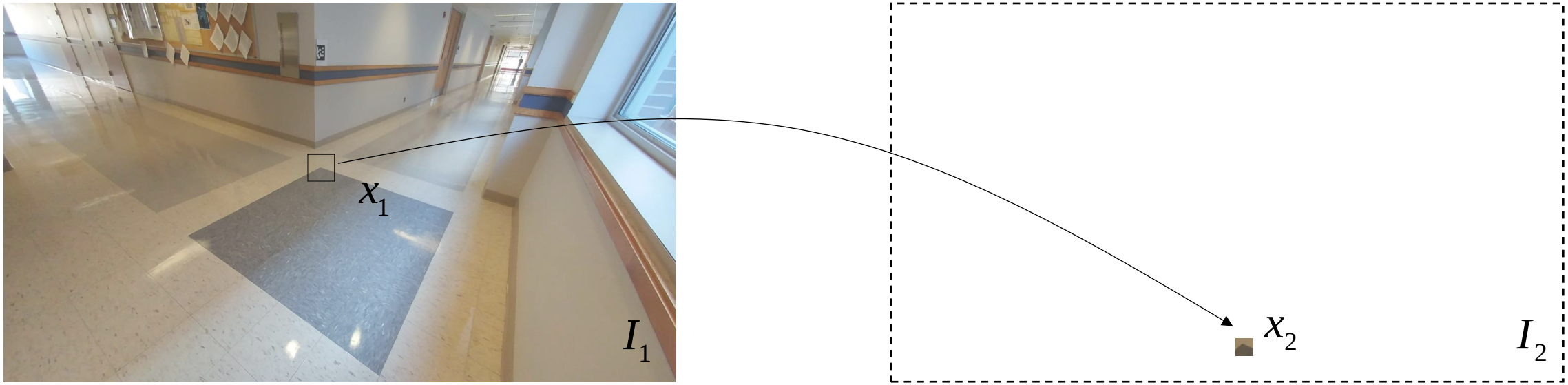
Image Transformation

Hyun Soo Park

GoPro
muzik

PIXEL TRANSPORT



Source image

Target image

$$x_2 = f(x_1)$$

Coordinate transform

PIXEL TRANSPORT

Source image

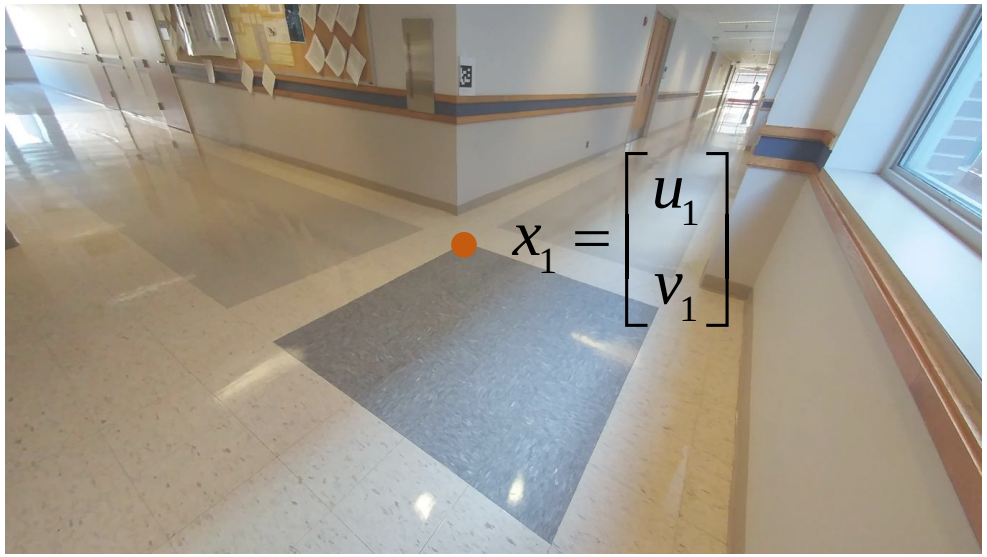
Target image

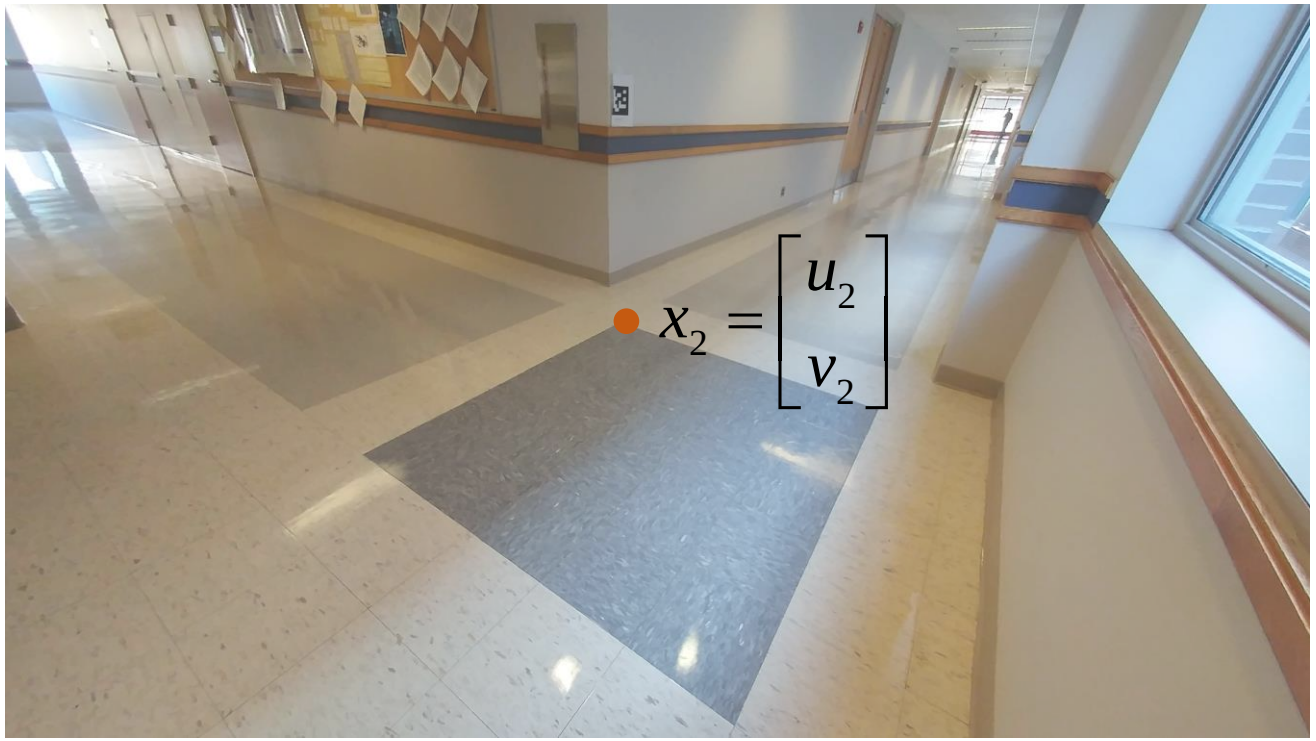
$$x_2 = f(x_1)$$

Coordinate transform

$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

Uniform Scaling

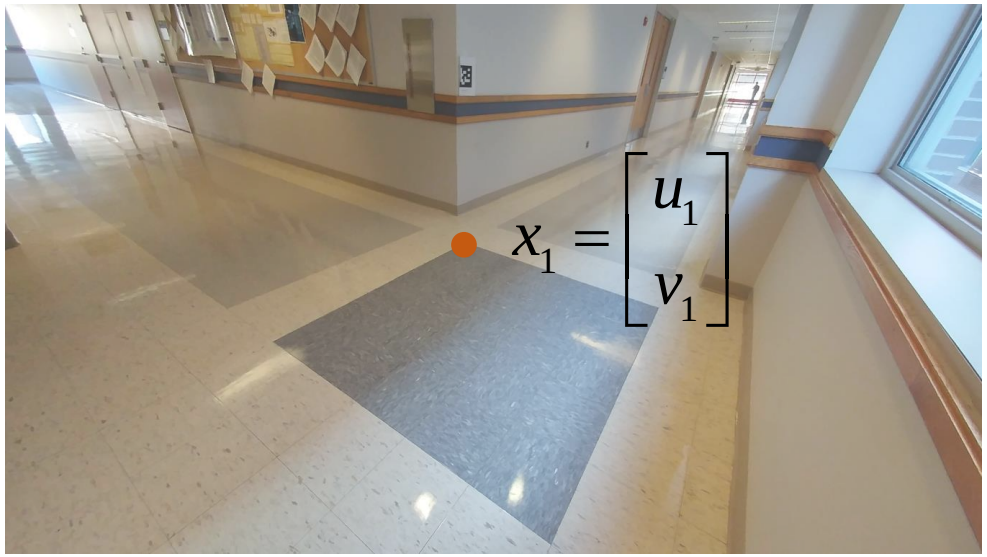


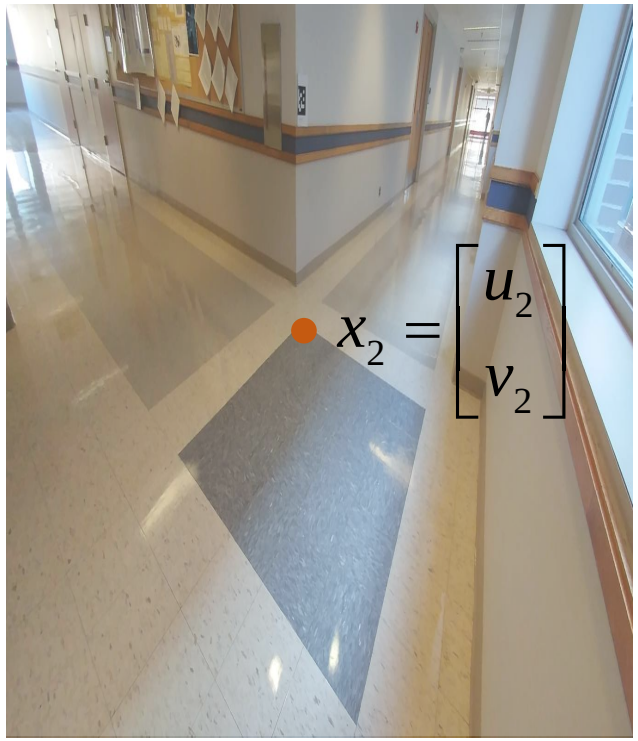


$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

$$x_2 = f(x_1) \longrightarrow \begin{aligned} u_2 &= s u_1 \\ v_2 &= s v_1 \end{aligned}$$

Non-uniform Scaling (Aspect Ratio Change)

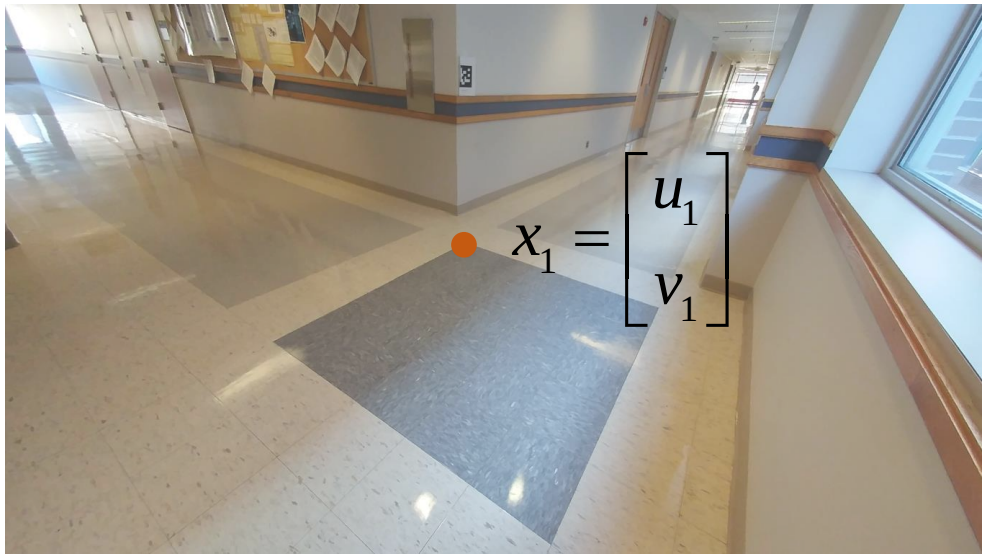


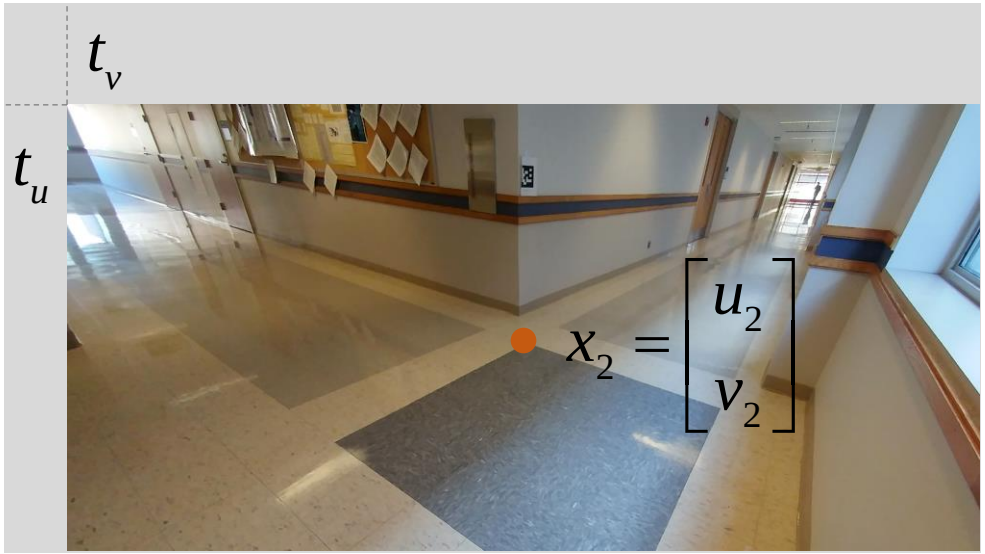


$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

$$x_2 = f(x_1) \longrightarrow \begin{aligned} u_2 &= s_u u_1 \\ v_2 &= s_v v_1 \end{aligned}$$

TRANSLATION

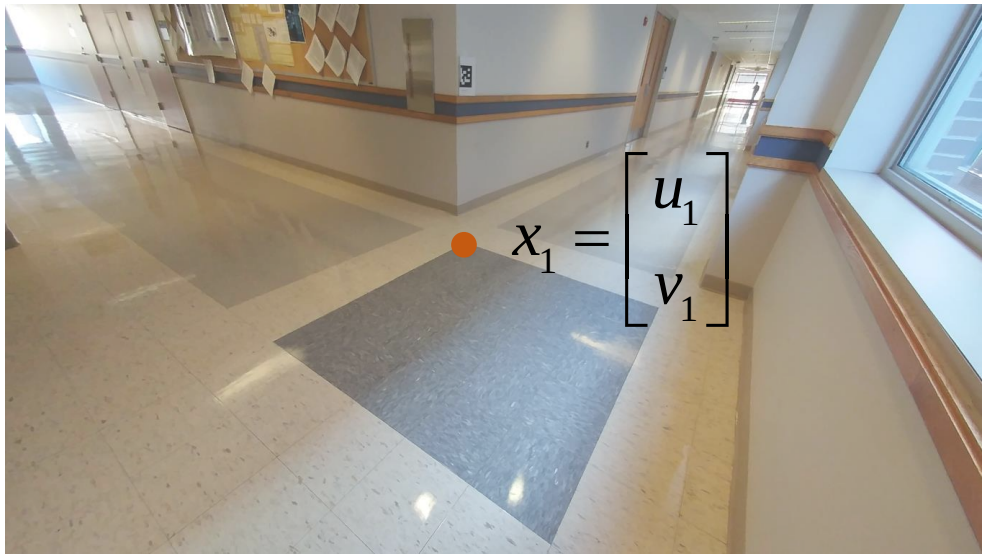


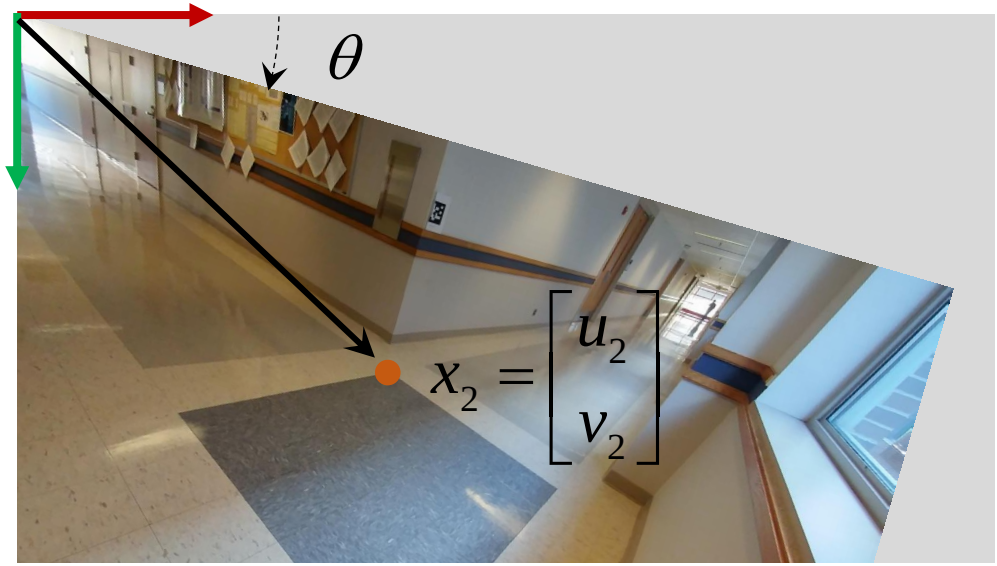


$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

$$x_2 = f(x_1) \longrightarrow \begin{aligned} u_2 &= u_1 + t_u \\ v_2 &= v_1 + t_v \end{aligned}$$

ROTATION

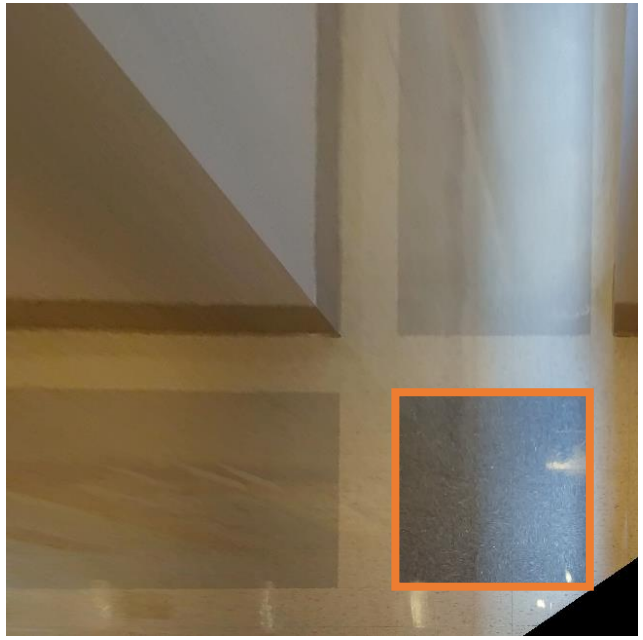


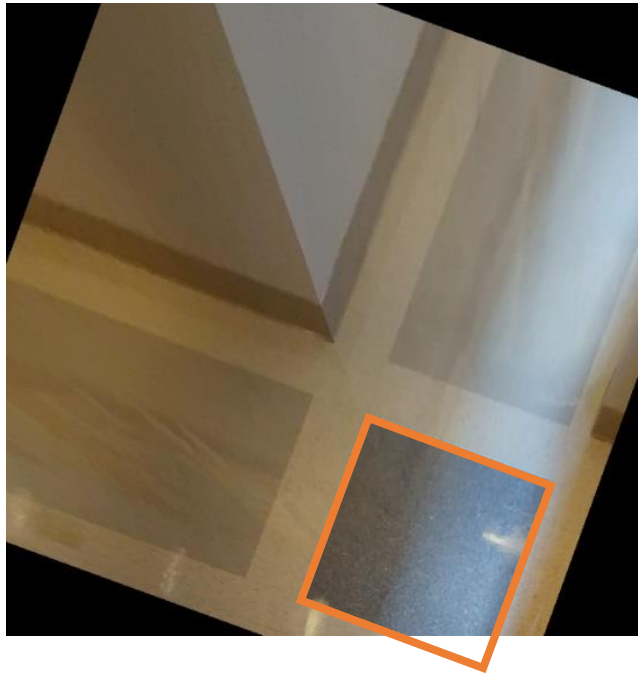


$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

$$x_2 = f(x_1) \longrightarrow \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \end{bmatrix}$$

EUCLIDEAN TRANSFORM SE(2) ~ TRANSLATION+ROTATION





Invariant properties

- Length
- Angle
- Area

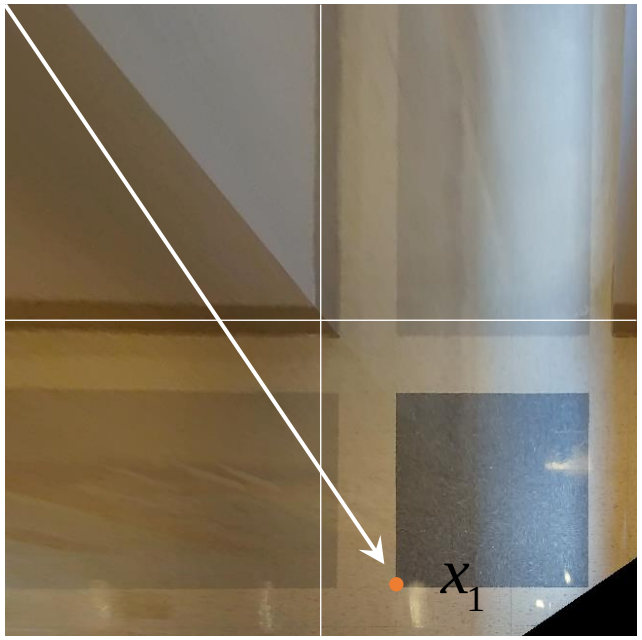
Degree of freedom

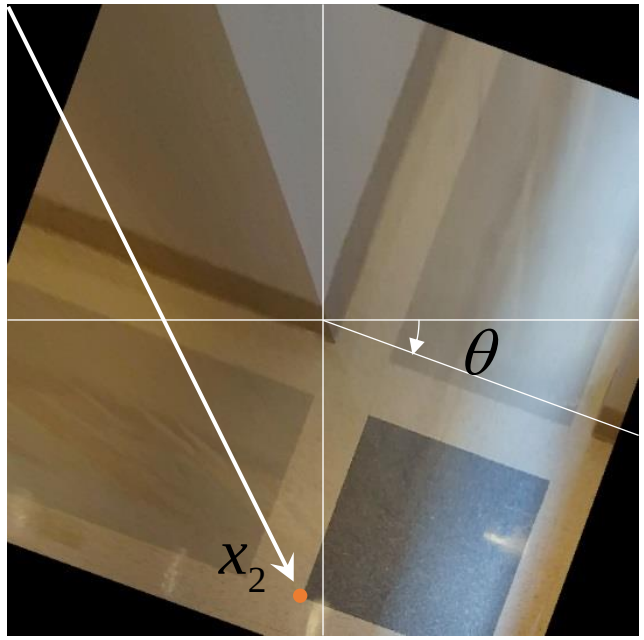
3 (2 translation+1 rotation)

$$I_2(x_2) = I_2(f(x_1)) = I_1(x_1)$$

$$x_2 = f(x_1) \longrightarrow \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_u \\ \sin\theta & \cos\theta & t_v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

EXAMPLE OF EUCLIDEAN TRANSFORM

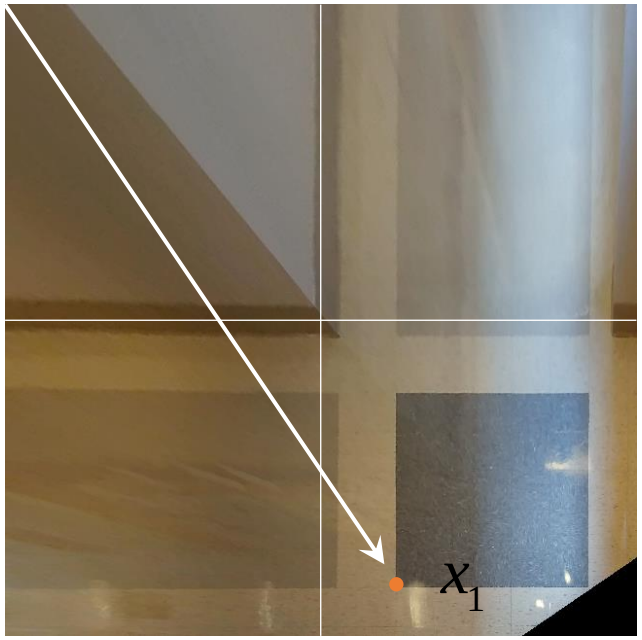


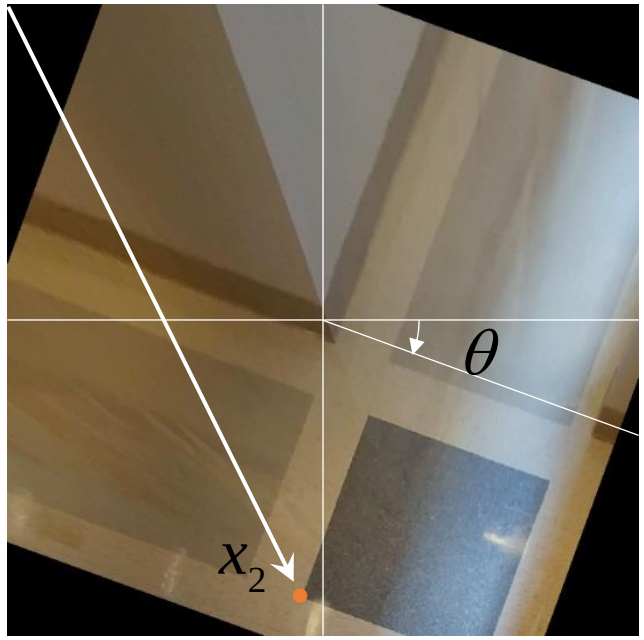


Rotate about the image center

$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_u \\ \sin\theta & \cos\theta & t_v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

Example of Euclidean Transform



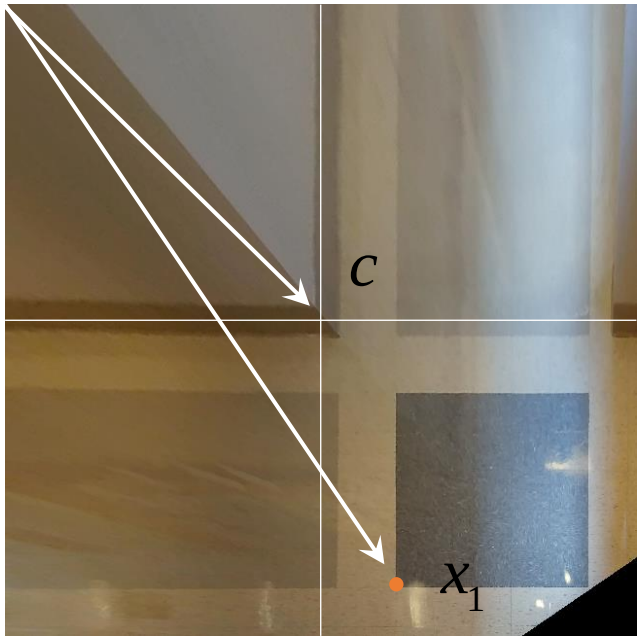


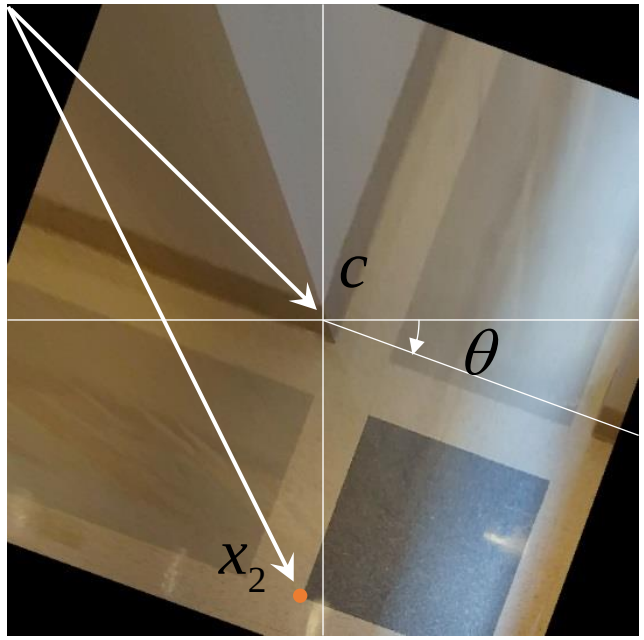
Rotate about the image center

$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix} \qquad t = ?$$

$$x_2 = Rx_1 + t$$

EXAMPLE OF EUCLIDEAN TRANSFORM





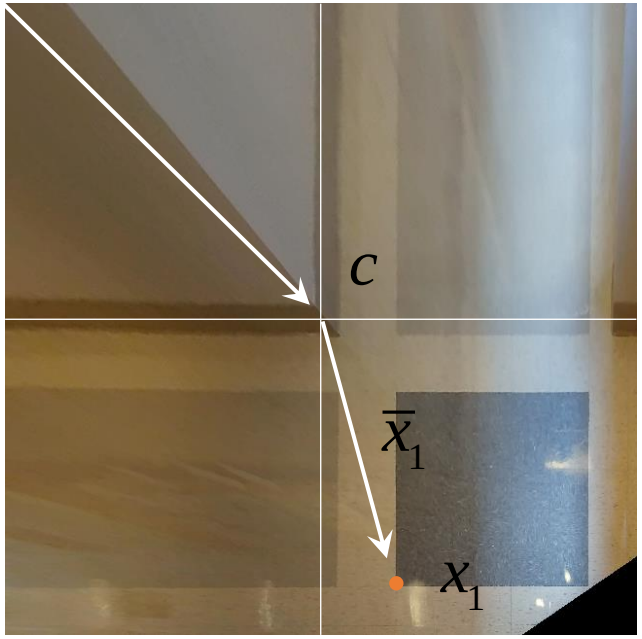
c : image center

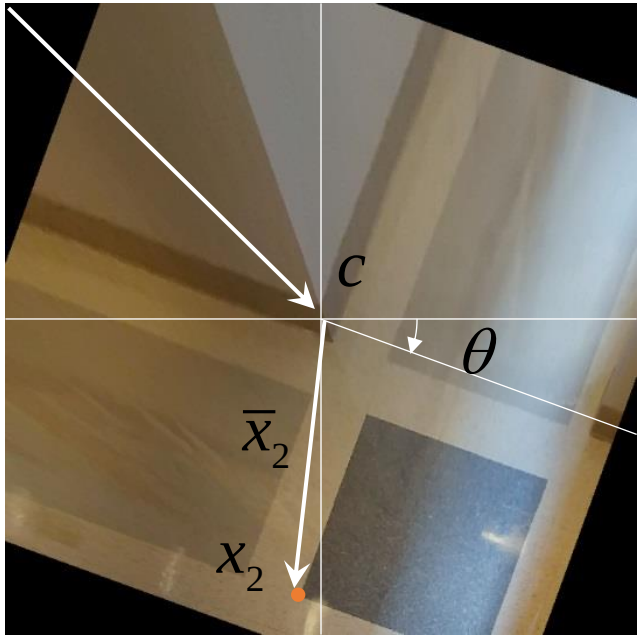
Rotate about the image center

$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix} \qquad t = ?$$

$$x_2 = Rx_1 + t$$

EXAMPLE OF EUCLIDEAN TRANSFORM





c : image center

Rotate about the image center

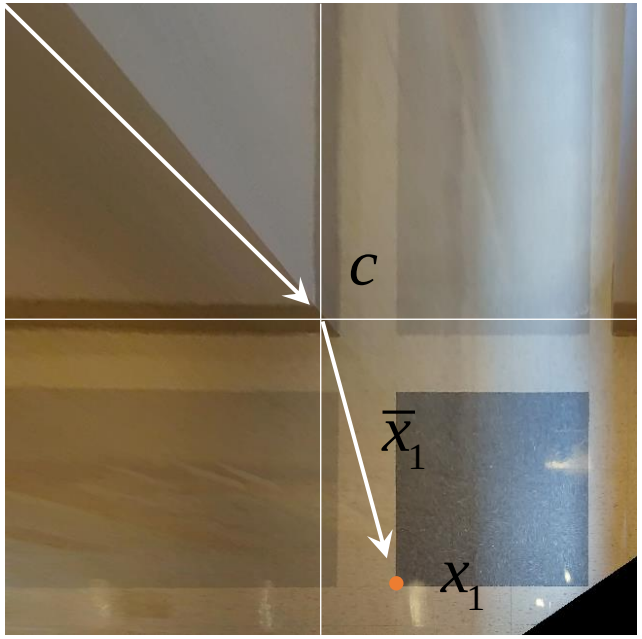
$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix} \qquad t = ?$$

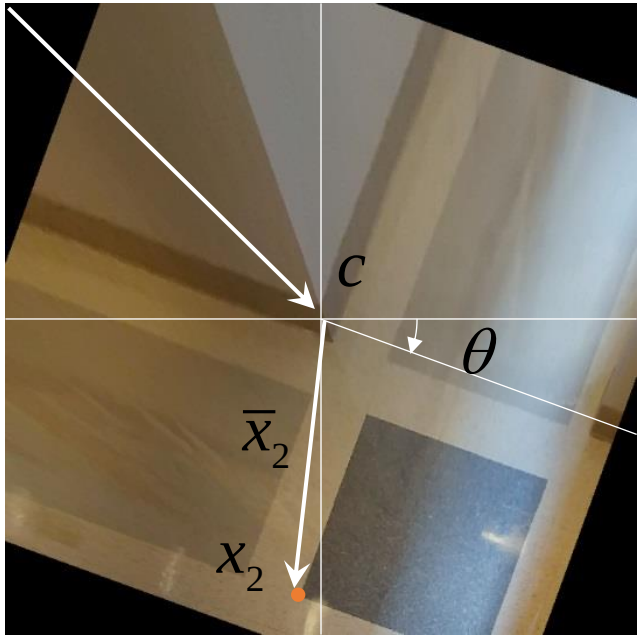
$$x_2 = Rx_1 + t$$

$$x_1 = c + \bar{x}_1$$

$$x_2 = c + \bar{x}_2$$

EXAMPLE OF EUCLIDEAN TRANSFORM





c : image center

Rotate about the image center

$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix} \qquad t = ?$$

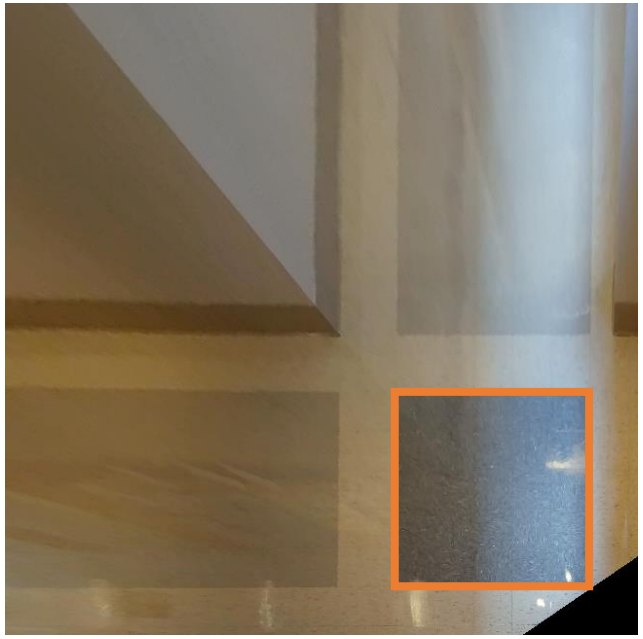
$$x_2 = Rx_1 + t$$

$$x_1 = c + \bar{x}_1$$

$$x_2 = c + \bar{x}_2$$

$$\longrightarrow \quad t = -Rc + c$$

Similarity Transform ~ Euclidean + Uniform Scaling



$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} sR & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix}$$

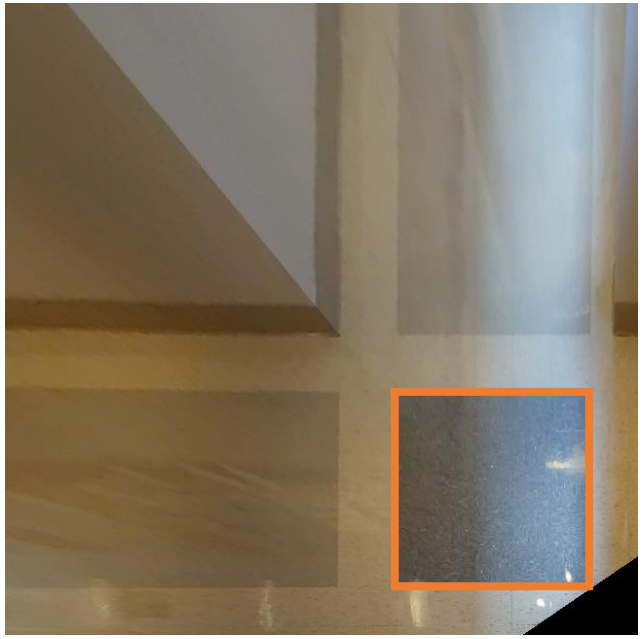
Invariant properties

- Length ratio
- Angle

Degree of freedom

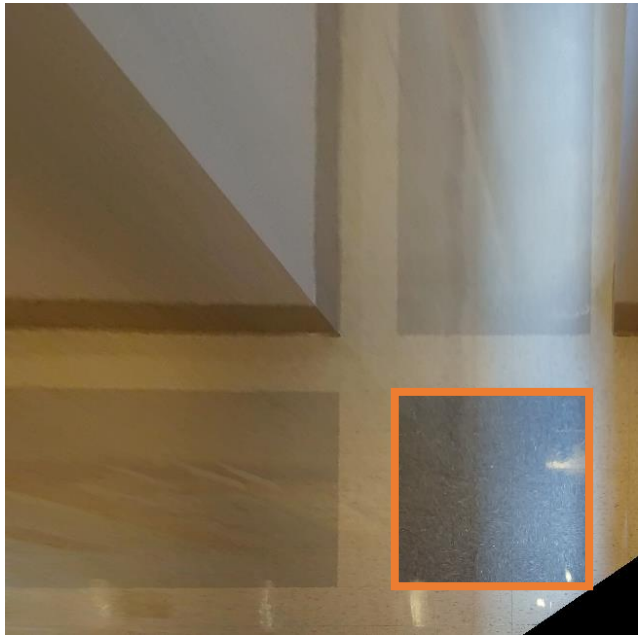
4 (2 translation+1 rotation+1 scale)

Similarity Transform ~ Euclidean + Uniform Scaling



$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta & t_u \\ s\sin\theta & s\cos\theta & t_v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

AFFINE TRANSFORM



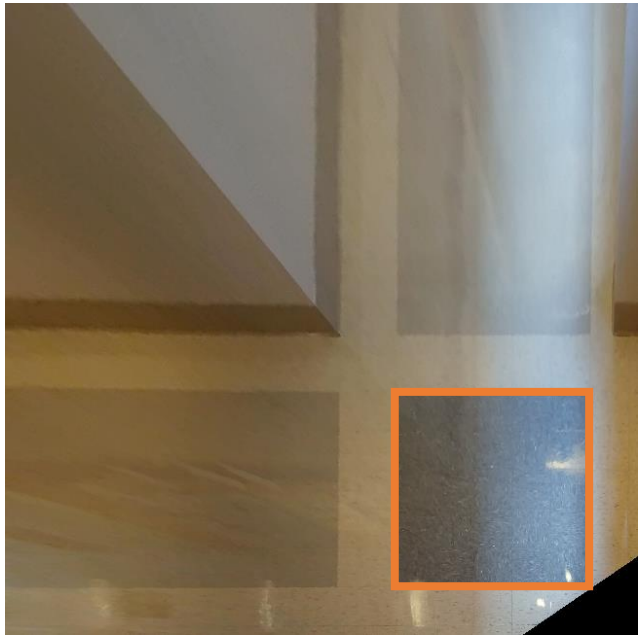
$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

Invariant properties

- Parallelism
- Ratio of area
- Ratio of length

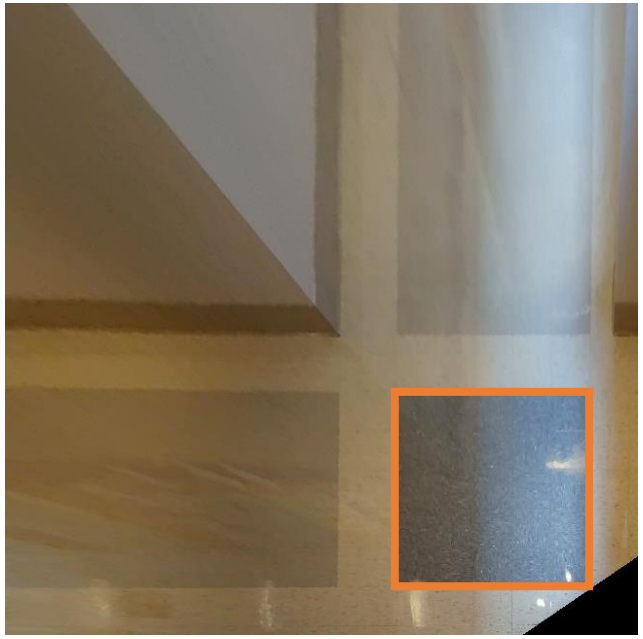
Degree of freedom

PERSPECTIVE TRANSFORM ~ HOMOGRAPHY



$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

PERSPECTIVE TRANSFORM ~ HOMOGRAPHY



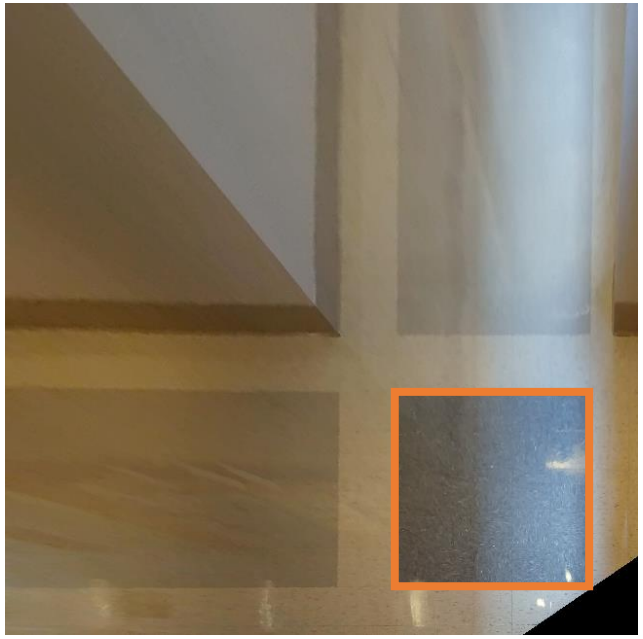
$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

$$u_2 = \frac{h_{11}u_1 + h_{12}v_1 + h_{13}}{h_{31}u_1 + h_{32}v_1 + h_{33}}$$

$$v_2 = \frac{h_{21}u_1 + h_{22}v_1 + h_{23}}{h_{31}u_1 + h_{32}v_1 + h_{33}}$$

: General form of plane to plane linear mapping

PERSPECTIVE TRANSFORM ~ HOMOGRAPHY



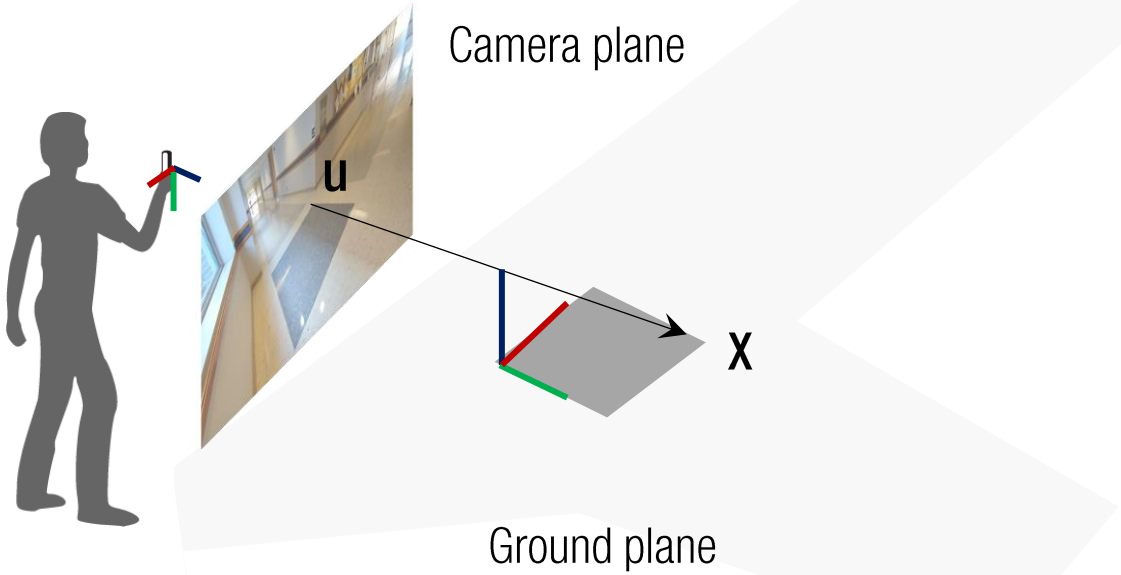
$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

$$u_2 = \frac{h_{11}u_1 + h_{12}v_1 + h_{13}}{h_{31}u_1 + h_{32}v_1 + 1}$$

$$v_2 = \frac{h_{21}u_1 + h_{22}v_1 + h_{23}}{h_{31}u_1 + h_{32}v_1 + 1}$$

: General form of plane to plane linear mapping

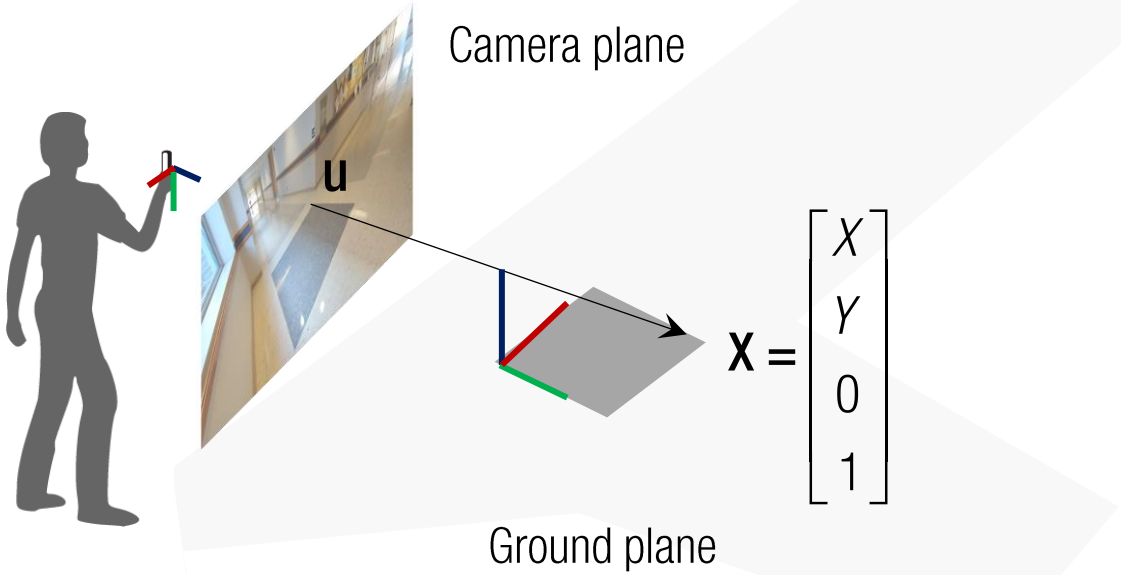
Homography I: 3D Plane-to-Image



$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

Camera plane Ground plane

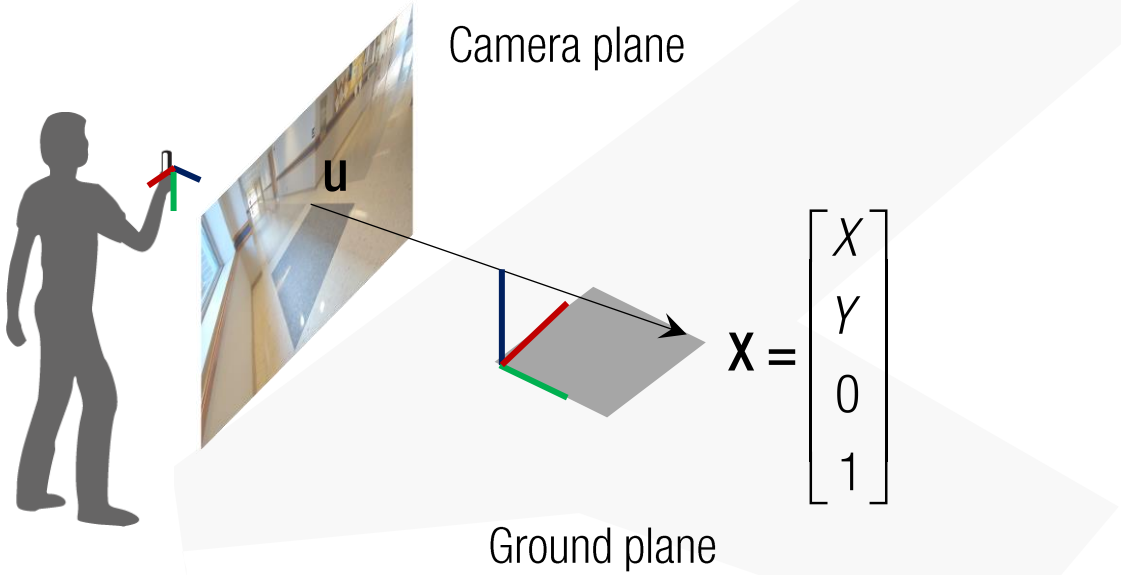
Homography I: 3D Plane-to-Image



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda \underbrace{\mathbf{u}}_{\text{Camera plane}} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \underbrace{\mathbf{X}}_{\text{Ground plane}}$$

Homography I: 3D Plane-to-Image

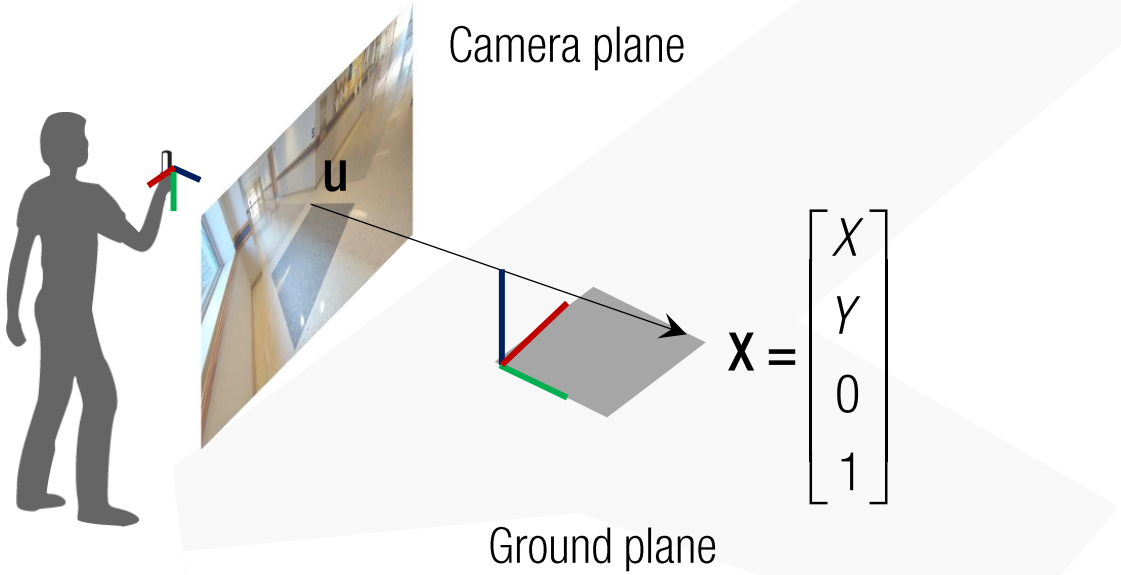


$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\underbrace{\lambda \mathbf{u}}_{\text{Camera plane}} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\underbrace{\mathbf{X}}_{\text{Ground plane}}$$

$$\longrightarrow \lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

Homography I: 3D Plane-to-Image



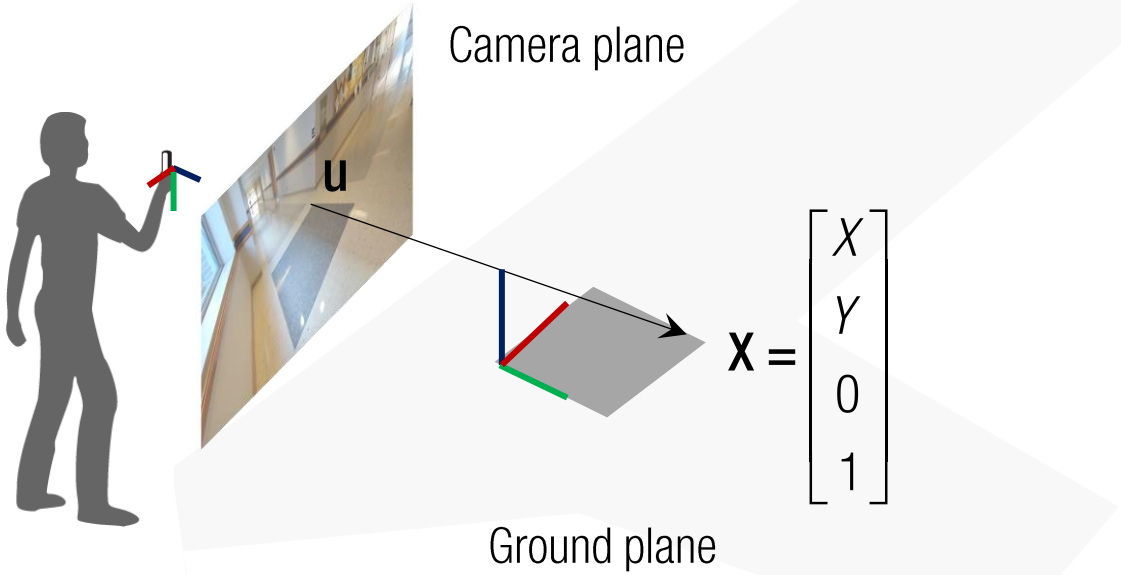
$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\underbrace{\lambda \mathbf{u}}_{\text{Camera plane}} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\underbrace{\mathbf{X}}_{\text{Ground plane}}$$

$$\longrightarrow \lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t}\end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t}\end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Homography I: 3D Plane-to-Image



$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

Camera plane Ground plane

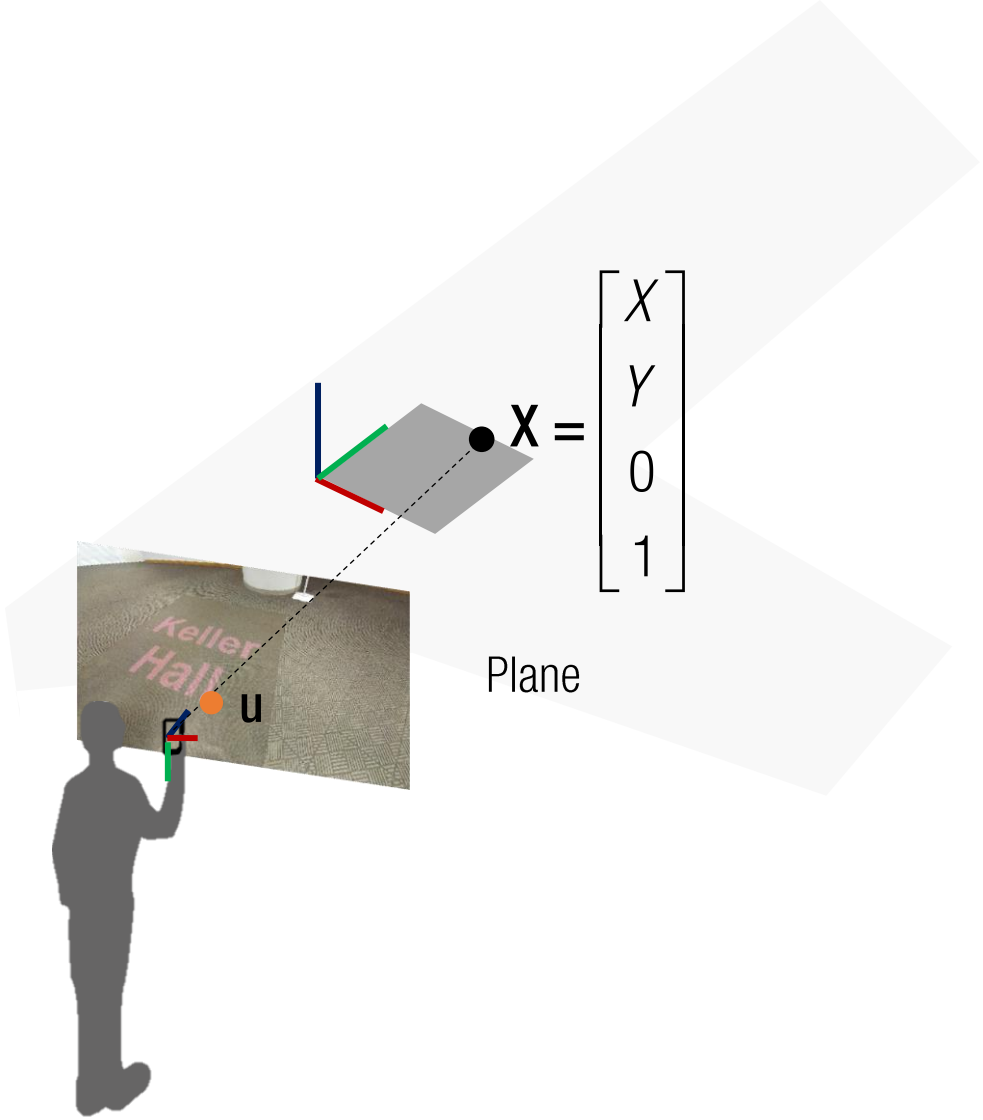
$$\longrightarrow \lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t}\end{bmatrix}\begin{bmatrix}X\\Y\\0\\1\end{bmatrix}$$

$$\longrightarrow \lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t}\end{bmatrix}\begin{bmatrix}X\\Y\\1\end{bmatrix}$$

$$\longrightarrow \lambda\begin{bmatrix}u_x\\u_y\\1\end{bmatrix} = \mathbf{H}\begin{bmatrix}X\\Y\\1\end{bmatrix}$$

Homography II: Image-to-Image through 3D Plane

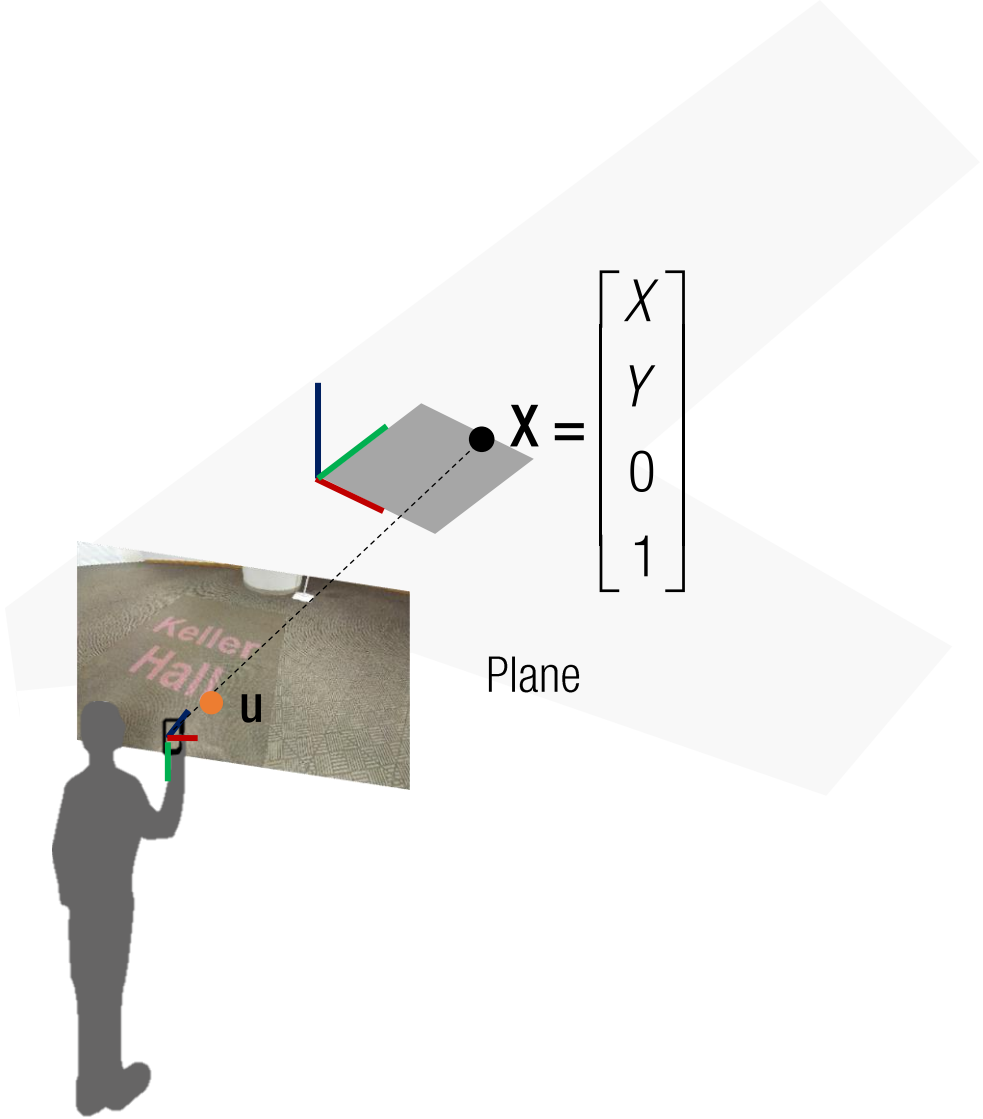
$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$



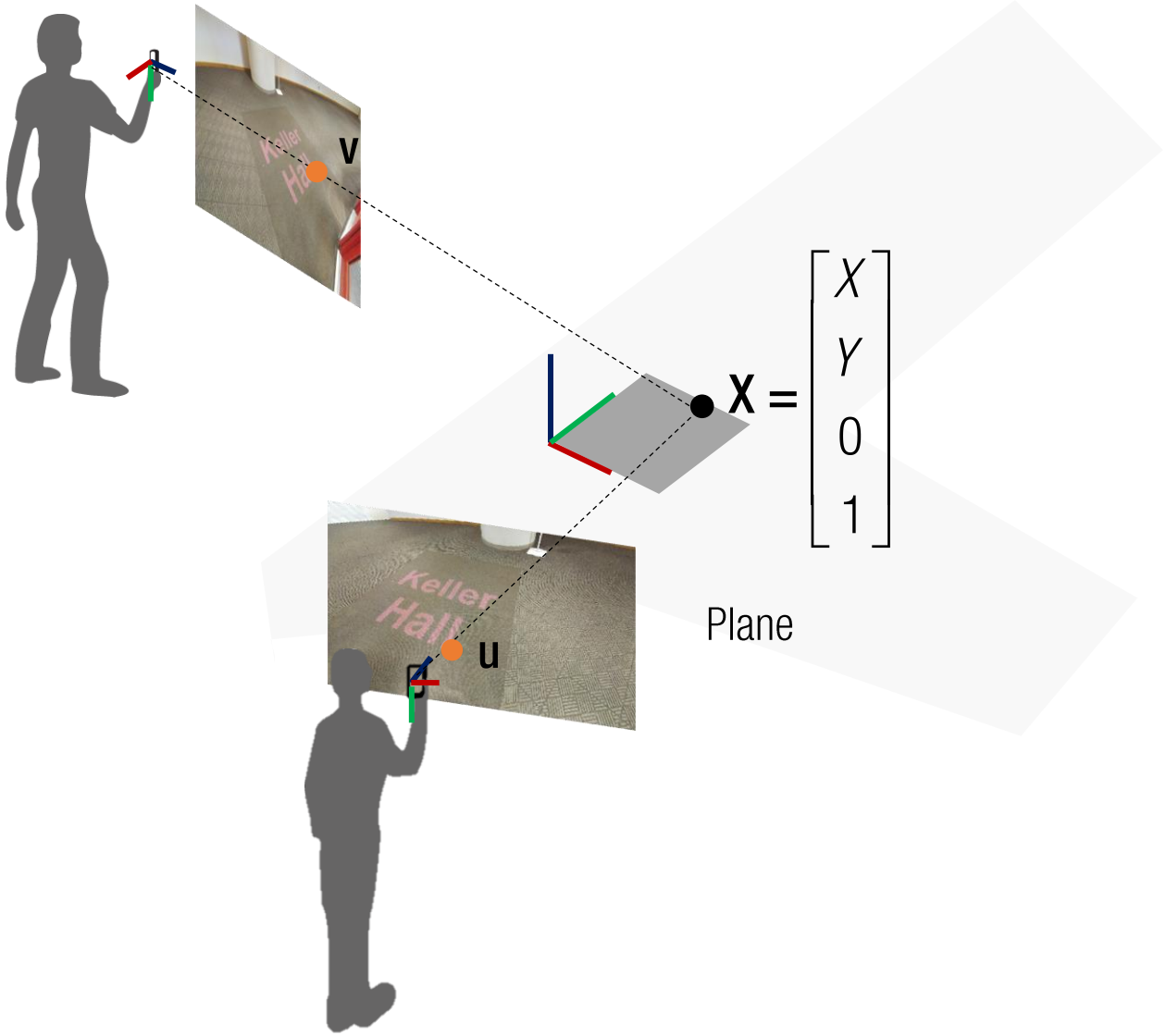
Homography II: Image-to-Image through 3D Plane

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\longrightarrow \quad \lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t}\end{bmatrix}\begin{bmatrix}X \\ Y \\ 1\end{bmatrix}$$



Homography II: Image-to-Image through 3D Plane



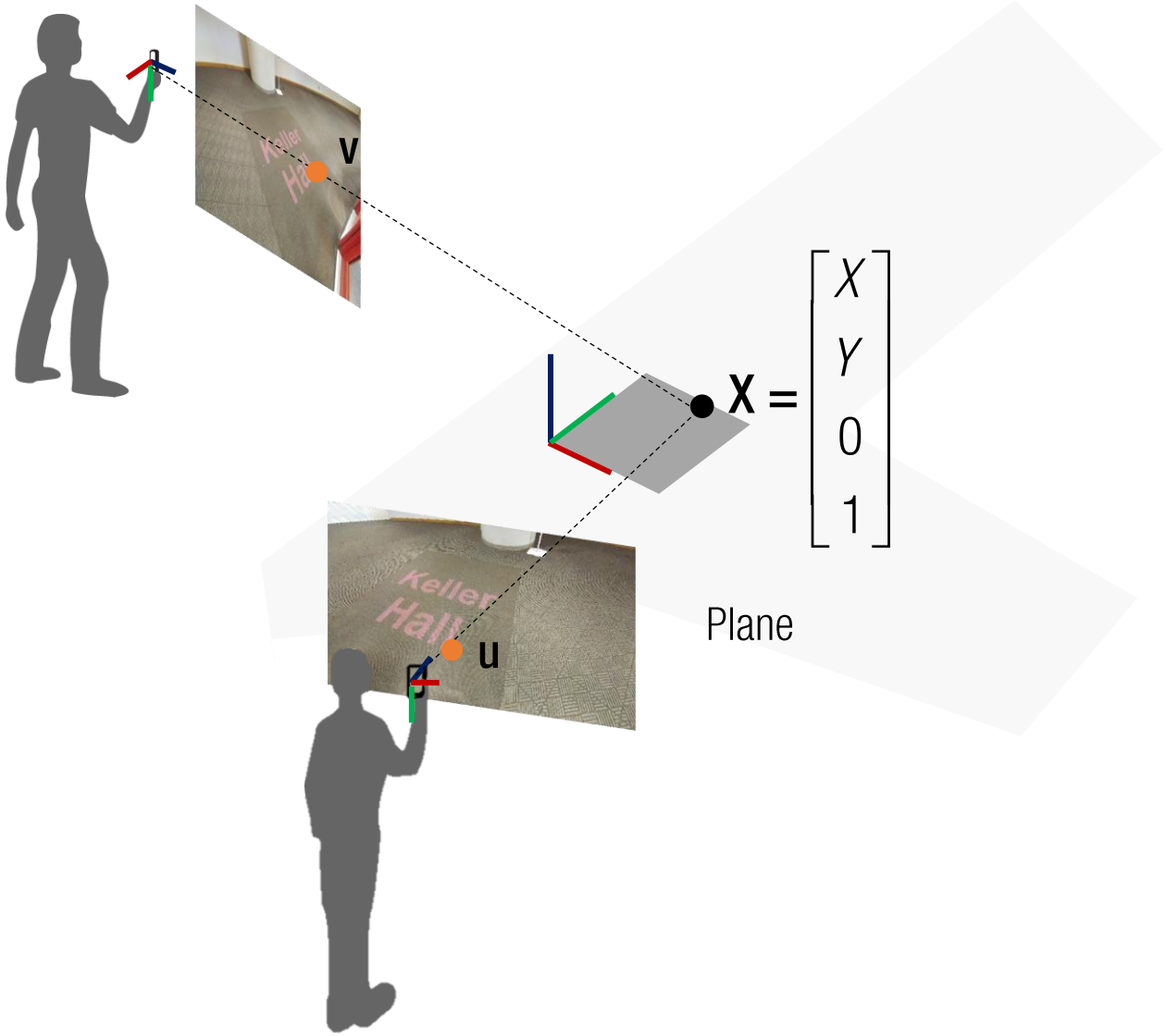
$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\mathbf{X}$$

$$\longrightarrow \quad \lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mu \mathbf{v} = \mathbf{K}'\begin{bmatrix} \mathbf{r}_1' & \mathbf{r}_2' & \mathbf{t}' \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Homography II: Image-to-Image through 3D Plane



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

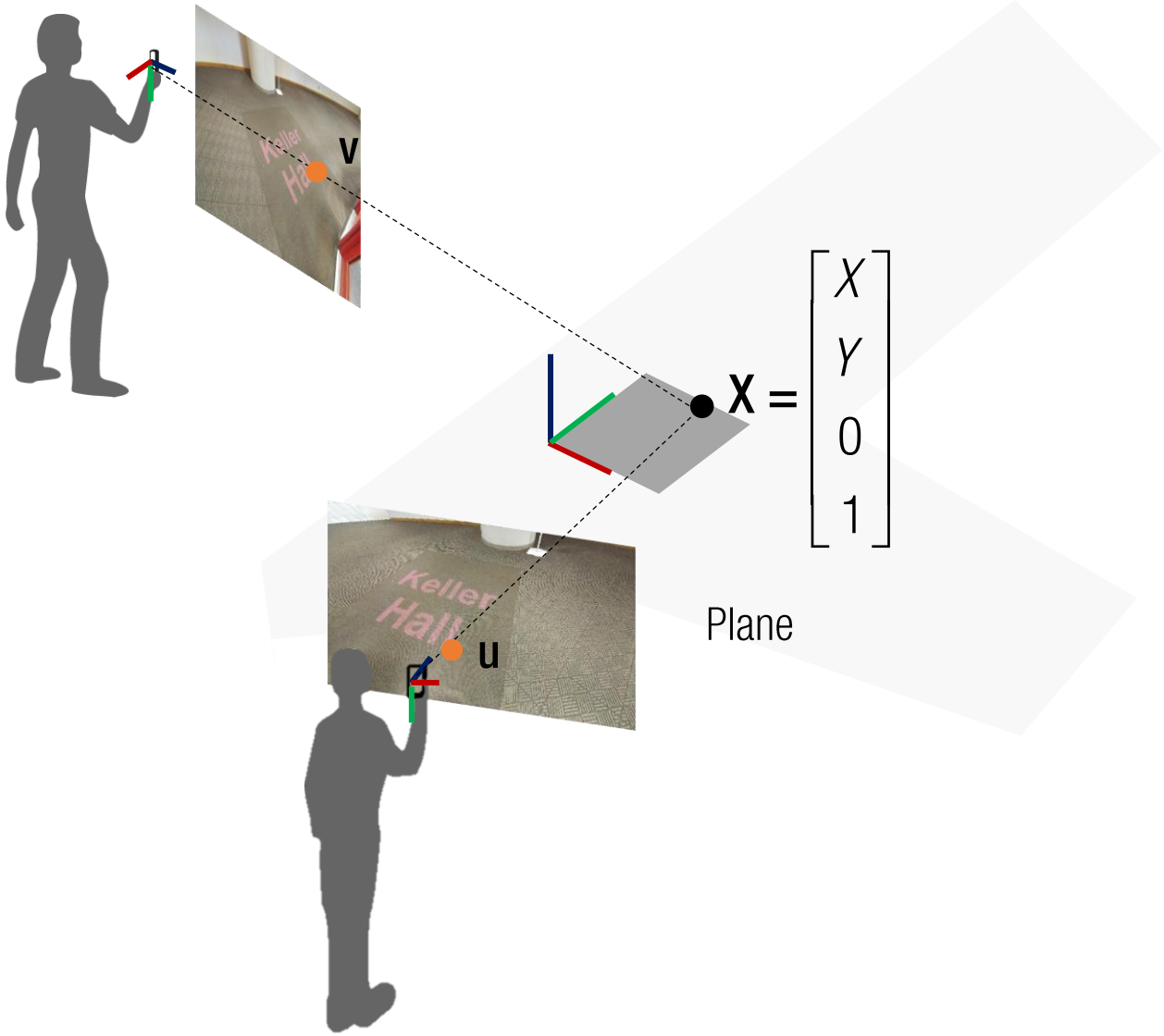
$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\mathbf{X}$$

$$\longrightarrow \quad \lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mu \mathbf{v} = \mathbf{K}'\begin{bmatrix} \mathbf{r}_1' & \mathbf{r}_2' & \mathbf{t}' \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

How are two image coordinates (**u**,**v**) related?

Homography II: Image-to-Image through 3D Plane



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\mathbf{X}$$

$$\longrightarrow \quad \lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mu \mathbf{v} = \mathbf{K}'\begin{bmatrix} \mathbf{r}_1' & \mathbf{r}_2' & \mathbf{t}' \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

How are two image coordinates (**u**,**v**) related?

$$\longrightarrow \quad \lambda \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}^{-1}\mathbf{K}^{-1}\mathbf{u} = \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mu \begin{bmatrix} \mathbf{r}_1' & \mathbf{r}_2' & \mathbf{t}' \end{bmatrix}^{-1}\mathbf{K}'^{-1}\mathbf{v}$$

$$\alpha \mathbf{v} = \mathbf{K}'\begin{bmatrix} \mathbf{r}_1' & \mathbf{r}_2' & \mathbf{t}' \end{bmatrix}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}^{-1}\mathbf{K}^{-1}\mathbf{u}$$

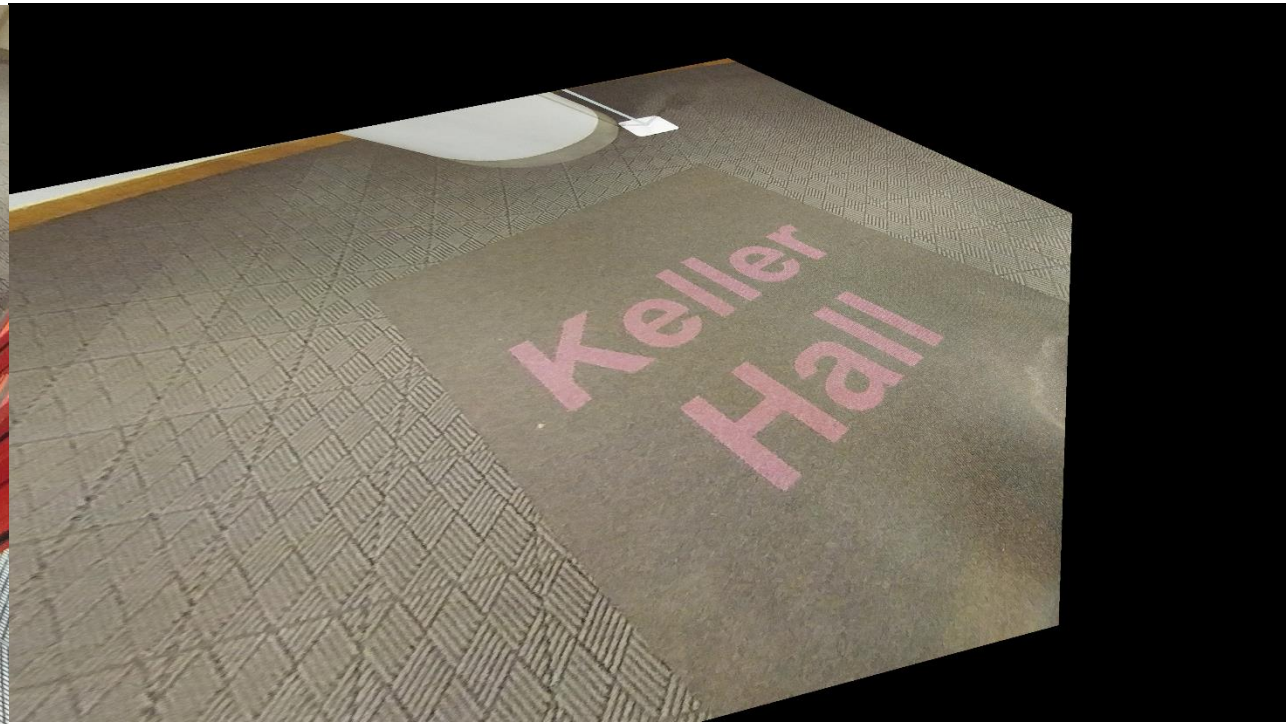
$$\alpha \mathbf{v} = \mathbf{H}\mathbf{u}$$

Homography II: Image-to-Image through 3D Plane

Keller entrance left

Keller entrance right

Keller entrance left



Right image to left

Keller
Hall

Left image to right

Right image to left

Keller
Hall

Homography II: Image-to-Image through 3D Plane

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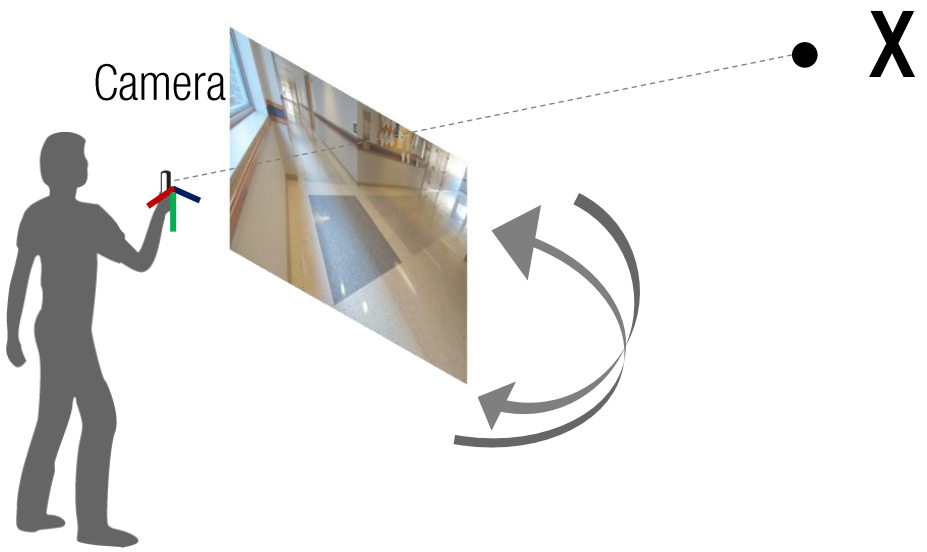
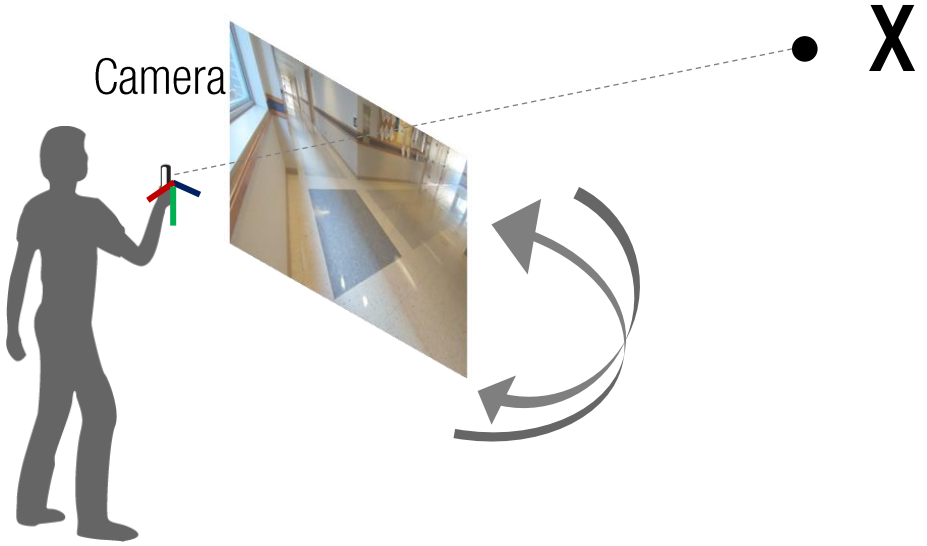
Homography III: 3D Pure Rotation

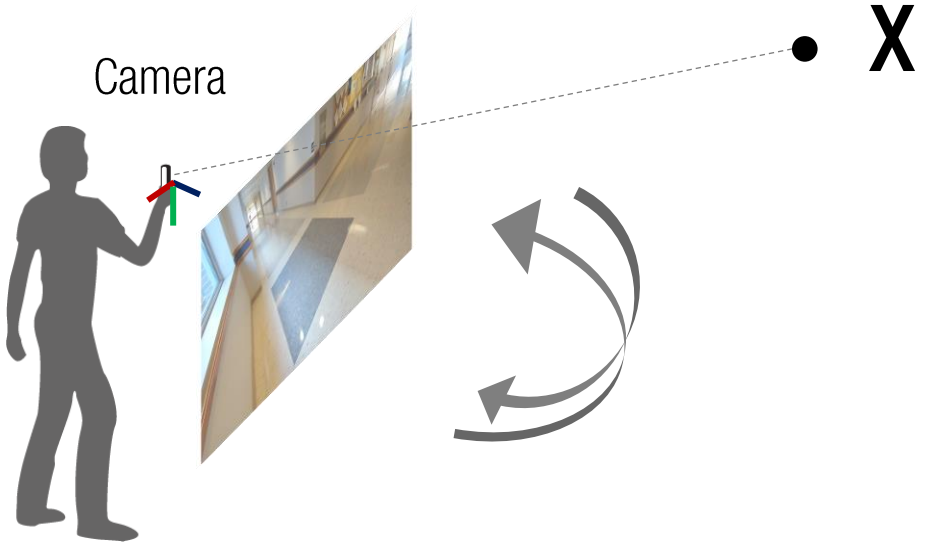


Homography III: 3D Pure Rotation



$$\lambda_1 \mathbf{u} = \mathbf{K}\mathbf{X}$$

Homography III: 3D Pure Rotation



$$\lambda_1 \mathbf{u} = \mathbf{K}\mathbf{X}$$

$$\lambda_2 \mathbf{v} = \mathbf{K}\mathbf{R}\mathbf{X}$$

Homography III: 3D Pure Rotation

$$\lambda_1 \mathbf{u} = \mathbf{K}\mathbf{X}$$

$$\lambda_2 \mathbf{v} = \mathbf{K}\mathbf{R}\mathbf{X}$$

$$\longrightarrow \quad \mathbf{X} = \lambda_1 \mathbf{K}^{-1}\mathbf{u} = \lambda_2 \mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v}$$

Homography III: 3D Pure Rotation

$$\lambda_1 \mathbf{u} = \mathbf{K}\mathbf{X}$$

$$\lambda_2 \mathbf{v} = \mathbf{K}\mathbf{R}\mathbf{X}$$

$$\longrightarrow \mathbf{X} = \lambda_1 \mathbf{K}^{-1}\mathbf{u} = \lambda_2 \mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v}$$

$$\longrightarrow \lambda \mathbf{v} = \mathbf{K}\mathbf{R}\mathbf{K}^{-1}\mathbf{u}$$

Homography III: 3D Pure Rotation

$$\lambda_1 \mathbf{u} = \mathbf{K}\mathbf{X}$$

$$\lambda_2 \mathbf{v} = \mathbf{K}\mathbf{R}\mathbf{X}$$

$$\longrightarrow \mathbf{X} = \lambda_1 \mathbf{K}^{-1}\mathbf{u} = \lambda_2 \mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v}$$

$$\longrightarrow \lambda \mathbf{v} = \mathbf{K}\mathbf{R}\mathbf{K}^{-1}\mathbf{u}$$

$$\longrightarrow \mathbf{H} = \mathbf{K}\mathbf{R}\mathbf{K}^{-1}$$

Homography III: 3D Pure Rotation

$$\lambda_1 \mathbf{u} = \mathbf{K}\mathbf{X}$$

$$\lambda_2 \mathbf{v} = \mathbf{K}\mathbf{R}\mathbf{X}$$

$$\longrightarrow \mathbf{X} = \lambda_1 \mathbf{K}^{-1}\mathbf{u} = \lambda_2 \mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v}$$

$$\longrightarrow \lambda \mathbf{v} = \mathbf{K}\mathbf{R}\mathbf{K}^{-1}\mathbf{u}$$

$$\longrightarrow \mathbf{H} = \mathbf{K}\mathbf{R}\mathbf{K}^{-1}$$

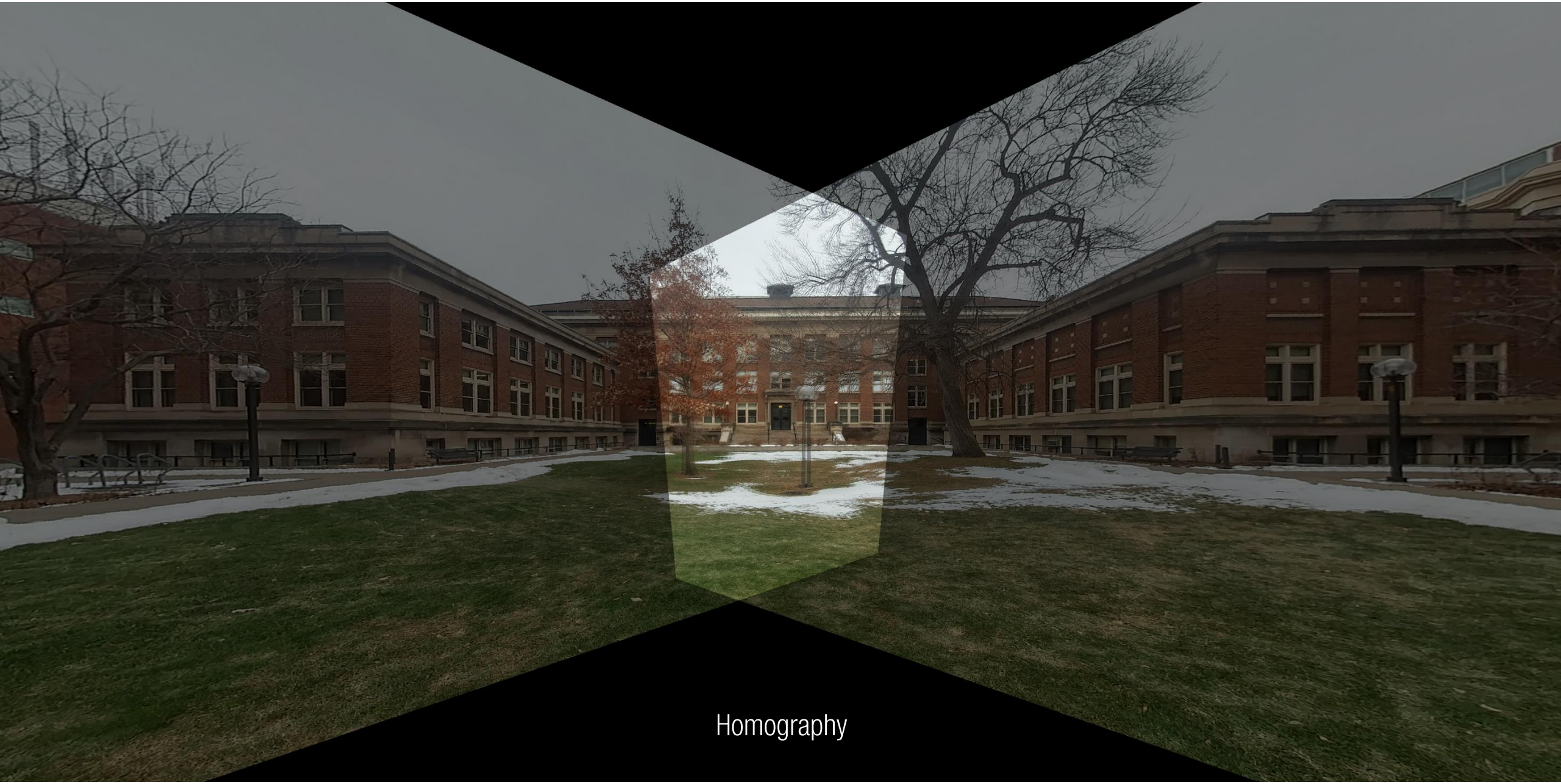
$$\longrightarrow \mathbf{R} = \mathbf{K}^{-1}\mathbf{H}\mathbf{K}$$

$$\lambda \mathbf{u} = \mathbf{H}\mathbf{v}$$

Lind Hall Left

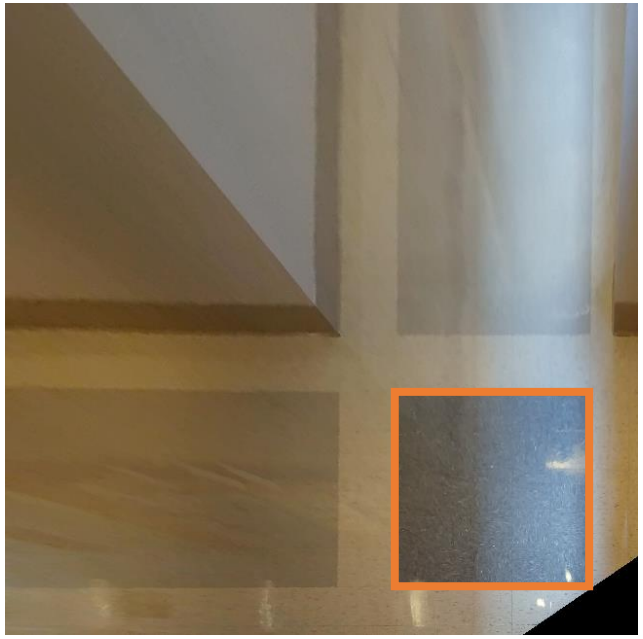
Lind Hall Right

Euclidean Transform (Translation)


Homography

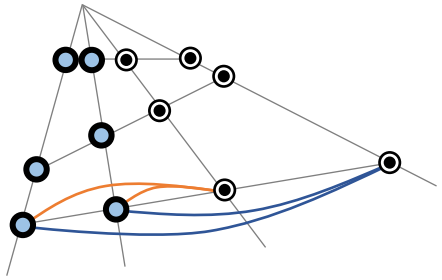
Why misalignment?
Homography

Perspective Transform ~ Homography



Invariant properties

- Cross ratio

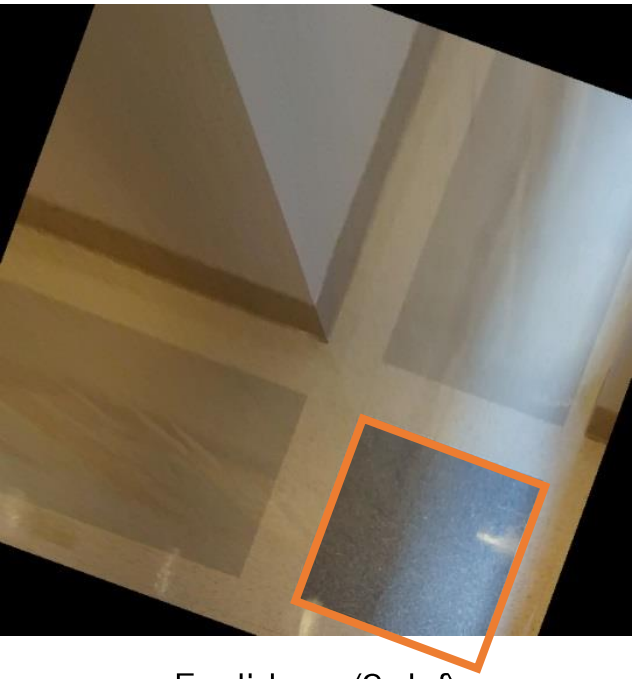


- Concurrency
- Colinearity

Degree of freedom

8 (9 variables – 1 scale)

Hierachy of Transformations



Euclidean (3 dof)

- Length
- Angle
- Area

$$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix}$$

Similarity (4 dof)

- Length ratio
- Angle

$$\begin{bmatrix} \alpha\cos\theta & -\alpha\sin\theta & t_x \\ \alpha\sin\theta & \alpha\cos\theta & t_y \\ & & 1 \end{bmatrix}$$

Affine (6 dof)

- Parallelism
- Ratio of area
- Ratio of length

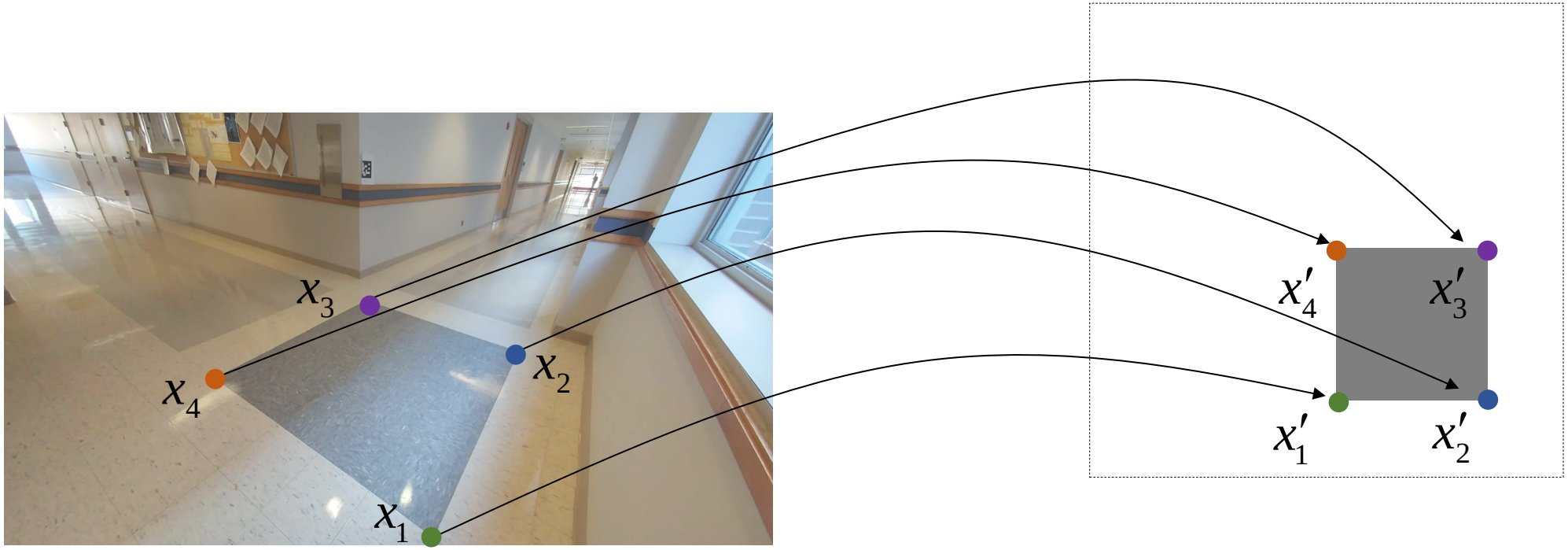
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Projective (8 dof)

- Cross ratio
- Concurrency
- Colinearity

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

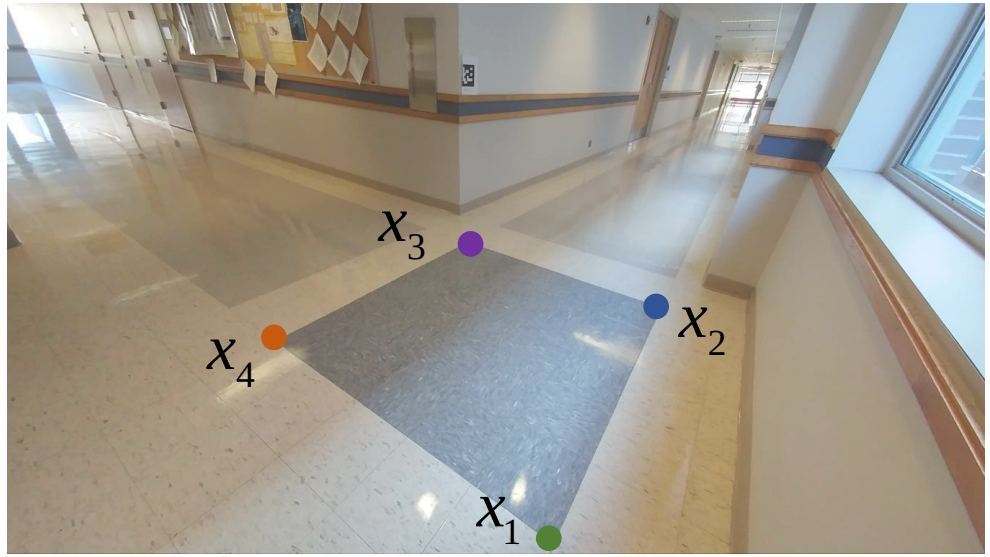
Homography Computation

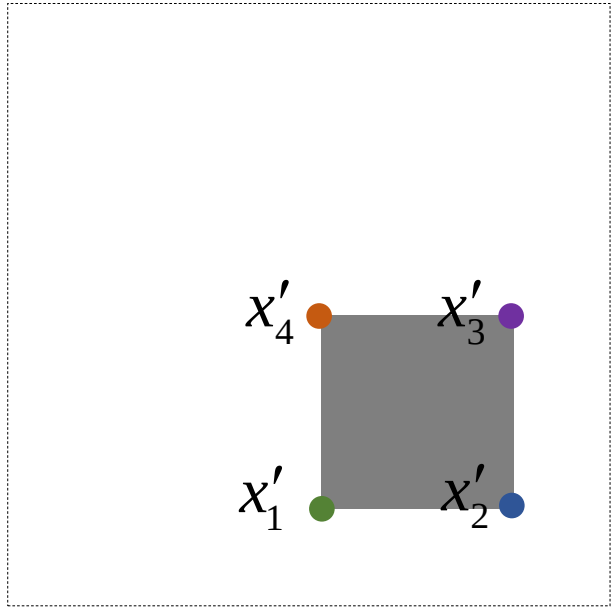


The image can be rectified as if it is seen from top view.

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

Homography Computation



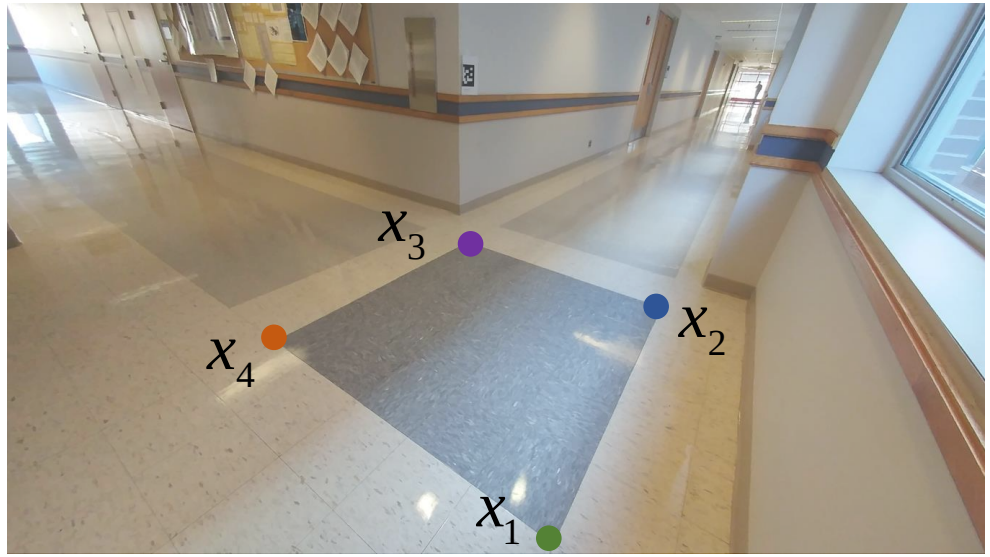


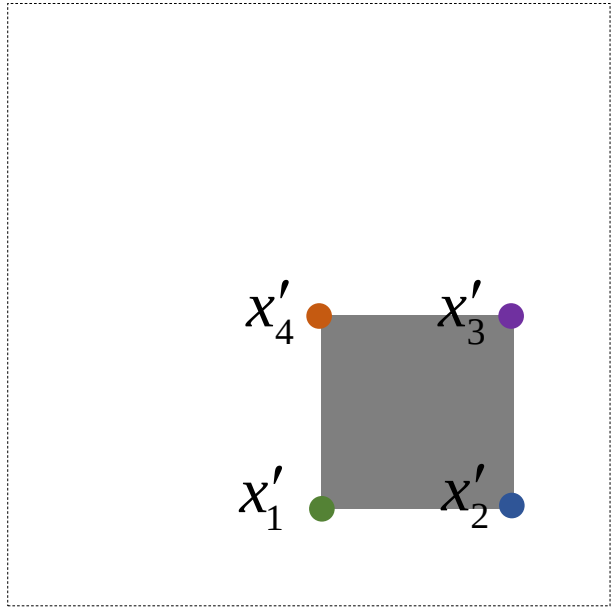
$$u'_1 = \frac{h_{11}u_1 + h_{12}v_1 + h_{13}}{h_{31}u_1 + h_{32}v_1 + 1}$$

$$v'_1 = \frac{h_{21}u_1 + h_{22}v_1 + h_{23}}{h_{31}u_1 + h_{32}v_1 + 1}$$

$$\longrightarrow \quad \begin{aligned} h_{11}u_1 + h_{12}v_1 + h_{13} - h_{31}u_1u'_1 - h_{32}v_1u'_1 - u'_1 &= 0 \\ h_{21}u_1 + h_{22}v_1 + h_{23} - h_{31}u_1v'_1 - h_{32}v_1v'_1 - v'_1 &= 0 \end{aligned}$$

Homography Computation



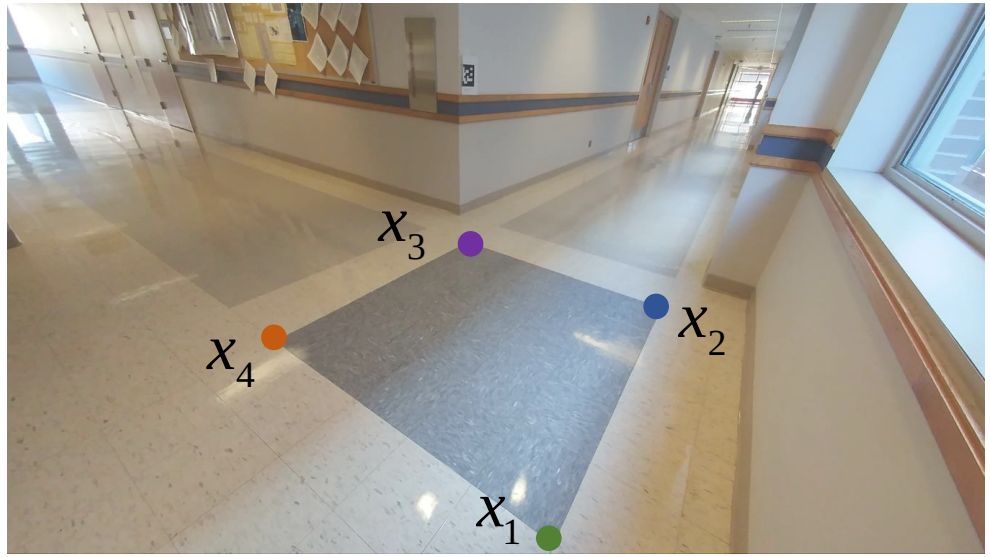


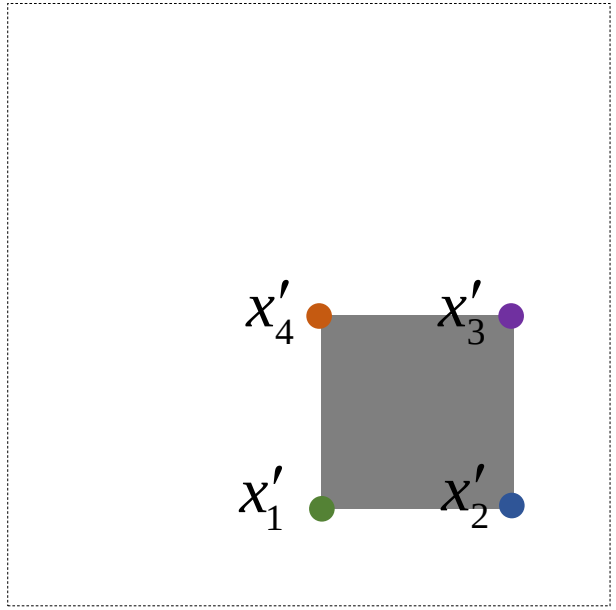
$$\begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1u' & -v_1u' \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1v' & -v_1v' \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} = \begin{bmatrix} u'_1 \\ v'_1 \end{bmatrix}$$

# of equations: 2
# of unknows: 8

How many correspondences are needed?

Homography Computation

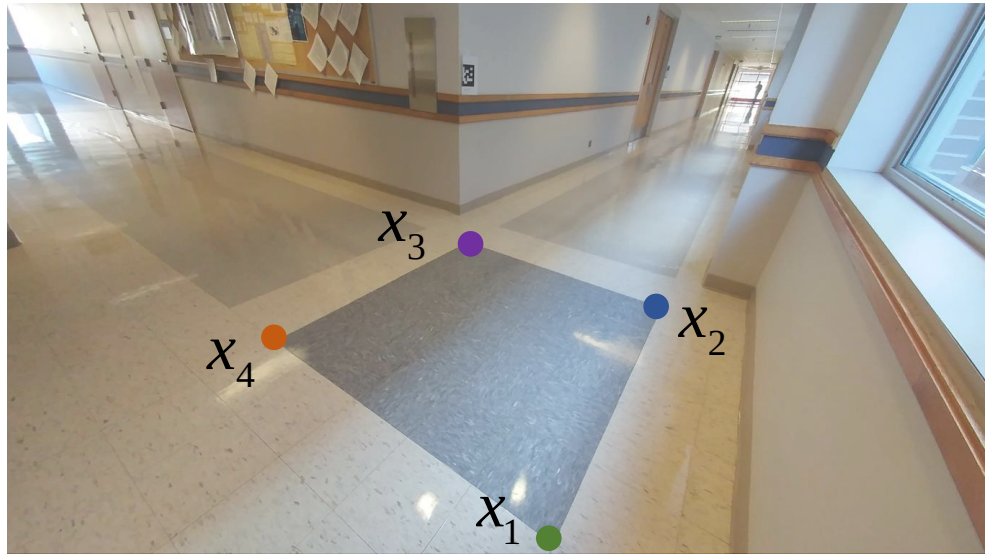


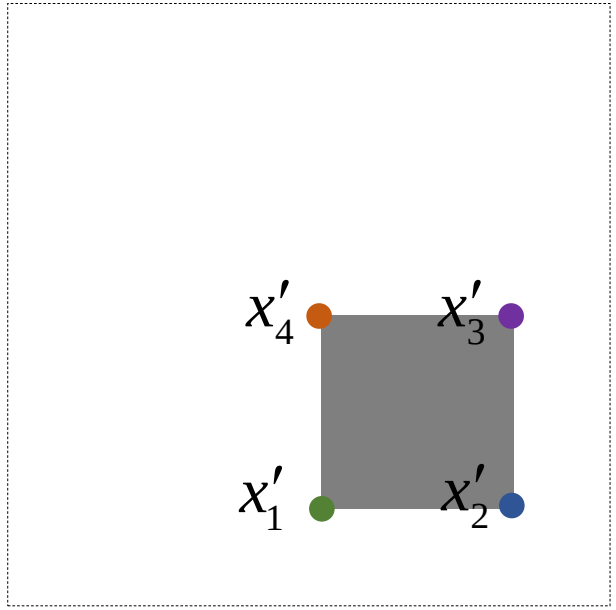


$$\begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1u'_1 & -v_1u'_1 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1v'_1 & -v_1v'_1 \\ & & & & \vdots & & & \\ u_4 & v_4 & 1 & 0 & 0 & 0 & -u_4u'_4 & -v_4u'_4 \\ 0 & 0 & 0 & u_4 & v_4 & 1 & -u_4v'_4 & -v_4v'_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} = \begin{bmatrix} u'_1 \\ v'_1 \\ \vdots \\ u'_4 \\ v'_4 \end{bmatrix}$$

# of equations: 2
# of unknows: 8

Homography Computation





$$\underbrace{\begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1u'_1 & -v_1u'_1 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1v'_1 & -v_1v'_1 \\ & & & & \vdots & & & \\ u_4 & v_4 & 1 & 0 & 0 & 0 & -u_4u'_4 & -v_4u'_4 \\ 0 & 0 & 0 & u_4 & v_4 & 1 & -u_4v'_4 & -v_4v'_4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ \vdots \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} u'_1 \\ v'_1 \\ \vdots \\ u'_4 \\ v'_4 \end{bmatrix}}_{b}$$

$$Ax = b \longrightarrow x = \left(A^T A\right)^{-1} A^T b$$

IMAGE WARPING

$$x_2 = f(x_1) \qquad I_2(x_2) = I_1(f(x_1))$$

```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x1 = [j, i];
        x2 = floor(mapping(x1));
        I2(x2) = I1(x1);
    end
end
```

Image Warping

$$x_2 = f(x_1) \qquad I_2(x_2) = I_1(f(x_1))$$

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Image Warping

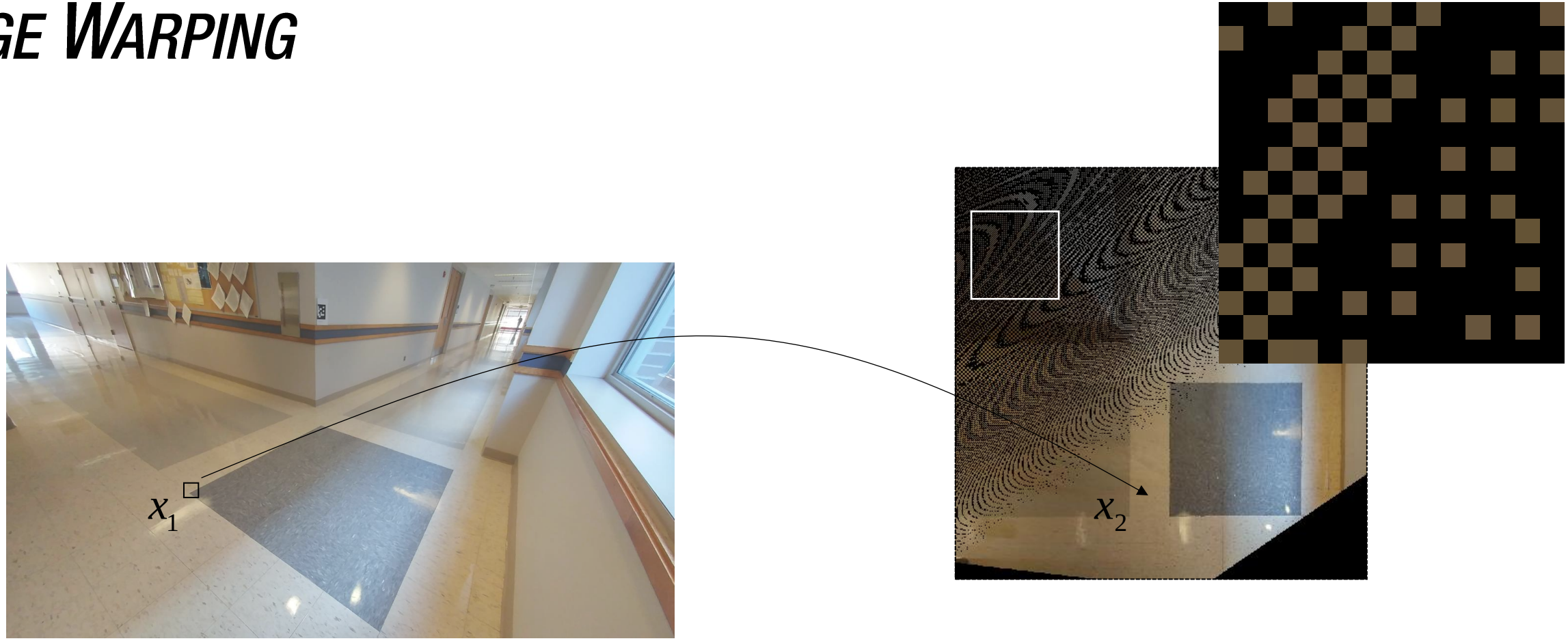

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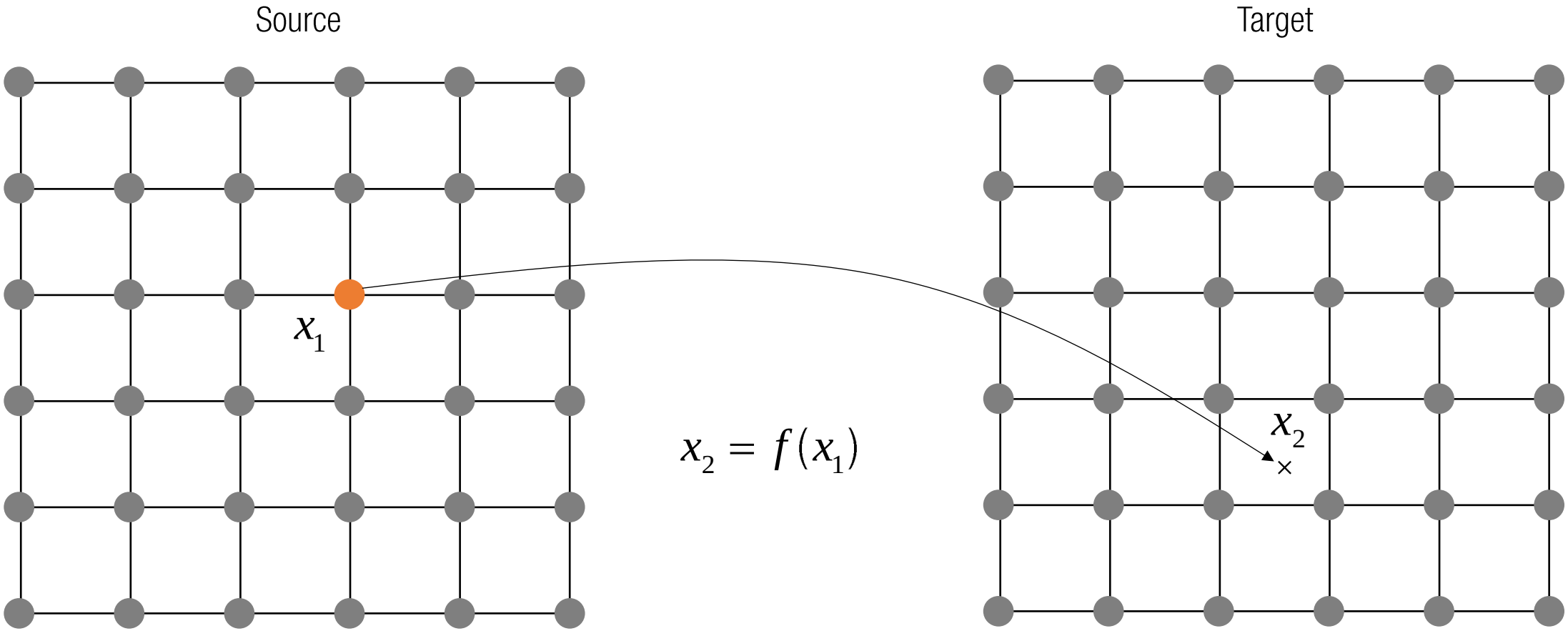
IMAGE WARPING



$$x_2 = f(x_1)$$

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    for j = 1 : size(im,2)
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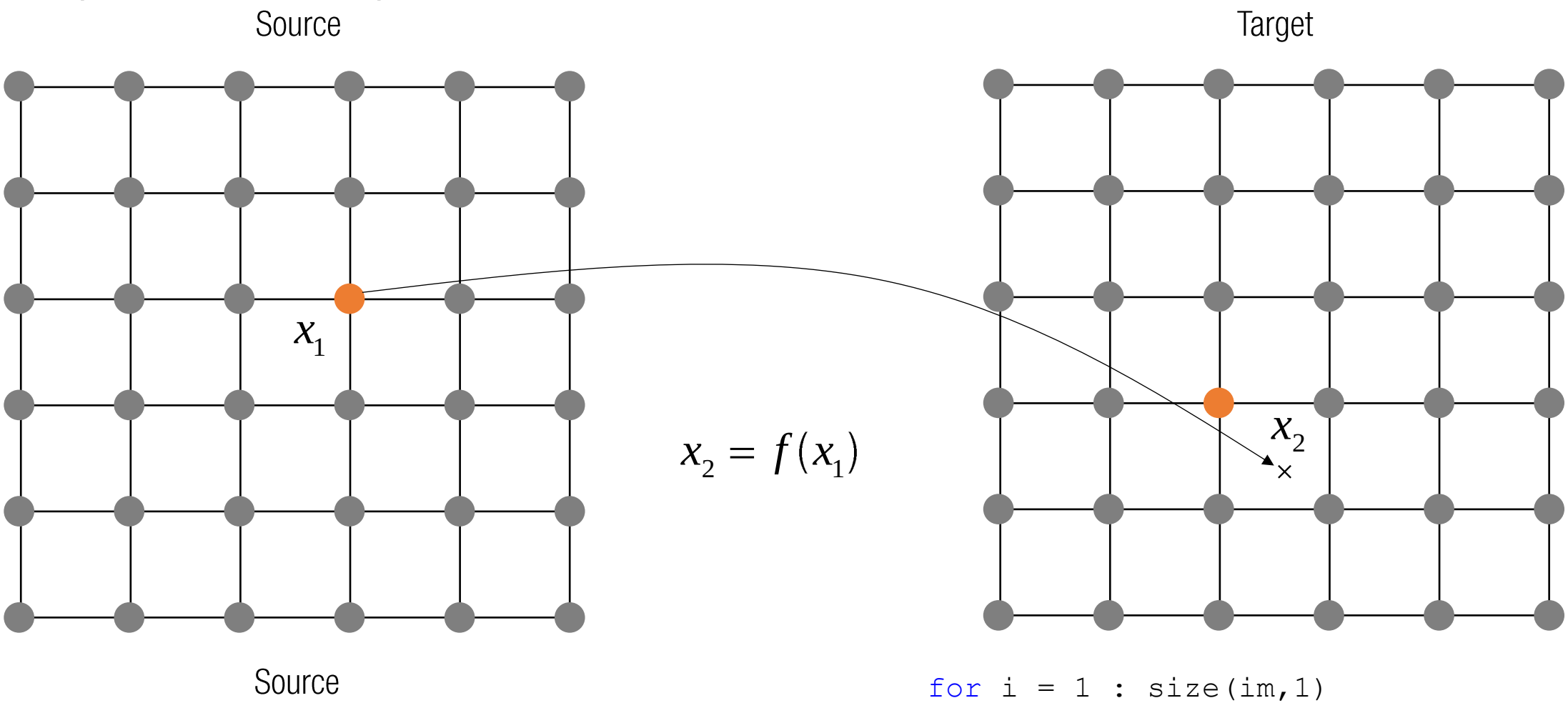
        x2 = floor(mapping(x1));
        I2(x2) = I1(x1);
    end
end
```

IMAGE WARPING

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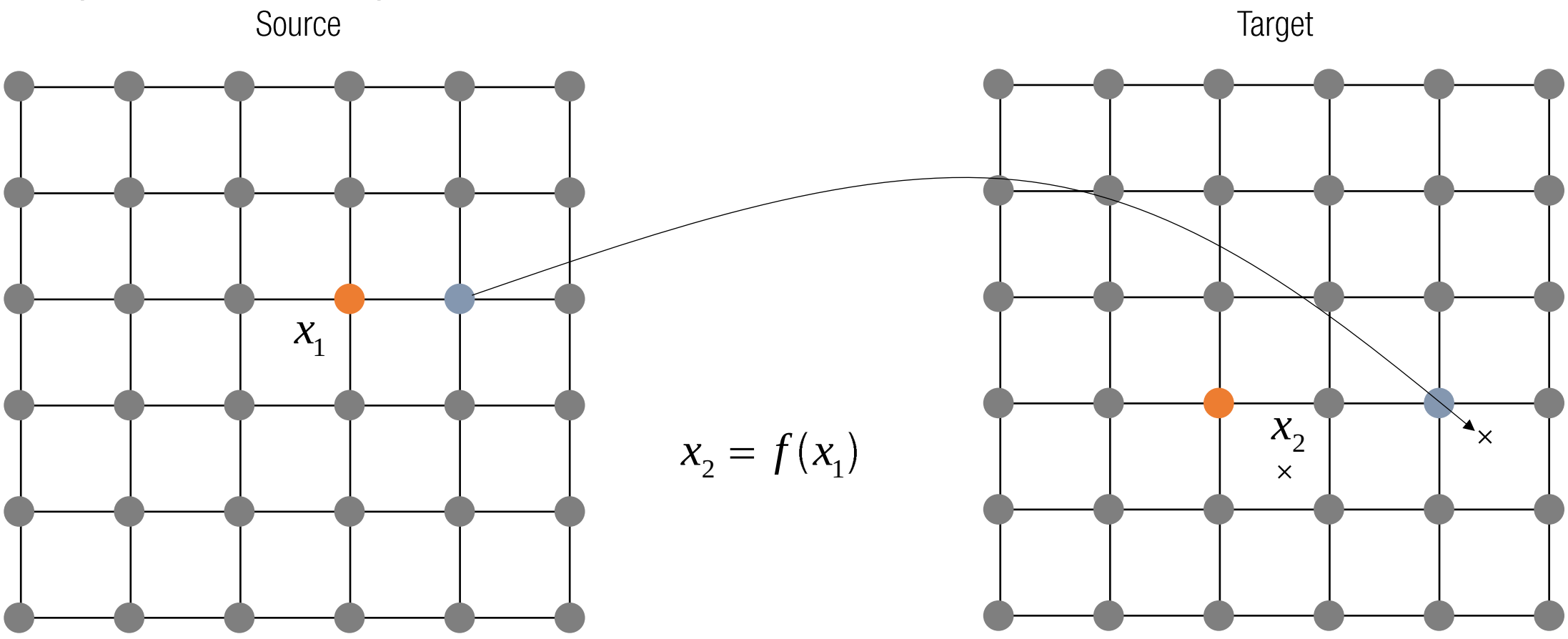
```

IMAGE WARPING

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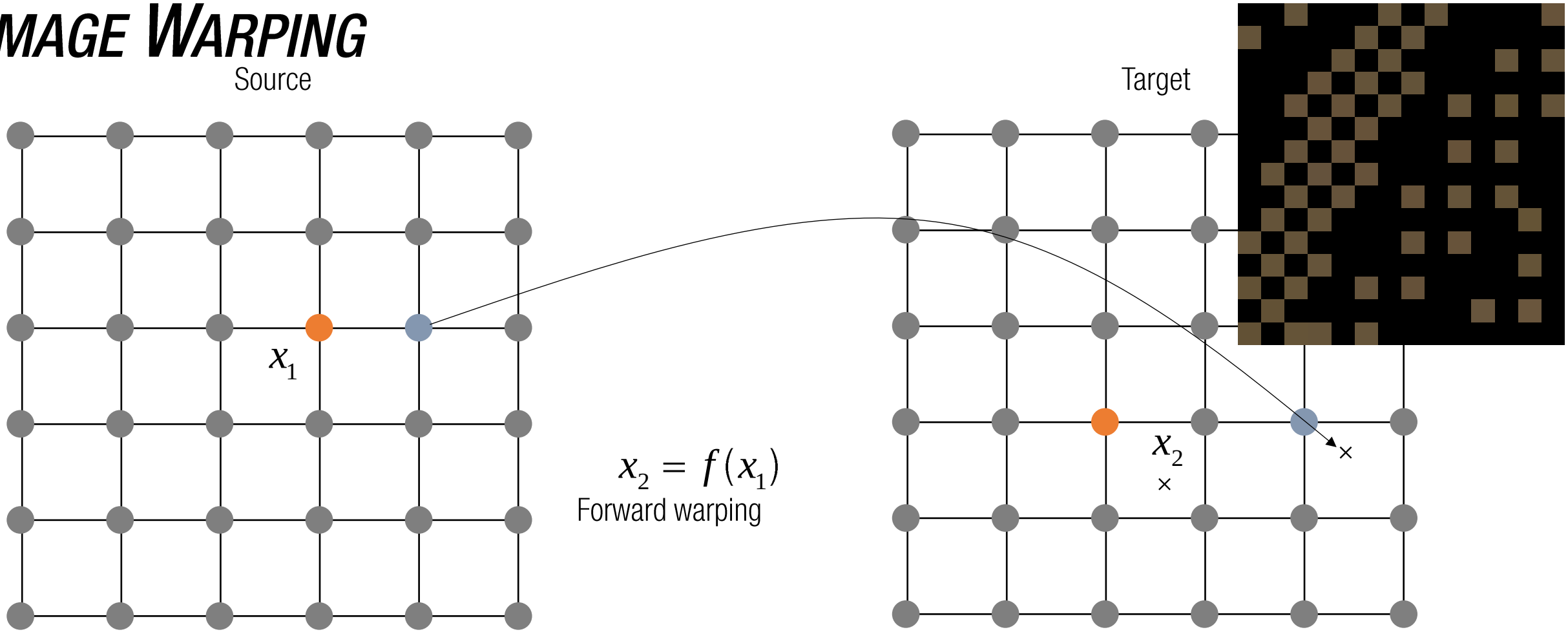
        I2(x2) = I1(x1);
    end
end
```

Image Warping




```
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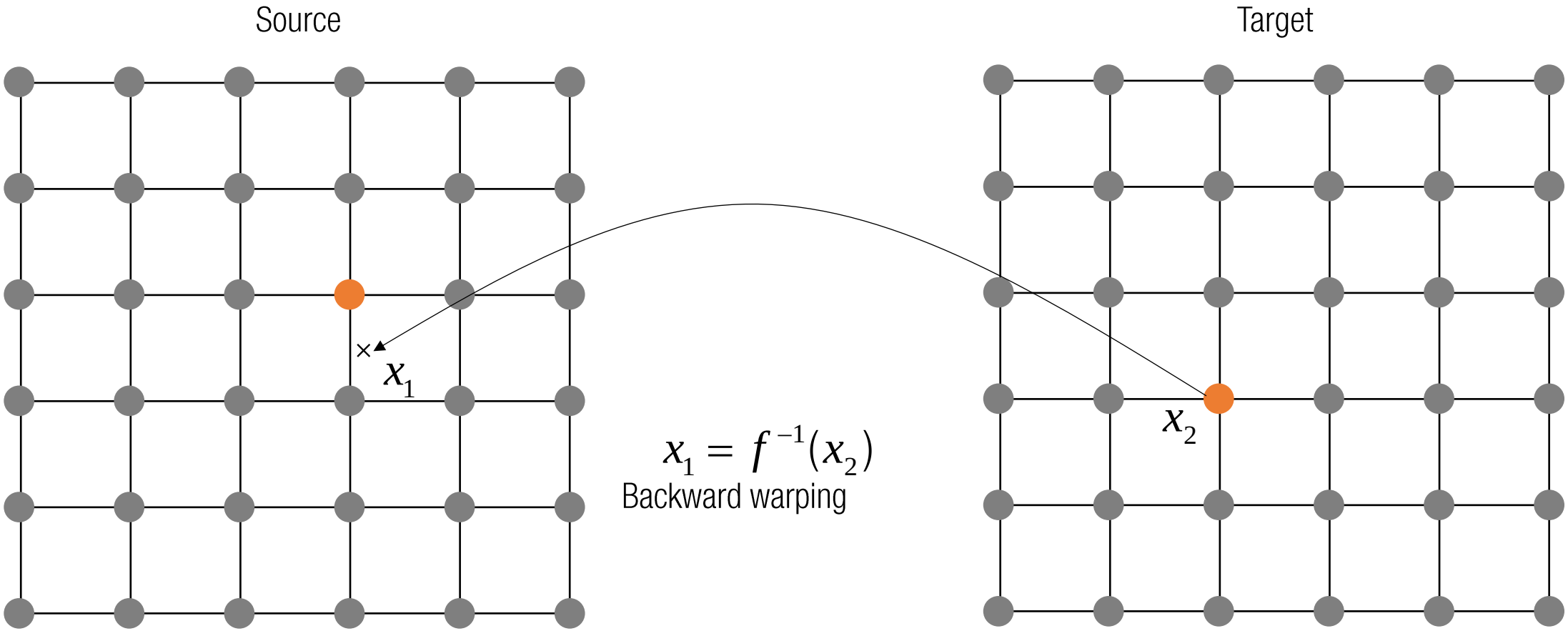
    for j = 1 : size(im,2)
        x1 = [j, i];
        x2 = floor(mapping(x1));
        I2(x2) = I1(x1);
    end
end
```

INVERSE WARPING

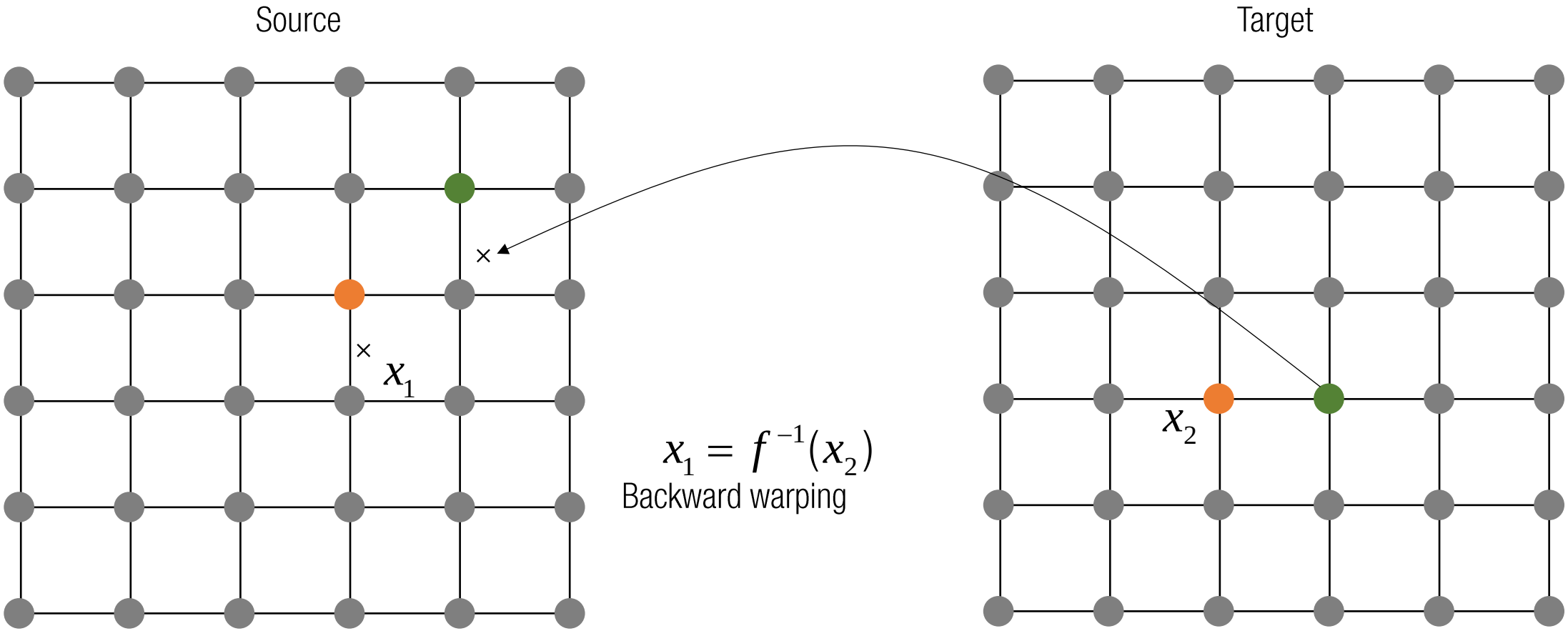


$$x_1 = f^{-1}(x_2)$$

Backward warping

```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x2 = [j, i];
        x1 = floor(inverse_mapping(x2));
        I2(x2) = I1(x1);
    end
end
```

Inverse Warping



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Backward warping

```
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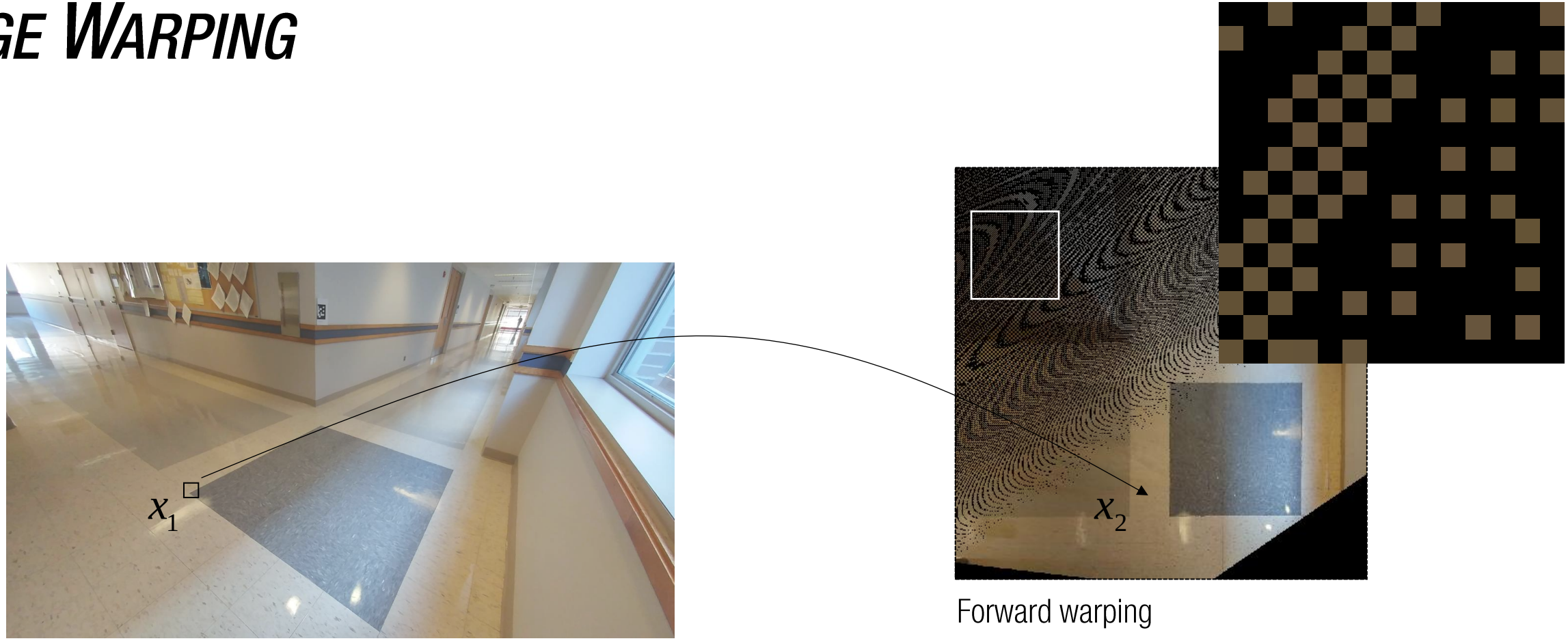
        x2 = [j, i];
        x1 = floor(inverse_mapping(x2));
        I2(x2) = I1(x1);
    end
end
```

Image Warping



Forward warping

$$x_2 = f(x_1) \qquad I_2(x_2) = I_1(f(x_1))$$

```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x1 = [j, i];
        x2 = floor(mapping(x1));
        I2(x2) = I1(x1);
    end
end
```

IMAGE WARPING

Backward warping

$$x_1 = f^{-1}(x_2) \qquad I_2(x_2) = I_1(f(x_1))$$

```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x2 = [j, i];
        x1 = floor(inverse_mapping(x2));
        I2(x2) = I1(x1);
    end
end
```

Image Warping

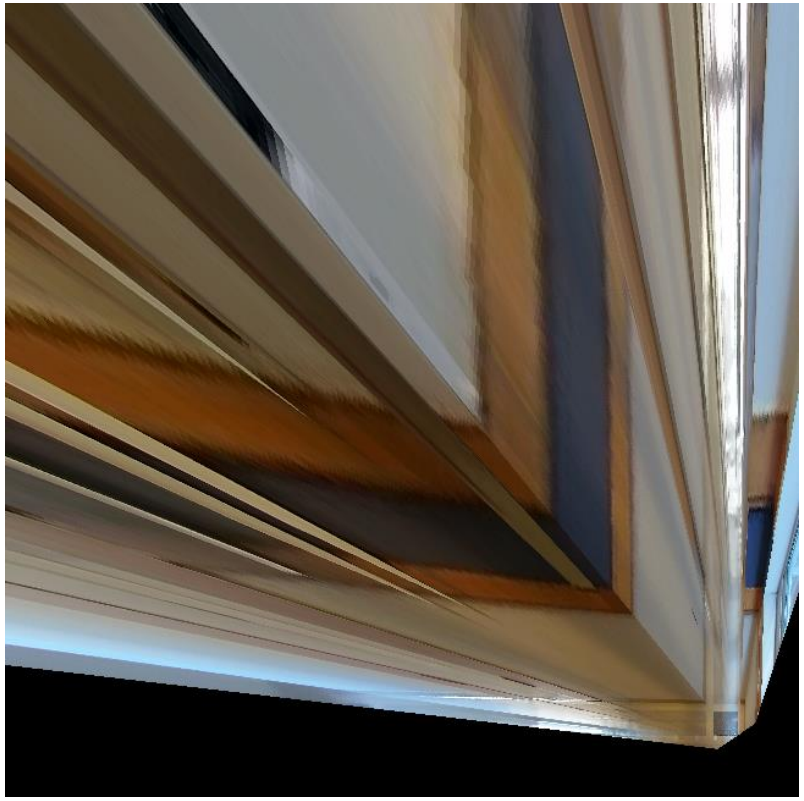


Image Warping

IMAGE WARPING

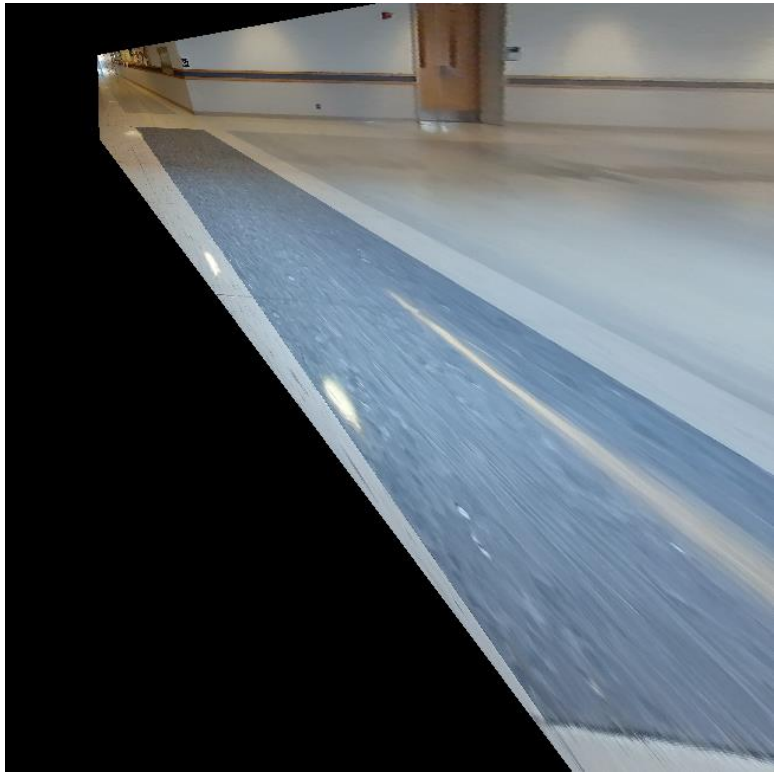


Image Panorama (Cylindrical Projection)

Image Panorama (Cylindrical Projection)

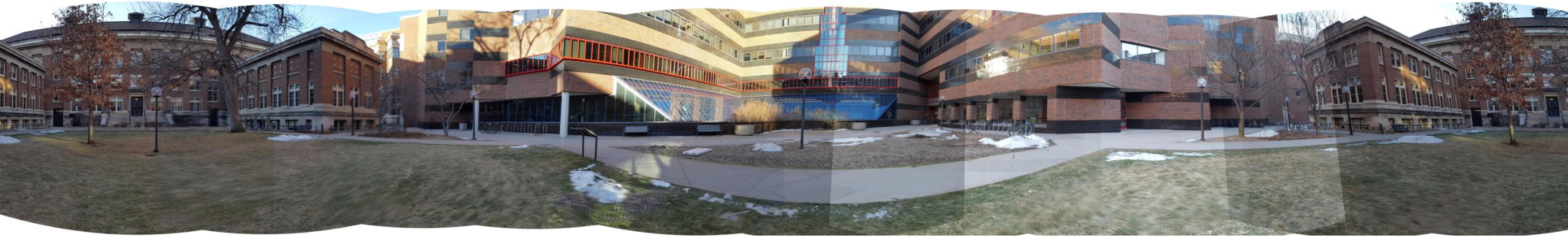
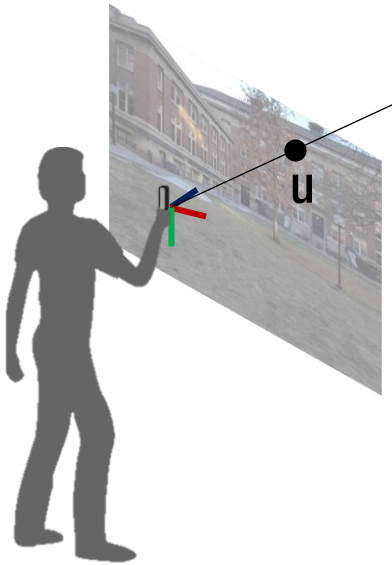


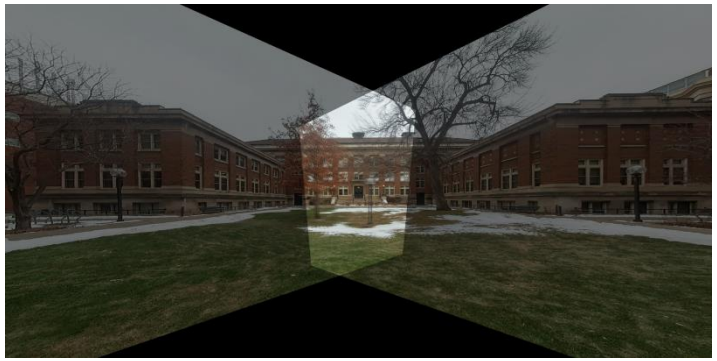
Image Panorama (Cylindrical Projection)

Image Panorama (Cylindrical Projection)

R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11}

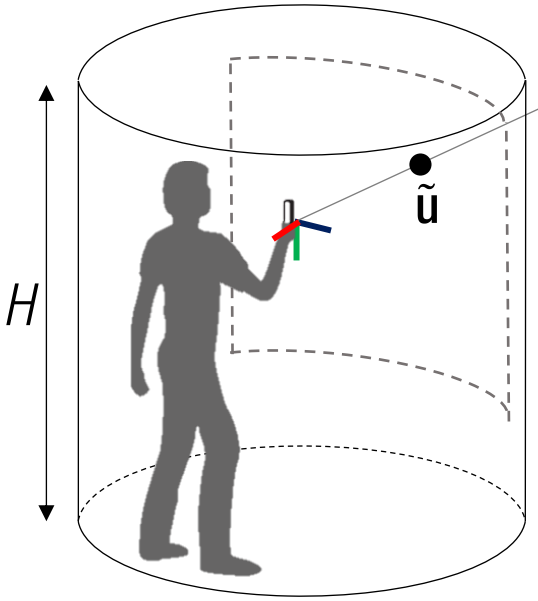


$$X = \lambda K^{-1} R^T u$$

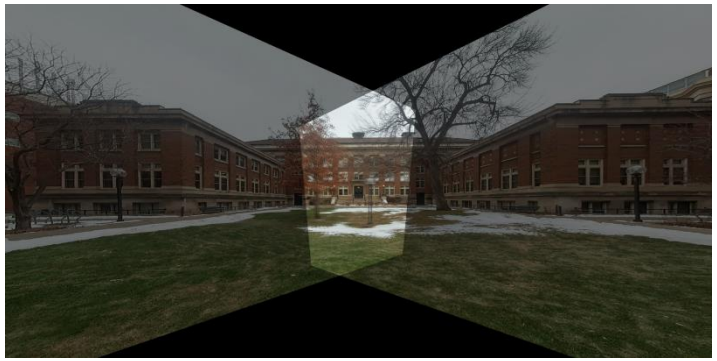


$$R = K^{-1} H K$$

Image Panorama (Cylindrical Projection)



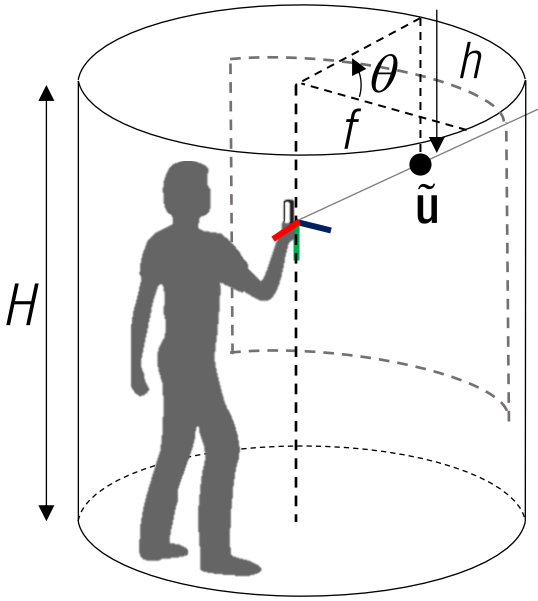
$$X = \lambda R^T K^{-1} u$$



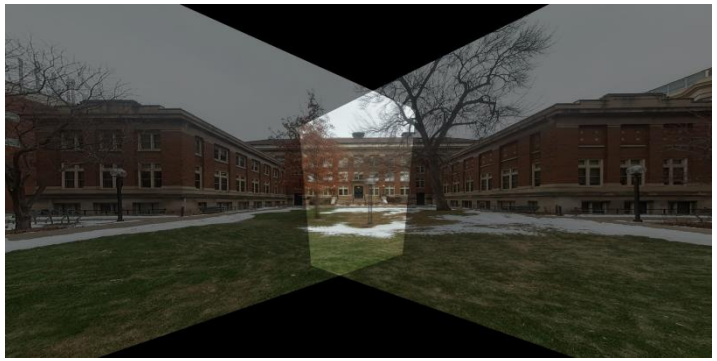
$$R = K^{-1} H K$$

$$\tilde{u} =$$

Image Panorama (Cylindrical Projection)



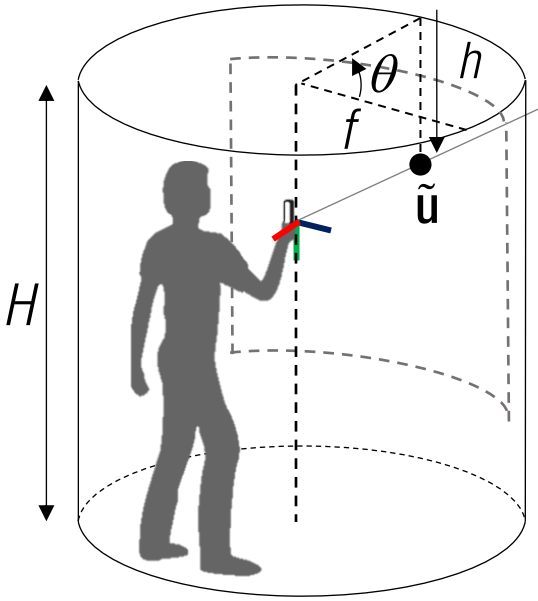
$$\mathbf{X} = \lambda \mathbf{R}^{\top} \mathbf{K}^{-1} \mathbf{u}$$



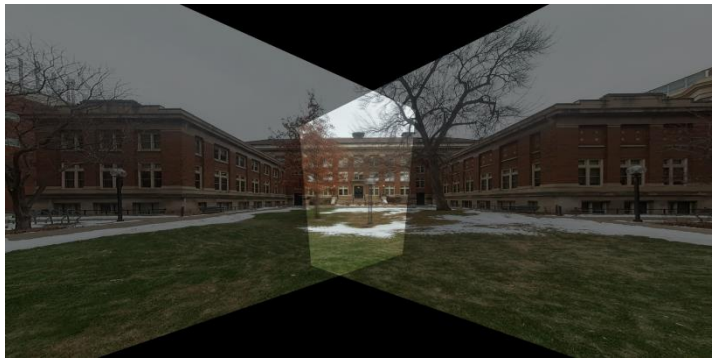
$$\mathbf{R} = \mathbf{K}^{-1} \mathbf{H} \mathbf{K}$$

$$\tilde{\mathbf{u}} = \begin{bmatrix} f \sin\theta \\ h - \frac{H}{2} \\ f \cos\theta \end{bmatrix}$$

Image Panorama (Cylindrical Projection)



$$\mathbf{X} = \lambda \mathbf{R}^{\mathsf{T}} \mathbf{K}^{-1} \mathbf{u}$$



$$\mathbf{R} = \mathbf{K}^{-1} \mathbf{H} \mathbf{K}$$

$$\tilde{\mathbf{u}} = \begin{bmatrix} f \sin\theta \\ h - \frac{H}{2} \\ f \cos\theta \end{bmatrix} = \lambda \mathbf{R}^{\mathsf{T}} \mathbf{K}^{-1} \mathbf{u}$$