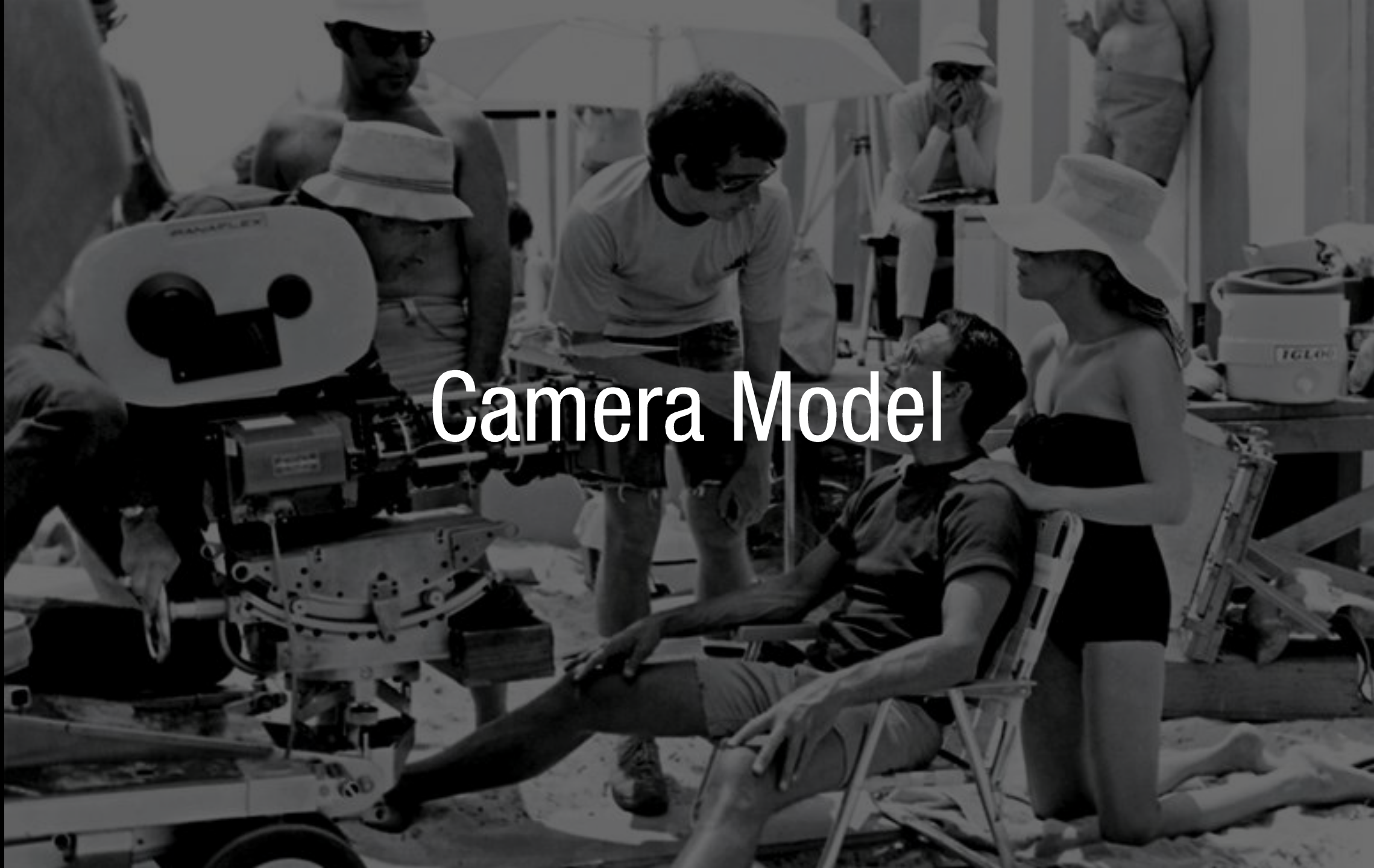


Camera Model



Paintings



Medieval painting

Paintings

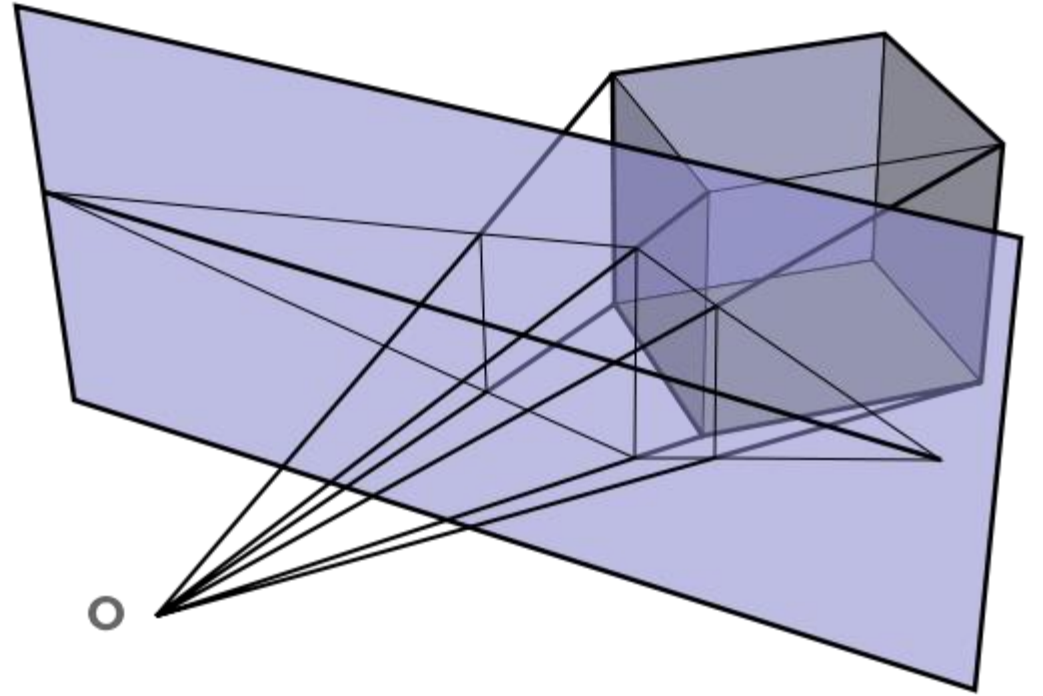


Medieval painting

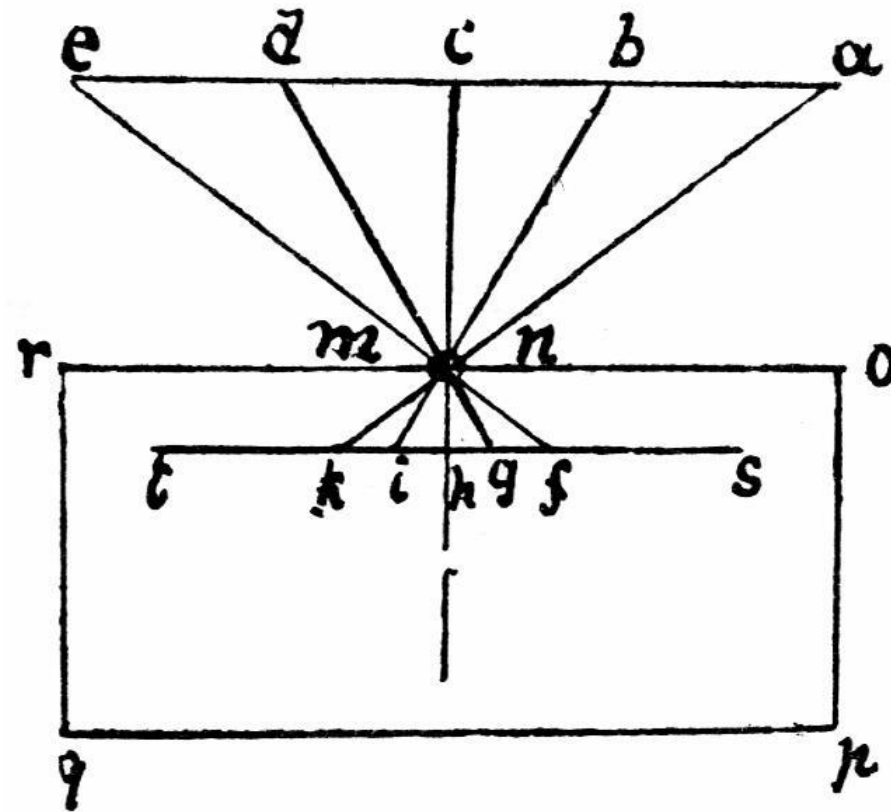


Pietro Perugino (Renaissance era)

Renaissance: Science into Art



Camera Obscura: Da Vinci



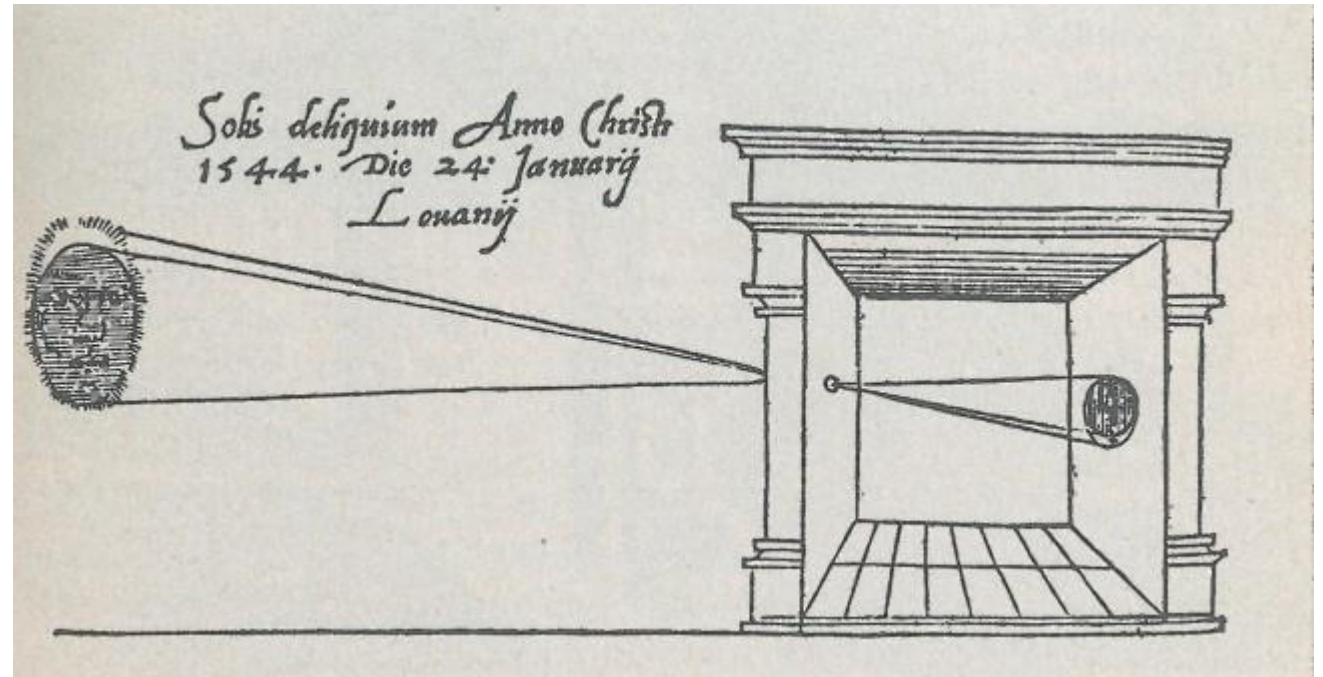
... all objects illuminated by the sun will send their images through this aperture and will appear, upside down, on the wall facing the hole.

- Codex Atlanticus

Camera Obscura: Frisius



Gemma Frisius



First camera obscura drawing



San Francisco, CA

CAMERA OBSCURA SAN FRANCISCO

1096 POINT LOBOS, SAN FRANCISCO, CA 94121, Tel: (415) 750-0415

HISTORY:

The word Camera Obscura is Latin and means translated the 'dark room'. Today it is not quite known, when and by whom exactly the Camera Obscura was invented. In the 11th. century the Arabian scholar Alhazen theorized that light waves travel in straight lines and he tried to prove it with a pinhole. What happened between then and the end of the 15th. century is obscure. The oldest forms of the Camera Obscura surviving, are sketches by Leonardo da Vinci and others, probably from around the late 15th. century. He was most likely not the only one, because around 1490 John Baptista della Porta revealed the phenomena in a book called 'Natural Magic'. While it was to the amusement of those scholars, other people condemned it as an invention of the devil. In the following centuries, the Camera Obscura became a wonderful tool for artists and astronomers. Artists, such as Vermeer, used the Camera Obscura as portable instruments for their paintings and portrait drawings.

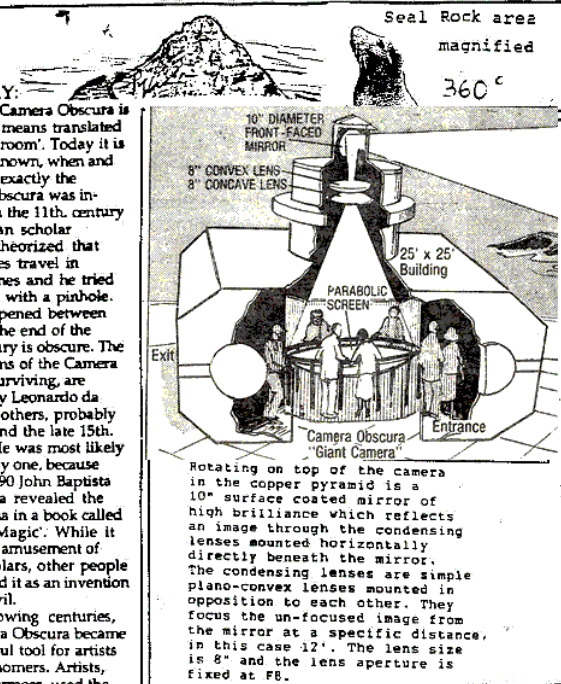
This rare optical device, shows you SEAL ROCK AREA in a new way. It produces a spectacular **LIVE IMAGE**, magnified seven times.

The Giant Camera is now a National Landmark, and is on the National Register of Historic Places. Now, you can share this Treasure Box with your family and friends. You have been exposed to the Camera Obscura Effect. Tell everyone to visit today.

The Camera is Always open from 11:00 am. till 5pm on beautiful days, and probably a little bit shorter on other days. To make sure, give us a call at (415) 750-0415.

General Admission - Three Dollars Each

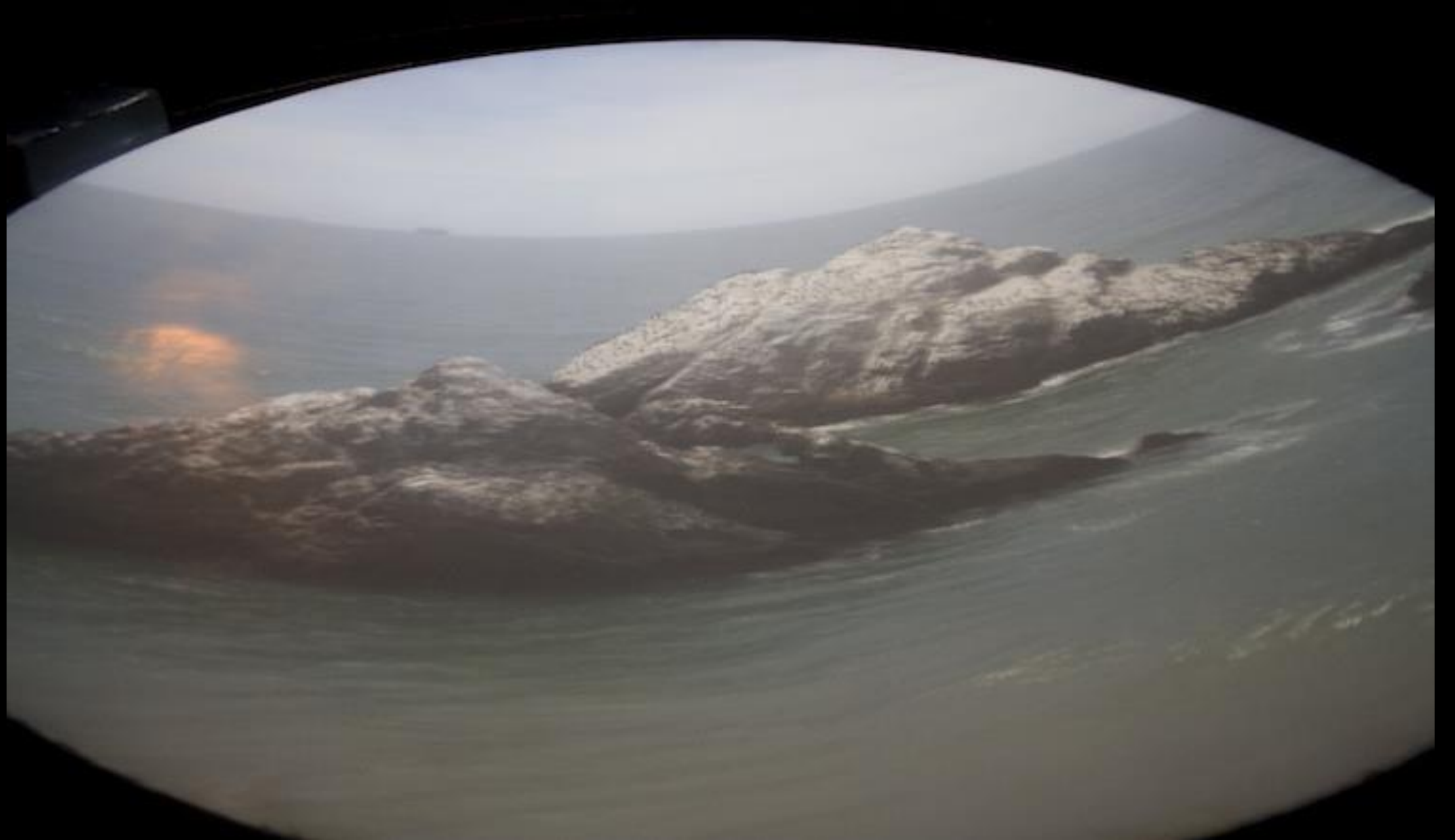
SUNSETS - Observe the sun with safety and amusement. See sunspots, solar flares and such phenomena as the **GREEN FLASH**.

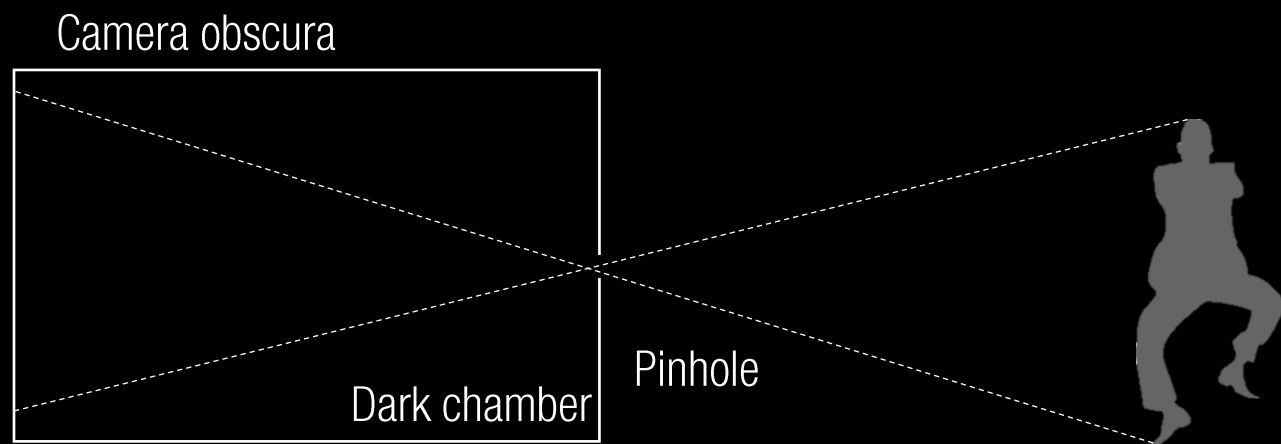


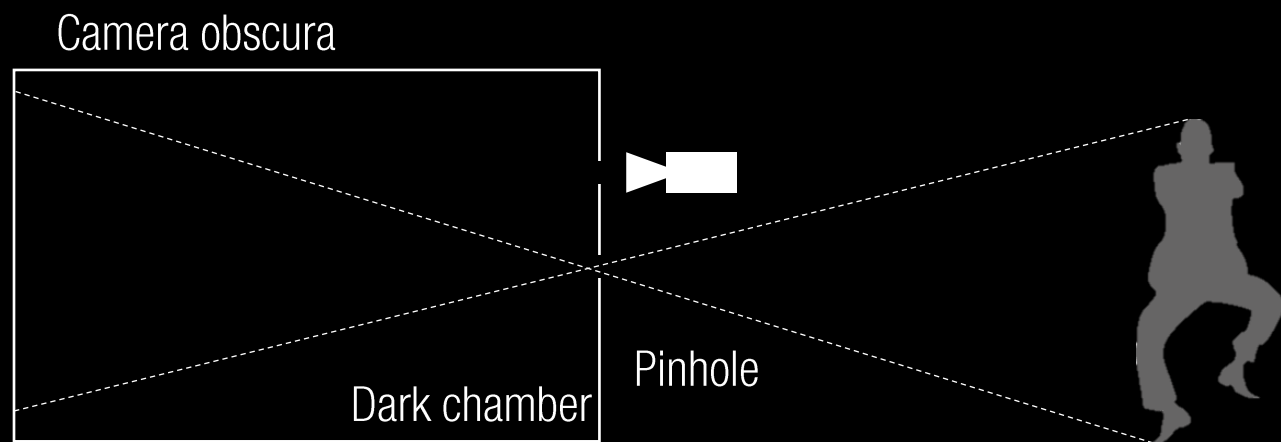
EXPERIENCE THE CAMERA OBSCURA EFFECT

History

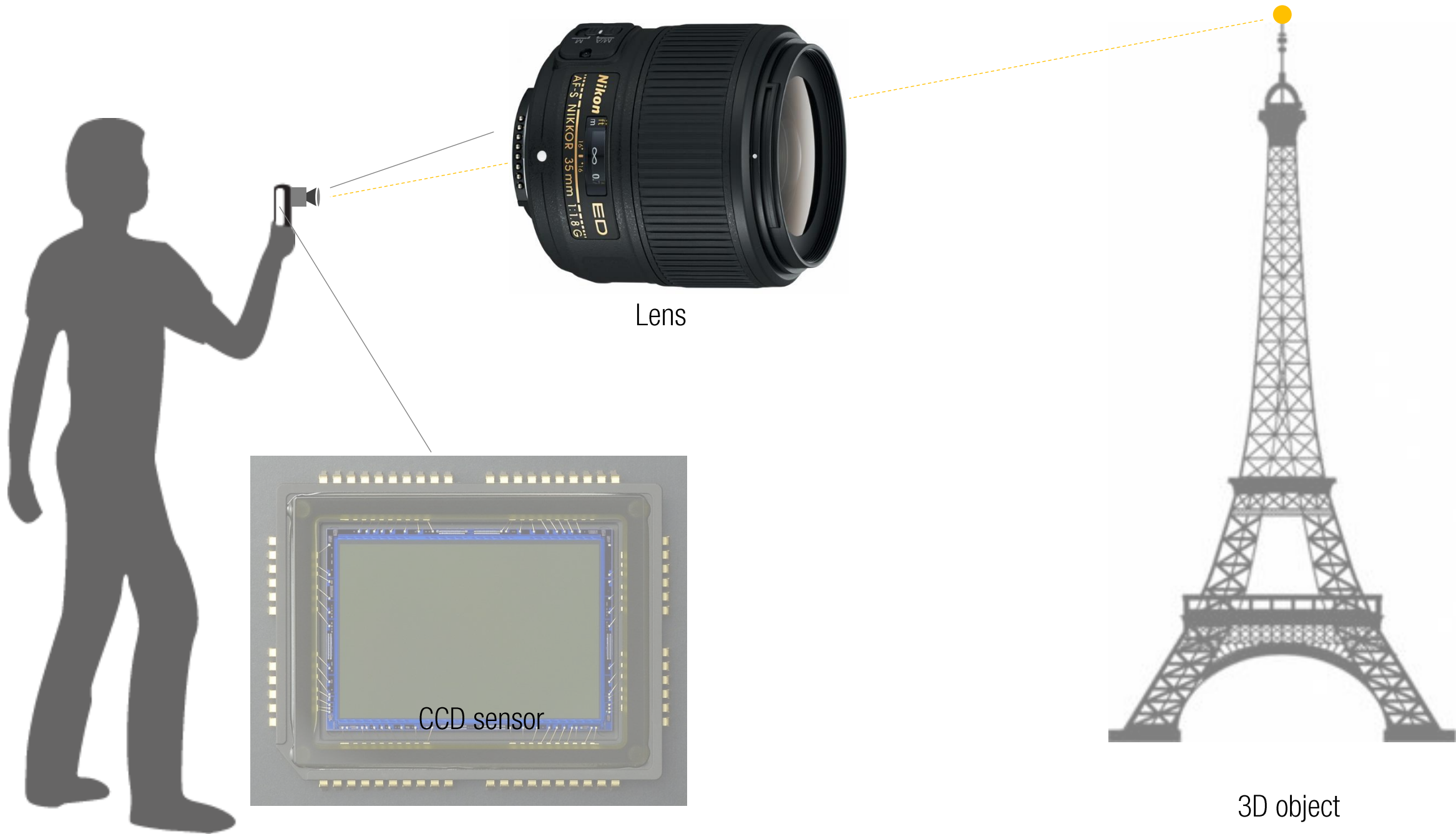
The Camera Obscura is the last remaining structure of the World Famous Playland at the Beach. Built by Floyd Jennings in 1946. It was built with the permission of George K. Whitney Sr., then owner of the Cliff House, Sutro Baths and Playland. Mr. Whitney later suggested, to making it look like a Camera, hence the name Giant Camera. This rare attraction is in keeping with Sutro's plan for recreational activities at Point Lobos. This structure provides scenic panoramic views, so spectacular with vivid colors. Making it a fun and learning experience. Walk through this optical instrument, which produces 360 degrees of spectacular **LIVE IMAGES** of the Seal Rock Area. Magnified Seven Times on a Six foot Parabolic Table. Now you can experience this **Special Effect**. You will be truly amazed, the Images standing up and coming at you. After this you will want to learn everything you can about the **CAMERA OBSCURA**. You will be telling your friends. Don't miss it.



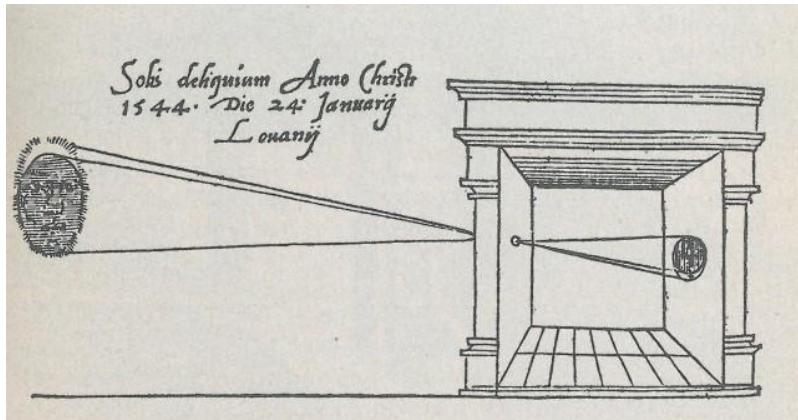
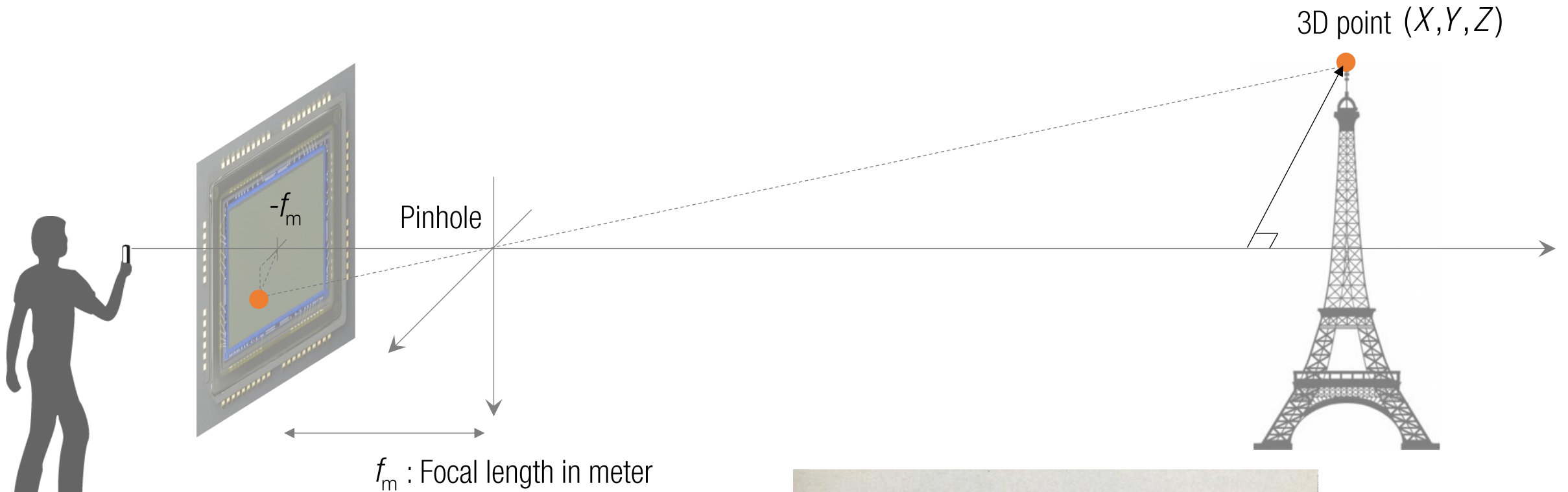




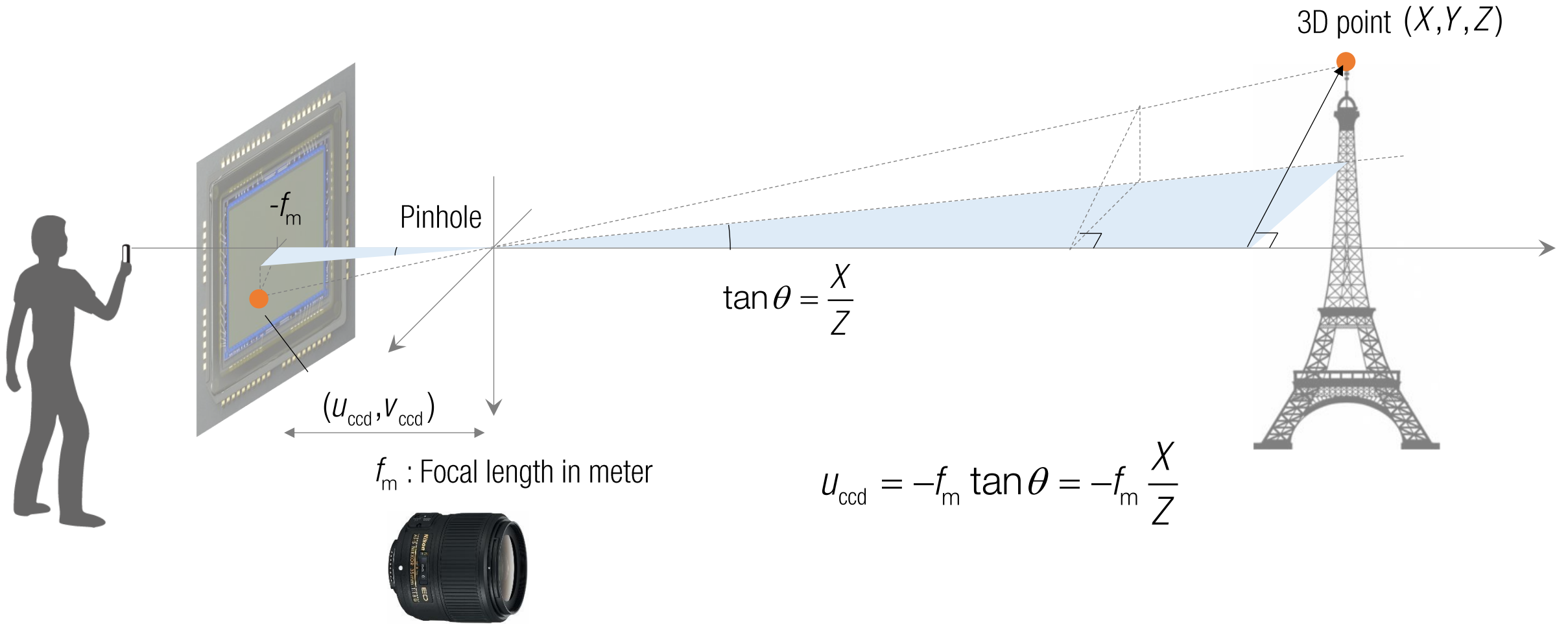




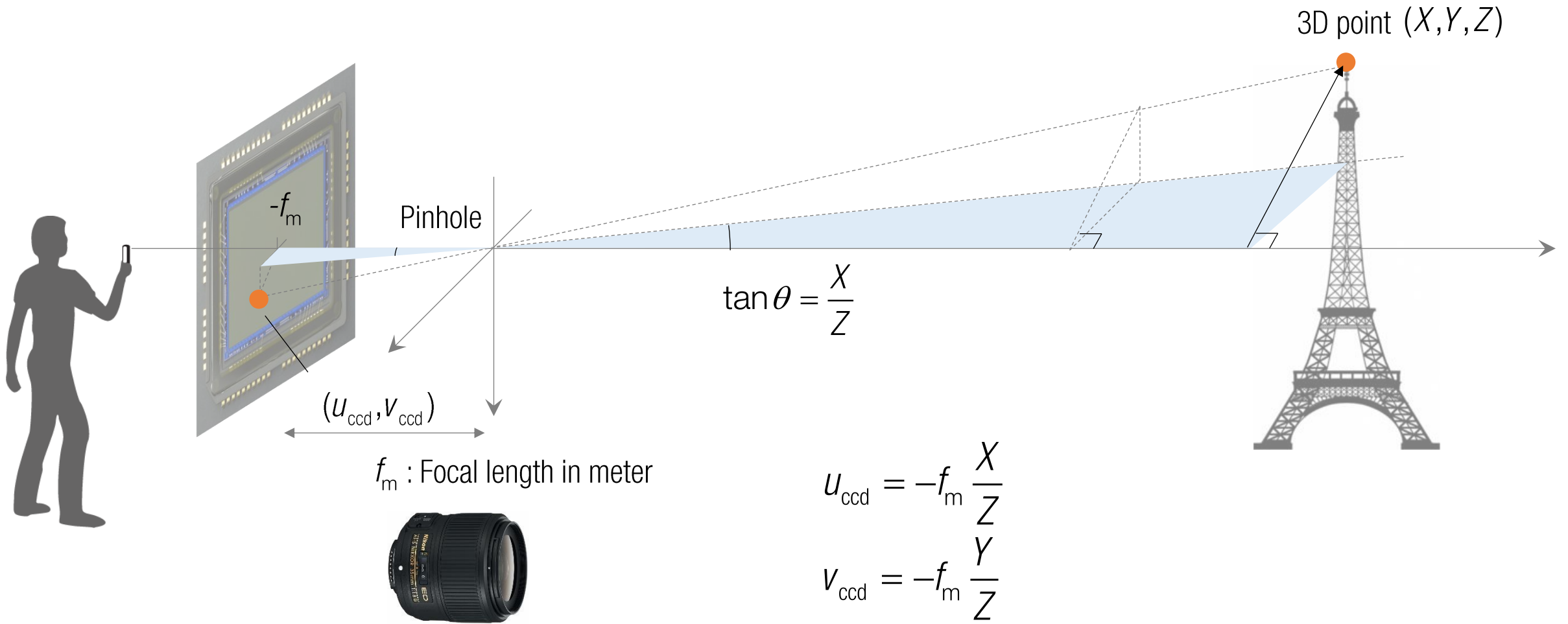
3D Point Projection (Metric Space)



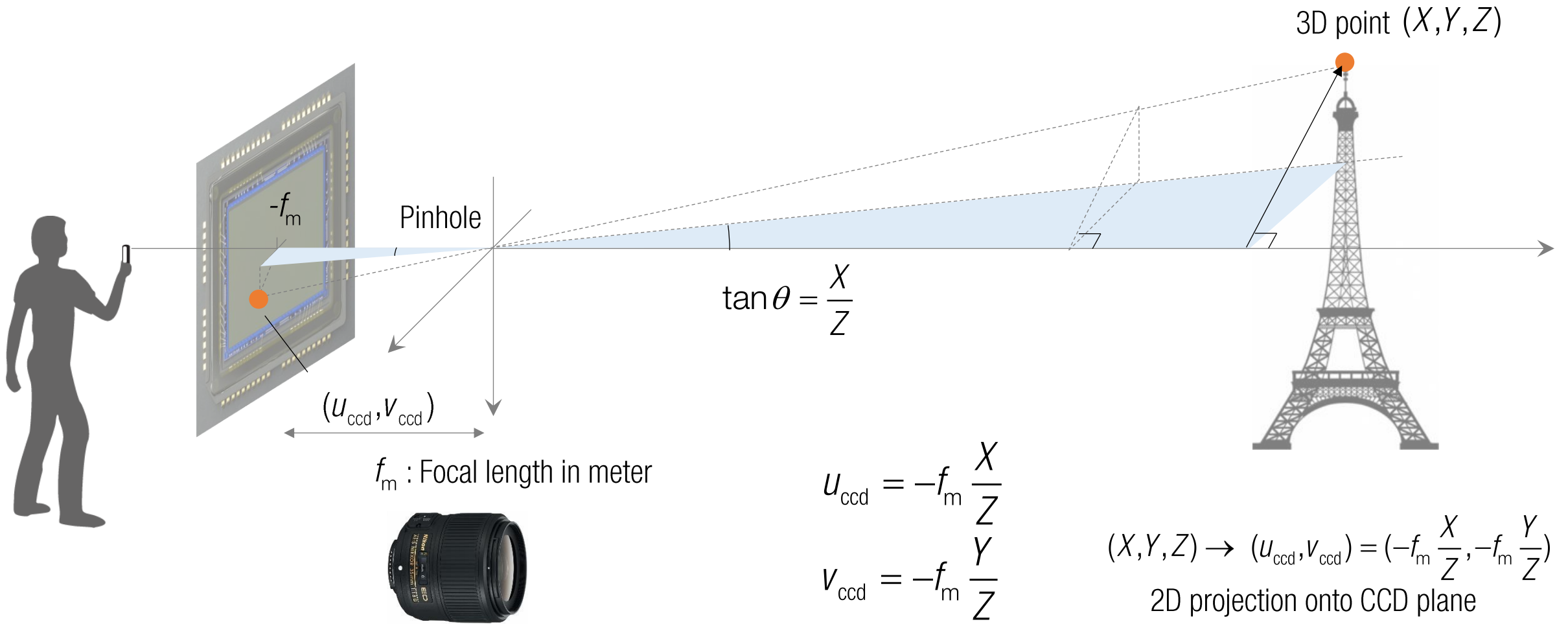
3D Point Projection (Metric Space)



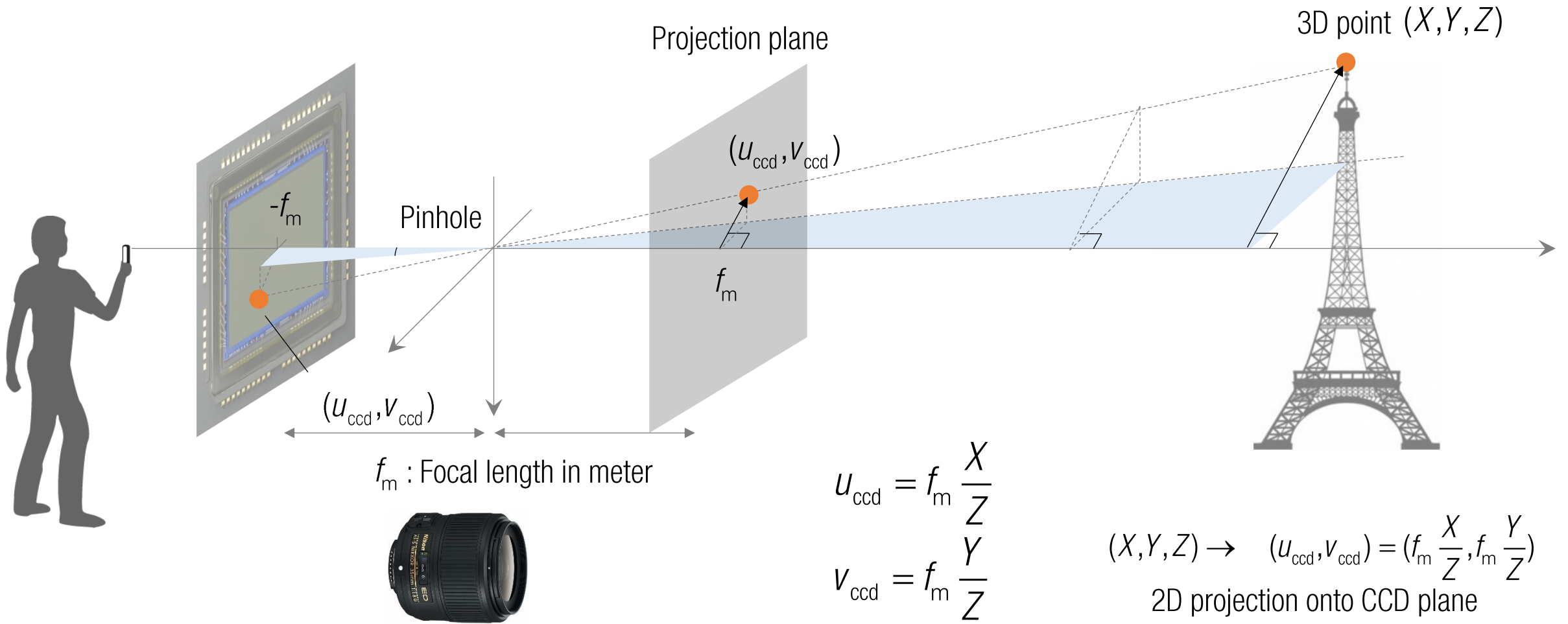
3D Point Projection (Metric Space)



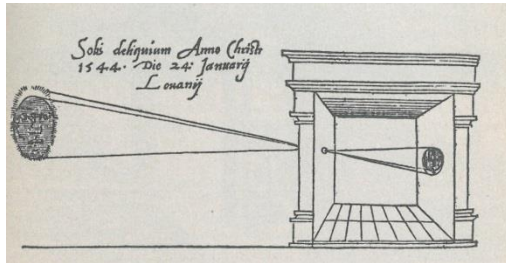
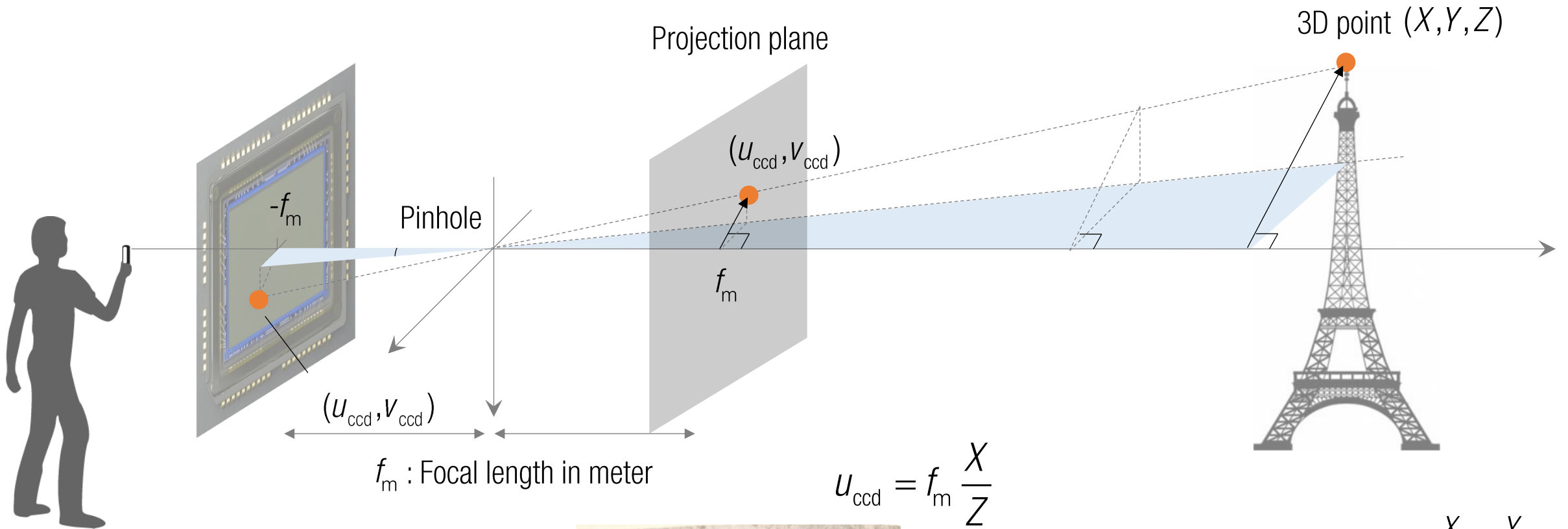
3D Point Projection (Metric Space)



3D Point Projection (Metric Space)



3D Point Projection (Metric Space)



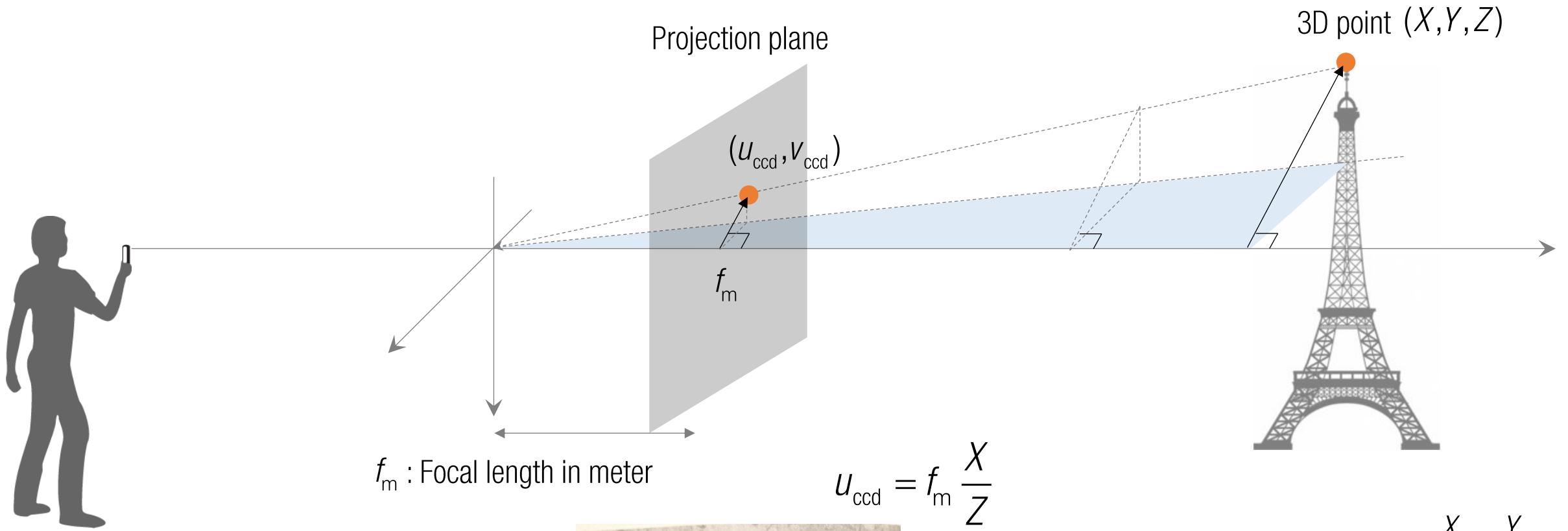
$$u_{ccd} = f_m \frac{X}{Z}$$

$$v_{ccd} = f_m \frac{Y}{Z}$$

$$(X, Y, Z) \rightarrow (u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$

2D projection onto CCD plane

3D Point Projection (Metric Space)



f_m : Focal length in meter

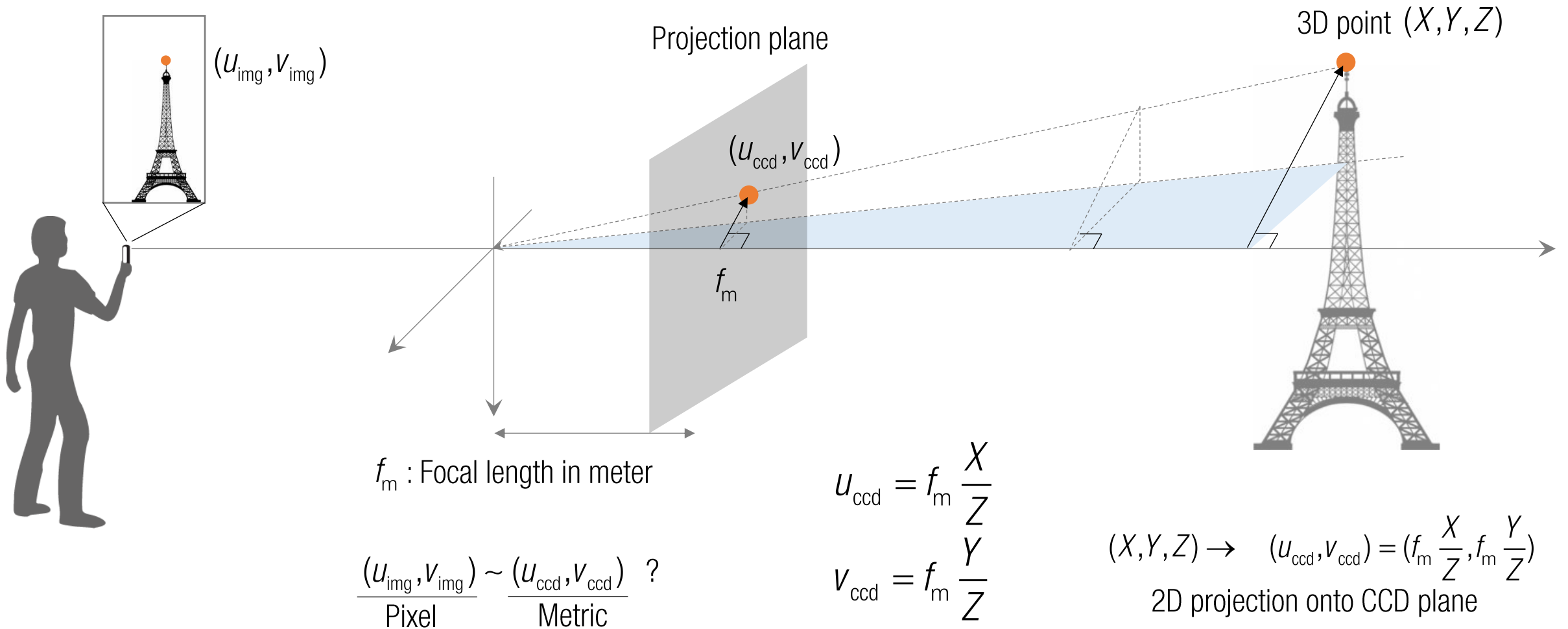
$$u_{\text{ccd}} = f_m \frac{X}{Z}$$
$$v_{\text{ccd}} = f_m \frac{Y}{Z}$$

$$(X, Y, Z) \rightarrow (u_{\text{ccd}}, v_{\text{ccd}}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z}\right)$$

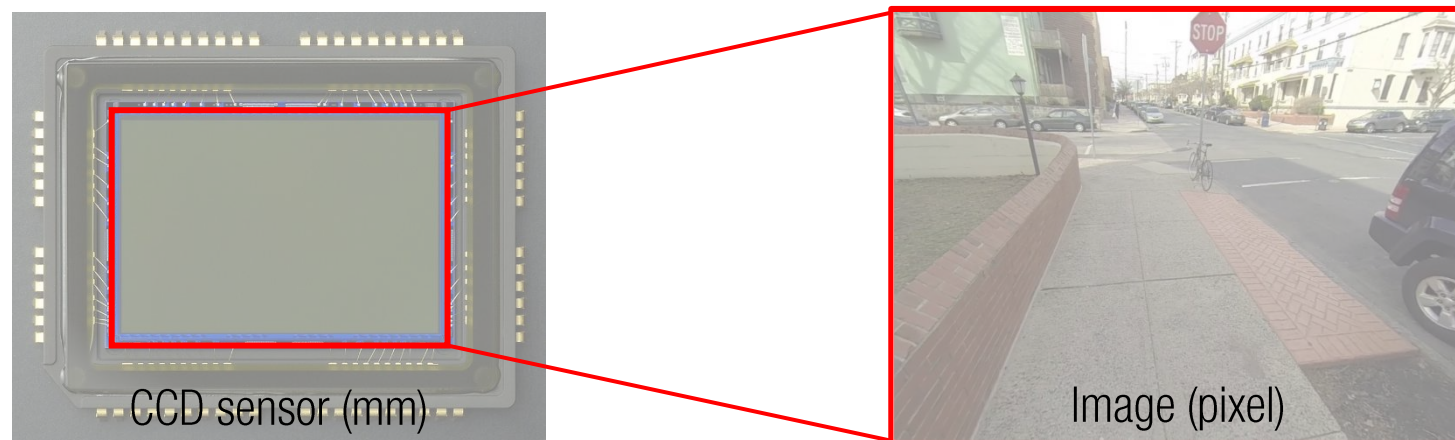
2D projection onto CCD plane



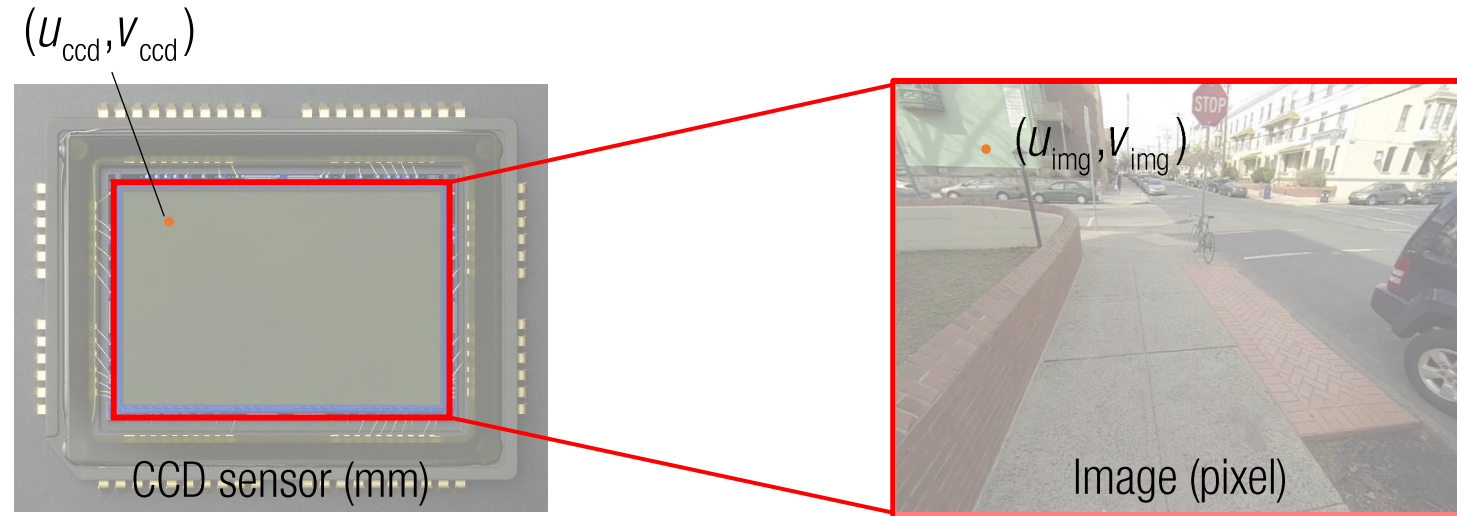
3D Point Projection (Metric Space)



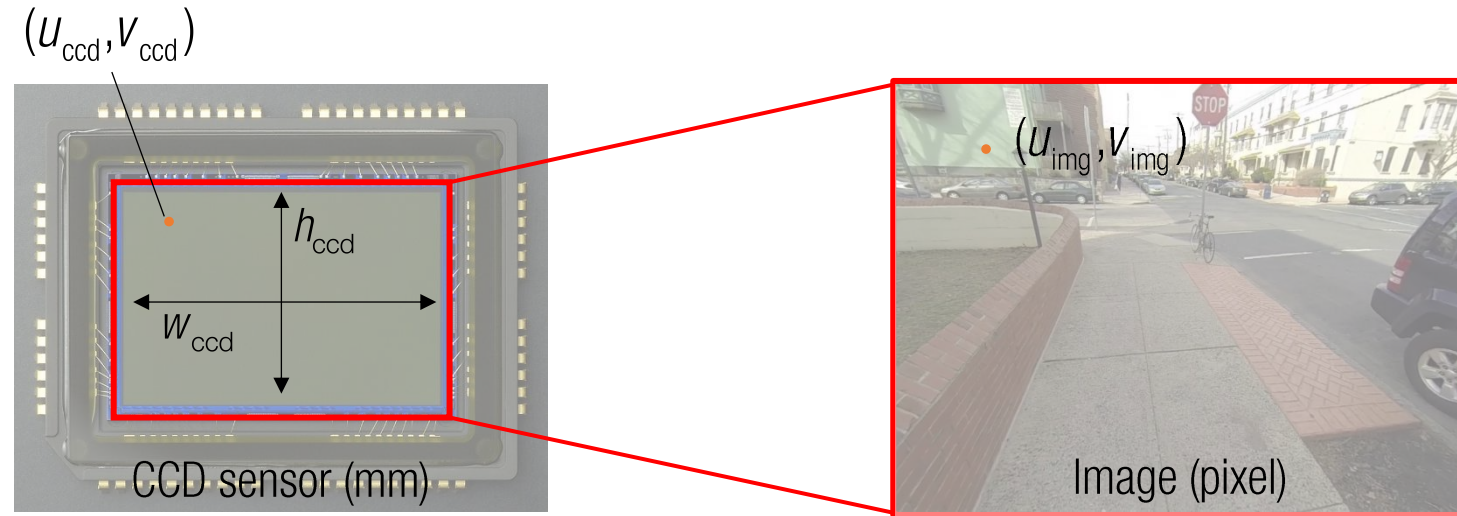
3D Point Projection (Pixel Space)

















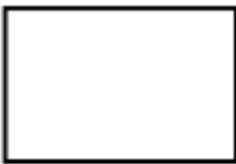
3D Point Projection (Pixel Space)



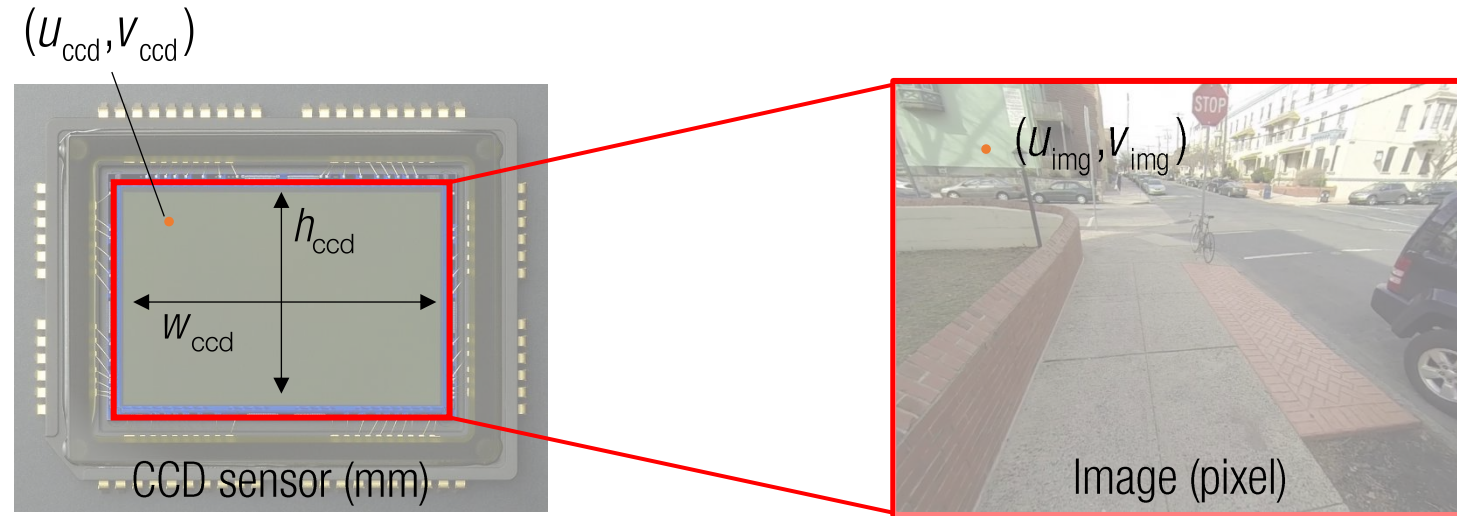
3D Point Projection (Pixel Space)



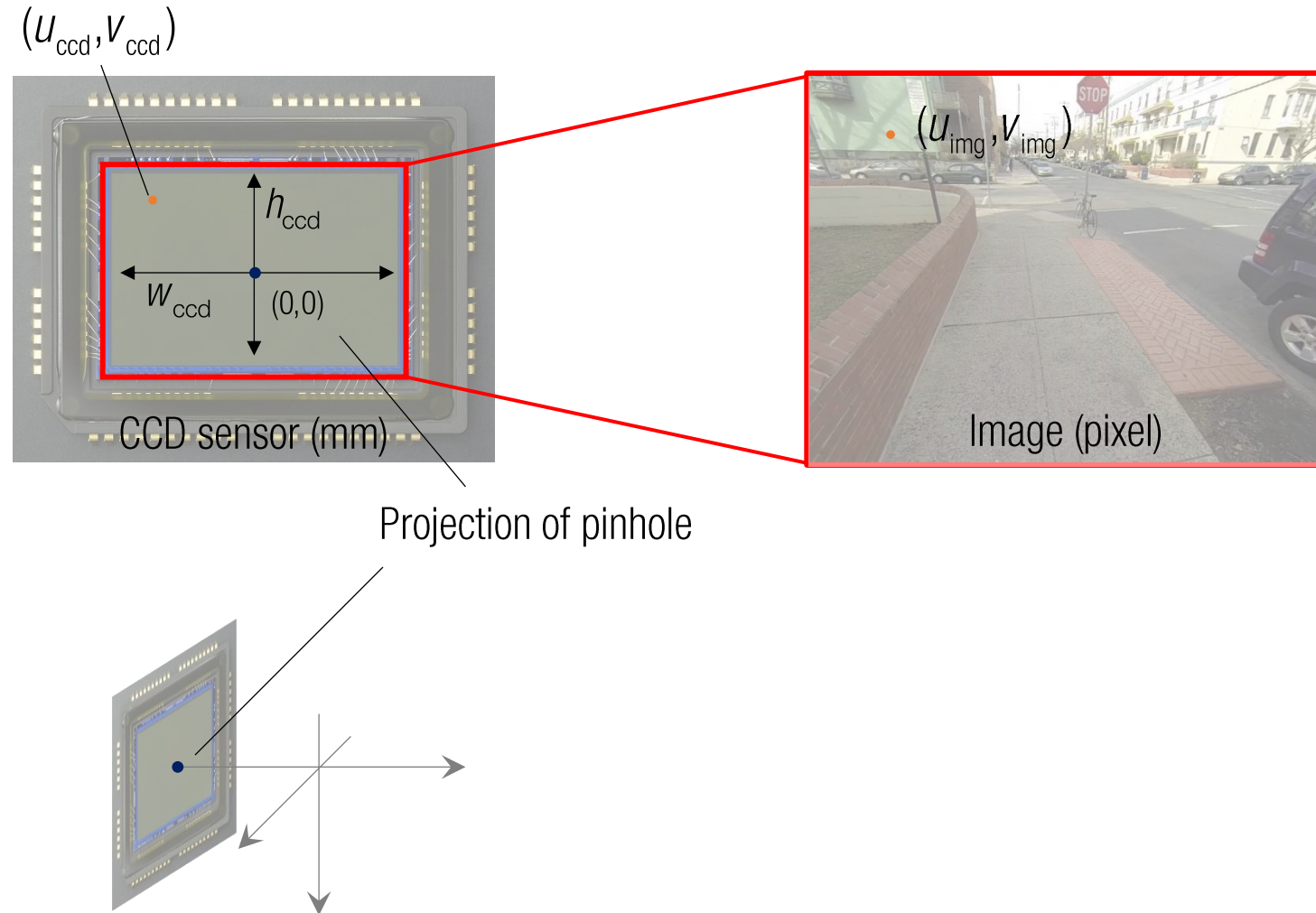
Imager Sizes	Formats (Type)	~Diag.	Uses
	1/7" - 1.85 x 1.39mm	2.3	Cell phones, web cams, etc....
	1/6" - 2.15 x 1.61mm	2.7	Cell phones, web cams, etc....
	1/5" - 2.55 x 1.91mm	3.2	Cell phones, web cams, etc....
	1/4" - 3.2 x 2.4mm	4.0	Cell phones, web cams, etc....
	1/3.6" - 4.0 x 3.0mm	5.0	P&S DSC
	1/3.2" - 4.536 x 3.416mm	5.678	P&S DSC
	1/3" - 4.8 x 3.6mm	6.0	Casio QV-8000SX (1.2MP), Epson PhotoPC 700 (1.2MP)
	1/2.7" - 5.27 x 3.96mm	6.592	Canon PowerShot A20 (1.92MP), HP PhotoSmart C618 (1.92)
	1/2" - 6.4 x 4.8mm	8.0	Olympus C-2100Z (1.92MP), Epson PhotoPC 850Z (1.92)
	1/1.8" - 7.176 x 5.319mm	8.932	Nikon Coolpix 995 (3.14MP), Olympus C-4040Z (3.9MP), Canon PowerShot G2 (3.8MP), Sony DSC-S85 (3.8MP)
	2/3" - 8.8 x 6.6mm	11.0	Nikon Coolpix 5000 (4.92MP), Sony DSC-F707 (4.92MP), Olympus E-10 (3.7MP), Minolta DiMAGE 7 (4.92MP)
	1" - 12.8 x 9.6mm	16.0	Not used in DSCs. Used in some high-end video cameras
	Kodak KAF-5100CE CCD 17.8 x 13.4mm (4/3")	22.28	Olympus announced development of a new camera and new lenses for this 4/3" size. 2614 x 1966 - 5.1MP - 6.8µm pixel
	Foveon X3 F7-35X3-A25B 20.7 x 13.8mm	24.9	Sigma SD9 (X3) 2268 x 1512 = 3.43MP - 9.12µm pixel 1.74x Focal Length Multiplier (35mm film)

	Canon D30 CMOS 21.8 x 14.5mm	26.2	Canon D30 2160 x 1440 = 3.11MP - 10.1µm pixel 1.65x Focal Length Multiplier (35mm film)
	Canon D60 CMOS 22.7 x 15.1mm	27.3	Canon D60 3072 x 2048 = 6.3MP - 7.4µm pixel 1.59x Focal Length Multiplier (35mm film)
	Nikon D100 CCD Nikon D1x CCD 23.7 x 15.6mm	28.2	Nikon D100 - 3008 x 2000 = 6.1MP - 7.8µm pixel Nikon D1x - 4024 x 1324 = 5.24MP - 5.9 x 11.7µm pixel 1.52x Focal Length Multiplier (35mm film)
	APS Film 25.1 x 16.7mm	30.148	APS cameras 1.44x Focal Length Multiplier (35mm film)
	Canon EOS-1D CCD 27.0 x 17.8mm	32.3	Canon EOS-1D 2464 x 1648 = 4.06MP - 10.8µm pixel 1.34x Focal Length Multiplier (35mm Film)
	Kodak KAF-6303CE CCD 27.8 x 18.5mm	33.4	Kodak 760 3088 x 2056 = 6.35MP - 9.0µm pixel 1.30x Focal Length Multiplier (35mm film)
	35mm Film Canon 1Ds Kodak 14n 36.0 x 24.0mm	43.27	35mm film cameras Canon 1Ds - 4064 x 2704 = 10.99MP - 8.85µm pixel Kodak DCS Pro 14n - 4536 x 3024 = 13.7MP - 7.94µm pixel

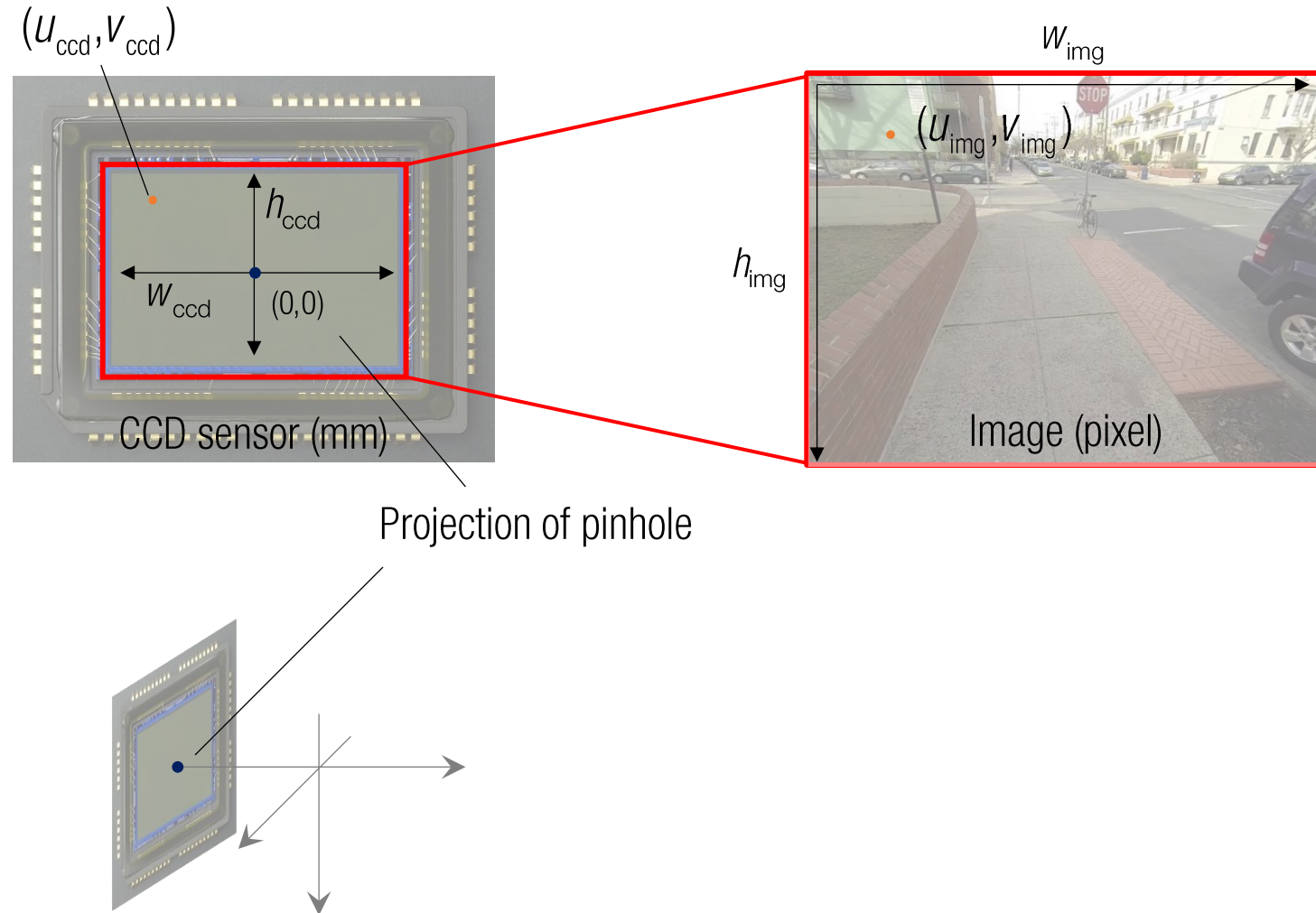
3D Point Projection (Pixel Space)



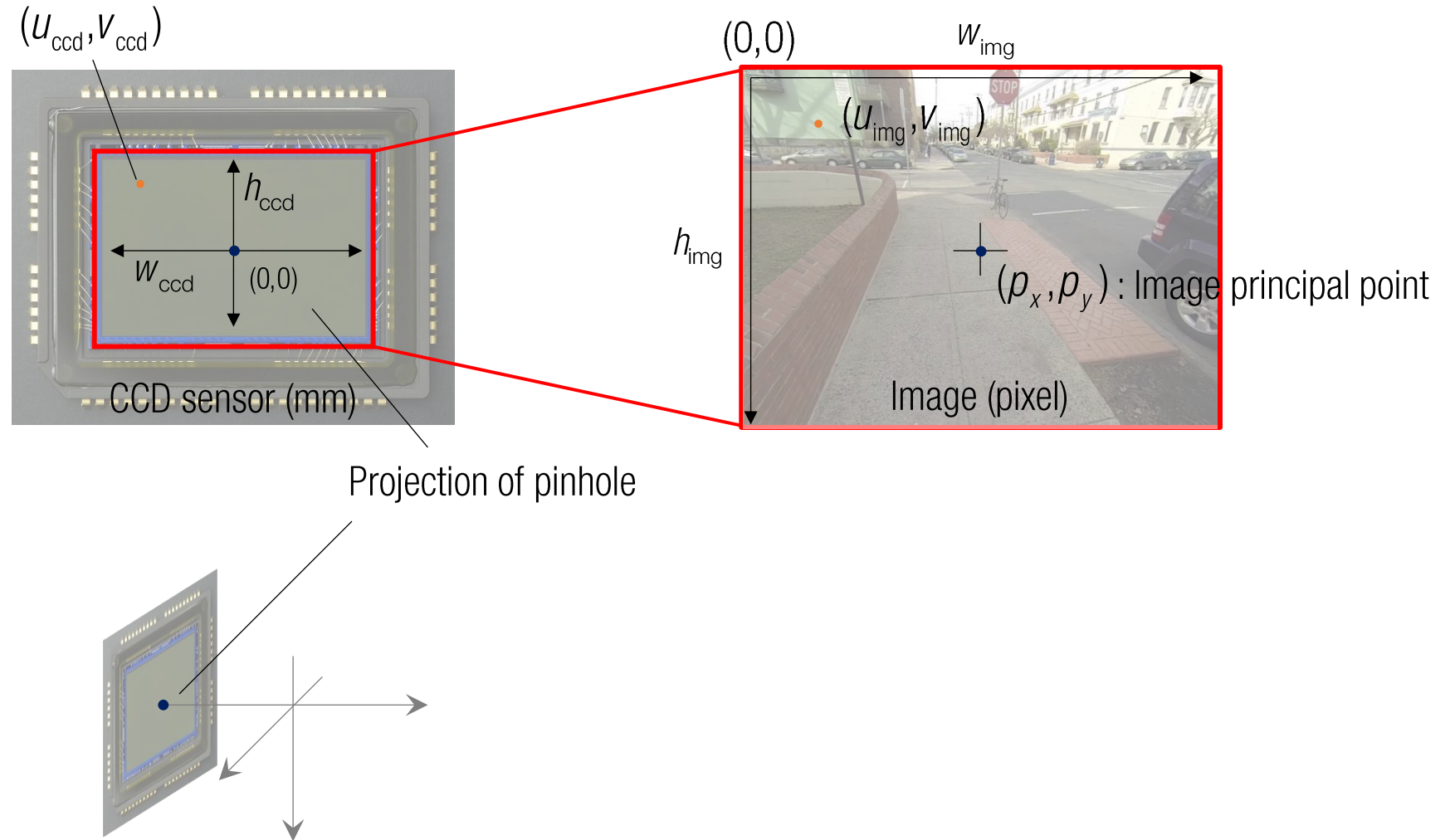
3D Point Projection (Pixel Space)



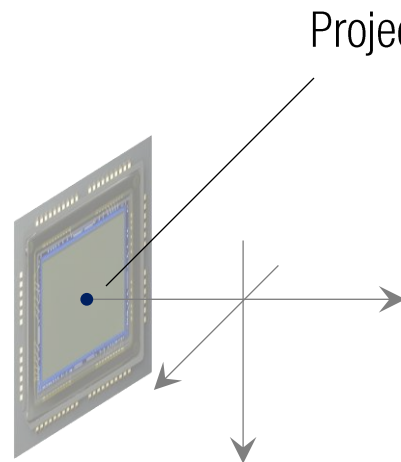
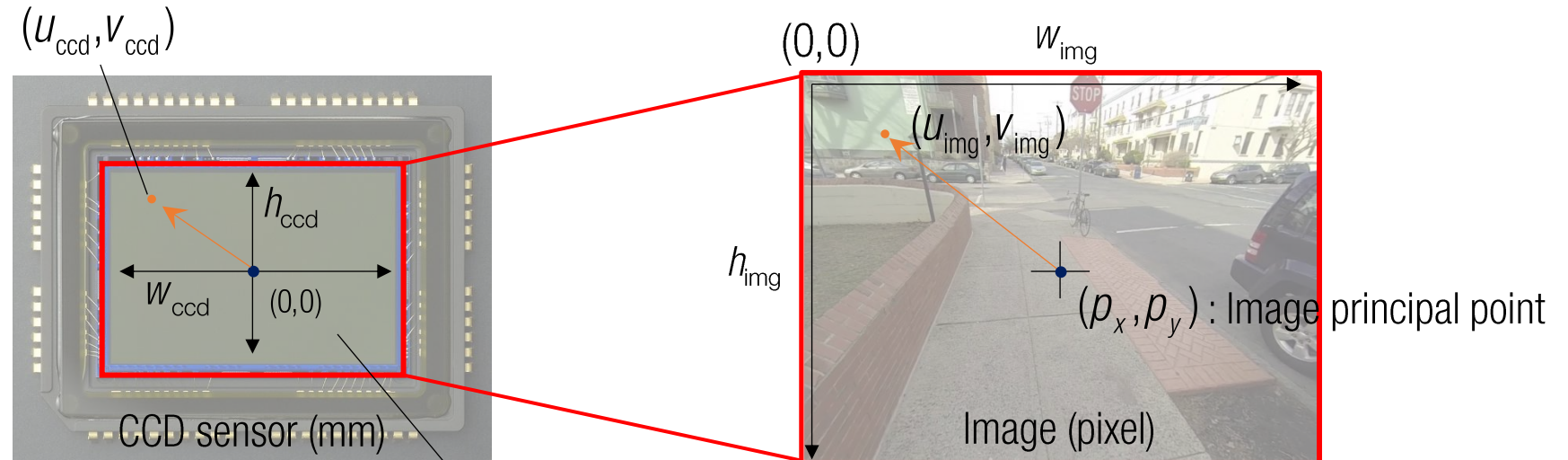
3D Point Projection (Pixel Space)



3D Point Projection (Pixel Space)

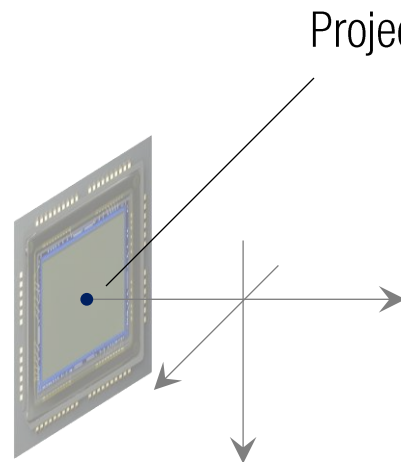
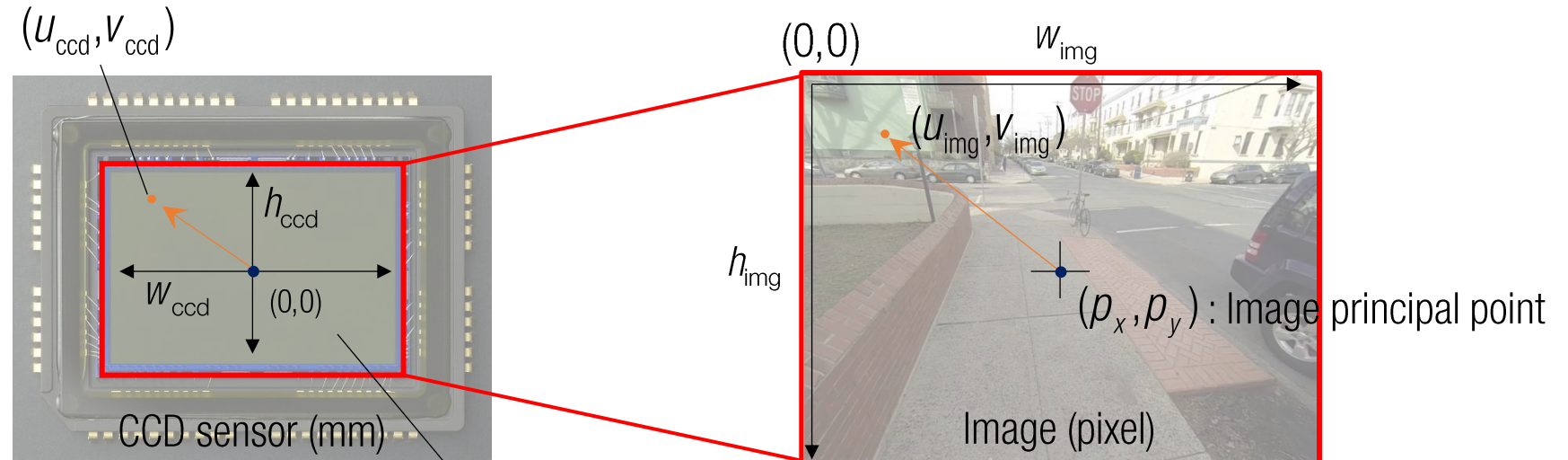


3D Point Projection (Pixel Space)



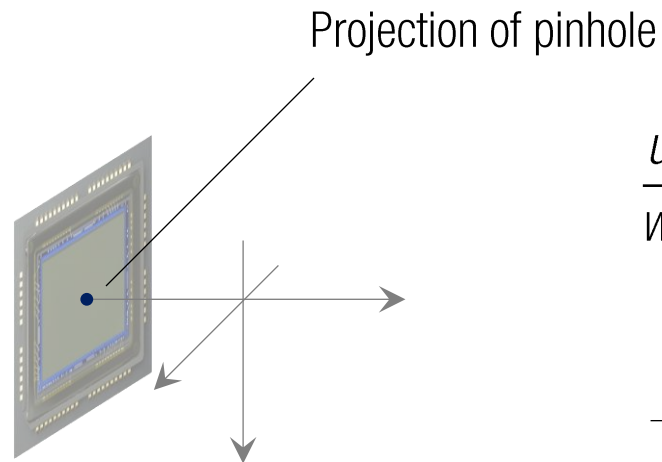
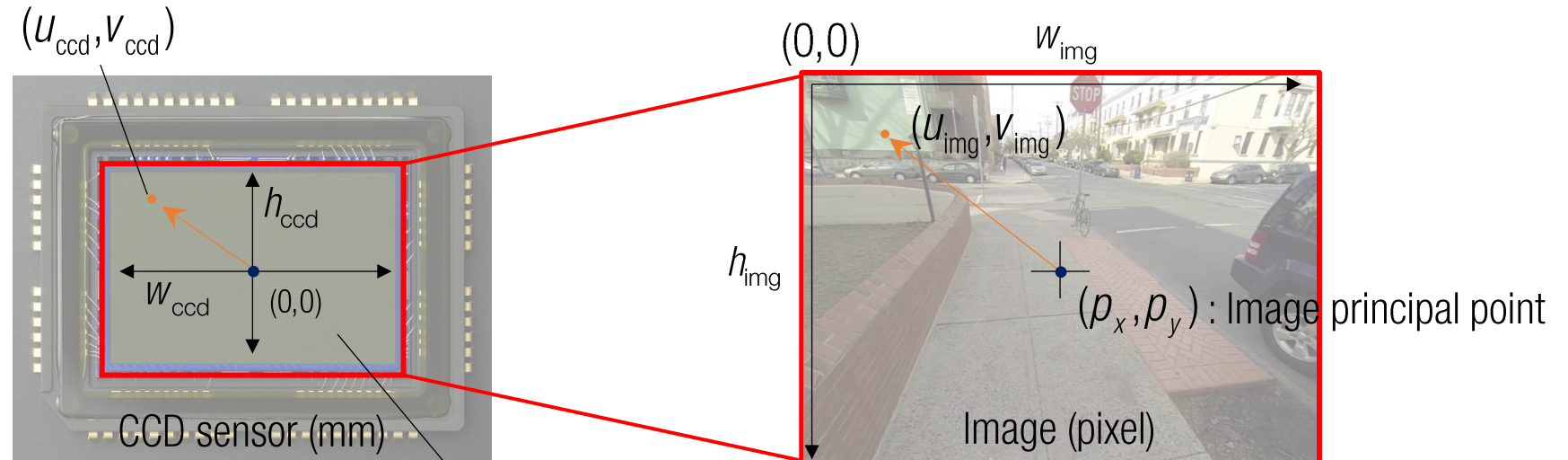
$$\frac{u_{\text{ccd}}}{w_{\text{ccd}}} = \frac{u_{\text{img}} - p_x}{w_{\text{img}}}$$

3D Point Projection (Pixel Space)



$$\frac{u_{\text{ccd}}}{w_{\text{ccd}}} = \frac{u_{\text{img}} - p_x}{w_{\text{img}}} \quad \frac{v_{\text{ccd}}}{h_{\text{ccd}}} = \frac{v_{\text{img}} - p_y}{h_{\text{img}}}$$

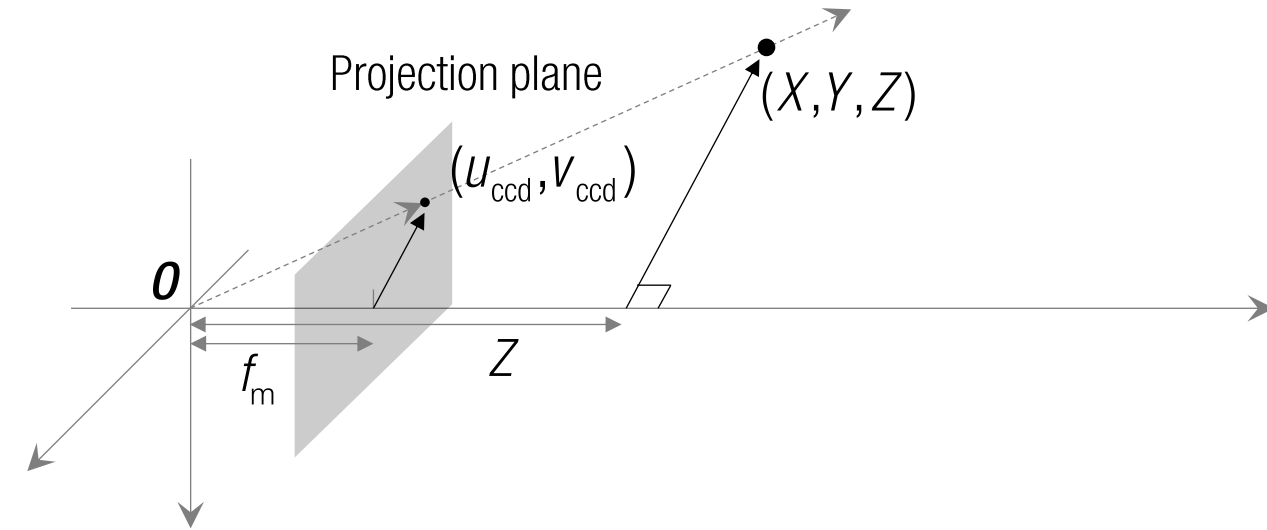
3D Point Projection (Pixel Space)



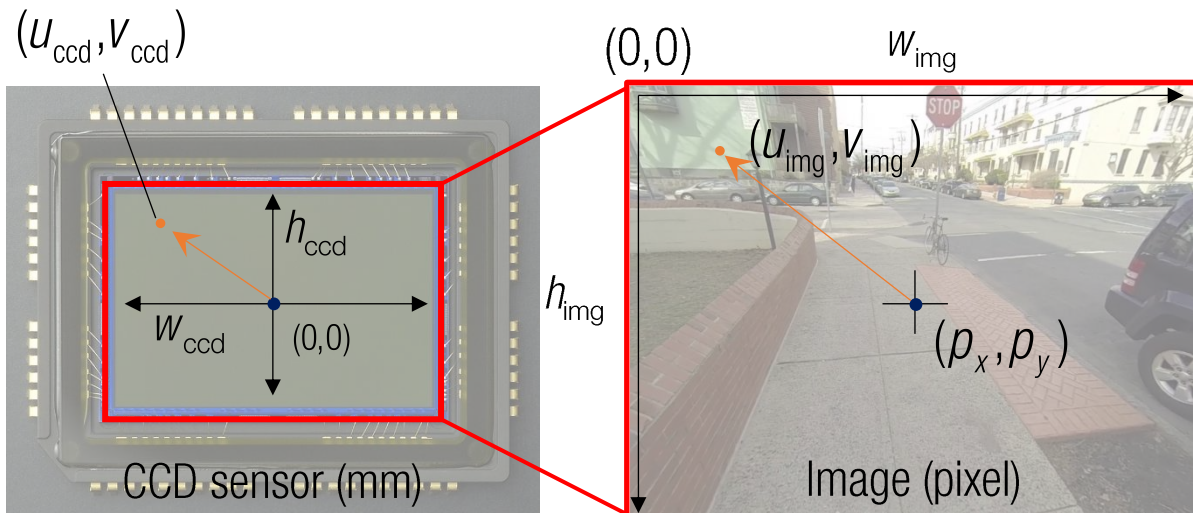
$$\frac{u_{\text{ccd}}}{w_{\text{ccd}}} = \frac{u_{\text{img}} - p_x}{w_{\text{img}}} \quad \frac{v_{\text{ccd}}}{h_{\text{ccd}}} = \frac{v_{\text{img}} - p_y}{h_{\text{img}}}$$

$$\longrightarrow u_{\text{img}} = u_{\text{ccd}} \frac{w_{\text{img}}}{w_{\text{ccd}}} + p_x \quad v_{\text{img}} = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y$$

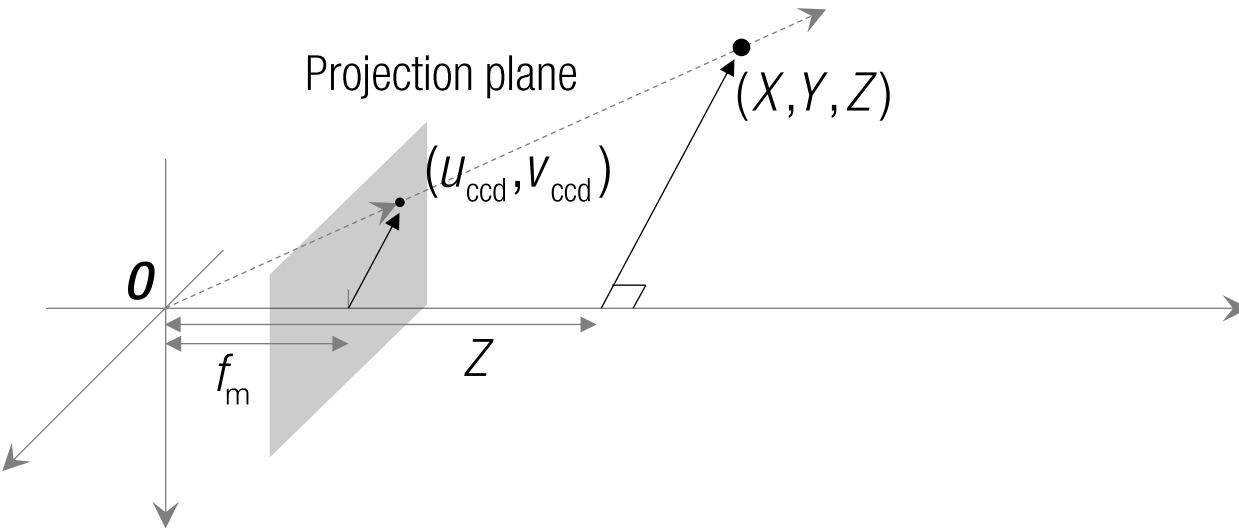
3D Point Projection (Pixel Space)



$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z}\right) \quad : \text{Metric projection}$$



3D Point Projection (Pixel Space)

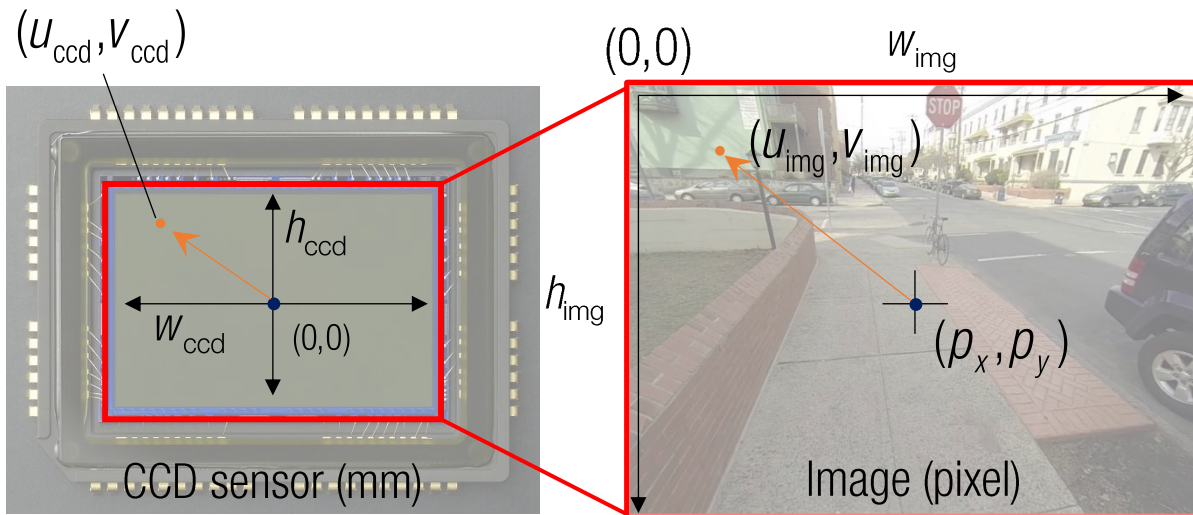


$$(u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}) \quad : \text{Metric projection}$$

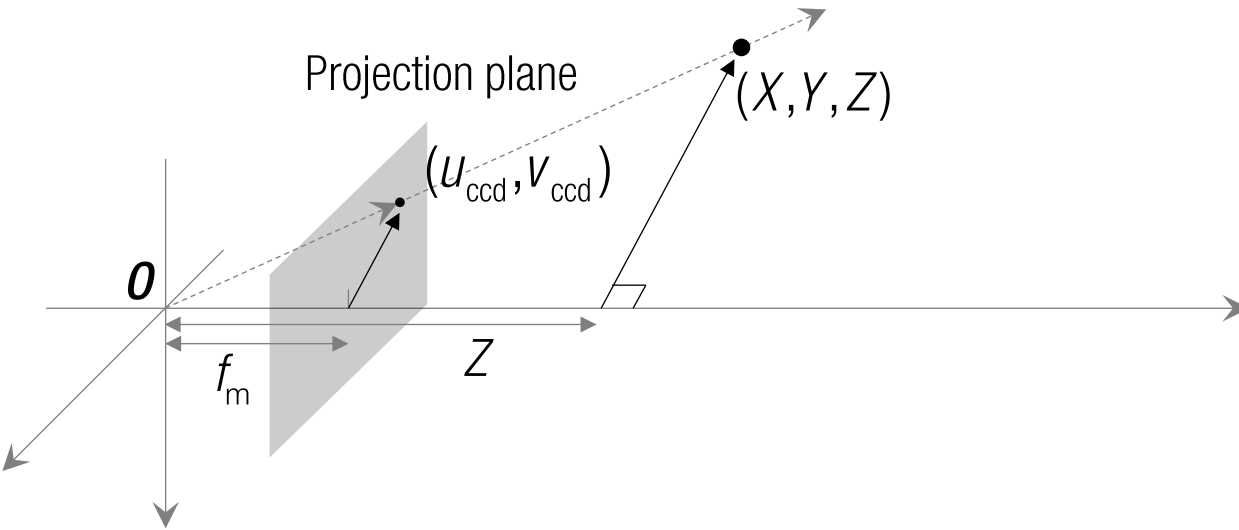
Pixel projection

$$\longrightarrow u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x$$

$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y$$



3D Point Projection (Pixel Space)

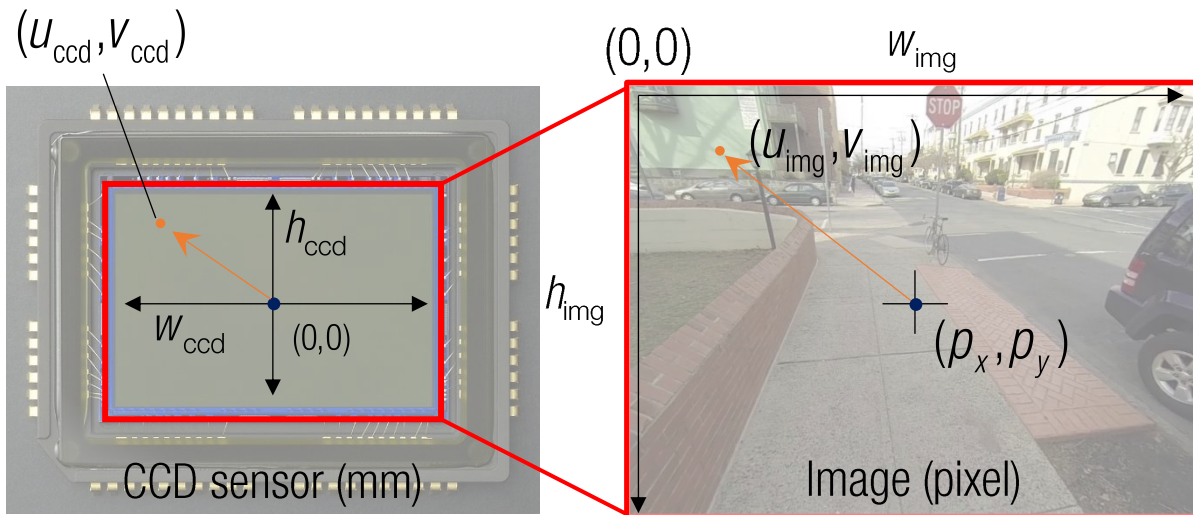


$$(u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}) \quad : \text{Metric projection}$$

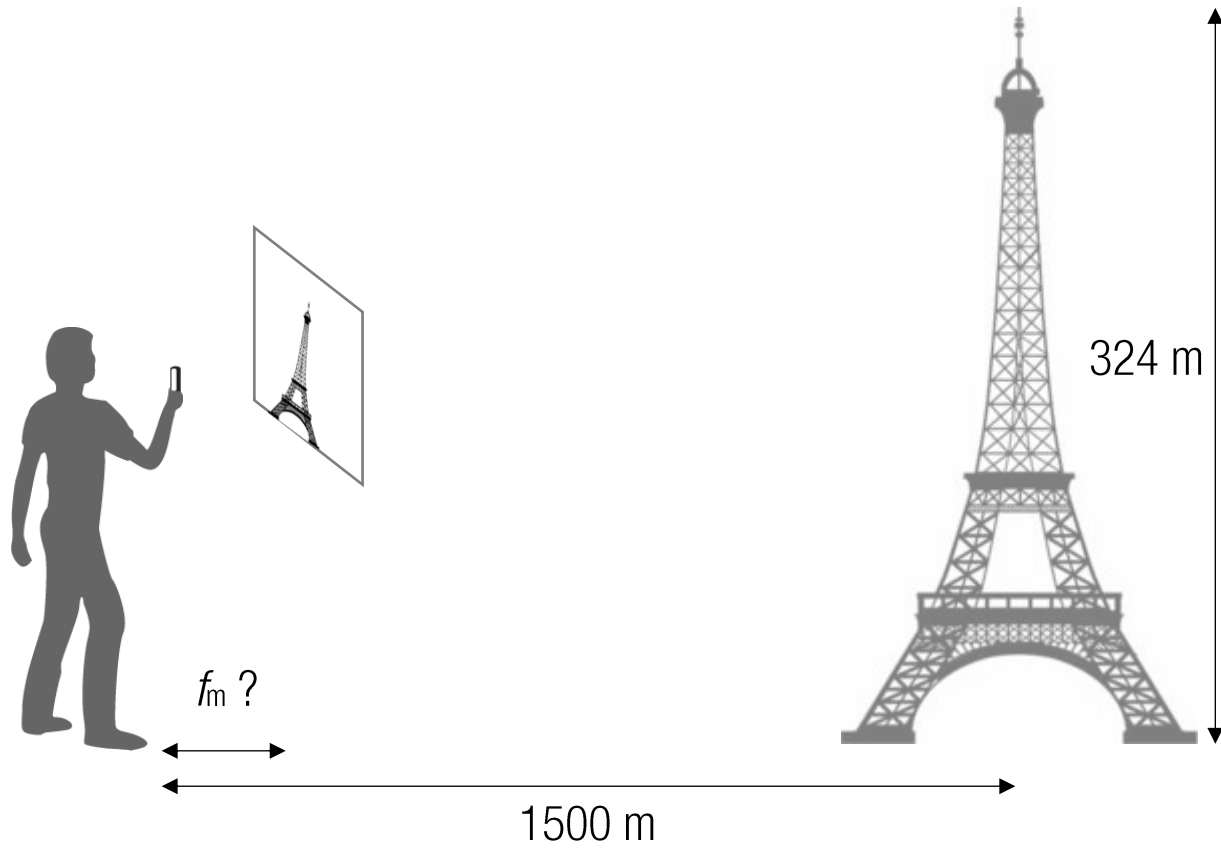
Pixel projection

$$\longrightarrow u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x = f_m \frac{w_{img}}{w_{ccd}} \frac{X}{Z} + p_x$$

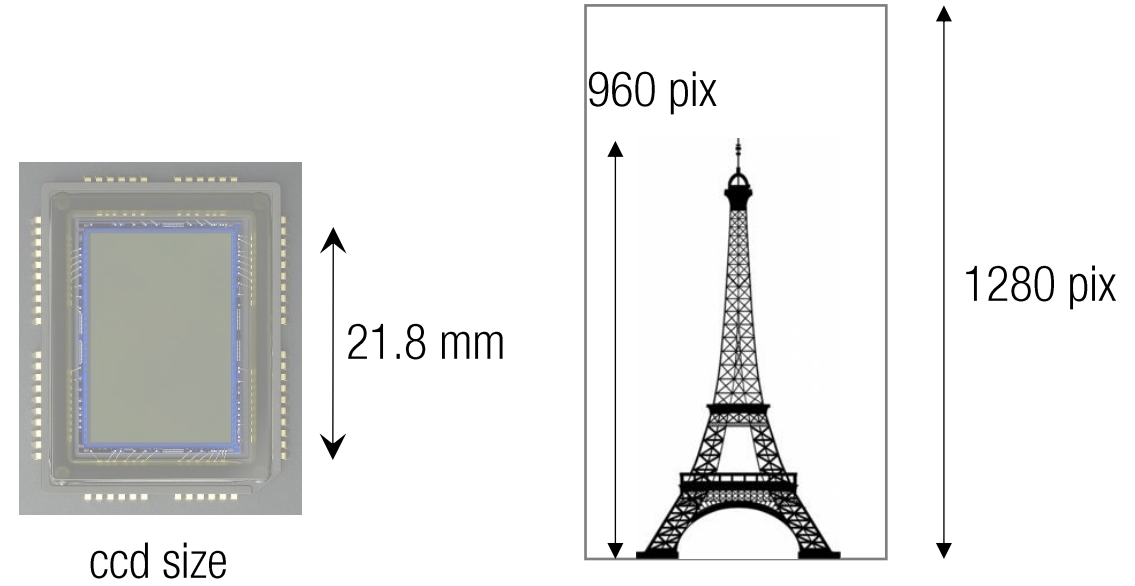
$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_m \frac{h_{img}}{h_{ccd}} \frac{Y}{Z} + p_y$$



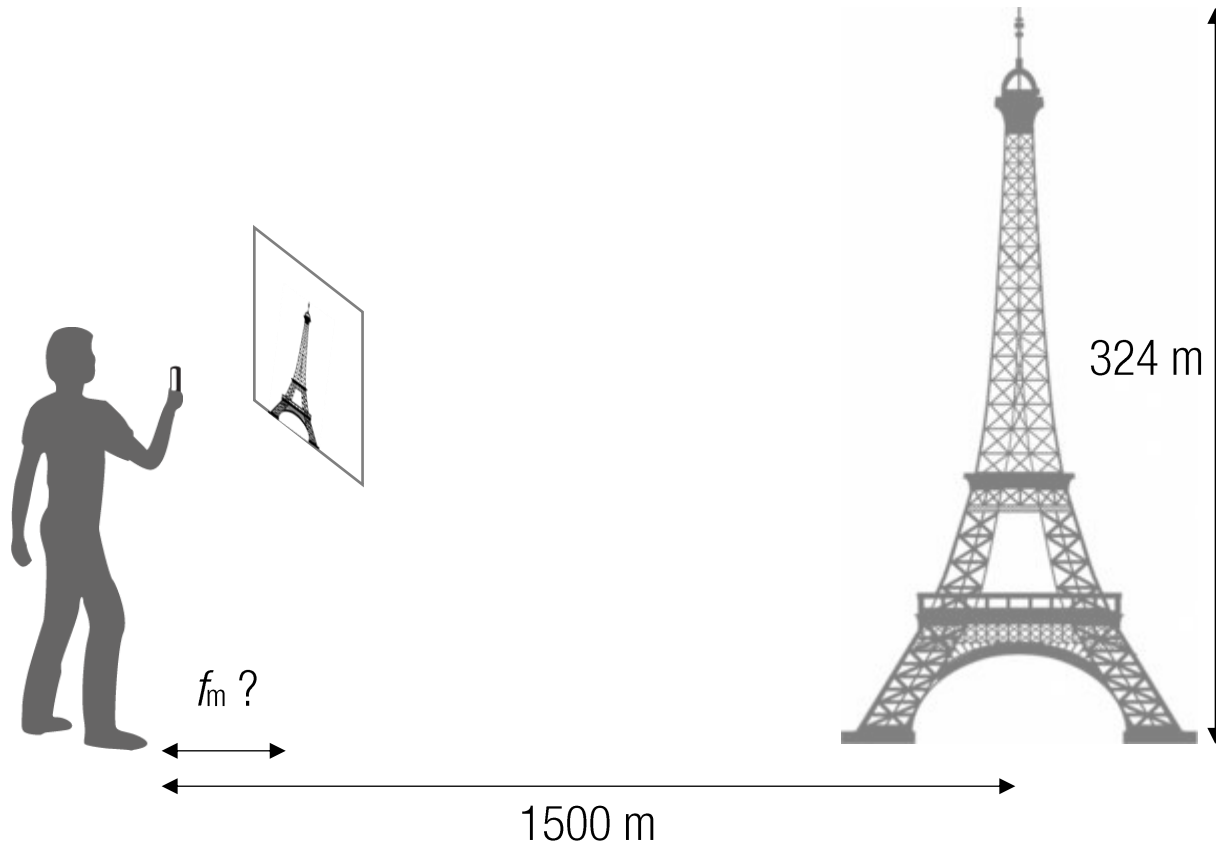
Exercise



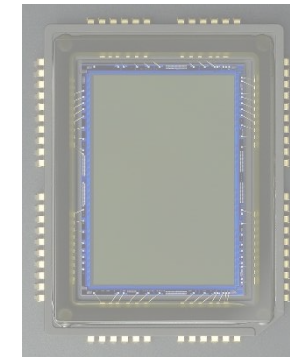
What f to make the height of Eifel tower appear 960 pixel distance?



Exercise

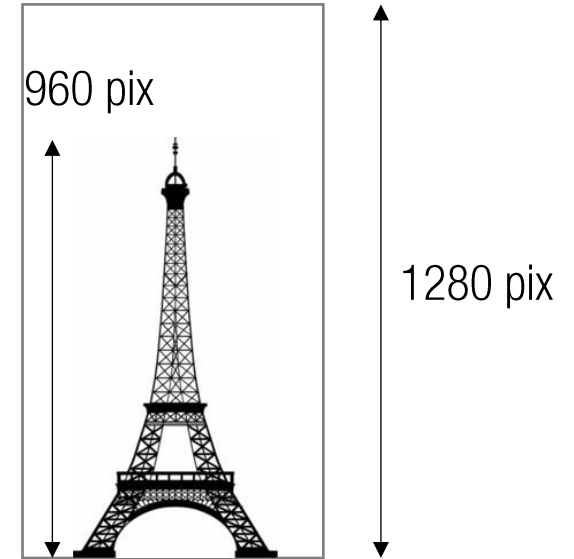


What f to make the height of Eifel tower appear 960 pixel distance?

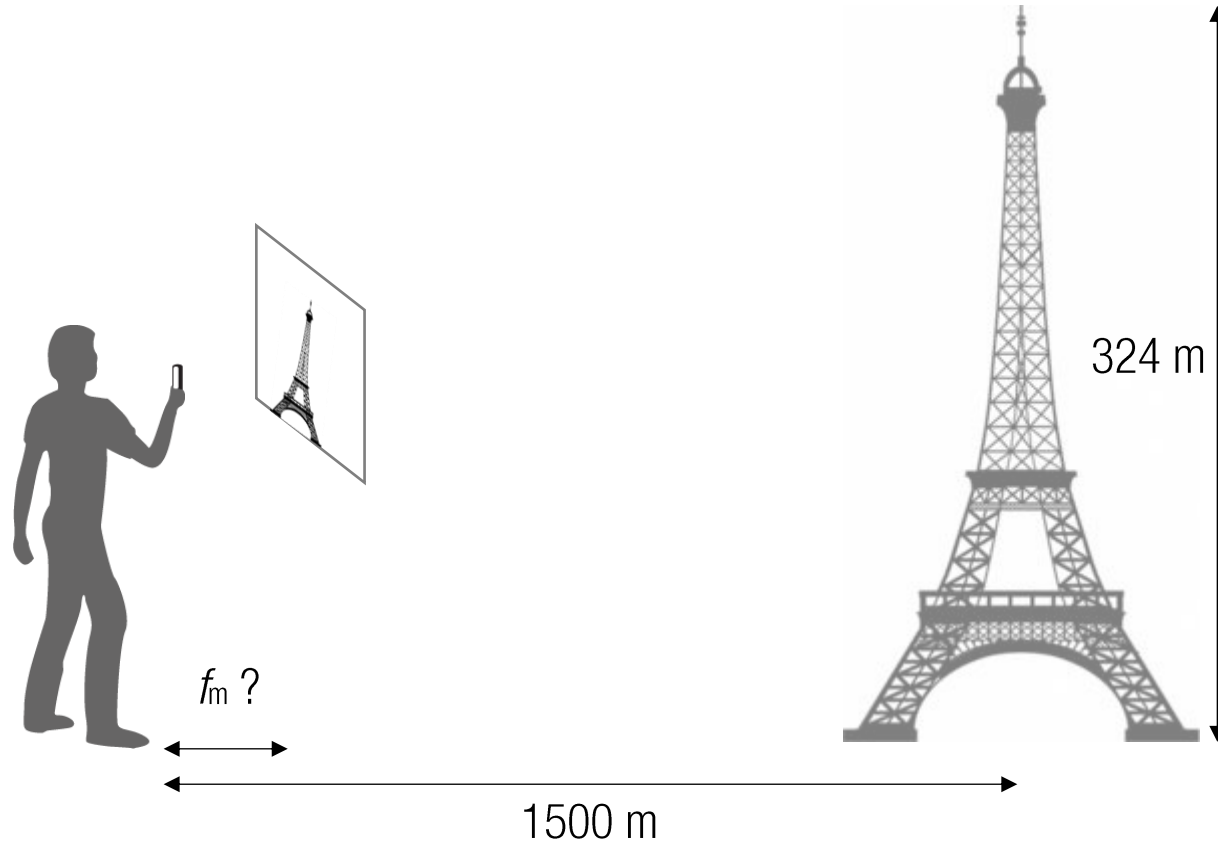


21.8 mm

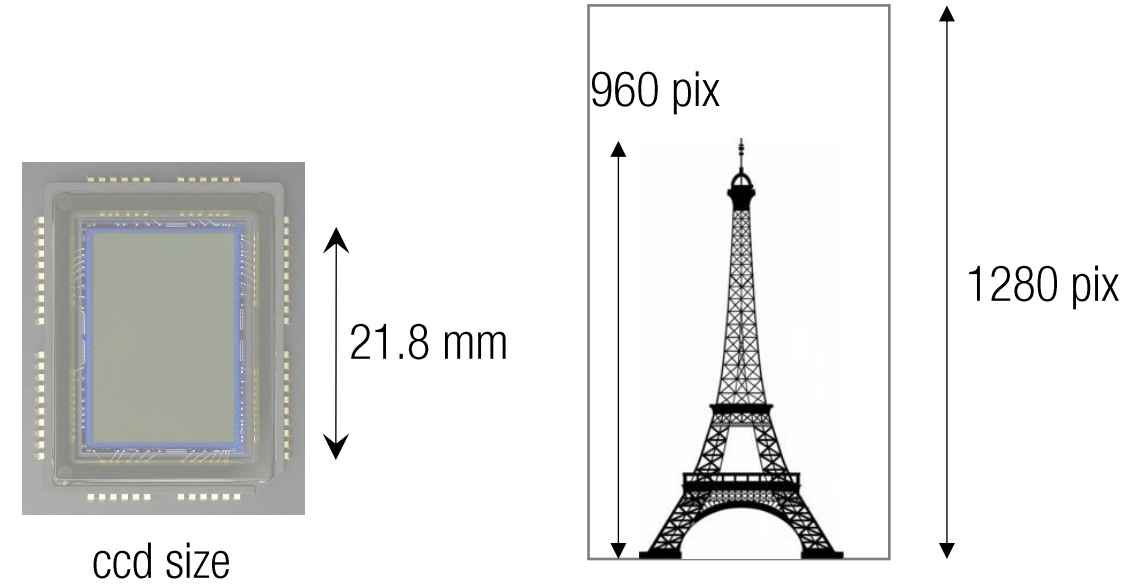
$$y_{\text{img}} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{\gamma}{Z}$$



Exercise



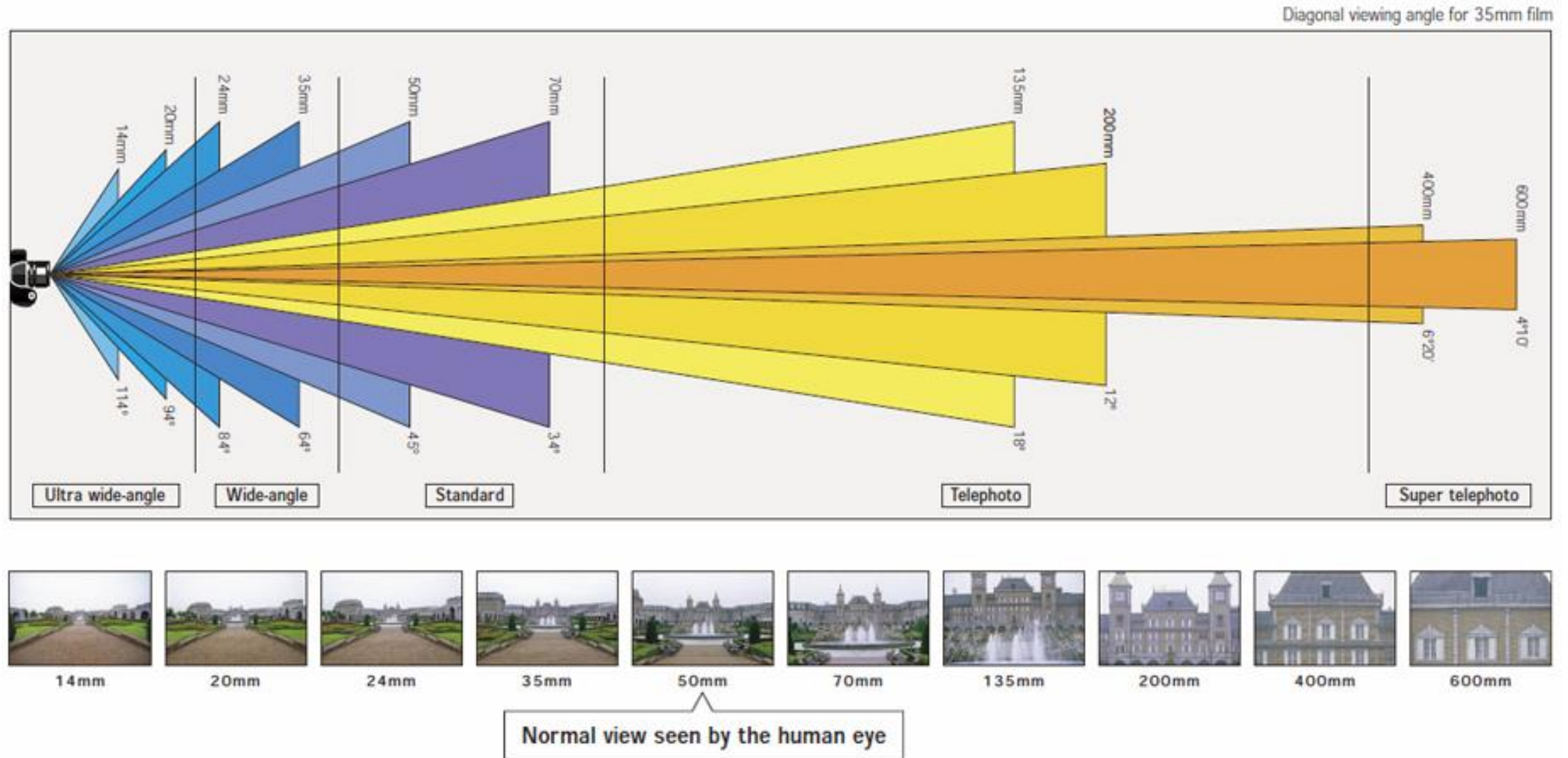
What f to make the height of Eiffel tower appear 960 pixel distance?



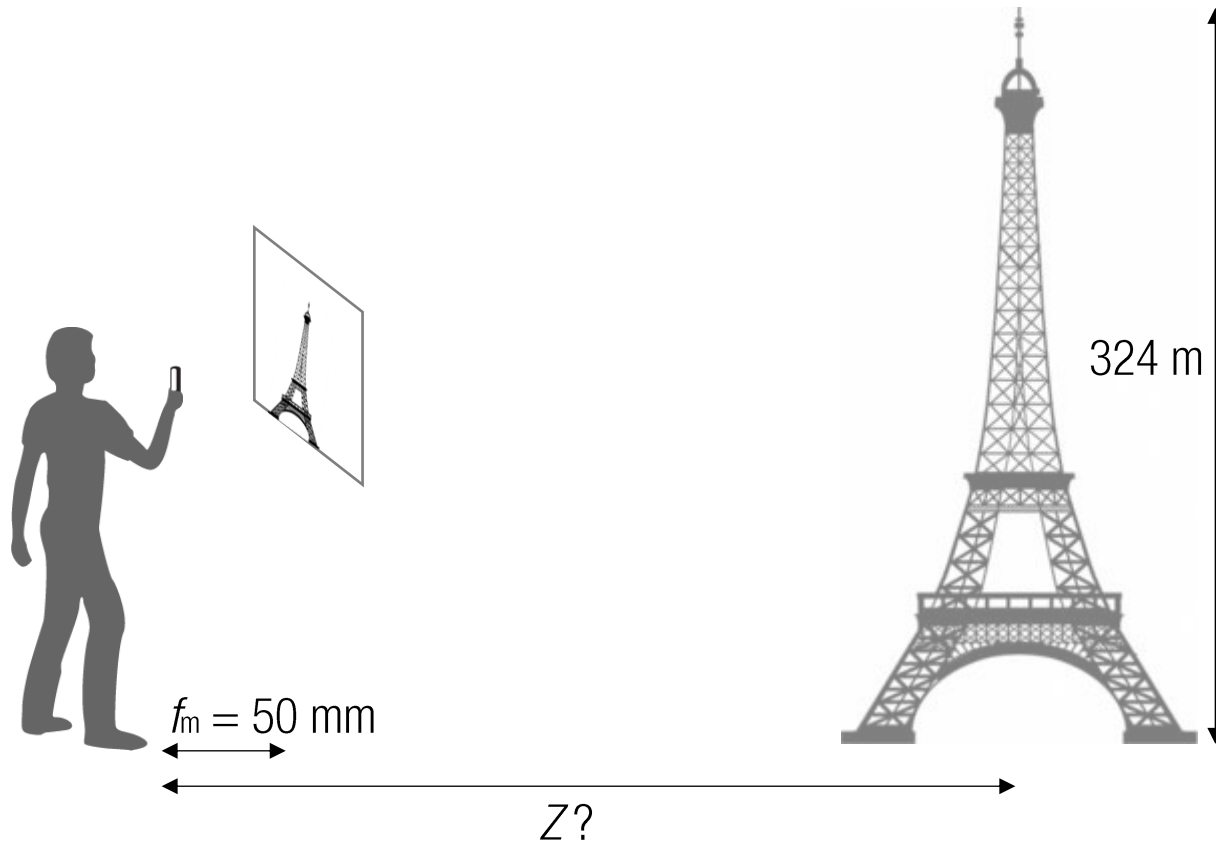
$$y_{\text{img}} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z}$$

$$960 = f_m \frac{1280}{0.0218} \frac{324}{1500} \rightarrow f_m = 0.0757 \text{ m}$$

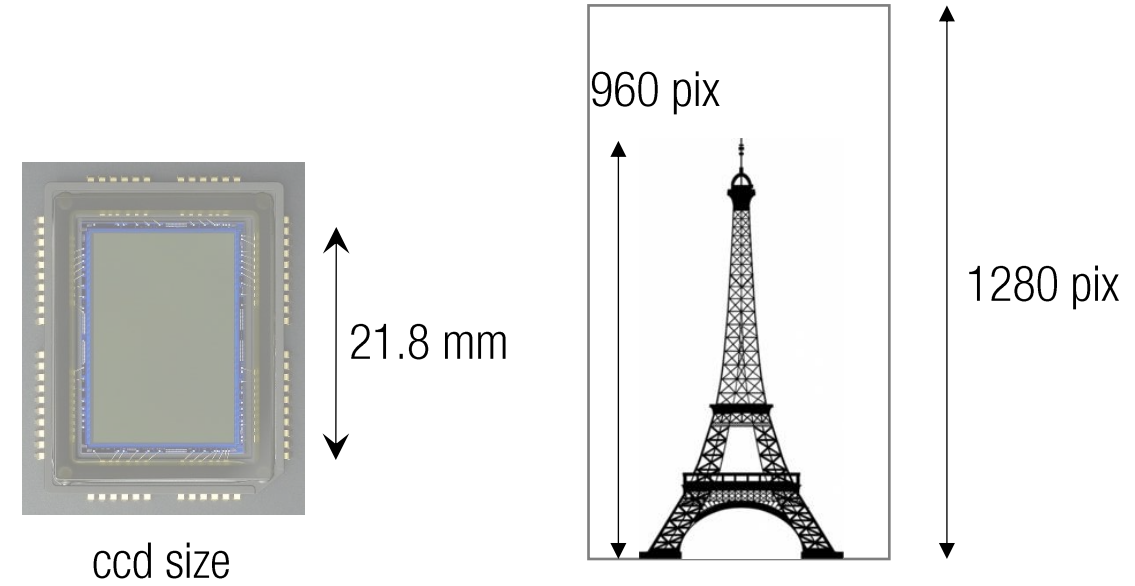
Focal Length



Exercise

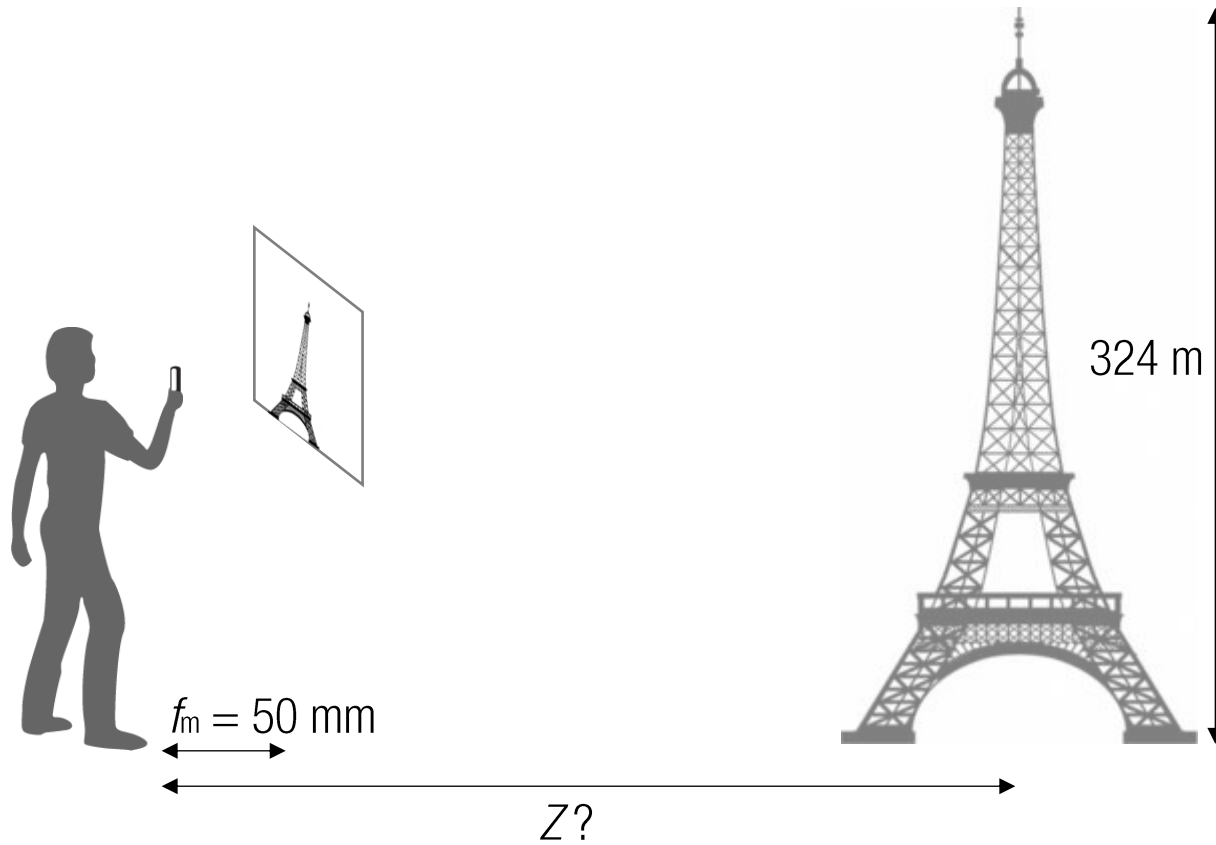


What Z to make the height of Eiffel tower appear 960 pixel distance?

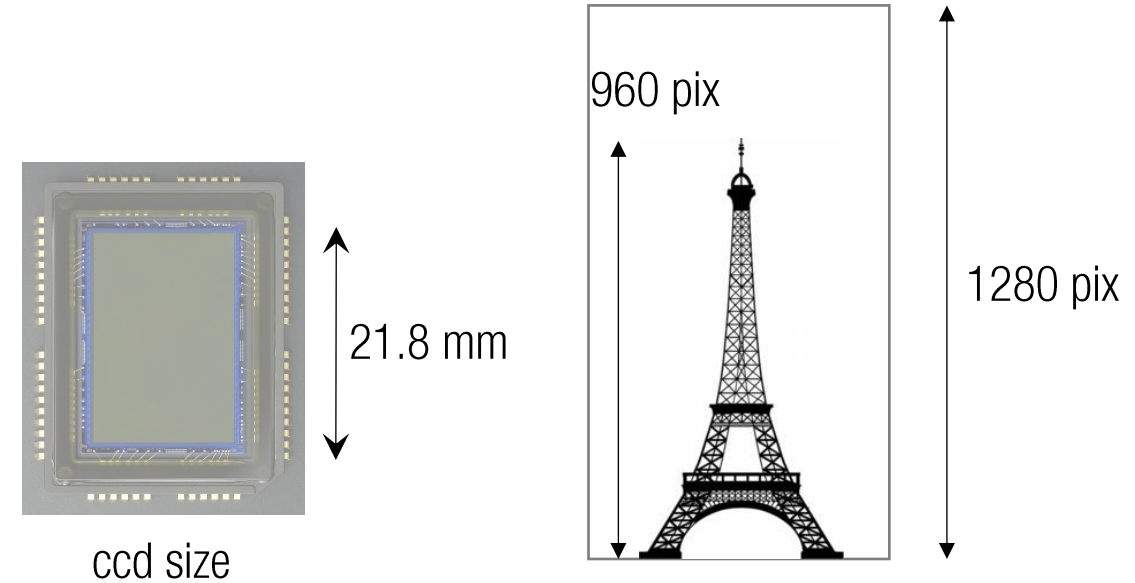


Do I need to move backward or forward?

Exercise



What Z to make the height of Eiffel tower appear 960 pixel distance?



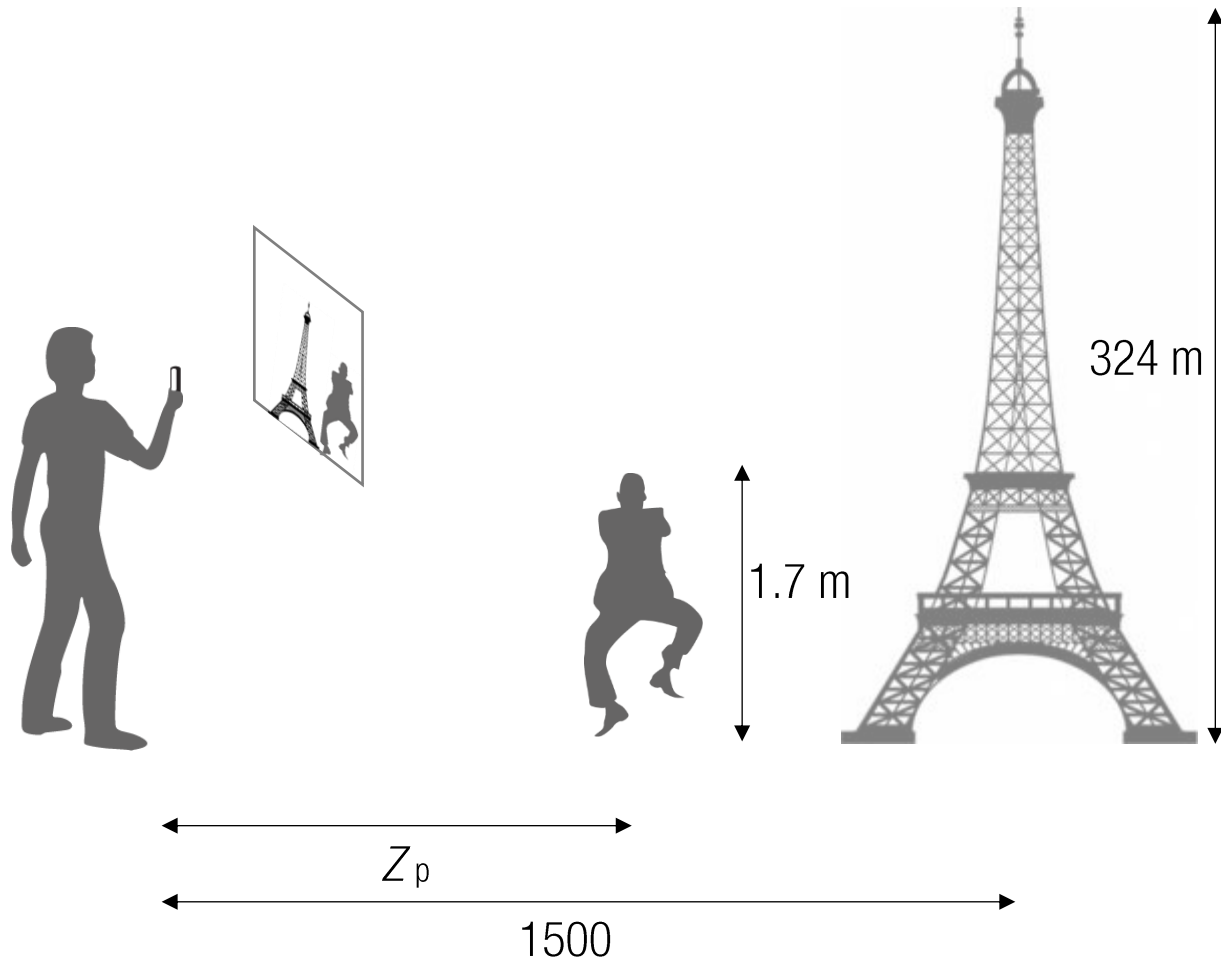
$$y_{\text{img}} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{\gamma}{Z}$$

$$960 = 0.05 \frac{1280}{0.0218} \frac{324}{Z} \rightarrow Z = 990.826 \text{ m}$$

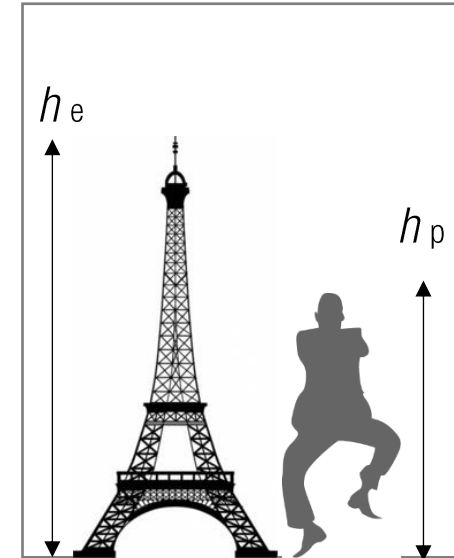
Do I need to move backward or forward?
Forward



Exercise

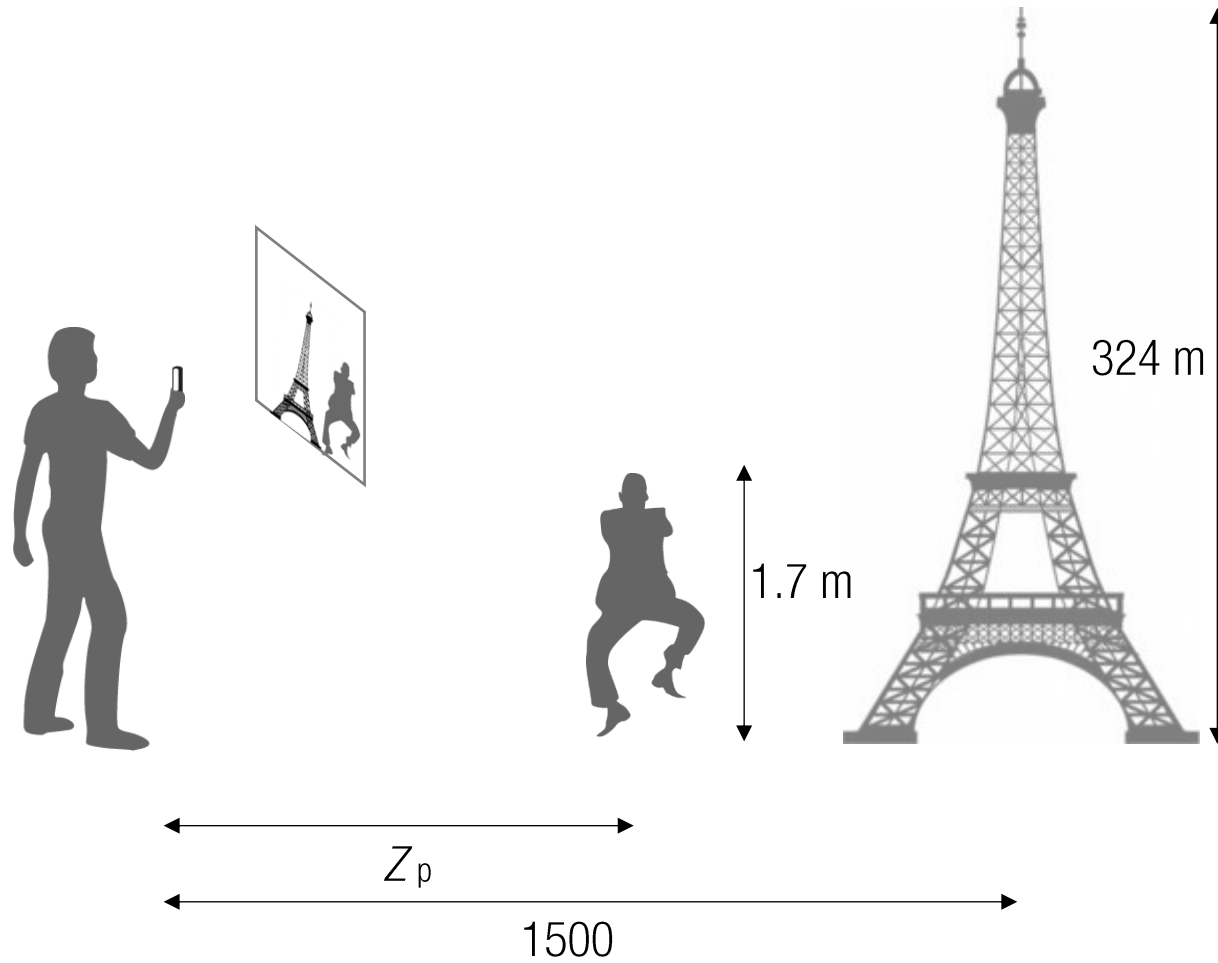


What Z_p to make the height of Eiffel tower appear twice of the person?

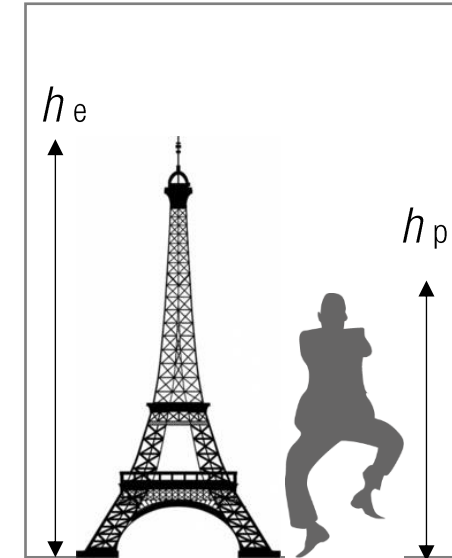


$$h_p = \frac{h_e}{2}$$

Exercise

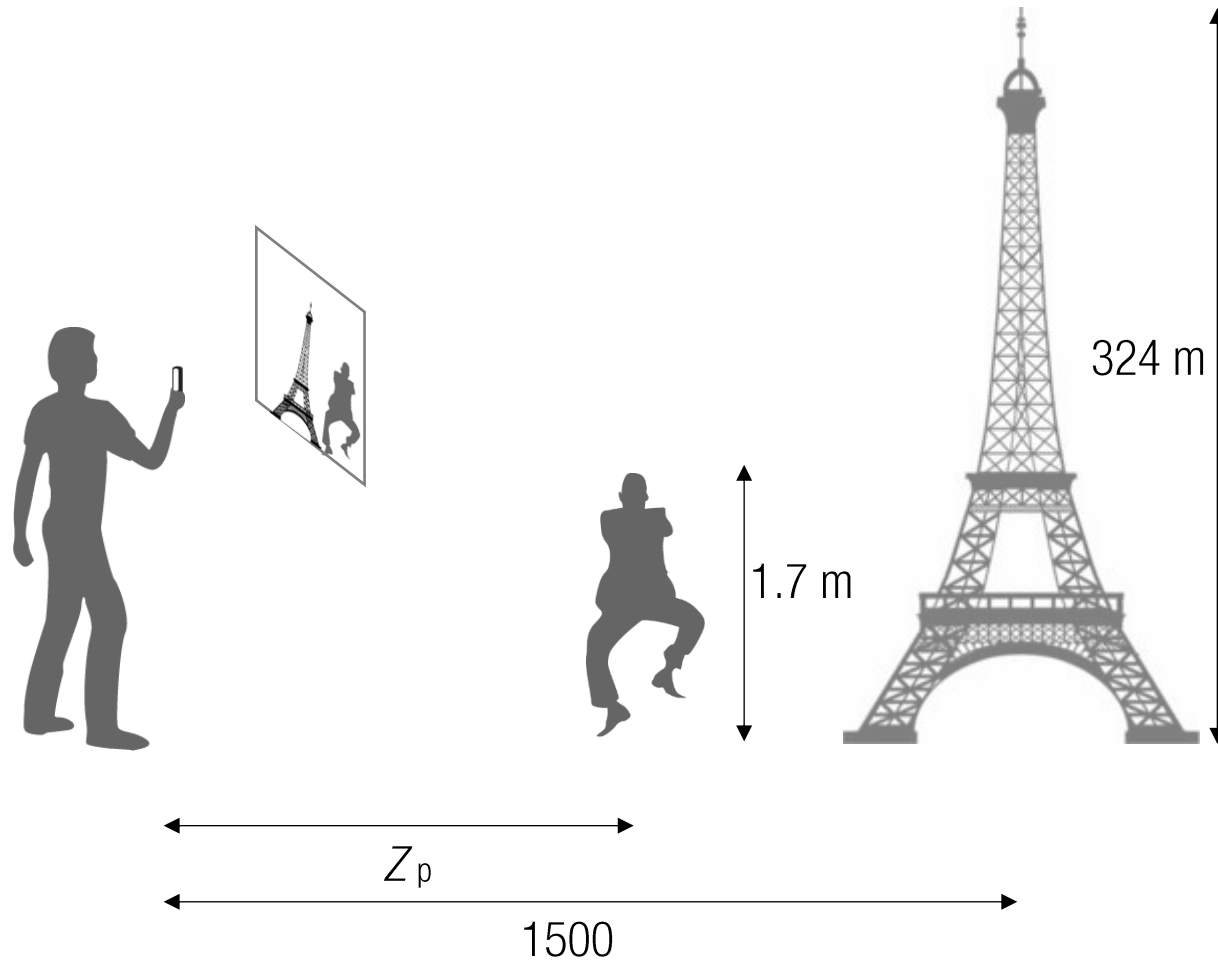


What Z_p to make the height of Eiffel tower appear twice of the person?

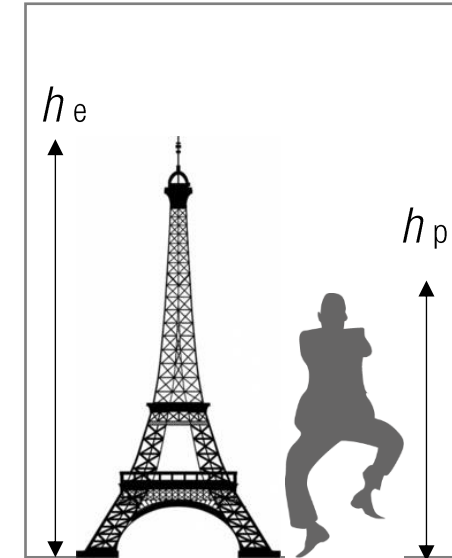


$$h_e = f \frac{Y}{Z} \quad h_p = f \frac{Y_p}{Z_p} \quad \text{s.t.} \quad h_p = \frac{h_e}{2}$$

Exercise



What Z_p to make the height of Eiffel tower appear twice of the person?



$$h_e = f \frac{Y}{Z} \quad h_p = f \frac{Y_p}{Z_p} \quad \text{s.t.} \quad h_p = \frac{h_e}{2}$$

$$f \frac{Y_p}{Z_p} = f \frac{Y}{2Z} \rightarrow Z_p = 2 \cdot 1500 \frac{1.7}{324} = 15.74 \text{ m}$$



VERTIGO (1958)





VERTIGO (1958)





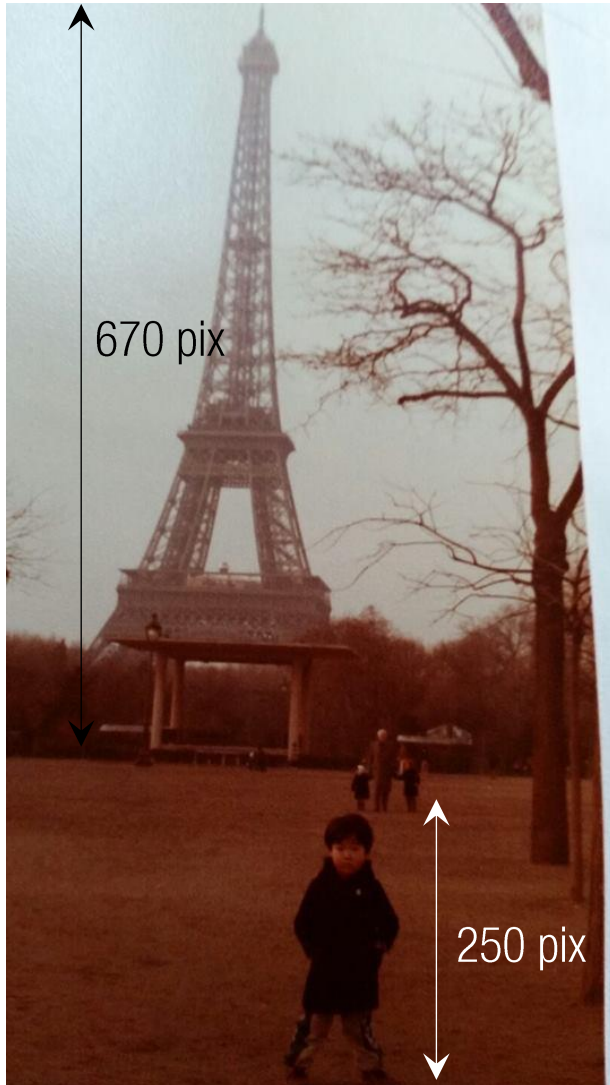
JAWS (1975)

Where Was I?



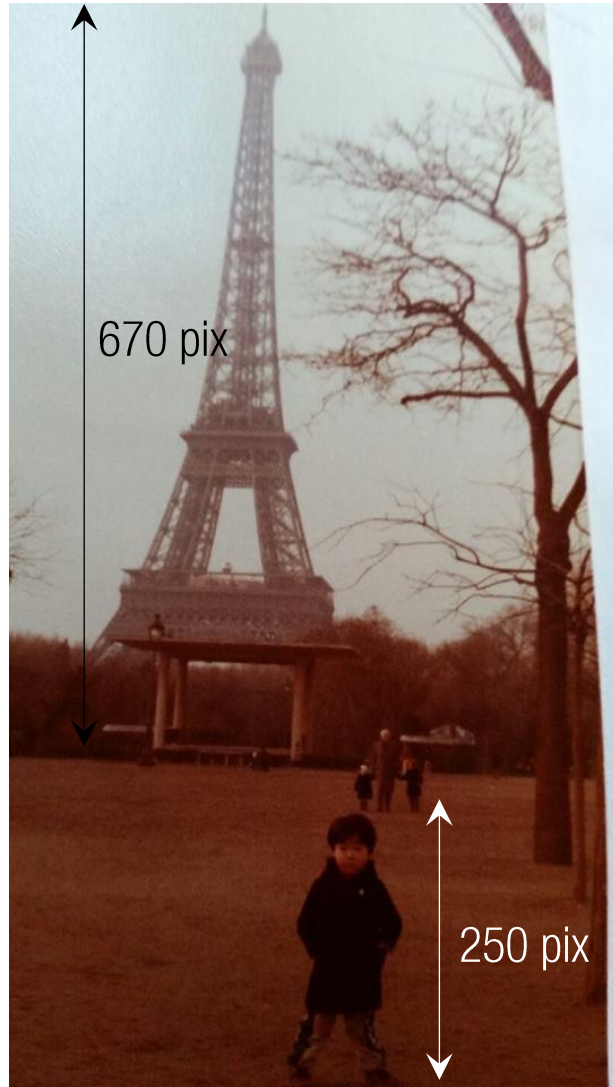
Circa 1984

Where Was I?

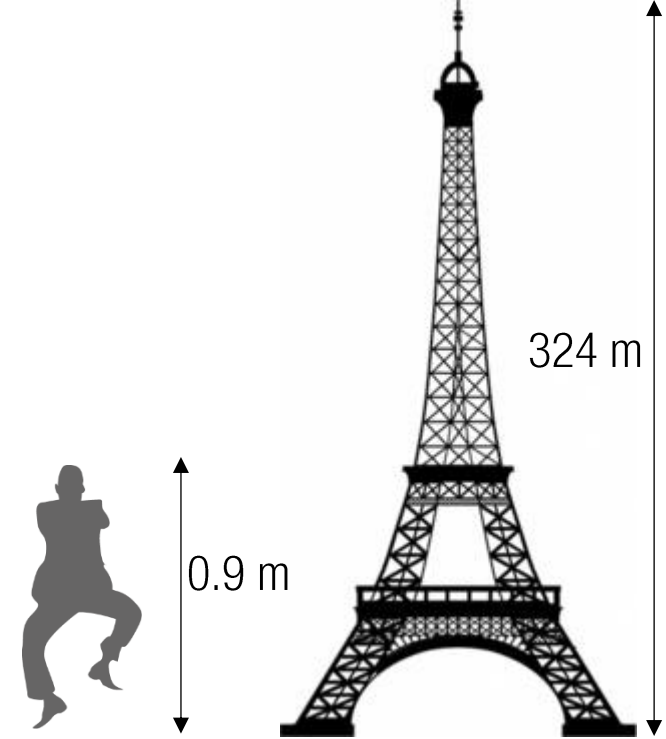
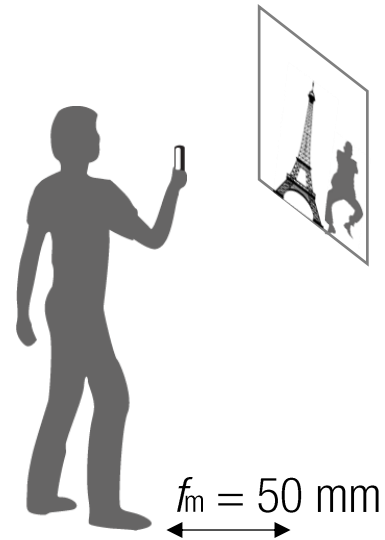


Circa 1984

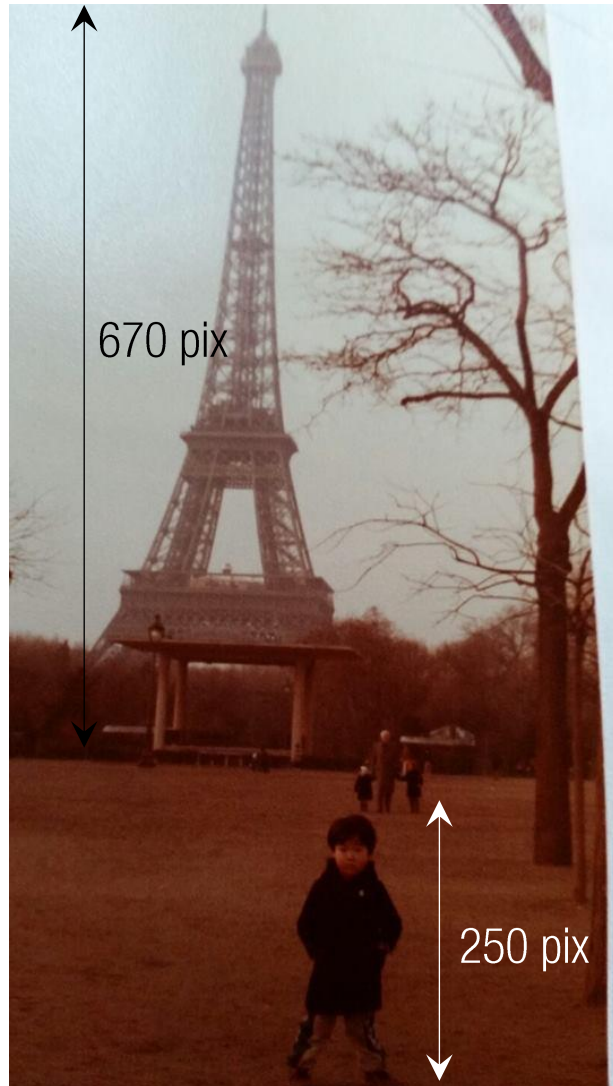
Where Was I?



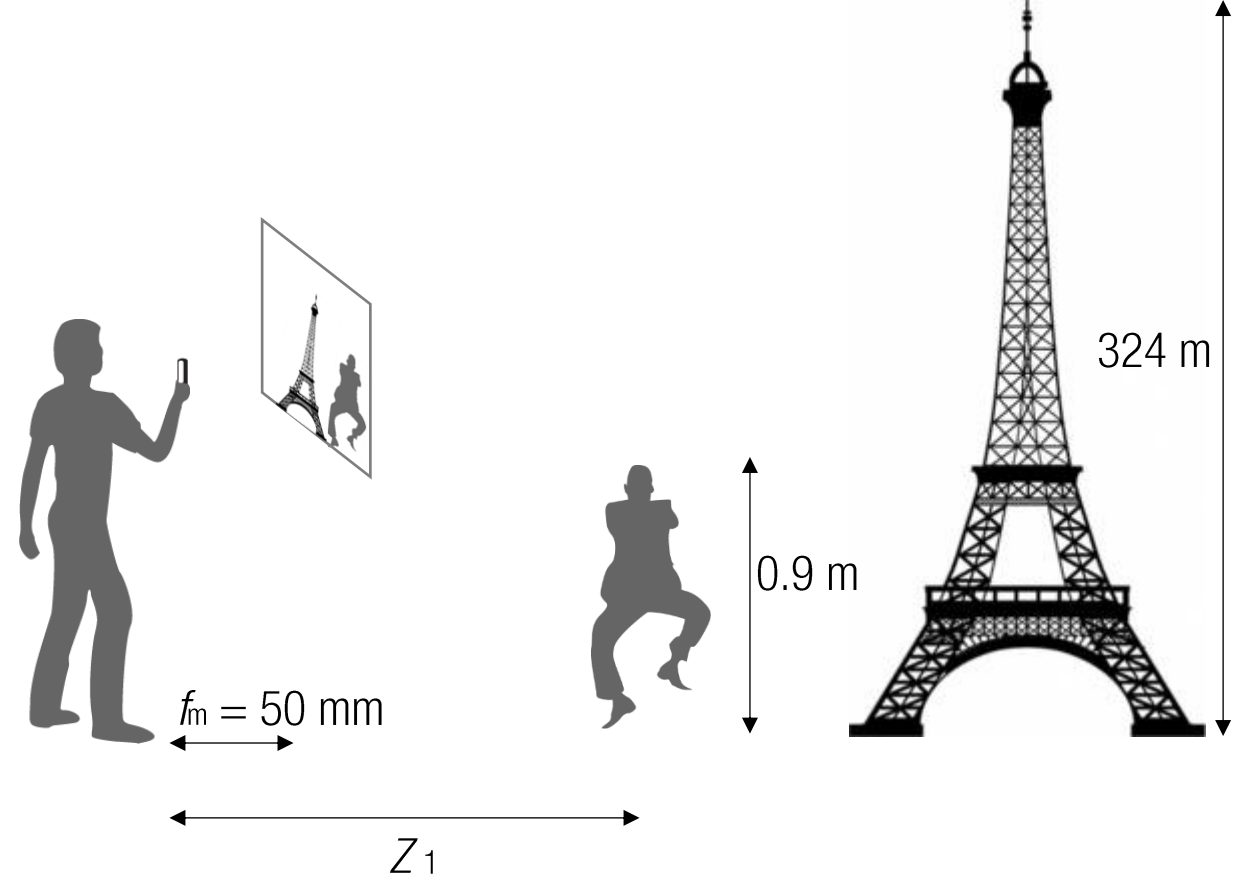
Circa 1984



Where Was I?

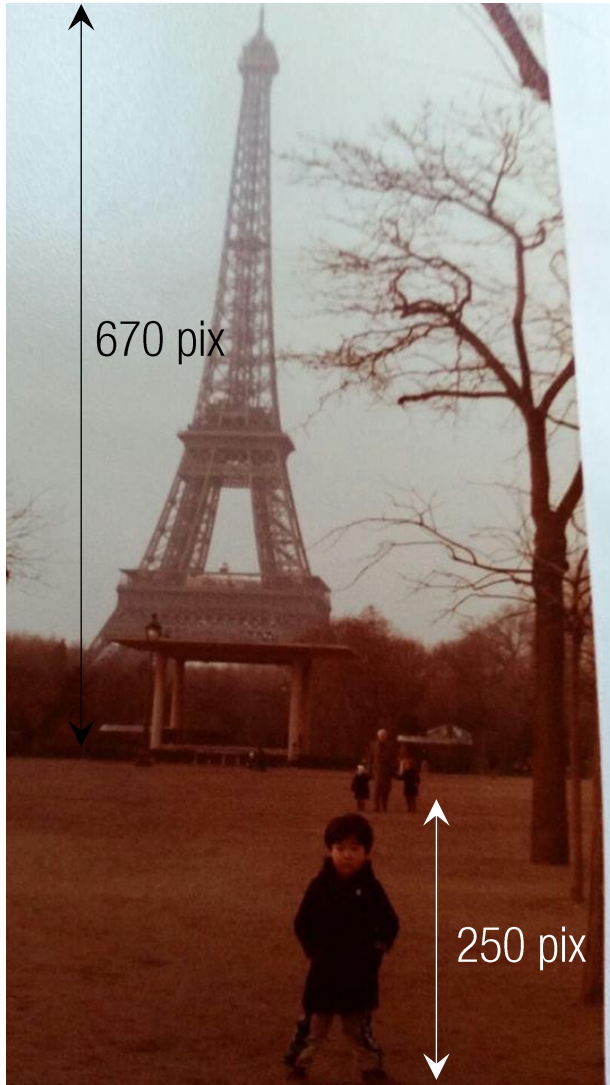


Circa 1984

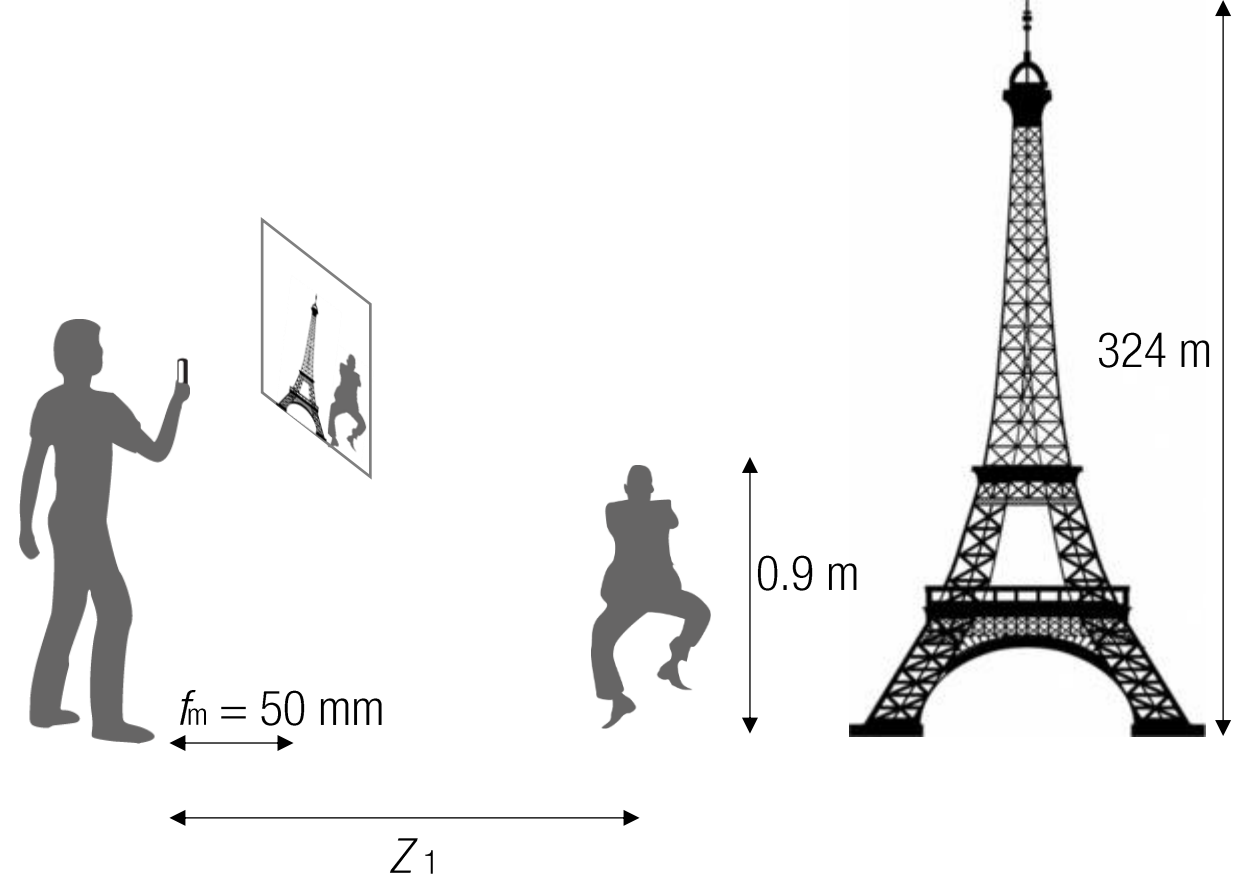


$$y_1 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{Z_1}.$$

Where Was I?

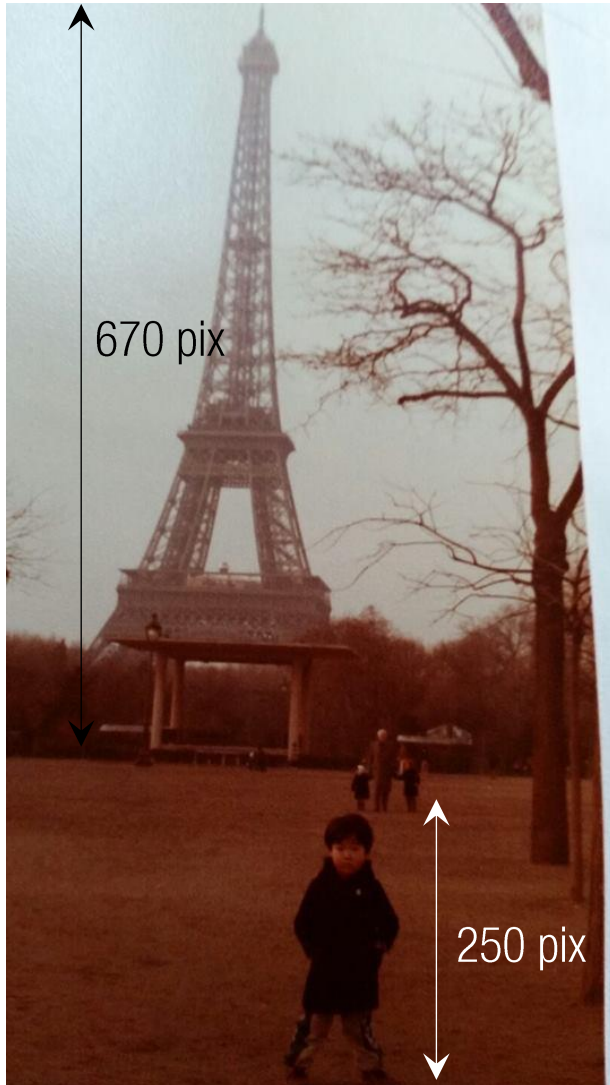


Circa 1984

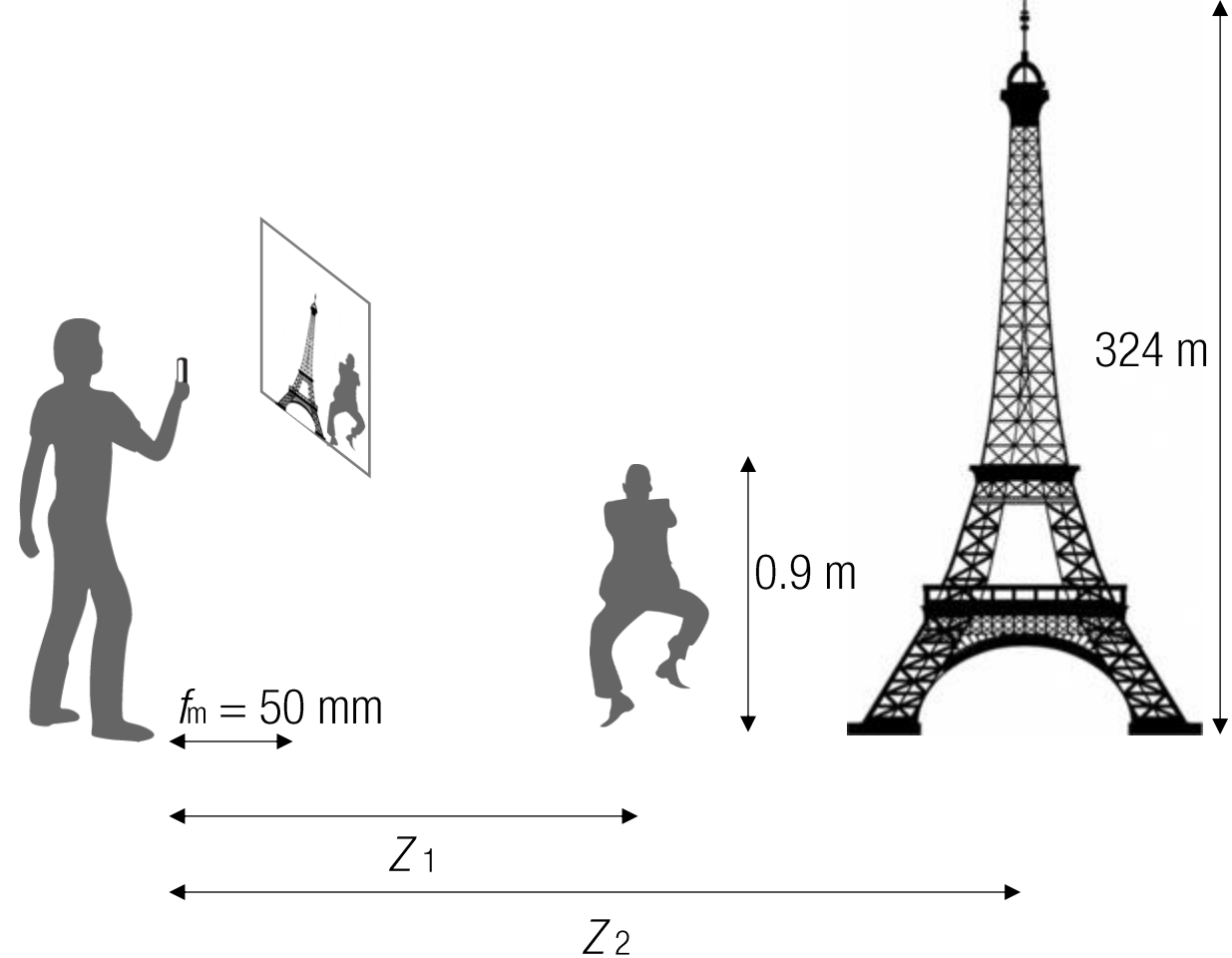


$$y_1 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{Z_1} \rightarrow Z_1 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{y_1} = 0.05 \frac{1280}{0.0218} \frac{0.9}{250} = 8.03 \text{ m}$$

Where Was I?



Circa 1984

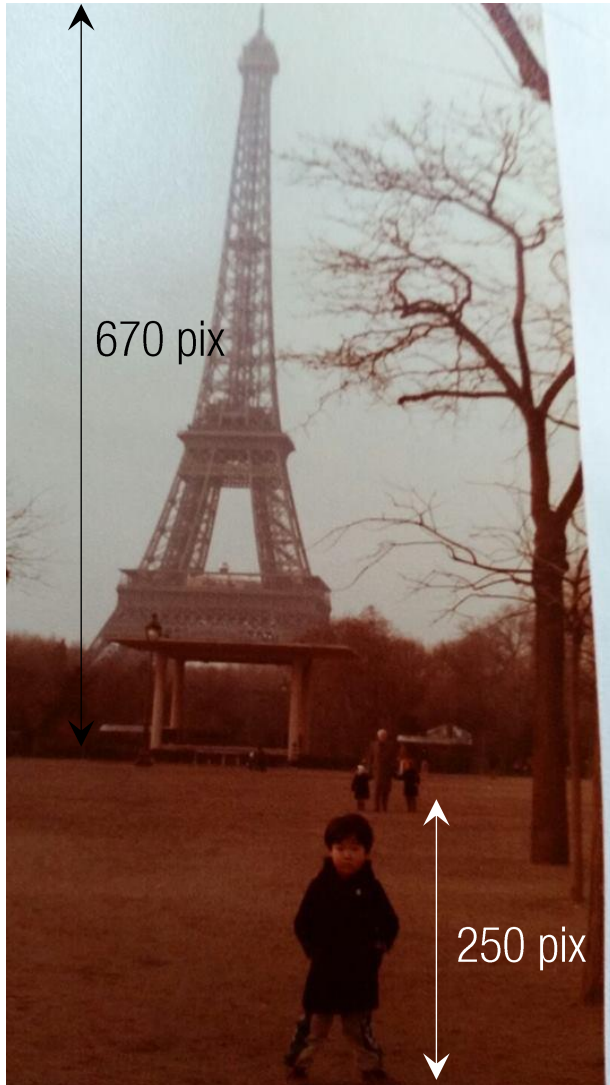


$$y_1 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{Z_1} \rightarrow Z_1 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{y_1} = 0.05 \frac{1280}{0.0218} \frac{0.9}{250} = 8.03 \text{ m}$$

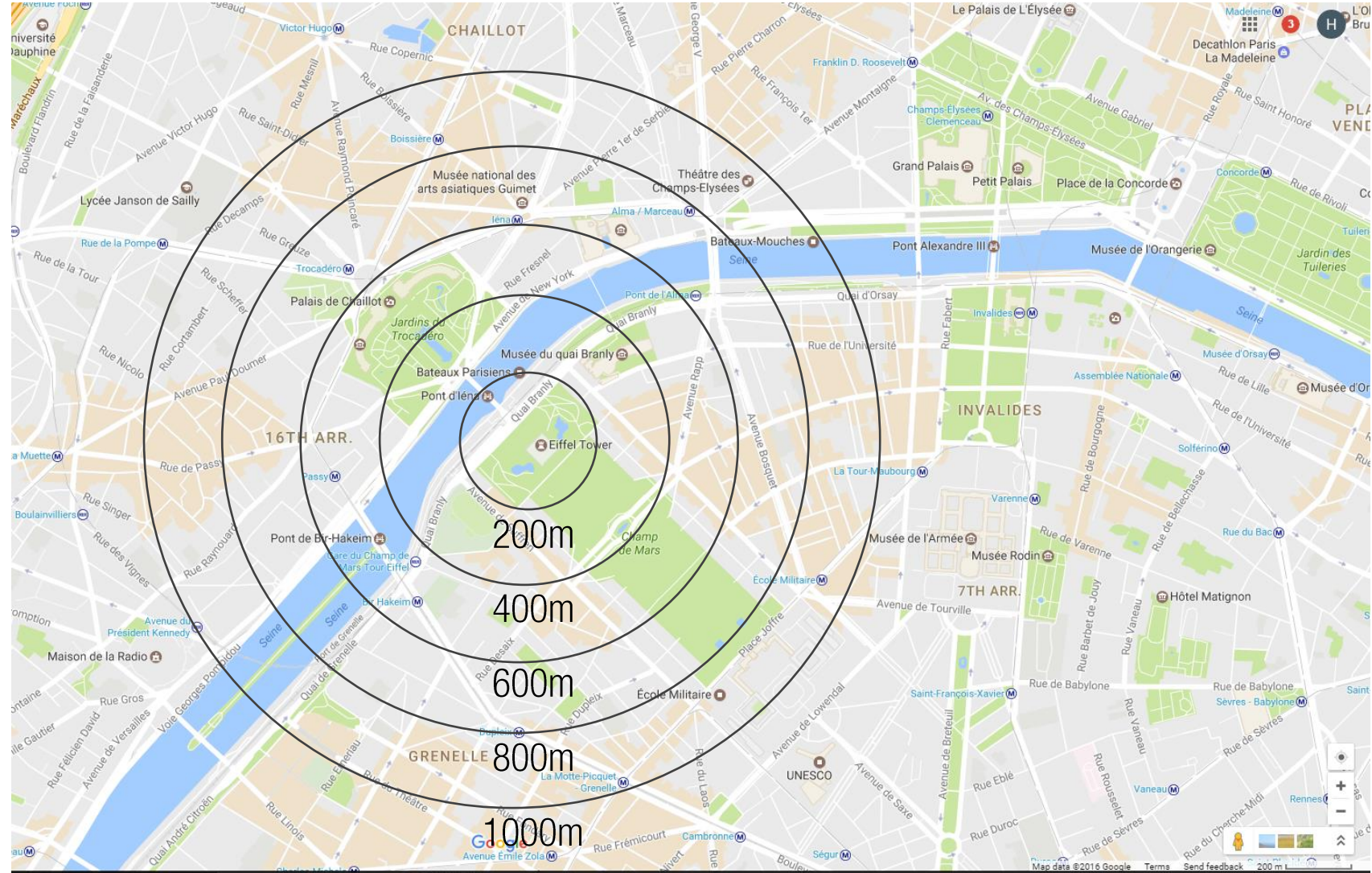
$$y_2 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{Z_2} \rightarrow Z_2 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{y_2} = 0.05 \frac{1280}{0.0218} \frac{324}{670} = 1079 \text{ m}$$

Where Was I?

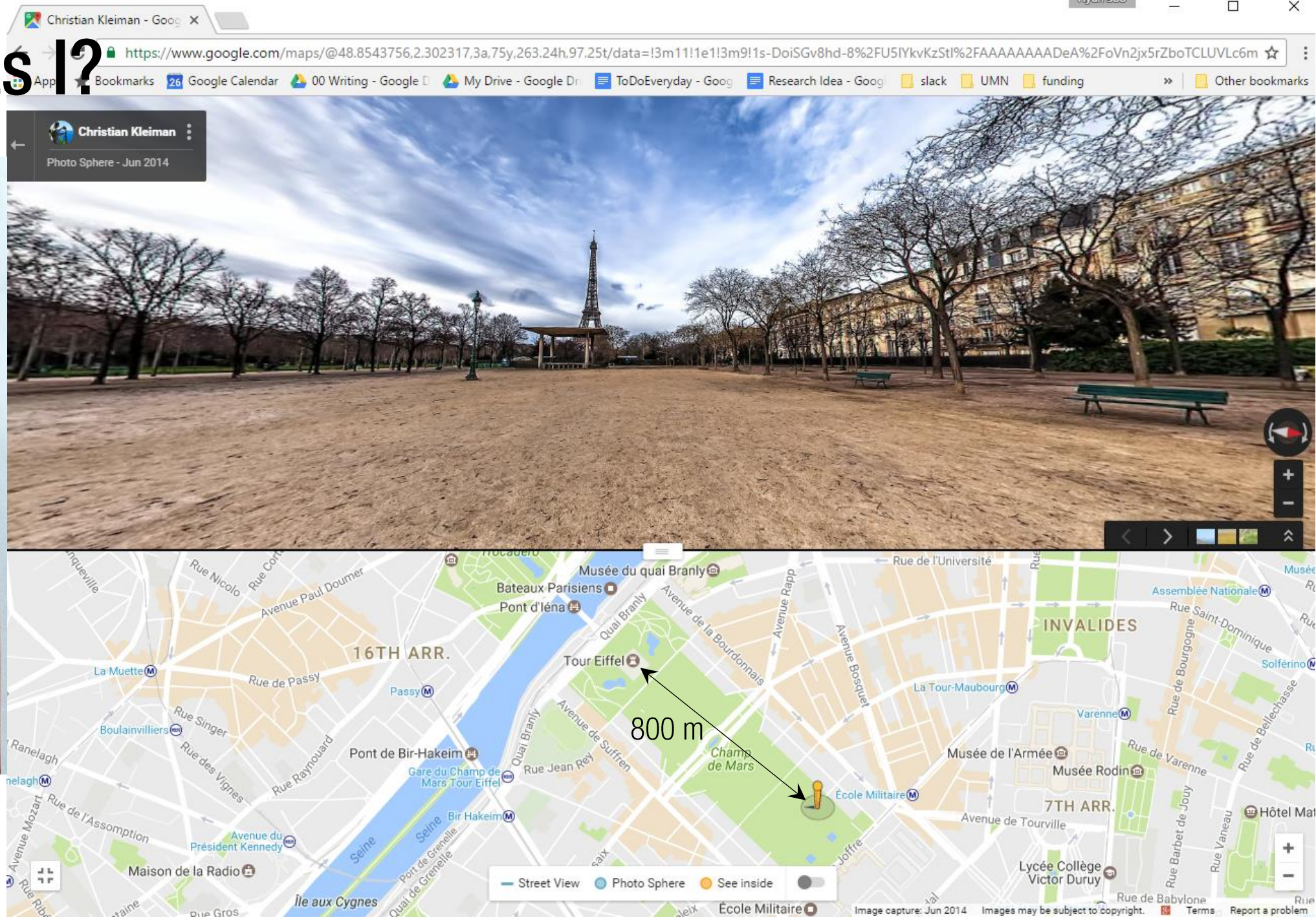
$$y_2 = f \frac{Y_2}{Z_2} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{Z_2} \rightarrow Z_2 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{y_2} = 0.05 \frac{1280}{0.0218} \frac{324}{670} = 1079\text{m}$$



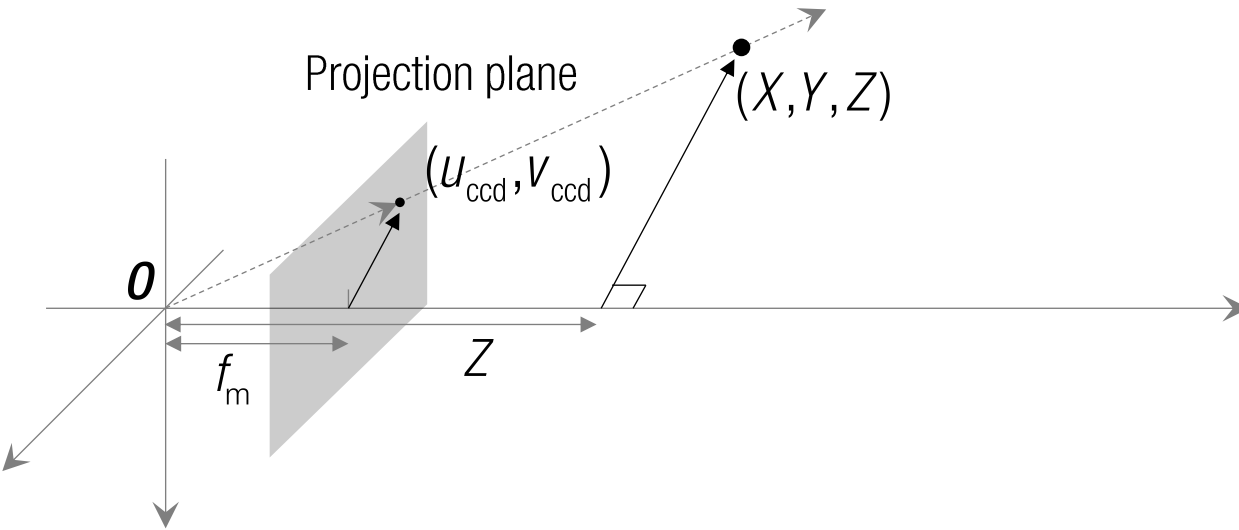
Circa 1984



Where Was I?



3D Point Projection (Pixel Space)

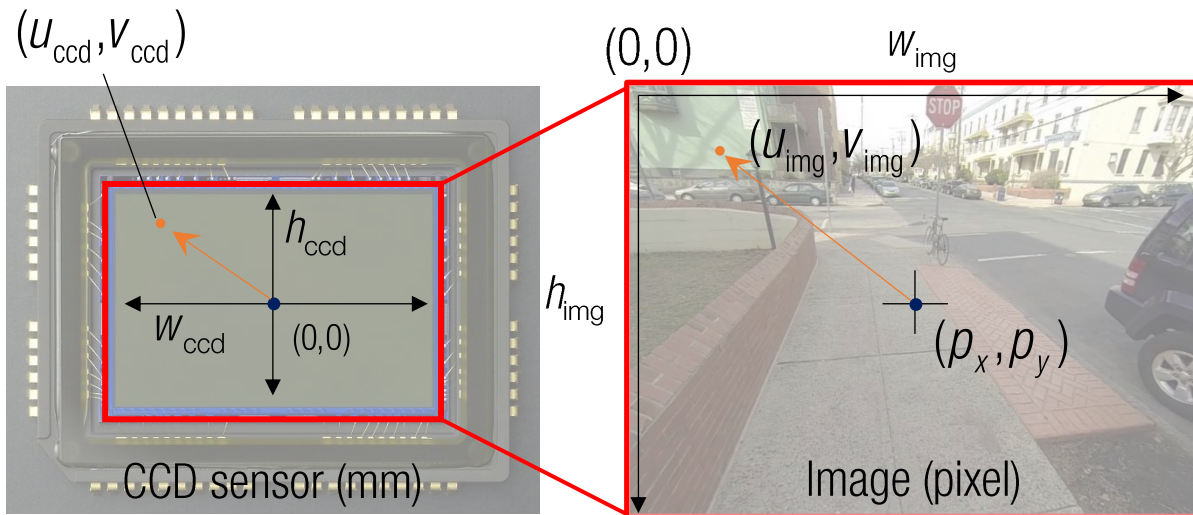


$$(u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}) \quad : \text{Metric projection}$$

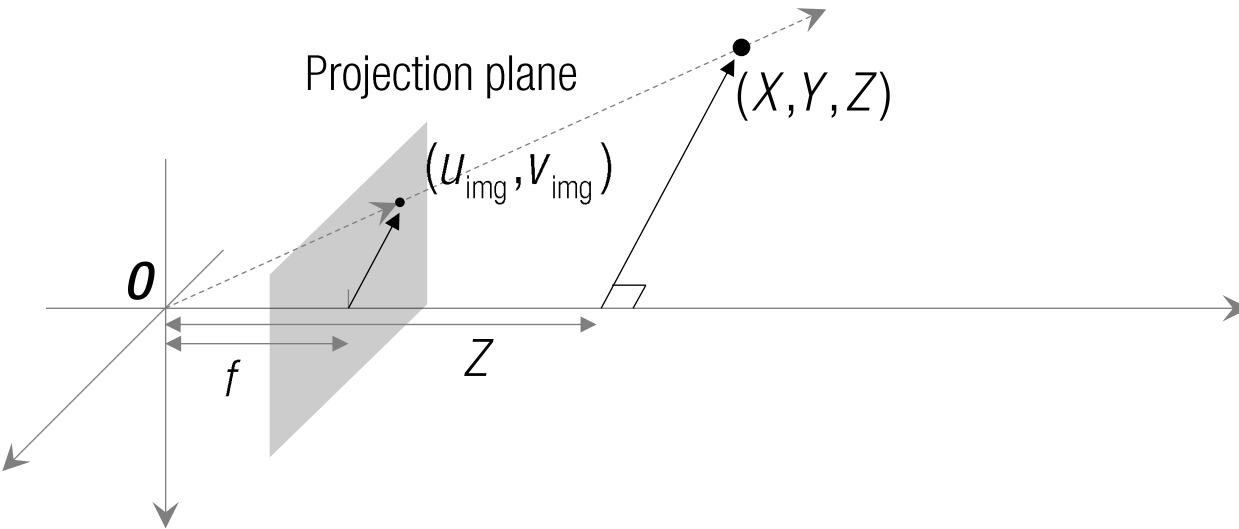
Pixel projection

$$\longrightarrow u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x = f_m \frac{w_{img}}{w_{ccd}} \frac{X}{Z} + p_x$$

$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_m \frac{h_{img}}{h_{ccd}} \frac{Y}{Z} + p_y$$



3D Point Projection (Pixel Space)



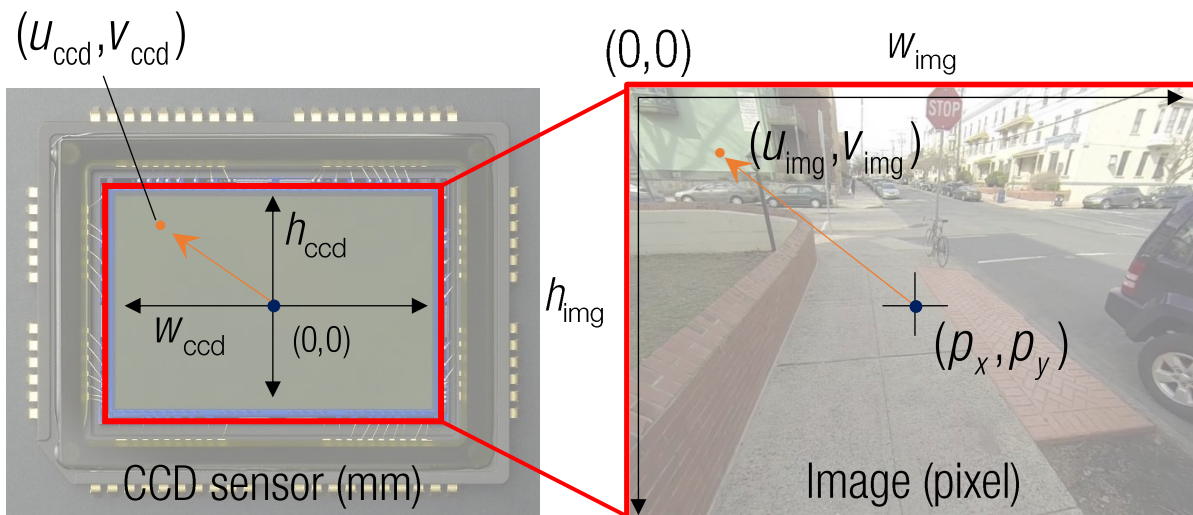
$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$

Pixel projection

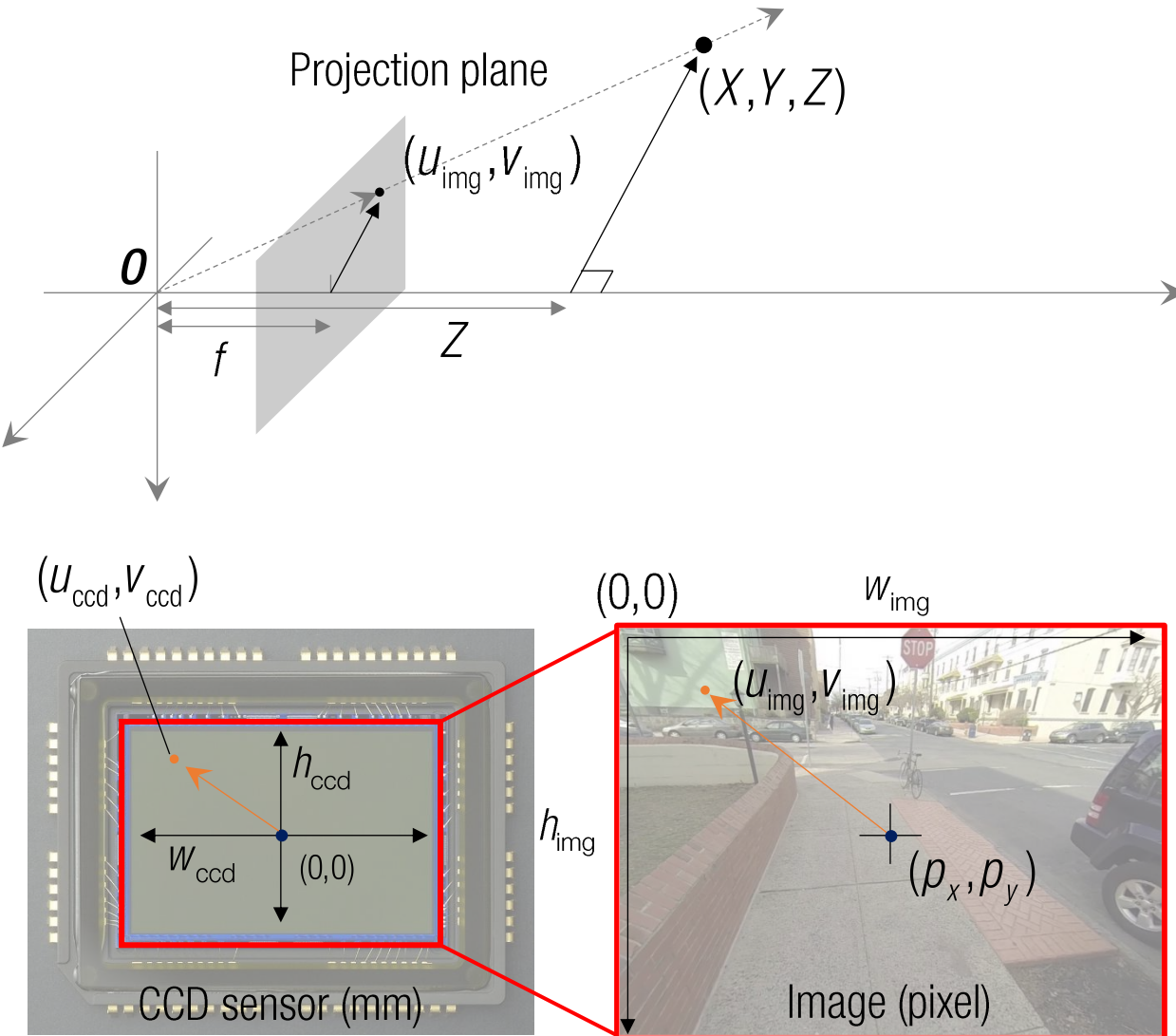
$$\longrightarrow u_{\text{img}} = u_{\text{ccd}} \frac{w_{\text{img}}}{w_{\text{ccd}}} + p_x = f_m \frac{w_{\text{img}}}{w_{\text{ccd}}} \frac{X}{Z} + p_x$$

$$v_{\text{img}} = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z} + p_y$$

Focal length in pixel



3D Point Projection (Pixel Space)



$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$

Pixel projection

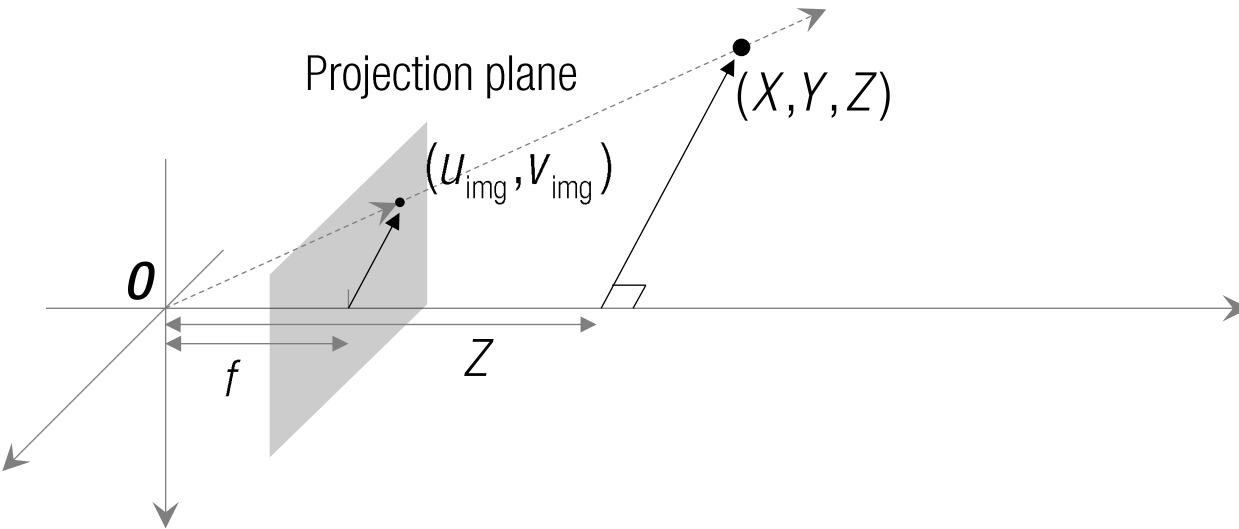
$$\longrightarrow u_{\text{img}} = u_{\text{ccd}} \frac{w_{\text{img}}}{w_{\text{ccd}}} + p_x = f_x \frac{X}{Z} + p_x$$

$$v_{\text{img}} = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y = f_y \frac{Y}{Z} + p_y$$

Focal length in pixel

$$\text{where } f_x = f_m \frac{w_{\text{img}}}{w_{\text{ccd}}} \quad f_y = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}}$$

3D Point Projection (Pixel Space)



$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z}\right) \quad : \text{Metric projection}$$

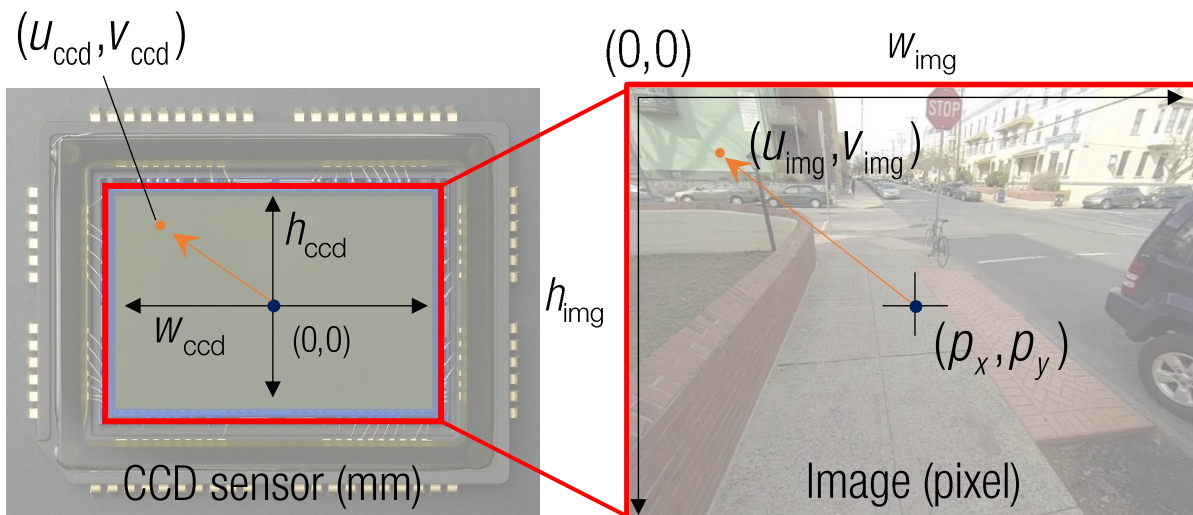
Pixel projection

$$\longrightarrow u_{\text{img}} = u_{\text{ccd}} \frac{w_{\text{img}}}{w_{\text{ccd}}} + p_x = \boxed{f} \frac{X}{Z} + p_x$$

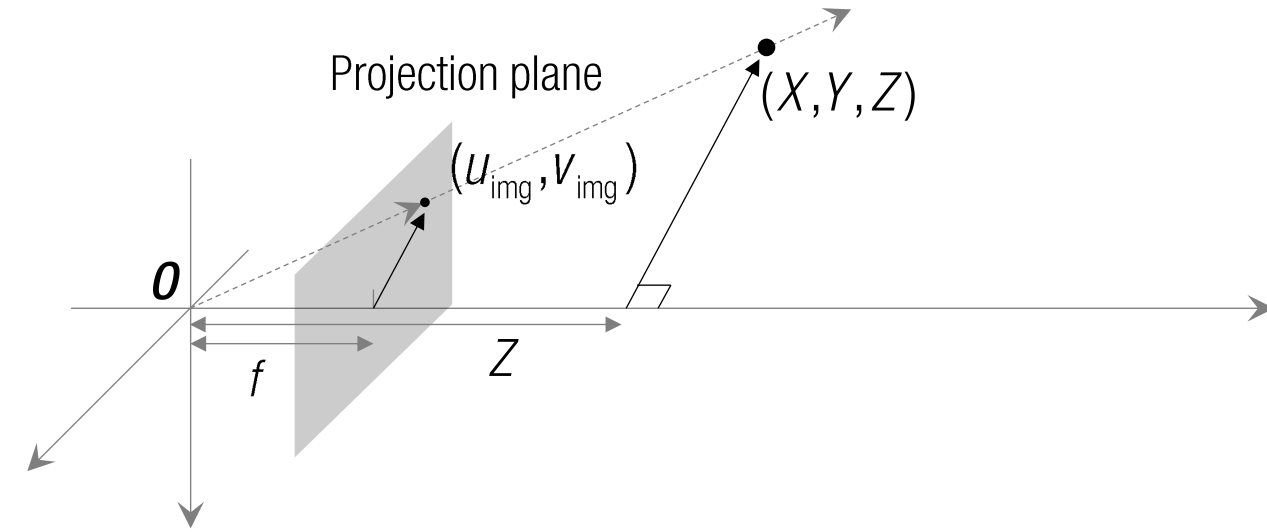
$$v_{\text{img}} = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y = \boxed{f} \frac{Y}{Z} + p_y$$

Focal length in pixel

$$\text{where } f = f_m \frac{w_{\text{img}}}{w_{\text{ccd}}} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \quad \text{if } \frac{w_{\text{img}}}{w_{\text{ccd}}} = \frac{h_{\text{img}}}{h_{\text{ccd}}}$$

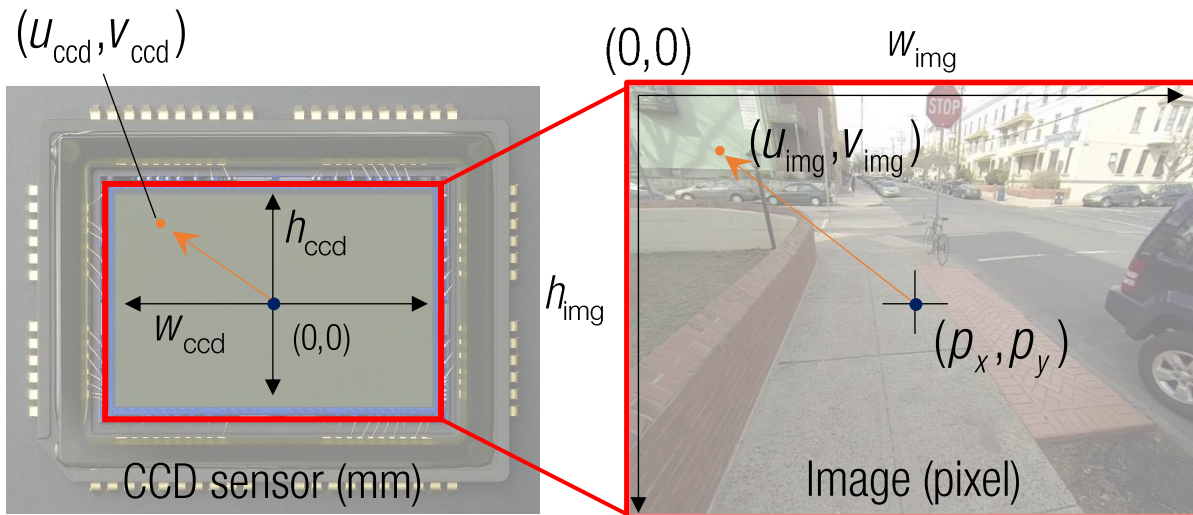


3D Point Projection (Pixel Space)

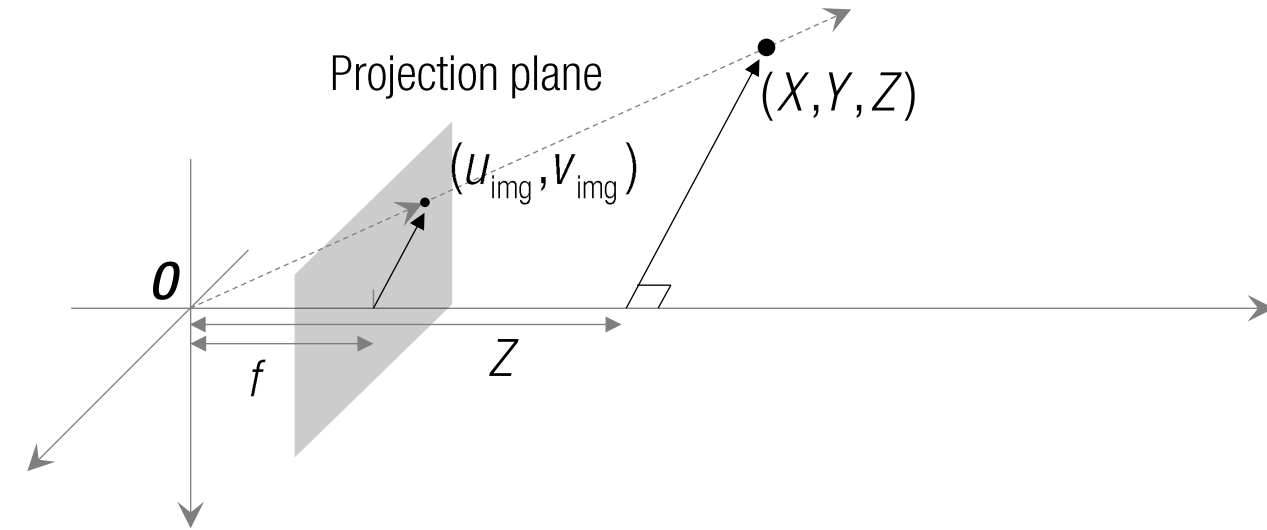


$$u_{\text{img}} = f \frac{X}{Z} + p_x$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y$$

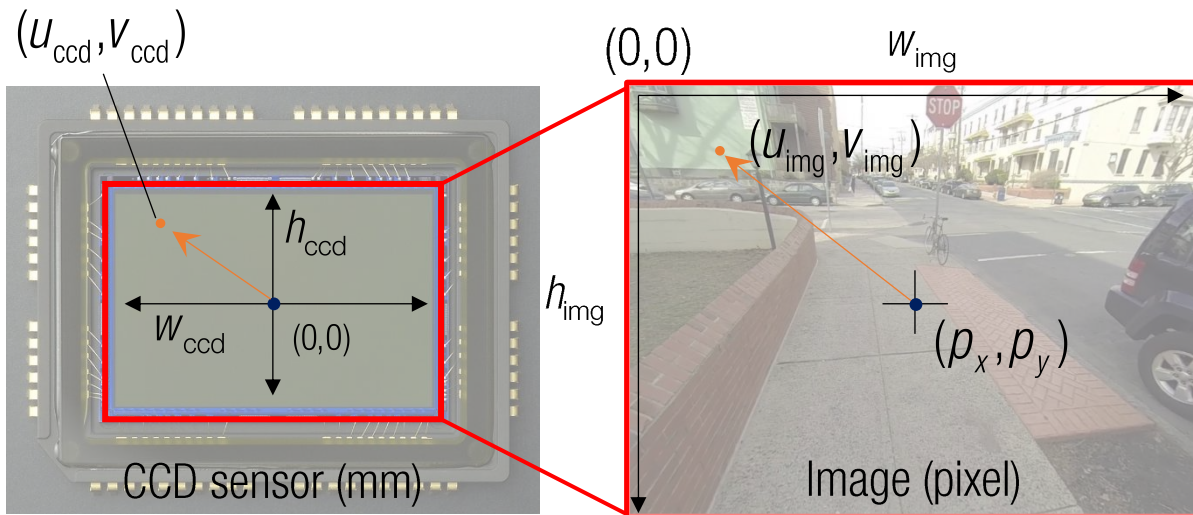


3D Point Projection (Pixel Space)

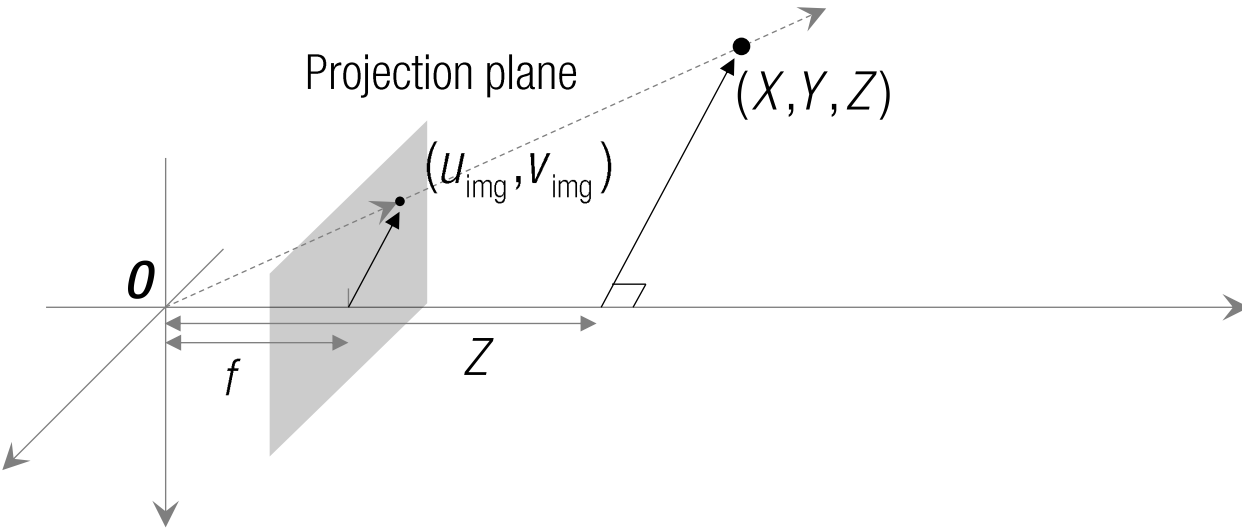


$$u_{\text{img}} = f \frac{X}{Z} + p_x \longrightarrow Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \longrightarrow Zv_{\text{img}} = fY + p_y Z$$



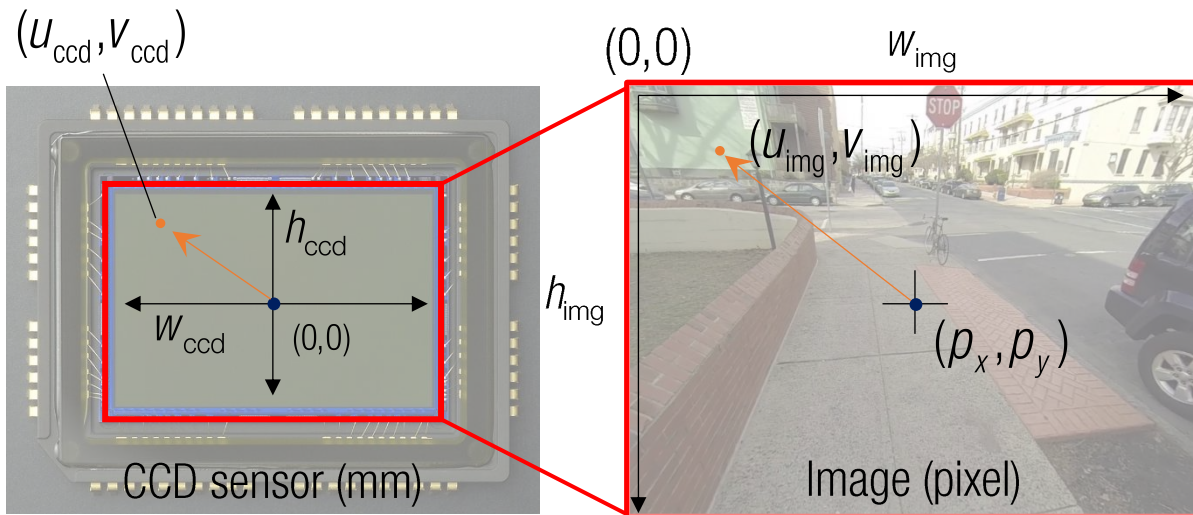
3D Point Projection (Pixel Space)



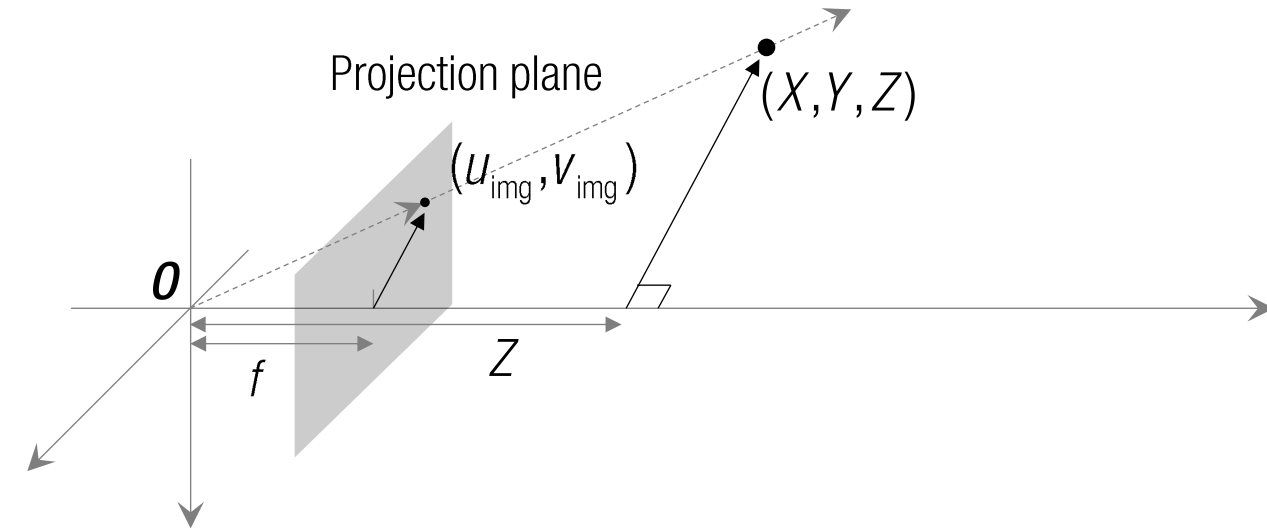
$$u_{\text{img}} = f \frac{X}{Z} + p_x \longrightarrow Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \longrightarrow Zv_{\text{img}} = fY + p_y Z$$

$$Z \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



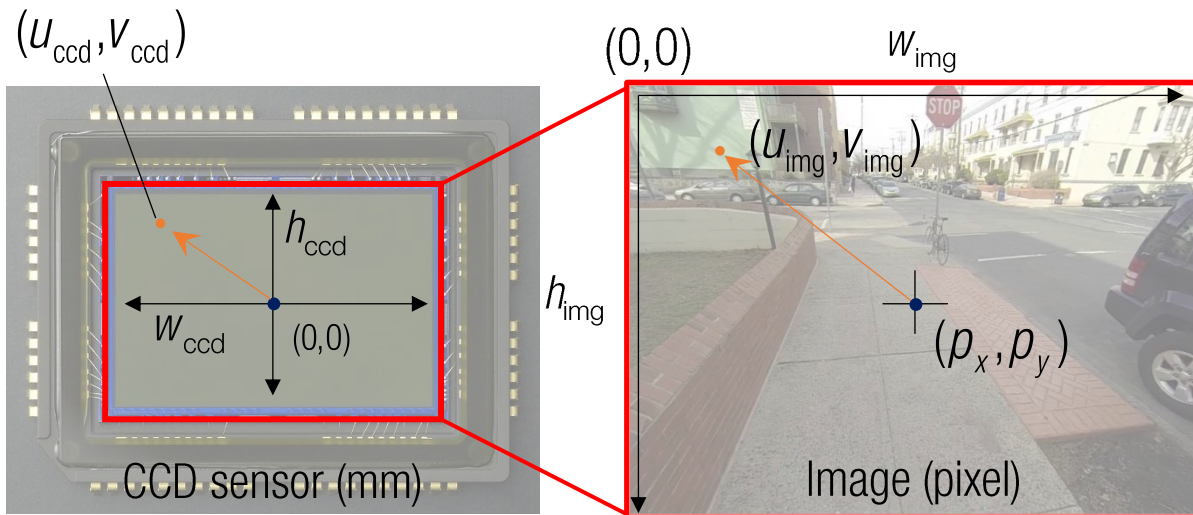
3D Point Projection (Pixel Space)



$$u_{\text{img}} = f \frac{X}{Z} + p_x \longrightarrow Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \longrightarrow Zv_{\text{img}} = fY + p_y Z$$

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Computer Graphics



$$u_{\text{img}} = f \frac{X}{Z} + p_x \longrightarrow Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \longrightarrow Zv_{\text{img}} = fY + p_y Z$$

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

← Graphics

Computer Vision = inv(Computer Graphics)



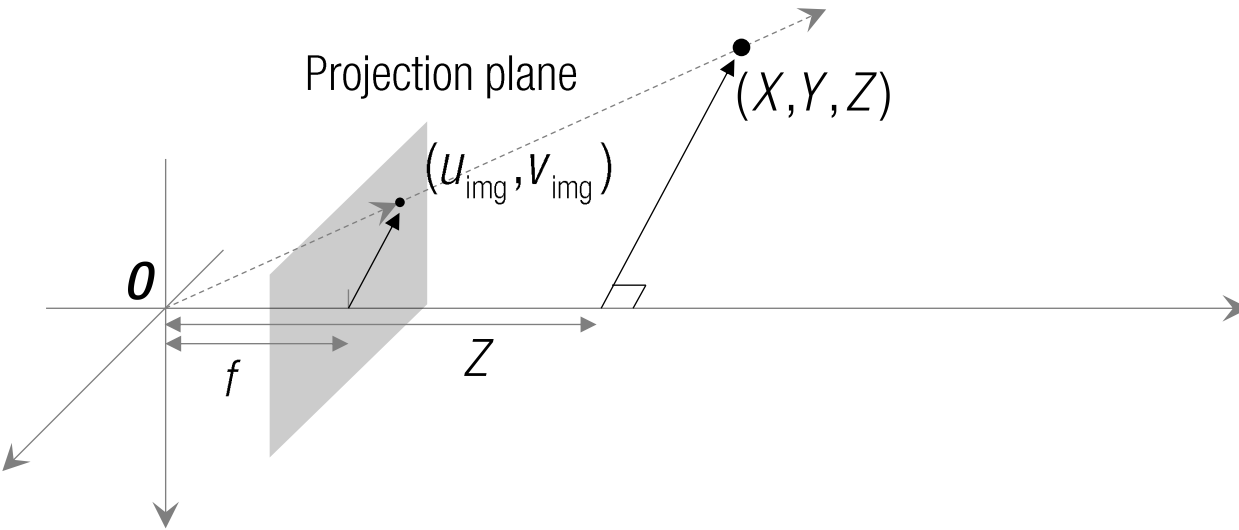
$$u_{\text{img}} = f \frac{X}{Z} + p_x \longrightarrow Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \longrightarrow Zv_{\text{img}} = fY + p_y Z$$

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

← Graphics
→ Vision

3D Point Projection (Pixel Space)



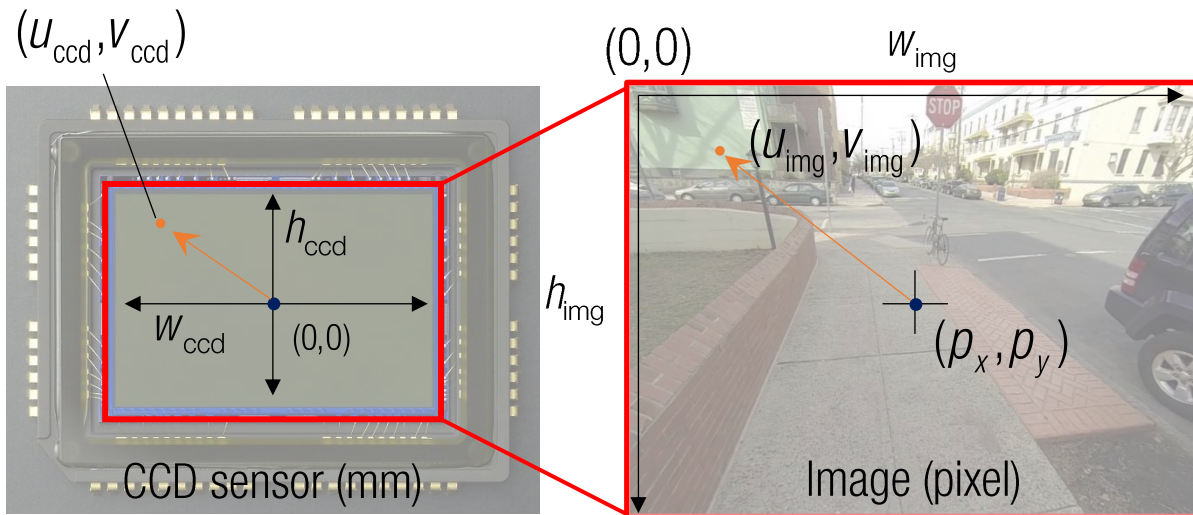
$$u_{\text{img}} = f \frac{X}{Z} + p_x \longrightarrow Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \longrightarrow Zv_{\text{img}} = fY + p_y Z$$

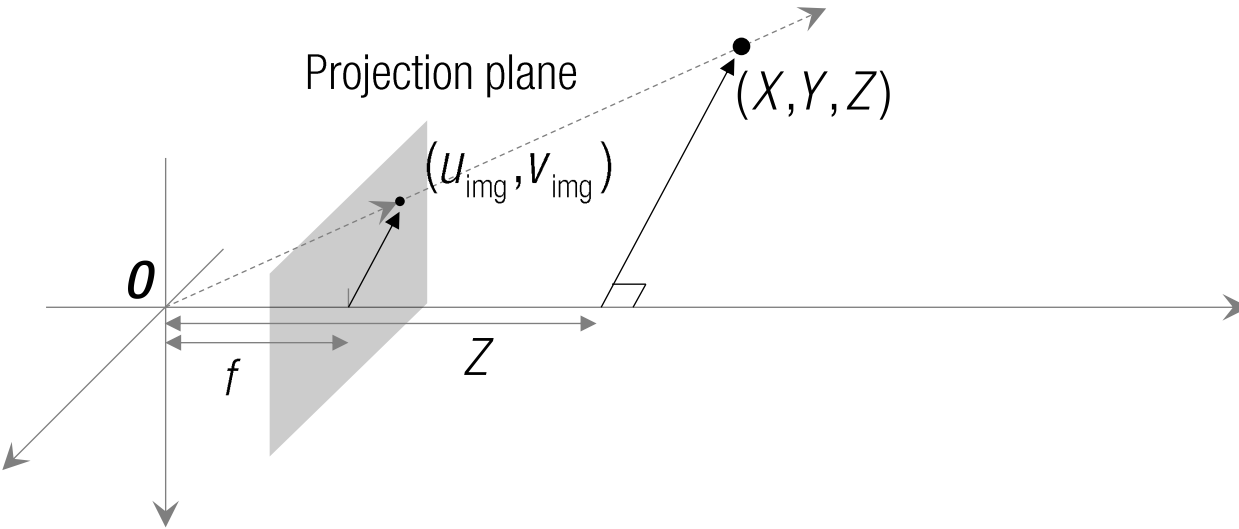
Pixel space

Metric space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



3D Point Projection (Pixel Space)



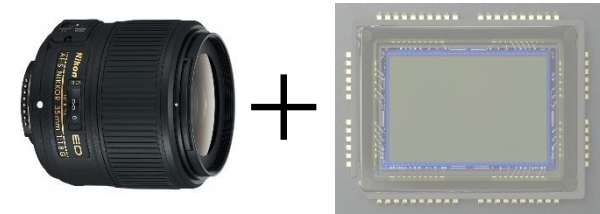
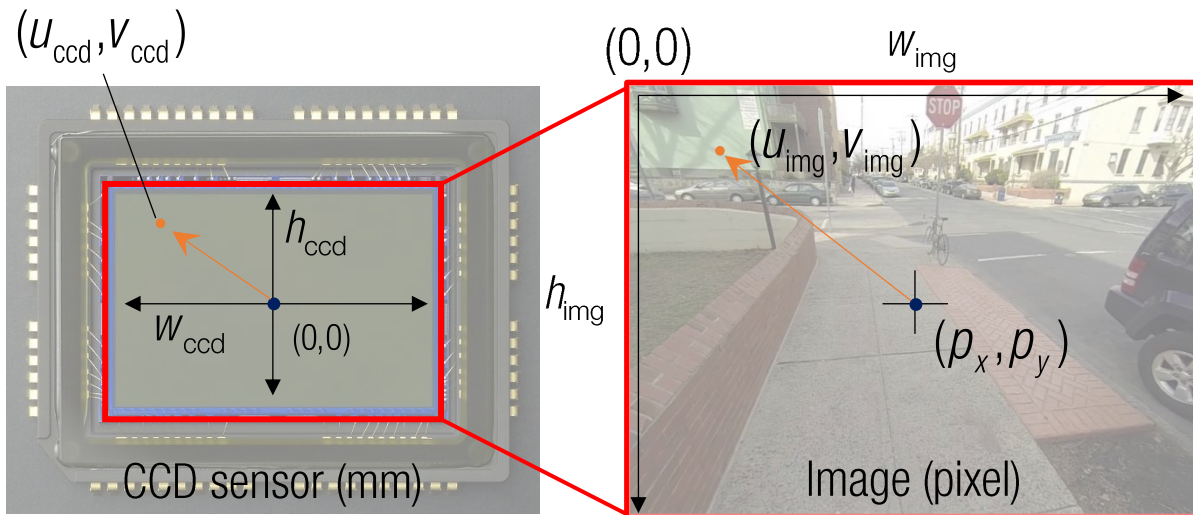
$$u_{\text{img}} = f \frac{X}{Z} + p_x \longrightarrow Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \longrightarrow Zv_{\text{img}} = fY + p_y Z$$

Pixel space

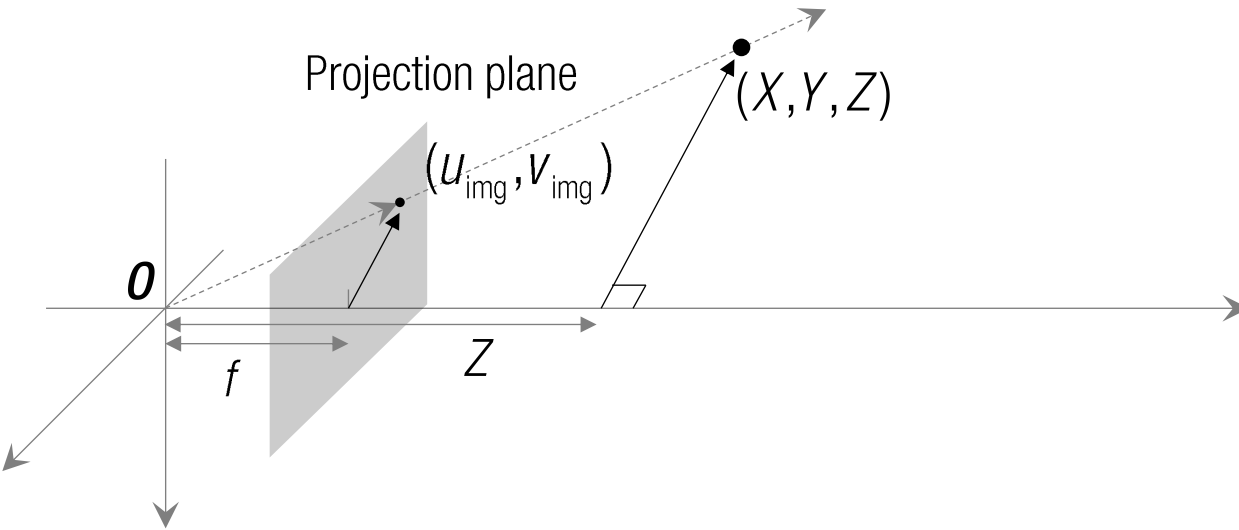
Metric space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Camera intrinsic parameter
: metric space to pixel space

3D Point Projection (Pixel Space)



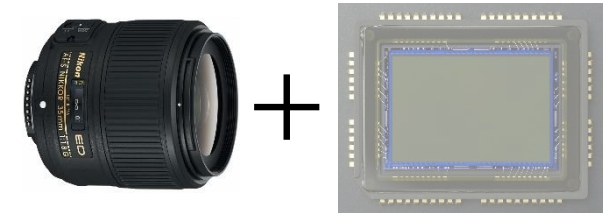
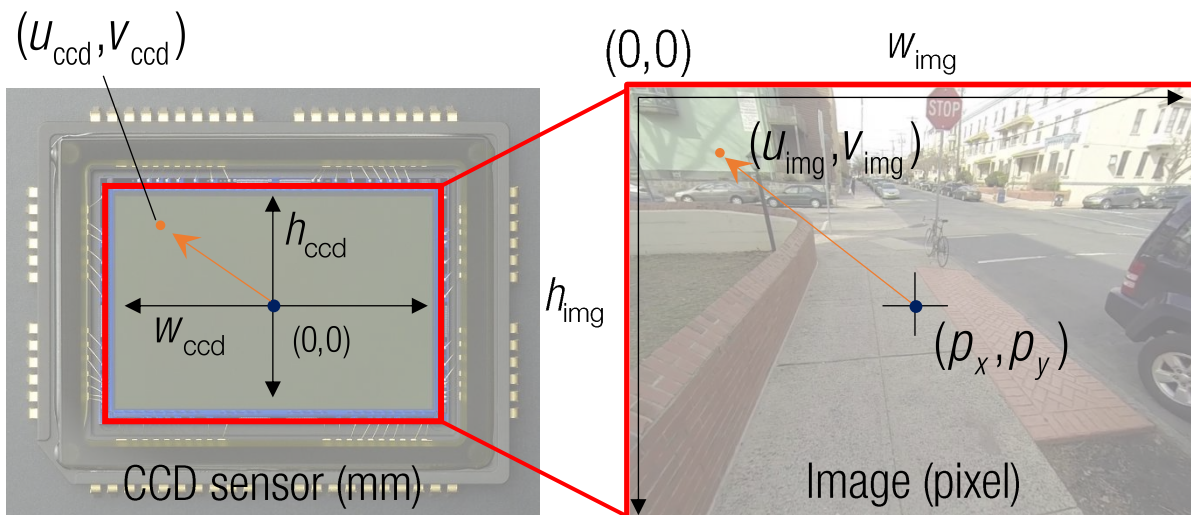
$$u_{\text{img}} = f \frac{X}{Z} + p_x \longrightarrow Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \longrightarrow Zv_{\text{img}} = fY + p_y Z$$

Pixel space

Metric space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ \mathbf{K} & p_y \\ & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

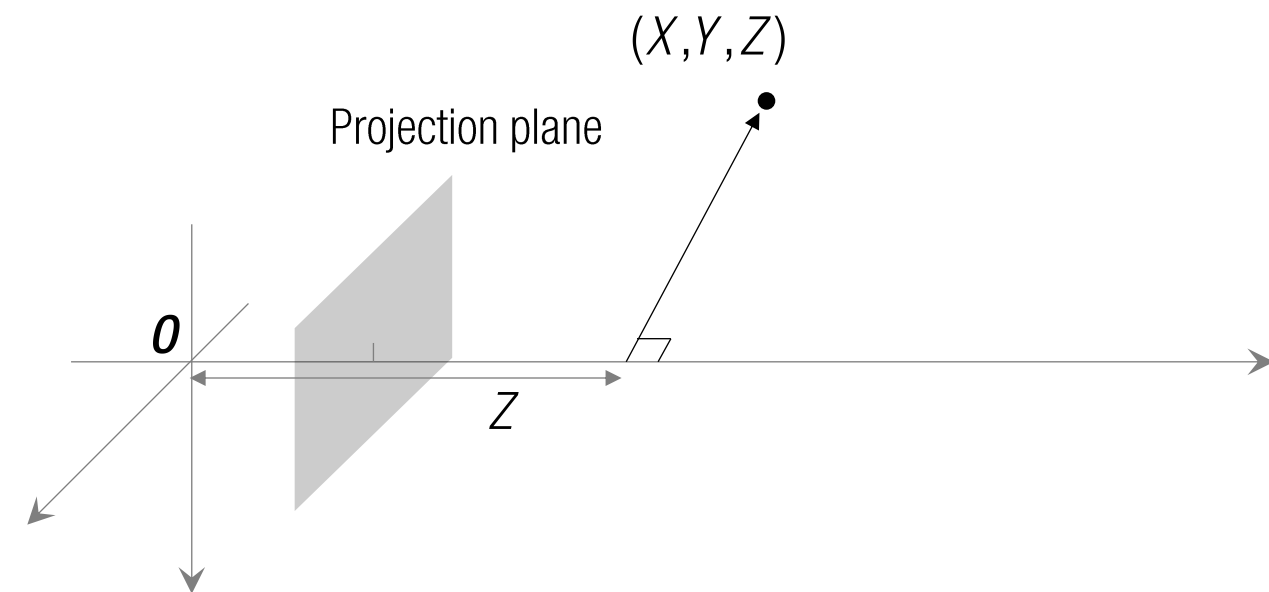


Camera intrinsic parameter
: metric space to pixel space

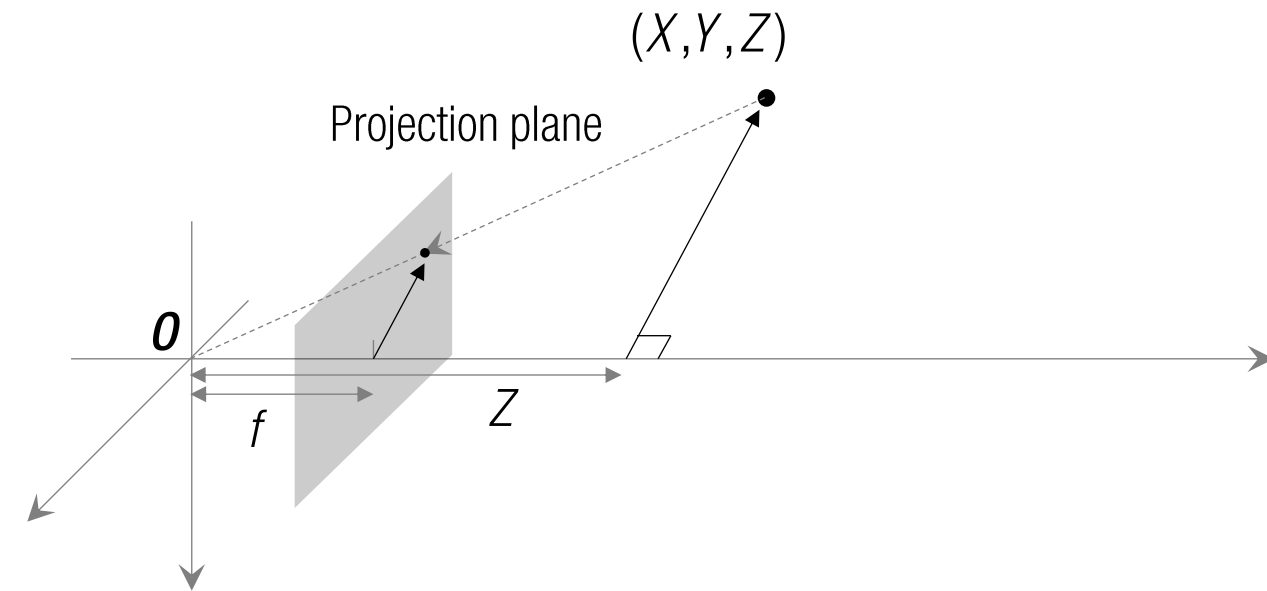
Camera Intrinsic Parameter

Metric space

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



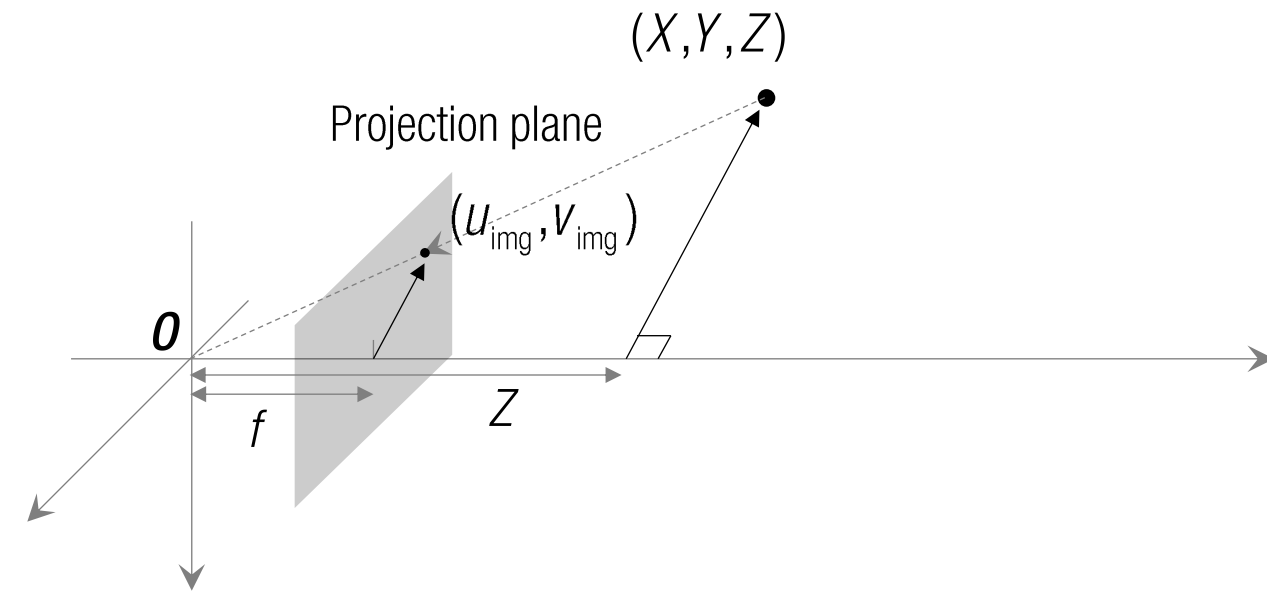
Camera Intrinsic Parameter



Metric space

$$\mathbf{K} \begin{bmatrix} f \\ p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Camera Intrinsic Parameter

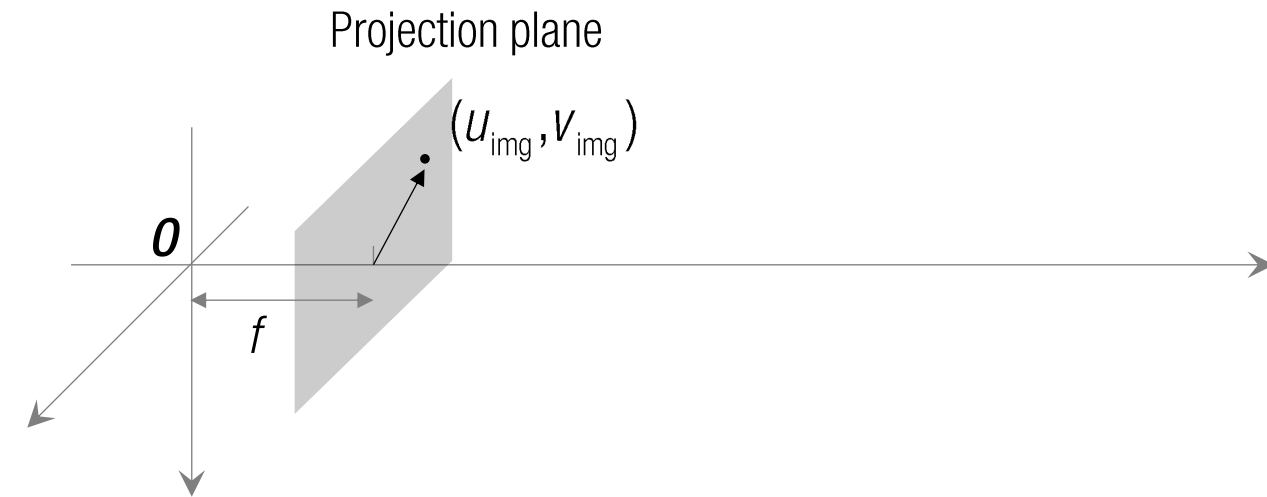


$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ p_x \\ p_y \\ 1 \end{bmatrix} \mathbf{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Pixel space

Metric space

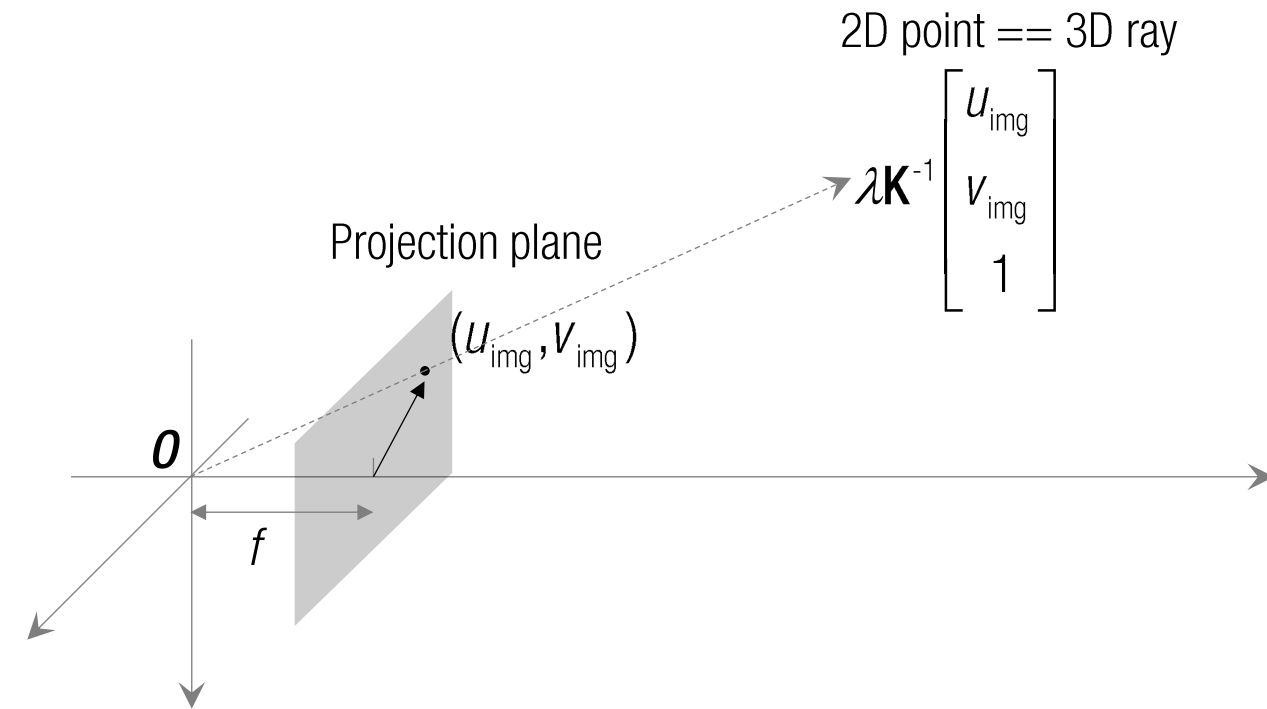
2D Inverse Projection



$$\begin{array}{c} \text{Pixel space} \\ \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ \end{bmatrix} \end{array} \quad \mathbf{K} \quad \begin{array}{c} \text{Metric space} \\ \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \end{array}$$

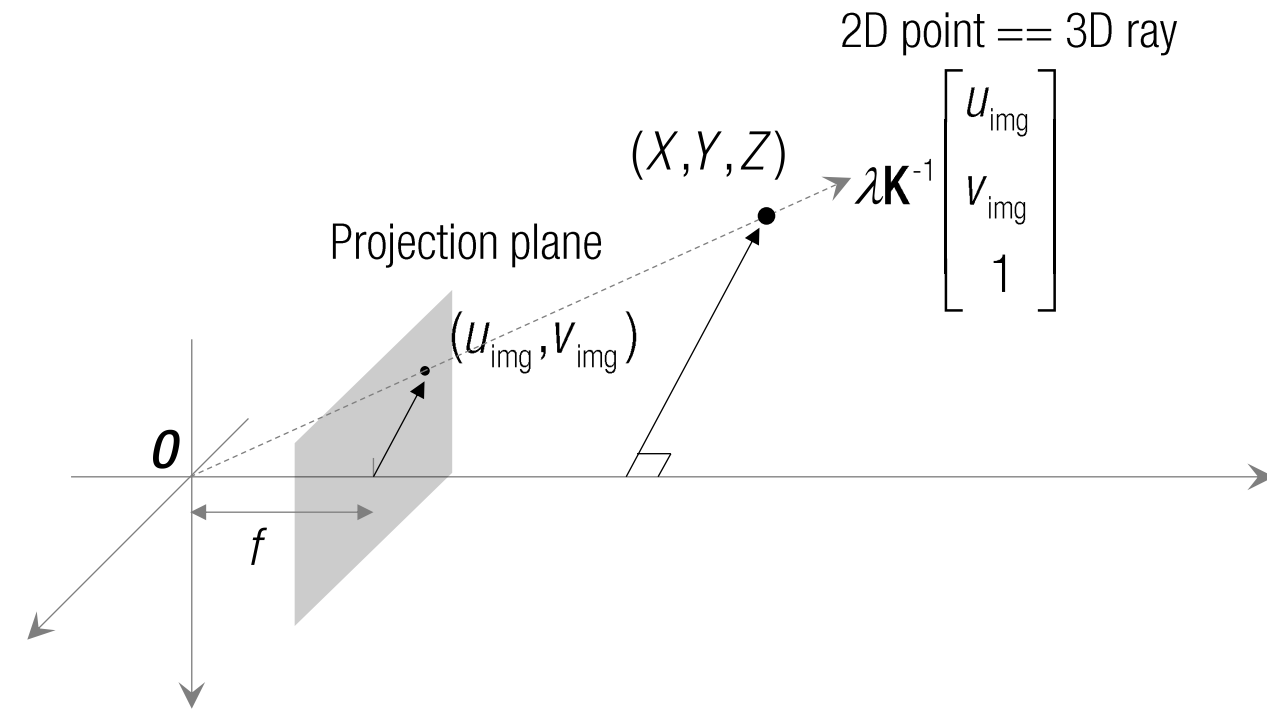
$$\begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix}$$

2D Inverse Projection



$$\begin{array}{c}
 \text{Pixel space} \\
 \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ \\ \end{bmatrix} \quad \mathbf{K} \quad \begin{array}{c} \text{Metric space} \\ \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \end{array} \\
 \hline
 \begin{array}{c} \lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} \\ \text{3D ray} \end{array}
 \end{array}$$

2D Inverse Projection

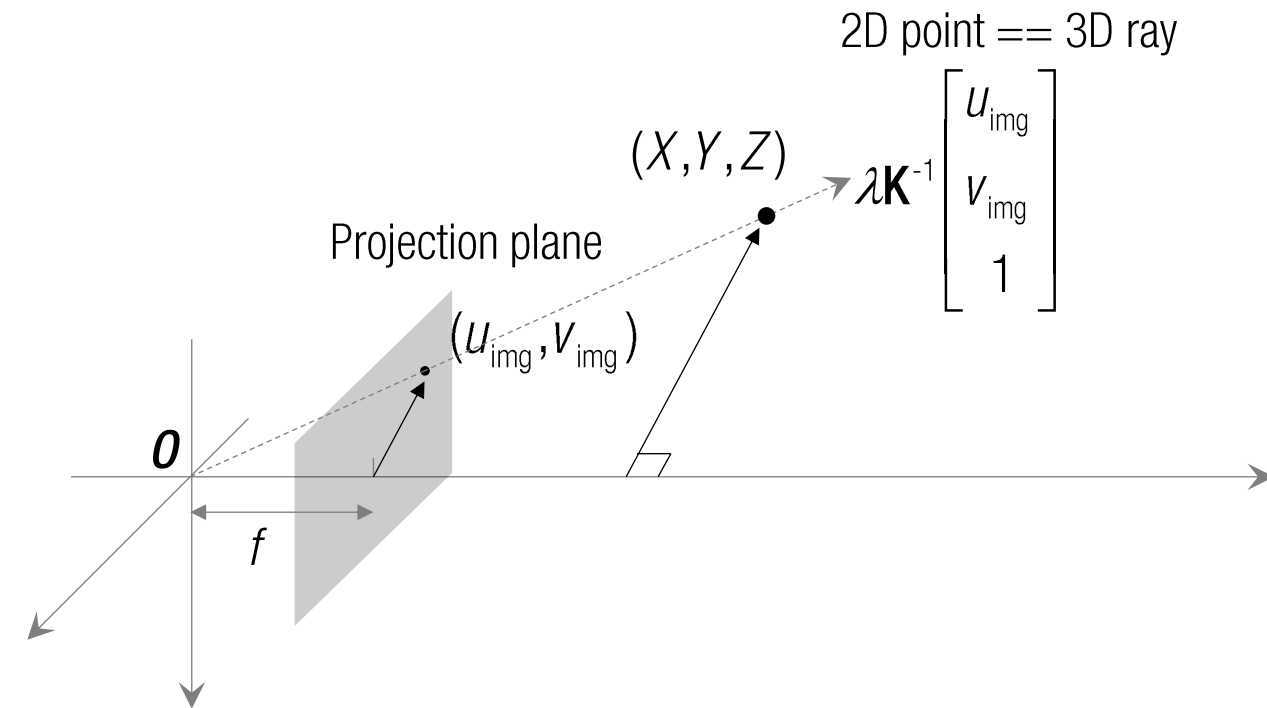


2D point == 3D ray

$$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix}$$

Pixel space		Metric space
$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \end{bmatrix}$	\mathbf{K}	$\begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$
$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$		
<div style="display: inline-block; width: 100%; border-top: 1px solid black; margin-bottom: 5px;"></div> 3D ray		

2D Inverse Projection



$$\begin{array}{c}
 \text{Pixel space} \qquad \qquad \qquad \text{Metric space} \\
 \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ \rho_x \\ \rho_y \\ 1 \end{bmatrix} \mathbf{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \\
 \\
 \underbrace{\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix}}_{\text{3D ray}} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
 \end{array}$$

The 3D point must lie in the 3D ray passing through the origin and 2D image point.