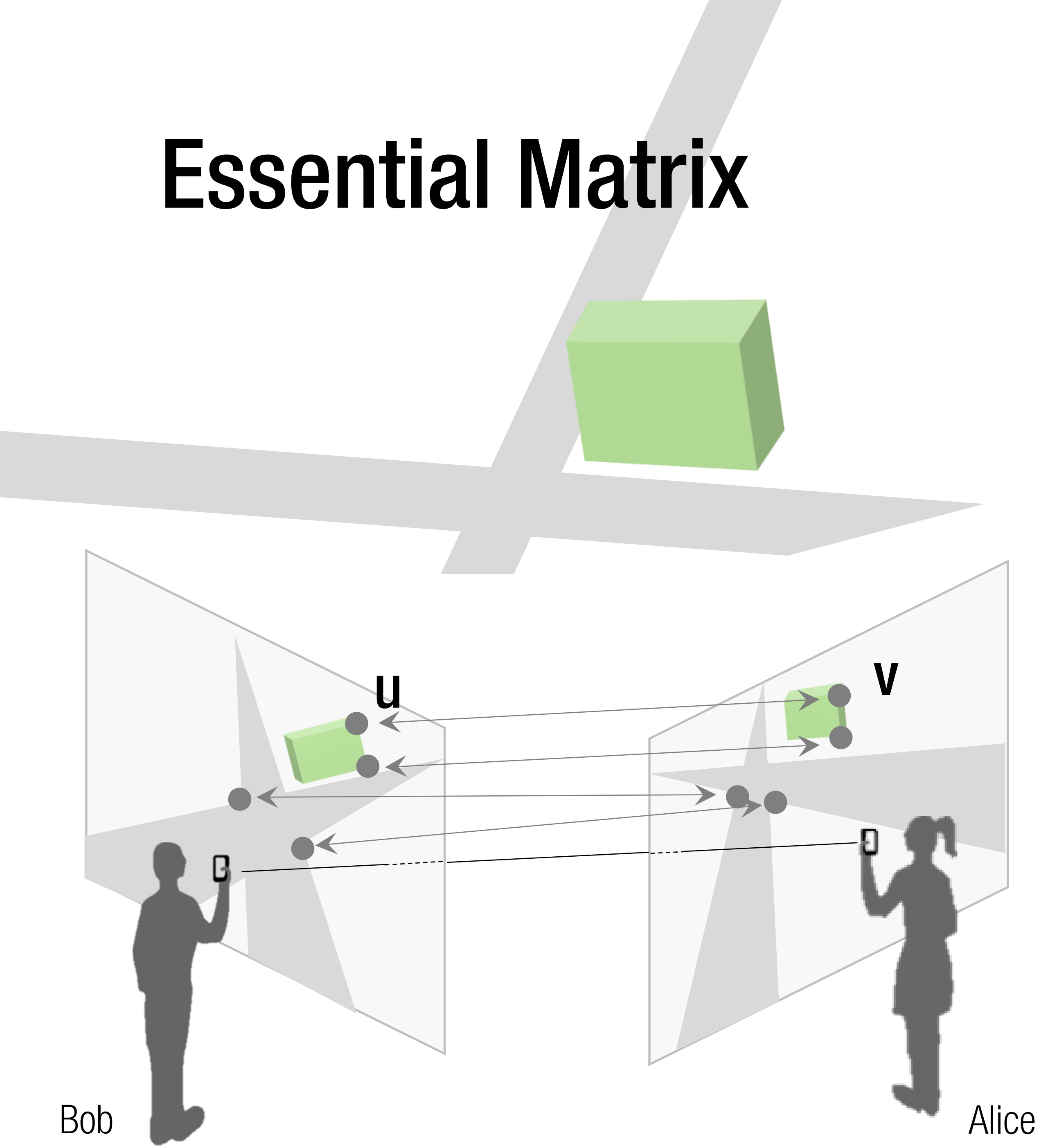
Essential Matrix



$$F = F(R, t)$$

$$= K^{-T} \left[t \right]_{\times} R K^{-1} = K^{-T} E K^{-1}$$

Essential Matrix



Essential Matrix:

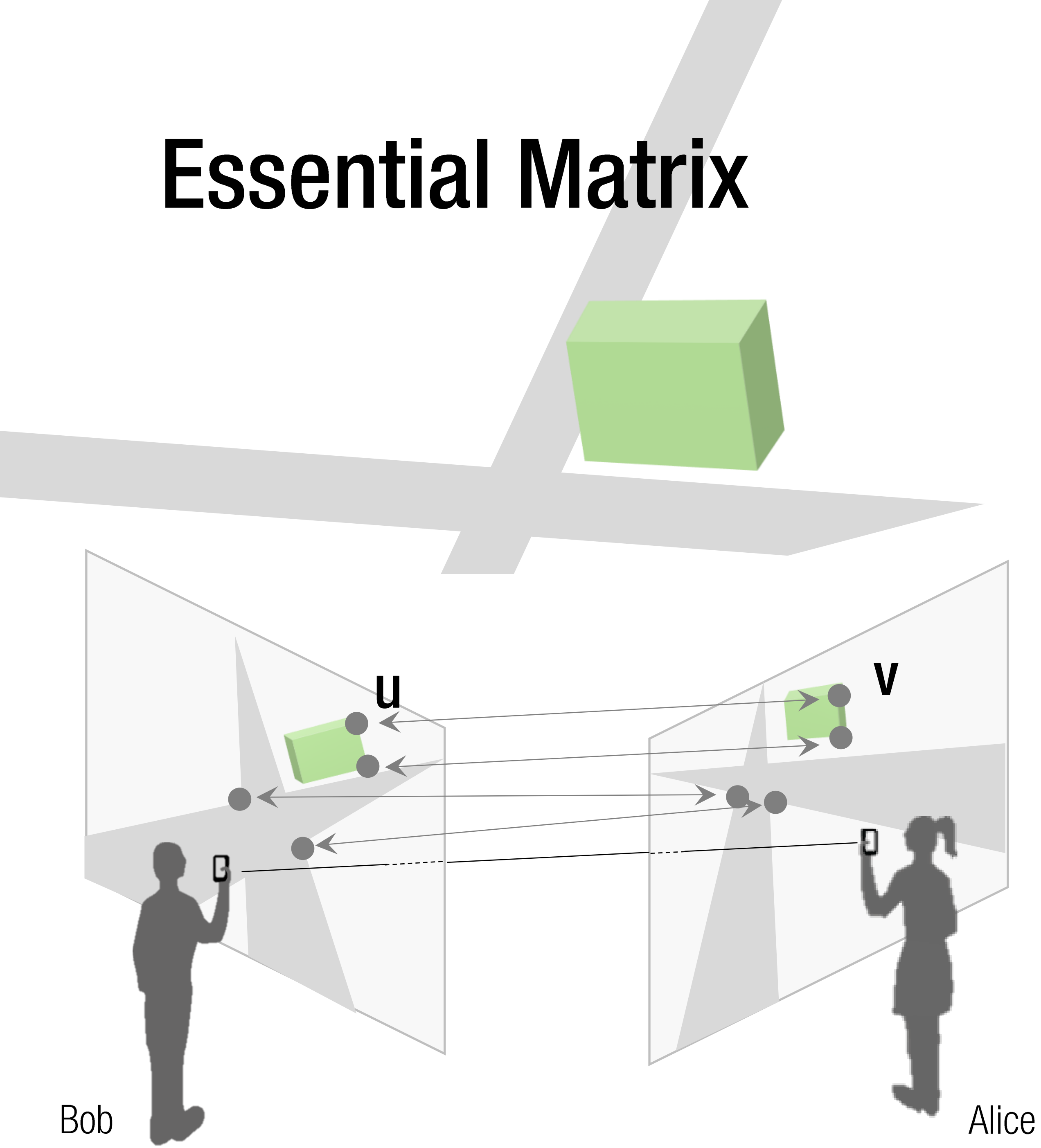
$$F = F(R,t)$$

$$= K^{-T}[t]_{\times}RK^{-1} = K^{-T}EK^{-1}$$

$$\longrightarrow E = \underline{K^{T}FK} \quad \text{where} \quad E = [t]_{\times}R$$

Calibrated fundamental matrix

Essential Matrix



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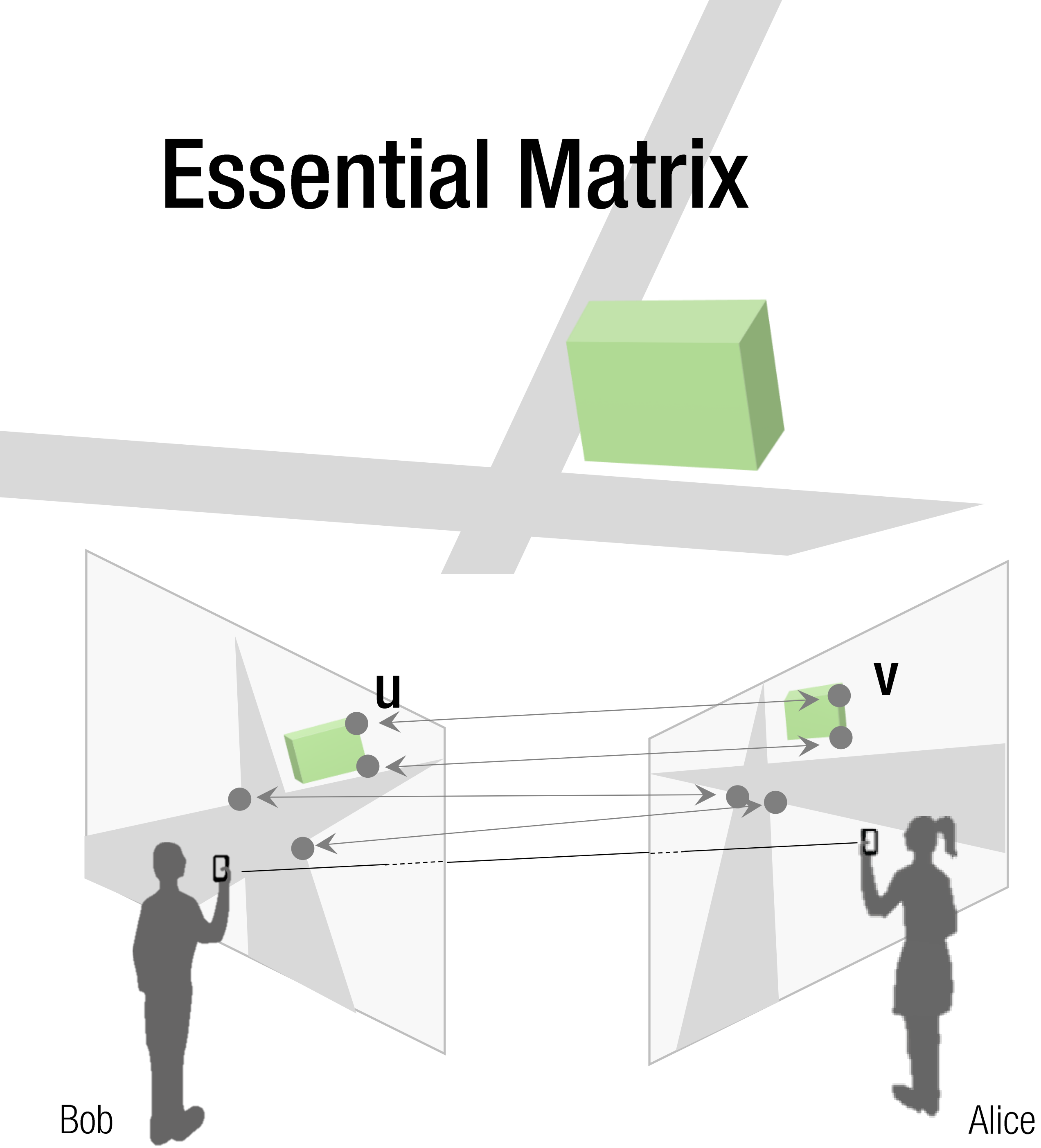
Calibrated fundamental matrix

Property of essential matrix:

$$E = UDV^{T} = U\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}V^{T}$$

Essential Matrix



Essential Matrix:

$$\mathbf{F} = \mathbf{F}(\mathbf{R},\mathbf{t})$$

$$= \mathbf{K}^{-T}\left[\mathbf{t}\right]_{\times}\mathbf{R}\mathbf{K}^{-1} = \mathbf{K}^{-T}\mathbf{E}\mathbf{K}^{-1}$$

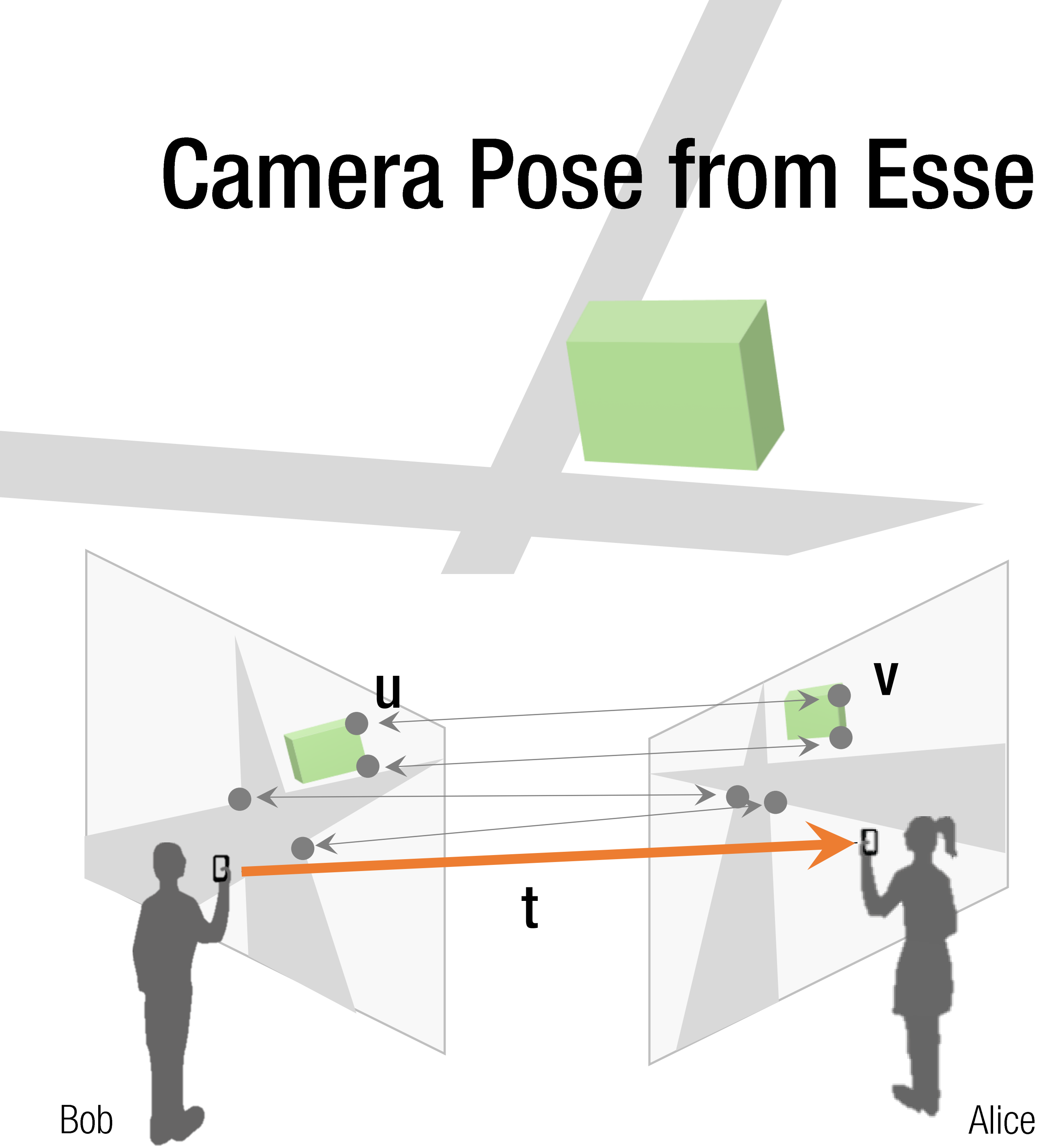
$$\rightarrow \mathbf{E} = \mathbf{K}^{T}\mathbf{F}\mathbf{K} \qquad \text{where} \quad \mathbf{E} = \left[\mathbf{t}\right]_{\times}\mathbf{R}$$

Calibrated fundamental matrix

Property of essential matrix:

$$\mathbf{E} = \mathbf{U}\mathbf{D}\mathbf{V}^{T} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^{T}$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

$$F = F(R, t)$$

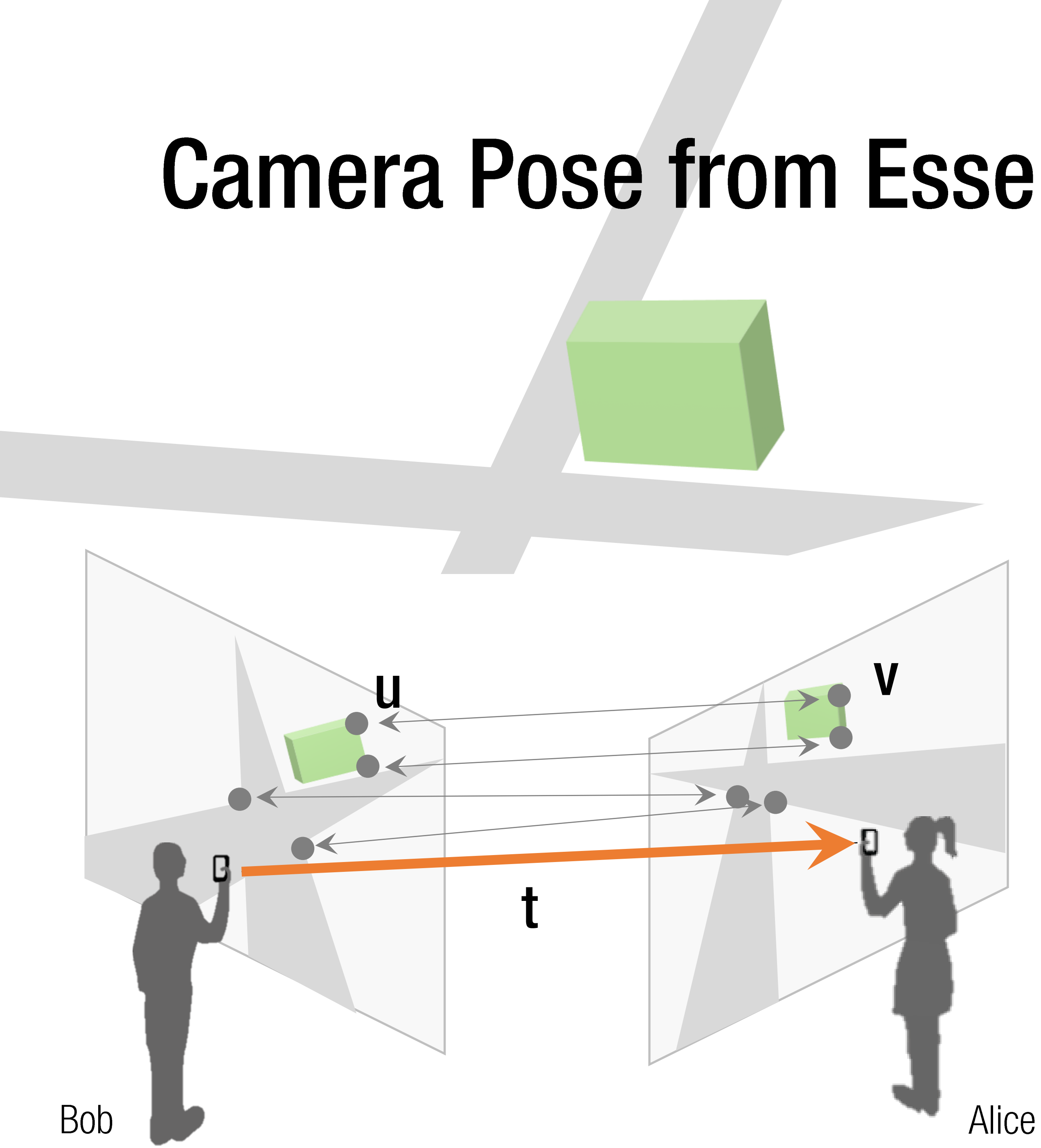
$$= K^{-T}[t]_{\times}RK^{-1} = K^{-T}EK^{-1}$$

$$\rightarrow E = \underline{K^{T}FK}$$ where $E = [t]_{\times}R$

Calibrated fundamental matrix

$$t =$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

$$F = F(R,t)$$

$$= K^{-T}[t]_{\times}RK^{-1} = K^{-T}EK^{-1}$$

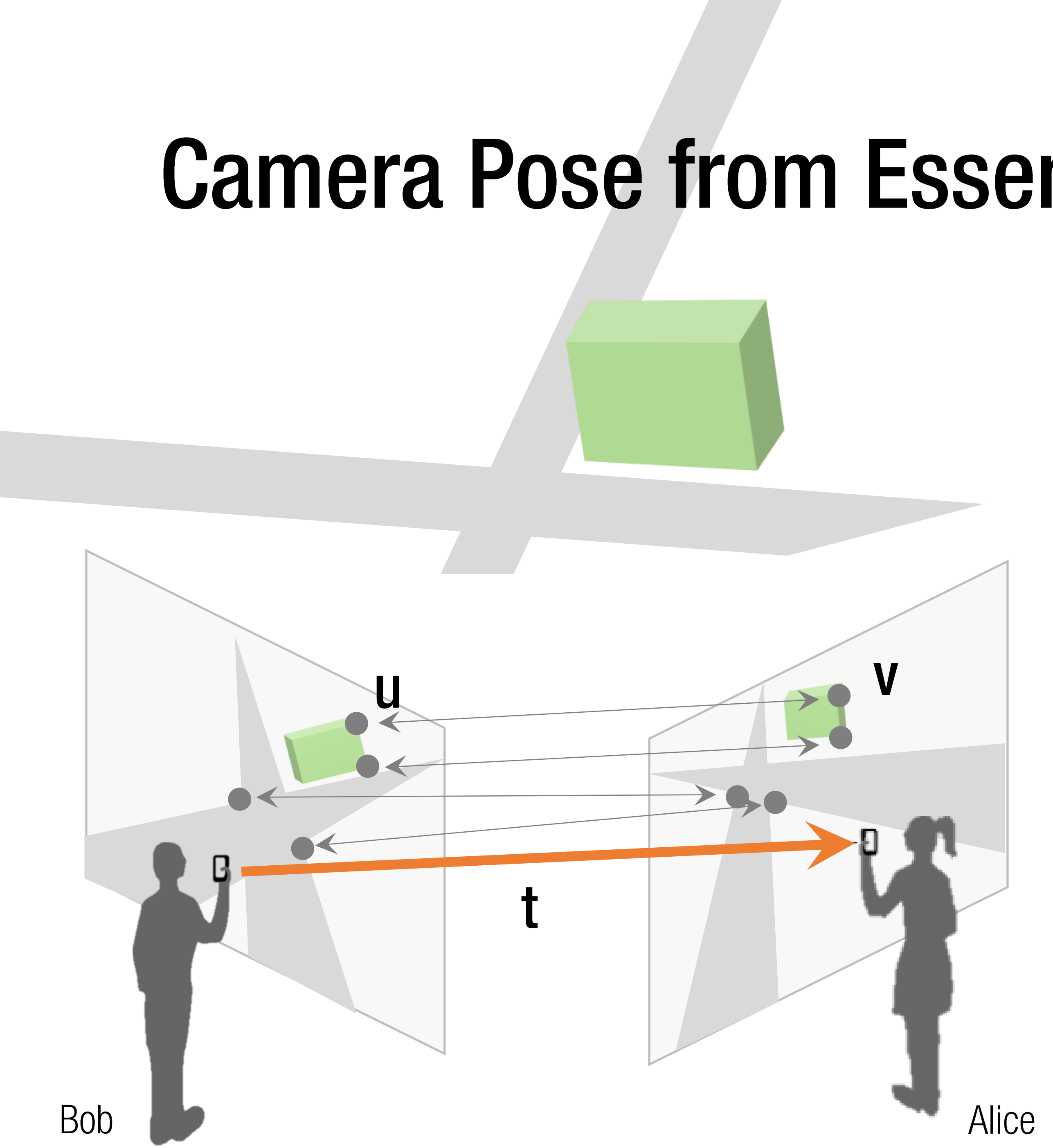
$$\longrightarrow E = \underline{K^{T}FK} \qquad \text{where} \quad E = [t]_{\times}R$$

Calibrated fundamental matrix

Left null space of E is translation vector, **t**:

$$\mathbf{t} =$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

$$\mathbf{F} = \mathbf{F}(\mathbf{R},\mathbf{t})$$

$$= \mathbf{K}^{-\mathrm{T}}[\mathbf{t}]_\times \mathbf{R}\mathbf{K}^{-1} = \mathbf{K}^{-\mathrm{T}}\mathbf{E}\mathbf{K}^{-1}$$

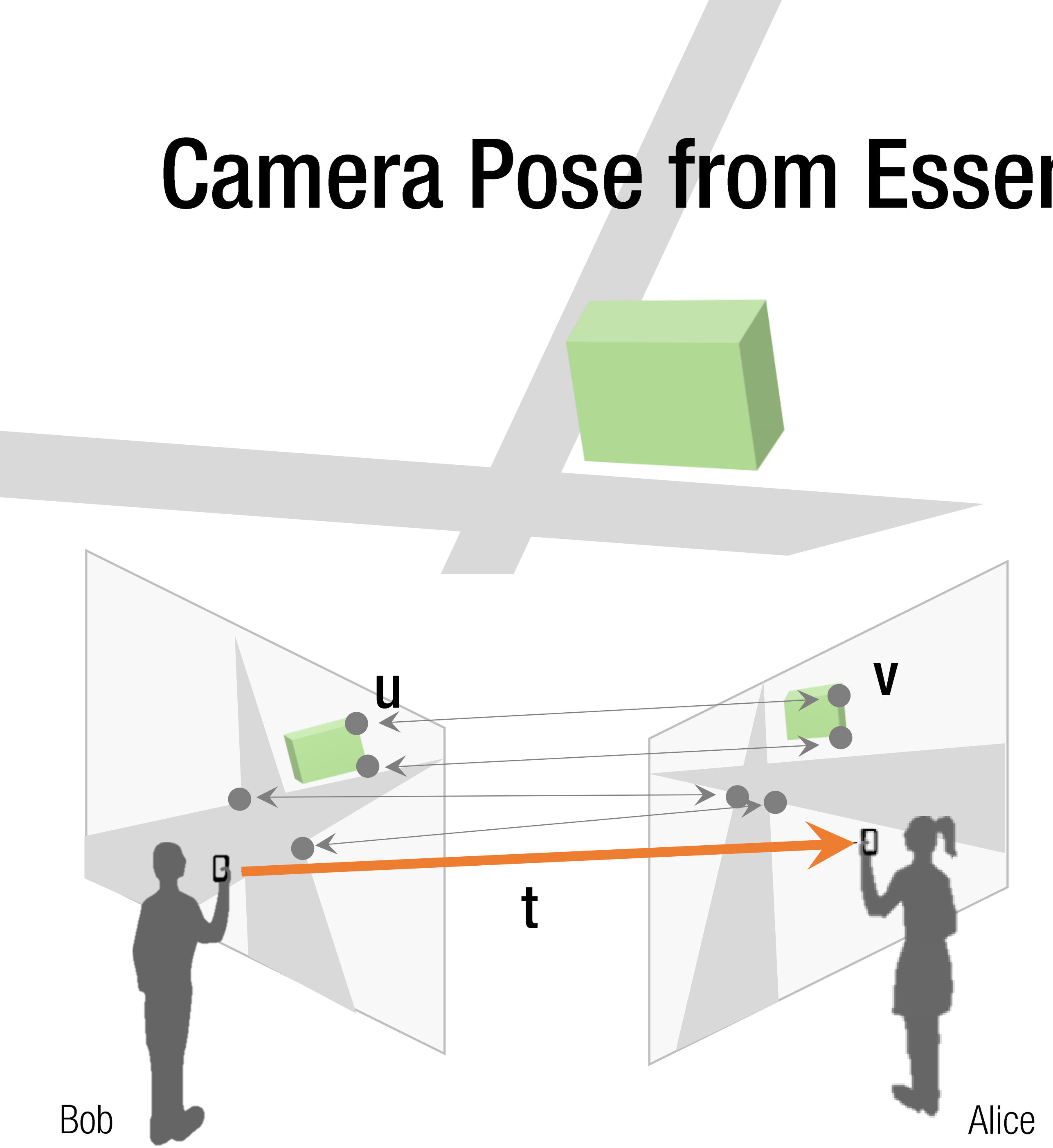
$$\longrightarrow \mathbf{E} = \underline{\mathbf{K}^{\mathrm{T}}\mathbf{F}\mathbf{K}} \quad \text{where} \quad \mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$$

Calibrated fundamental matrix

Left null space of E is translation vector, **t**:

$$\mathbf{t} = \pm\text{null}(\mathbf{E}^{\mathrm{T}}) = \pm\text{null}\left(\left([\mathbf{t}]_\times \mathbf{R}\right)^{\mathrm{T}}\right)$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

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$$\longrightarrow E = \underline{K^{T}FK} \quad \text{where} \quad E = [t]_\times R$$

Calibrated fundamental matrix

Left null space of E is translation vector, **t**:

$$\mathbf{t} = \pm \mathrm{null}(E^T) = \pm \mathrm{null}\left(\left([t]_\times R\right)^T\right)$$

$$\because t^T [t]_\times R = -\left([t]_\times t\right)^T R = -\underline{\left(t \times t\right)^T} R = 0$$

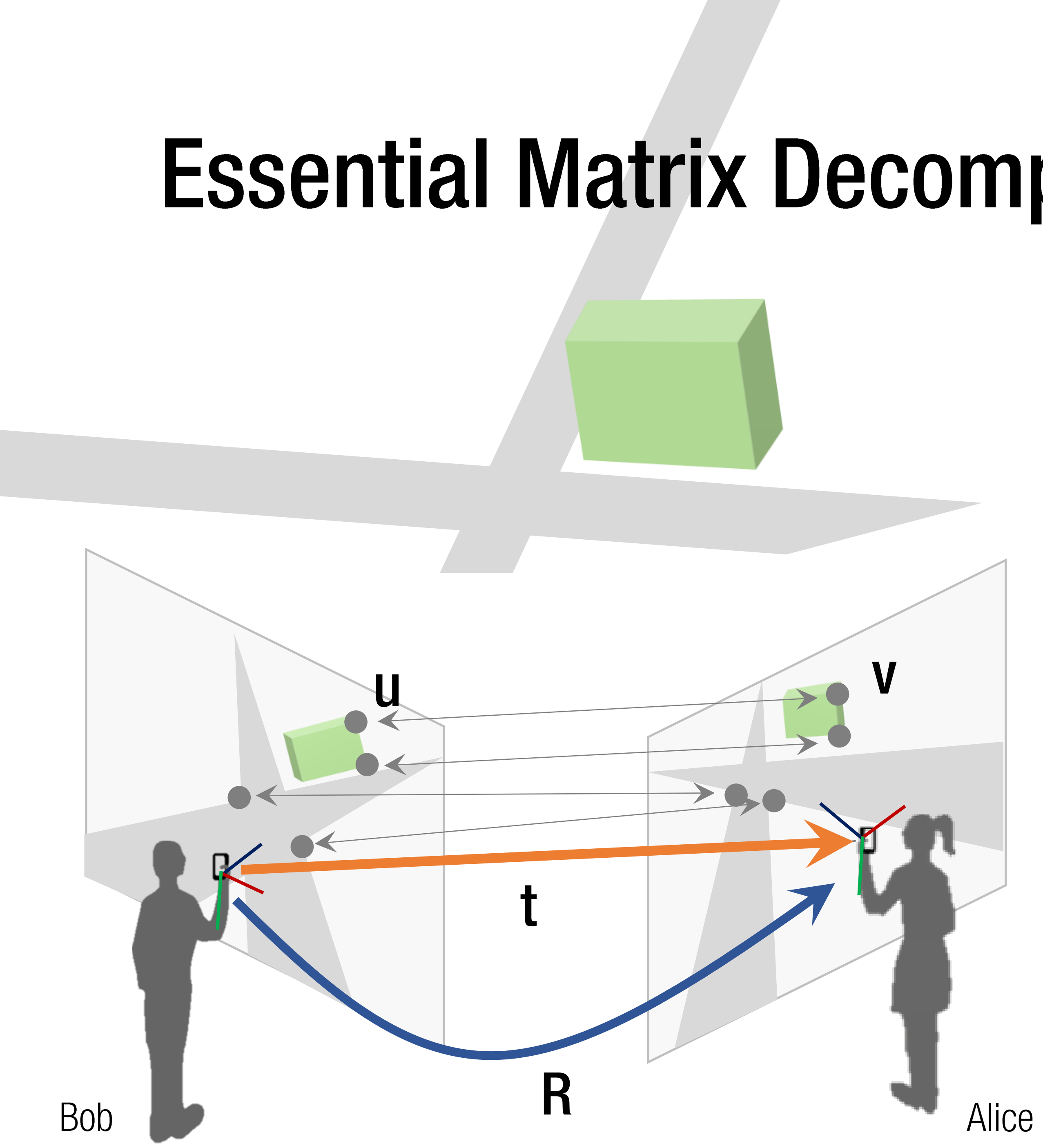
Self-cross product

Essential Matrix Decomposition

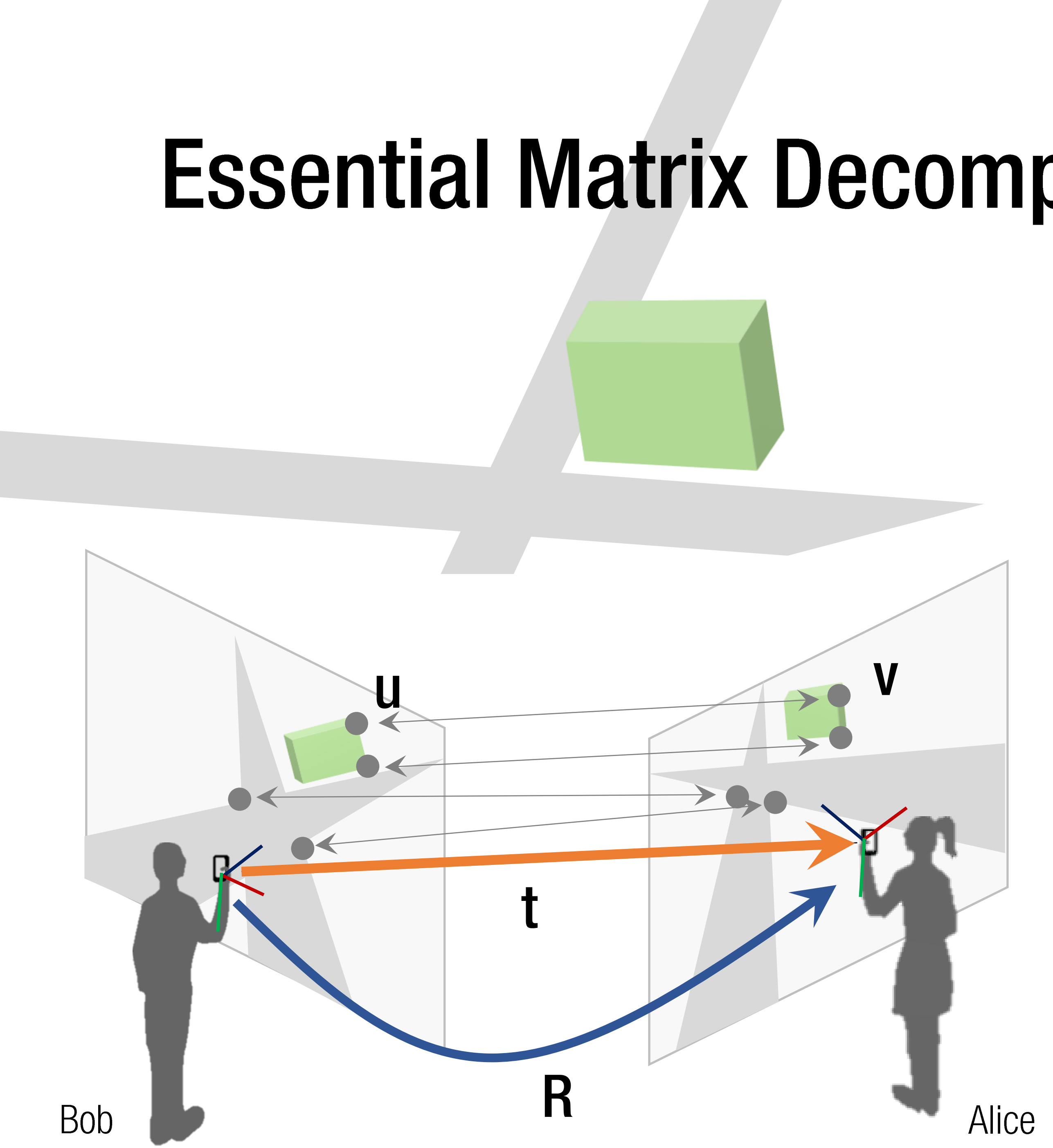
Left null space of E is translation vector, **t**:

$$\mathbf{t} = \text{null}(\mathbf{E}^{\mathsf{T}}) = \text{null}\left(\left([\mathbf{t}]_{\times}\mathbf{R}\right)^{\mathsf{T}}\right)$$

Can I invert $[\mathbf{t}]_{\times}$?



Essential Matrix Decomposition



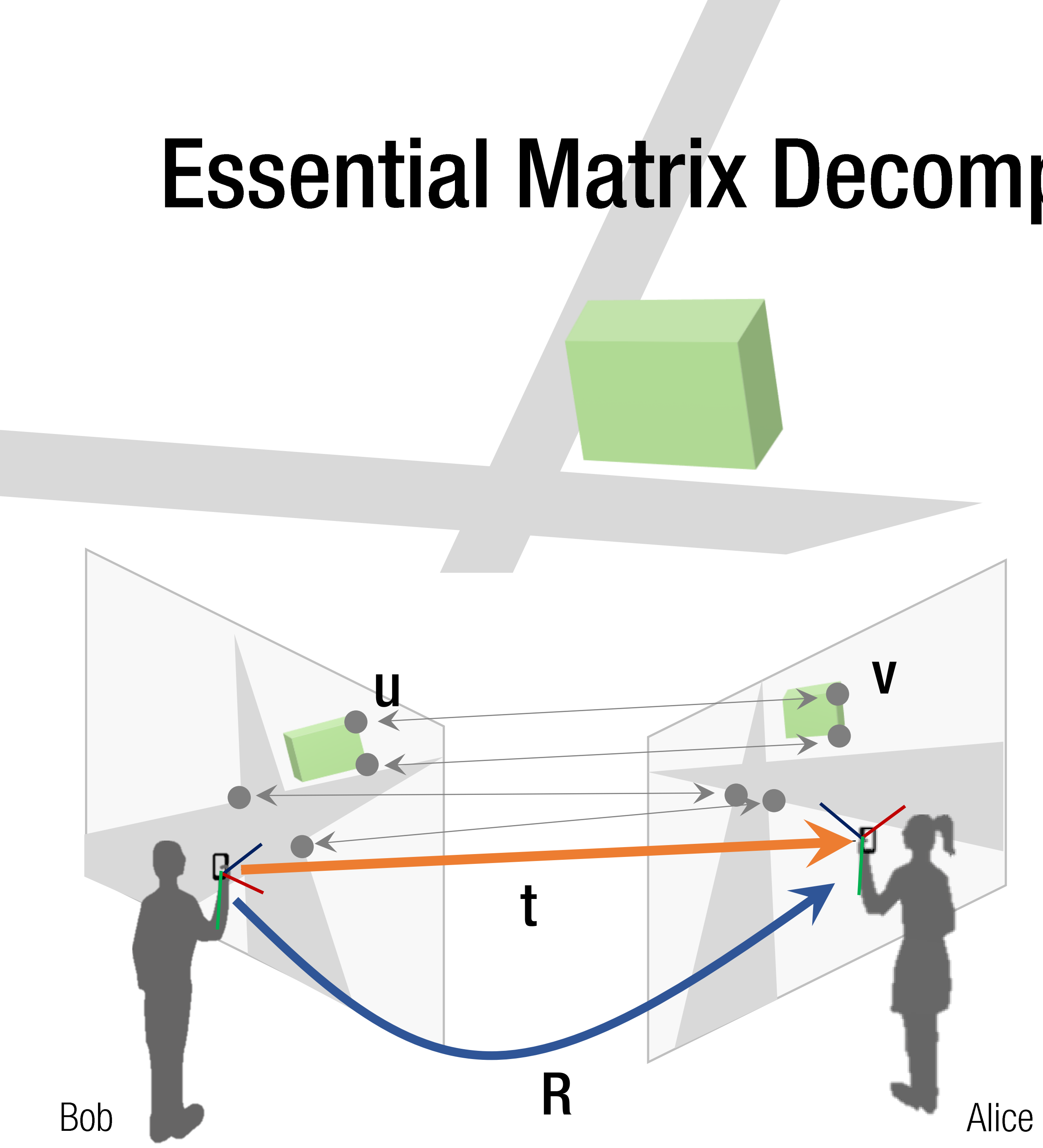
Left null space of E is translation vector, **t**:

$$\mathbf{t} = \mathrm{null}(\mathbf{E}^{\mathsf{T}}) = \mathrm{null}\left(\left([\mathbf{t}]_{\times}\mathbf{R}\right)^{\mathsf{T}}\right)$$

$$\longrightarrow \quad \mathbf{t} = \mathbf{u}_3 \qquad \text{where} \quad \mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}$$

$$\mathbf{E} = \mathbf{UDV}^{\mathsf{T}} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^{\mathsf{T}}$$

Essential Matrix Decomposition



Left null space of E is translation vector, **t**:

$$\mathbf{t} = \text{null}(\mathbf{E}^{\mathsf{T}}) = \text{null}\left(\left([\mathbf{t}]_{\times}\mathbf{R}\right)^{\mathsf{T}}\right)$$

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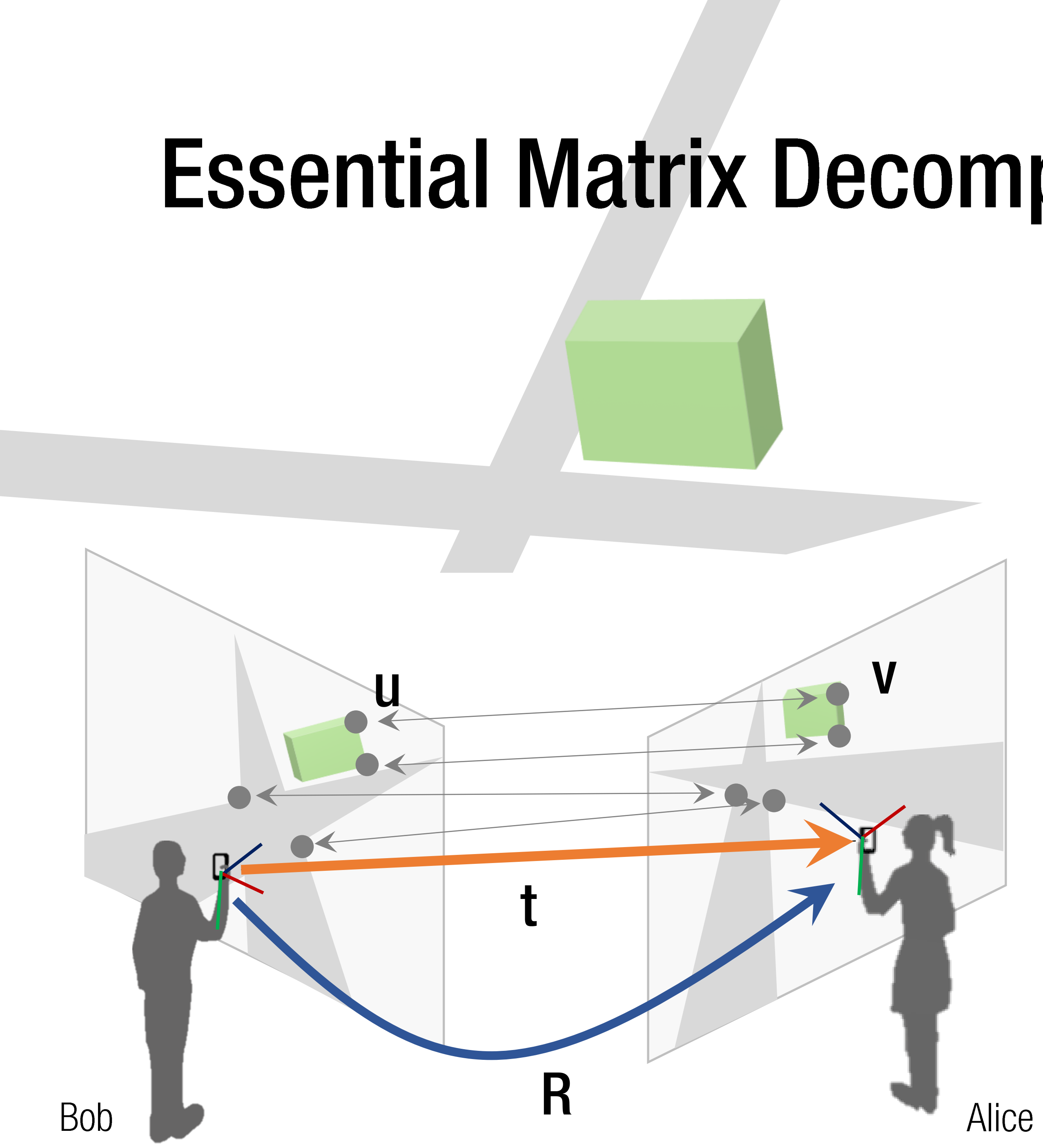
$$\mathbf{E} = \mathbf{UDV}^{\mathsf{T}} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^{\mathsf{T}}$$

$\longrightarrow$ $\mathbf{t} = \mathbf{u}_1 \times \mathbf{u}_2$ (orthogonal matrix, **U**)

Essential Matrix Decomposition



Left null space of E is translation vector, **t**:

$$\mathbf{t} = \text{null}(\mathbf{E}^T) = \text{null}\left(\left([\mathbf{t}]_\times \mathbf{R}\right)^T\right)$$

$\longrightarrow$ $\mathbf{t} = \mathbf{u}_3$ where $\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}$

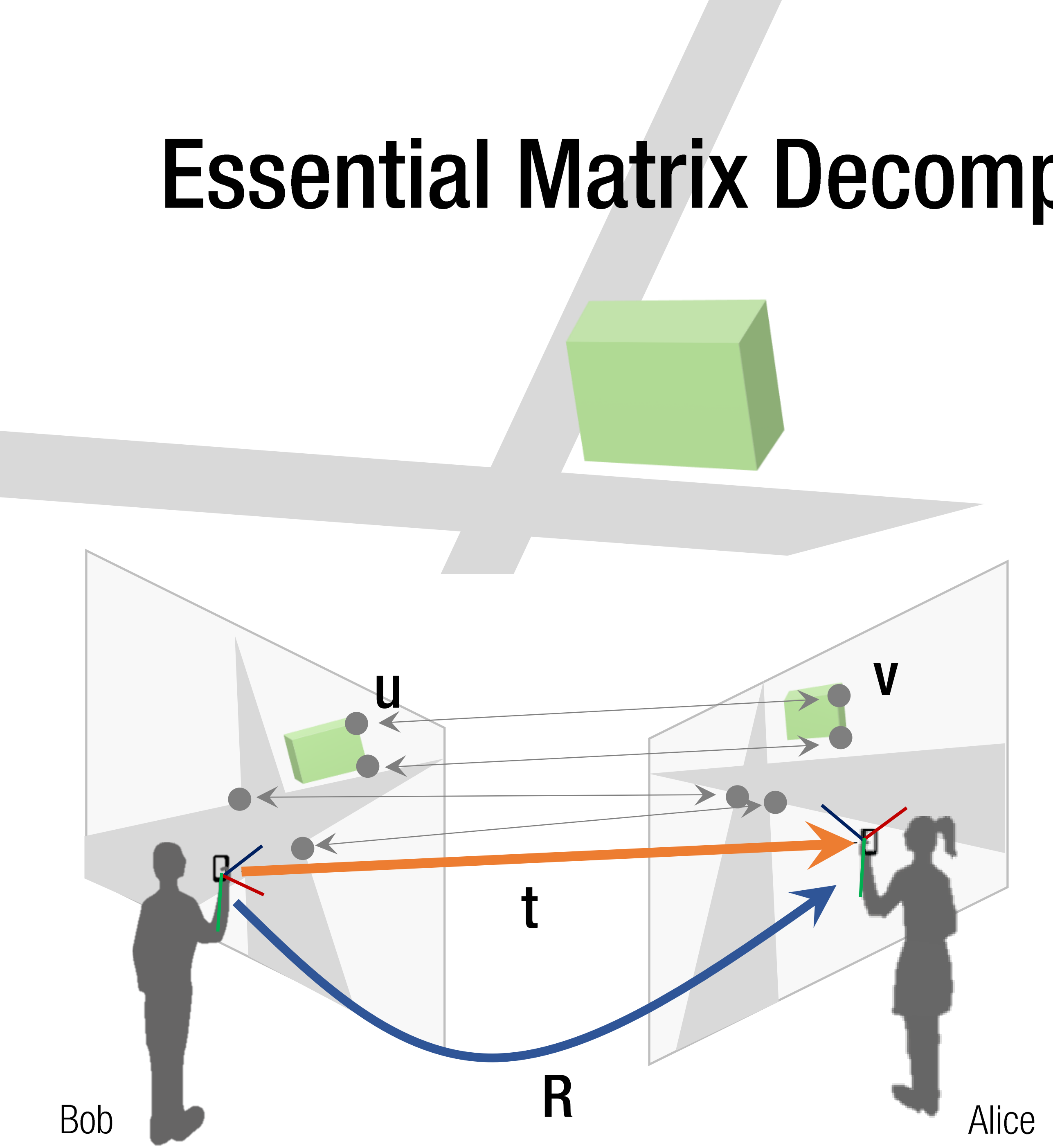
$$\mathbf{E} = \mathbf{U}\mathbf{D}\mathbf{V}^T = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^T$$

$\longrightarrow$ $\mathbf{t} = \mathbf{u}_1 \times \mathbf{u}_2$ (orthogonal matrix, **U**)

$$[\mathbf{t}]_\times = [\mathbf{u}_1 \times \mathbf{u}_2]_\times = \mathbf{u}_2\mathbf{u}_1^T - \mathbf{u}_1\mathbf{u}_2^T$$

:

Essential Matrix Decomposition



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$$\mathbf{t} = \text{null}(\mathbf{E}^T) = \text{null}\left(\left([\mathbf{t}]_\times \mathbf{R}\right)^T\right)$$

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$$\mathbf{E} = \mathbf{UDV}^T = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^T$$

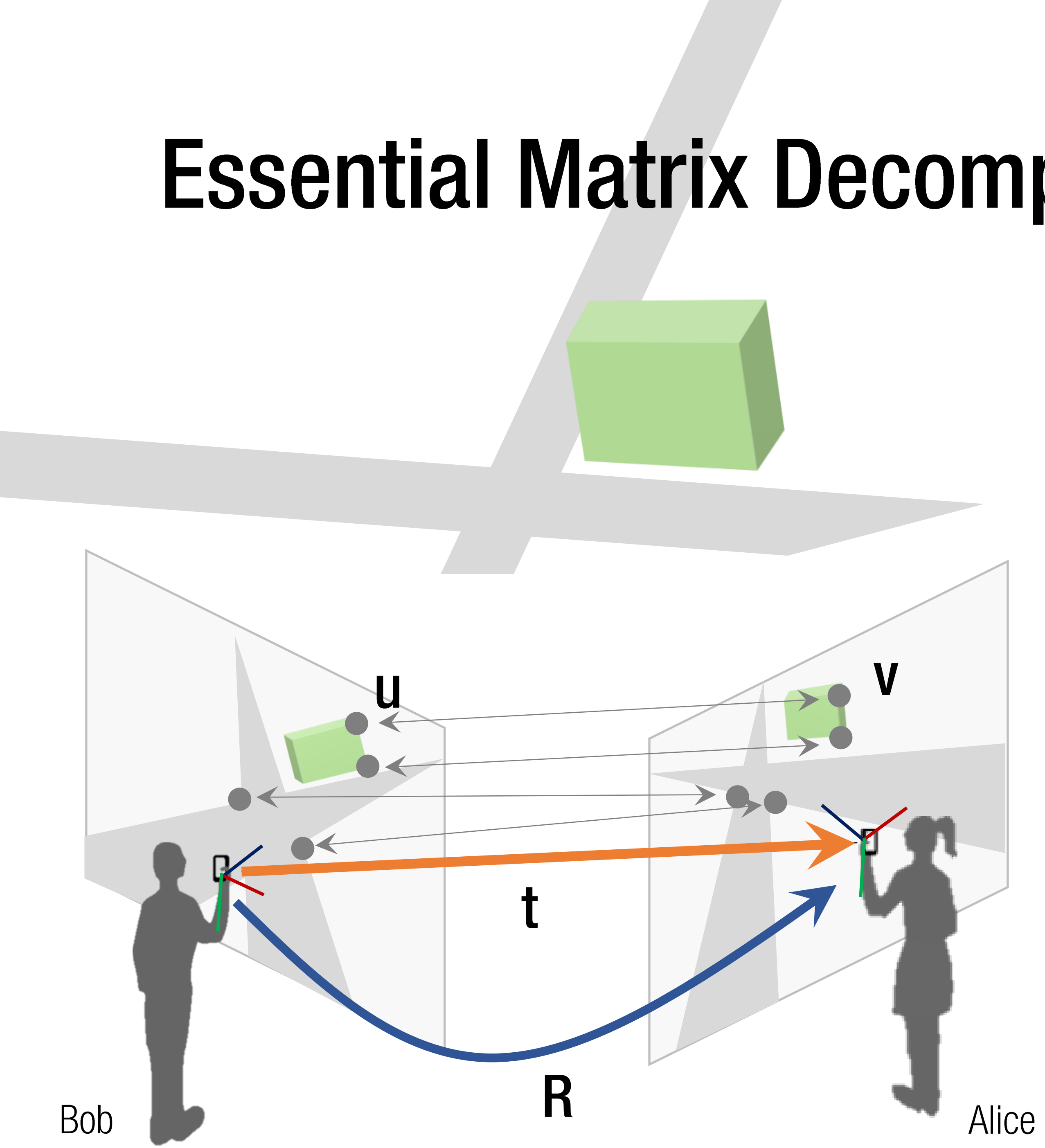
$\longrightarrow$ $\mathbf{t} = \mathbf{u}_1 \times \mathbf{u}_2$ (orthogonal matrix, **U**)

$$[\mathbf{t}]_\times = [\mathbf{u}_1 \times \mathbf{u}_2]_\times = \mathbf{u}_2\mathbf{u}_1^T - \mathbf{u}_1\mathbf{u}_2^T$$

$$= \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^T$$

Prove!

Essential Matrix Decomposition



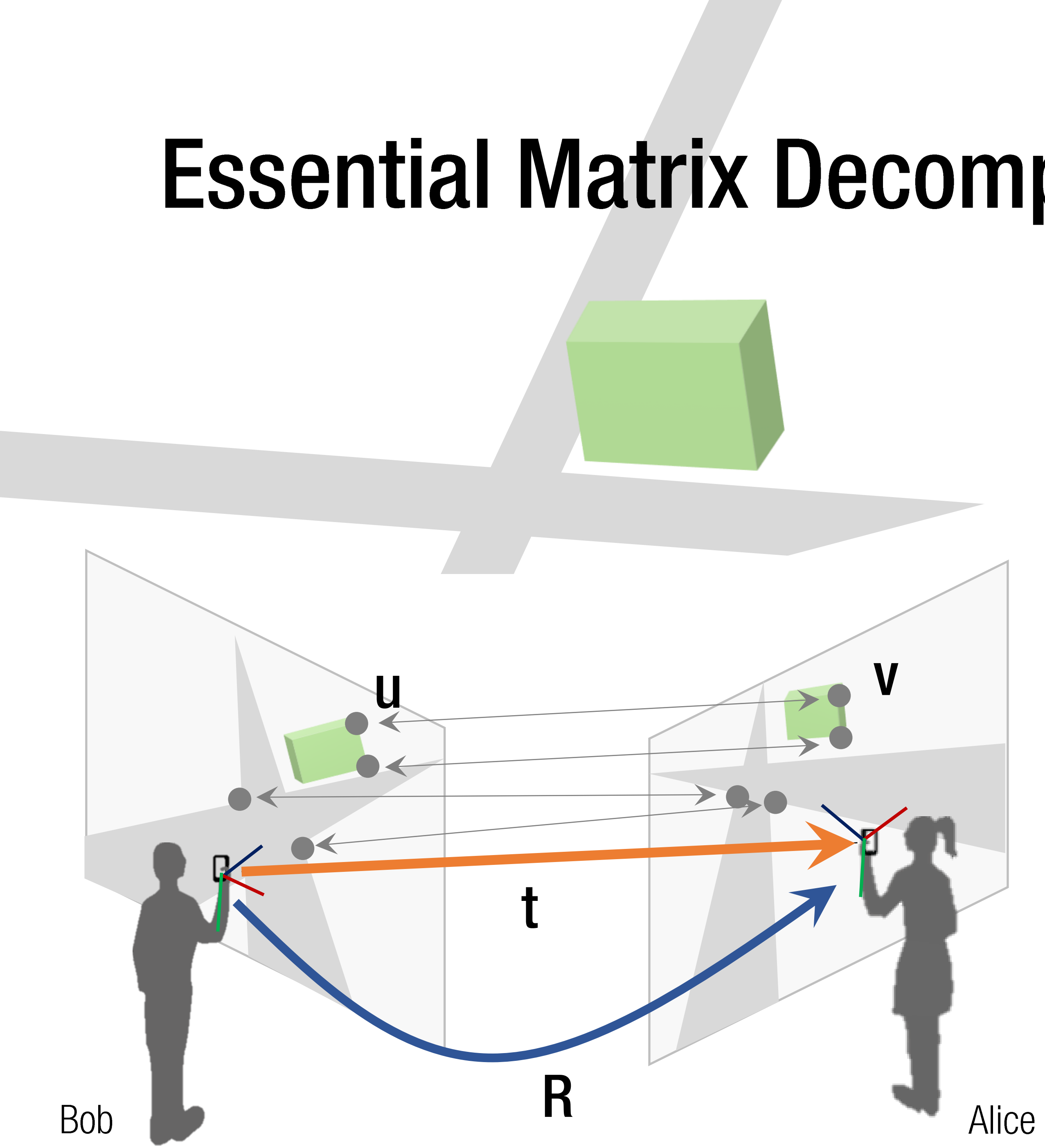
$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^T\mathbf{R}$$

where $\mathbf{R} \in SO(3)$

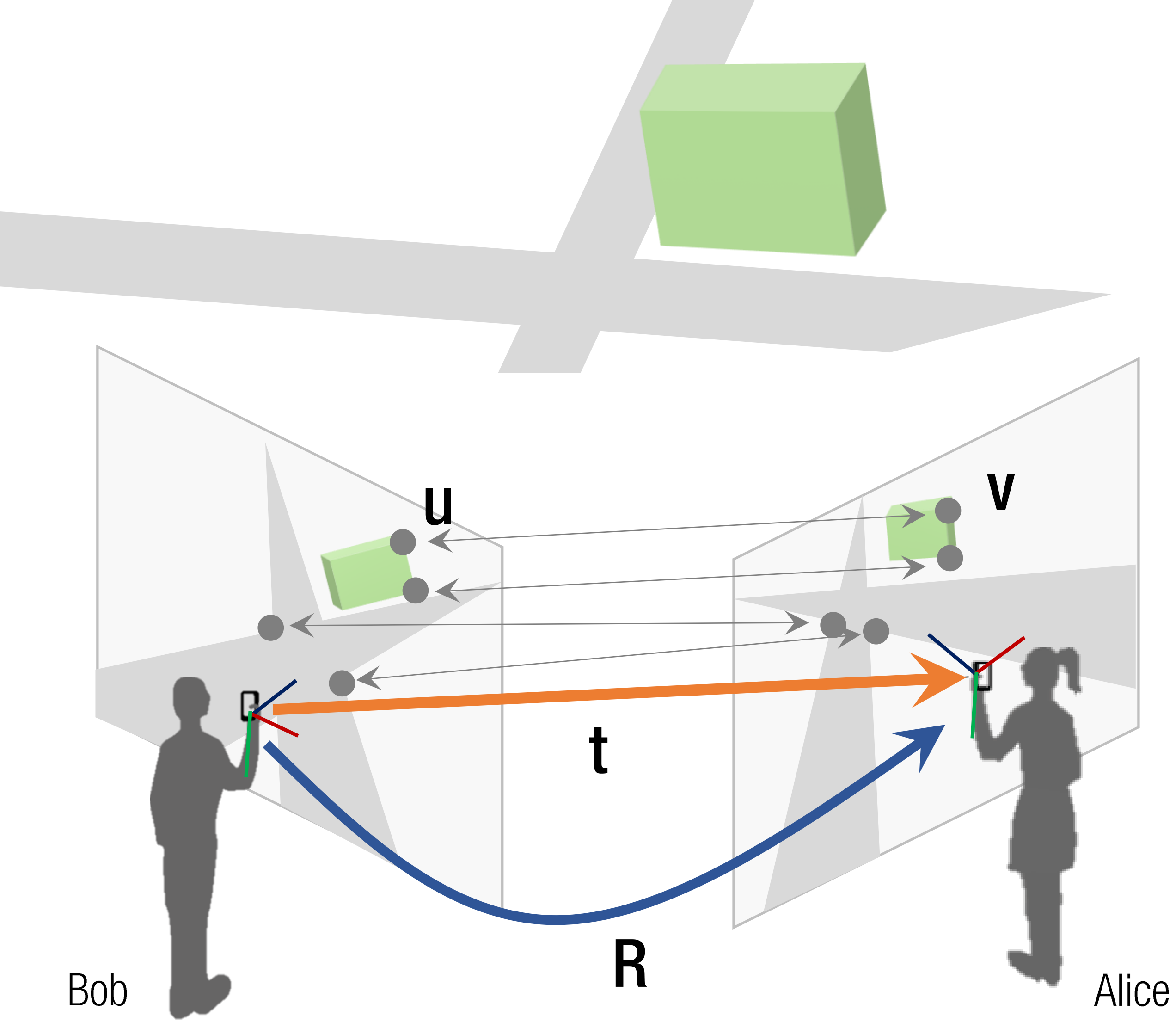
Essential Matrix Decomposition

$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^{\mathsf{T}}\mathbf{R} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^{\mathsf{T}}$$

where $\mathbf{R} \in SO(3)$



Essential Matrix Decomposition



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Define $\mathbf{R} = \mathbf{U}\underline{\mathbf{W}}\mathbf{V}^\mathsf{T}$

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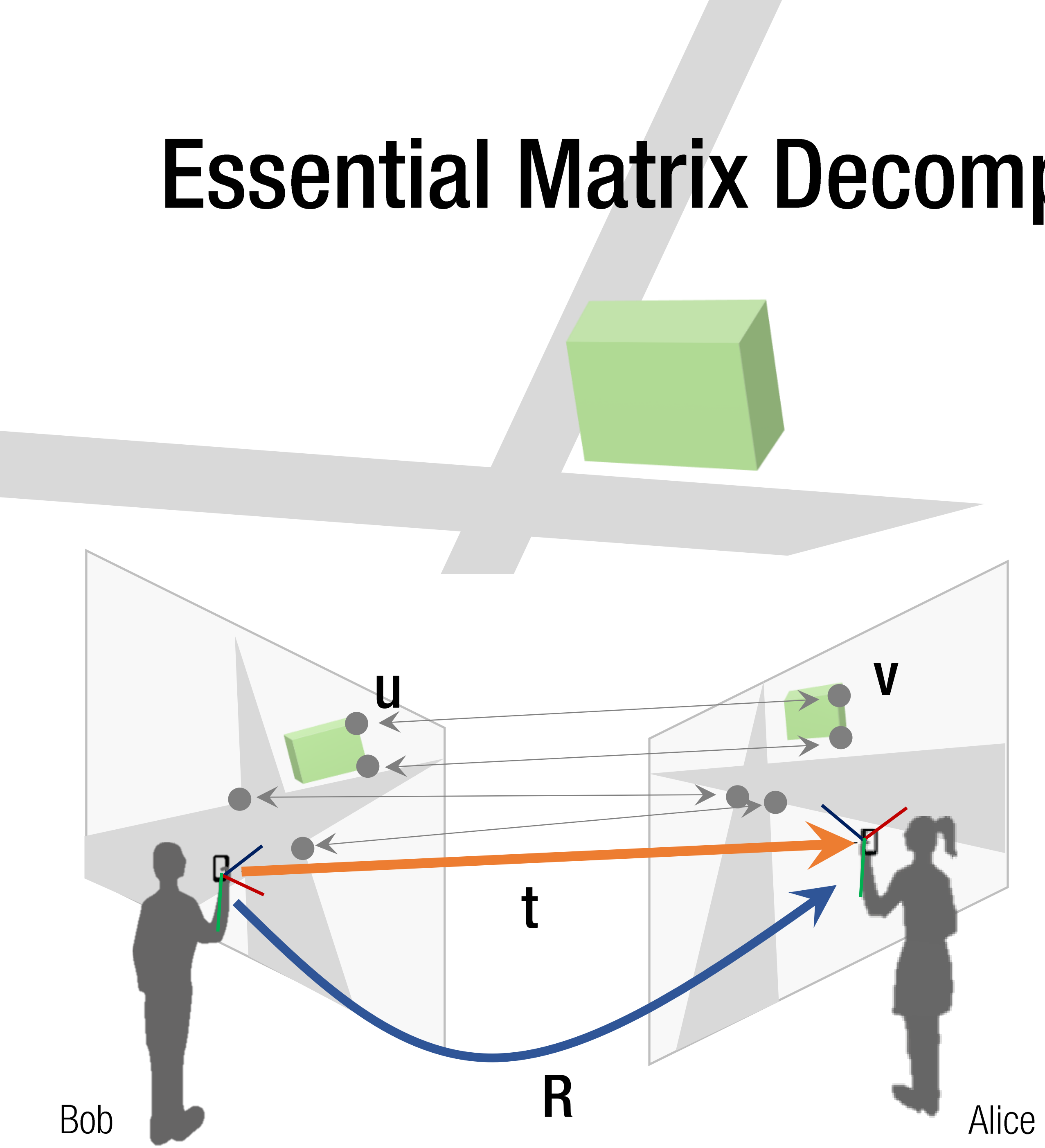
Essential Matrix Decomposition



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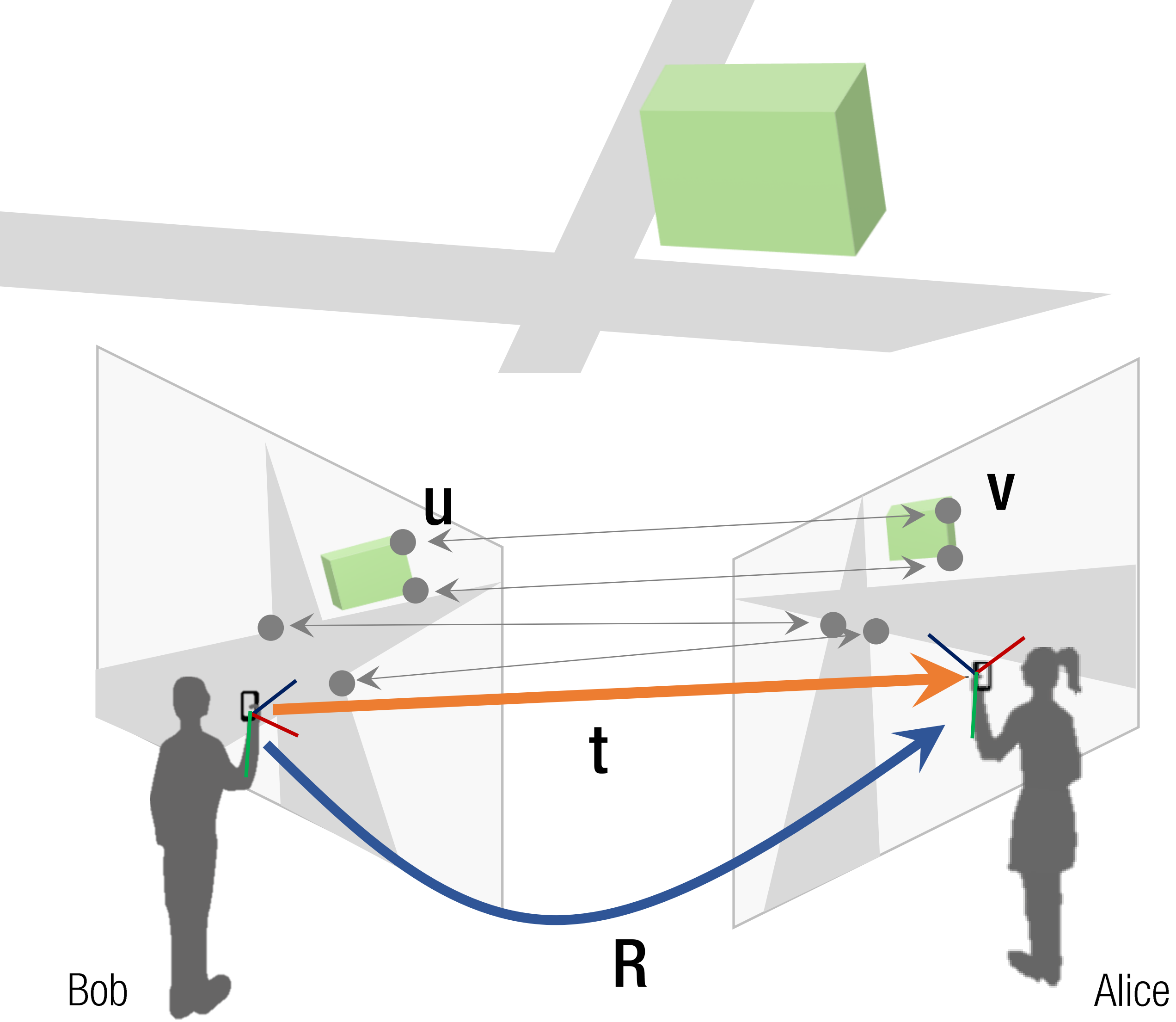
where $\mathbf{R} \in SO(3)$

Define $\mathbf{R} = \mathbf{UWV}^\mathsf{T}$

$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^\mathsf{T}\mathbf{UWV}^\mathsf{T} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{WV}^\mathsf{T}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{W}$$

Essential Matrix Decomposition



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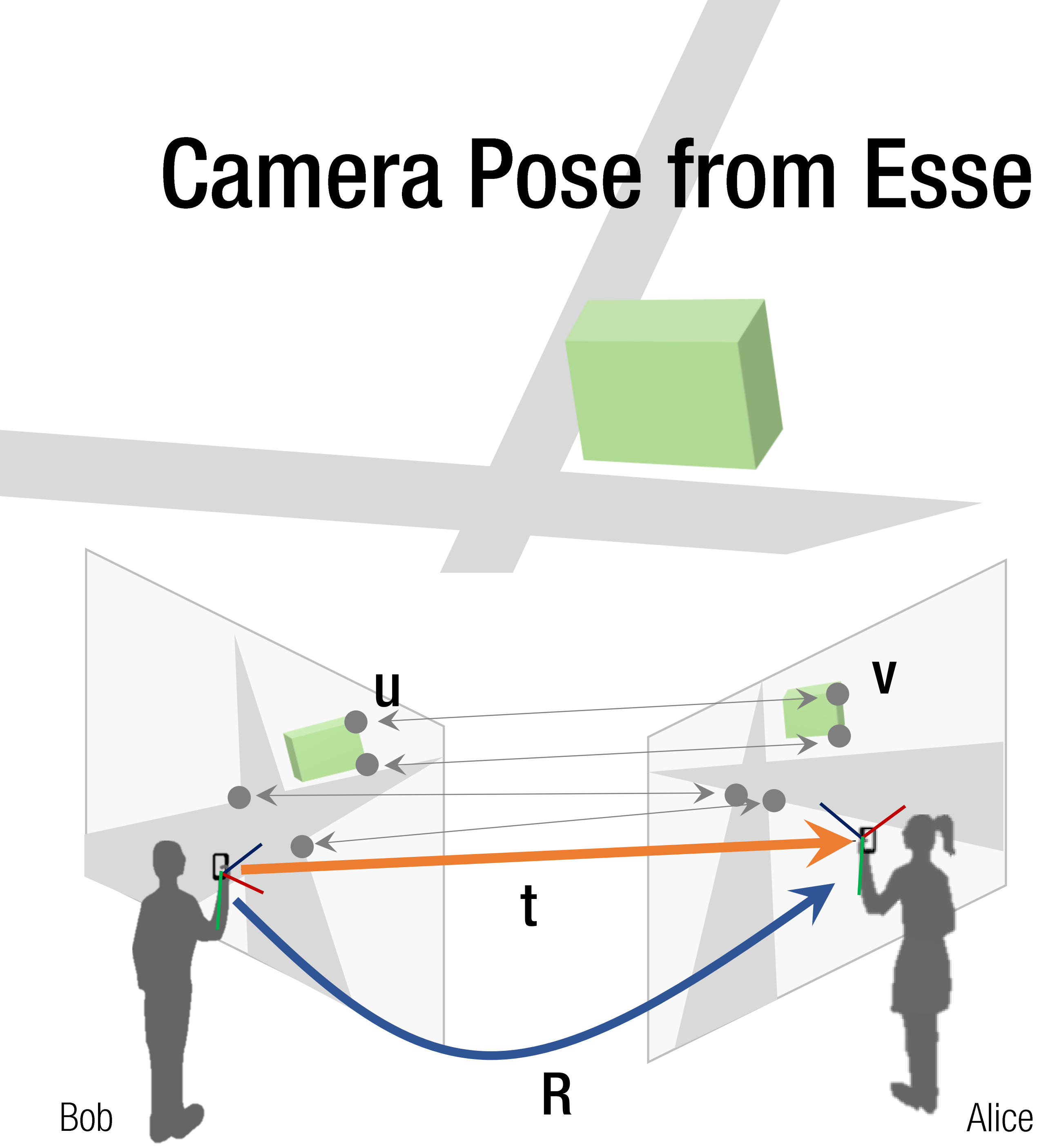
$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{W} \qquad \mathbf{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ or } \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Camera Pose from Essential Matrix (Rotation)

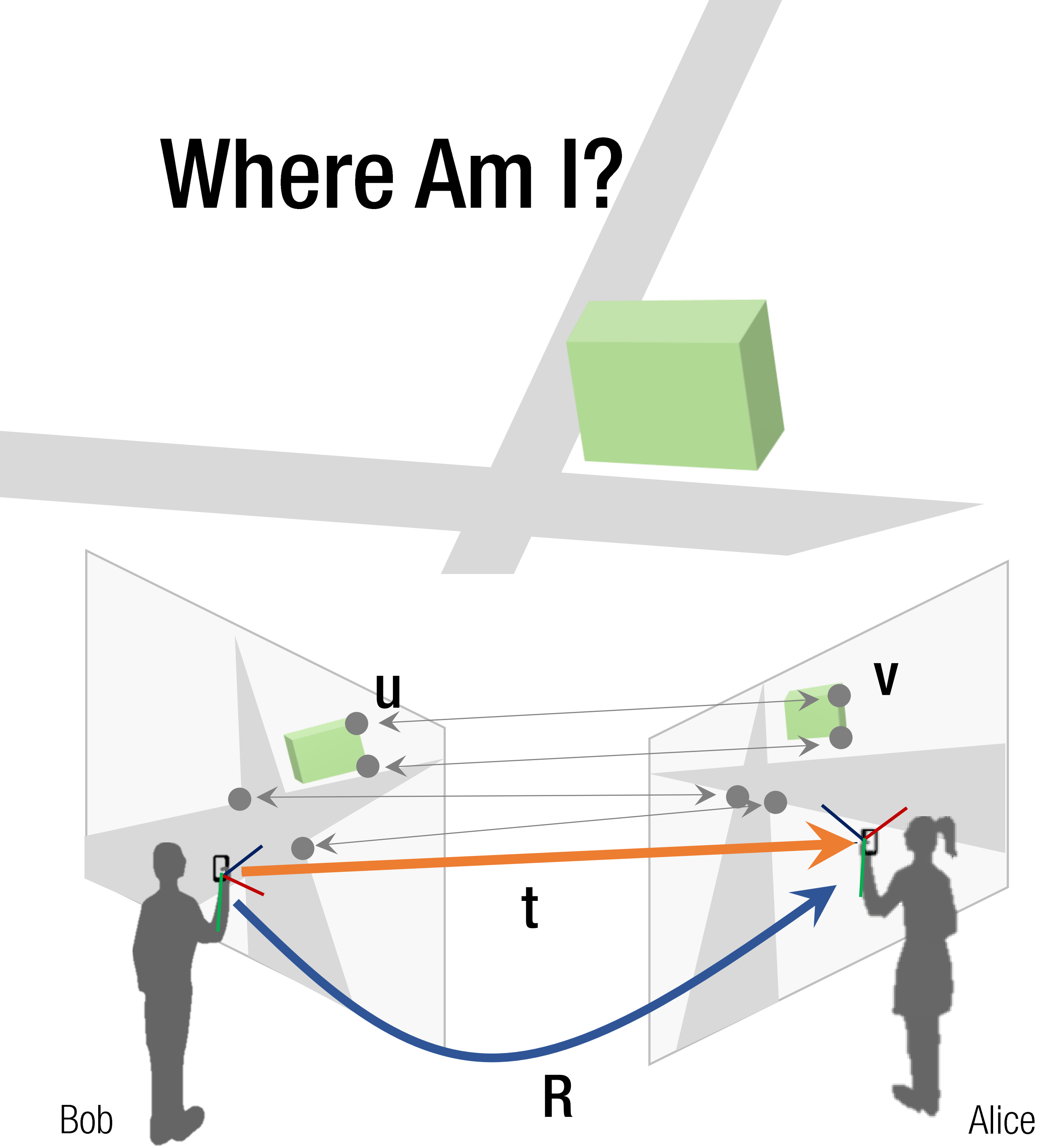


$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^T\mathbf{R} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^T$$

where $\mathbf{R} \in SO(3)$

$$\mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^T, \text{ or } \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^T$$

Where Am I?



$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^\mathsf{T}\mathbf{R} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^\mathsf{T}$$

$$\mathbf{t} = \pm\mathbf{u}_3$$

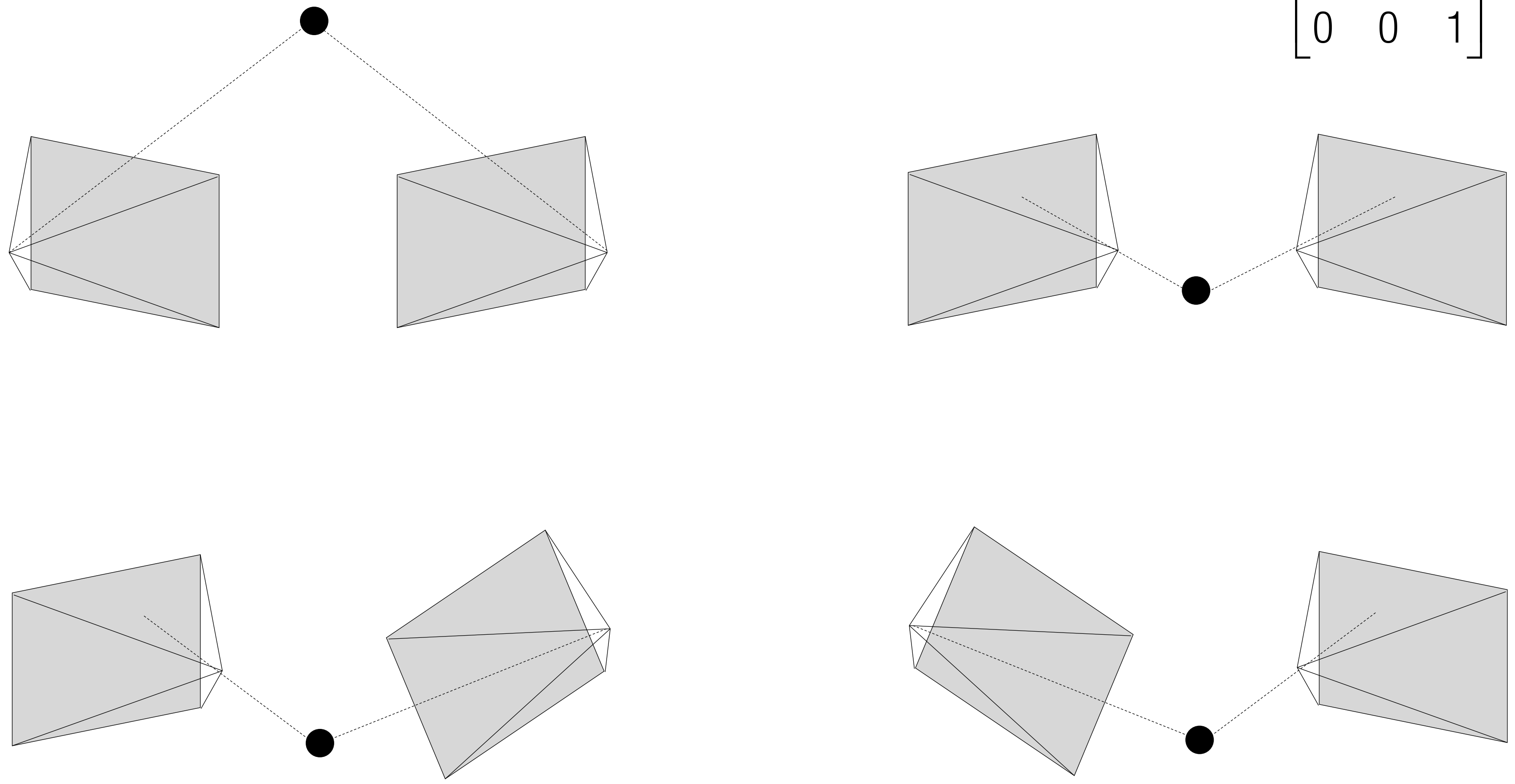
$$\mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^\mathsf{T}, \text{ or } \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^\mathsf{T}$$

→ Four configurations

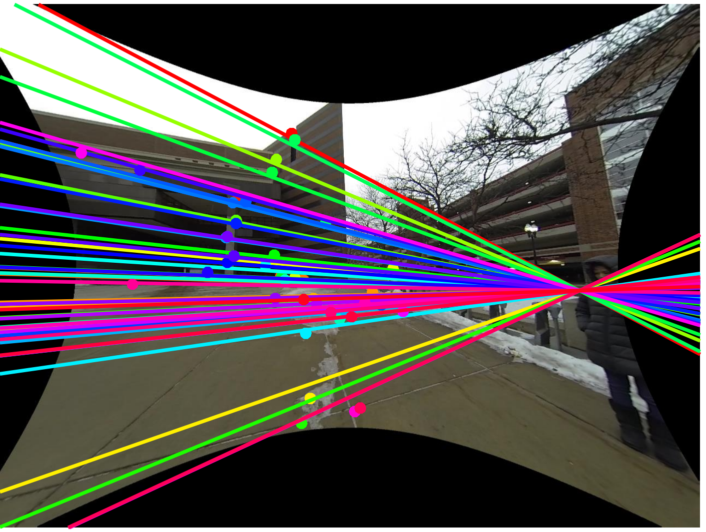
Four Configurations

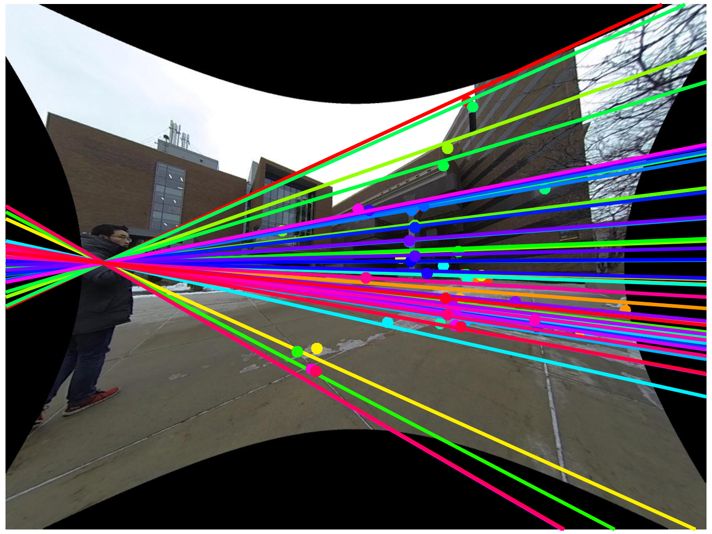
$\mathbf{t} = \pm \mathbf{u}_3$

$$\mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^{\mathsf{T}}, \text{ or } \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^{\mathsf{T}}$$



Camera Pose Estimation





$$E = K^T F K$$

```
function E = ComputeEssentialMatrix(F, K)

E = K' * F * K;

[u d v] = svd(E);

d(1,1) = 1;
d(2,2) = 1;
d(3,3) = 0;

E = u * d * v';
```

SVD cleanup

D =

1.0468	0	0
0	0.9975	0
0	0	0.0000

Before cleanup

D =

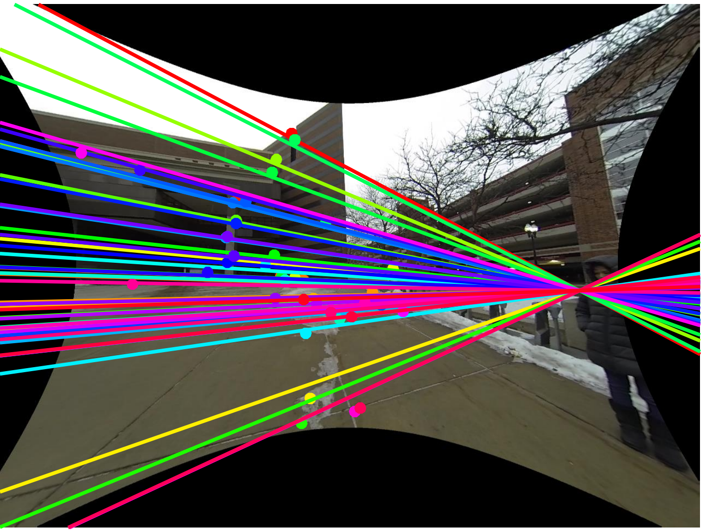
1.0000	0	0
0	1.0000	0
0	0	0.0000

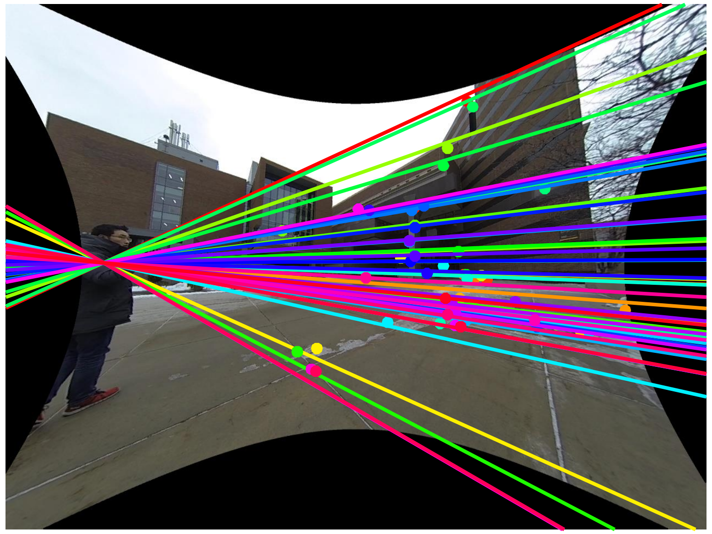
After cleanup

Camera Pose Estimation

$$\mathbf{t} = \pm \mathbf{u}_3$$

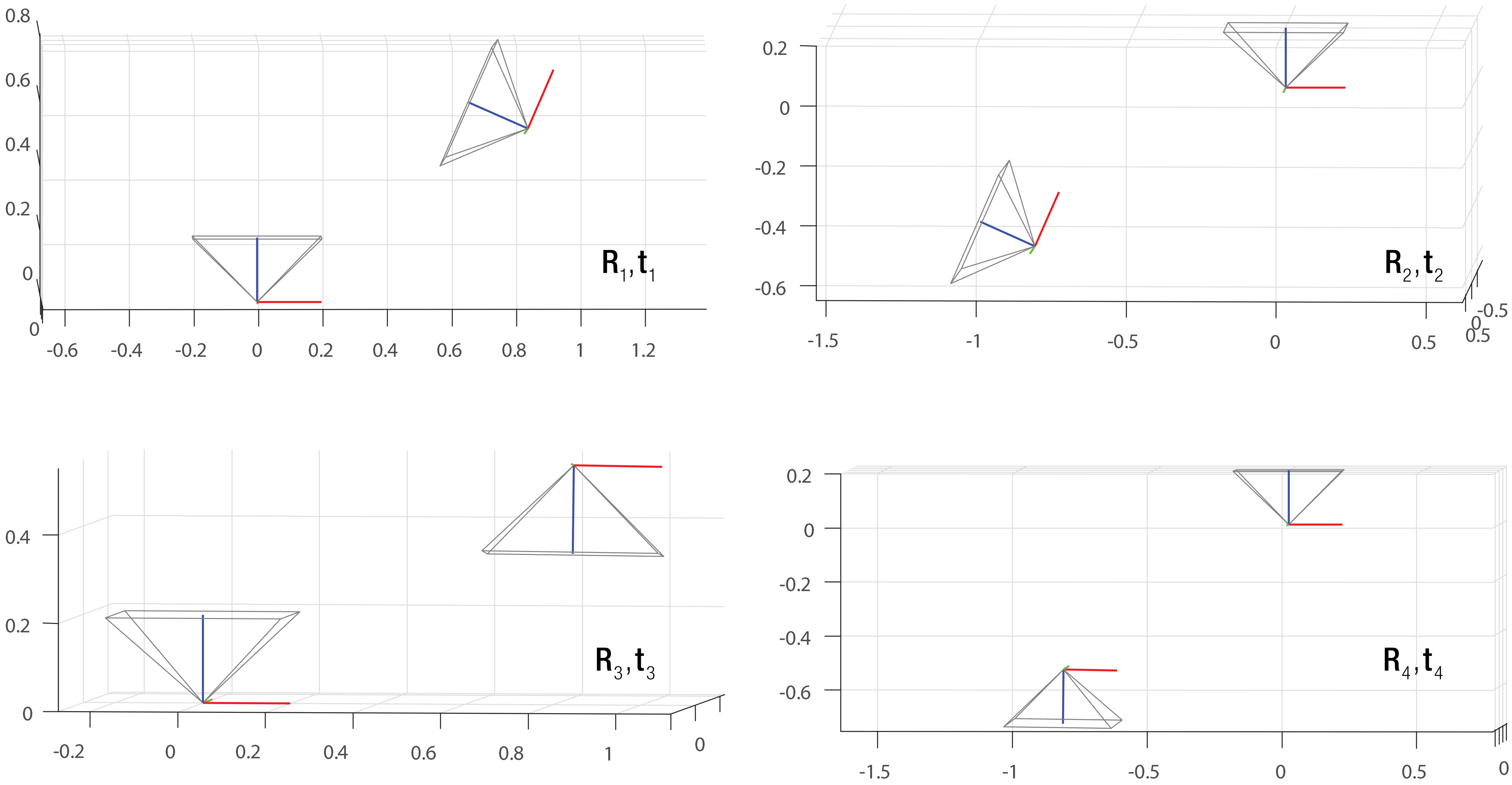
$$\mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^{\mathsf{T}}, \text{ or } \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^{\mathsf{T}}$$





```
function [R1 t1, R2, t2, R3, t3, R4, t4] = ...
CameraPoseFromEssentialMatrix(E)

[U D V] = svd(E);

W = [0 -1 0;
    1 0 0;
    0 0 1];

t1 = U(:,3);
R1 = U * W * V';
if det(R1) < 0
  t1 = -t1;  R1 = -R1;
end

...
```

$$\det(\mathbf{R}) = 1$$


0.8
0.6
0.4
0.2
0
0
-0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1 1.2
R_1, t_1
0.2
0
-0.2
-0.4
-0.6
-1.5 -1 -0.5 0 0.5
-0.5 0 0.5
R_2, t_2
0.4
0.2
0
-0.2 0 0.2 0.4 0.6 0.8 1
0
R_3, t_3
0.2
0
-0.2
-0.4
-0.6
-1.5 -1 -0.5 0 0.5
0
R_4, t_4

Camera Image Projection