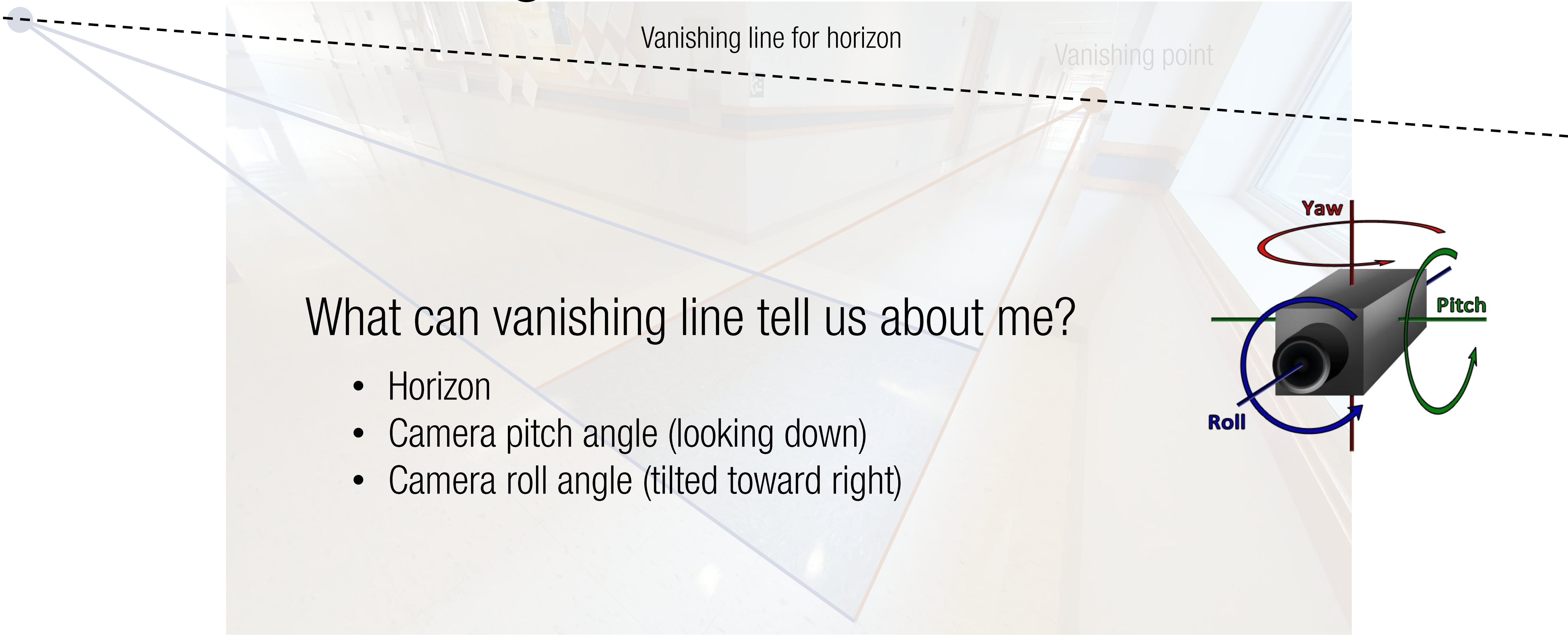
Perspective-n-Point

Hyun Soo Park

Recall: Vanishing Line

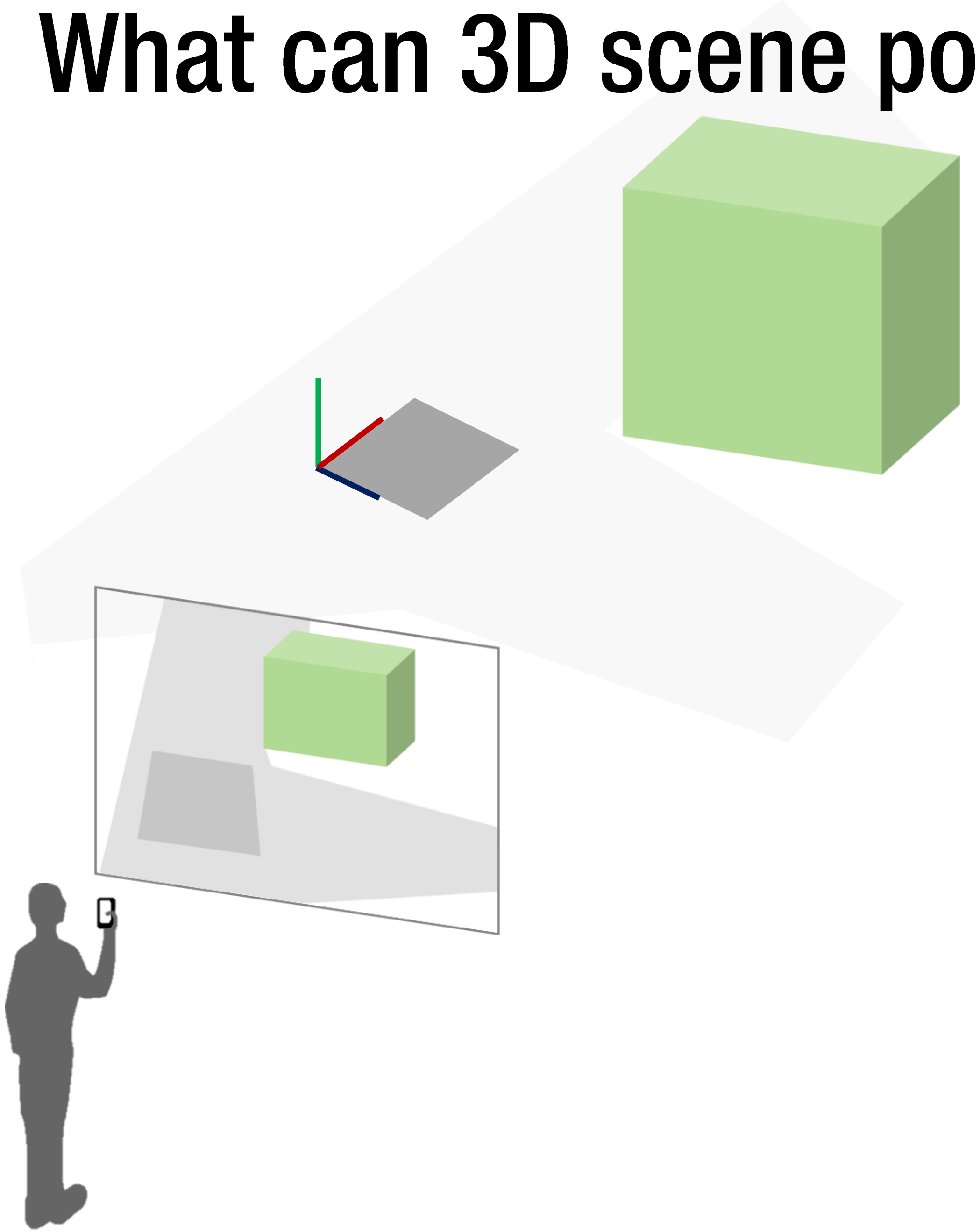


What can vanishing line tell us about me?

- Horizon
- Camera pitch angle (looking down)
- Camera roll angle (tilted toward right)

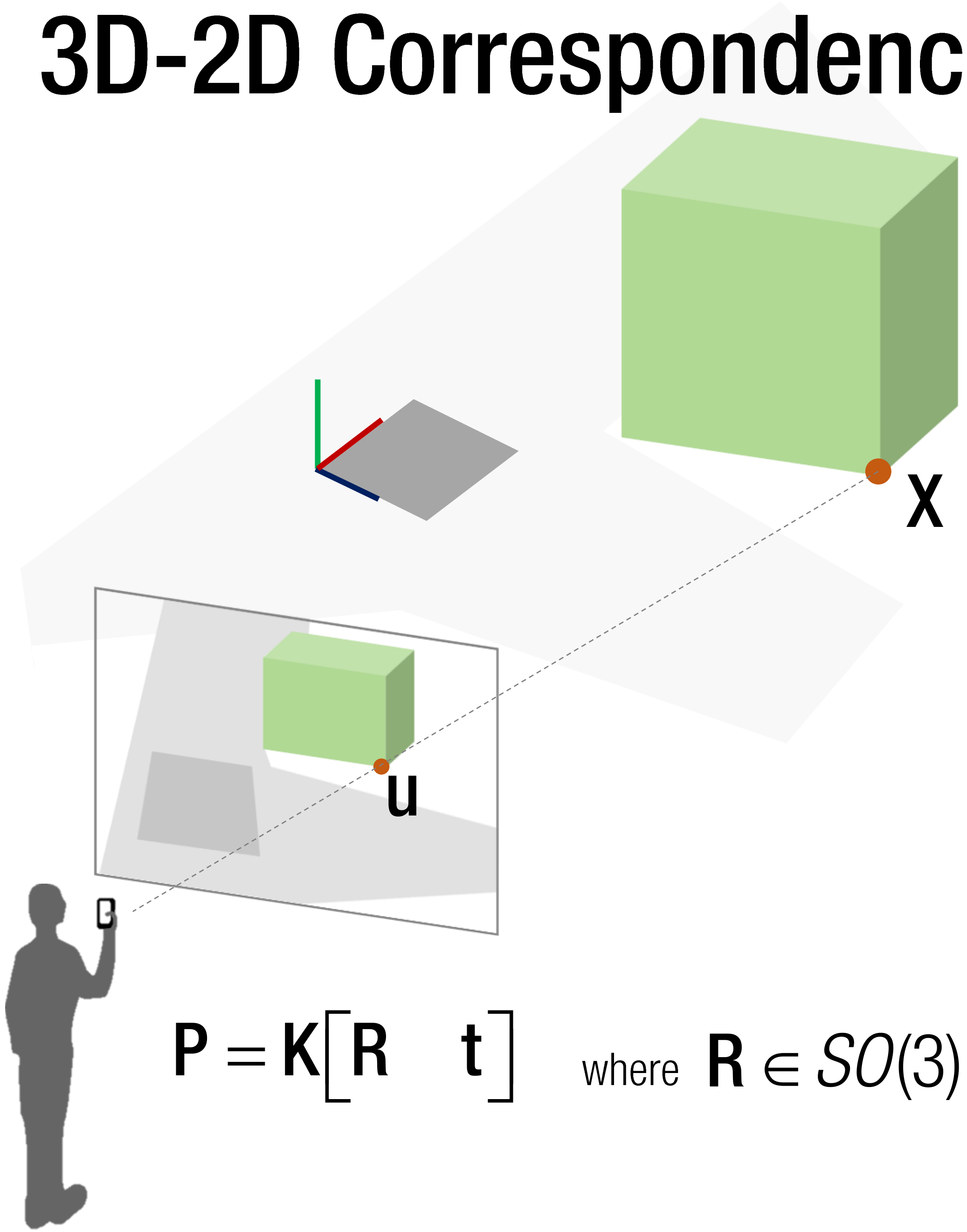
Keller Hall

What can 3D scene points tell us about?



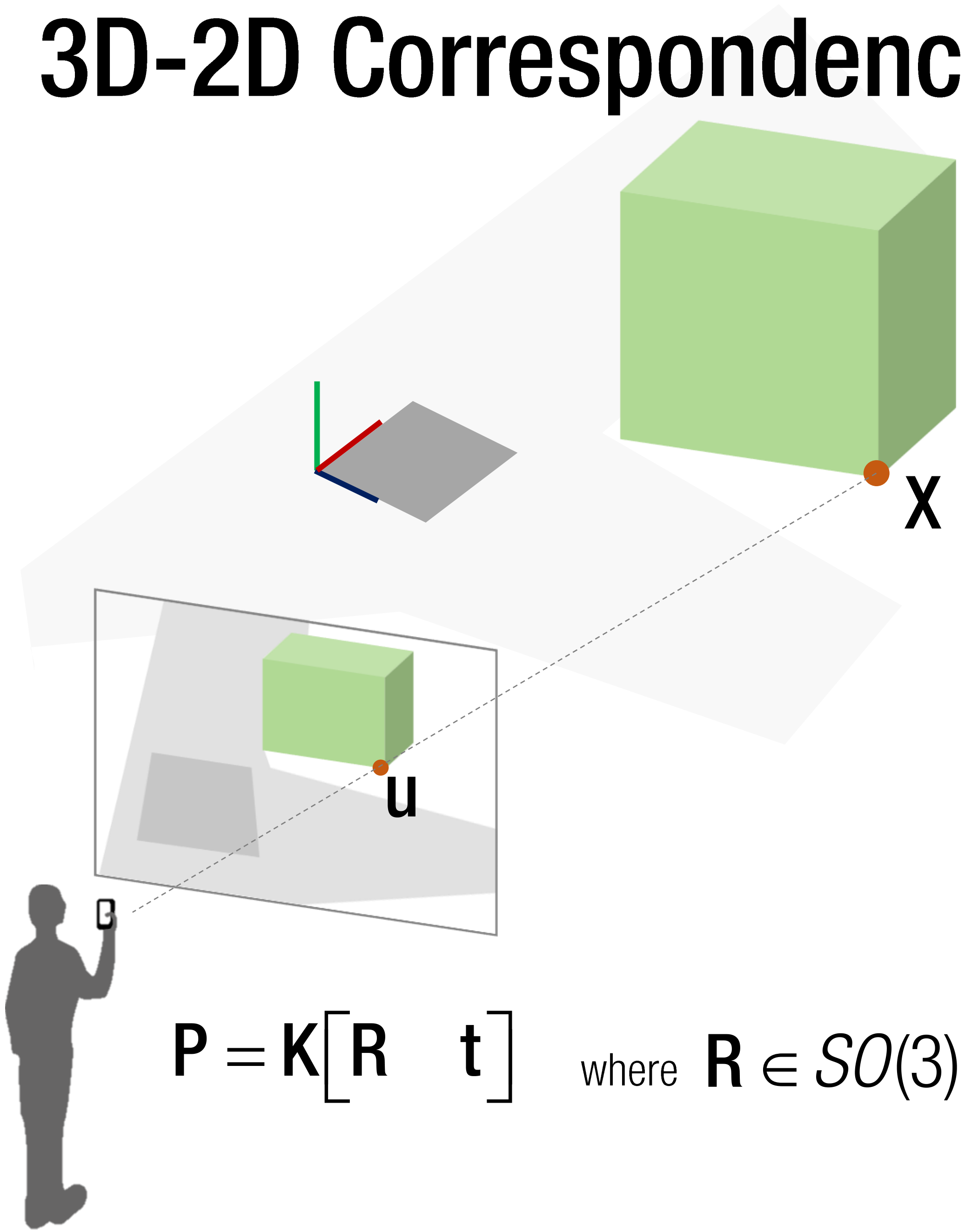
http://www.wired.com/gadgetlab/2010/07/camera-software-lets-you-see-into-the-past/

3D-2D Correspondence



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D Correspondence



3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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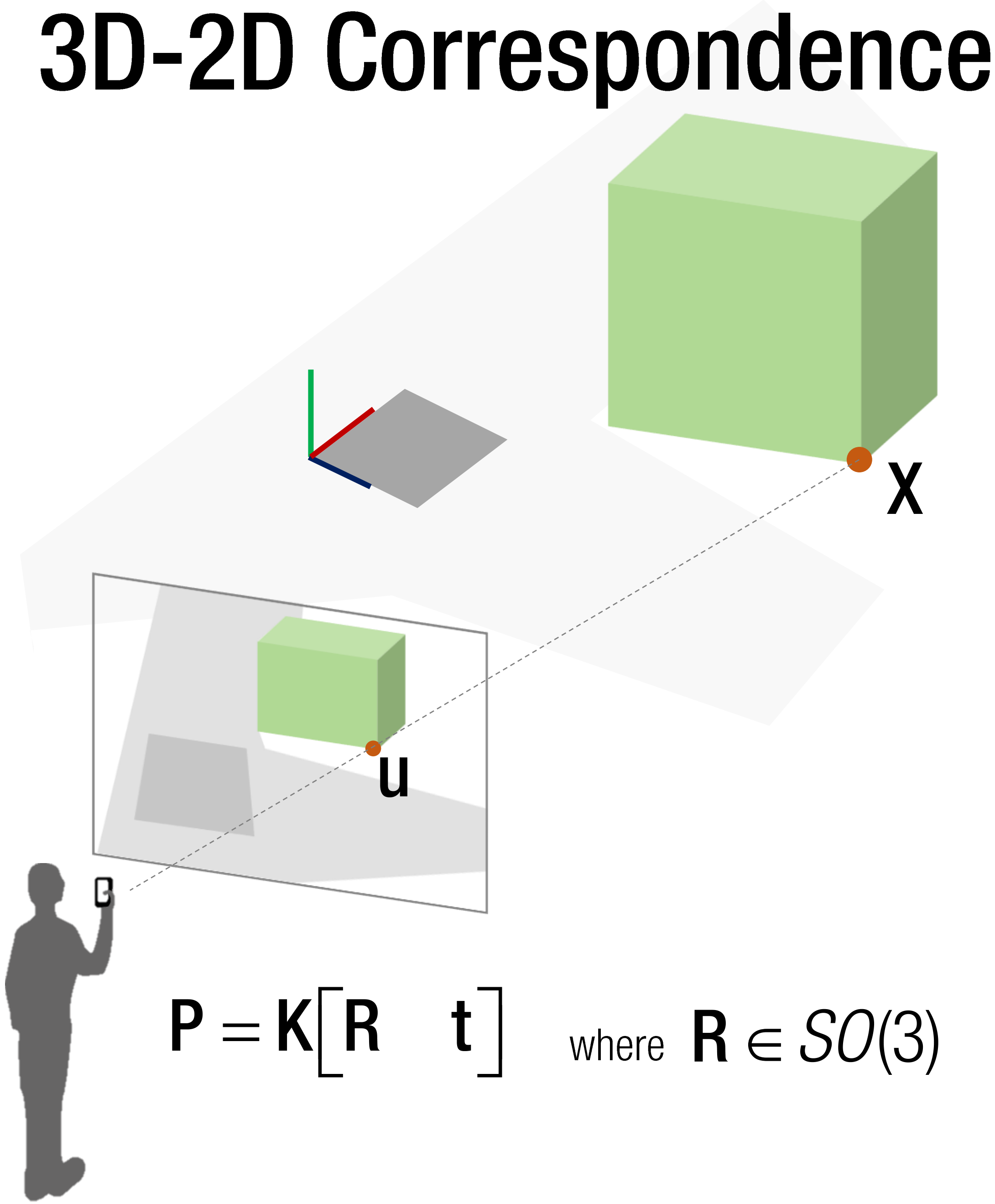

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Linear in camera matrix

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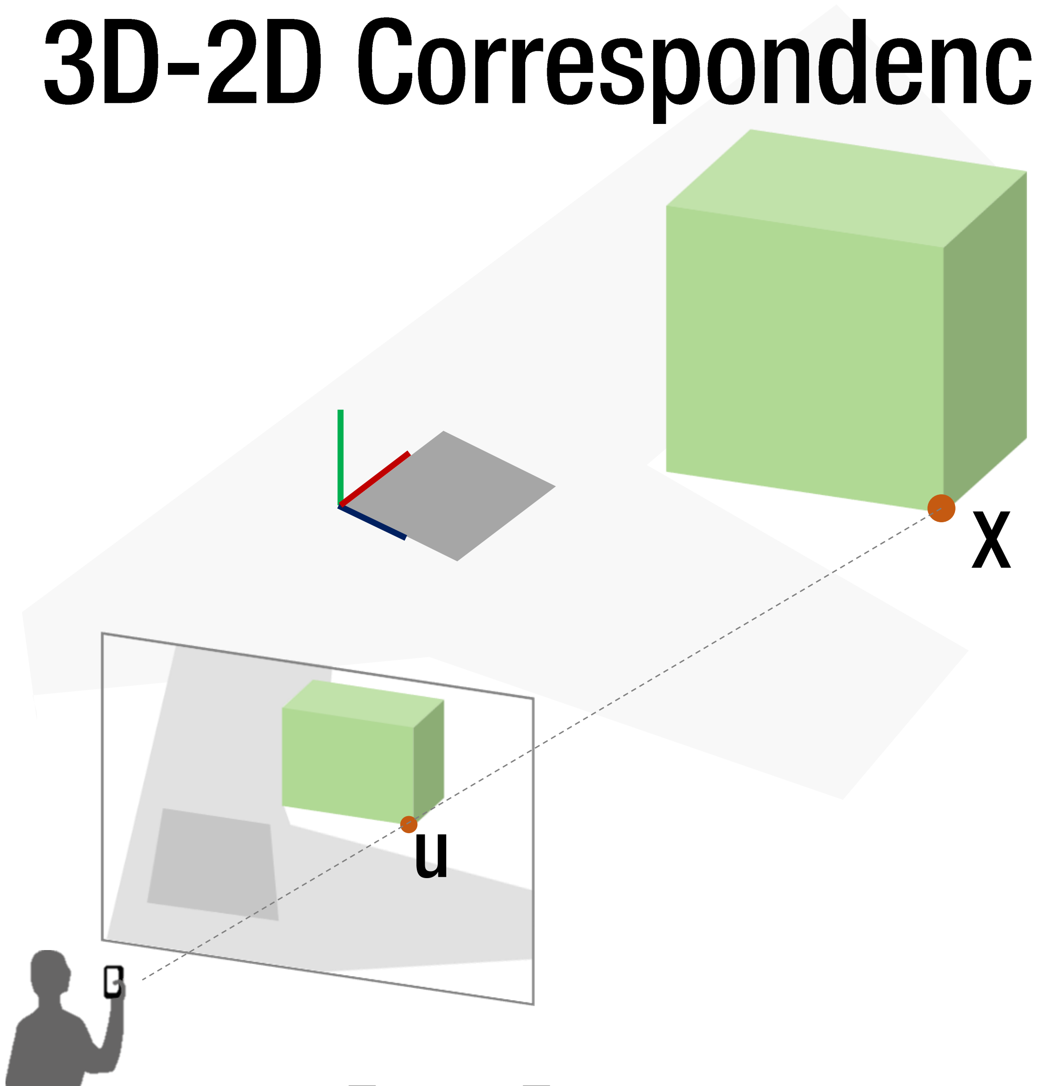

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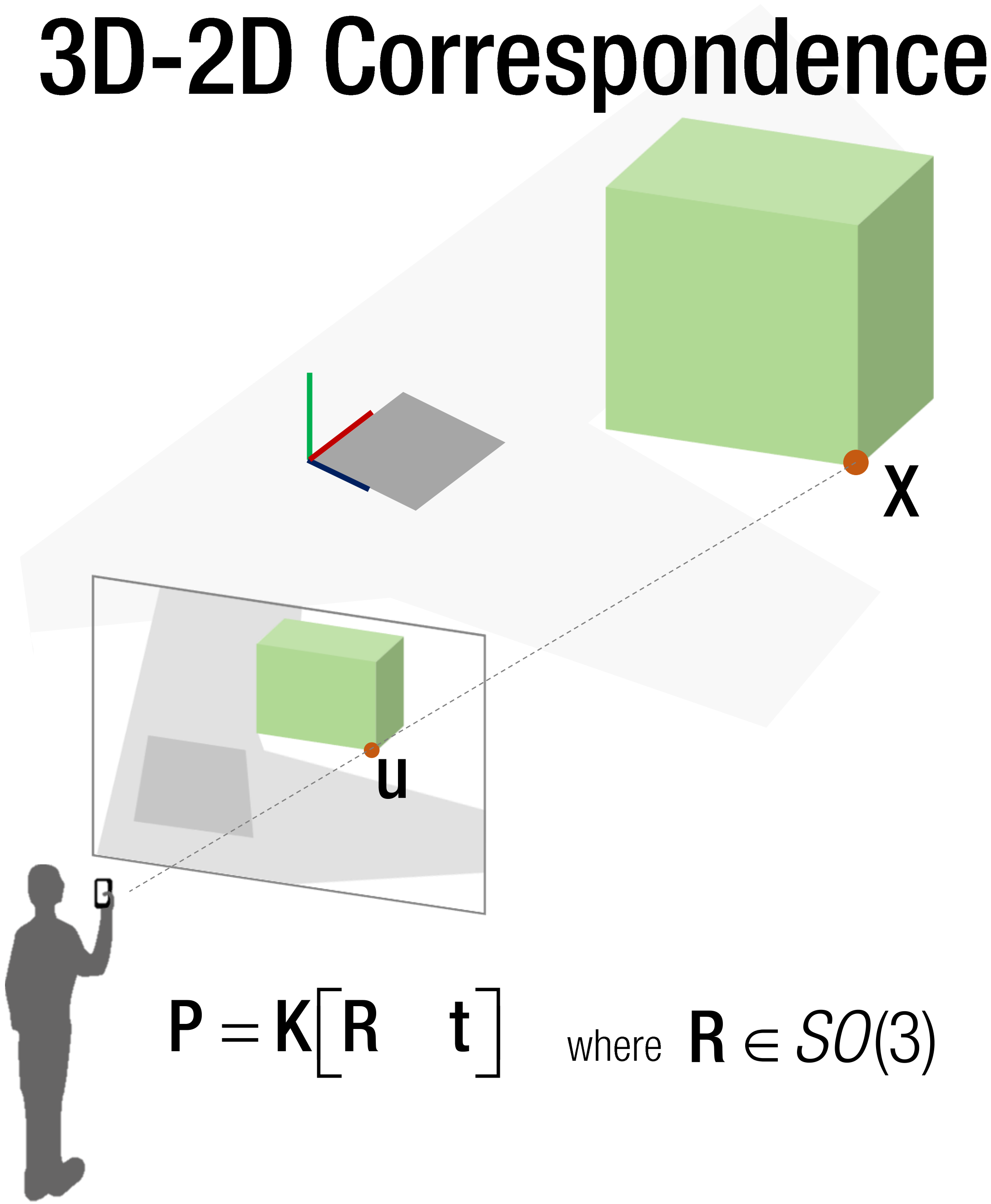
# of unknowns: 11 = 12 (3x4 matrix) – 1 (scale)
# of equations per correspondence: 2

3D-2D Correspondence



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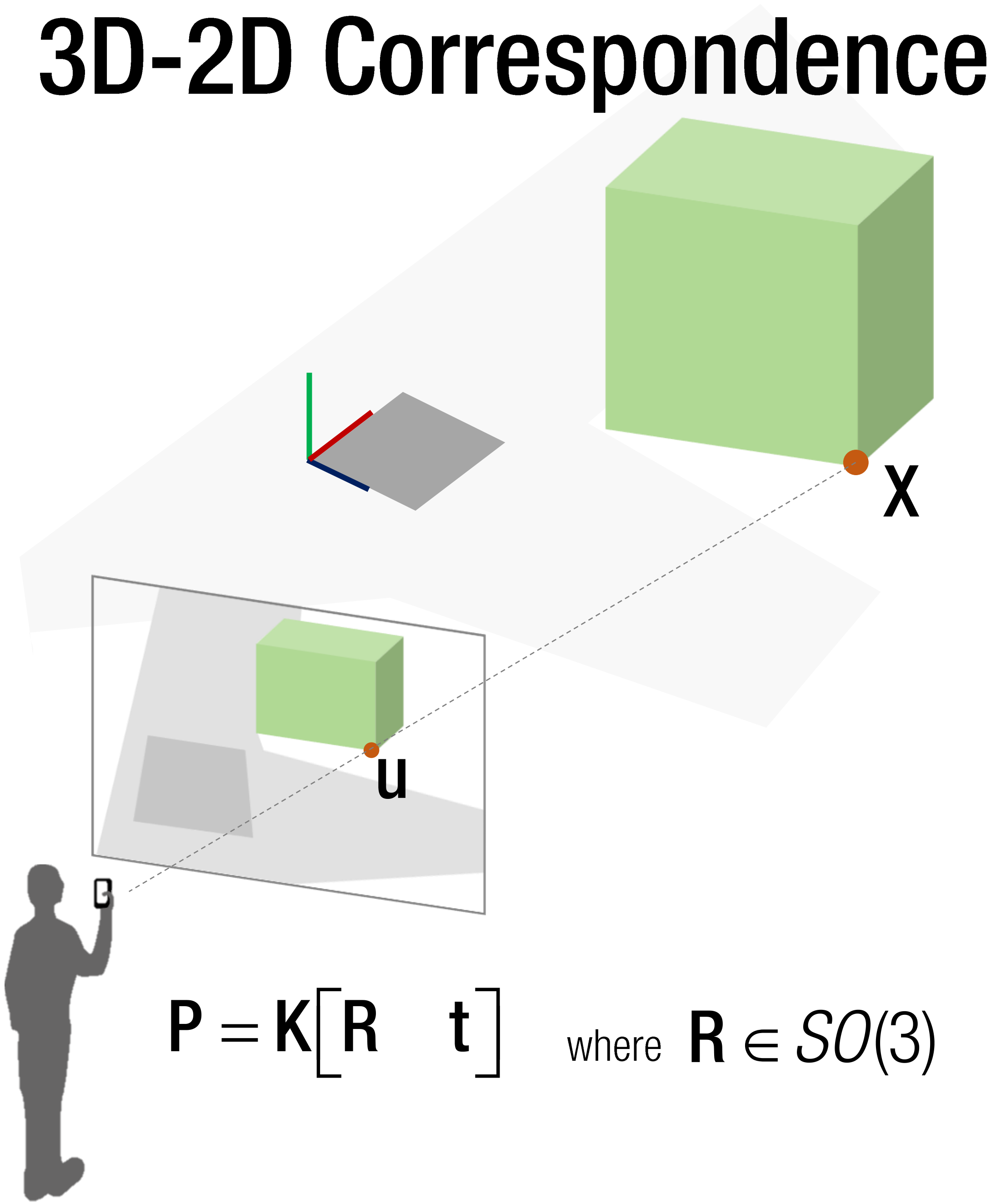

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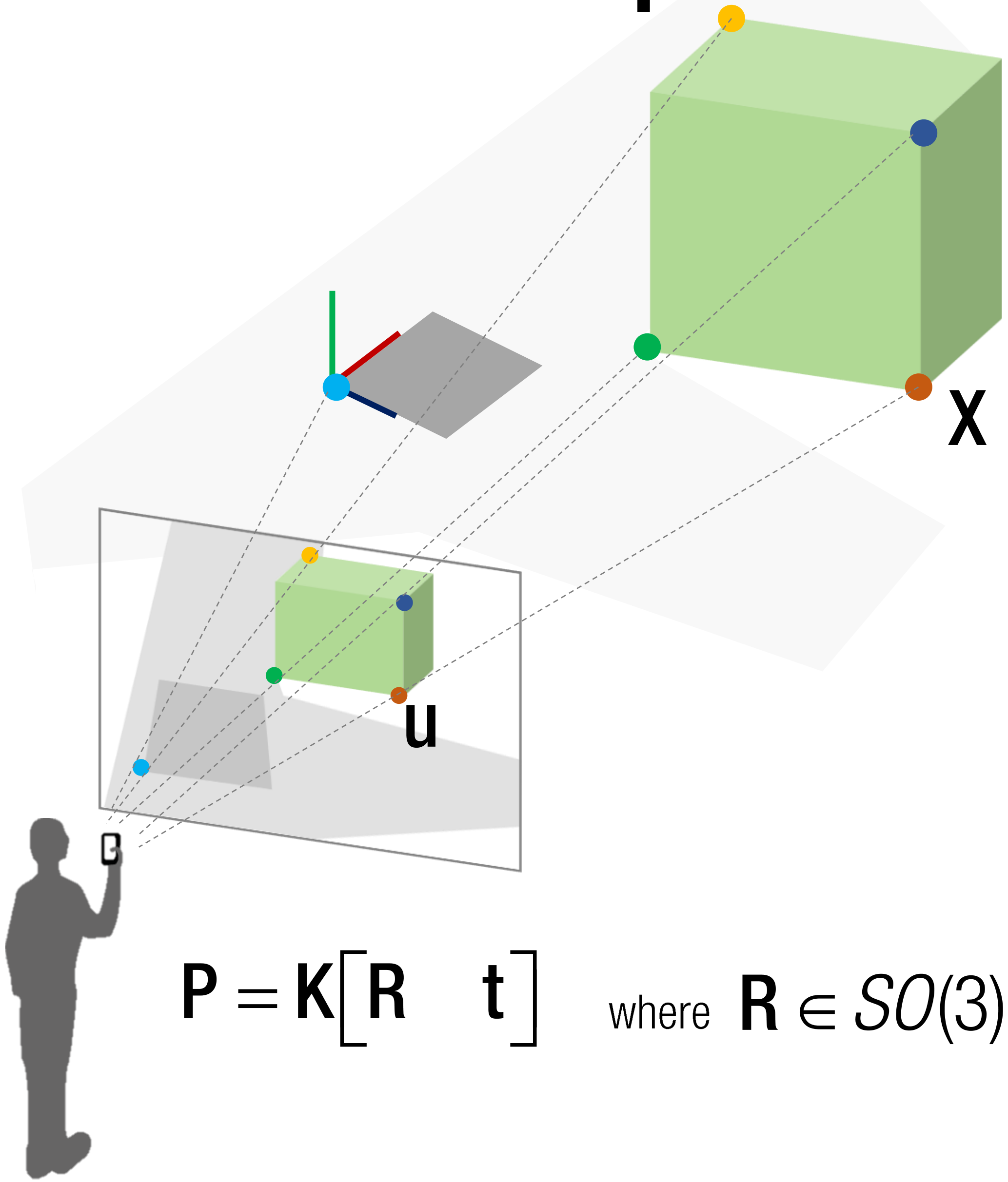
$$\begin{bmatrix} X & Y & Z & 1 & & & & & -u^xX & -u^xY & -u^xZ & -u^x \\ & & & & X & Y & Z & 1 & -u^yX & -u^yY & -u^yZ & -u^y \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2x12

3D-2D Correspondence



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$2m$x12

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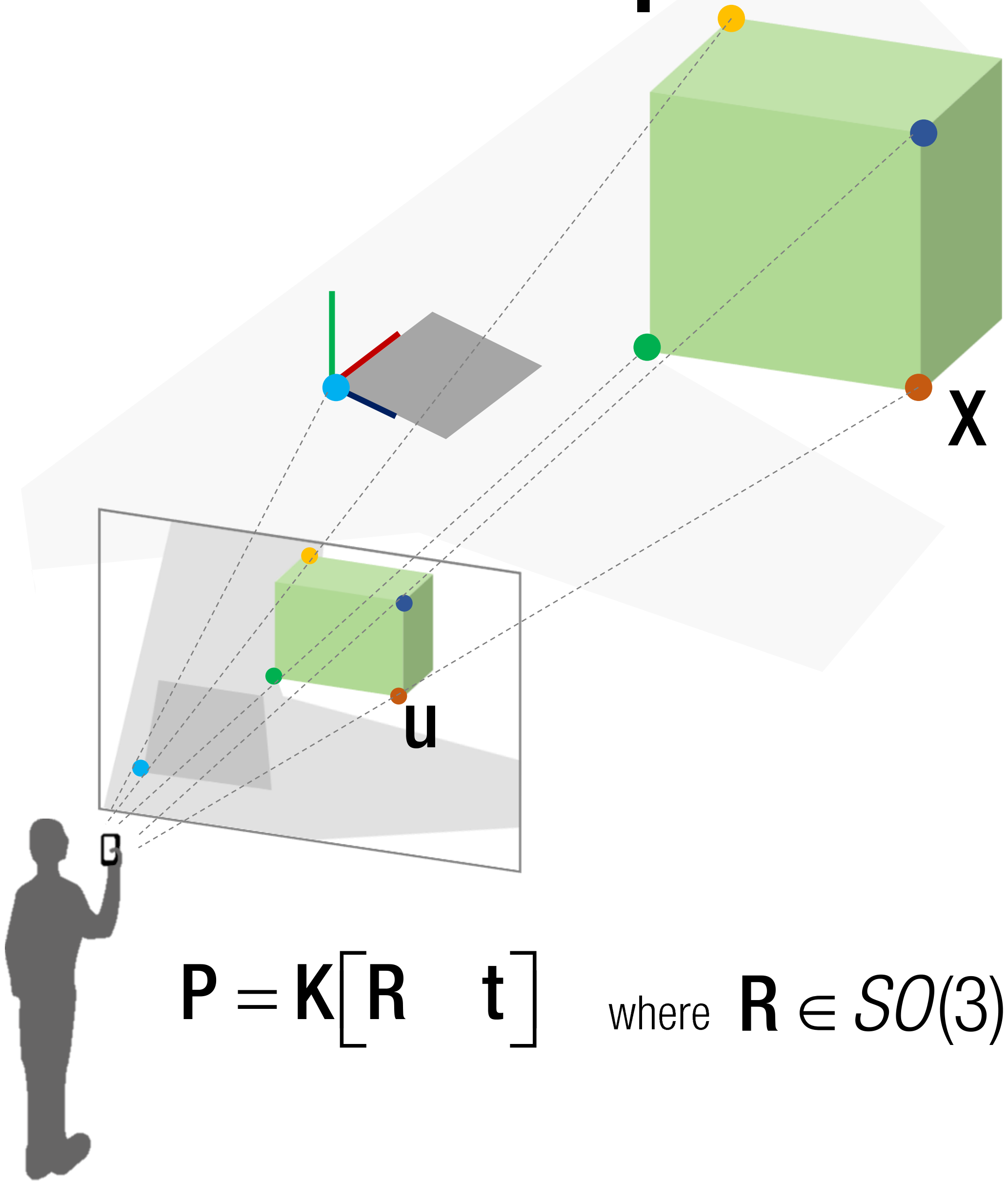

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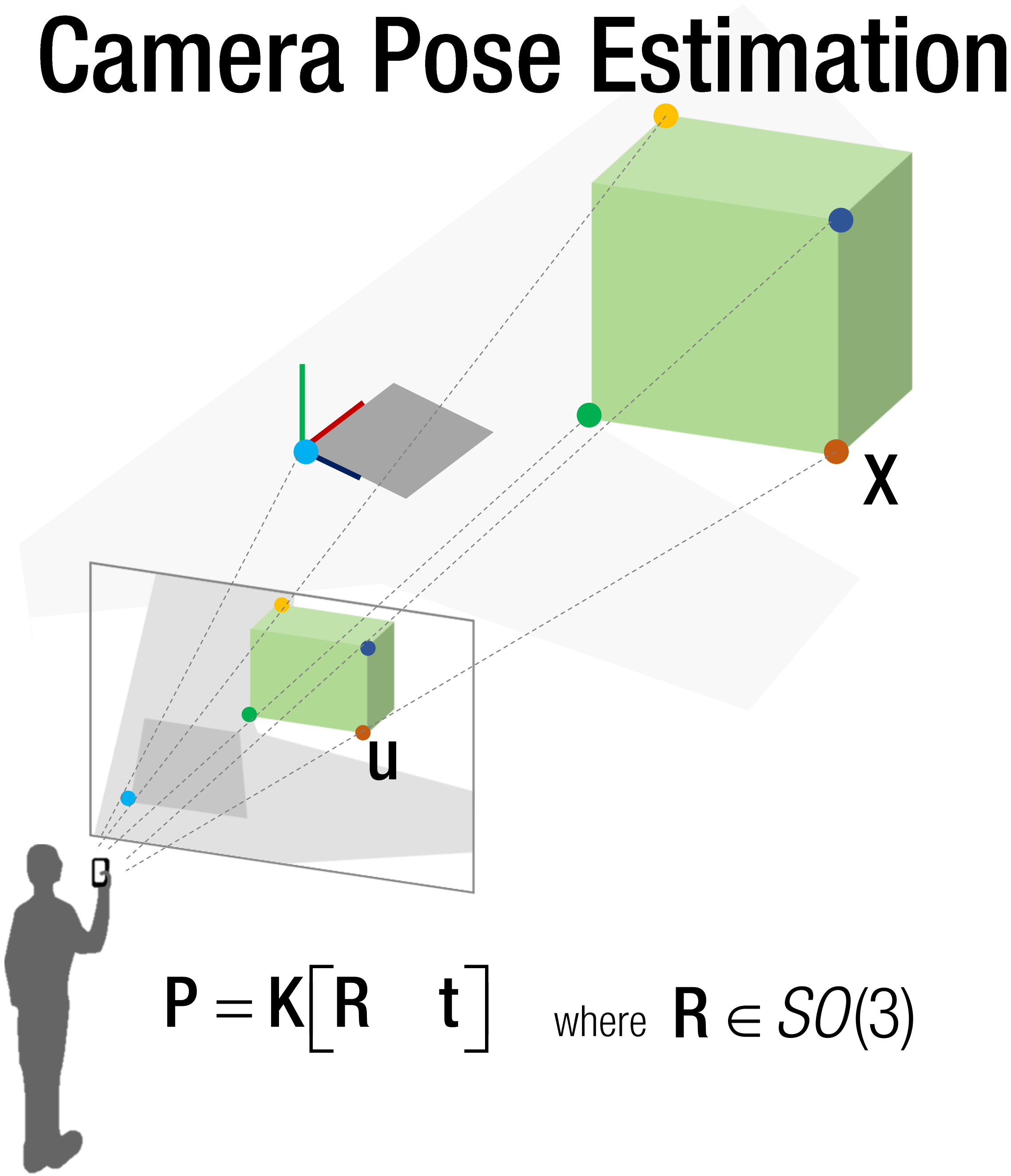
$2m$x12

Camera Pose Estimation



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

If **K** is given,

$$\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} = \gamma\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4\end{bmatrix}$$

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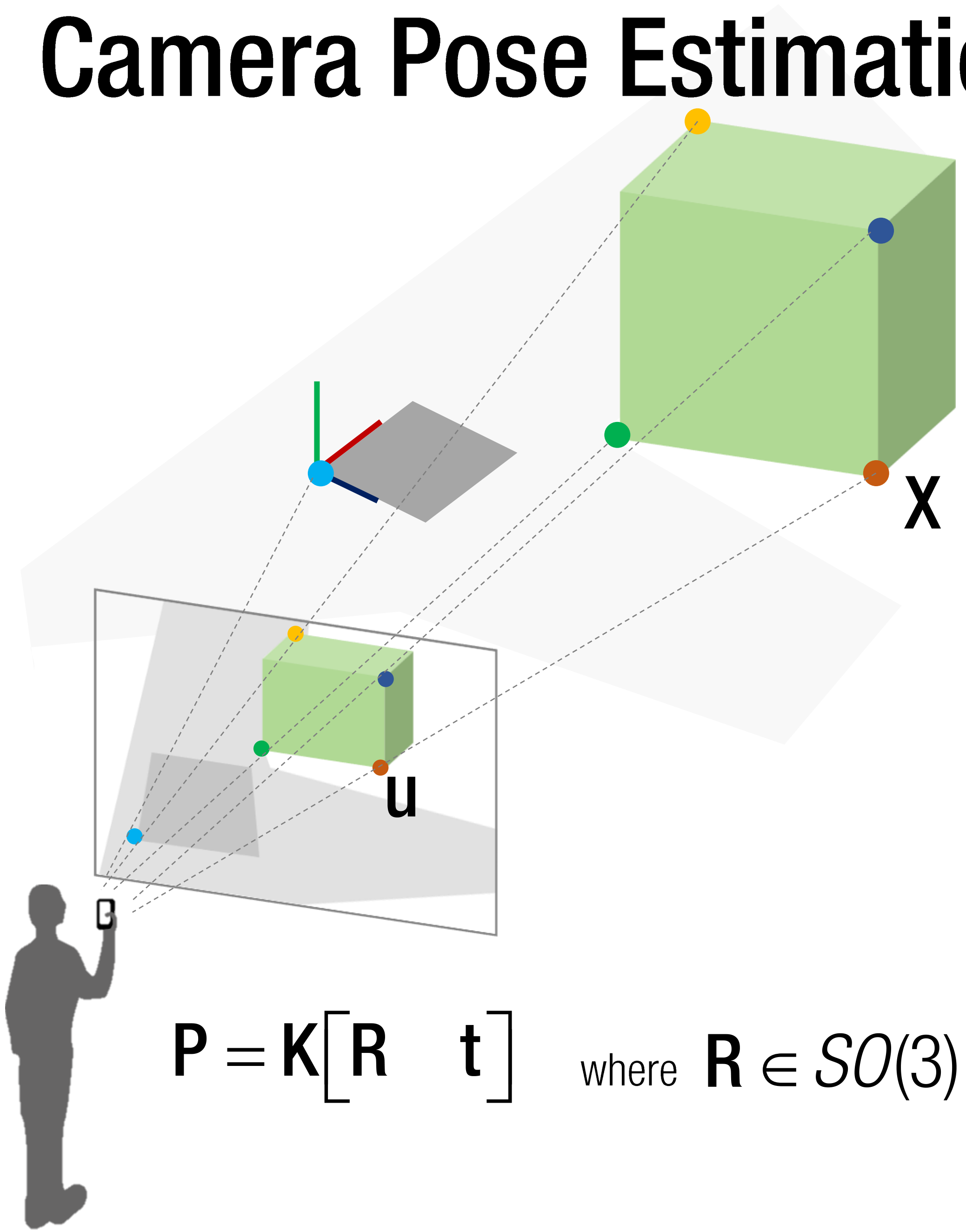

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$$\longrightarrow \gamma\mathbf{R} = \mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix}$$

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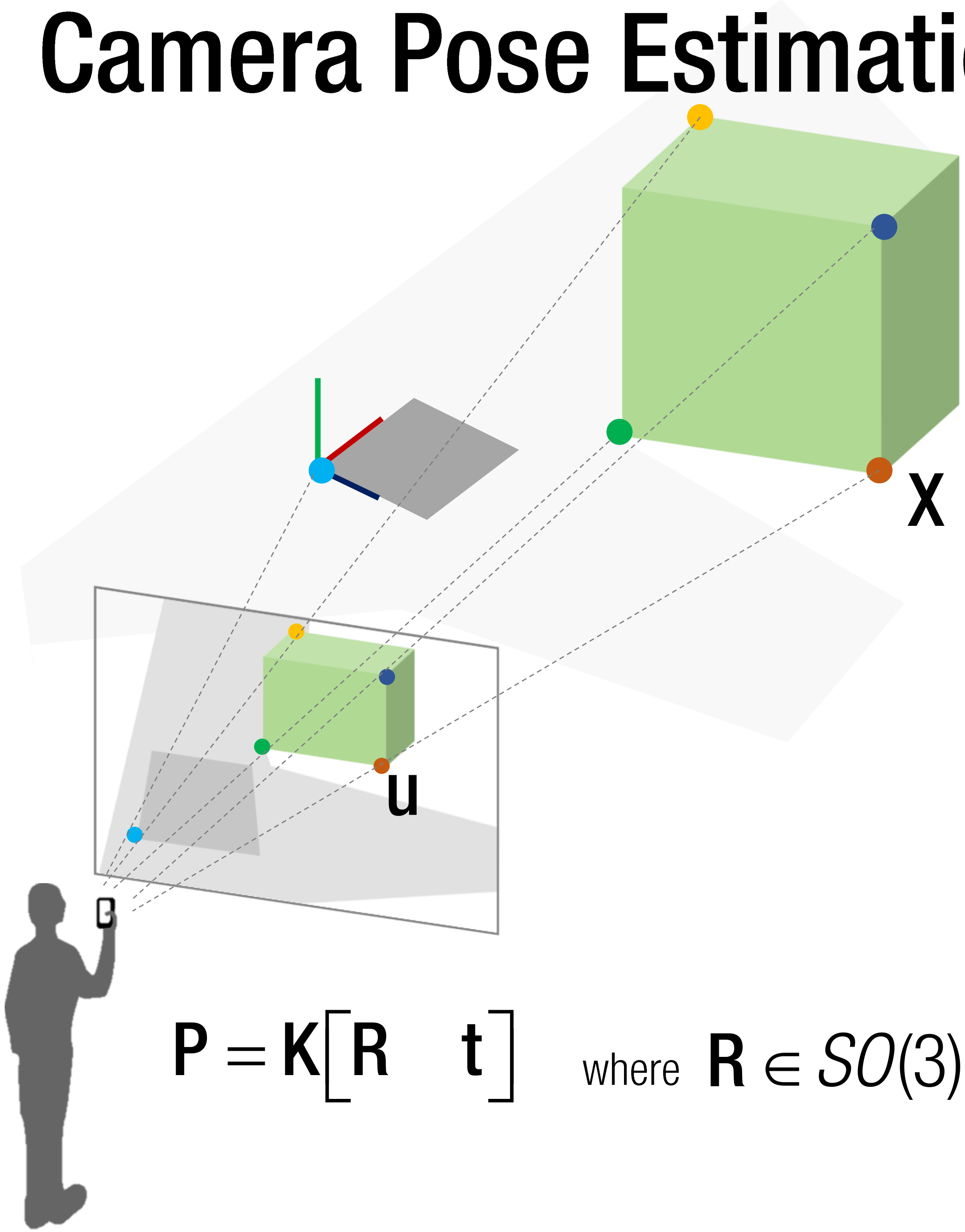

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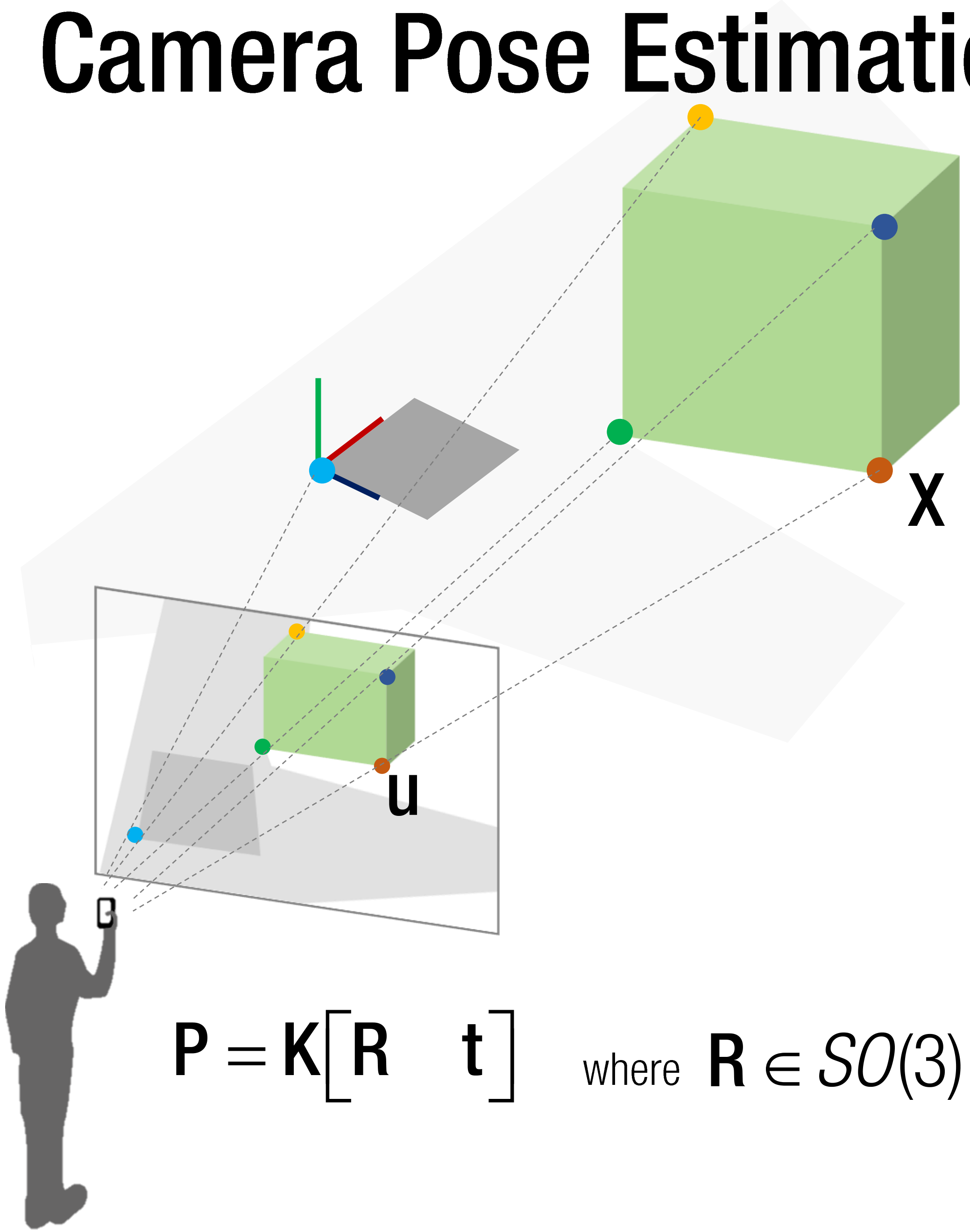

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$$\mathbf{R} = \mathbf{U}\mathbf{V}^{\mathsf{T}}$$: SVD cleanup

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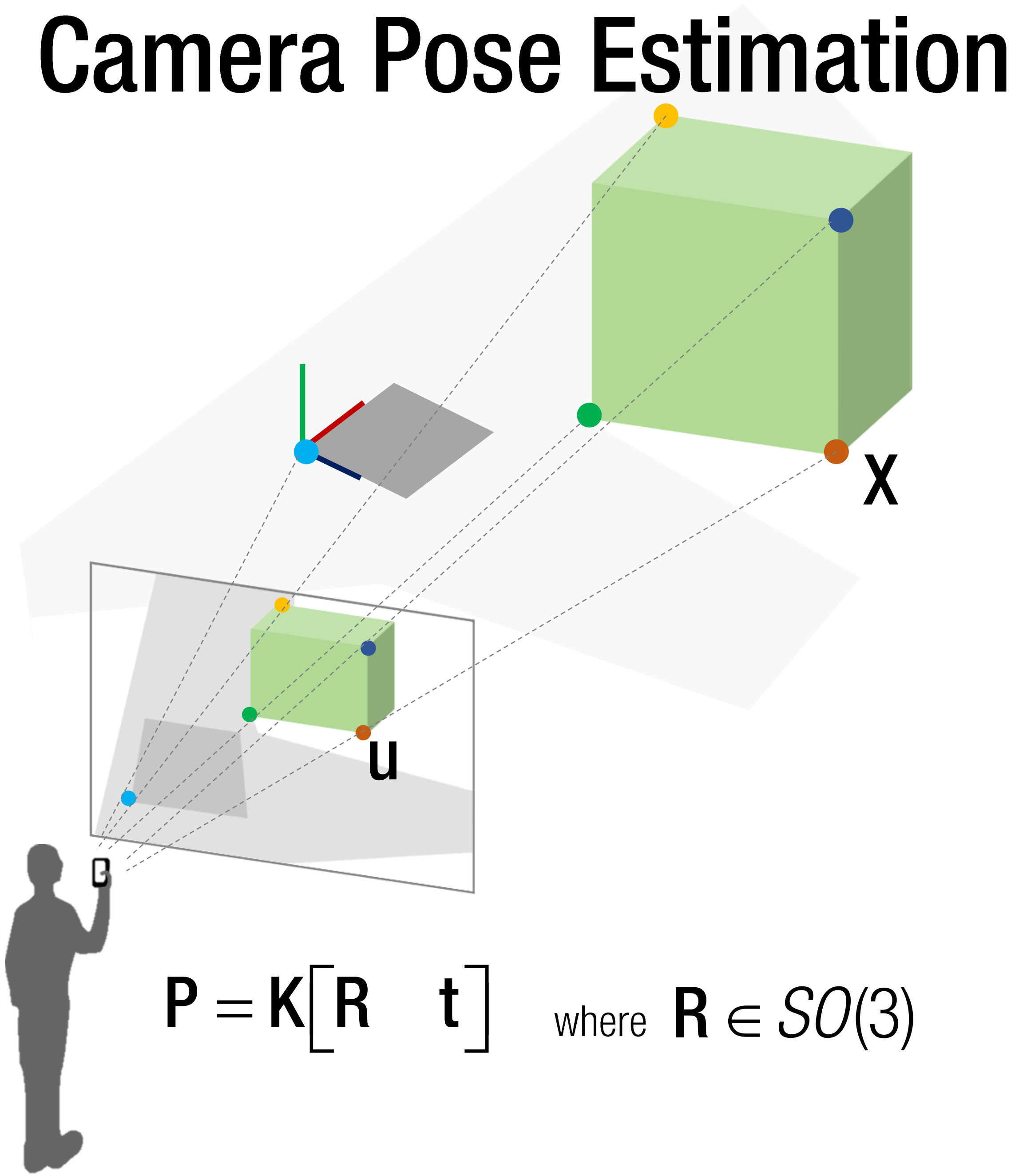

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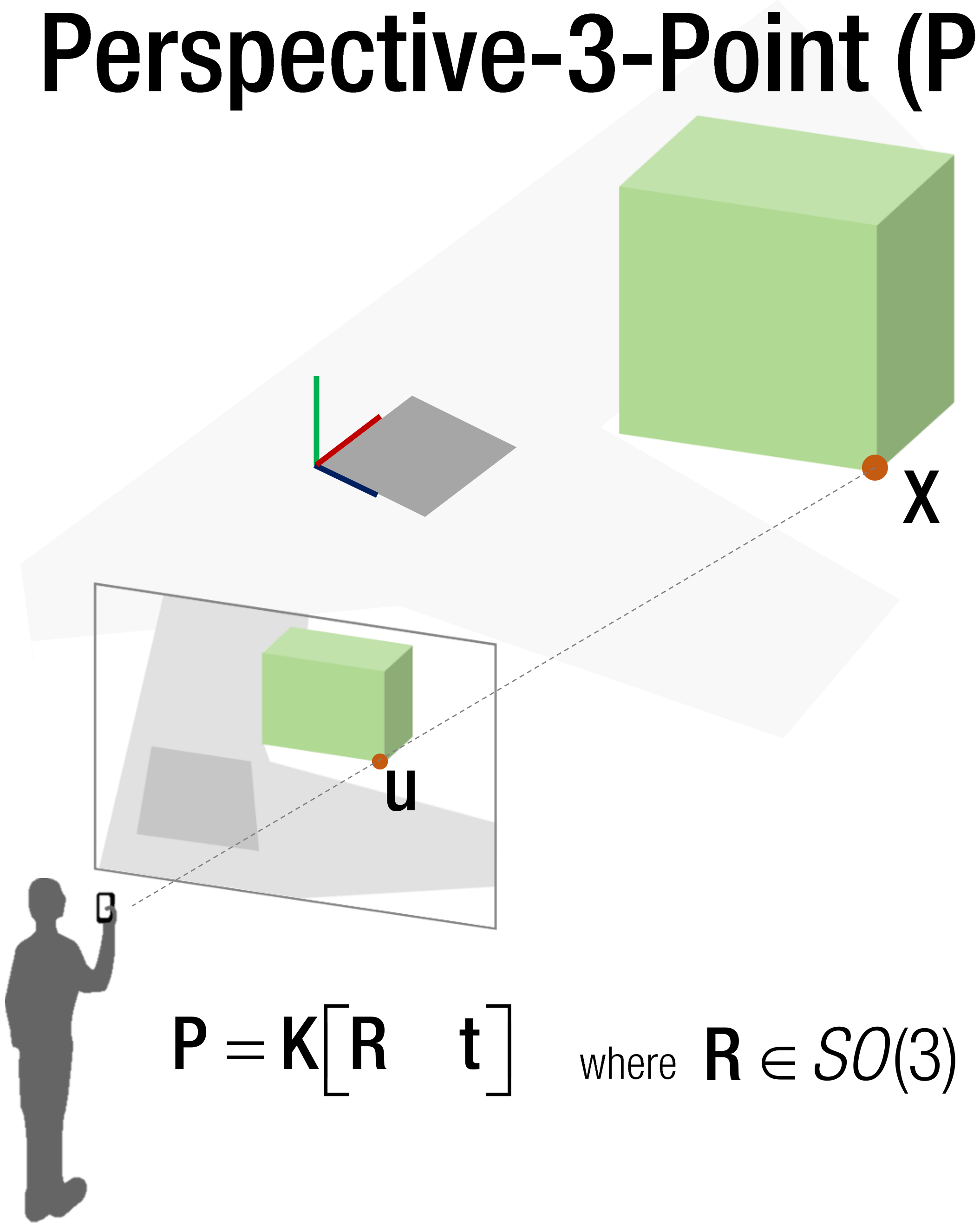
$\longrightarrow \mathbf{t} = \dfrac{\mathbf{K}^{-1}\mathbf{p}_4}{d_{11}}$: Translation and scale recovery

Perspective-3-Point (P3P)



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Linear in camera matrix

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6 dof when **K** is known.

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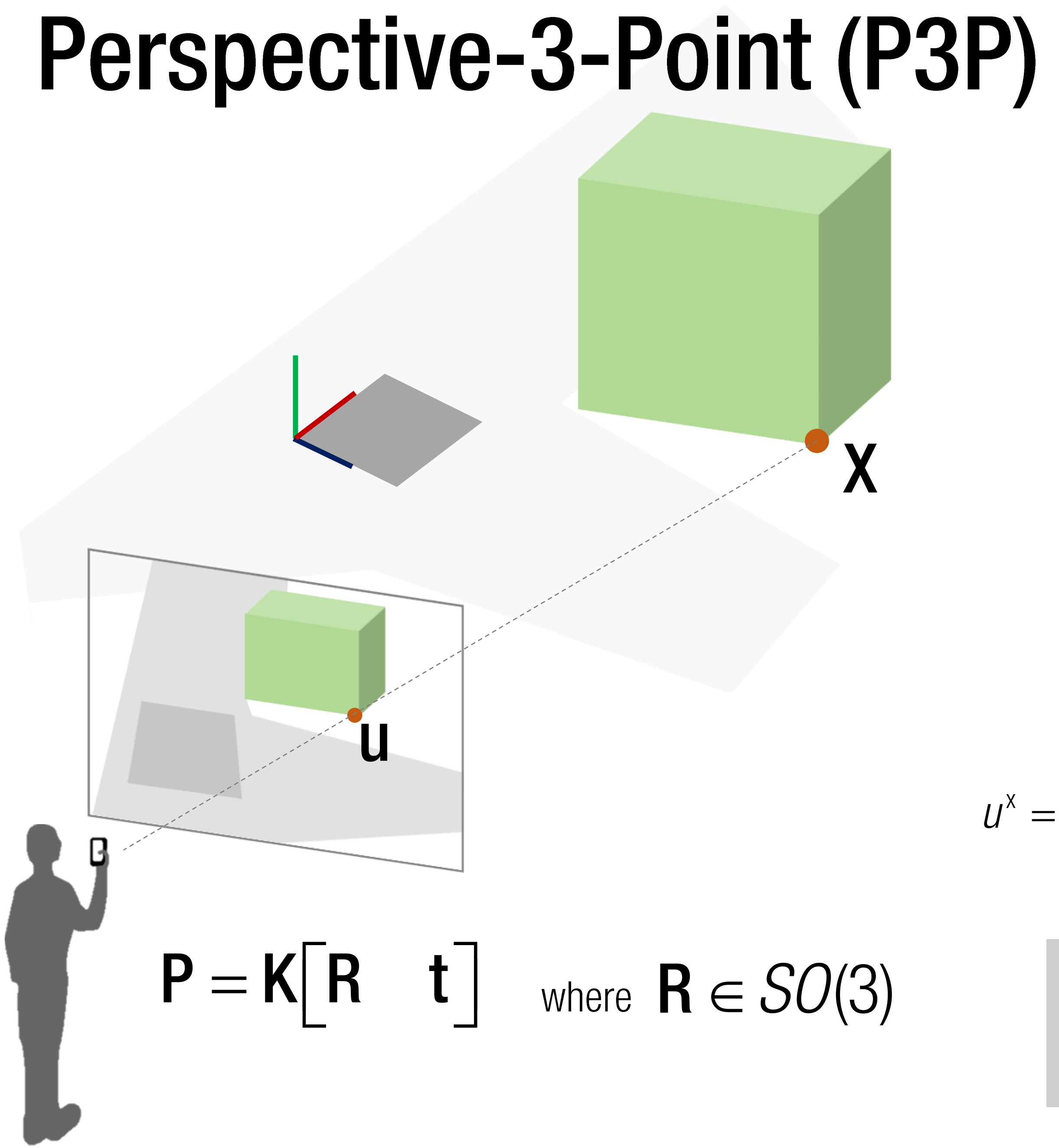

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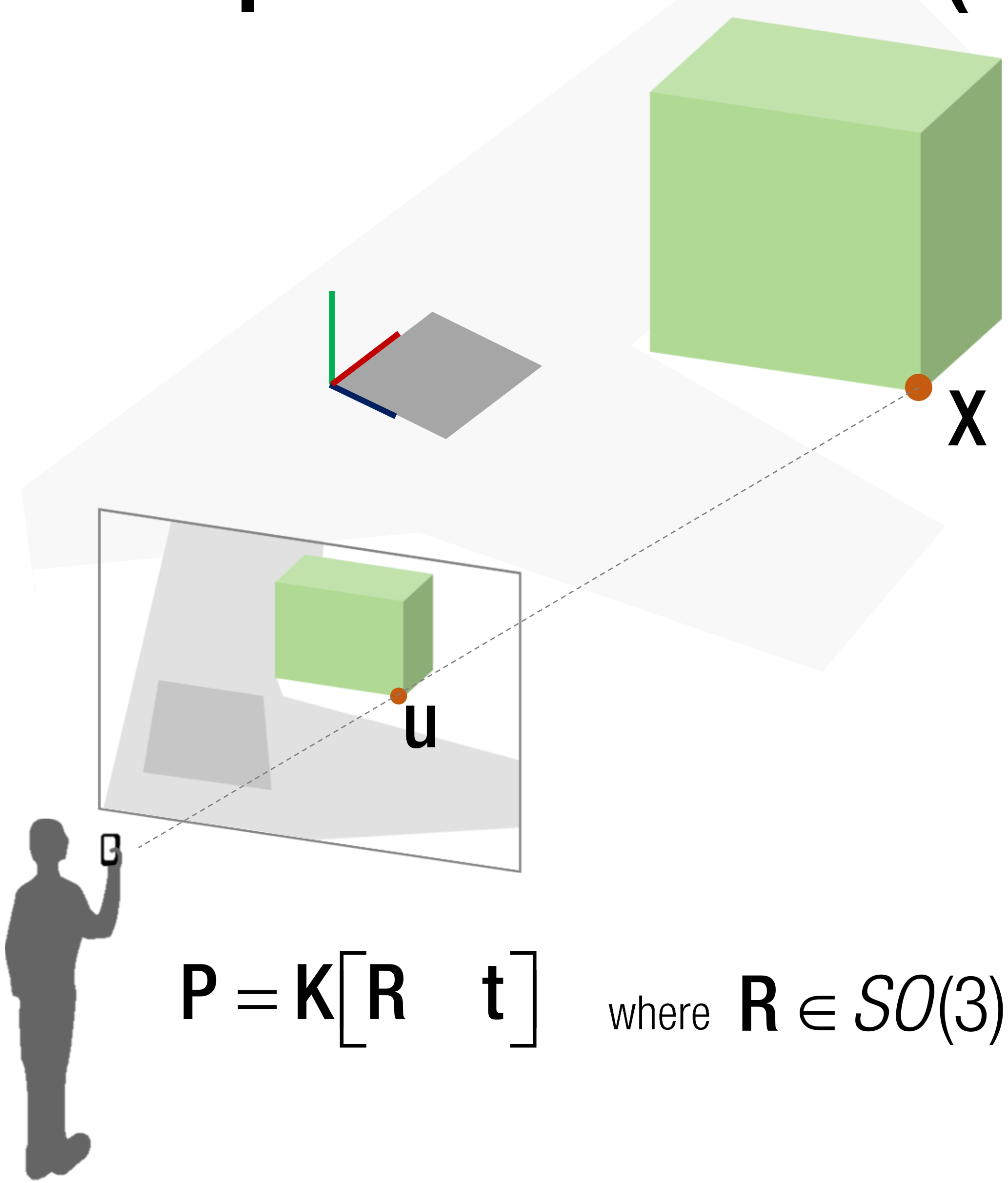

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3 correspondences should be enough.

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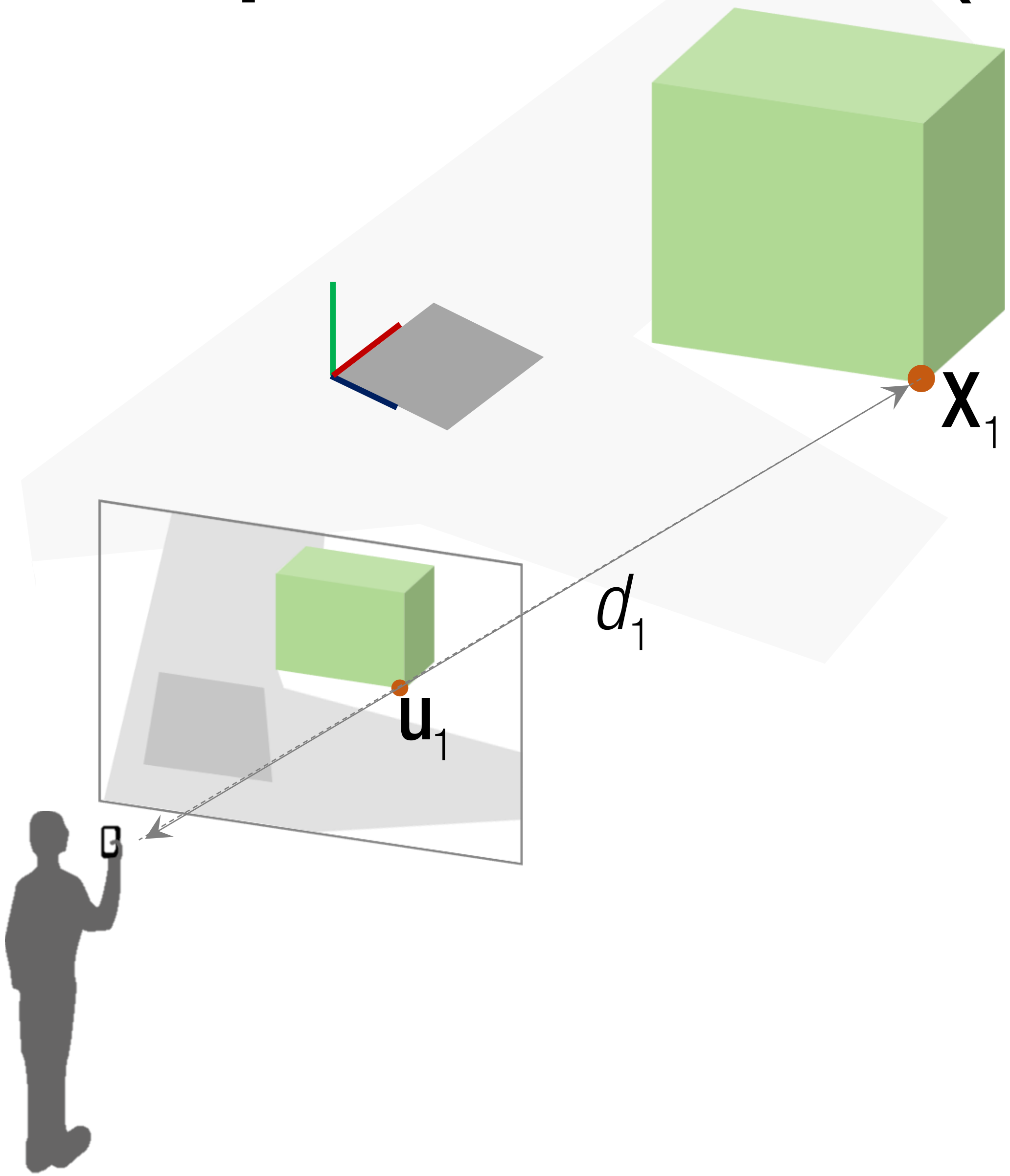
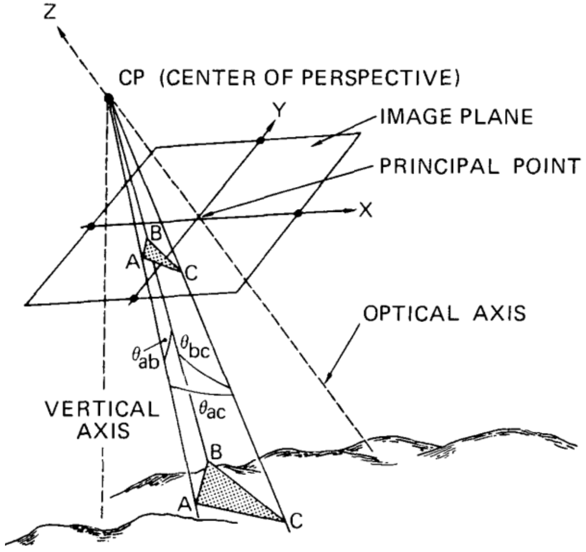


RANSAC with PnP

Graphics and Image Processing — J. D. Foley, Editor

Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography

Martin A. Fischler and Robert C. Bolles
SRI International



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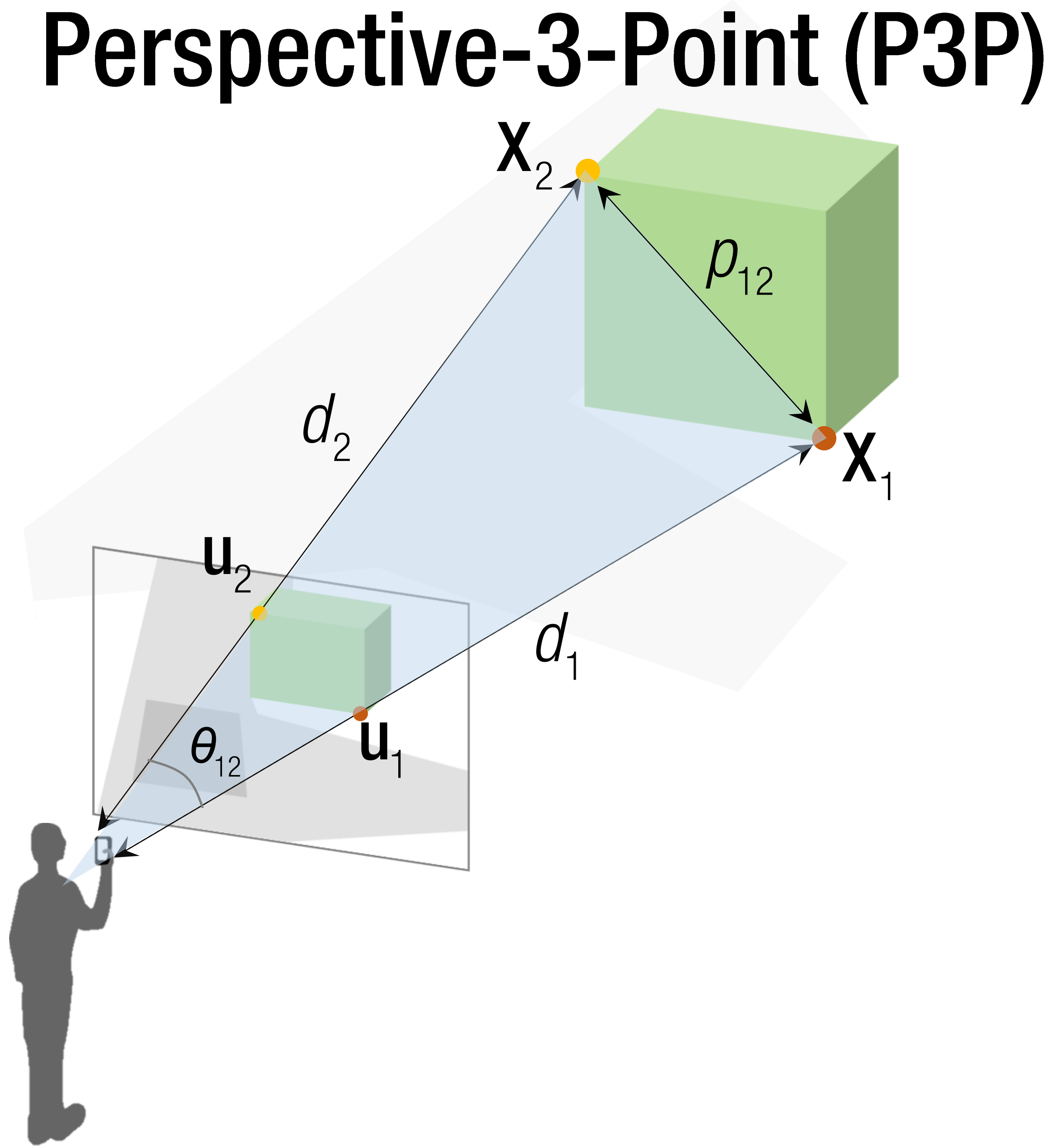
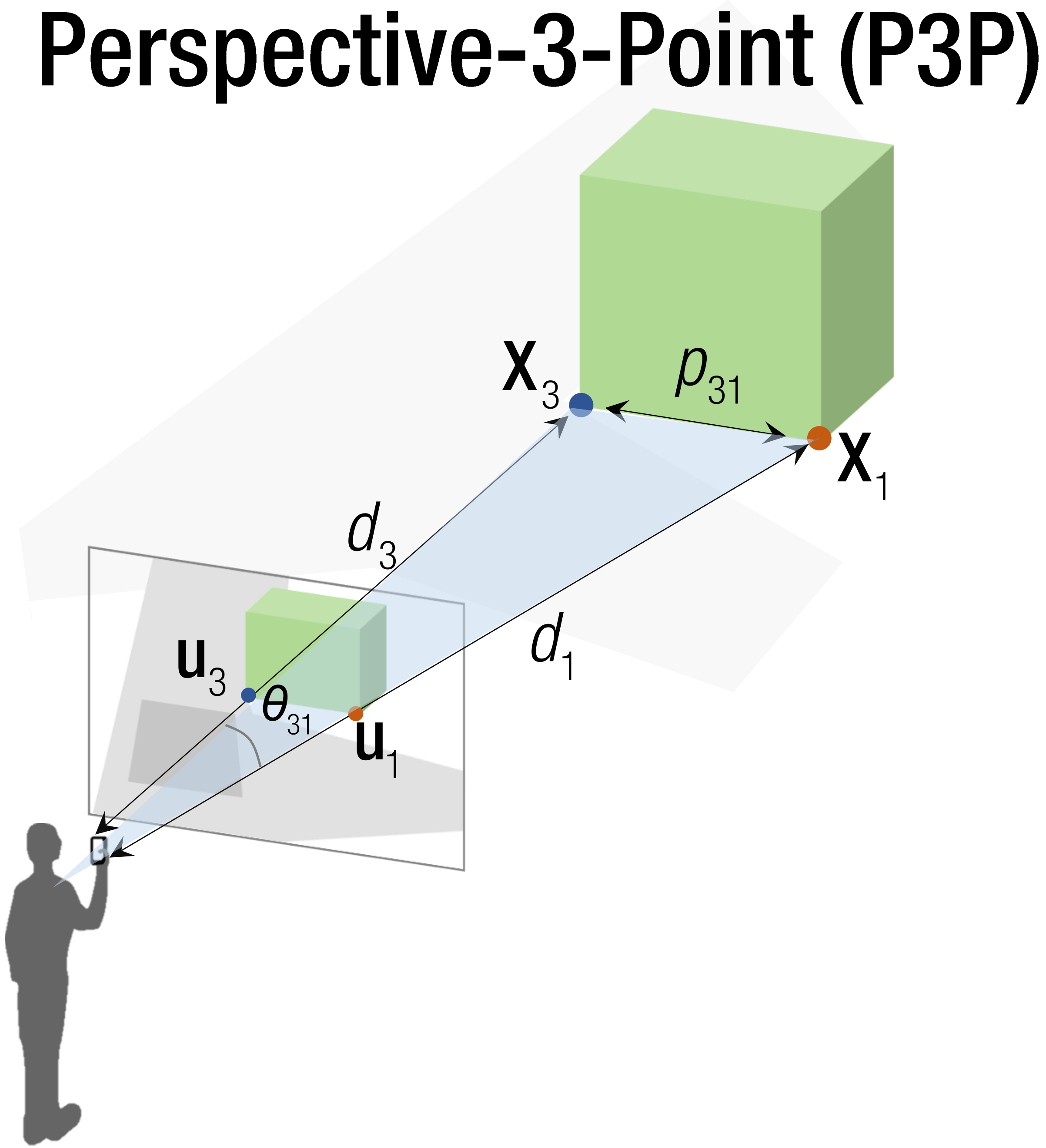


2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\boldsymbol{\theta}_{12} = p_{12}^2$$

Perspective-3-Point (P3P)



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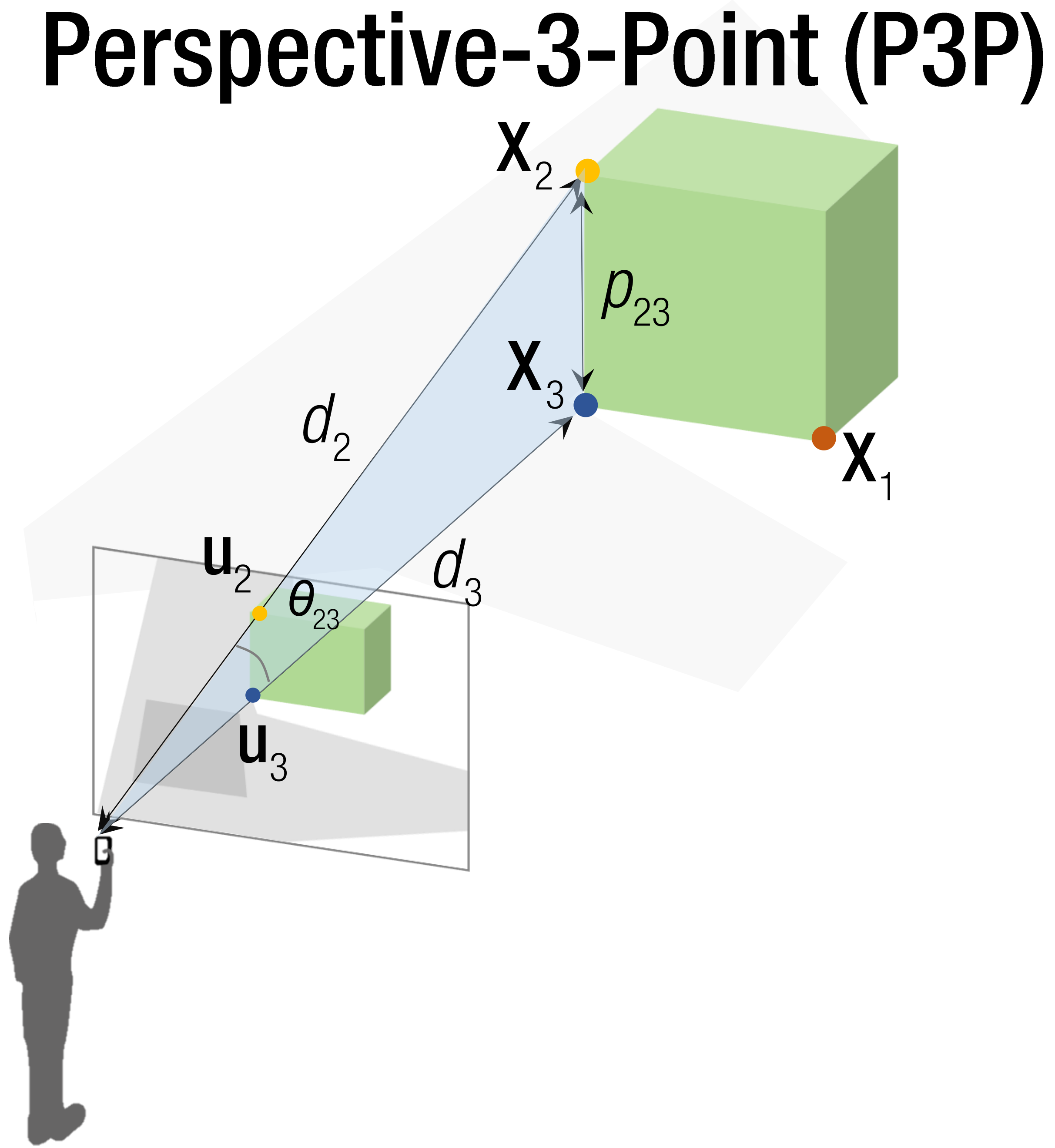

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$$d_2^2 + d_3^2 - 2d_2d_3\cos\theta_{23} = p_{23}^2$$

3 equations

Unknowns: d_1, d_2, d_3

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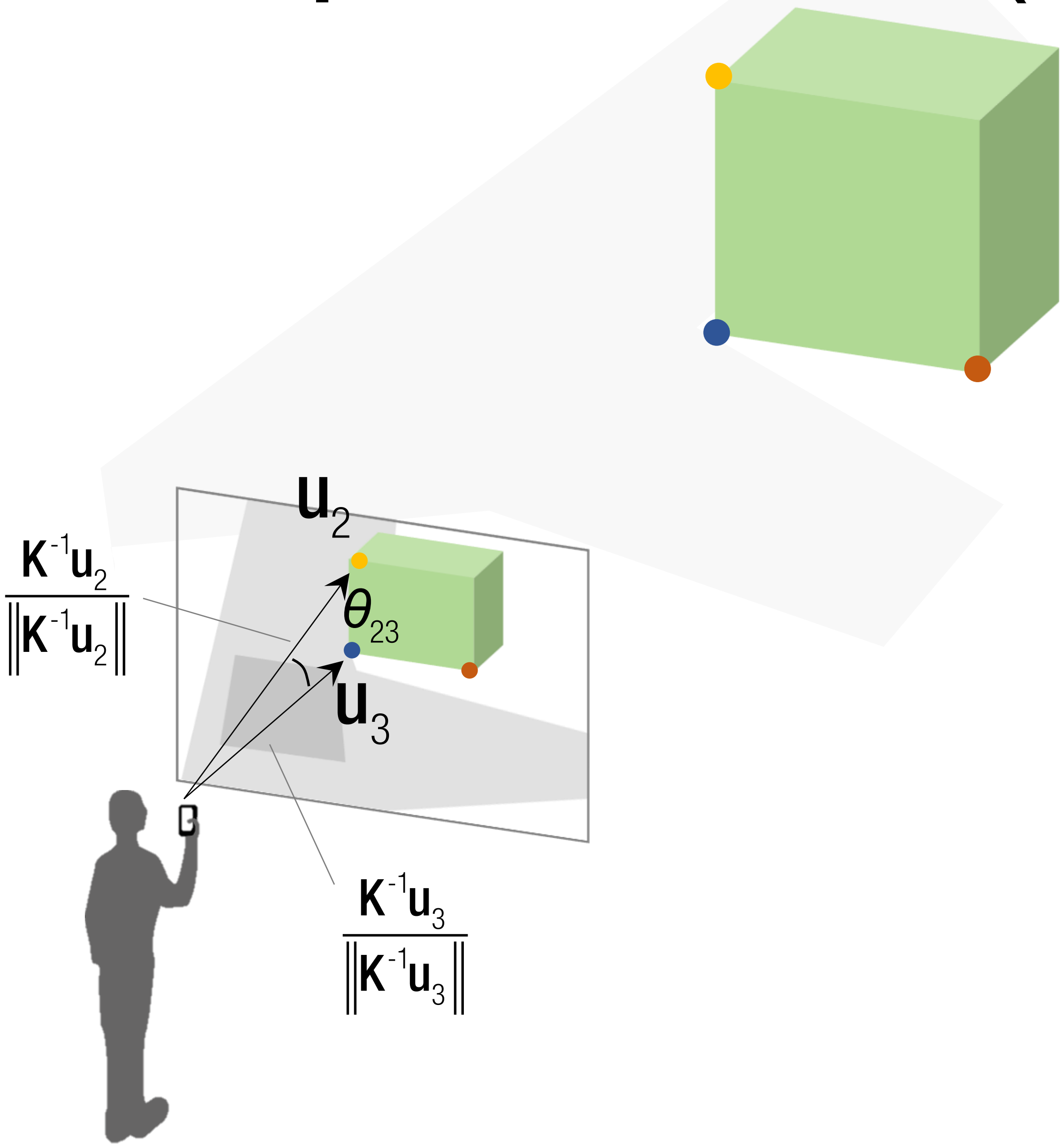


2nd Cosine law:

$$
\left.
\begin{aligned}
d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} &= p_{12}^2 \\
d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} &= p_{31}^2 \\
d_2^2 + d_3^2 - 2d_2d_3\cos\theta_{23} &= p_{23}^2
\end{aligned}
\right| \quad \text{3 equations}
$$

Unknowns: d_1, d_2, d_3

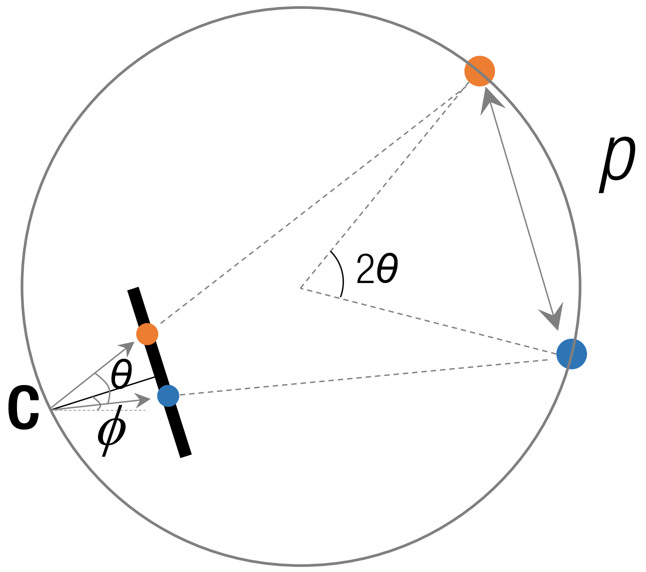
Note:

$$\cos\theta_{12} = \frac{\left(\mathbf{K}^{-1}\mathbf{u}_1\right)^{\mathrm{T}}\left(\mathbf{K}^{-1}\mathbf{u}_2\right)}{\left\|\mathbf{K}^{-1}\mathbf{u}_1\right\|\left\|\mathbf{K}^{-1}\mathbf{u}_2\right\|} \qquad \cos\theta_{23} = \frac{\left(\mathbf{K}^{-1}\mathbf{u}_2\right)^{\mathrm{T}}\left(\mathbf{K}^{-1}\mathbf{u}_3\right)}{\left\|\mathbf{K}^{-1}\mathbf{u}_2\right\|\left\|\mathbf{K}^{-1}\mathbf{u}_3\right\|}$$

$$\cos\theta_{31} = \frac{\left(\mathbf{K}^{-1}\mathbf{u}_1\right)^{\mathrm{T}}\left(\mathbf{K}^{-1}\mathbf{u}_3\right)}{\left\|\mathbf{K}^{-1}\mathbf{u}_1\right\|\left\|\mathbf{K}^{-1}\mathbf{u}_3\right\|}$$

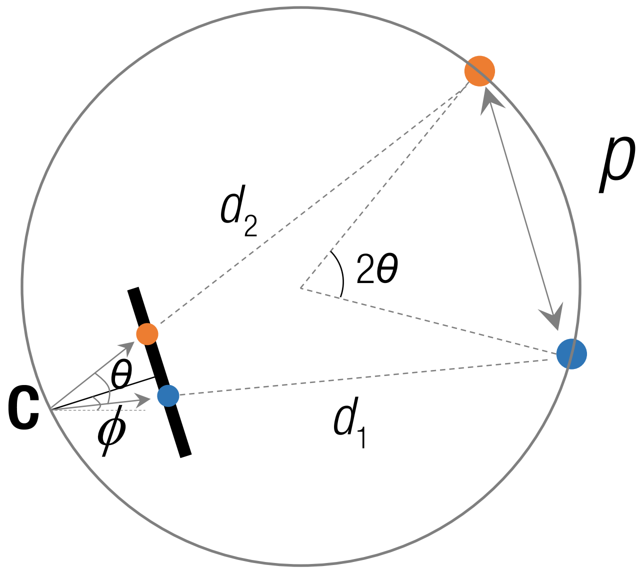
Geometric Interpretation: 1D Camera

Geometric Interpretation: 1D Camera



Property of inscribed angle

Geometric Interpretation: 1D Camera

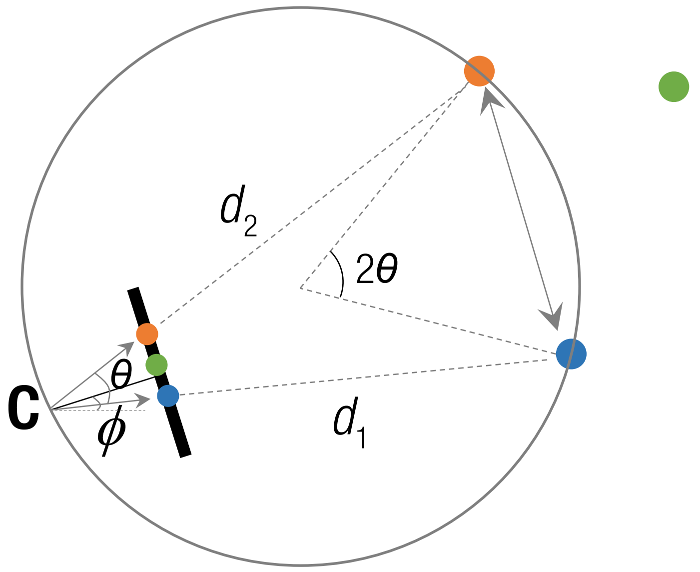


2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta = p^2$$

Infinite number of solutions

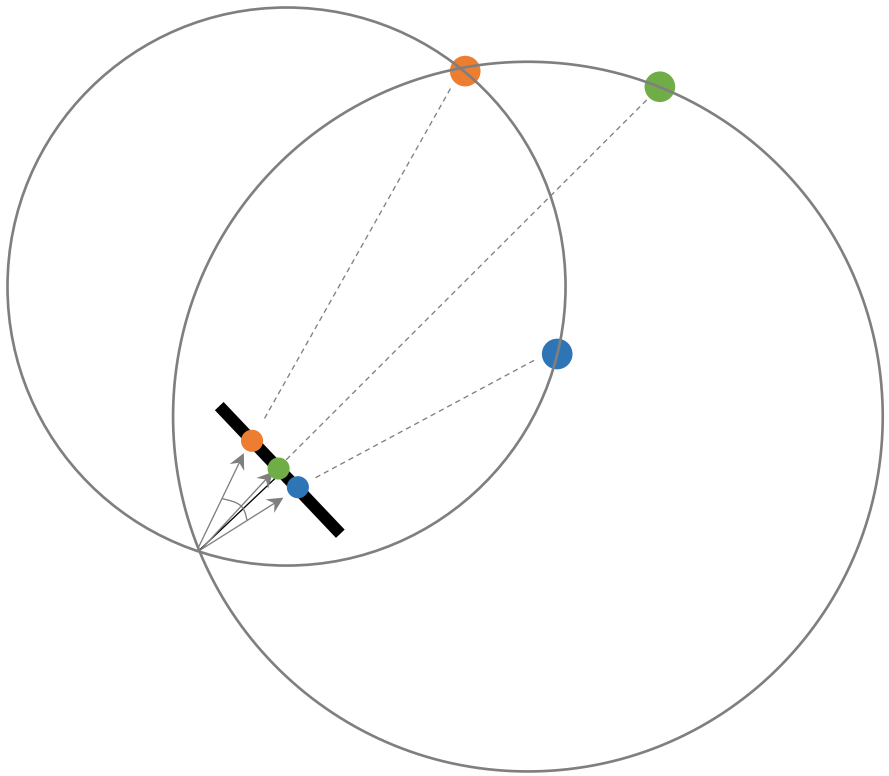
Geometric Interpretation: 1D Camera



2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\boldsymbol{\theta} = p^2$$

Geometric Interpretation: 1D Camera

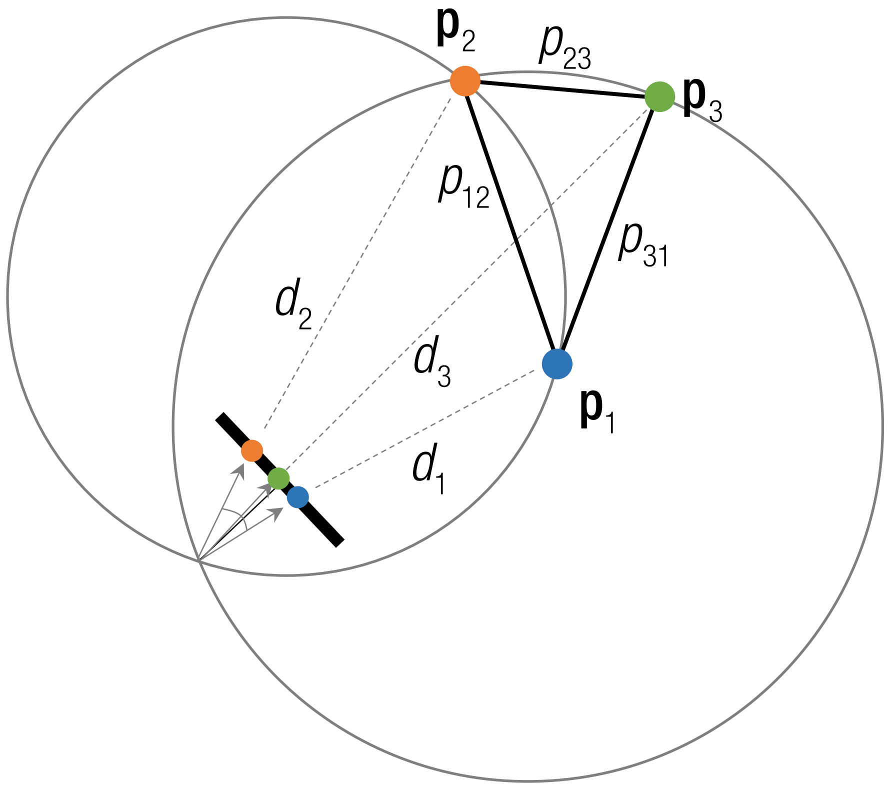


2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta = p^2$$

Finite number of solutions

Geometric Interpretation: 1D Camera

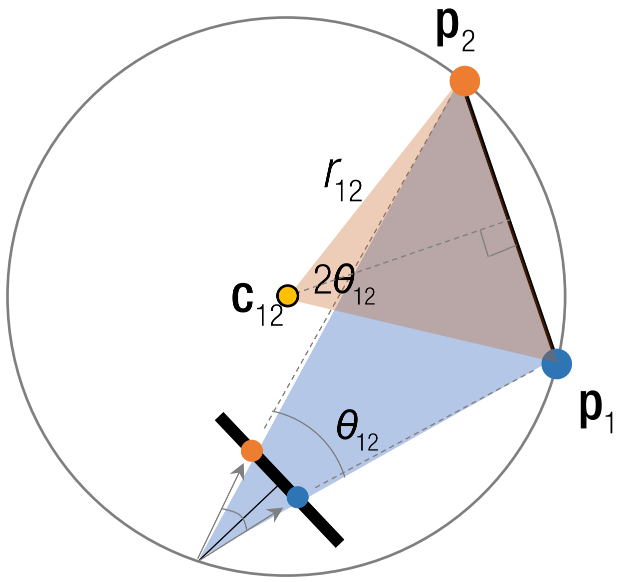


2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_2^2 + d_3^2 - 2d_2d_3\cos\theta_{23} = p_{23}^2$$

Finite number of solutions

Geometric Interpretation: 1D Camera

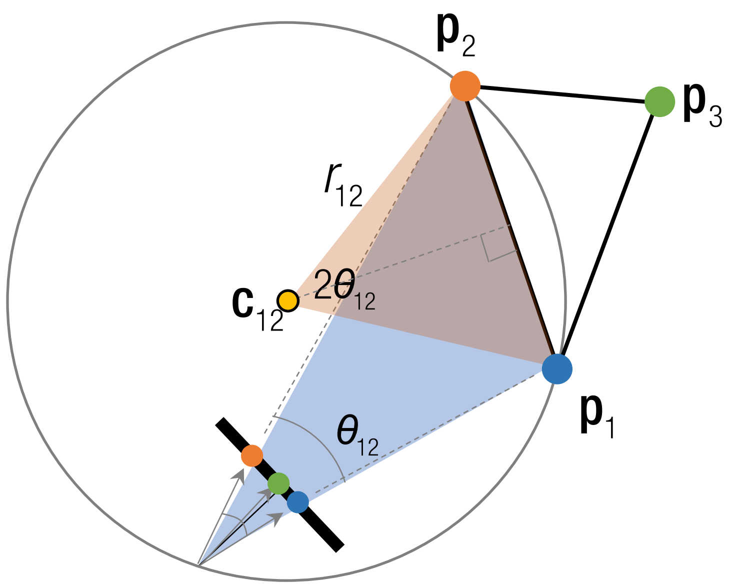


$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \mathbf{u}_{12}$$

$$\mathbf{u}_{12} \perp \mathbf{p}_2 - \mathbf{p}_1$$

where $r_{12} = \dfrac{\|\mathbf{p}_2 - \mathbf{p}_1\|}{2\sin\theta_{12}}$

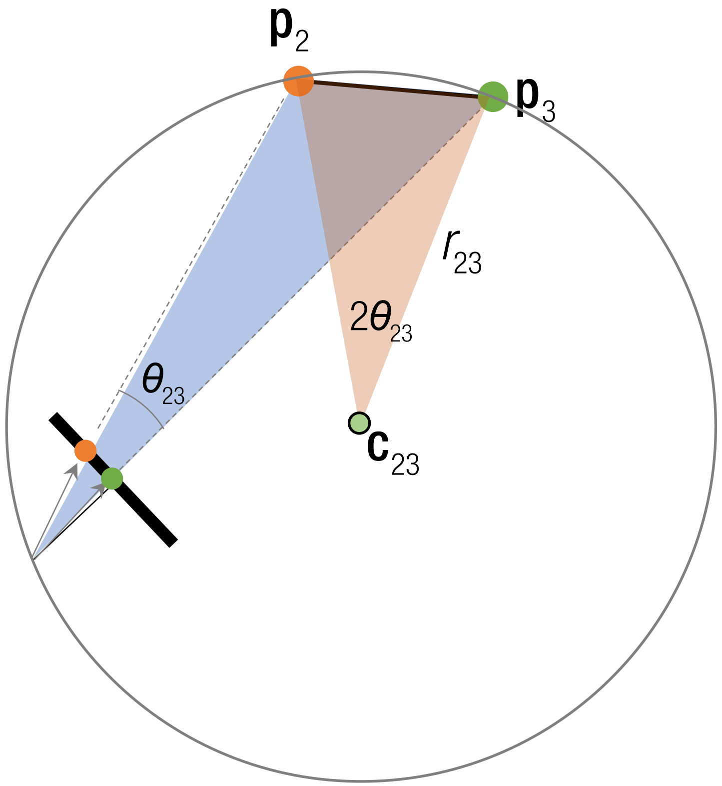
Geometric Interpretation: 1D Camera



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \mathbf{u}_{12} \qquad \mathbf{u}_{12} \perp \mathbf{p}_2 - \mathbf{p}_1$$

where $r_{12} = \dfrac{\left\| \mathbf{p}_2 - \mathbf{p}_1 \right\|}{2\sin\theta_{12}}$

Geometric Interpretation: 1D Camera



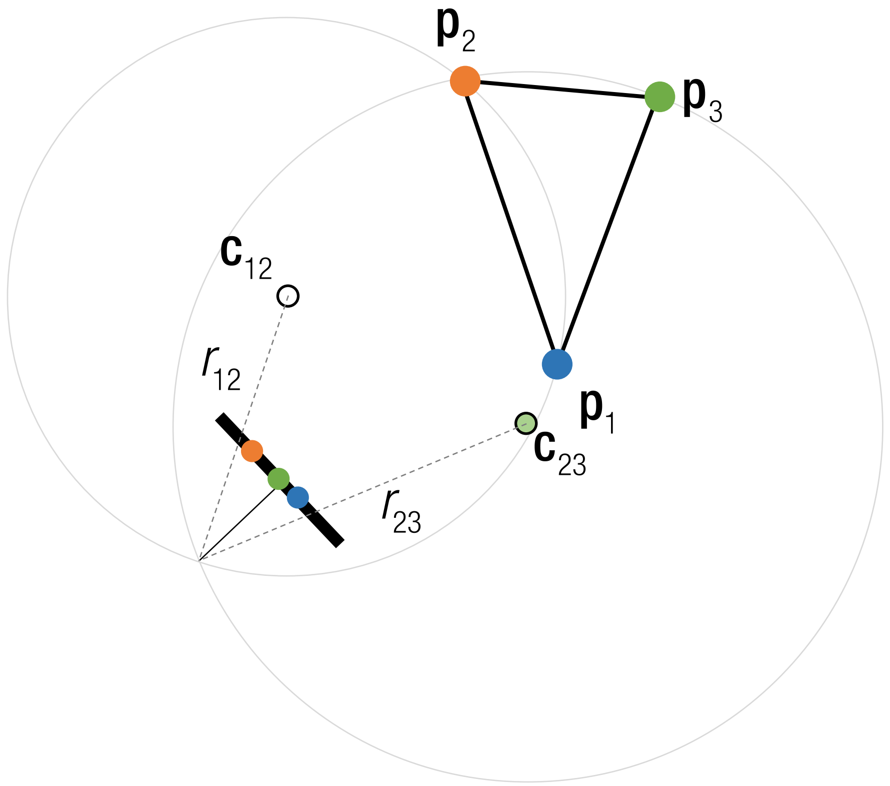
$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \mathbf{u}_{12} \qquad \mathbf{u}_{12} \perp \mathbf{p}_2 - \mathbf{p}_1$$

where $r_{12} = \dfrac{\|\mathbf{p}_2 - \mathbf{p}_1\|}{2\sin\theta_{12}}$

$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \mathbf{u}_{23} \qquad \mathbf{u}_{23} \perp \mathbf{p}_3 - \mathbf{p}_2$$

where $r_{23} = \dfrac{\|\mathbf{p}_3 - \mathbf{p}_2\|}{2\sin\theta_{23}}$

Geometric Interpretation: 1D Camera



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \mathbf{u}_{12} \qquad \mathbf{u}_{12} \perp \mathbf{p}_2 - \mathbf{p}_1$$

$$\text{where} \quad r_{12} = \frac{\|\mathbf{p}_2 - \mathbf{p}_1\|}{2\sin\theta_{12}}$$

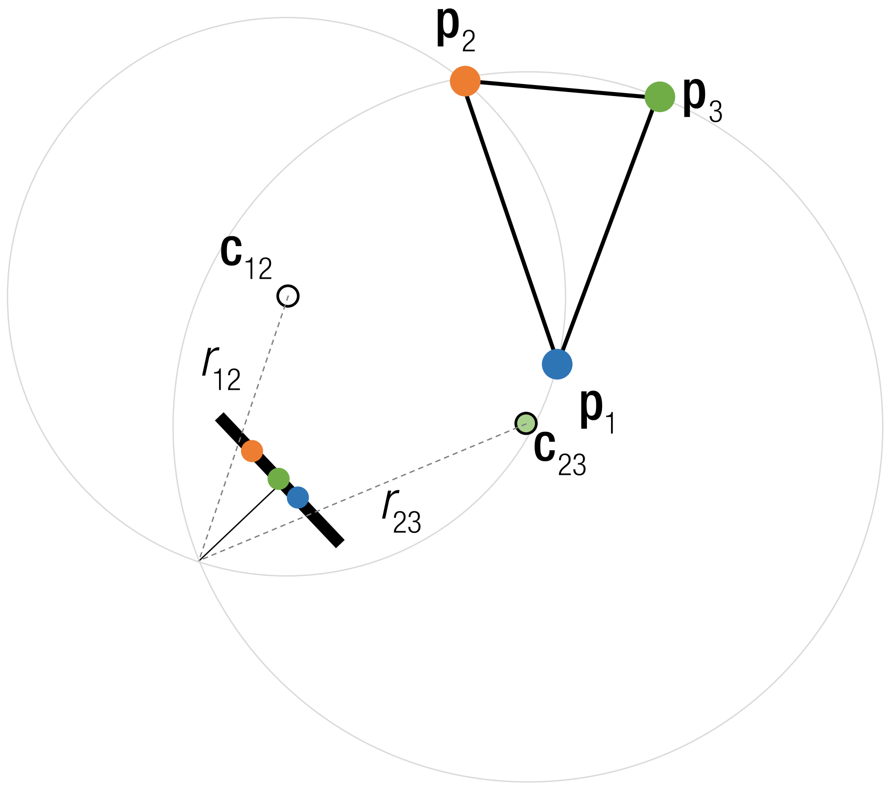
$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \mathbf{u}_{23} \qquad \mathbf{u}_{23} \perp \mathbf{p}_3 - \mathbf{p}_2$$

$$\text{where} \quad r_{23} = \frac{\|\mathbf{p}_3 - \mathbf{p}_2\|}{2\sin\theta_{23}}$$

$$\|\mathbf{x} - \mathbf{c}_{12}\|^2 = r_{12}^2 \qquad \|\mathbf{x} - \mathbf{c}_{23}\|^2 = r_{23}^2$$

HW: Drive **x** and orientation.

Geometric Interpretation: Family of Solutions



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \mathbf{u}_{12} \qquad \mathbf{u}_{12} \perp \mathbf{p}_2 - \mathbf{p}_1$$

where $r_{12} = \dfrac{\|\mathbf{p}_2 - \mathbf{p}_1\|}{2\sin\theta_{12}}$

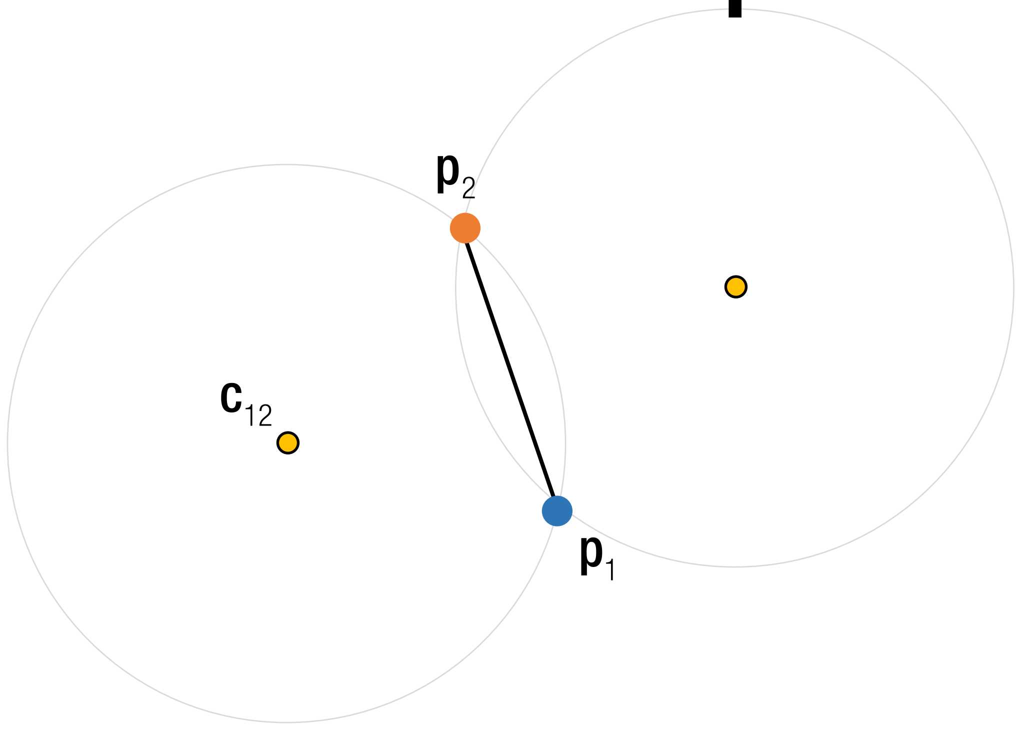
$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \mathbf{u}_{23} \qquad \mathbf{u}_{23} \perp \mathbf{p}_3 - \mathbf{p}_2$$

where $r_{23} = \dfrac{\|\mathbf{p}_3 - \mathbf{p}_2\|}{2\sin\theta_{23}}$

$$\|\mathbf{x} - \mathbf{c}_{12}\|^2 = r_{12}^2 \qquad \|\mathbf{x} - \mathbf{c}_{23}\|^2 = r_{23}^2$$

HW: Drive **x** and orientation.

Geometric Interpretation: Family of Solutions

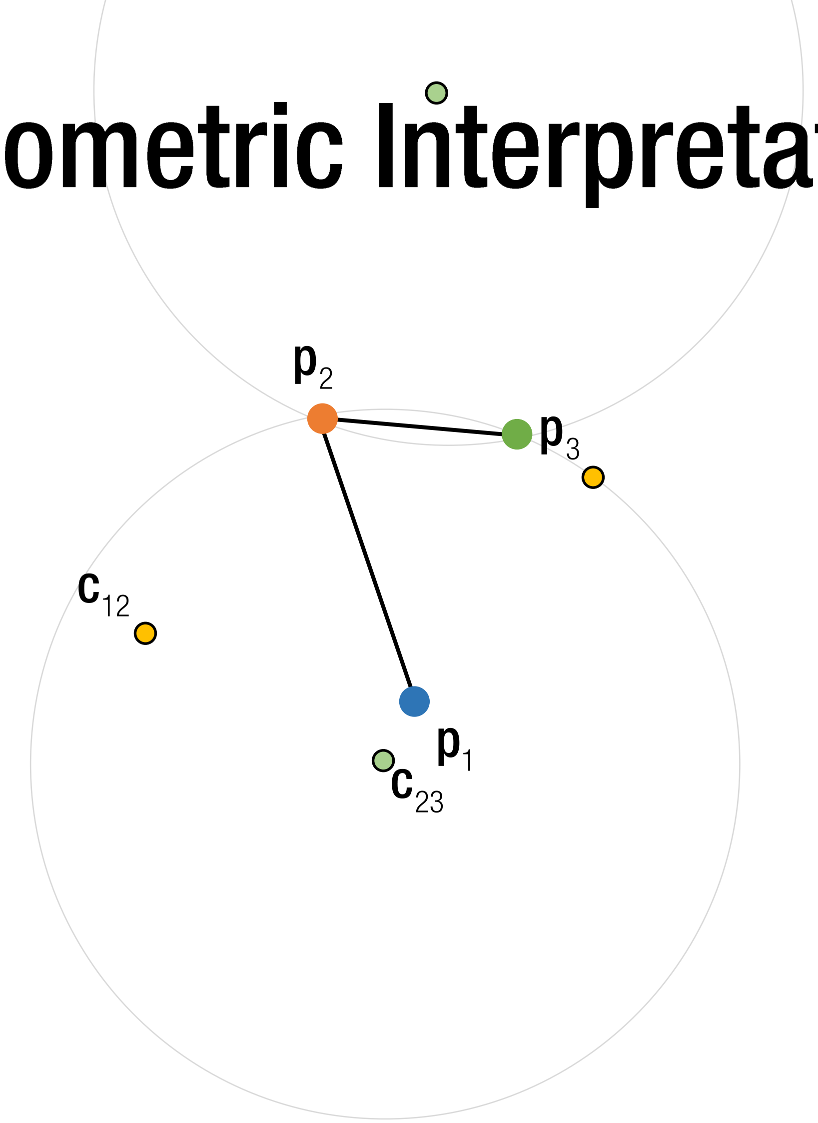


$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \mathbf{u}_{12}$$

$$\mathbf{u}_{12} \perp \mathbf{p}_2 - \mathbf{p}_1$$

where $r_{12} = \dfrac{\left\| \mathbf{p}_2 - \mathbf{p}_1 \right\|}{2\sin\theta_{12}}$

Geometric Interpretation: Family of Solutions



4 combinations

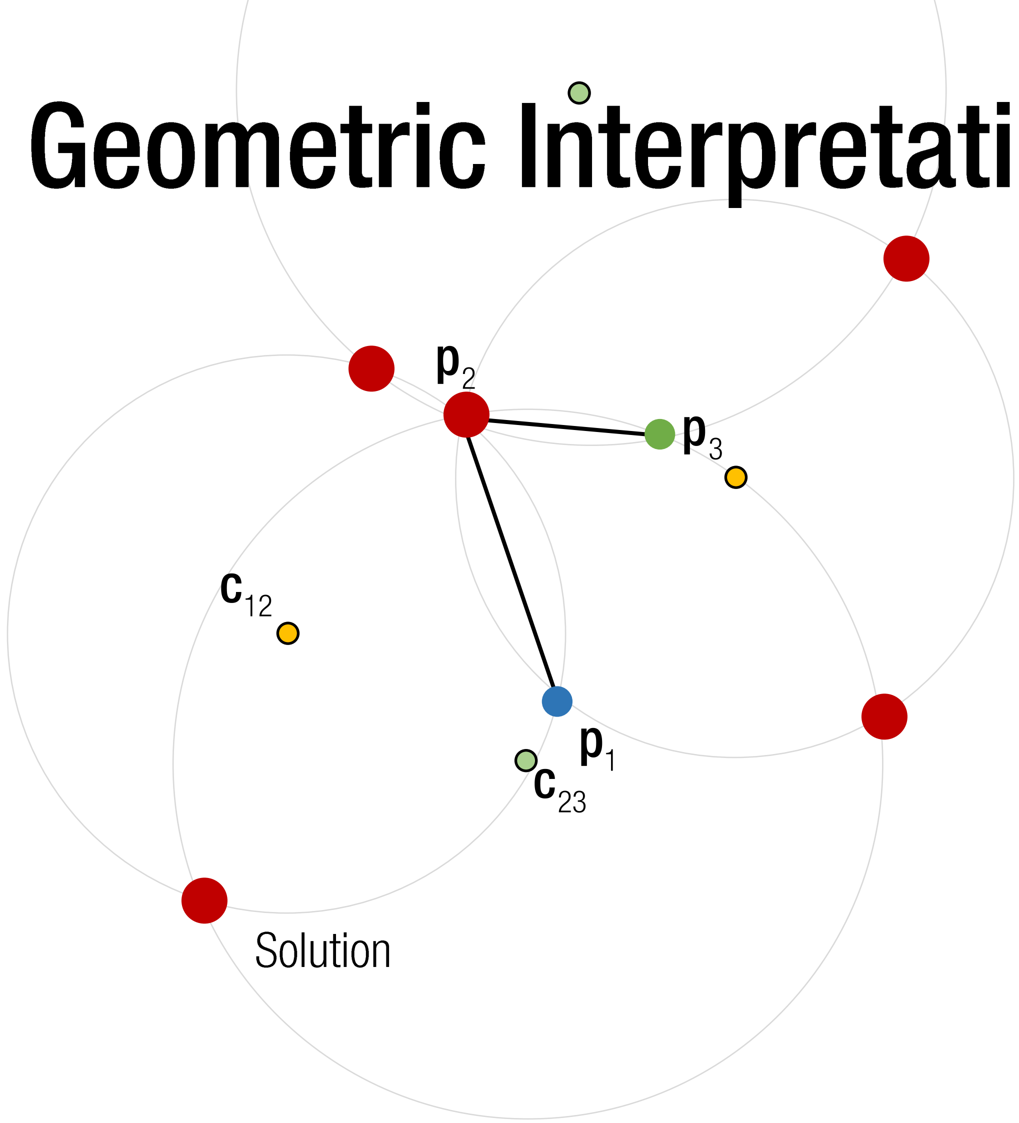
$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \mathbf{u}_{12} \qquad \mathbf{u}_{12} \perp \mathbf{p}_2 - \mathbf{p}_1$$

where $r_{12} = \dfrac{\|\mathbf{p}_2 - \mathbf{p}_1\|}{2\sin\theta_{12}}$

$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \mathbf{u}_{23} \qquad \mathbf{u}_{23} \perp \mathbf{p}_3 - \mathbf{p}_2$$

where $r_{23} = \dfrac{\|\mathbf{p}_3 - \mathbf{p}_2\|}{2\sin\theta_{23}}$

Geometric Interpretation: Family of Solutions



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \mathbf{u}_{12} \qquad \mathbf{u}_{12} \perp \mathbf{p}_2 - \mathbf{p}_1$$

$$\text{where} \quad r_{12} = \frac{\|\mathbf{p}_2 - \mathbf{p}_1\|}{2\sin\theta_{12}}$$

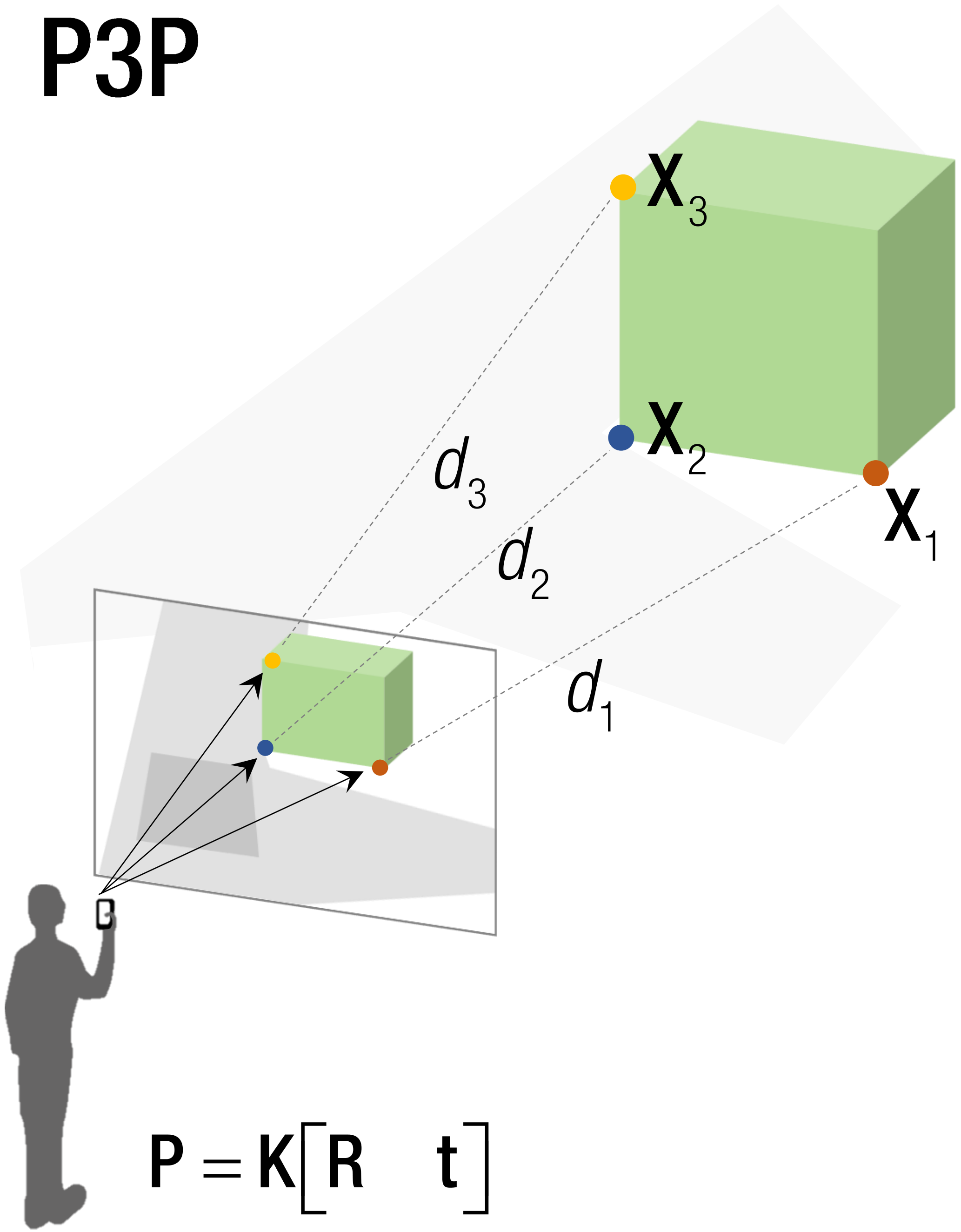
$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \mathbf{u}_{23} \qquad \mathbf{u}_{23} \perp \mathbf{p}_3 - \mathbf{p}_2$$

$$\text{where} \quad r_{23} = \frac{\|\mathbf{p}_3 - \mathbf{p}_2\|}{2\sin\theta_{23}}$$

4 combinations of circle centers

→ 4 solutions except for $\mathbf{p}_2$ ($\mathbf{p}_2$ is counted four times.).

P3P



$$P = K\begin{bmatrix} R & t \end{bmatrix}$$

2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} = p_{23}^2$$

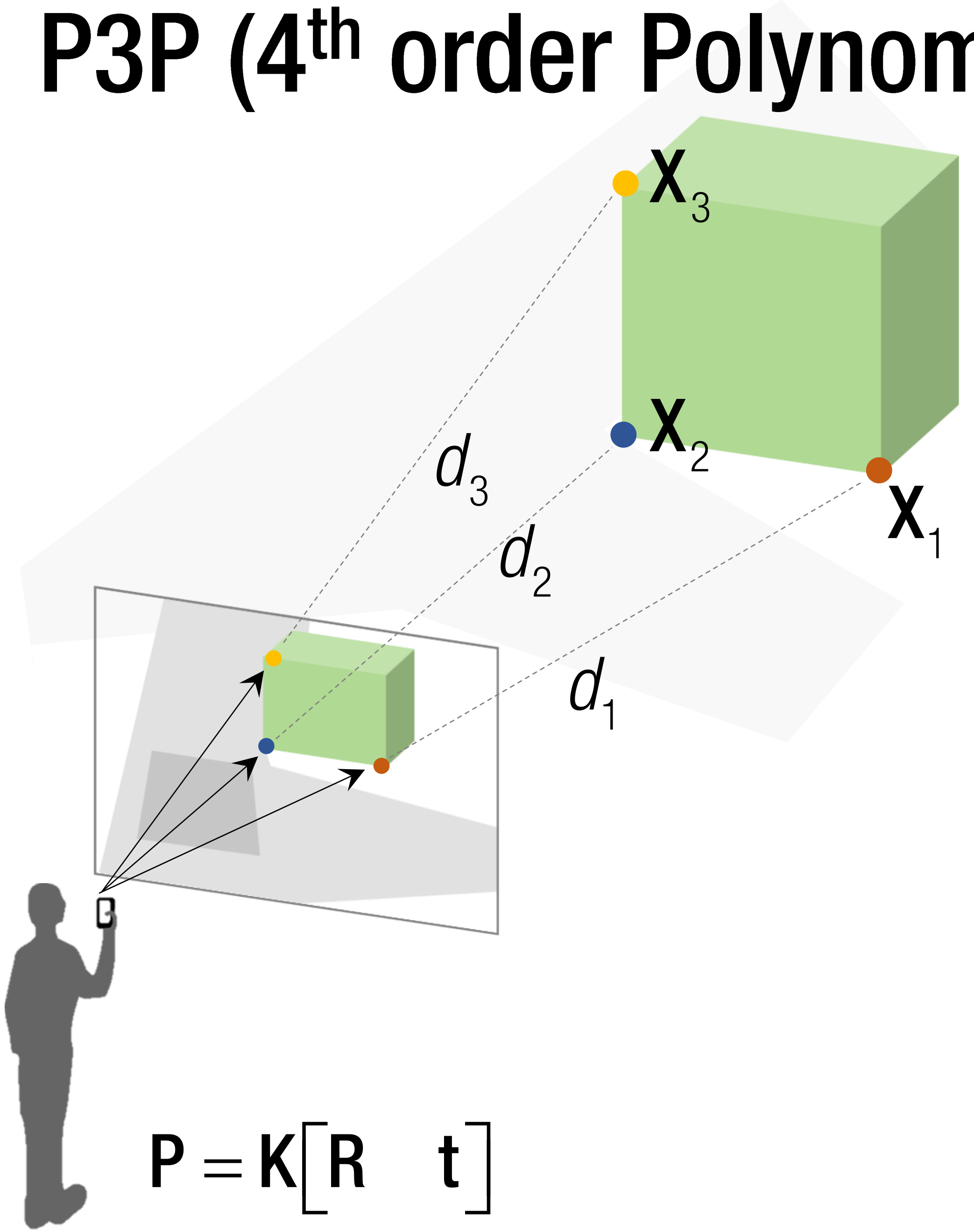
3 equations

The number of possible solutions: 8 = 2x2x2

$d_1 > 0 \quad d_2 > 0 \quad d_3 > 0$: 4 = 2x2x2/2

⟶ requires additional fourth point to verify the solution.

P3P (4th order Polynomial)



$$\mathrm{P} = \mathrm{K}\begin{bmatrix}\mathrm{R} & \mathrm{t}\end{bmatrix}$$

2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} = p_{23}^2$$

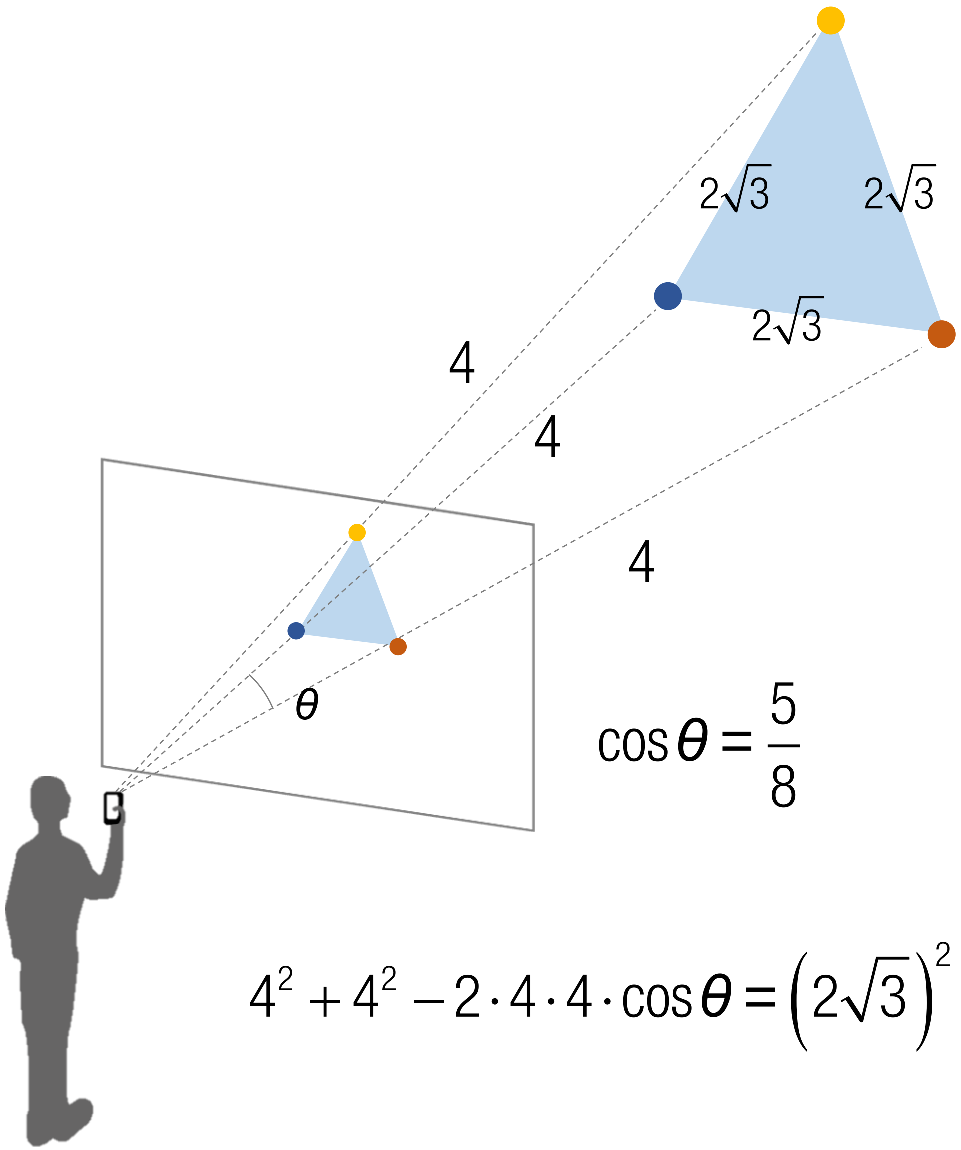
3 equations

4th order polynomial:

$$\rightarrow a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$$

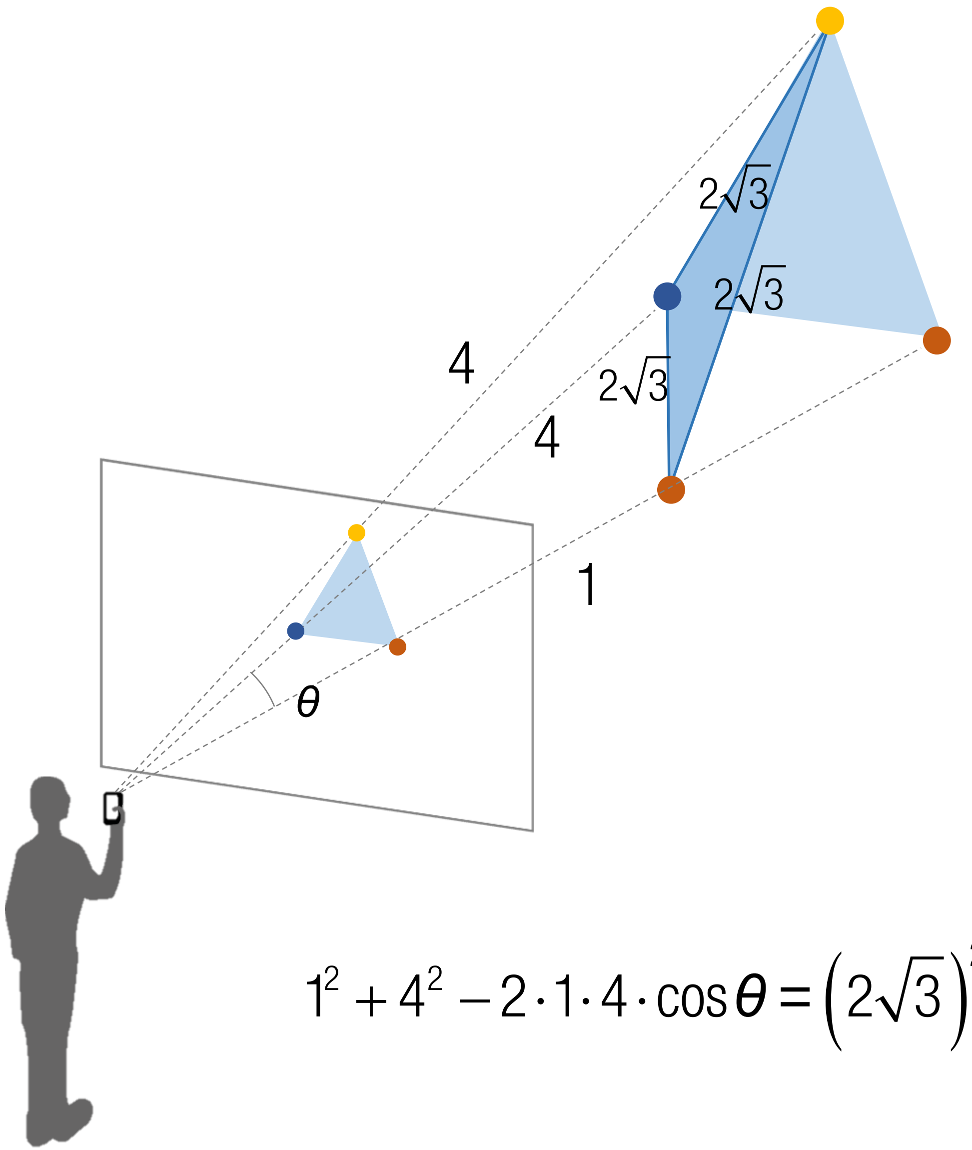
Closed form solutions exist.

Four Solution Example



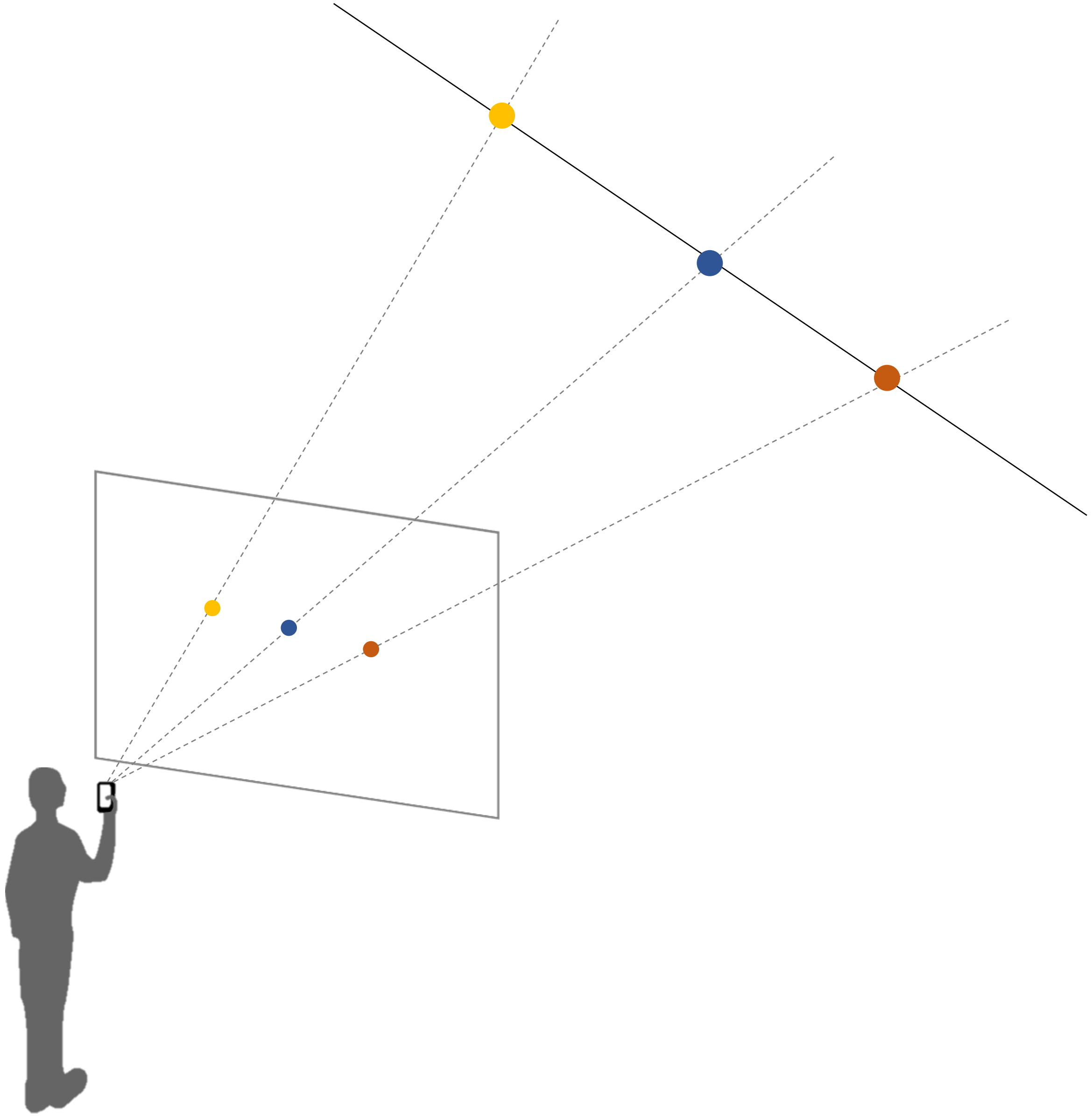
$$\cos\theta = \frac{5}{8}$$

$$4^2 + 4^2 - 2\cdot4\cdot4\cdot\cos\theta = \left(2\sqrt{3}\right)^2$$

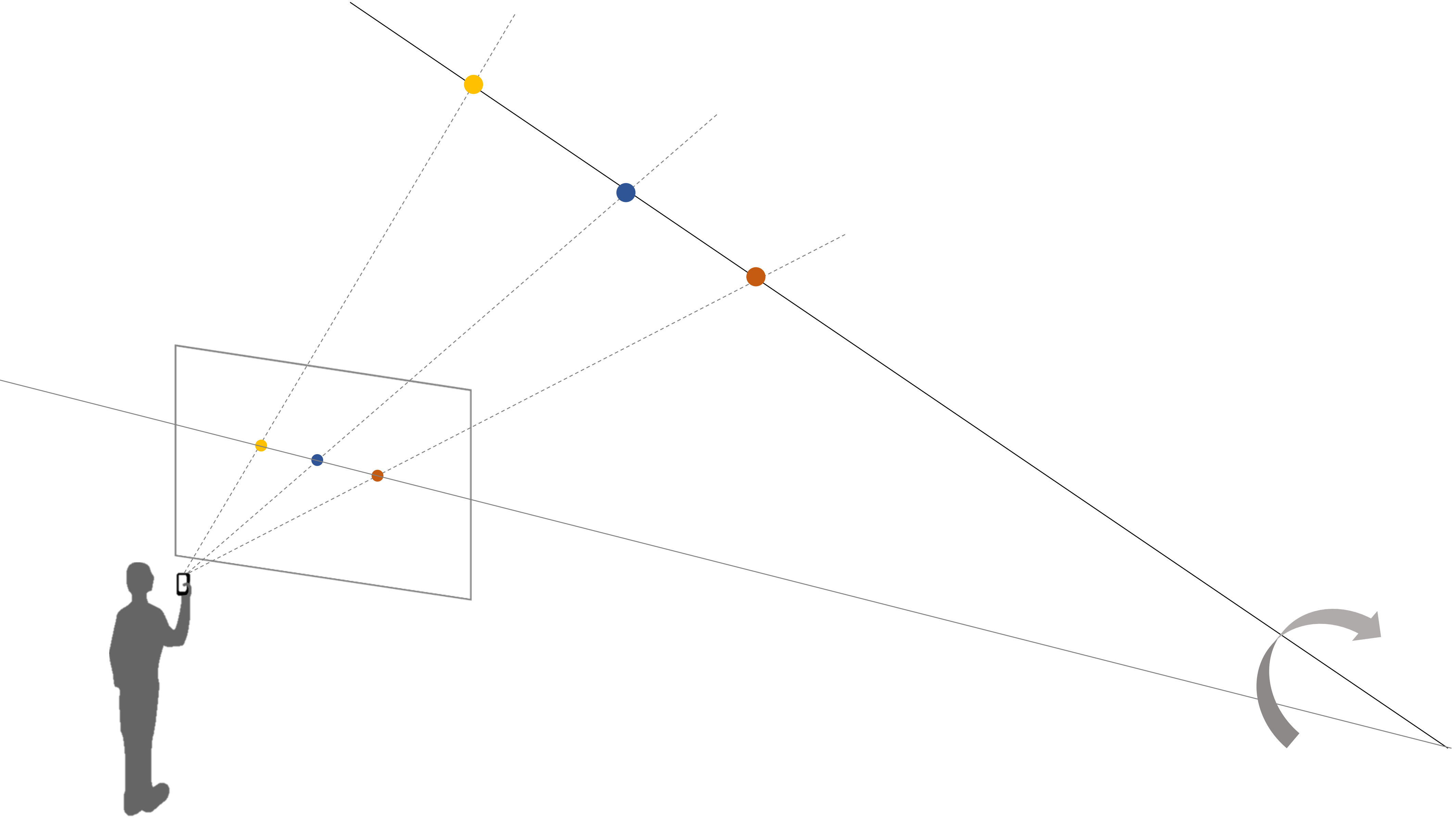


$$1^2 + 4^2 - 2\cdot1\cdot4\cdot\cos\theta = \left(2\sqrt{3}\right)^2$$

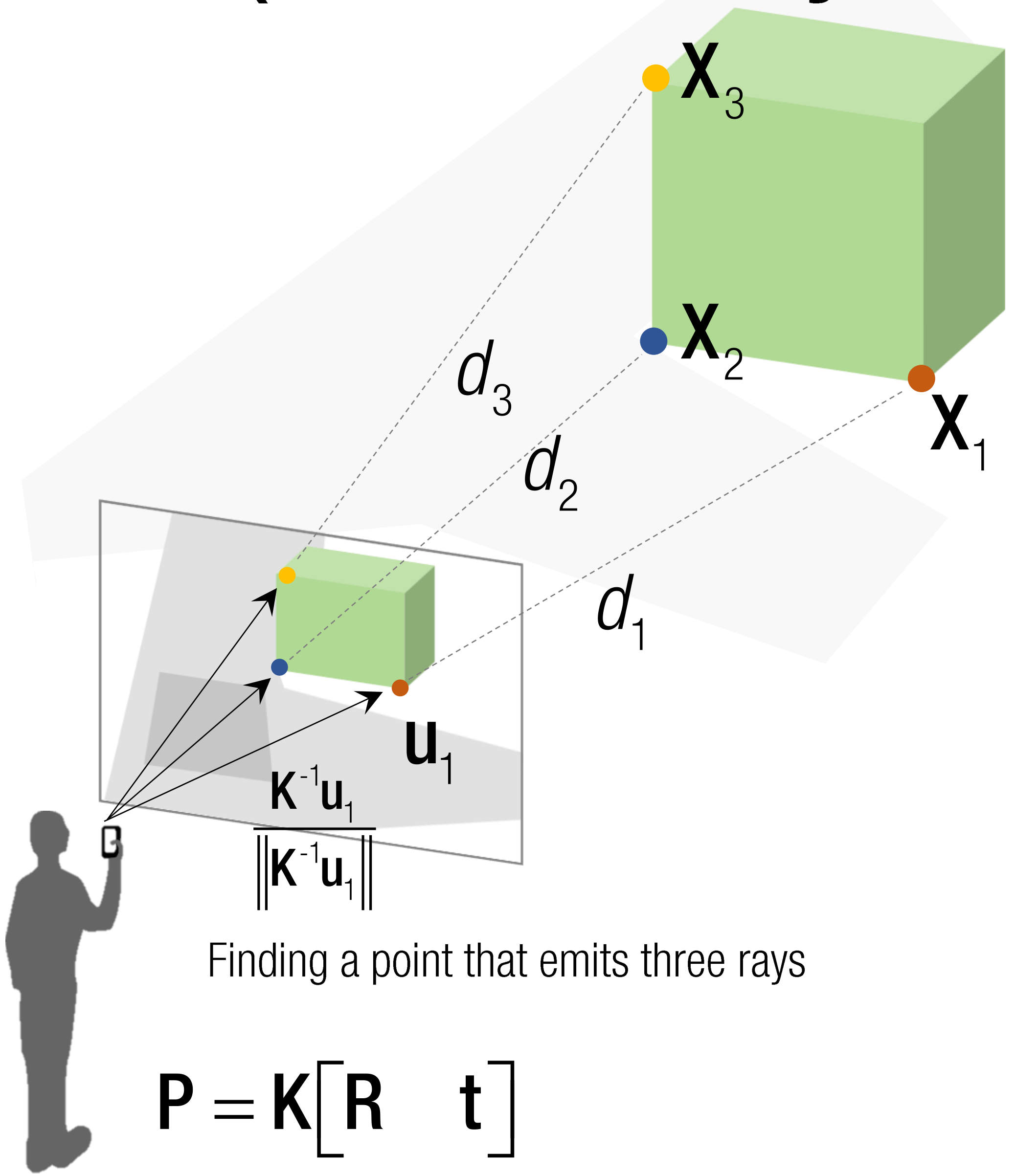
Ambiguity



Ambiguity (Colinear points)



P3P (4th order Polynomial)



Finding a point that emits three rays

$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} = p_{23}^2$$

3 equations

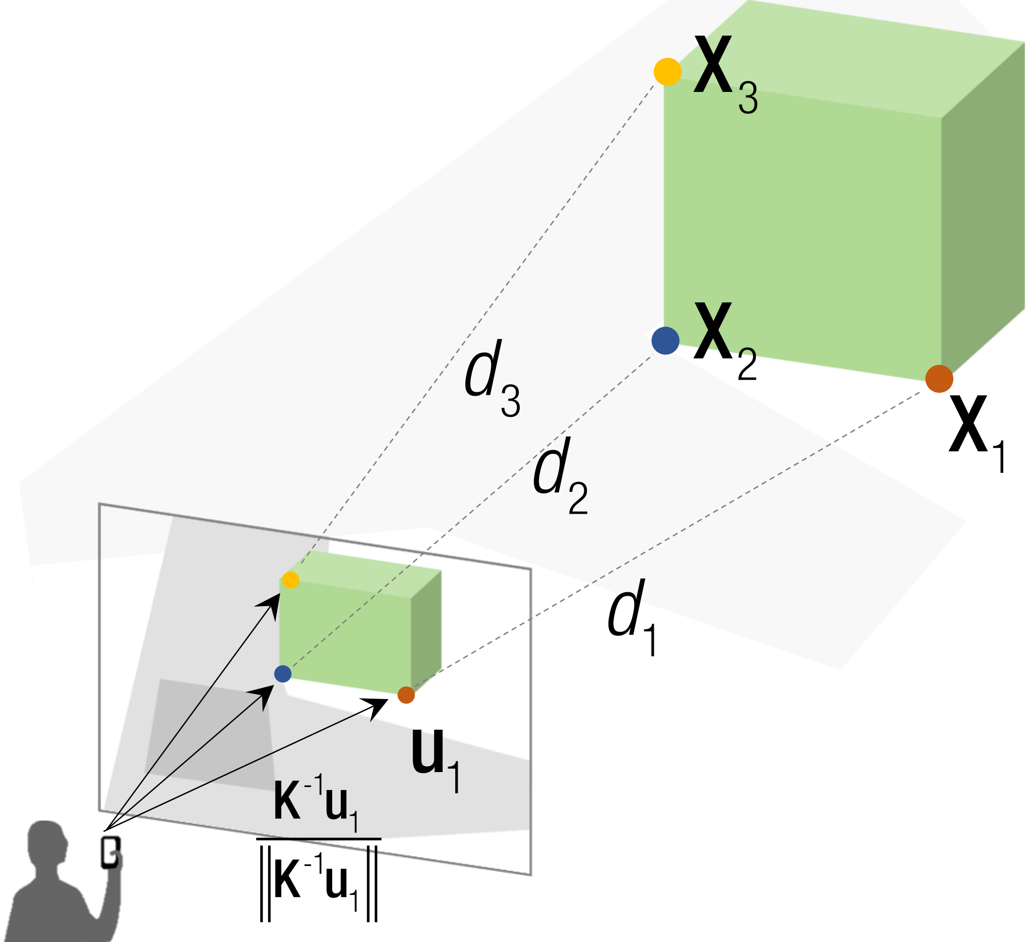
4th order polynomial:

→ $a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$

Closed form solutions exist.

→ Compute **t** using $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$, d_1, d_2, and d_3.

P3P (4^{th} order Polynomial)



Finding a point that emits three rays

$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

2^{nd} Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
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$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} = p_{23}^2$$

3 equations

4^{th} order polynomial:

$$\rightarrow a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$$

Closed form solutions exist.

→ Compute **t** using $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$, d_1, d_2, and d_3.

$$\rightarrow \begin{bmatrix}\tilde{\mathbf{X}}_1 & \tilde{\mathbf{X}}_2 & \tilde{\mathbf{X}}_3\end{bmatrix} = \mathbf{R}\begin{bmatrix}\mathbf{X}_1 & \mathbf{X}_2 & \mathbf{X}_3\end{bmatrix}$$

Rotation matrix computation

where $\tilde{\mathbf{X}}_1 = d_1\frac{\mathbf{K}^{-1}\mathbf{u}_1}{\|\mathbf{K}^{-1}\mathbf{u}_1\|}$ $\tilde{\mathbf{X}}_2 = d_2\frac{\mathbf{K}^{-1}\mathbf{u}_2}{\|\mathbf{K}^{-1}\mathbf{u}_2\|}$ $\tilde{\mathbf{X}}_3 = d_3\frac{\mathbf{K}^{-1}\mathbf{u}_3}{\|\mathbf{K}^{-1}\mathbf{u}_3\|}$