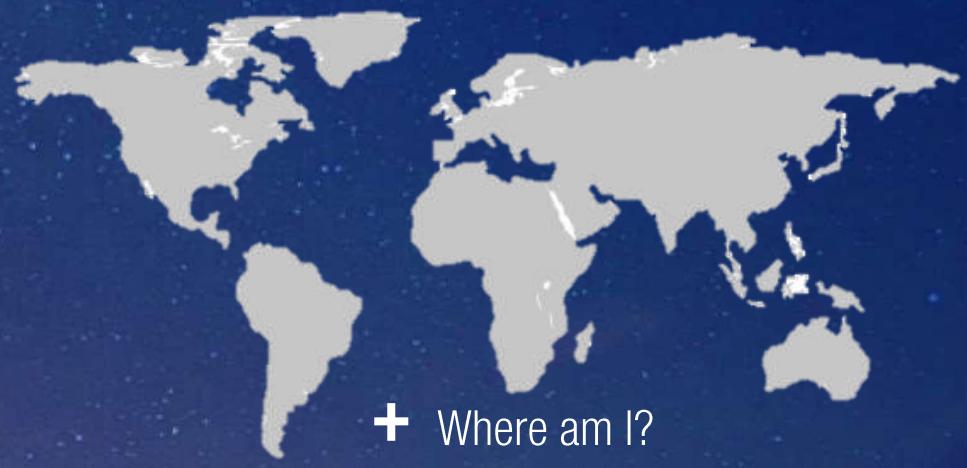


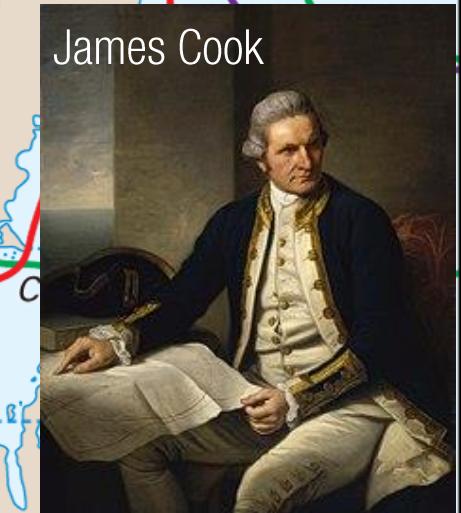
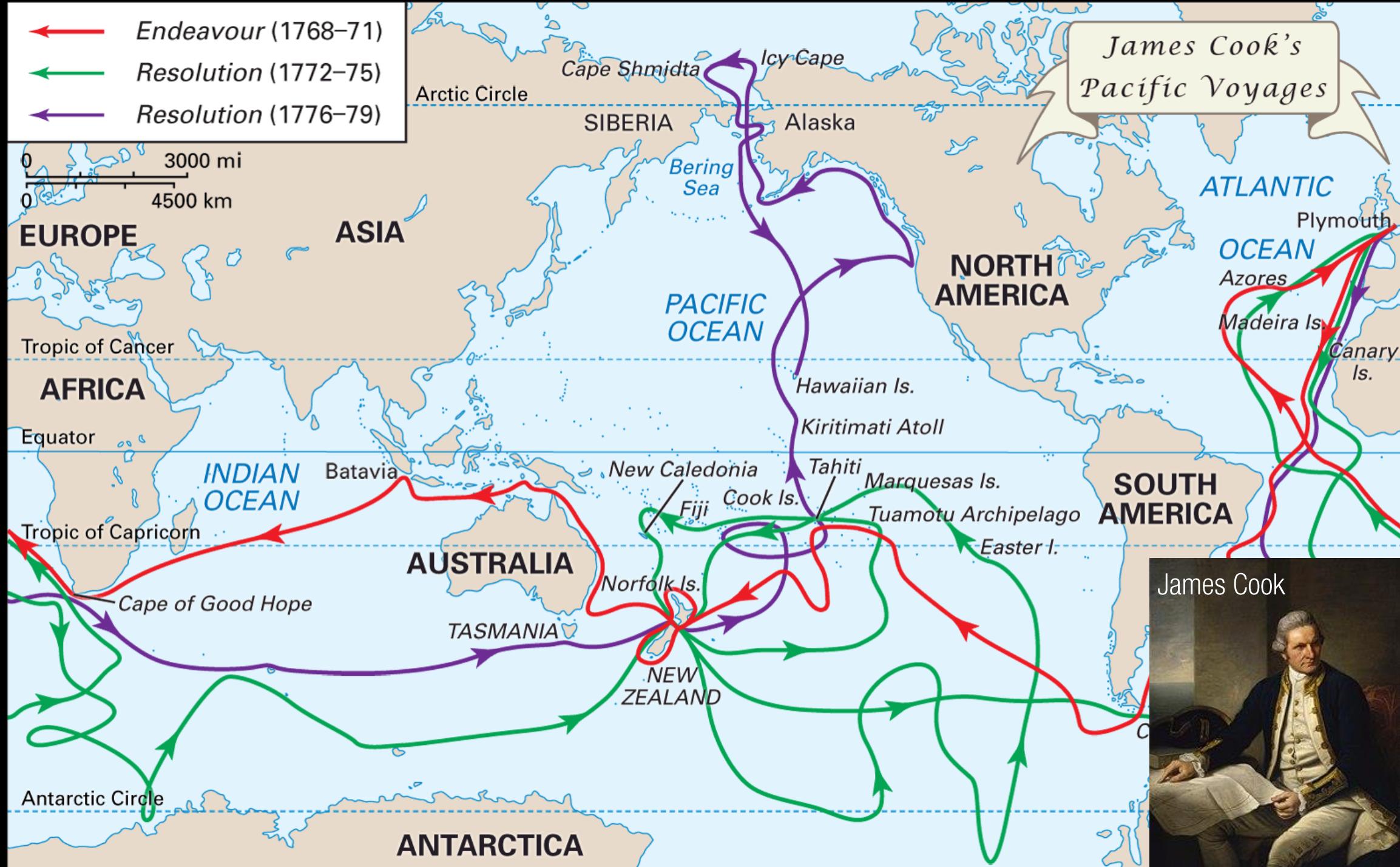
A photograph of a woman with short dark hair, wearing a light-colored jacket over a patterned top, pointing her right index finger towards a multi-story building. The building has a sign that reads "Studios de París". The perspective of the building creates a clear vanishing point. The foreground shows a paved area with some yellow lines.

Camera Localization using Vanishing Points

Hyun Soo Park



+ Where am I?



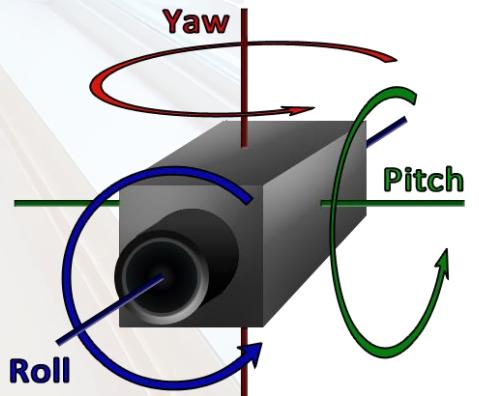
Vanishing point

Vanishing line for horizon

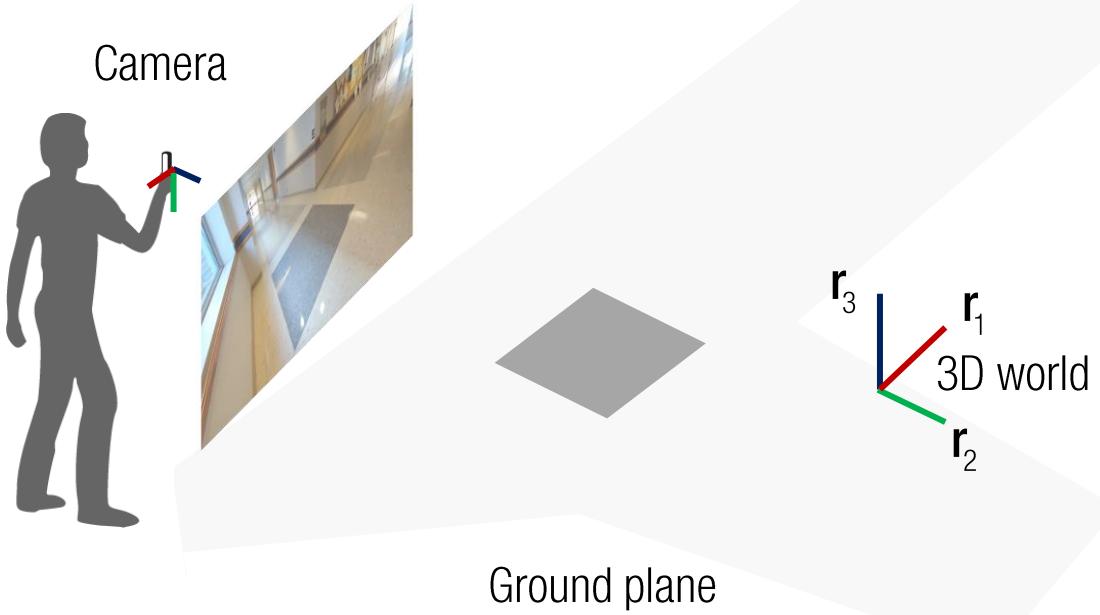
Vanishing point

What can vanishing line tell us about me?

- Horizon
- Camera pitch angle (looking down)
- Camera roll angle (tilted toward right)



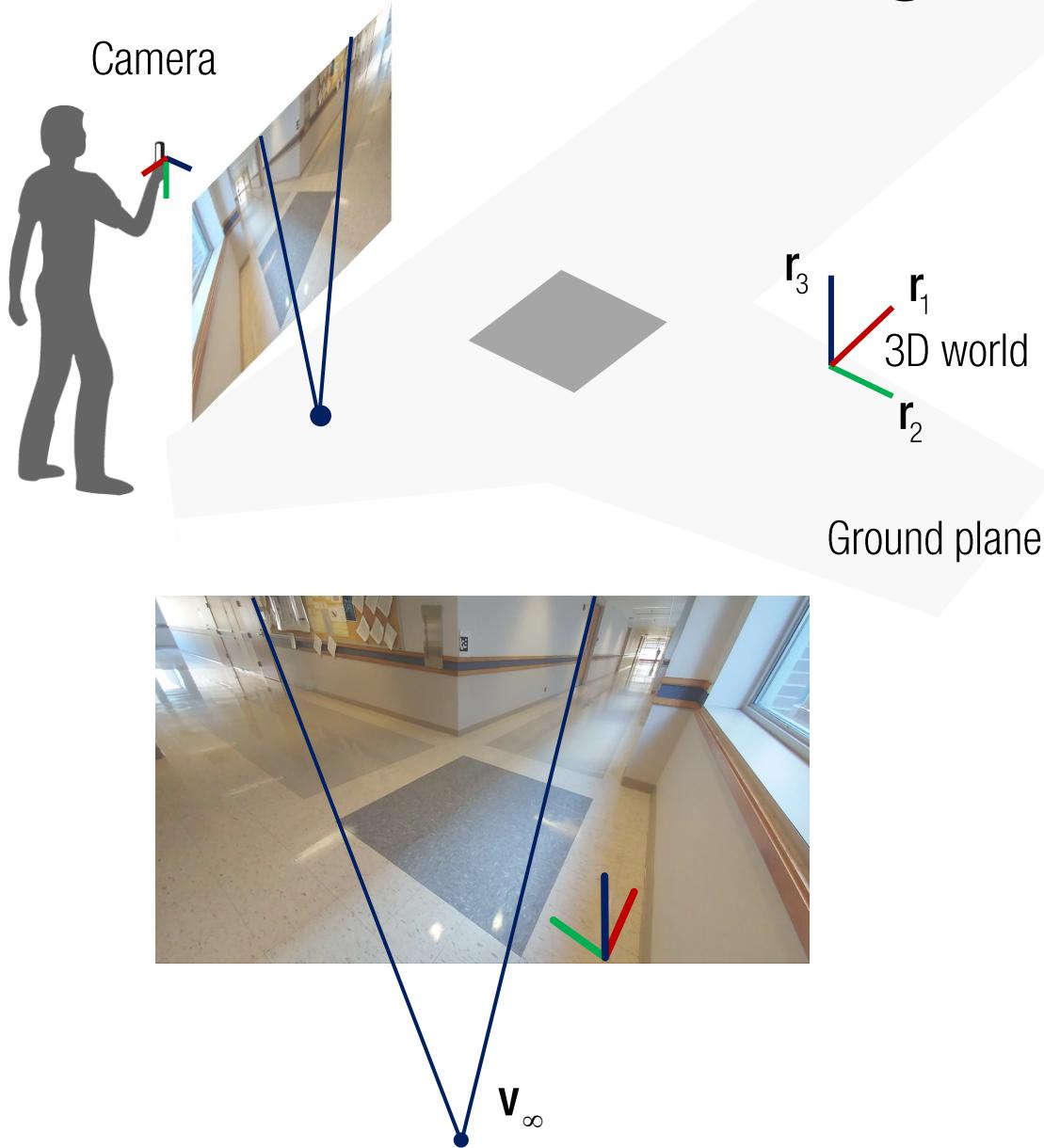
Where am I w.r.t. Ground Plane?



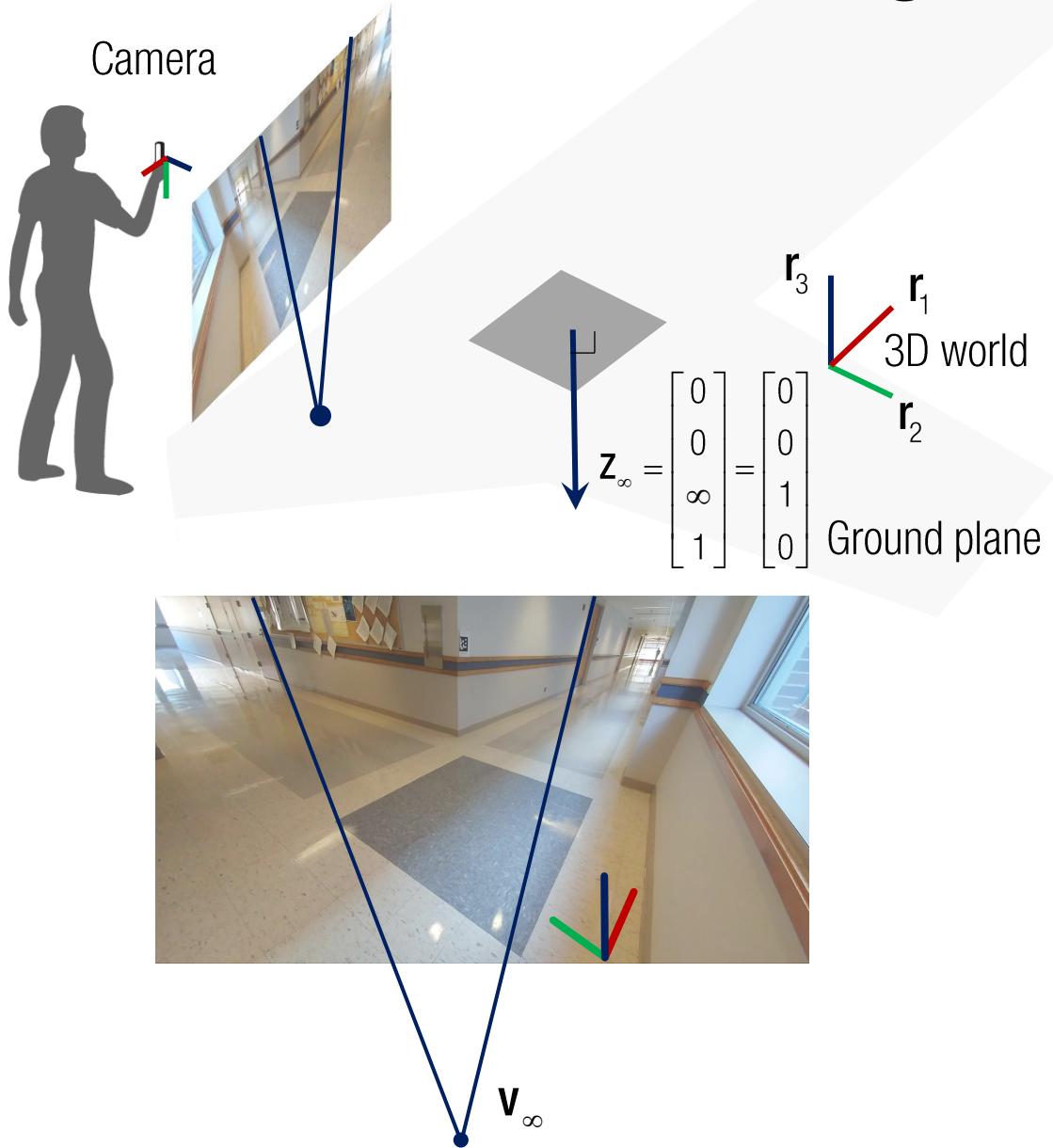
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{K} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t}_w^c \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

How to compute?

What can a Vanishing Point tell us about?



What can a Vanishing Point tell us about?



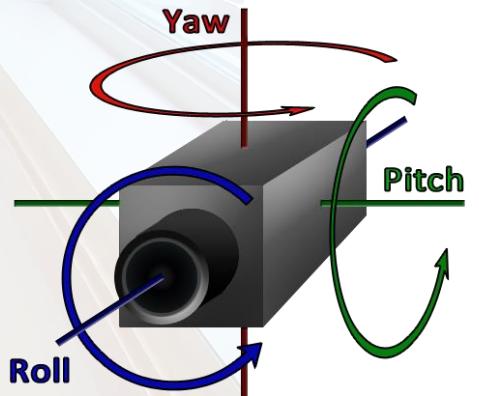
Vanishing point

Vanishing line for horizon

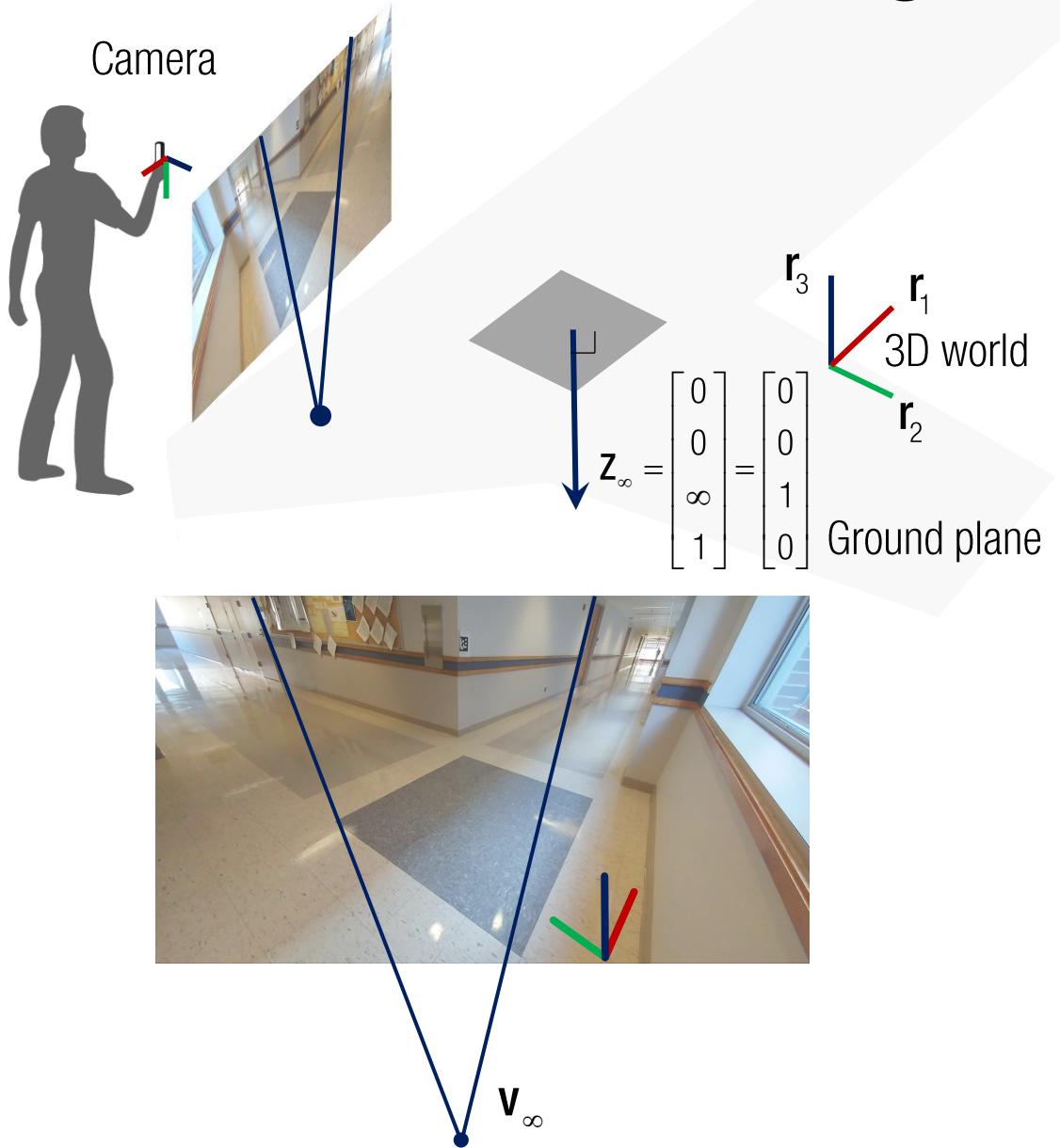
Vanishing point

What can vanishing line tell us about me?

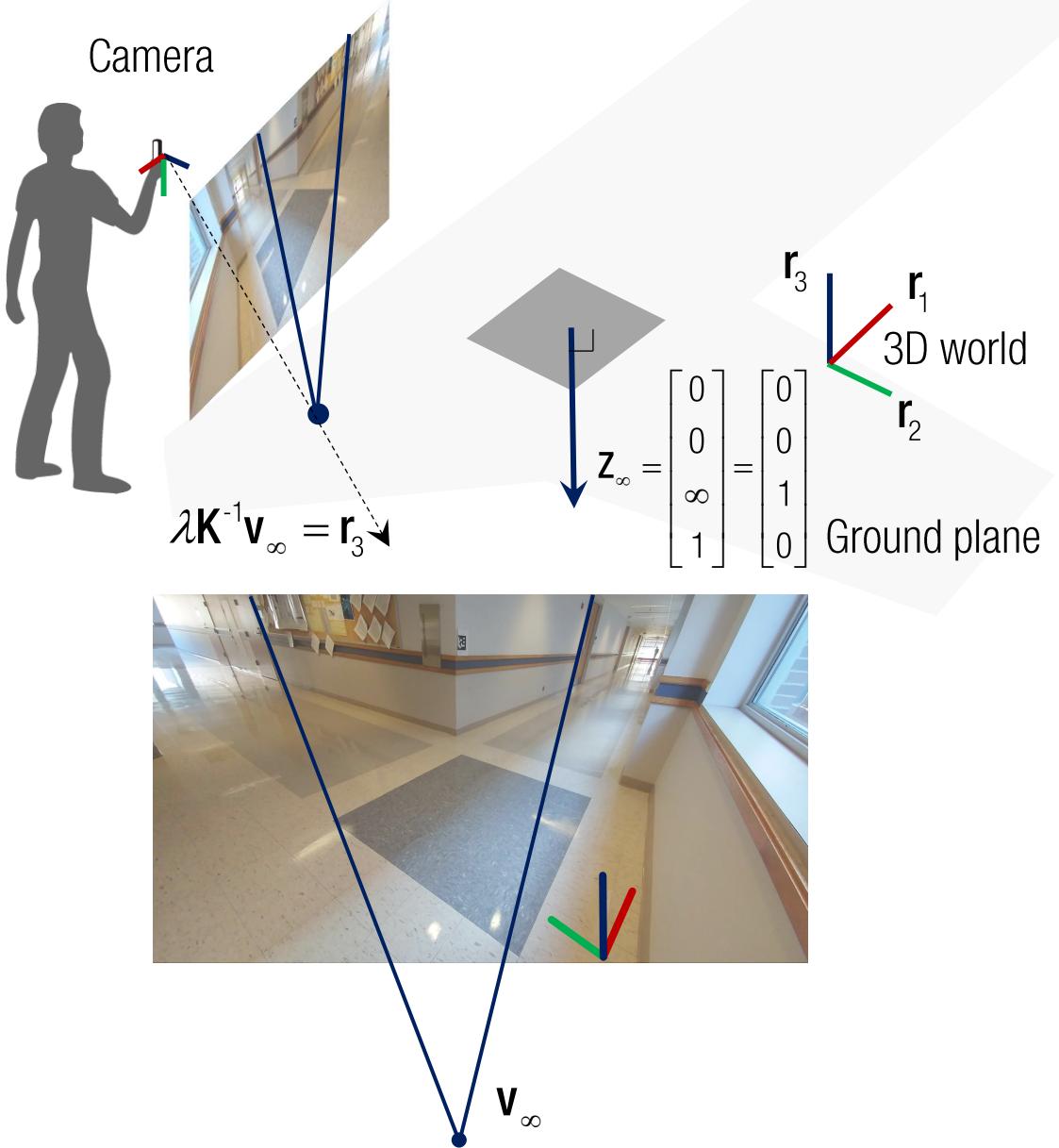
- Horizon
- Camera pitch angle (looking down)
- Camera roll angle (tilted toward right)



What can a Vanishing Point tell us about?



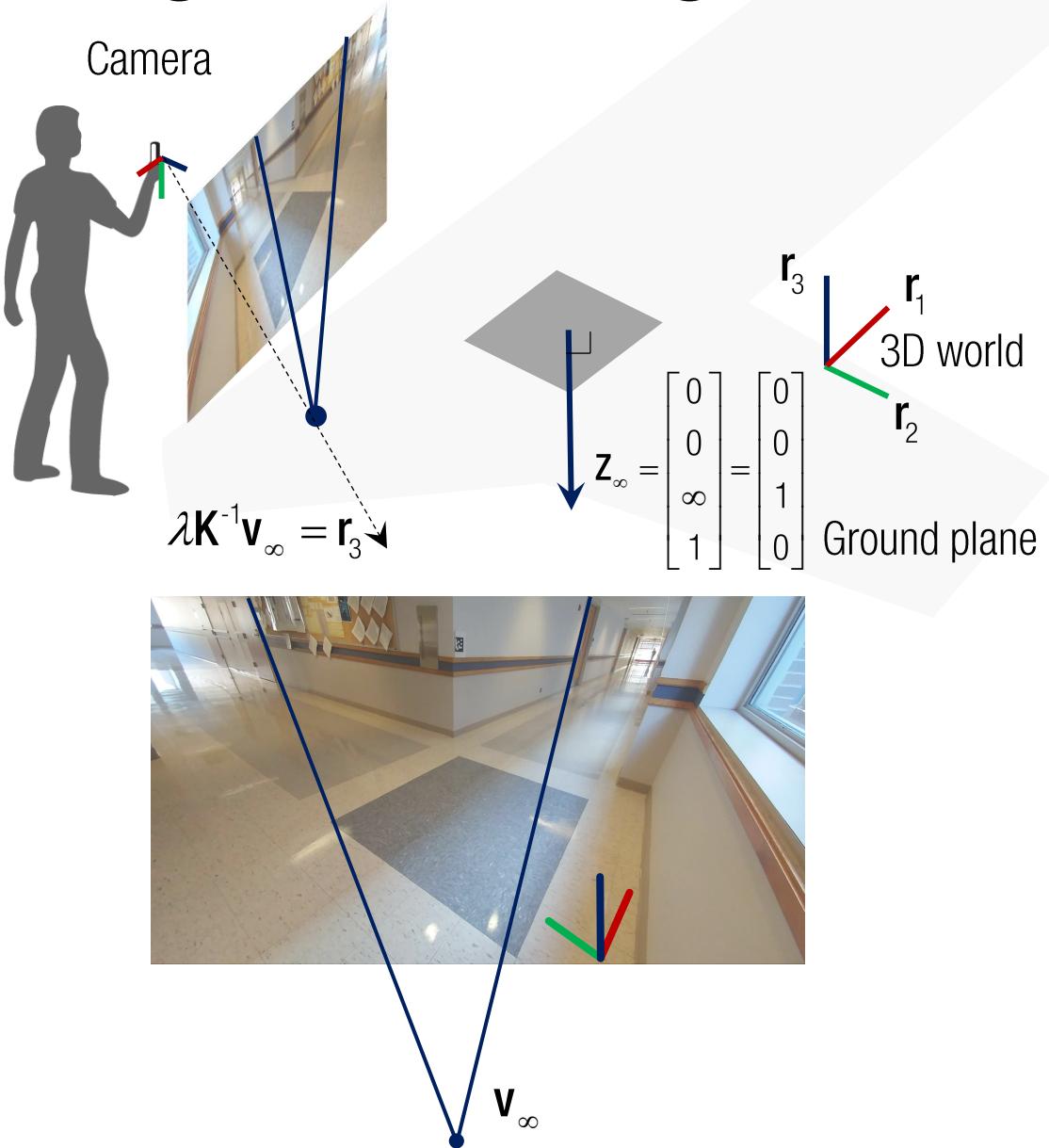
Single Vanishing Point



$$\lambda v_\infty = K \begin{bmatrix} r_1 & r_2 & r_3 & t_w^C \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda v_\infty = Kr_3$$

Single Vanishing Point



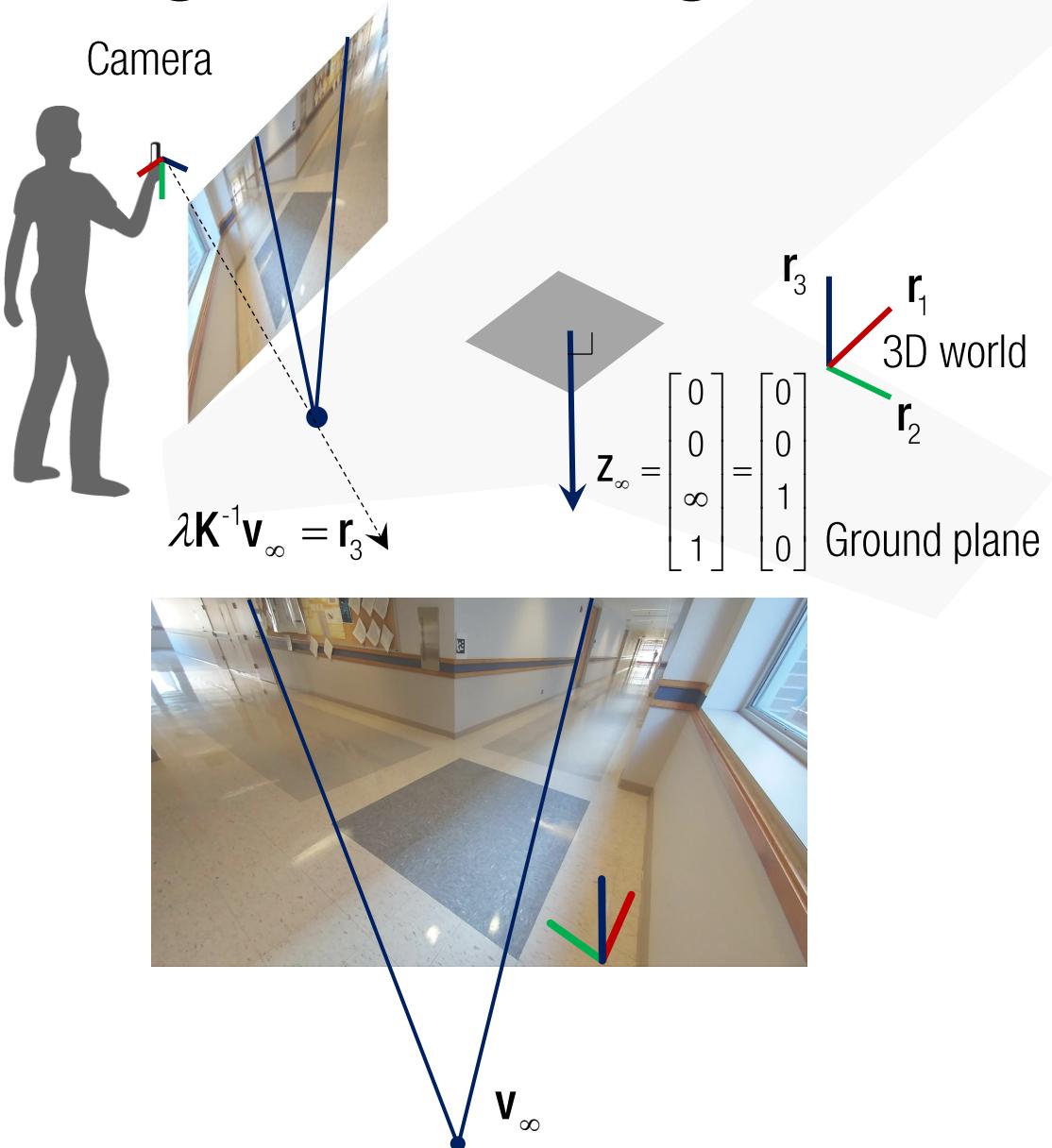
$$\lambda v_\infty = K \begin{bmatrix} r_1 & r_2 & r_3 & t_w^C \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda v_\infty = Kr_3$$

$$\rightarrow r_3 = \frac{K^{-1}v_\infty}{\|K^{-1}v_\infty\|} \quad \text{because } r_3 \text{ is a unit vector.}$$

Z vanishing point tells us about the surface normal of the ground plane

Single Vanishing Point



$$\lambda \mathbf{v}_\infty = \mathbf{K} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t}_w^C \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

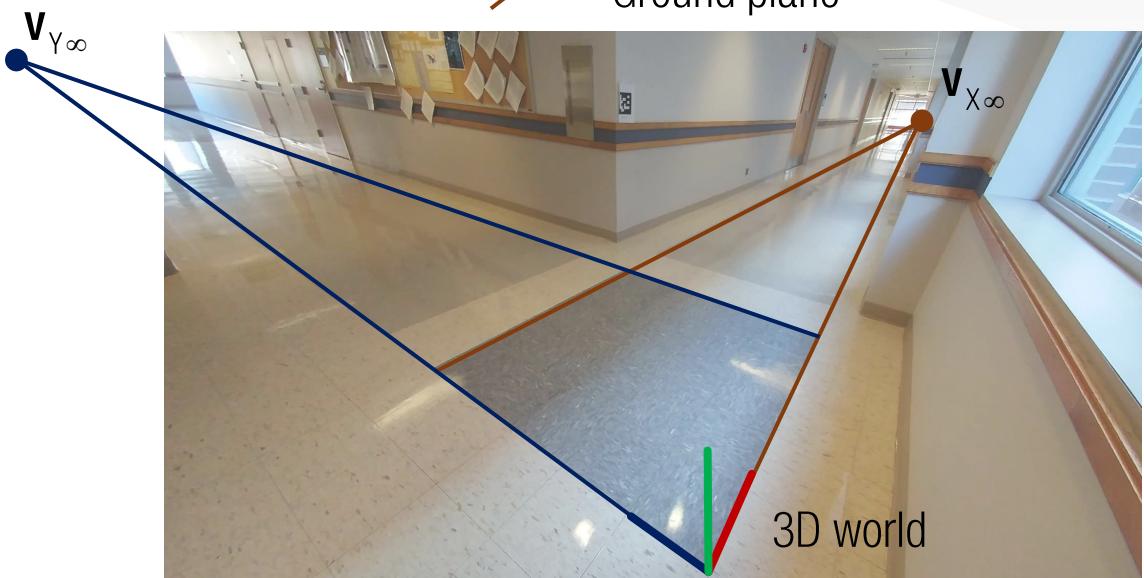
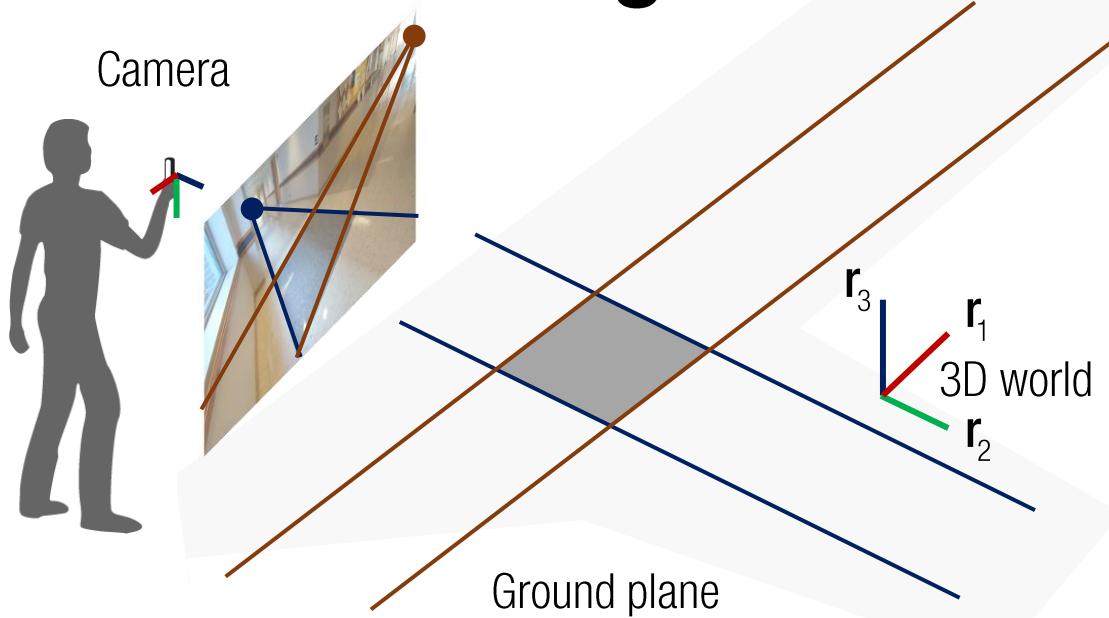
$$\lambda \mathbf{v}_\infty = \mathbf{K} \mathbf{r}_3$$

$$\rightarrow \mathbf{r}_3 = \frac{\mathbf{K}^{-1} \mathbf{v}_\infty}{\|\mathbf{K}^{-1} \mathbf{v}_\infty\|}$$
 because \mathbf{r}_3 is a unit vector.

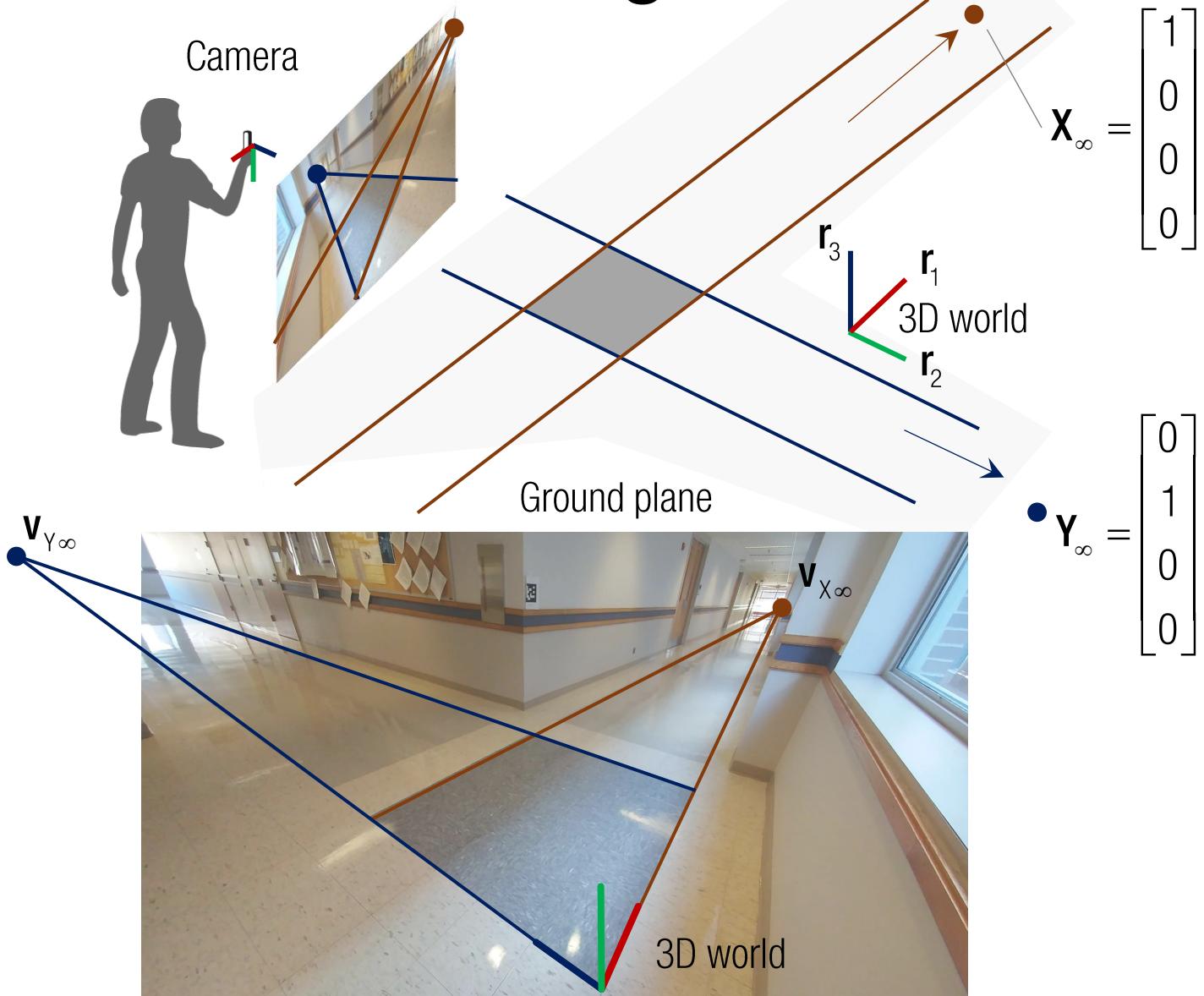
Z vanishing point tells us about the surface normal of the ground plane

Rotation ambiguity

Two Vanishing Points



Two Vanishing Points



$$x_\infty = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y_\infty = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

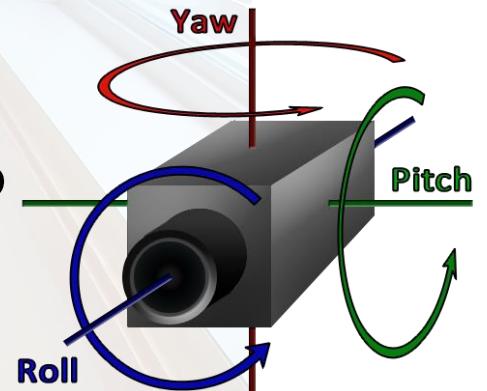
Vanishing point

Vanishing line for horizon

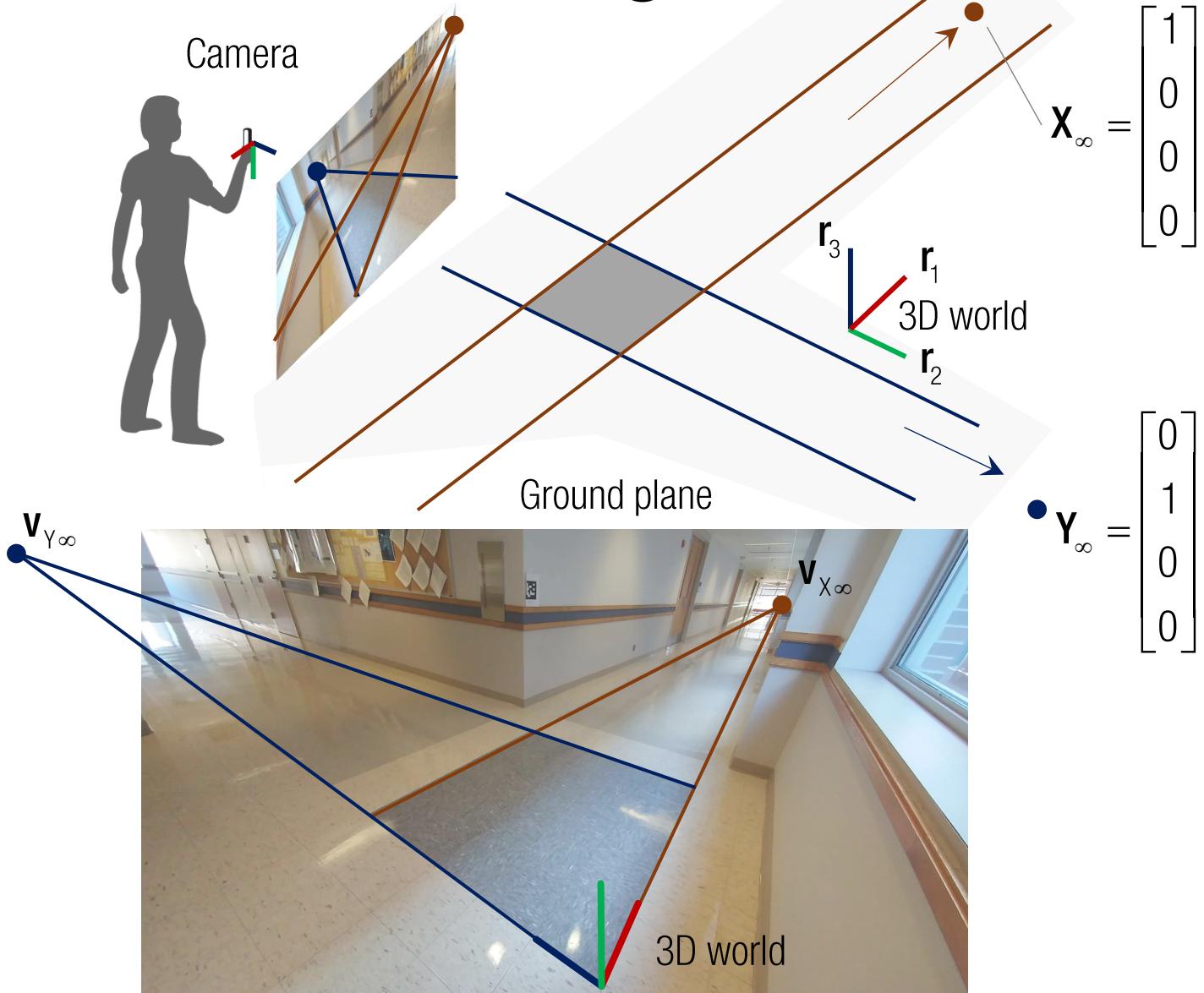
Vanishing point

What can two vanishing points tell us about me?

- Horizon
- Camera pitch angle (looking down)
- Camera roll angle (tilted toward right)
- Camera yaw angle



Two Vanishing Points

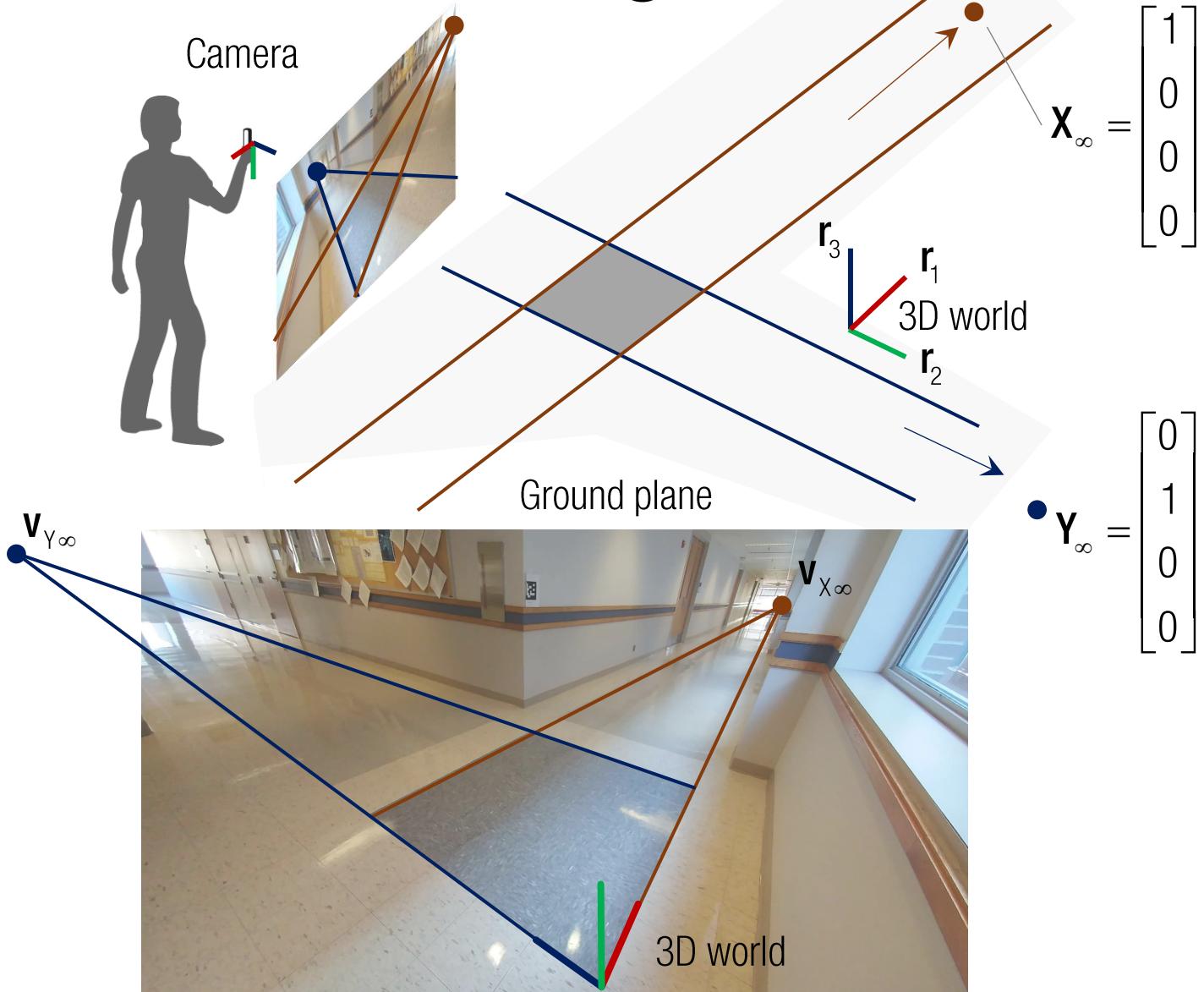


$$x_\infty = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y_\infty = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned}\lambda v_{x\infty} &= K \begin{bmatrix} r_1 & r_2 & r_3 & t_w^c \end{bmatrix} x_\infty \\ &= Kr_1\end{aligned}$$

Two Vanishing Points



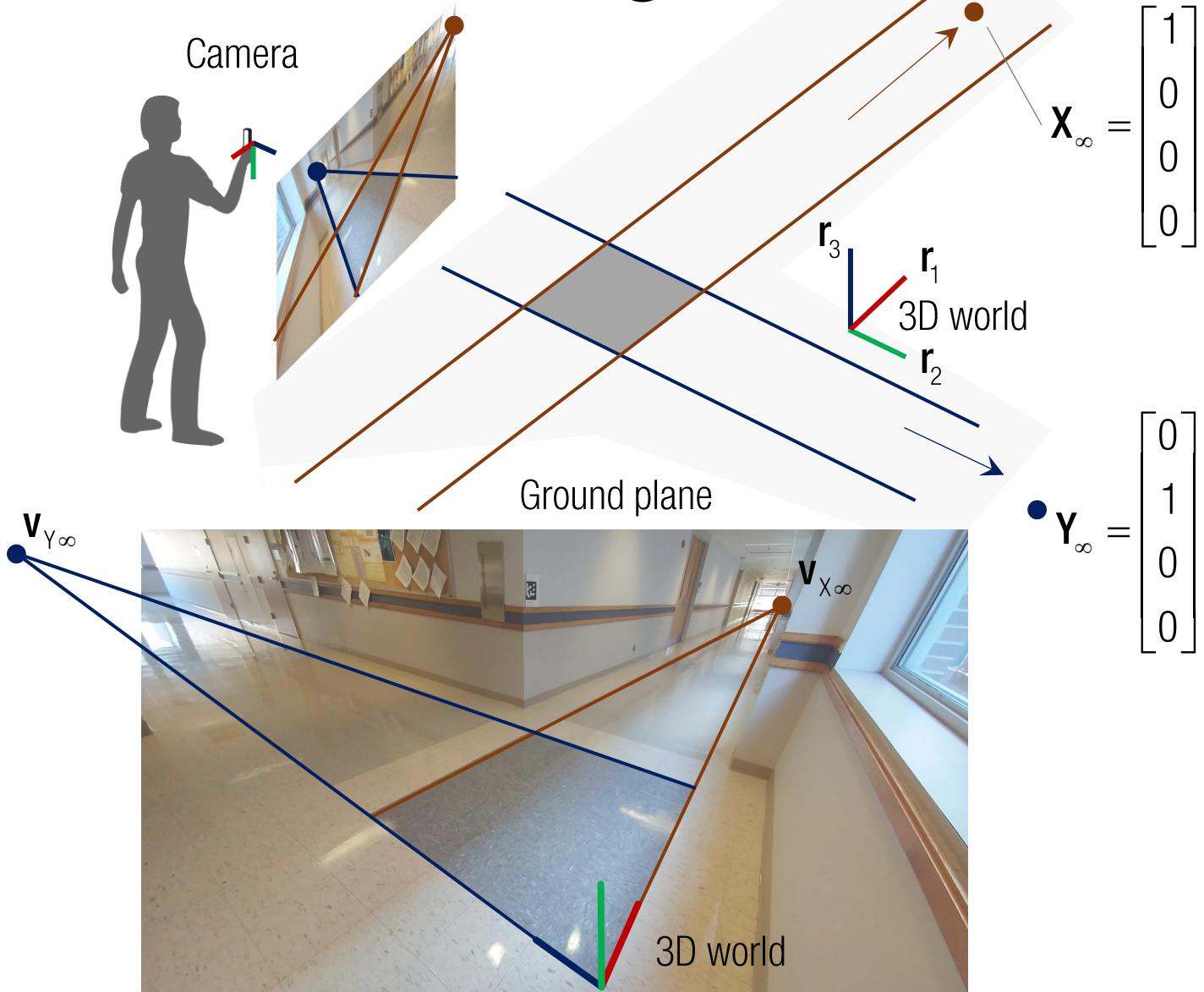
$$x_\infty = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y_\infty = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda v_{X\infty} = K \begin{bmatrix} r_1 & r_2 & r_3 & t_w^c \end{bmatrix} x_\infty \\ = Kr_1$$

$$\lambda v_{Y\infty} = K \begin{bmatrix} r_1 & r_2 & r_3 & t_w^c \end{bmatrix} y_\infty \\ = Kr_2$$

Two Vanishing Points



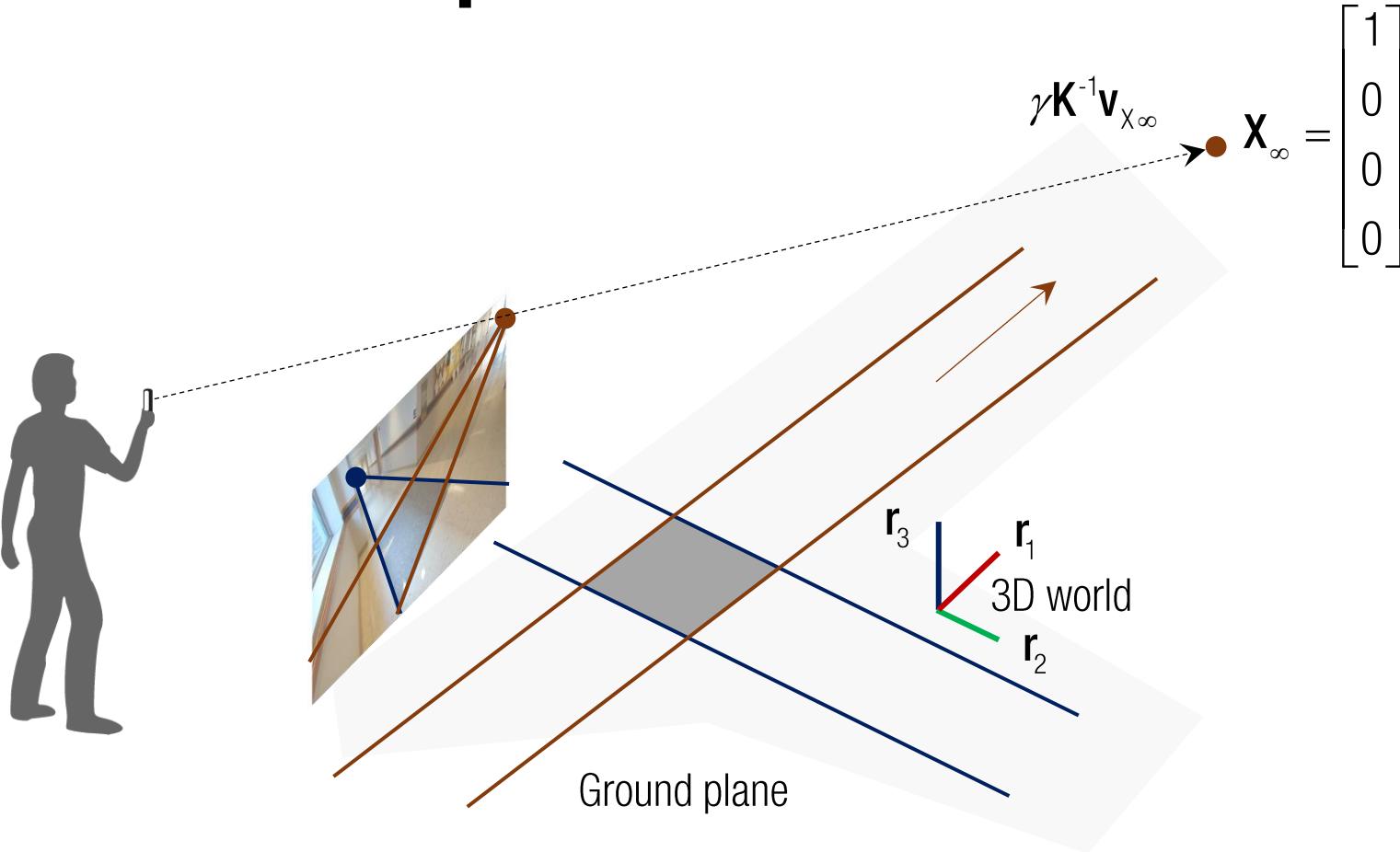
$$\begin{aligned} \lambda v_{X_\infty} &= K [r_1 \ r_2 \ r_3 \ t_w^C] X_\infty \\ &= Kr_1 \end{aligned}$$

$$\begin{aligned} \lambda v_{Y_\infty} &= K [r_1 \ r_2 \ r_3 \ t_w^C] Y_\infty \\ &= Kr_2 \end{aligned}$$

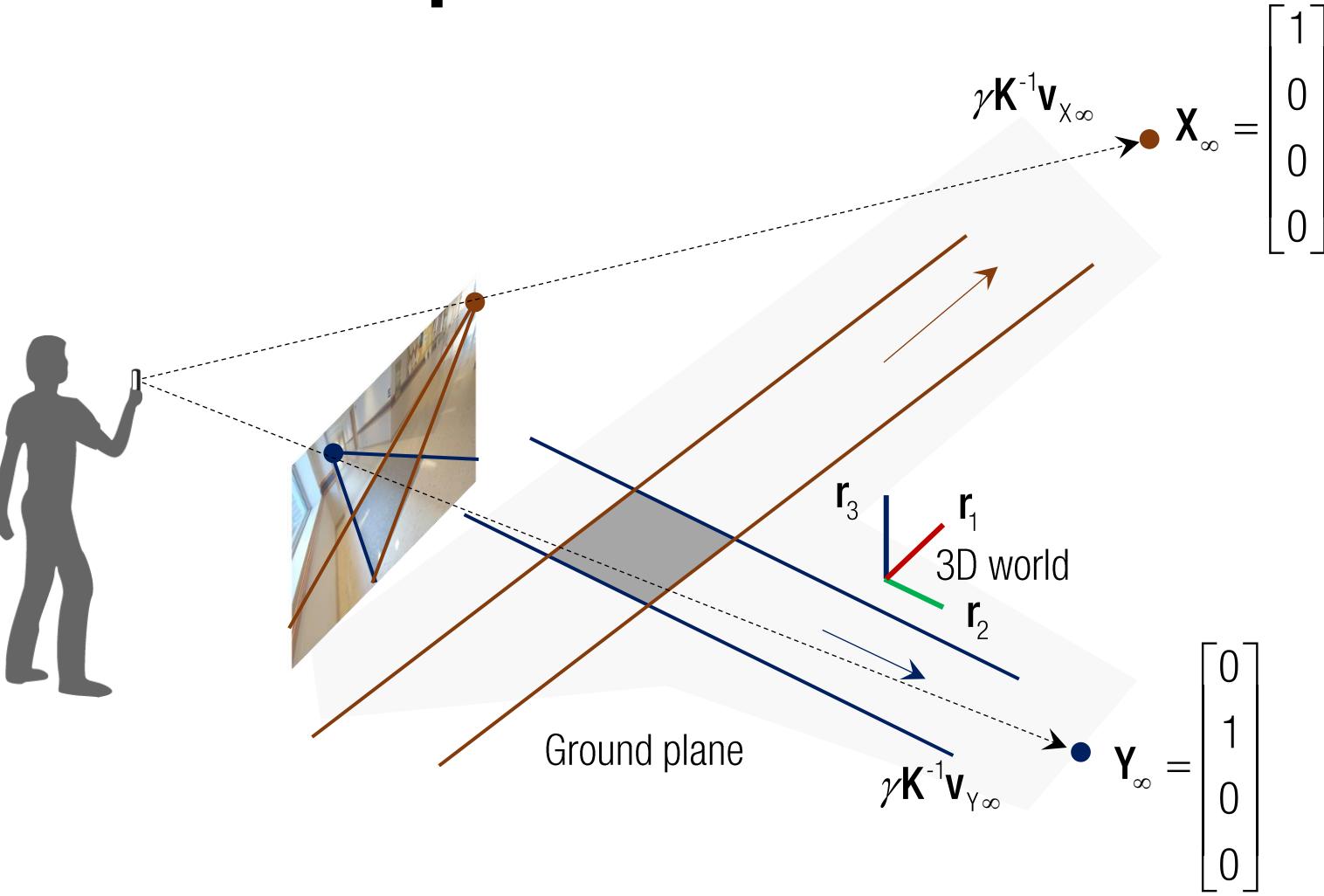
$$\rightarrow r_1 = \frac{K^{-1} v_{X_\infty}}{\|K^{-1} v_{X_\infty}\|}, \quad r_2 = \frac{K^{-1} v_{Y_\infty}}{\|K^{-1} v_{Y_\infty}\|}$$

$r_3 = r_1 \times r_2$: Orthogonality constraint

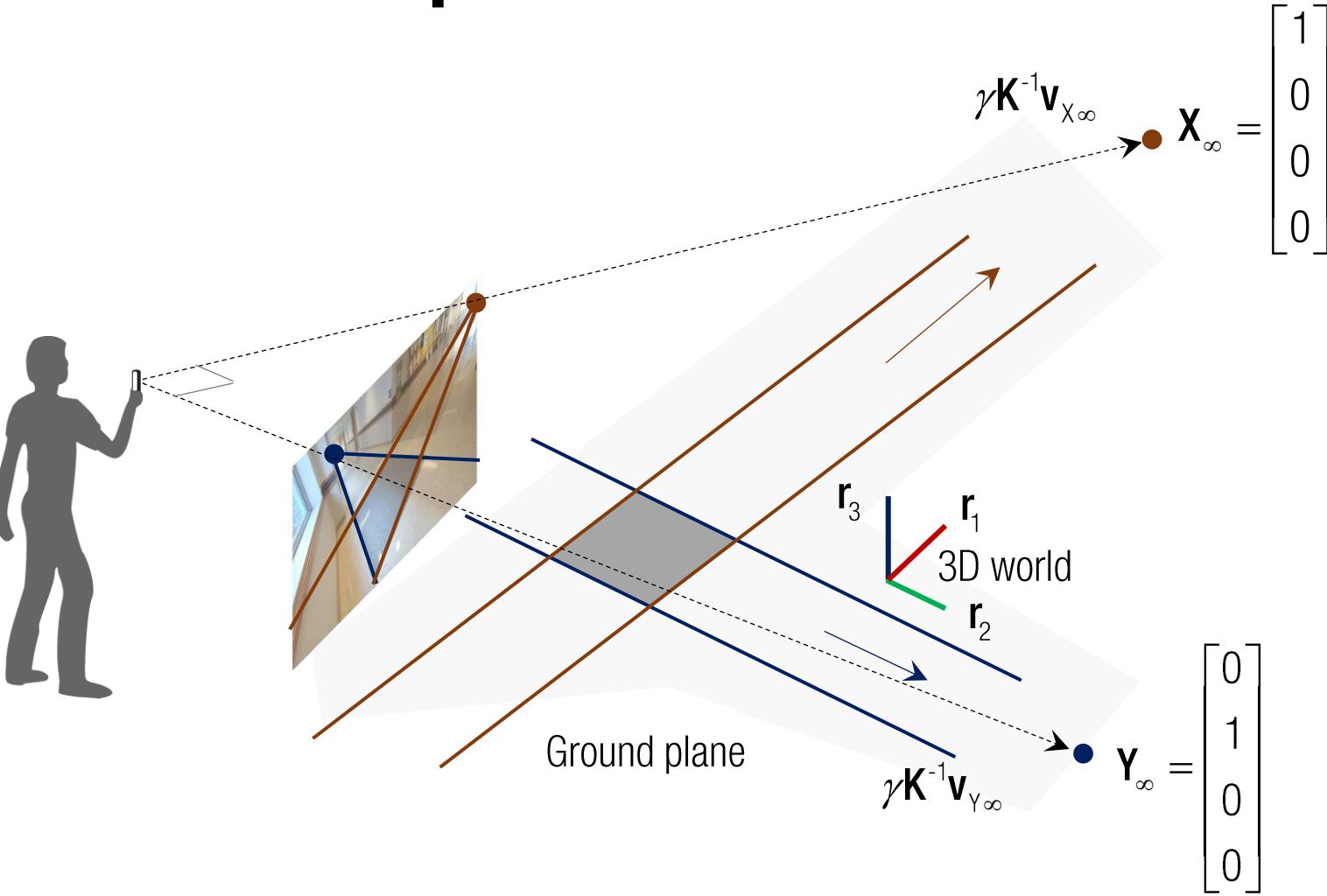
Geometric Interpretation



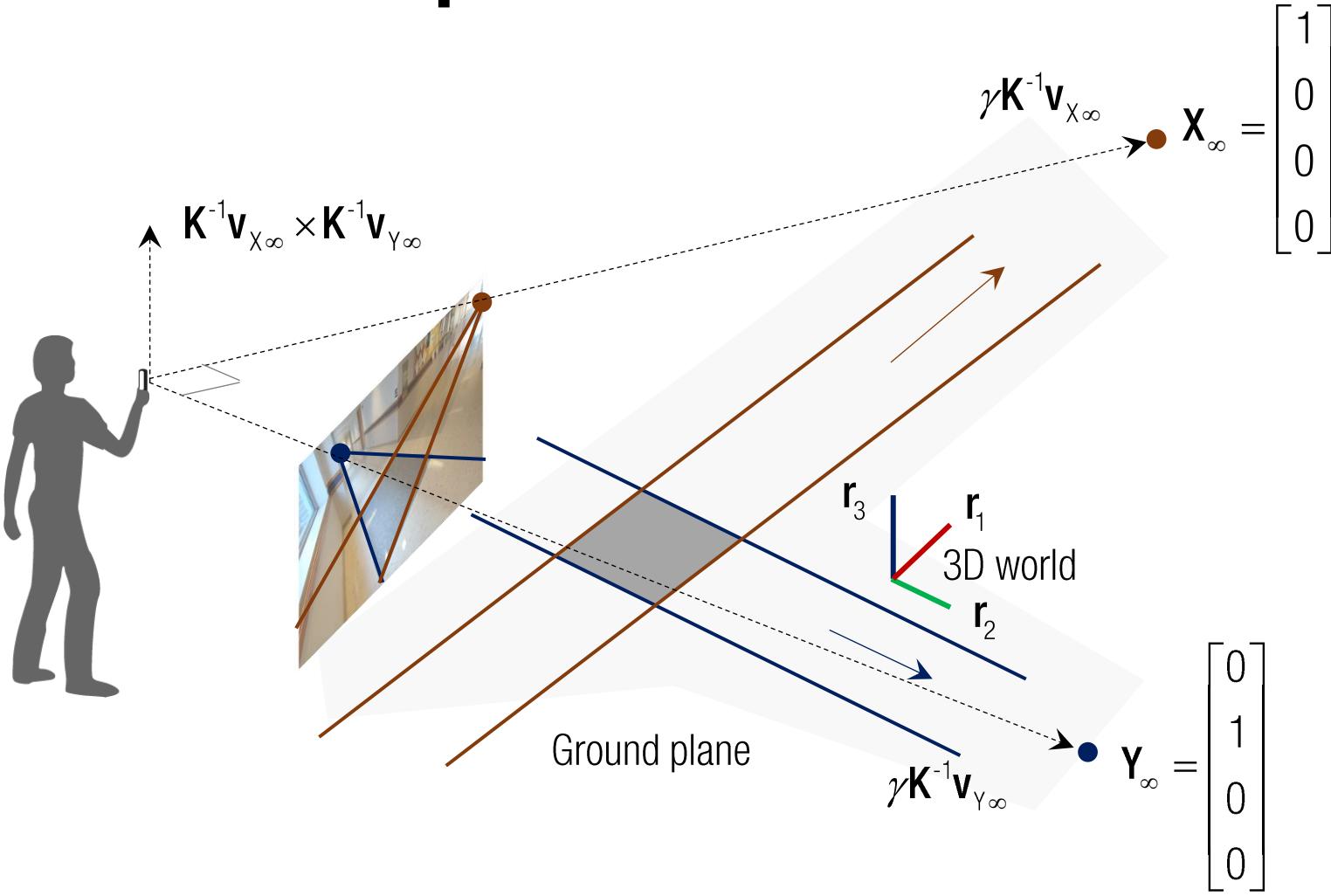
Geometric Interpretation



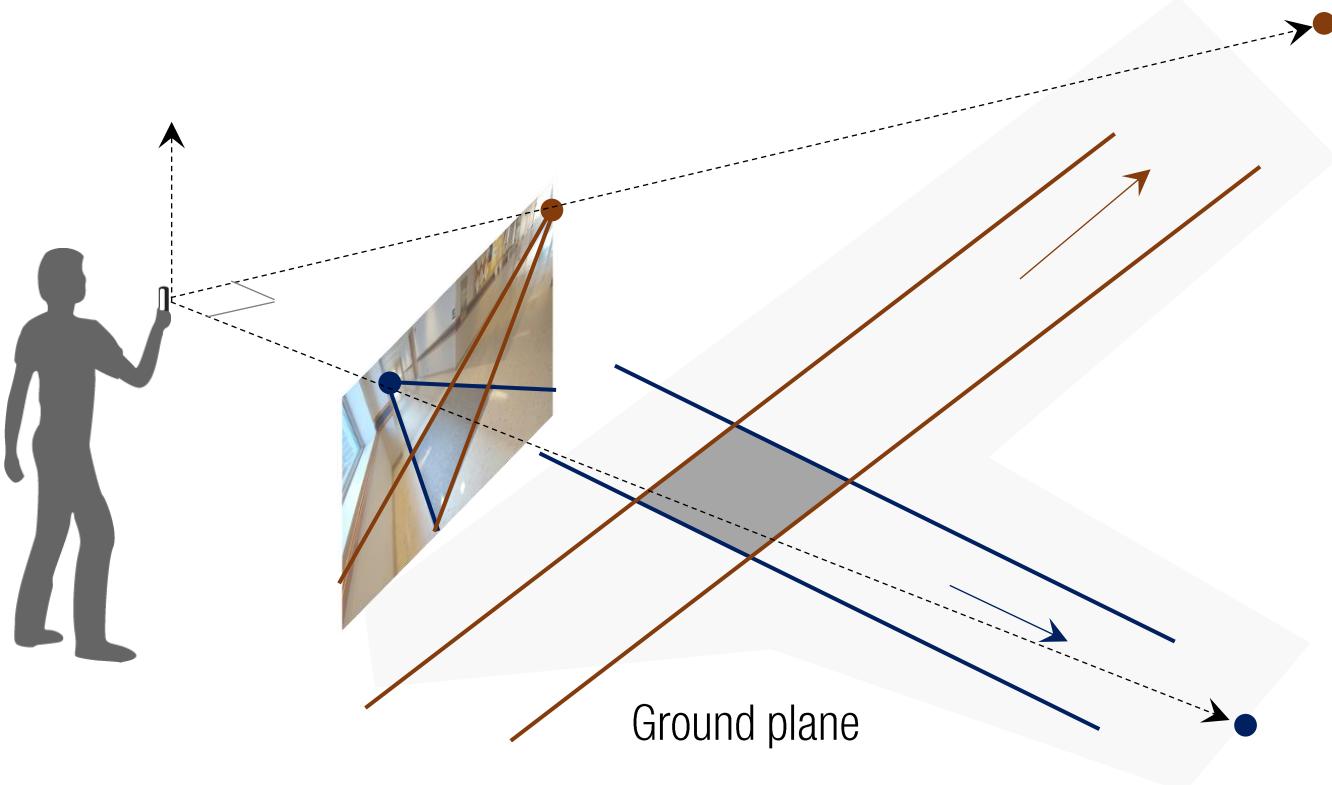
Geometric Interpretation



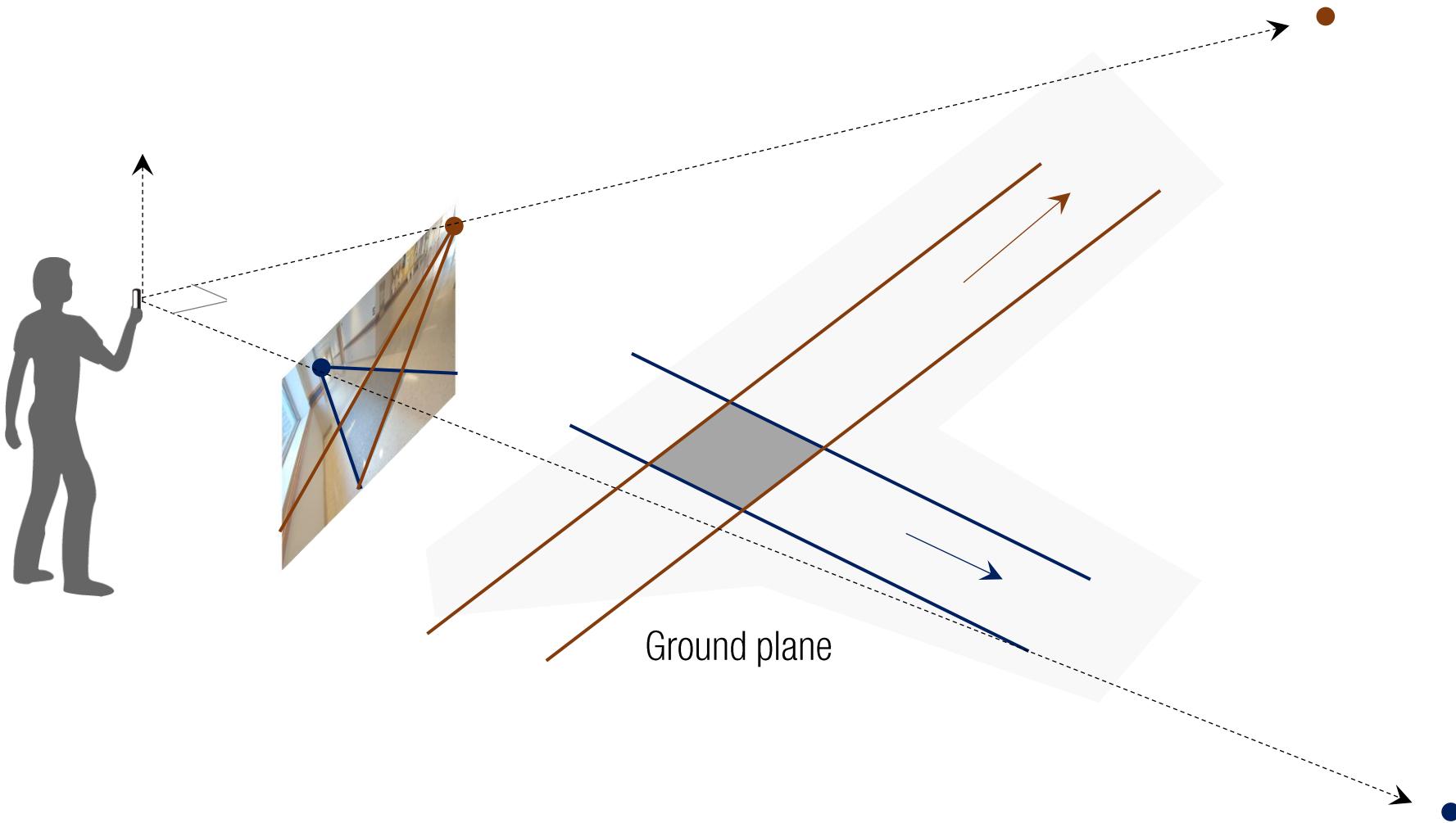
Geometric Interpretation



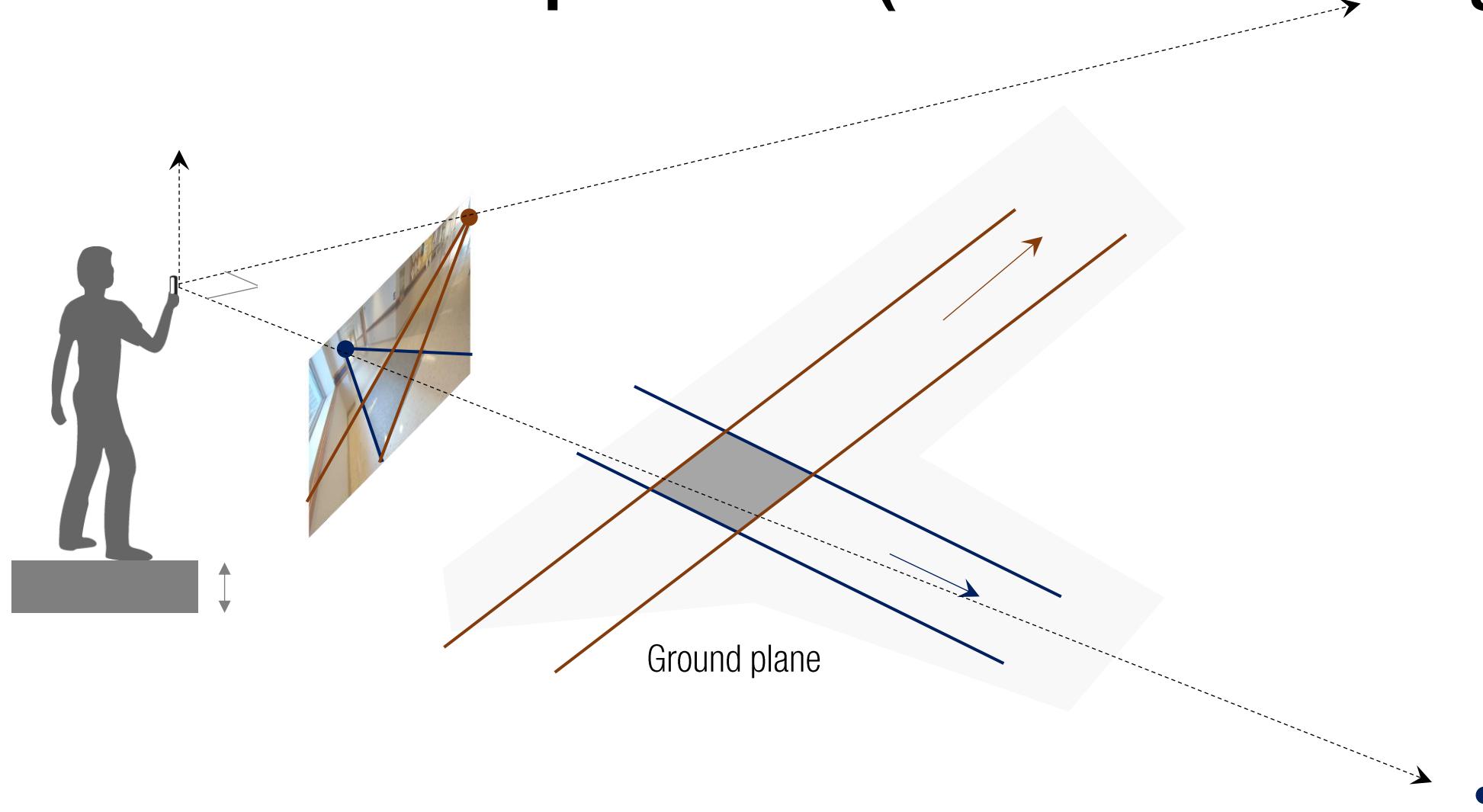
Geometric Interpretation (Translation Ambiguity)



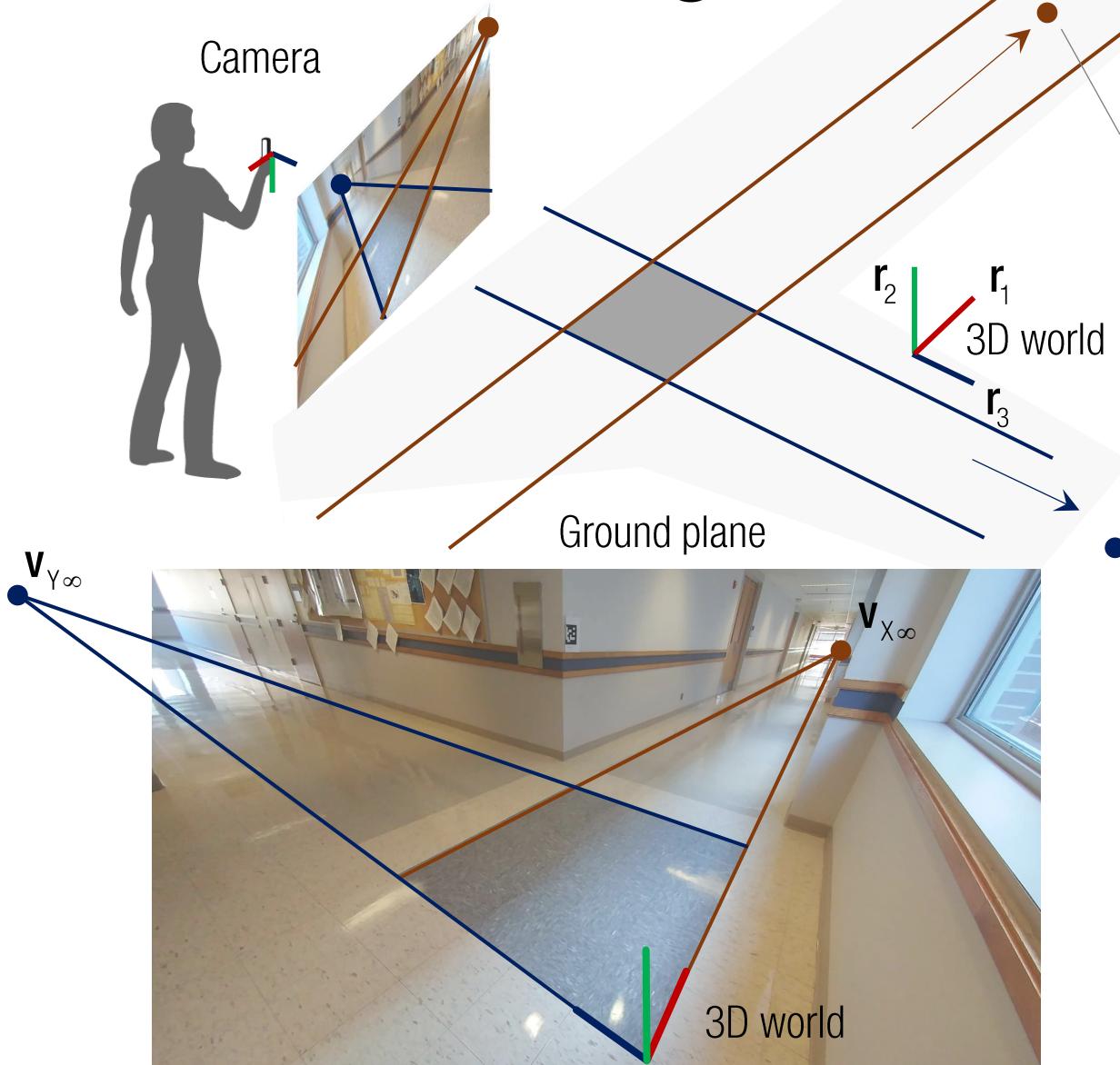
Geometric Interpretation (Translation Ambiguity)



Geometric Interpretation (Translation Ambiguity)



Two Vanishing Points



```
f = 4000;
K = [f 0 size(im,2)/2;
      0 f size(im,1)/2;
      0 0 1];
```

```
|l11 = GetLineFromTwoPoints(m11,m12);
l12 = GetLineFromTwoPoints(m13,m14);
```

```
|l21 = GetLineFromTwoPoints(m21,m22);
l22 = GetLineFromTwoPoints(m23,m24);
```

```
v1 = GetPointFromTwoLines(l11,l12);
v2 = GetPointFromTwoLines(l21,l22);
```

```
r1 = inv(K)*v1/norm(inv(K)*v1);
r2 = inv(K)*v2/norm(inv(K)*v2);
```

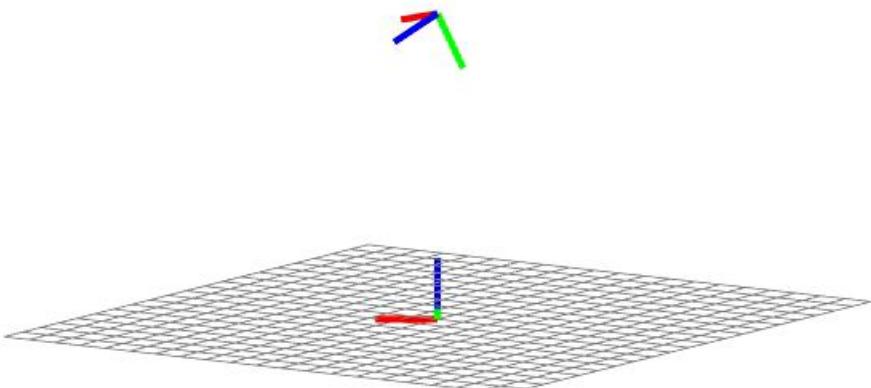
```
r3 = Vec2Skew(r1)*r2;
```

$R = \begin{bmatrix} 0.2448 & -0.5178 & 0.0424 \\ -0.1737 & -0.1960 & -0.6978 \\ 0.9539 & 0.8327 & -0.1379 \end{bmatrix}$
Not orthogonal matrix!

$\det(R) = 0.5077$

$R'^*R = \begin{bmatrix} 0.3299 & 0.0294 & -0.2036 \\ 0.0294 & 0.5555 & -0.2327 \\ -0.2036 & -0.2327 & 1.6224 \end{bmatrix}$

Two Vanishing Points



f = 1224; ← Change focal length
K = [f 0 size(im,2)/2;
0 f size(im,1)/2;
0 0 1];

|l11 = GetLineFromTwoPoints(m11,m12);
l12 = GetLineFromTwoPoints(m13,m14);

|l21 = GetLineFromTwoPoints(m21,m22);
l22 = GetLineFromTwoPoints(m23,m24);

v1 = GetPointFromTwoLines(l11,l12);
v2 = GetPointFromTwoLines(l21,l22);

r1 = inv(K)*v1/norm(inv(K)*v1);
r2 = inv(K)*v2/norm(inv(K)*v2);

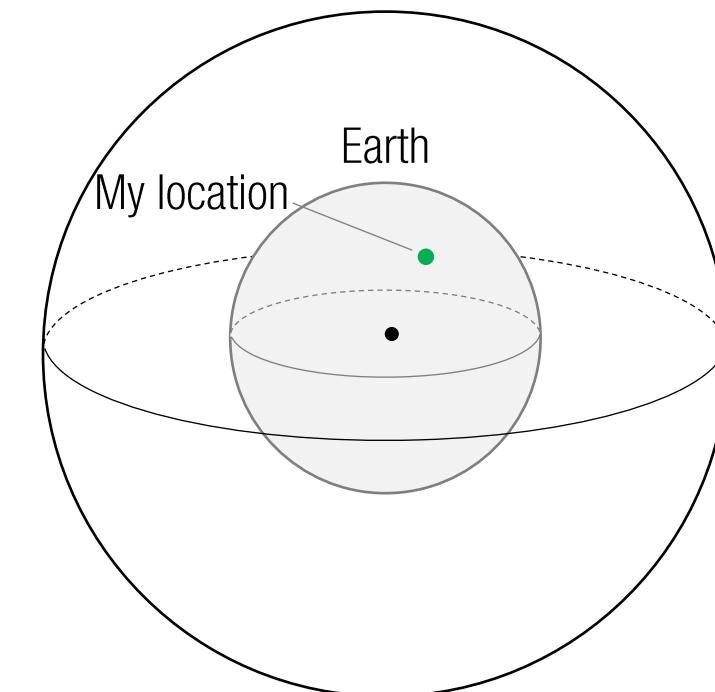
r3 = Vec2Skew(r1)*r2;

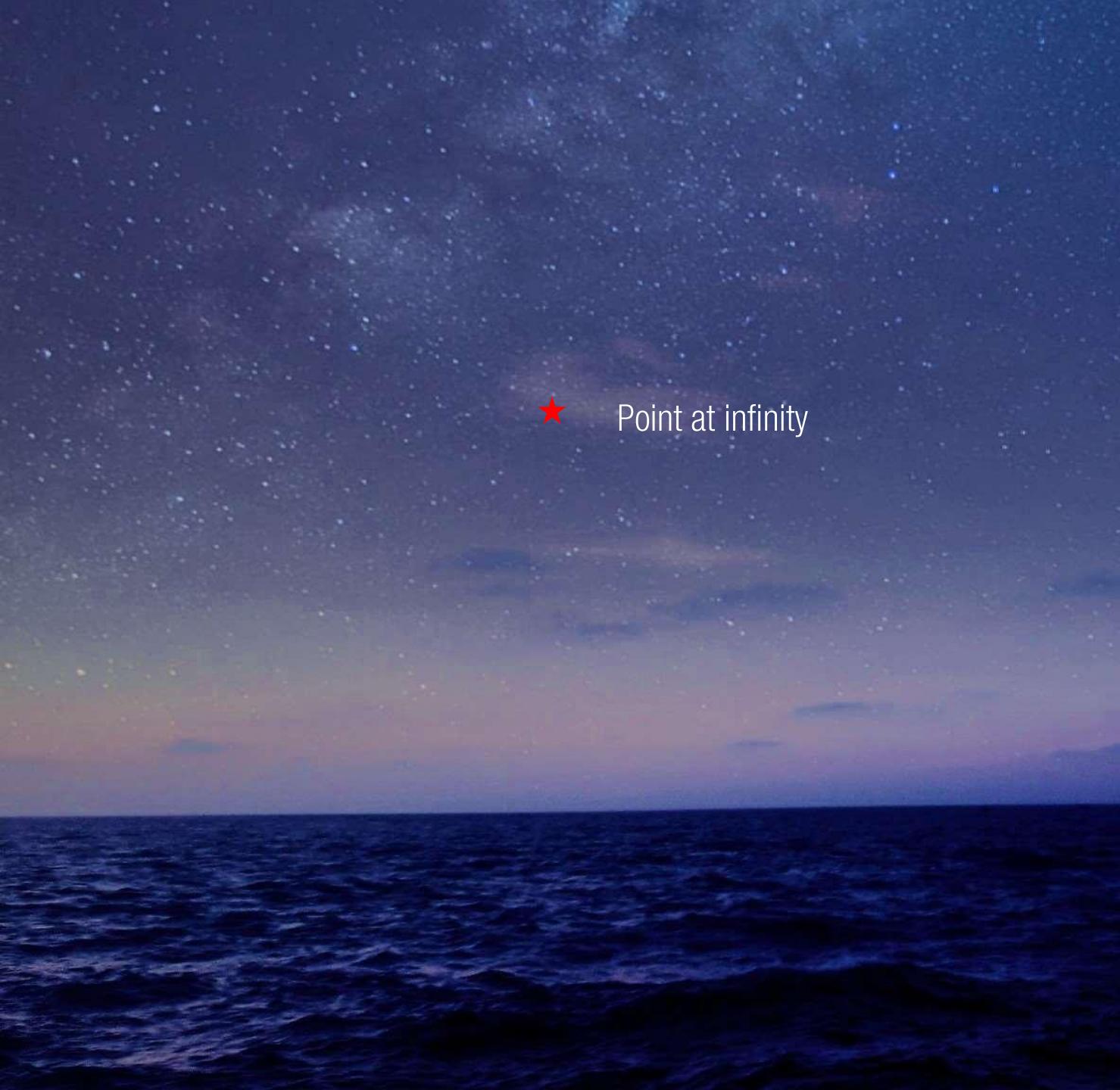
Orthogonal matrix!

R =
0.5846 -0.8496 0.0508
-0.4149 -0.3216 -0.8367
0.6972 0.4180 -0.5405

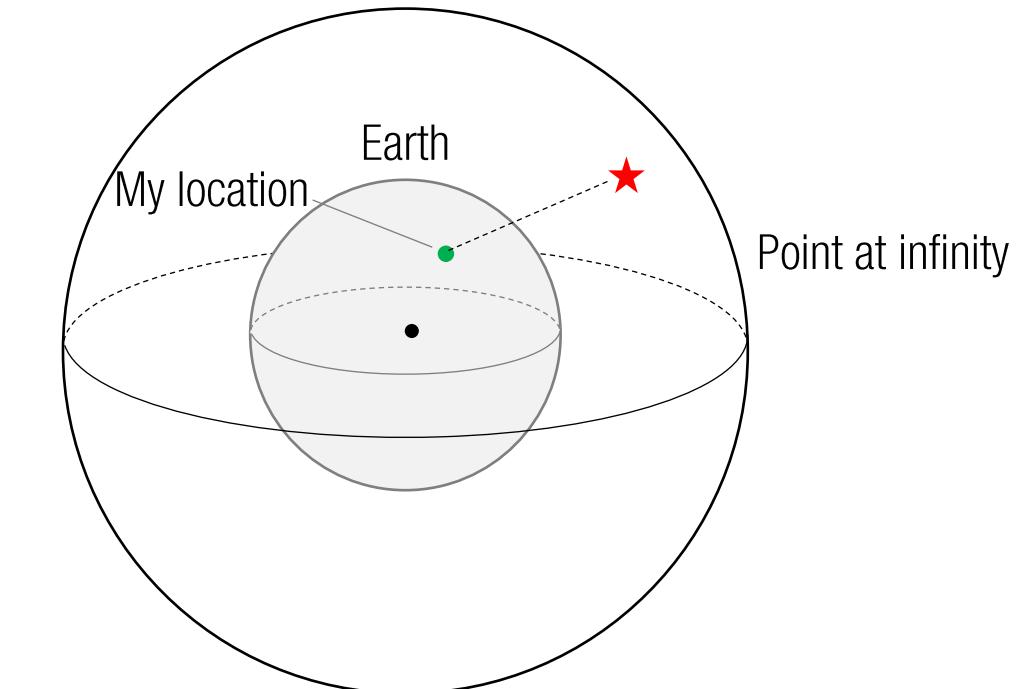
det(R) =
0.9948

R'*R =
1.0662 -0.0118 0.0250
-0.0118 0.9757 0.0285
0.0250 0.0285 0.9530





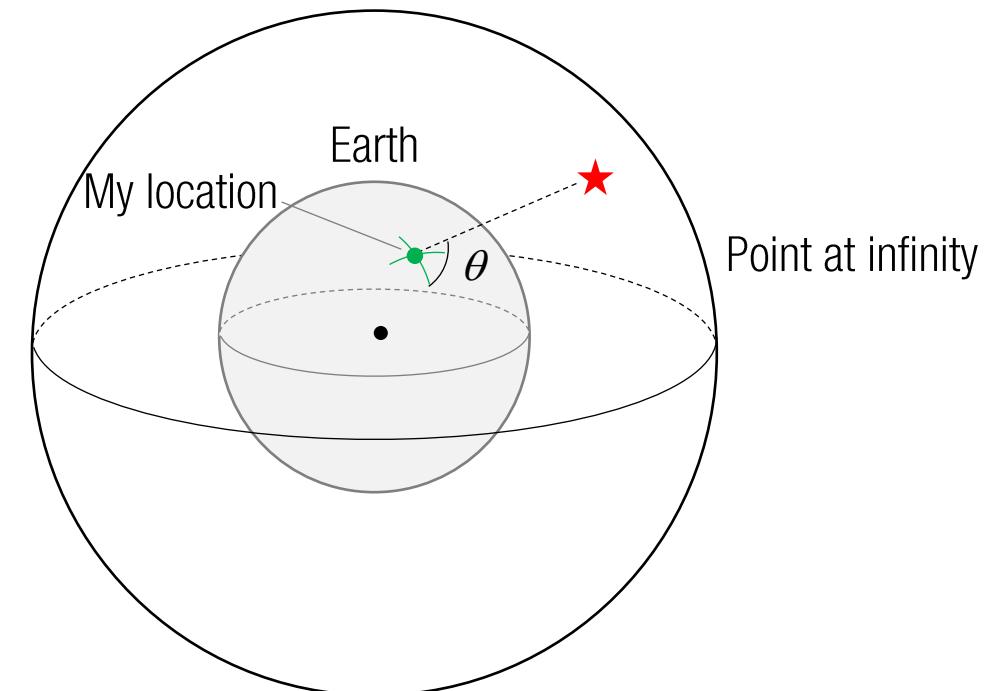
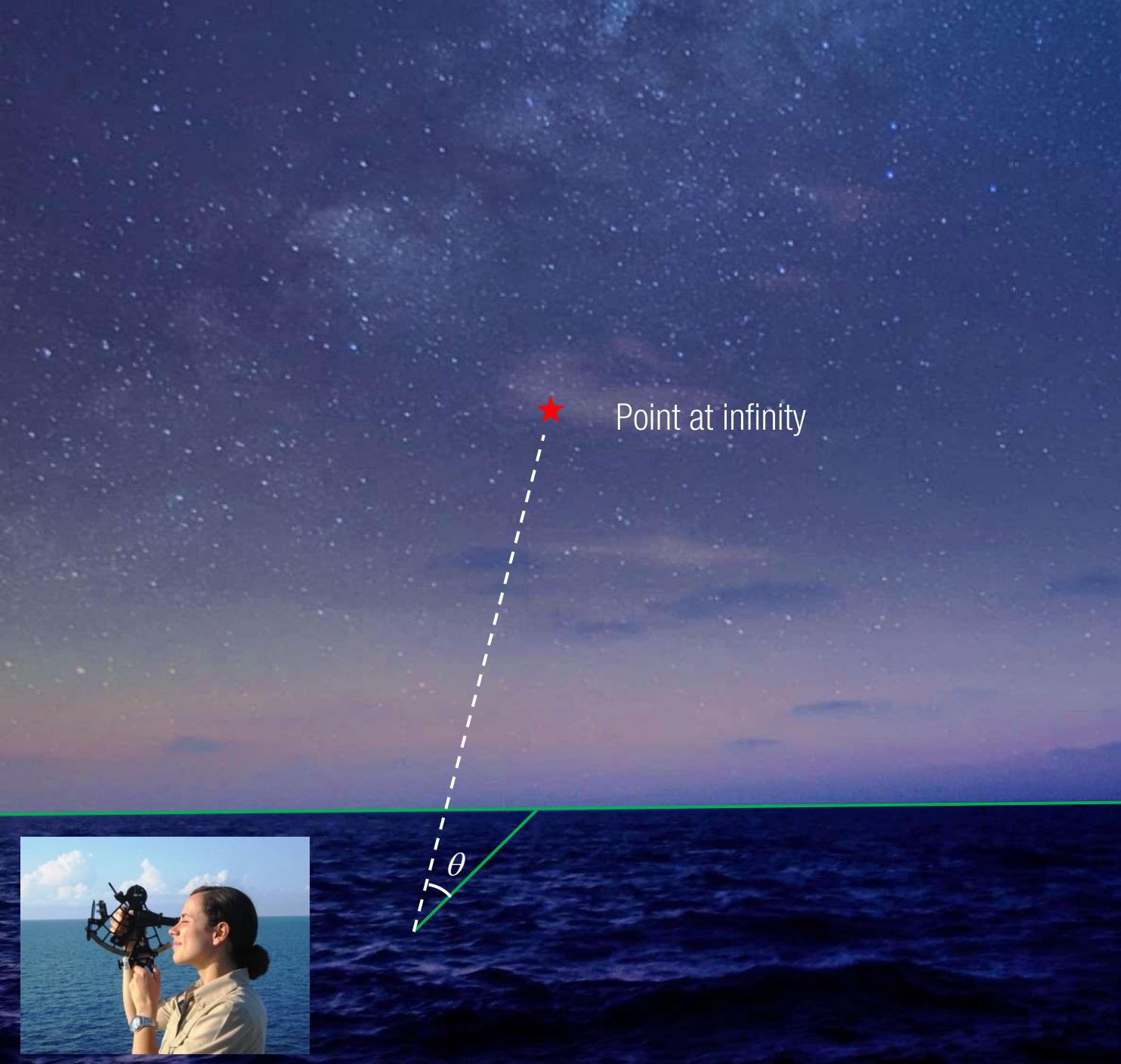
Point at infinity

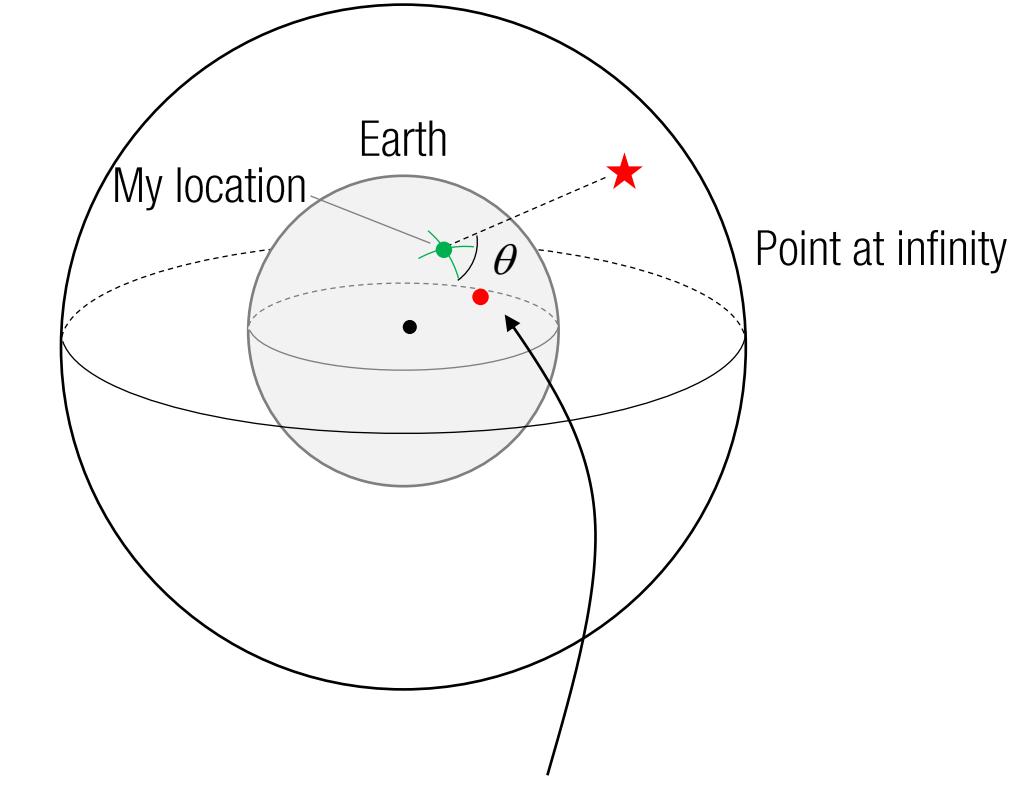
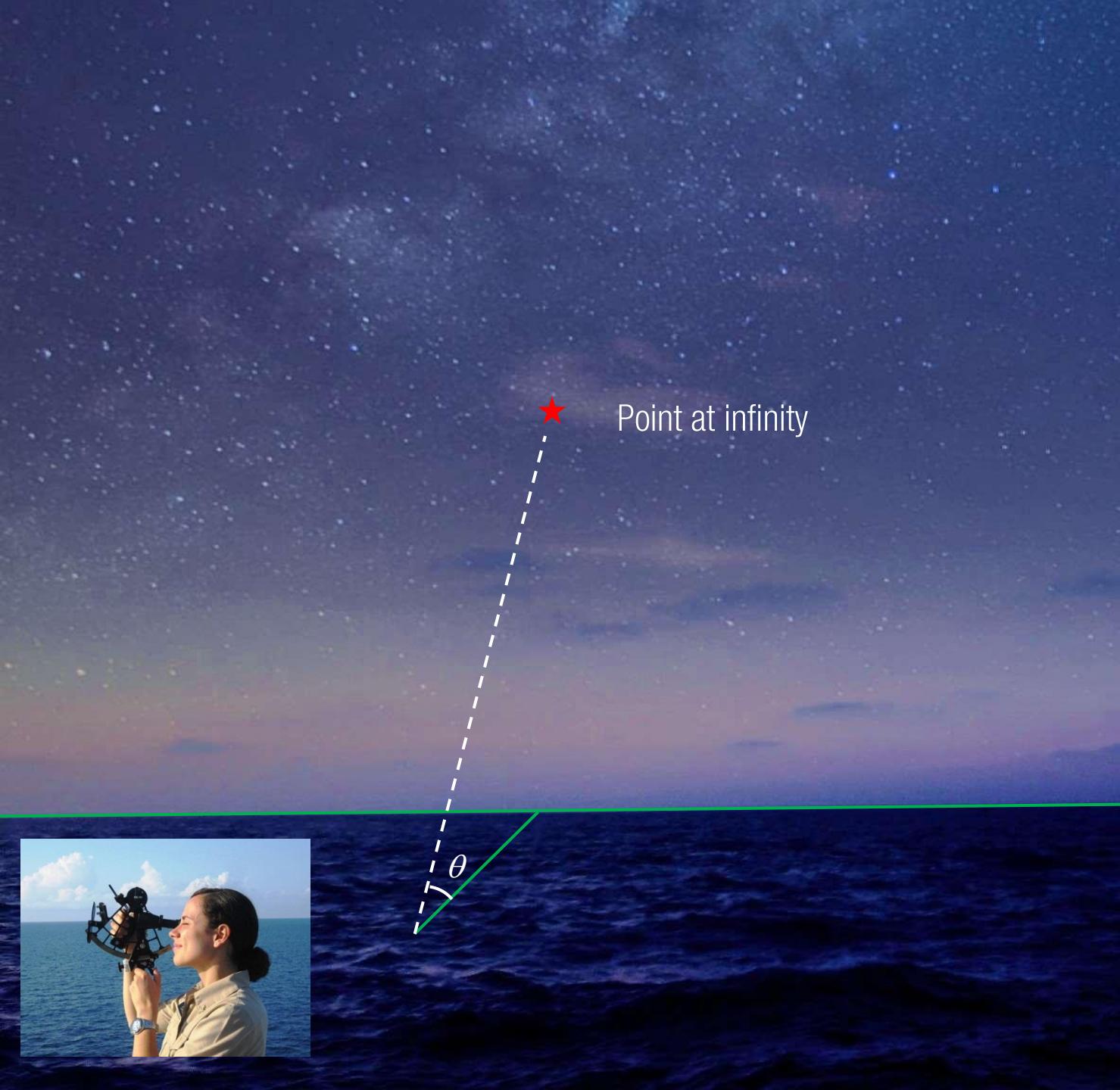


Earth

My location

Point at infinity



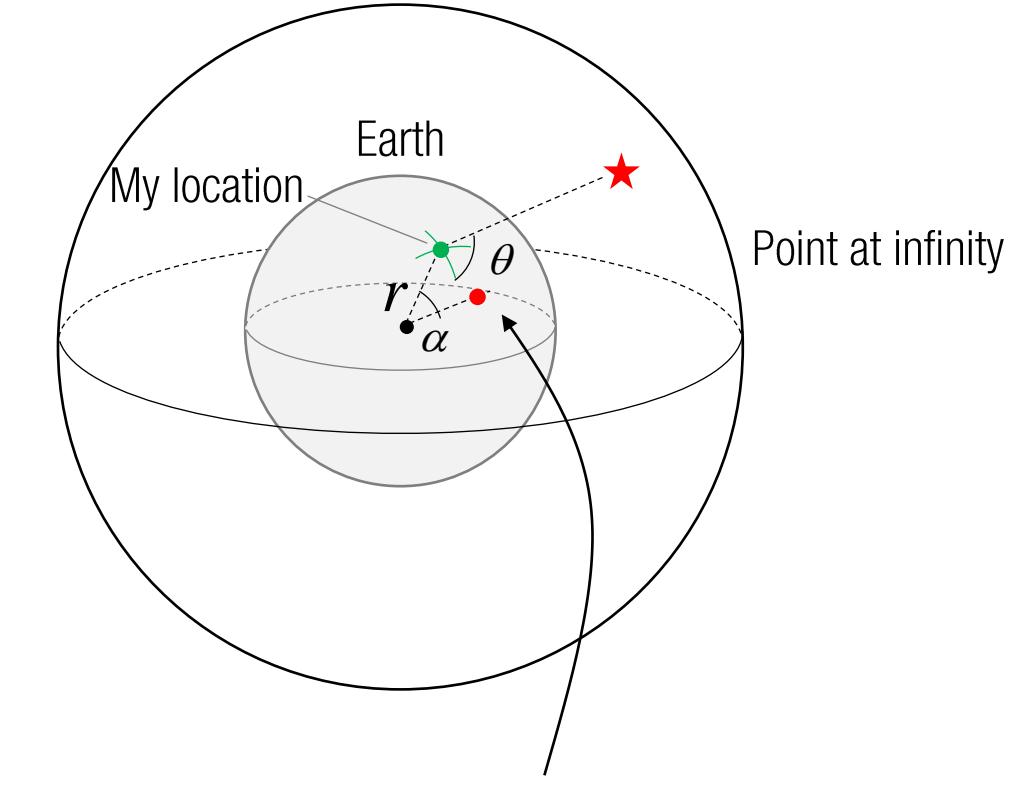
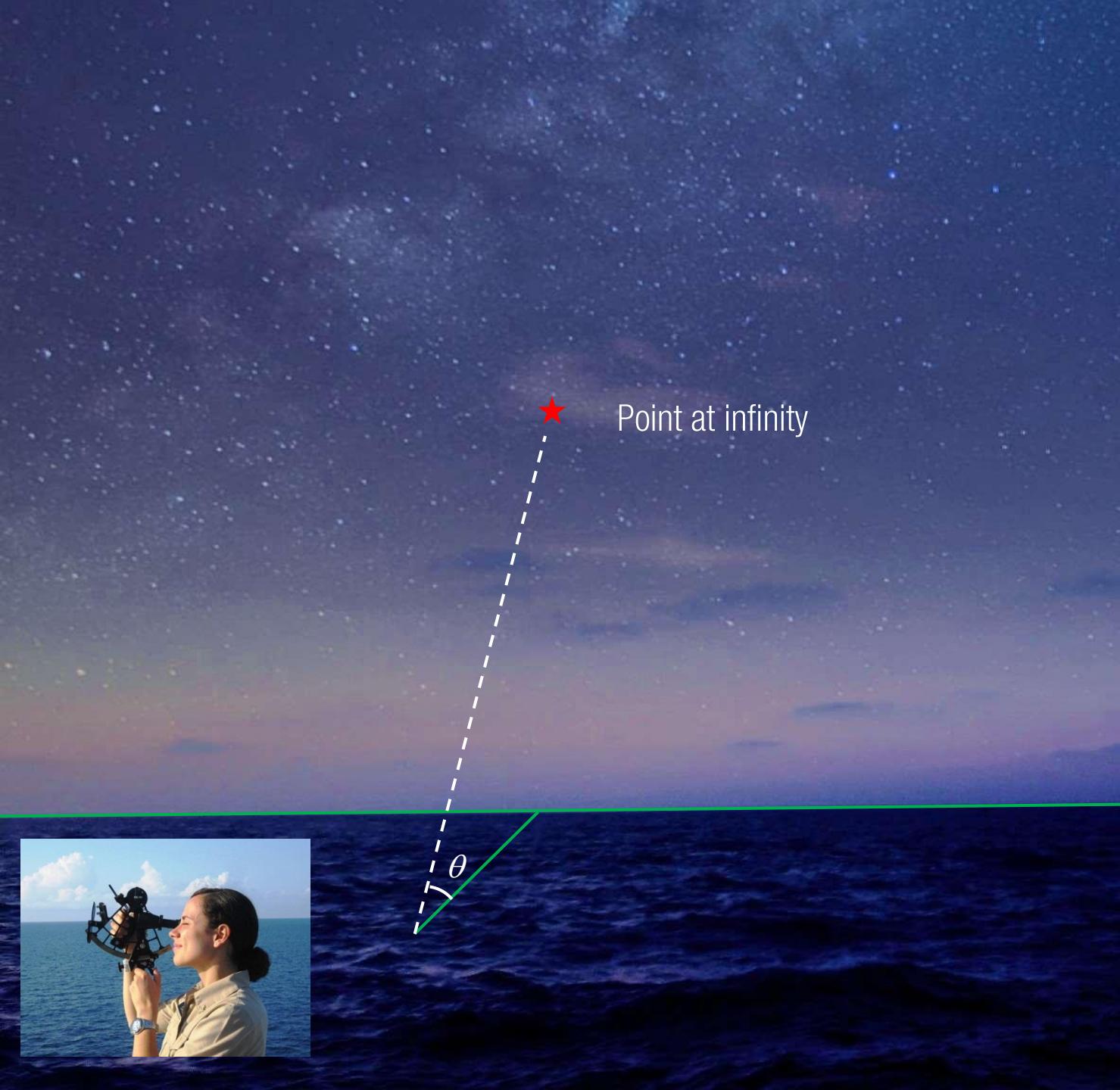


Reference coordinate of the star

292

GREENWICH P. M. 1942 MAY 26 (TUESDAY)

GCT	SUN GHA Dec.	T GHA	VENUS-1&2 GHA Dec.	MARS-1.9 GHA Dec.	JUPITER-1.5 GHA Dec.	MOON GHA Dec.	Lat. Sun-rise h m m Twil. h m m Moon-rise h m m Dif.
12 00	0 47 N21 04	63 25 ° 41 45 N 6 53'	313 11 ° 23 31'	337 04 N23 14'	230 23 S 2 ° 34'		
10	3 17	65 56 44 15	315 41	339 34	232 48	36	
20	5 47	68 26 46 45	318 11	342 04	235 13	38	
30	8 17	70 57 49 15	320 41	344 35	237 38	40	
40	10 47	73 27 51 45	323 11	347 05	240 03	41	
50	13 17	75 57 54 15	325 41	349 35	242 27	43	N
13 00	15 47 N21 04	78 28 56 45 N 6 54	328 11 N23 31	352 06 N23 14	244 52 S 2 45	60	3 00 75 15 08 87
10	18 17	80 58 59 15	330 42	354 36	247 17	47 58	
20	20 47	83 29 61 45	333 12	357 06	249 42	49 56	30 55 04 83
30	23 17	85 59 64 15	335 42	359 37	252 07	51 54	42 50 03 80
40	25 47	88 30 66 45	338 12	2 07	254 32	53 52	3 52 45 02 78
50	28 17	91 00 69 15	340 42	4 37	256 56	54 50	4 02 42 15 01 76
14 00	30 47 N21 05	93 30 71 45 N 6 55	343 12 N23 31	7 08 N23 14	259 21 S 2 56	45	21 36 14 58 72

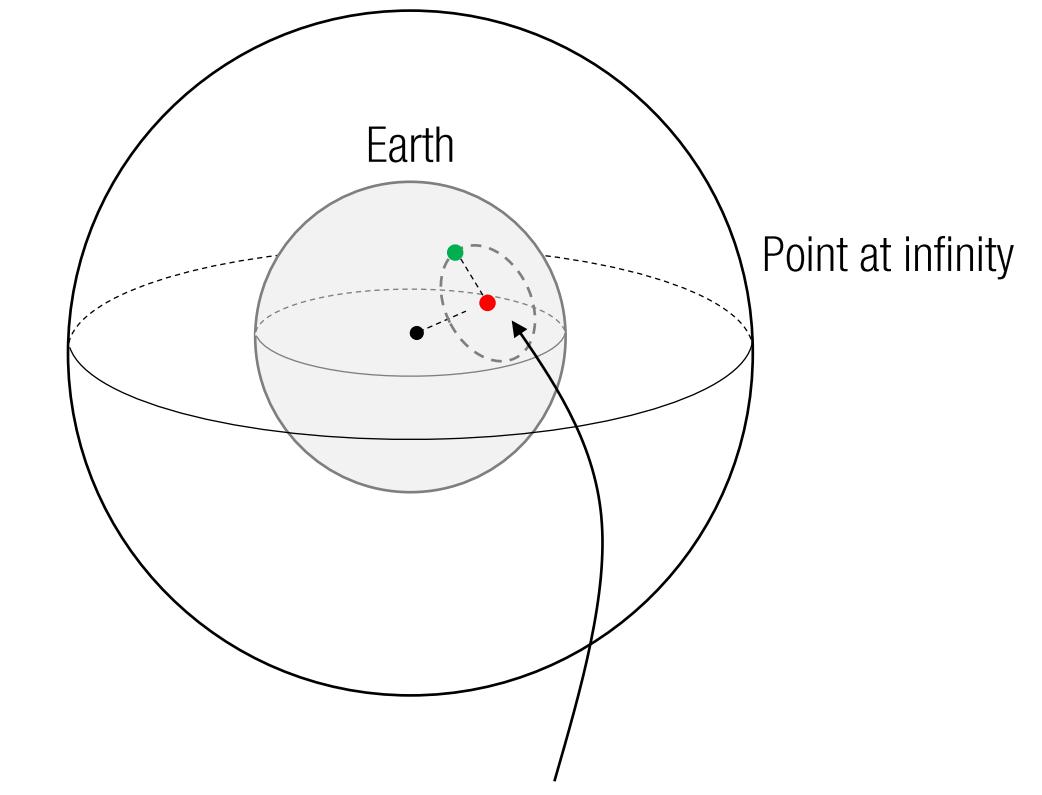
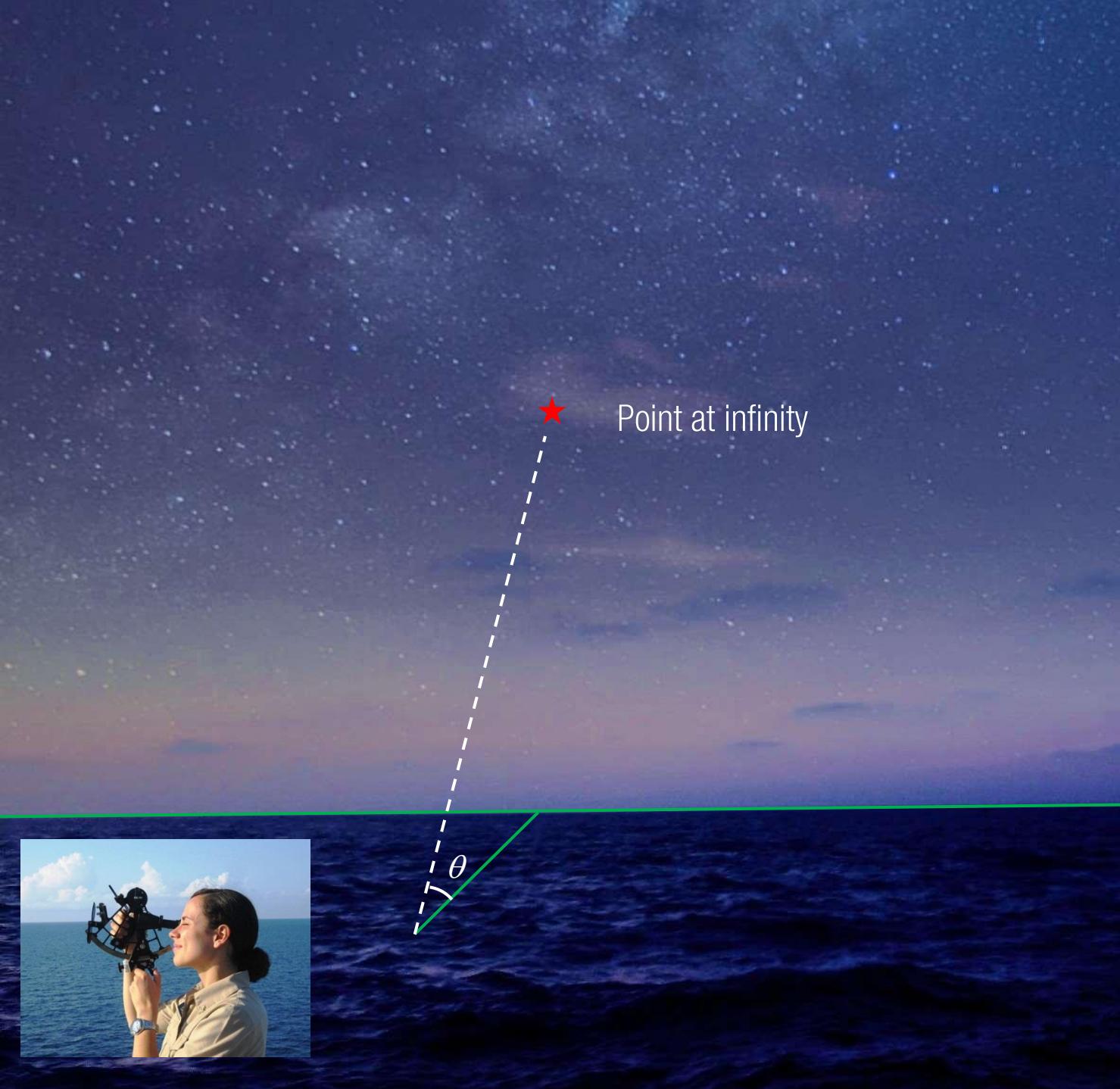


Reference coordinate of the star

292

GREENWICH P. M. 1942 MAY 26 (TUESDAY)

GCT	SUN GHA Dec.	T GHA	VENUS-1&2 GHA Dec.	MARS-1.5 GHA Dec.	JUPITER-1.5 GHA Dec.	MOON GHA Dec.	Lat. Sun- rise	Long. Twil-	Long. Moon- rise	Dif.
12 00	0° 47' N 21° 04'	63° 25' 0"	41° 45' N 6° 53'	313° 11' N 23° 31'	337° 04' N 23° 14'	230° 23' S 2° 34'				
10	3 17	65 56 44 15	315 41	339 34	232 48	36				
20	5 47	68 26 46 45	318 11	342 04	235 13	38				
30	8 17	70 57 49 15	320 41	344 35	237 38	40				
40	10 47	73 27 51 45	323 11	347 05	240 03	41				
50	13 17	75 57 54 15	325 41	349 35	242 27	43	N			
13 00	15 47 N 21 04	78 28 56 45 N 6 54	328 11 N 23 31	352 06 N 23 14	244 52 S 2 45	60	3 00	75	15 08	87
10	18 17	80 58 59 15	330 42	354 36	247 17	47	58	17	64	06 85
20	20 47	83 29 61 45	333 12	357 06	249 42	49	56	30	55	04 83
30	23 17	85 59 64 15	335 42	359 37	252 07	51	54	42	50	03 80
40	25 47	88 30 66 45	338 12	2 07	254 32	53	52	3 52	45	02 78
50	28 17	91 00 69 15	340 42	4 37	256 56	54	50	4 02	42	15 01 76
14 00	30 47 N 21 05	93 30 71 45 N 6 55	343 12 N 23 31	7 08 N 23 14	259 21 S 2 56	45	21	36	14 58	72

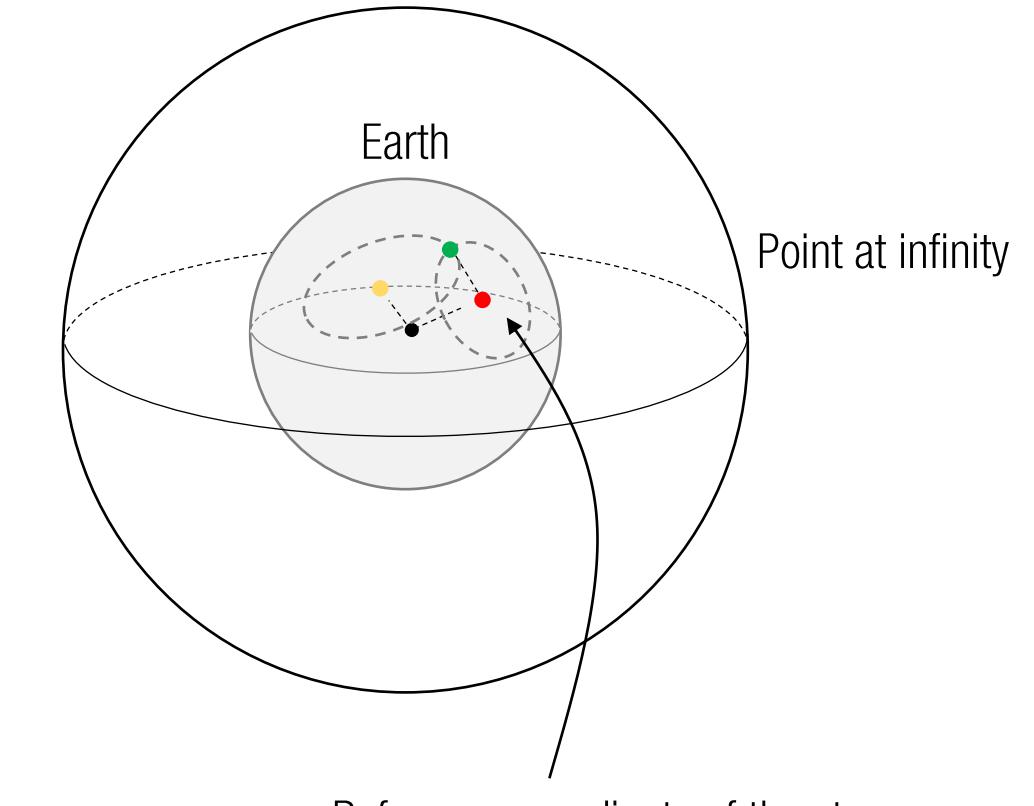
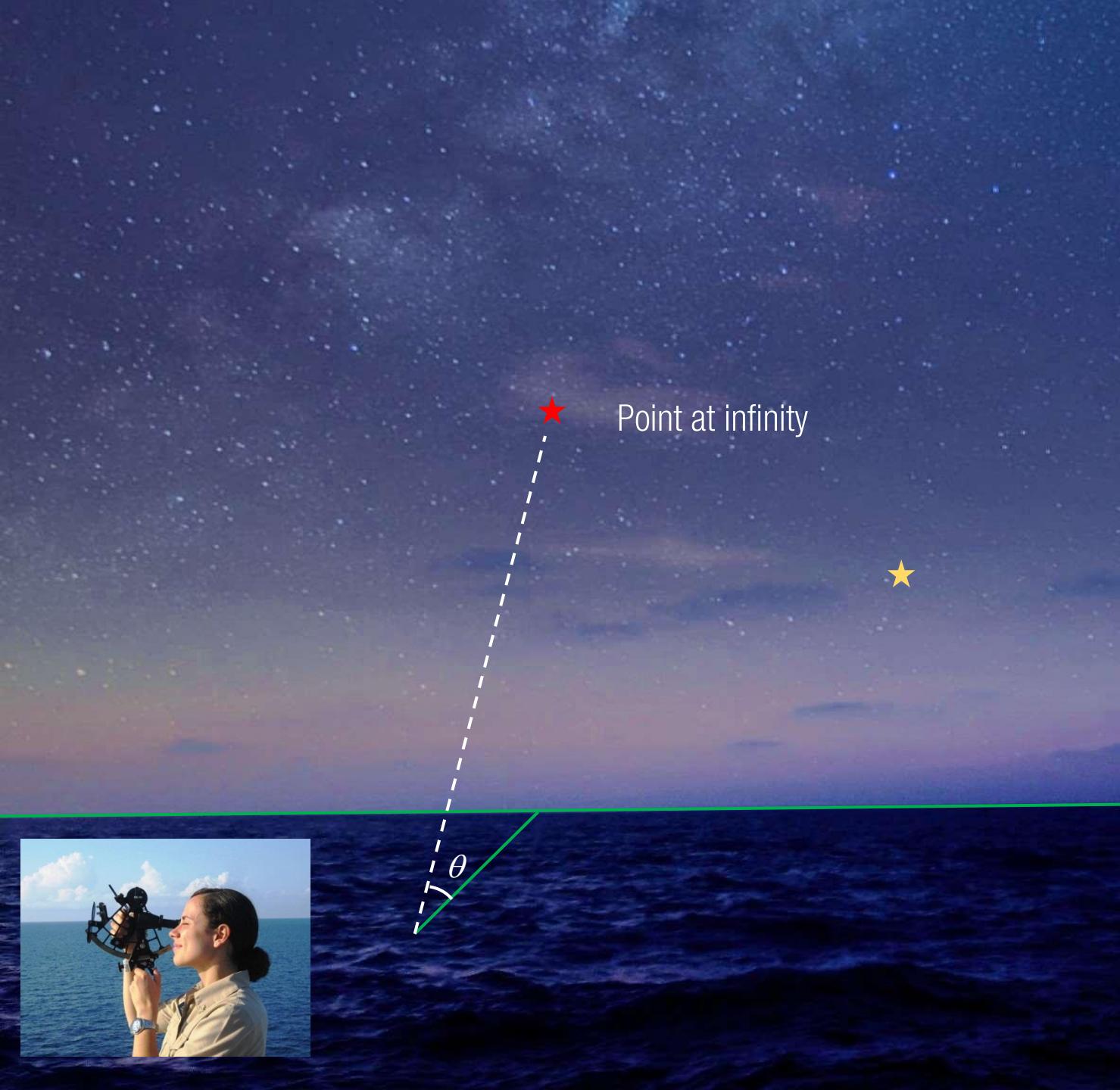


Reference coordinate of the star

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GREENWICH P. M. 1942 MAY 26 (TUESDAY)

GCT	SUN		VENUS-1.5		MARS 1.9		JUPITER-1.5		MOON		Lat.	Sunrise	Twil.	Moonrise	Diff.	
	GHA	Dec.	GHA	Dec.	GHA	Dec.	GHA	Dec.	GHA	Dec.						
12 00	0° 47' N21 04'	63 25	0° 41 45 N 6 53	313 11 N23 31	0° 337 04 N23 14	230 23 S 2 34	N	o	h	m	h	m	h	m	h	m
	3 17	65 56	44 15	315 41	339 34	232 48										
	5 47	68 26	46 45	318 11	342 04	235 13										
	8 17	70 57	49 15	320 41	344 35	237 38										
	10 47	73 27	51 45	323 11	347 05	240 03										
	13 17	75 57	54 15	325 41	349 35	242 27										
	15 47 N21 04	78 28	56 45 N 6 54	328 11 N23 31	356 06 N23 14	244 52 S 2 45										
	18 17	80 58	59 15	330 42	354 36	247 17										
	20 47	83 29	61 45	333 12	357 06	249 42										
	23 17	85 59	64 15	335 42	359 37	252 07										
13 00	25 47	88 30	66 45	338 12	2 07	254 32	53 52	3 52	45	02	15 02	03 50	03 50	02	78	02 78
	28 17	91 00	69 15	340 42	4 37	256 56	54 50	4 02	42	15	01 50	01 50	01 50	01	76	01 76
	30 47 N21 05	93 30	71 45 N 6 55	343 12 N23 31	7 08 N23 14	259 21 S 2 56	45 21	16 14	58	45	21	16 14	58	21	58	21



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GREENWICH P. M. 1942 MAY 26 (TUESDAY)

GCT	SUN GHA Dec.	VENUS-1&2 GHA Dec.	MARS 1.9 GHA Dec.	JUPITER-1.5 GHA Dec.	MOON GHA Dec.	Lat. Sun- rise	Long. Twil-	Lat. Moon- rise	Diff.
12 00	0° 47' N 21° 04'	63° 25' 41° 45' N 6° 53'	313° 11' N 23° 31'	337° 04' N 23° 14'	230° 23' S 2° 34'	60° 3' 00"	75° 15' 08"	87°	
10	3 17	65 56 44 15	315 41	339 34	232 48	36			
20	5 47	68 26 46 45	318 11	342 04	235 13	38			
30	8 17	70 57 49 15	320 41	344 35	237 38	40			
40	10 47	73 27 51 45	323 11	347 05	240 03	41			
50	13 17	75 57 54 15	325 41	349 35	242 27	43			
13 00	15 47 N 21 04	78 28 56 45 N 6 54	328 11 N 23 31	352 06 N 23 14	244 52 S 2 45	60° 3' 00"	75° 15' 08"	87°	
10	18 17	80 58 59 15	330 42	354 36	247 17	47 58			
20	20 47	83 29 61 45	333 12	357 06	249 42	49 56	30 55	04 83	
30	23 17	85 59 64 15	335 42	359 37	252 07	51 54	42 50	03 80	
40	25 47	88 30 66 45	338 12	2 07	254 32	53 52	3 52 45	02 78	
50	28 17	91 00 69 15	340 42	4 37	256 56	54 50	4 02 42	15 01 76	
14 00	30 47 N 21 05	93 30 71 45 N 6 55	343 12 N 23 31	7 08 N 23 14	259 21 S 2 56	45° 21' 36"	14 58' 72"		