Single View Camera Calibration

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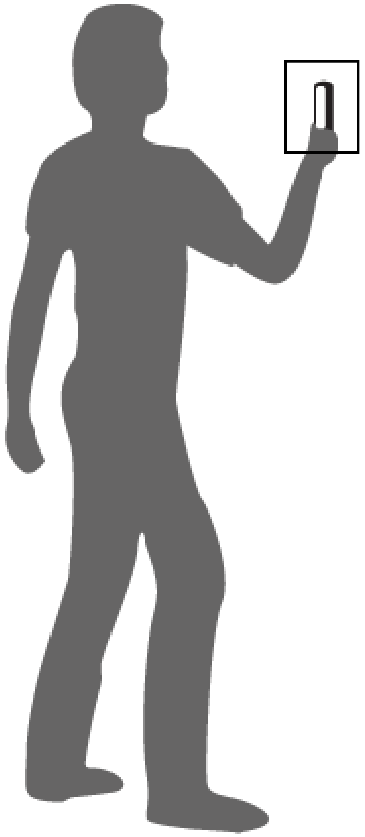
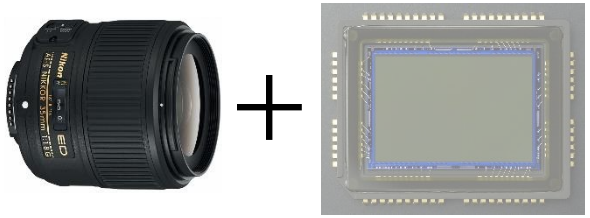
Camera Intrinsic Parameter



Pixel space

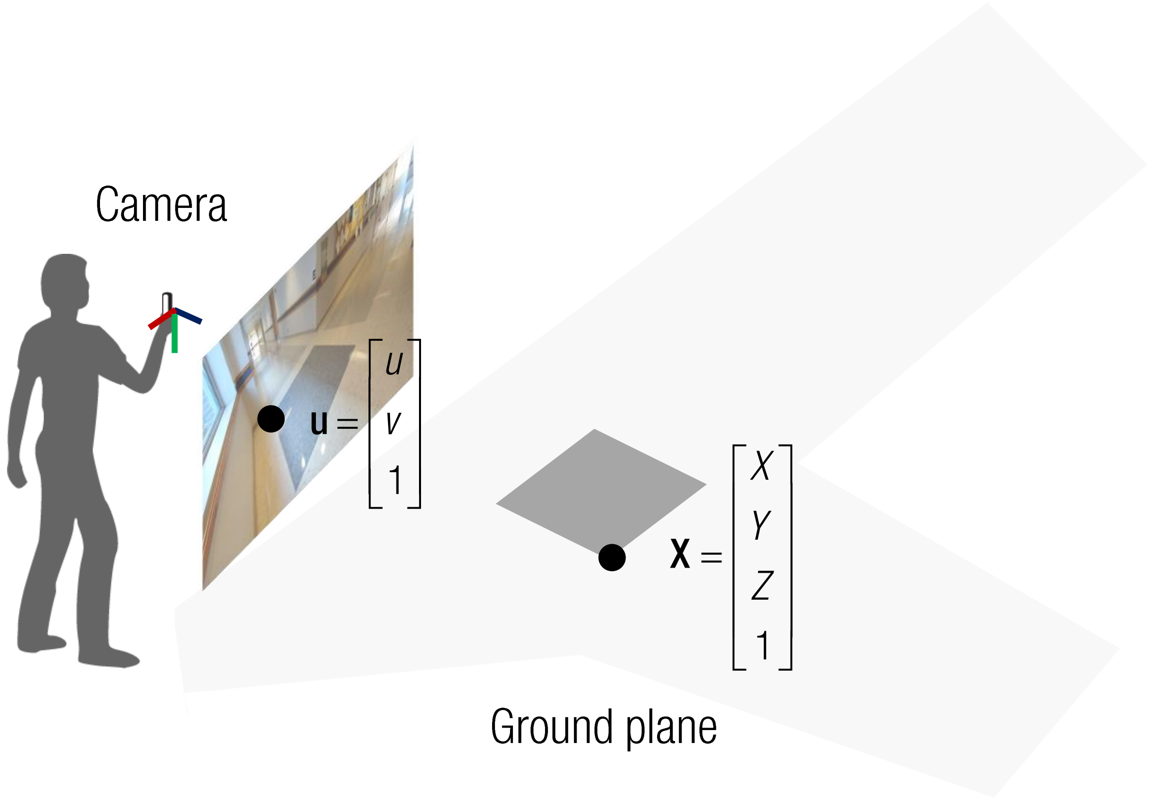
Metric space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Camera intrinsic parameter
: metric space to pixel space

Camera Calibration in Pixel Space

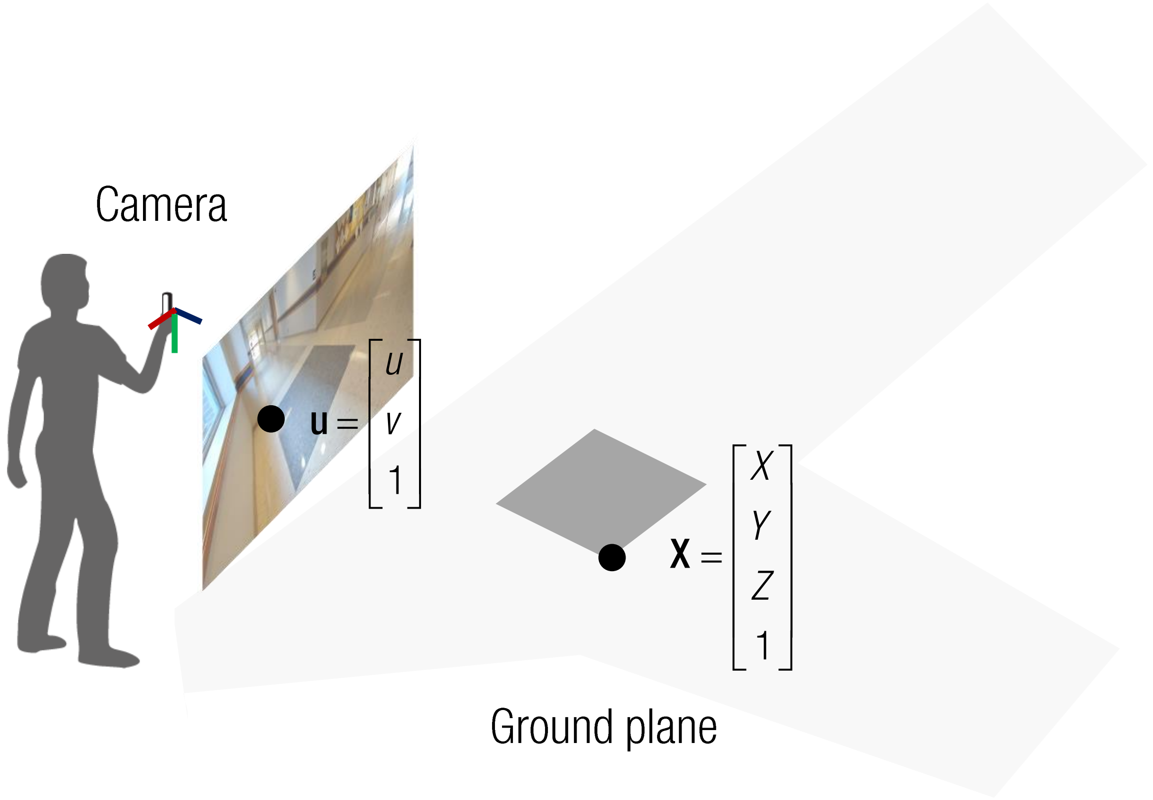


$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns:

# of equations:

Camera Calibration in Pixel Space

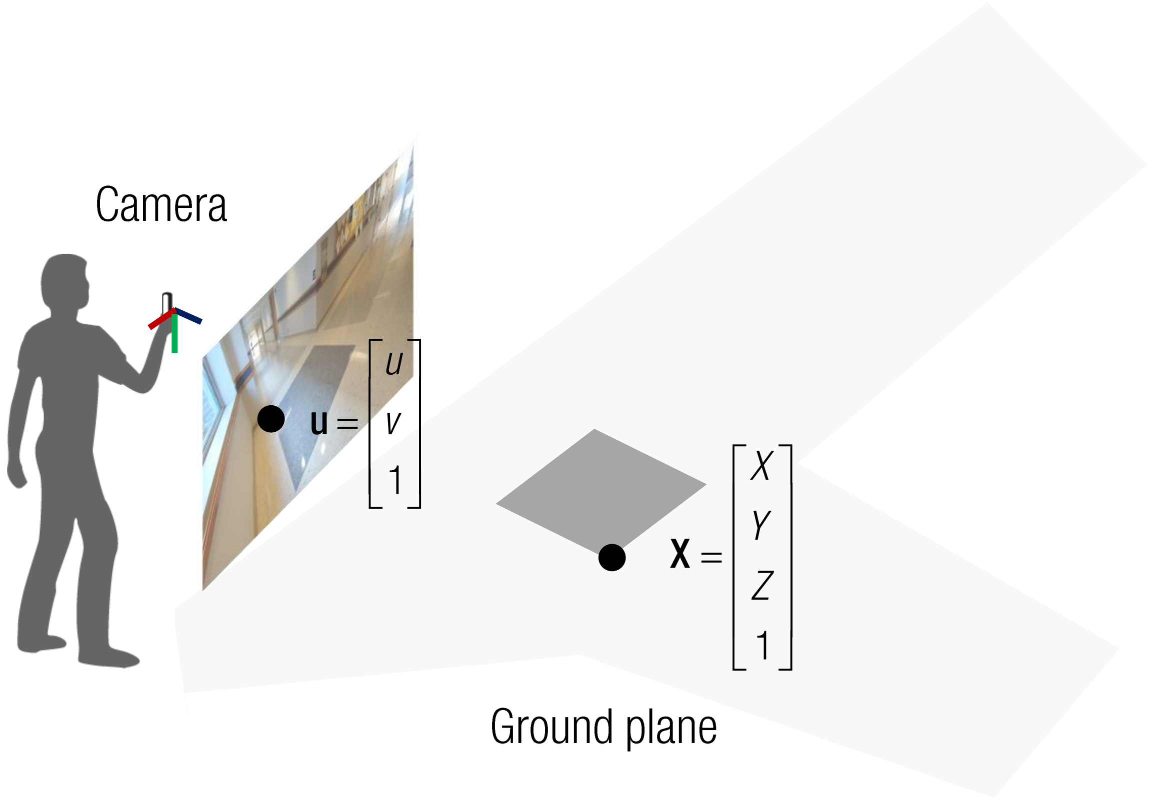


$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (**K**) + 6 (**R** and **t**) + 3 (**X**)

# of equations:

Camera Calibration in Pixel Space

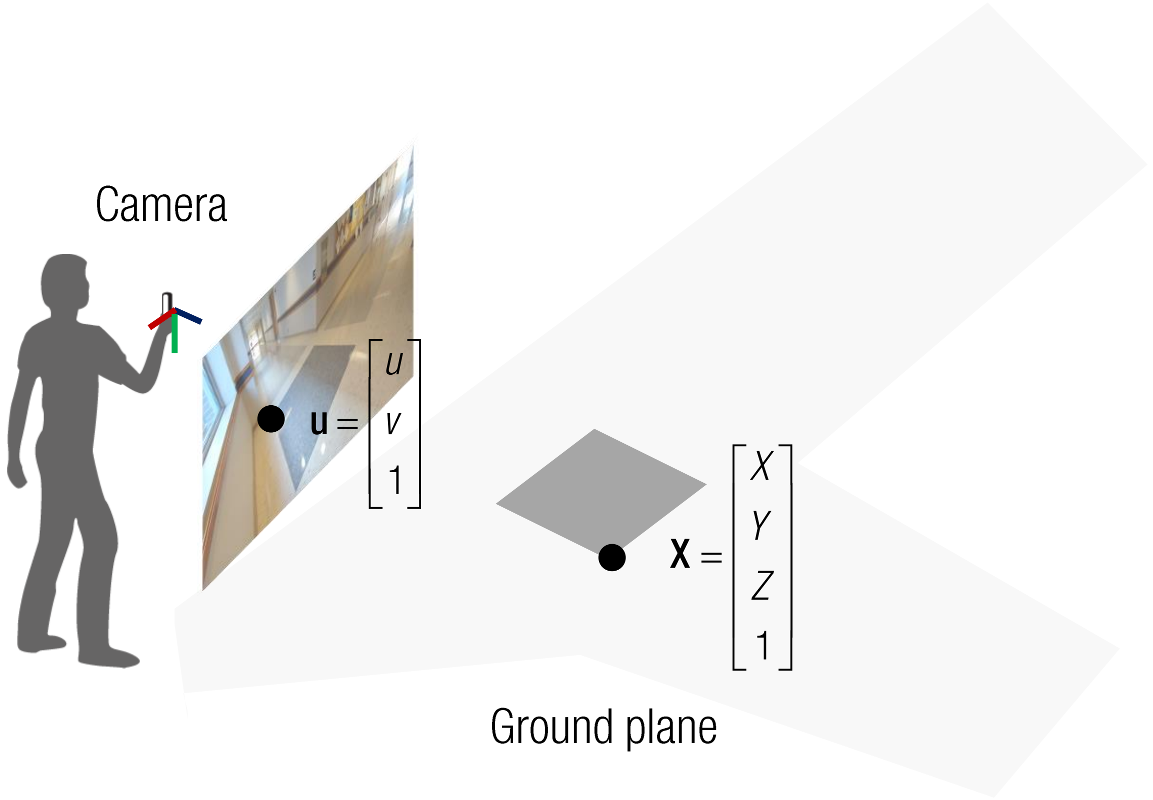


$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (**K**) + 6 (**R** and **t**) + 3 (**X**)

# of equations: 2

Camera Calibration in Pixel Space



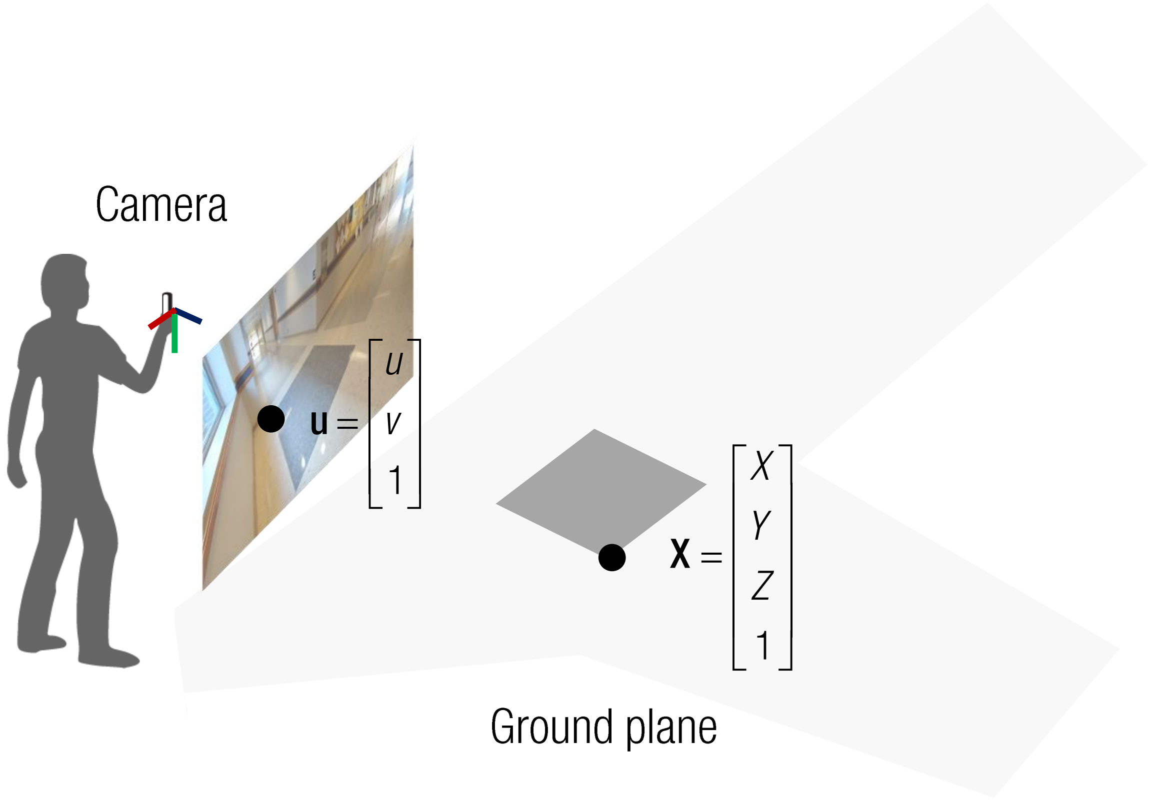
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (**K**) + 6F (**R** and **t**) + 3P (**X**)

# of equations: 2P

where F is # of images and P is # of points.

Camera Calibration in Pixel Space



$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

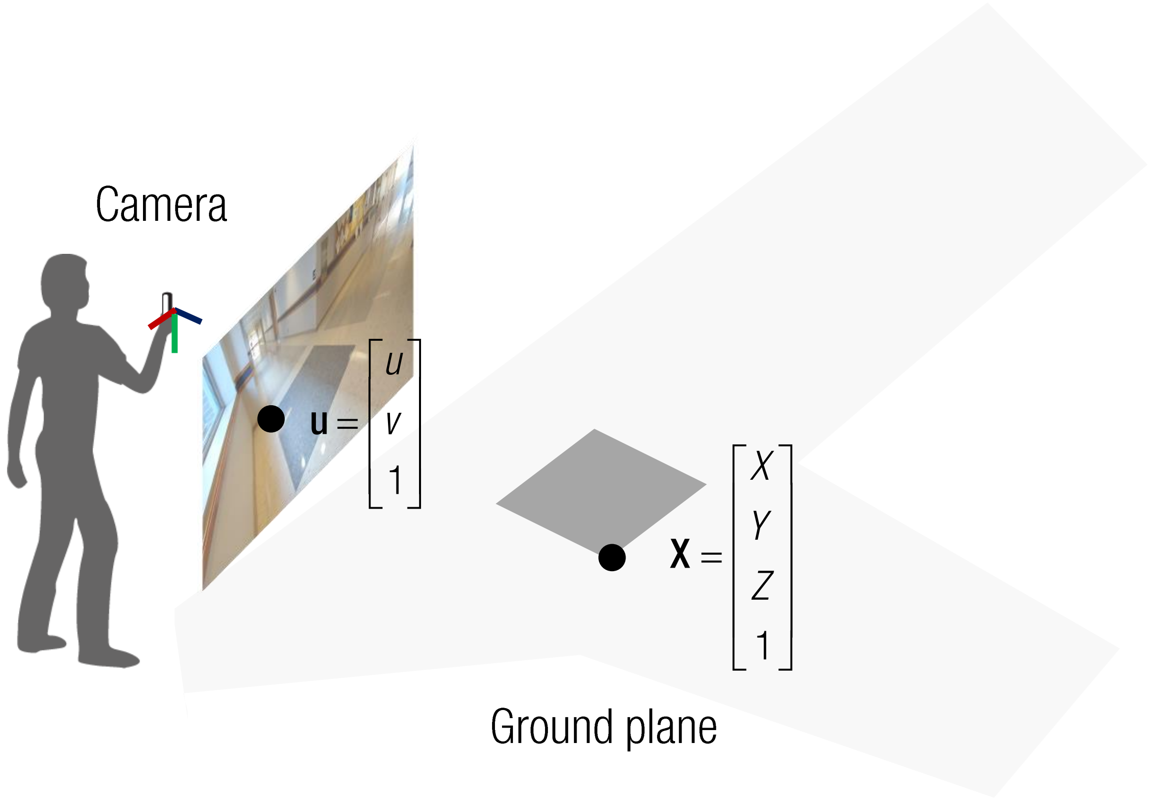
# of unknowns: 3 (**K**) + 6F (**R** and **t**) + 3P (**X**)

# of equations: 2P

where F is # of images and P is # of points.

# of unknowns > # of equations

Camera Calibration in Pixel Space



$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (**K**) + 6*F* (**R** and **t**) + 3*P* (**X**)

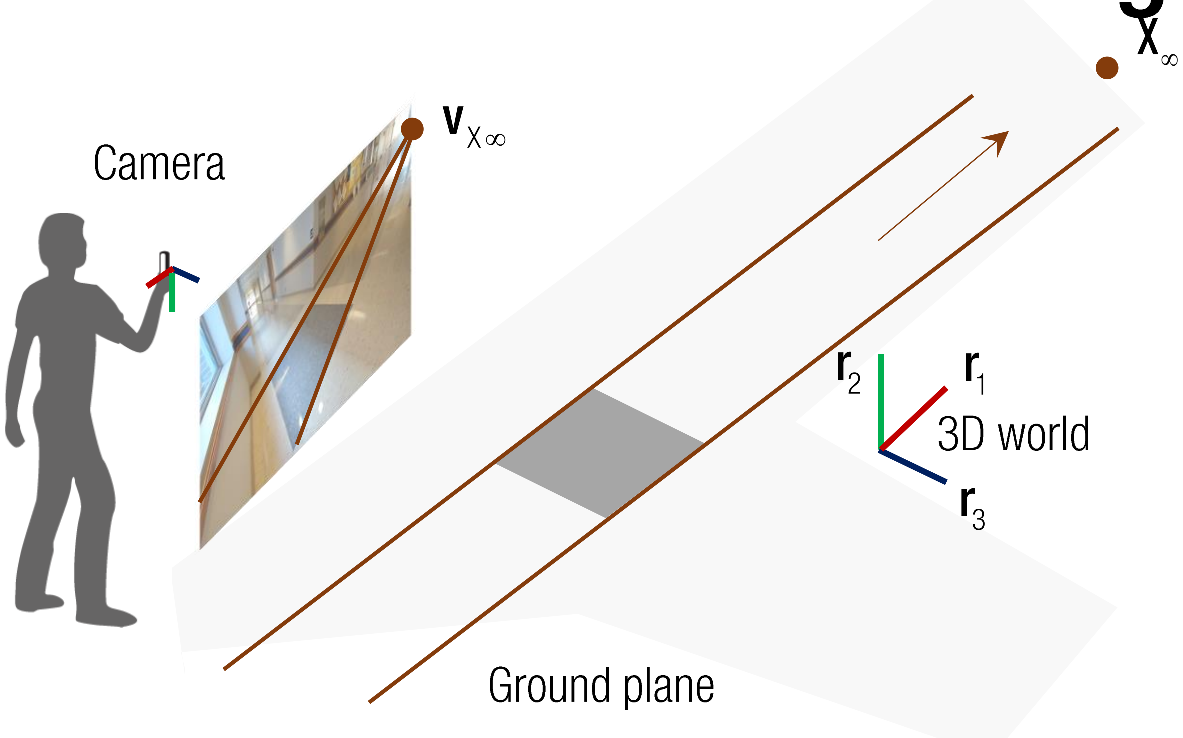
# of equations: 2*P*

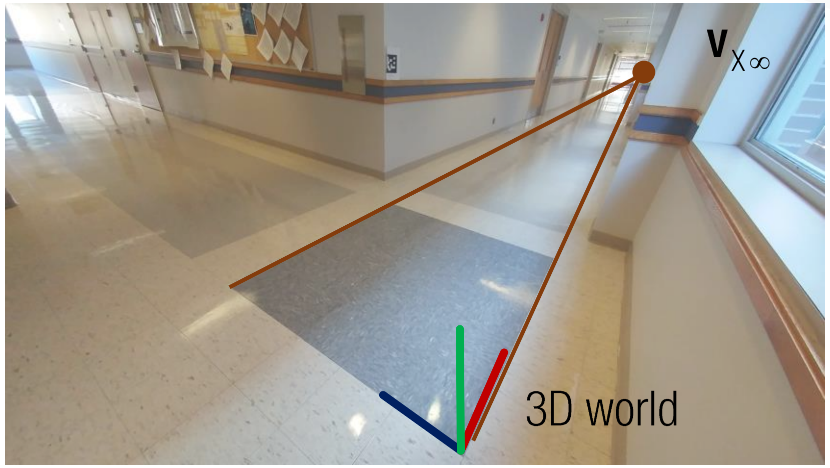
where *F* is # of images and *P* is # of points.

# of unknowns > # of equations

What do we know about the scene?

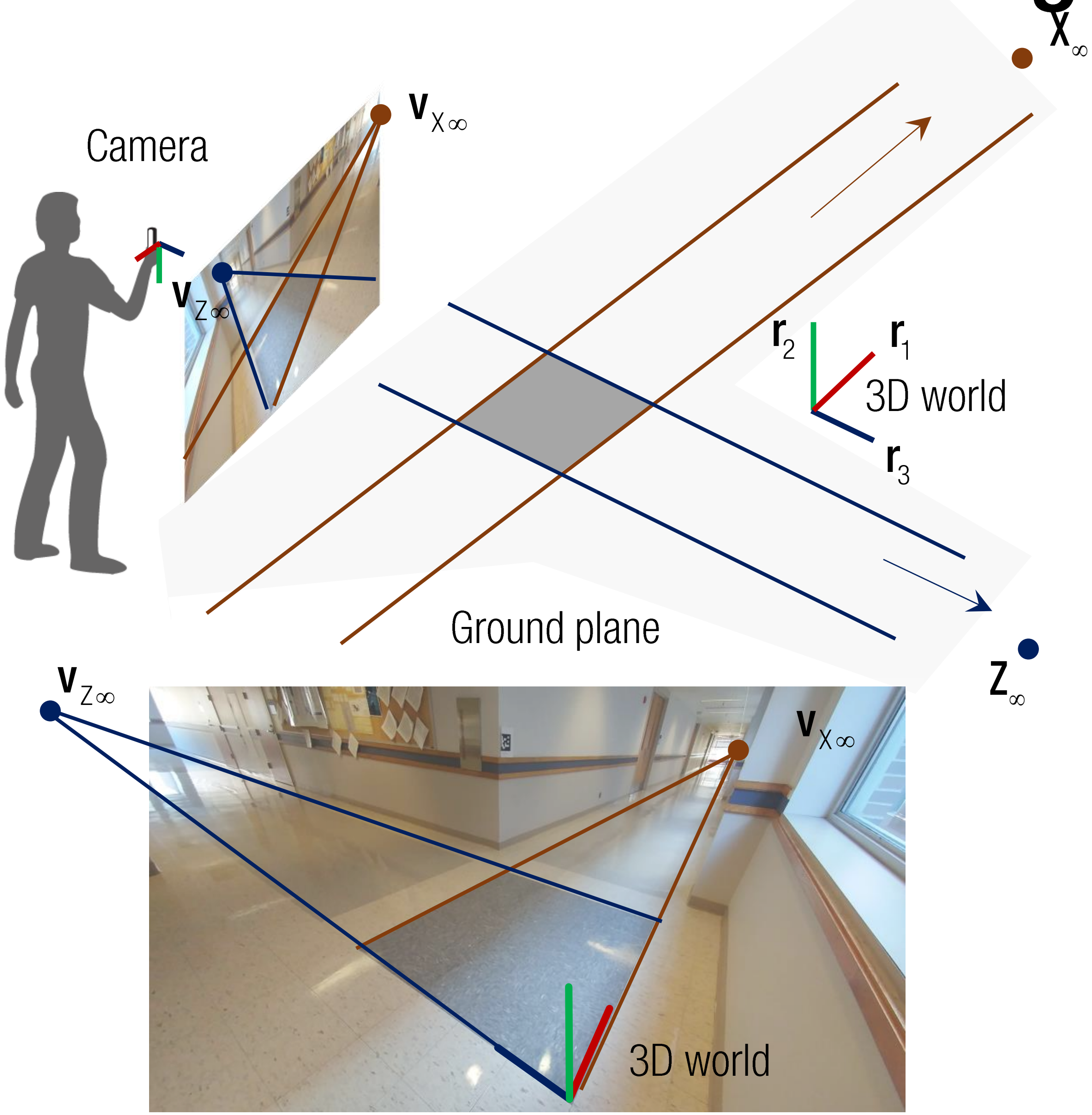
Camera Calibration using Vanishing Points





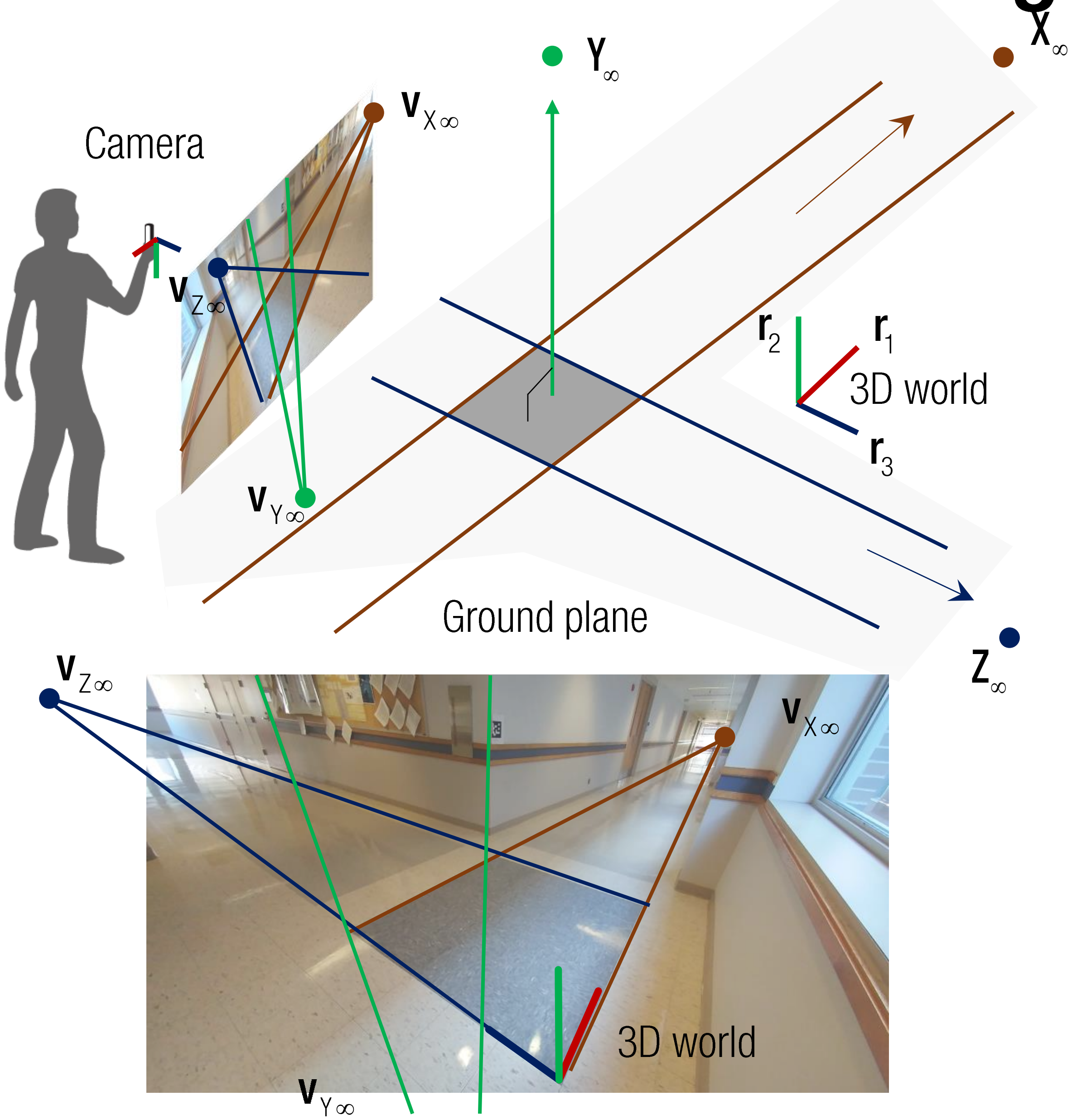
$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty$$

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty$$

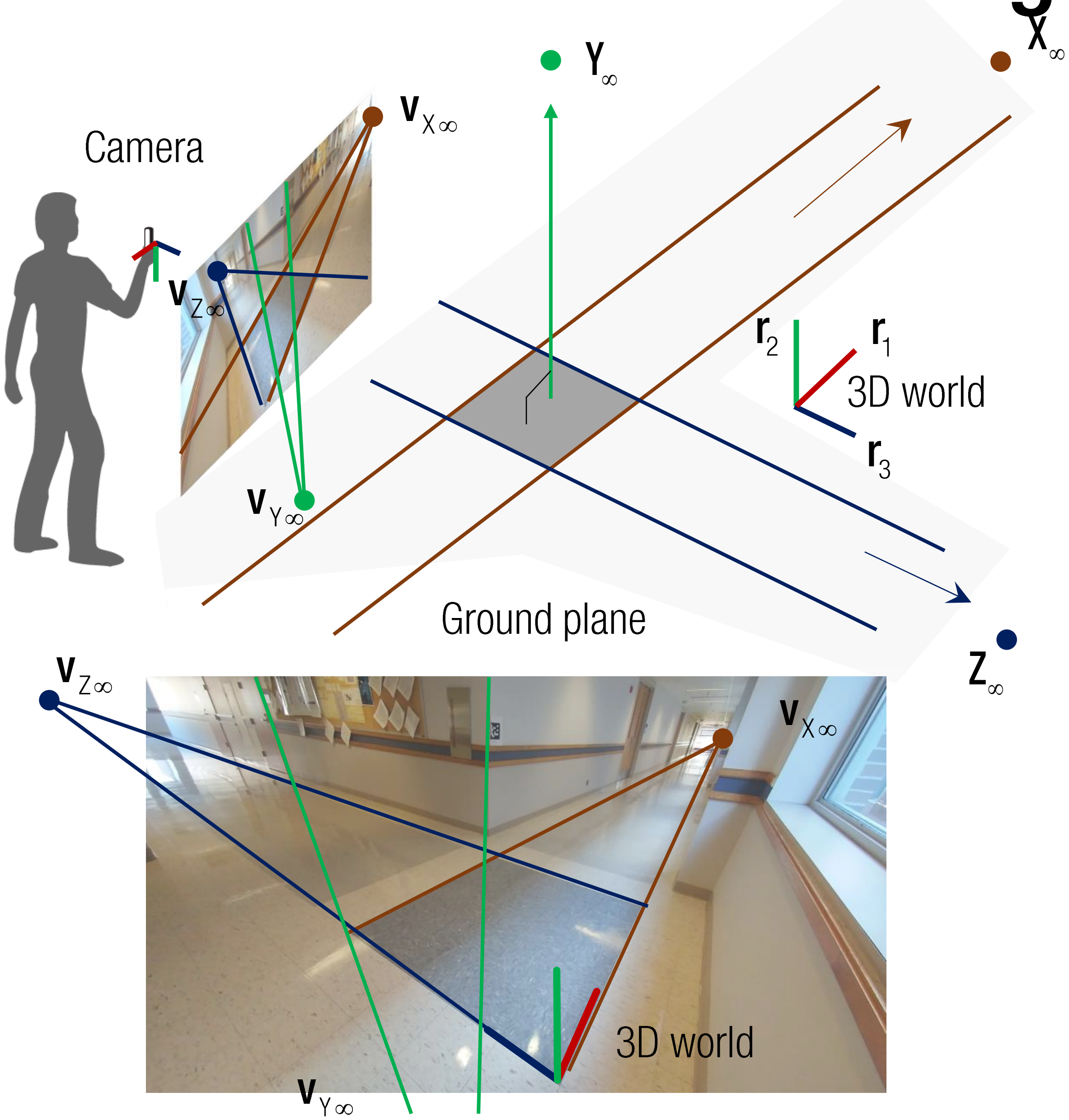
Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

Note that the camera extrinsic is still unknown (**R** and **t**).

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

Note that the camera extrinsic is still unknown (**R** and **t**).

Known property of points at infinity:

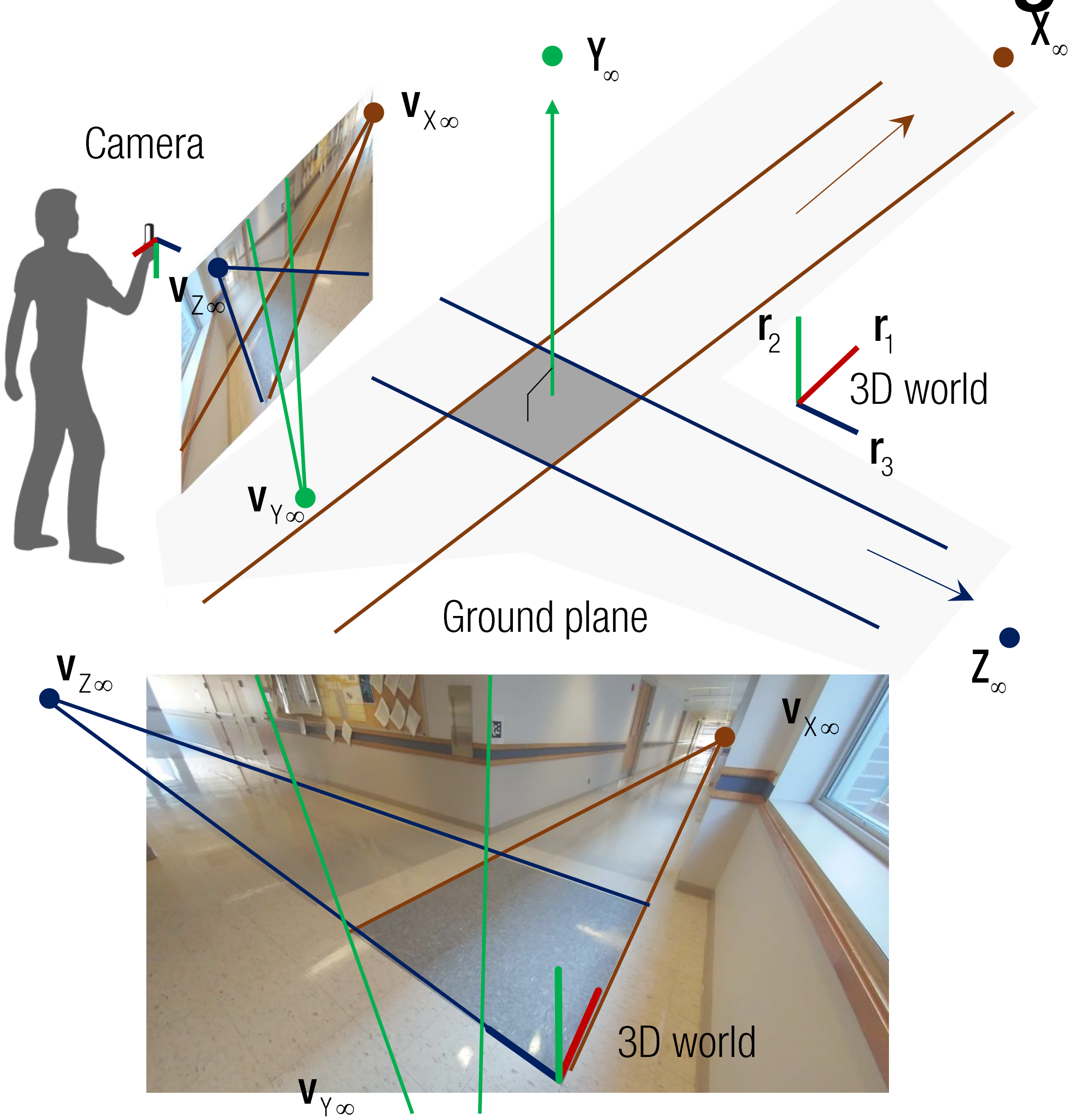
$$(\mathbf{X}_\infty)^{\mathsf{T}}(\mathbf{Y}_\infty) = 0$$

$$(\mathbf{Y}_\infty)^{\mathsf{T}}(\mathbf{Z}_\infty) = 0$$

$$(\mathbf{Z}_\infty)^{\mathsf{T}}(\mathbf{X}_\infty) = 0$$

These axes are perpendicular to each other.

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

Note that the camera extrinsic is still unknown (**R** and **t**).

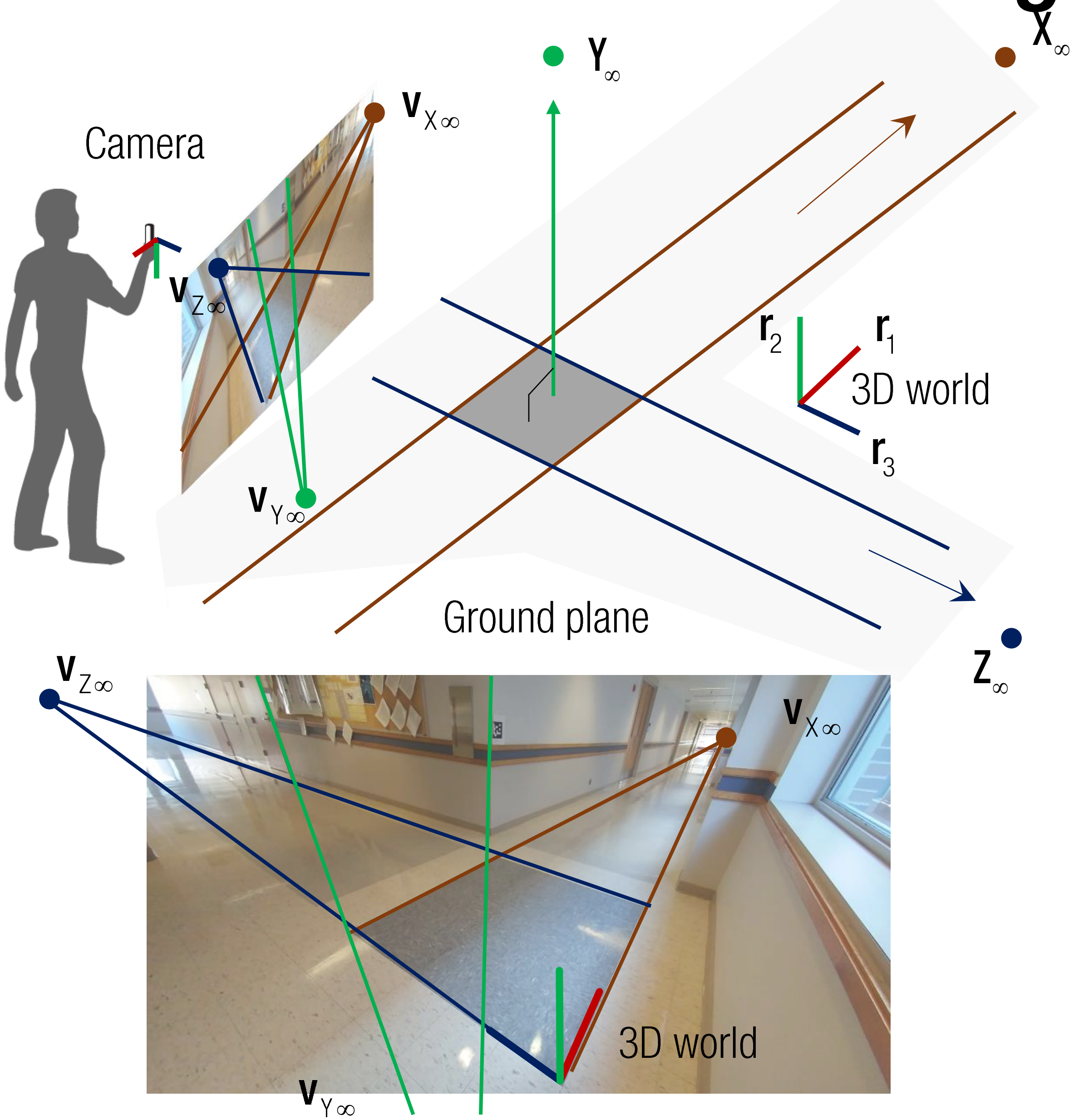
Known property of points at infinity:

$$(\mathbf{X}_\infty)^\mathsf{T}(\mathbf{Y}_\infty) = 0 \qquad (\mathbf{R}\mathbf{X}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{Y}_\infty) = 0$$

$$(\mathbf{Y}_\infty)^\mathsf{T}(\mathbf{Z}_\infty) = 0 \quad \longleftrightarrow \quad (\mathbf{R}\mathbf{Y}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{Z}_\infty) = 0$$

$$(\mathbf{Z}_\infty)^\mathsf{T}(\mathbf{X}_\infty) = 0 \qquad (\mathbf{R}\mathbf{Z}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{X}_\infty) = 0$$

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

$$\lambda \mathbf{K}^{-1}\mathbf{v}_{X\infty} = \mathbf{R}\mathbf{X}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Y\infty} = \mathbf{R}\mathbf{Y}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Z\infty} = \mathbf{R}\mathbf{Z}_\infty$$

Note that the camera extrinsic is still unknown (**R** and **t**).

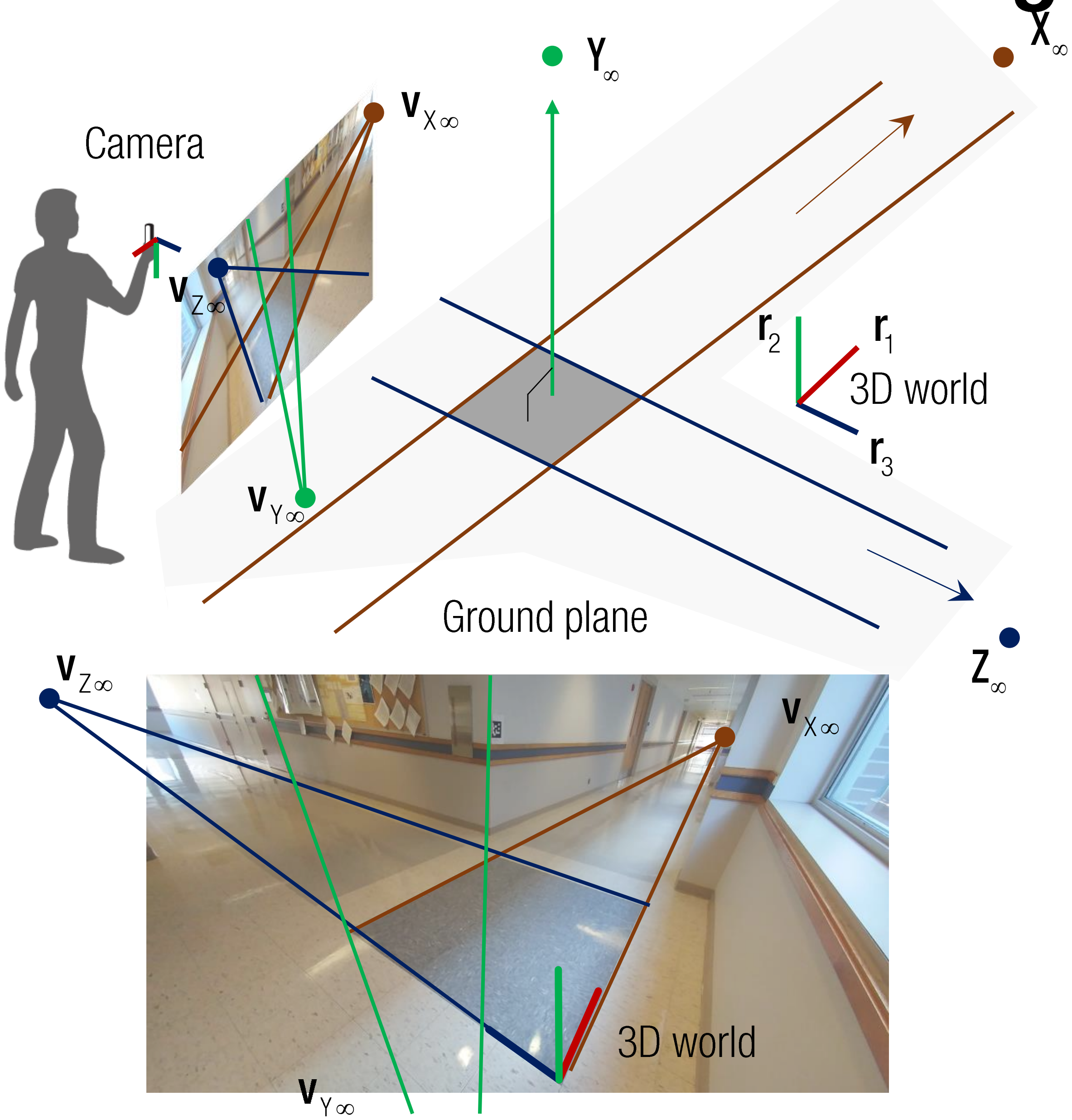
Known property of points at infinity:

$$(\mathbf{X}_\infty)^\mathsf{T}(\mathbf{Y}_\infty) = 0 \qquad (\mathbf{R}\mathbf{X}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{Y}_\infty) = 0$$

$$(\mathbf{Y}_\infty)^\mathsf{T}(\mathbf{Z}_\infty) = 0 \quad \longleftrightarrow \quad (\mathbf{R}\mathbf{Y}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{Z}_\infty) = 0$$

$$(\mathbf{Z}_\infty)^\mathsf{T}(\mathbf{X}_\infty) = 0 \qquad (\mathbf{R}\mathbf{Z}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{X}_\infty) = 0$$

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

$$\lambda \mathbf{K}^{-1}\mathbf{v}_{X\infty} = \mathbf{R}\mathbf{X}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Y\infty} = \mathbf{R}\mathbf{Y}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Z\infty} = \mathbf{R}\mathbf{Z}_\infty$$

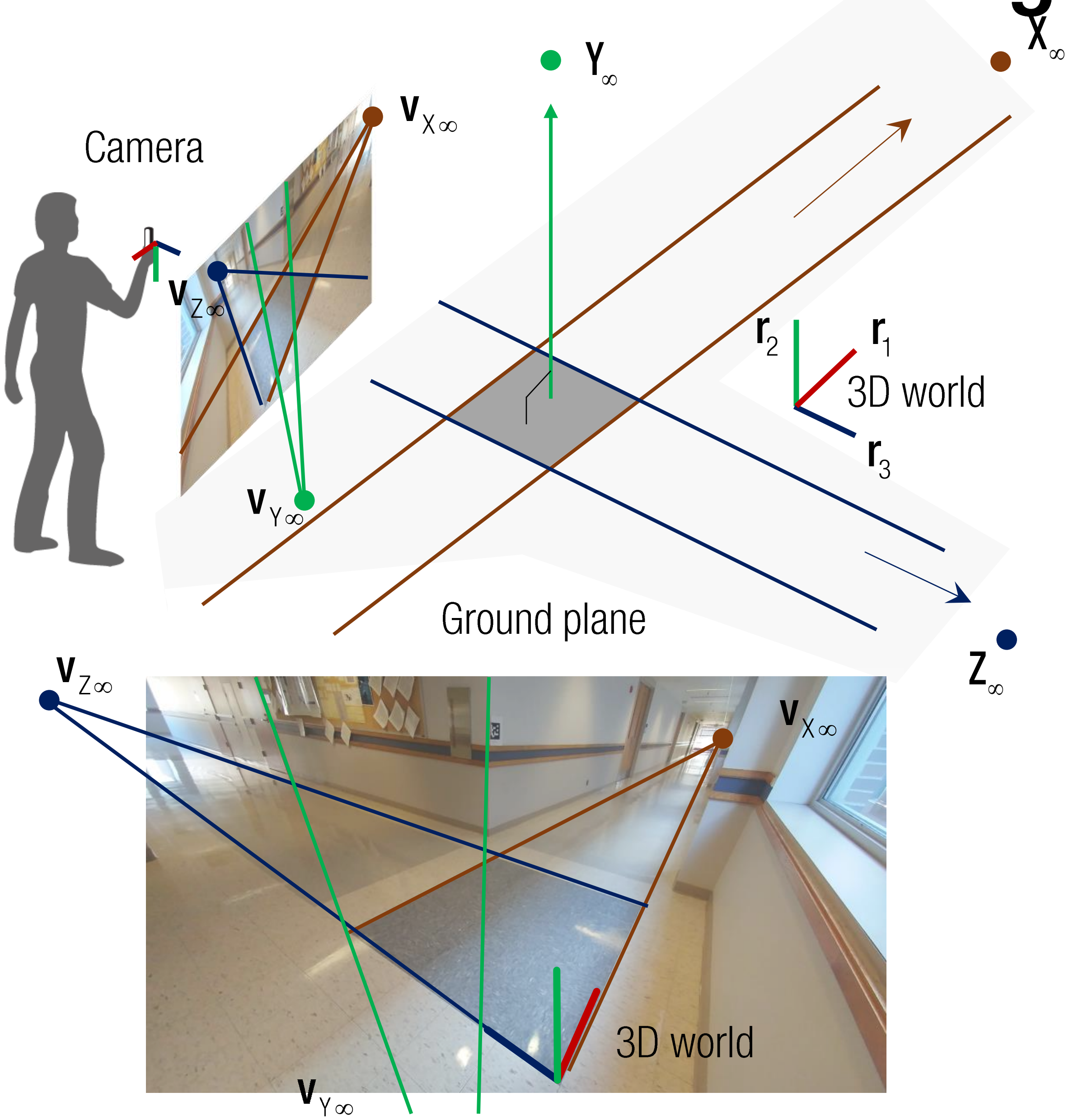
Note that the camera extrinsic is still unknown (**R** and **t**).

Known property of points at infinity:

$$(\mathbf{X}_\infty)^T(\mathbf{Y}_\infty) = 0 \qquad (\mathbf{R}\mathbf{X}_\infty)^T(\mathbf{R}\mathbf{Y}_\infty) = 0$$
$$(\mathbf{Y}_\infty)^T(\mathbf{Z}_\infty) = 0 \quad \longleftrightarrow \quad (\mathbf{R}\mathbf{Y}_\infty)^T(\mathbf{R}\mathbf{Z}_\infty) = 0$$
$$(\mathbf{Z}_\infty)^T(\mathbf{X}_\infty) = 0 \qquad (\mathbf{R}\mathbf{Z}_\infty)^T(\mathbf{R}\mathbf{X}_\infty) = 0$$

$$(\mathbf{K}^{-1}\mathbf{v}_{X\infty})^T(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Y\infty})^T(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Z\infty})^T(\mathbf{K}^{-1}\mathbf{v}_{X\infty}) = 0$$

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

$$\lambda \mathbf{K}^{-1}\mathbf{v}_{X\infty} = \mathbf{R}\mathbf{X}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Y\infty} = \mathbf{R}\mathbf{Y}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Z\infty} = \mathbf{R}\mathbf{Z}_\infty$$

Note that the camera extrinsic is still unknown (**R** and **t**).

Known property of points at infinity:

$$(\mathbf{X}_\infty)^\mathsf{T}(\mathbf{Y}_\infty) = 0 \qquad (\mathbf{R}\mathbf{X}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{Y}_\infty) = 0$$

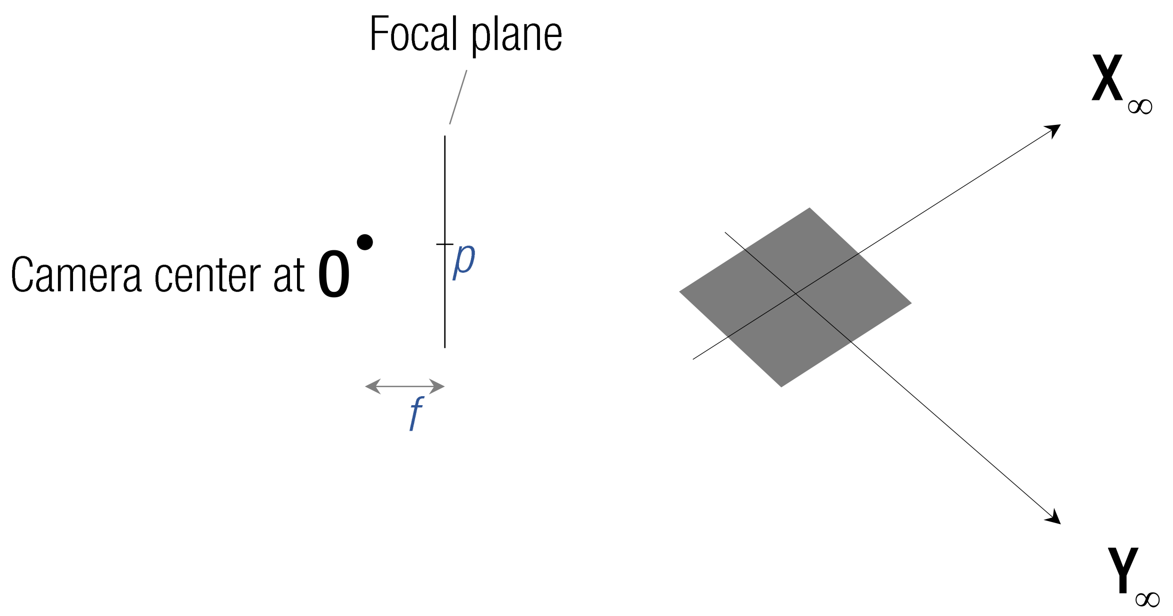
$$(\mathbf{Y}_\infty)^\mathsf{T}(\mathbf{Z}_\infty) = 0 \quad \longleftrightarrow \quad (\mathbf{R}\mathbf{Y}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{Z}_\infty) = 0$$

$$(\mathbf{Z}_\infty)^\mathsf{T}(\mathbf{X}_\infty) = 0 \qquad (\mathbf{R}\mathbf{Z}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{X}_\infty) = 0$$

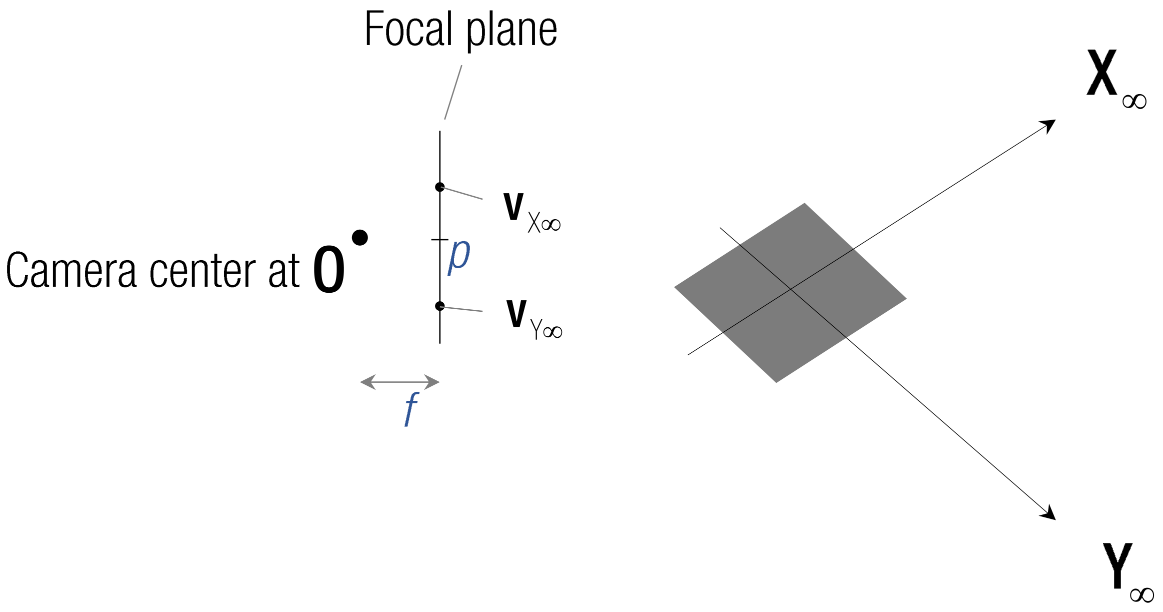
$$(\mathbf{K}^{-1}\mathbf{v}_{X\infty})^\mathsf{T}(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Y\infty})^\mathsf{T}(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Z\infty})^\mathsf{T}(\mathbf{K}^{-1}\mathbf{v}_{X\infty}) = 0$$

: 3 unknowns and 3 equations

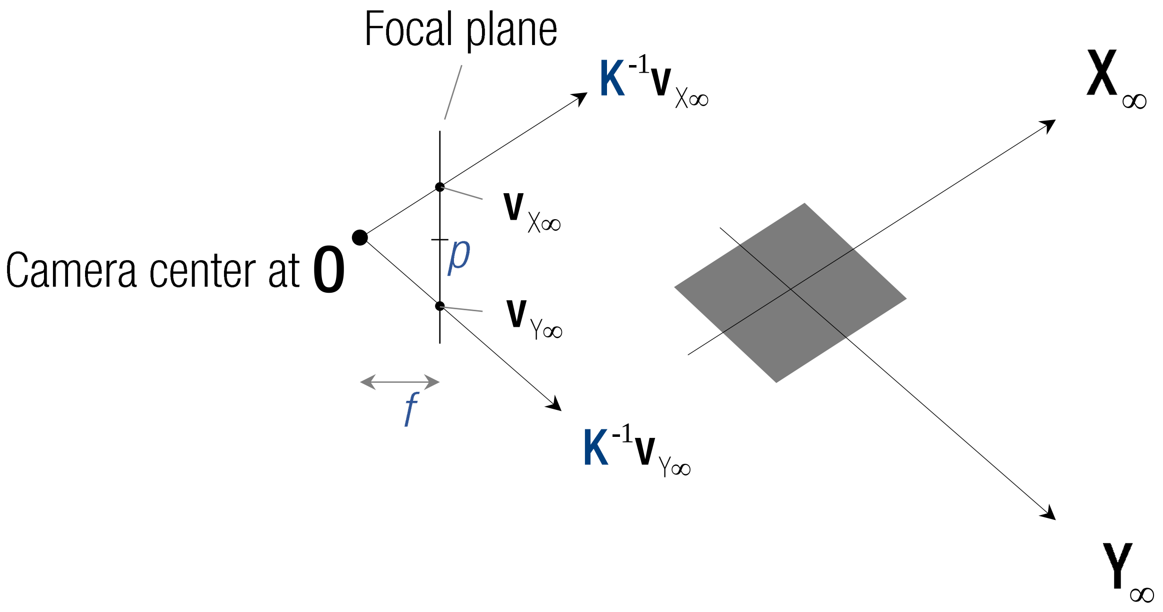
Geometric Interpretation with 1D Camera



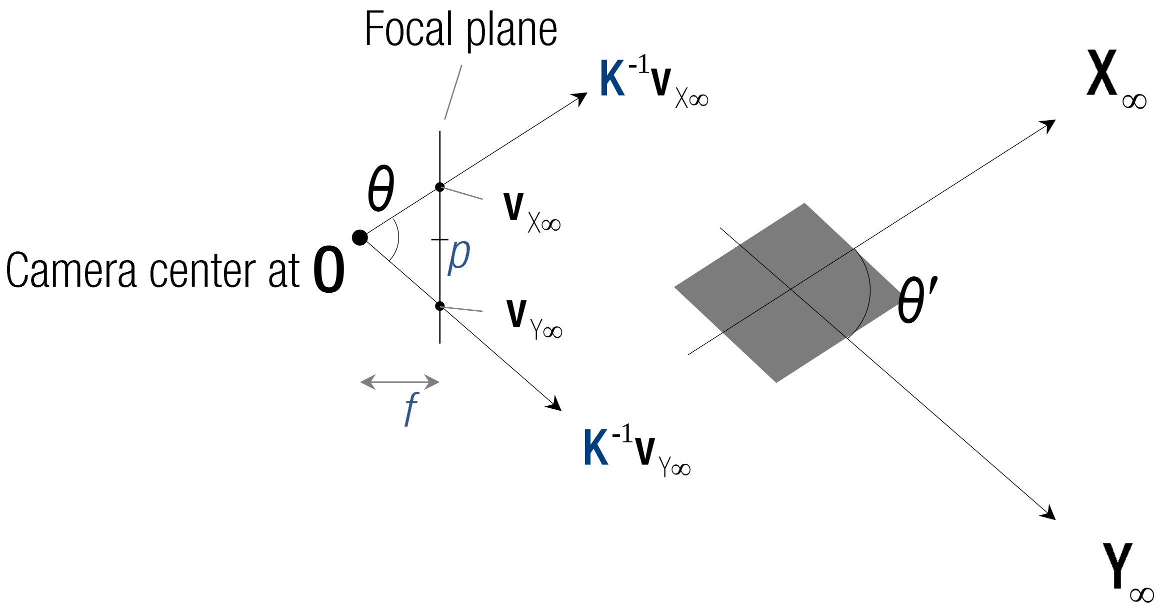
Geometric Interpretation with 1D Camera



Geometric Interpretation with 1D Camera

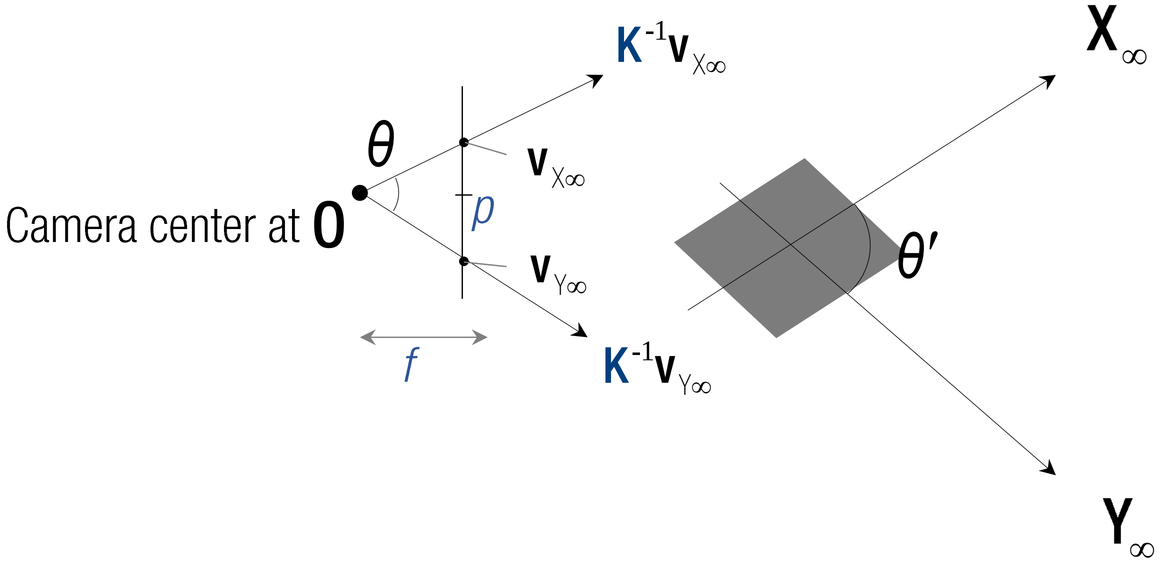


Geometric Interpretation with 1D Camera



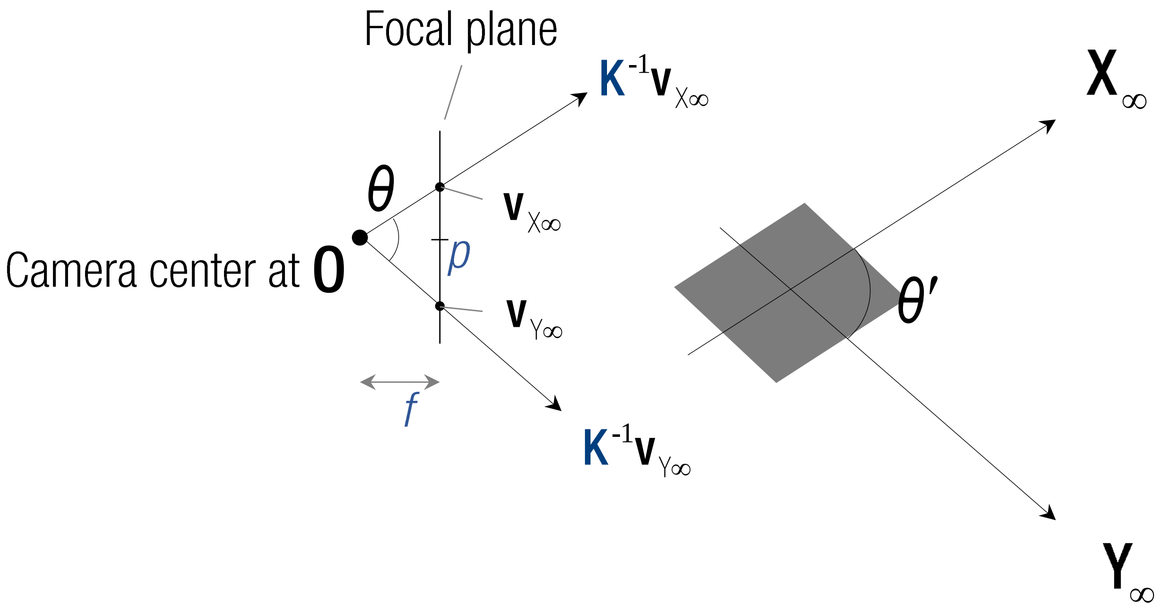
$\theta = \theta'$

Geometric Interpretation with 1D Camera



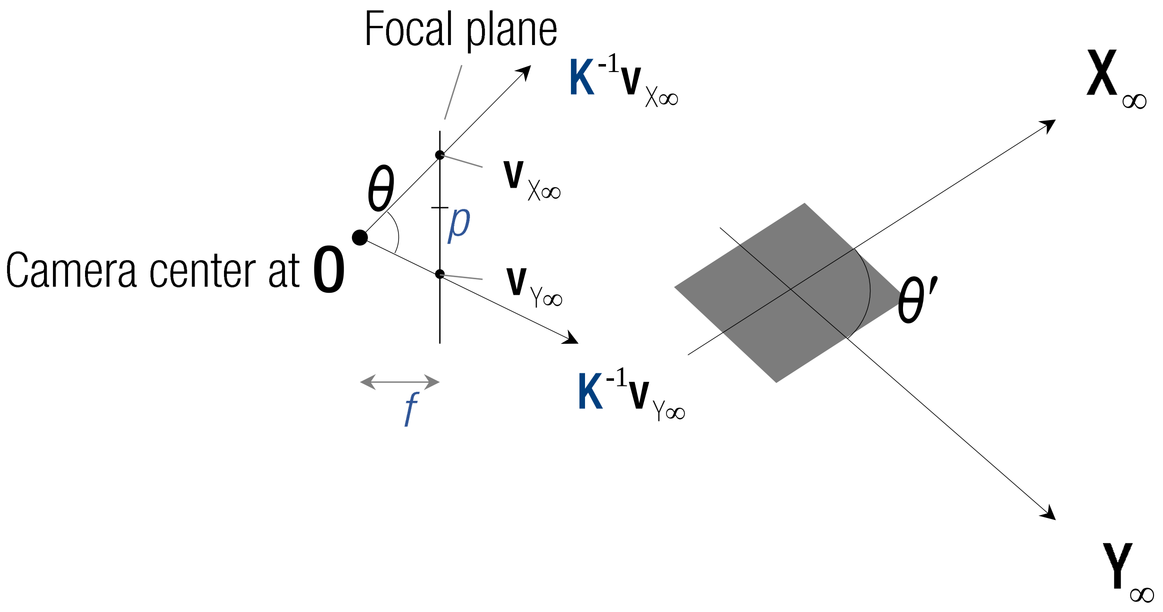
$\theta \neq \theta'$

Geometric Interpretation with 1D Camera



$\theta = \theta'$

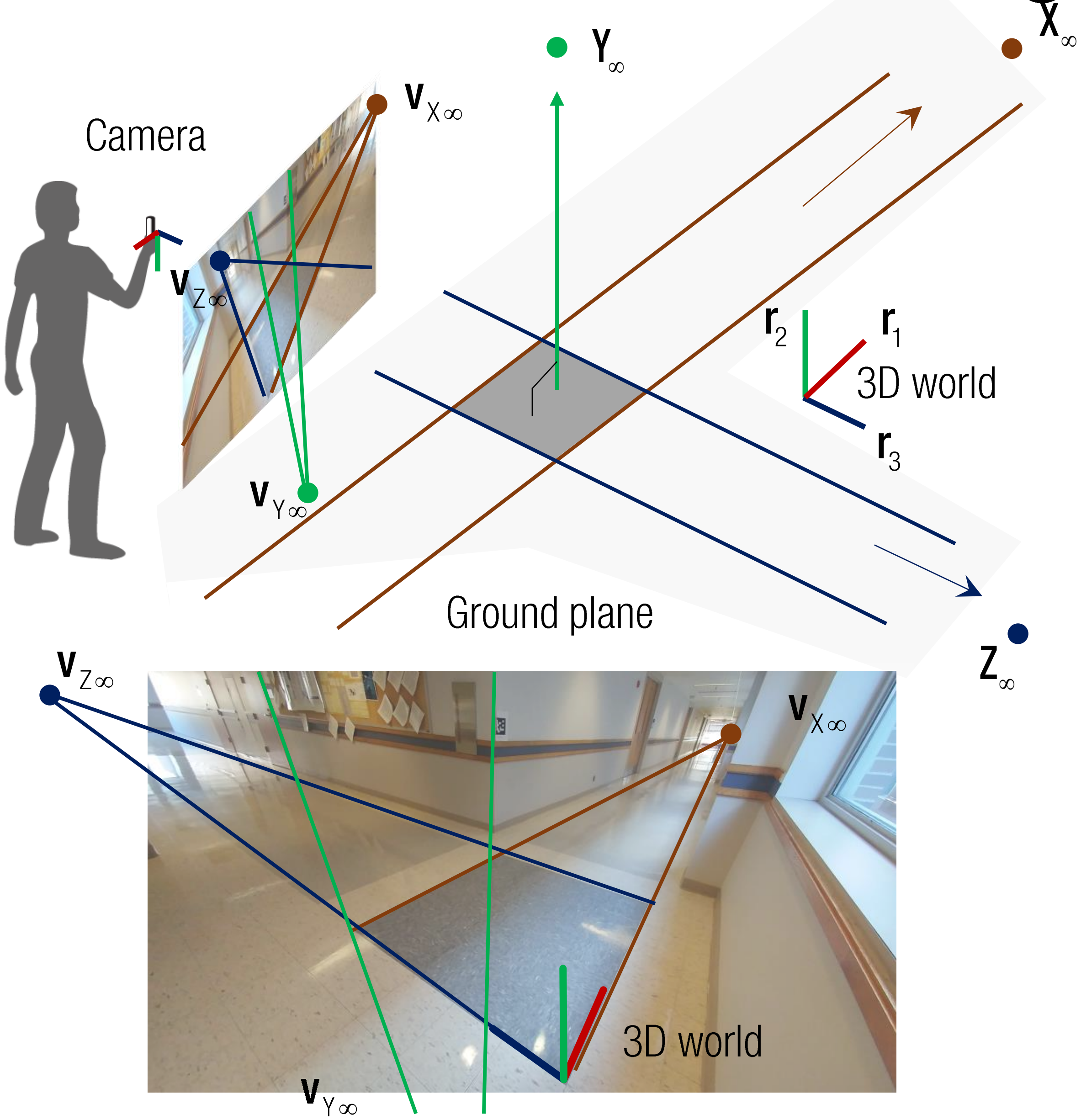
Geometric Interpretation with 1D Camera



$$\theta \neq \theta'$$

Given two vanishing points, the focal length and principal point are uniquely defined.
For the 2D camera case, another vanishing point is needed to uniquely define f, p_x, and p_y.

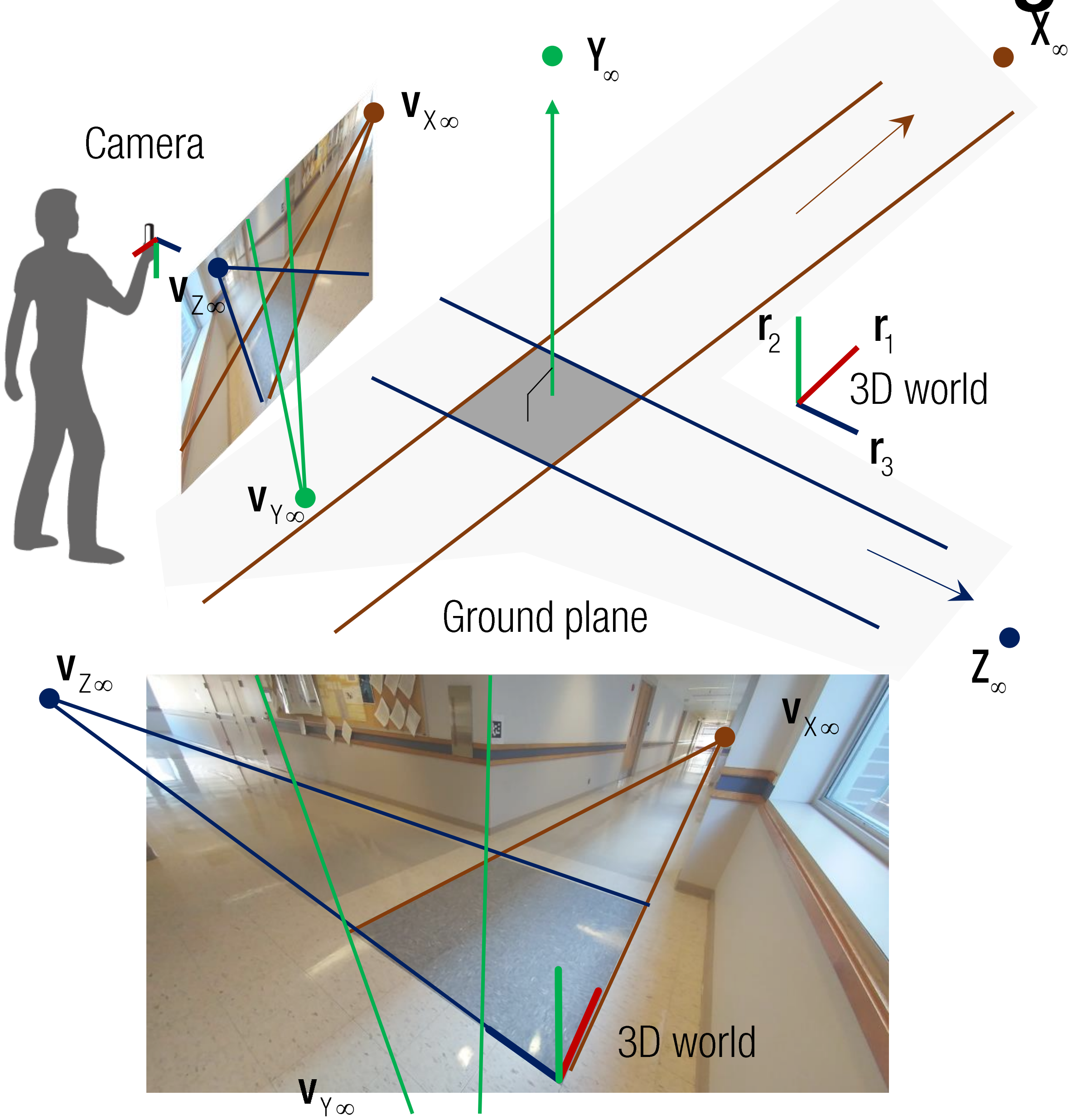
Camera Calibration using Vanishing Points



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

Camera Calibration using Vanishing Points

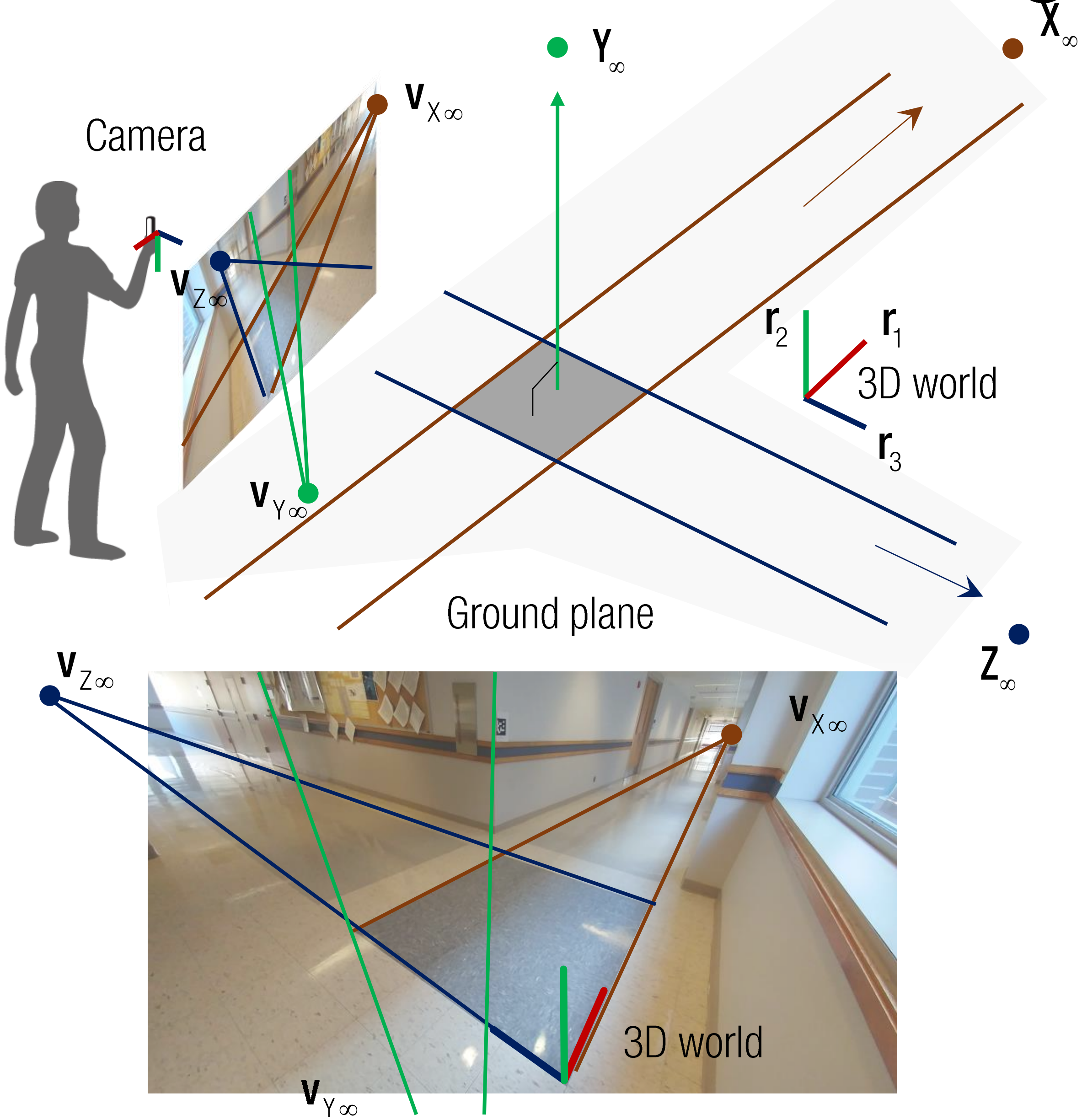


$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{\mathrm{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{\mathrm{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{\mathrm{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

$$\longrightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_i\right)^{\mathrm{T}}\left(\mathbf{K}^{-1}\mathbf{v}_j\right)=\mathbf{v}_i^{\mathrm{T}}\mathbf{K}^{-\mathrm{T}}\mathbf{K}^{-1}\mathbf{v}_j \qquad i,j=X,Y,Z, \text{ and } i\neq j$$

Camera Calibration using Vanishing Points



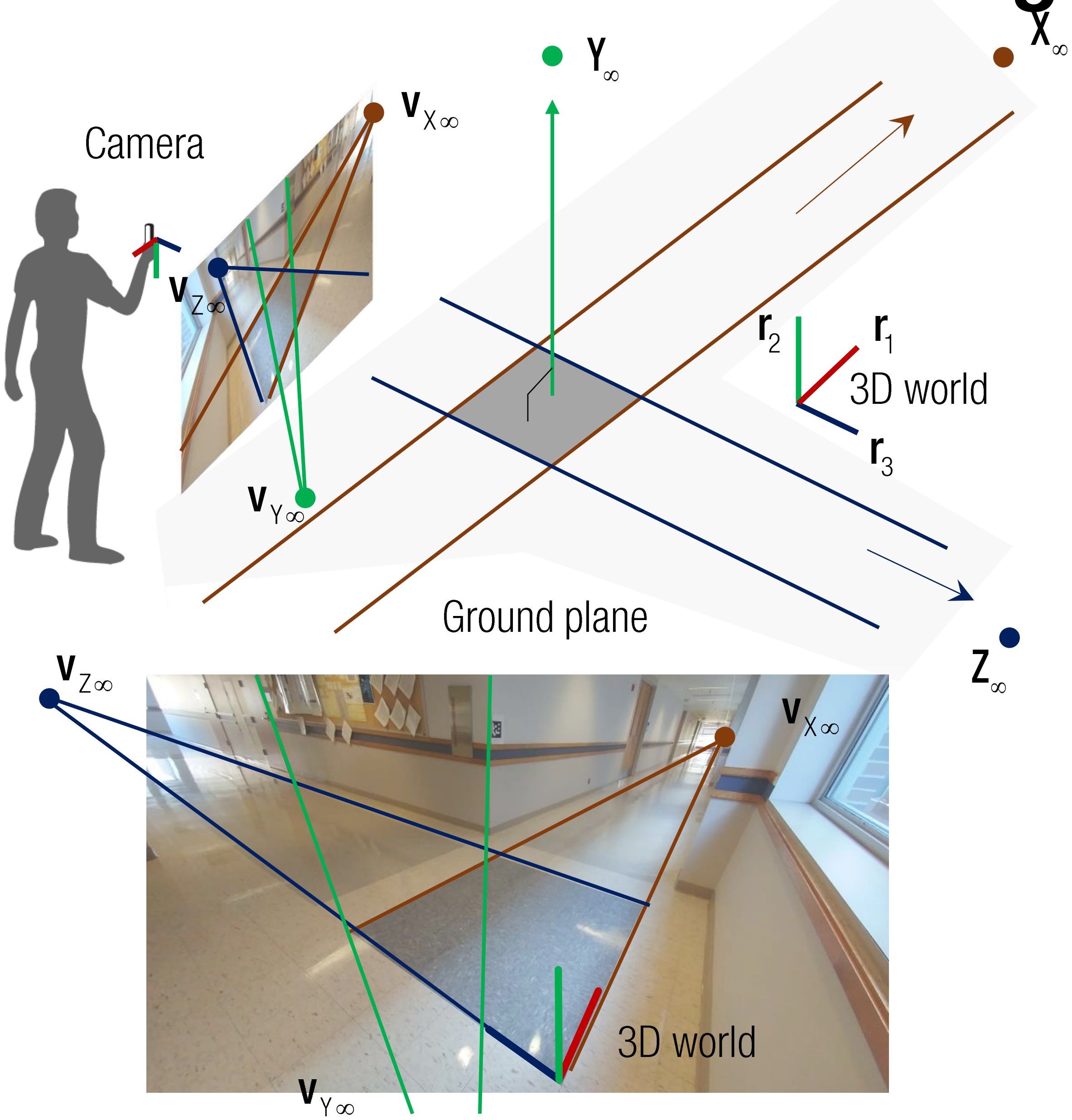
$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

$$\longrightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_{i}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{j}\right)=\mathbf{v}_i^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_j \qquad i,j=X,Y,Z,\ \text{and}\ i\neq j$$

$$\mathbf{K}^{-T}\mathbf{K}^{-1}=\begin{bmatrix}1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1\end{bmatrix}\begin{bmatrix}1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1\end{bmatrix}$$

Camera Calibration using Vanishing Points



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

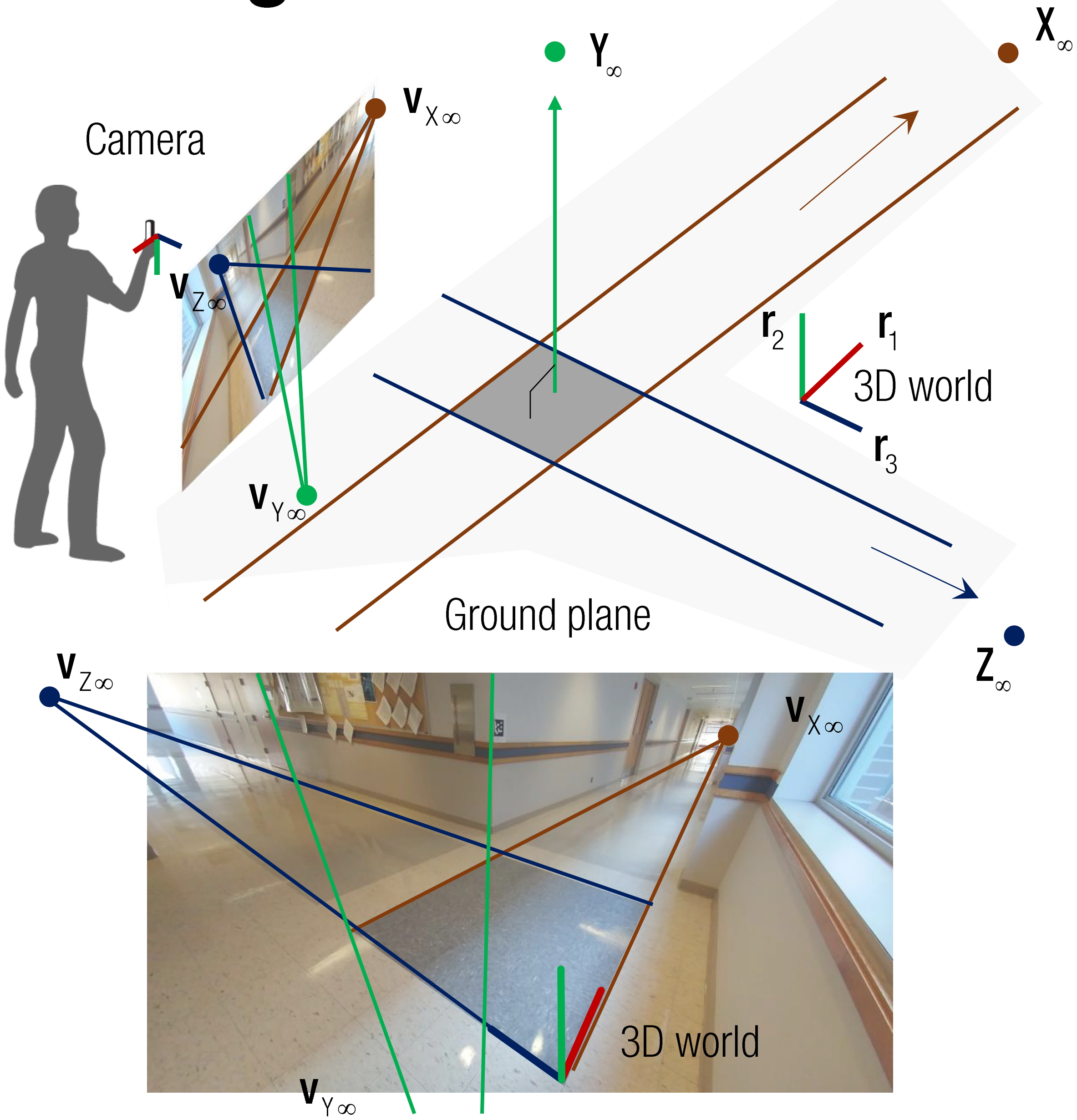
: 3 unknowns and 3 equations

$$\longrightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_{i}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{j}\right)=\mathbf{v}_{i}^{T}\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_{j} \qquad i,j=X,Y,Z,\ \text{and}\ i\neq j$$

$$\mathbf{K}^{-T}\mathbf{K}^{-1}=\begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix}\begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1}{f^2} & & -\frac{p_x}{f^2} \\ & \frac{1}{f^2} & -\frac{p_y}{f^2} \\ -\frac{p_x}{f^2} & -\frac{p_y}{f^2} & \frac{p_x^{\,2}}{f^2}+\frac{p_y^2}{f^2}+1 \end{bmatrix}$$

Change of Variables



$$\left(K^{-1}v_{X\infty}\right)^T\left(K^{-1}v_{Y\infty}\right)=\left(K^{-1}v_{Y\infty}\right)^T\left(K^{-1}v_{Z\infty}\right)=\left(K^{-1}v_{Z\infty}\right)^T\left(K^{-1}v_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

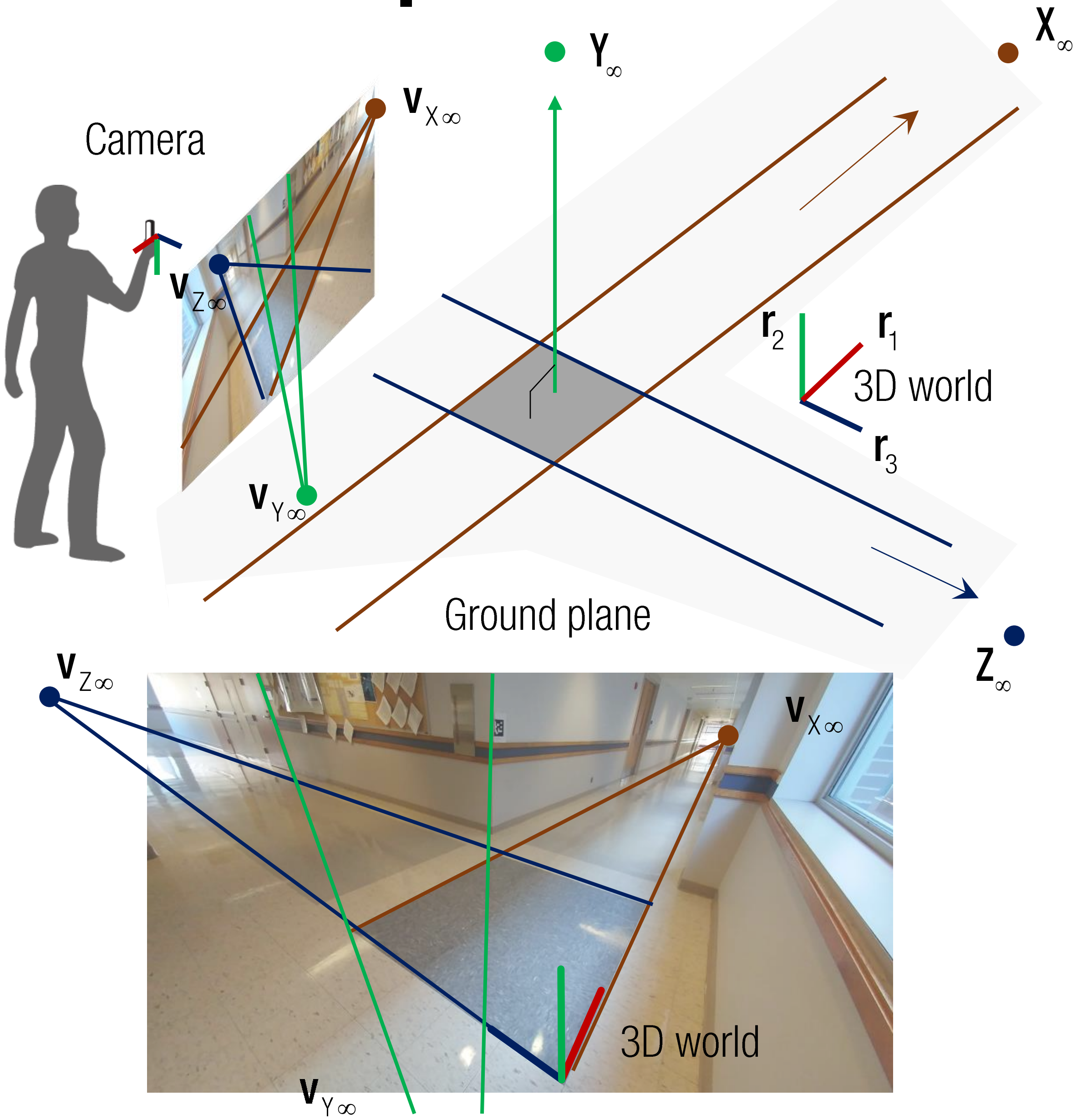
$$\longrightarrow \quad \left(K^{-1}v_i\right)^T\left(K^{-1}v_j\right)=v_i^T K^{-T}K^{-1}v_j \qquad i,j=X,Y,Z,\ \text{and } i\neq j$$

$$K^{-T}K^{-1}=\begin{bmatrix}1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1\end{bmatrix}\begin{bmatrix}1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{f^2} & & -\frac{p_x}{f^2} \\ & \frac{1}{f^2} & -\frac{p_y}{f^2} \\ -\frac{p_x}{f^2} & -\frac{p_y}{f^2} & \frac{p_x^2}{f^2}+\frac{p_y^2}{f^2}+1\end{bmatrix}=\begin{bmatrix}b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4\end{bmatrix}$$

where $b_1=\frac{1}{f^2},\quad b_2=-\frac{p_x}{f^2},\quad b_3=-\frac{p_y}{f^2},\quad b_4=\frac{p_x^2}{f^2}+\frac{p_y^2}{f^2}+1$

Linear Equation



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^T\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right) = \left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^T\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right) = \left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^T\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right) = 0$$

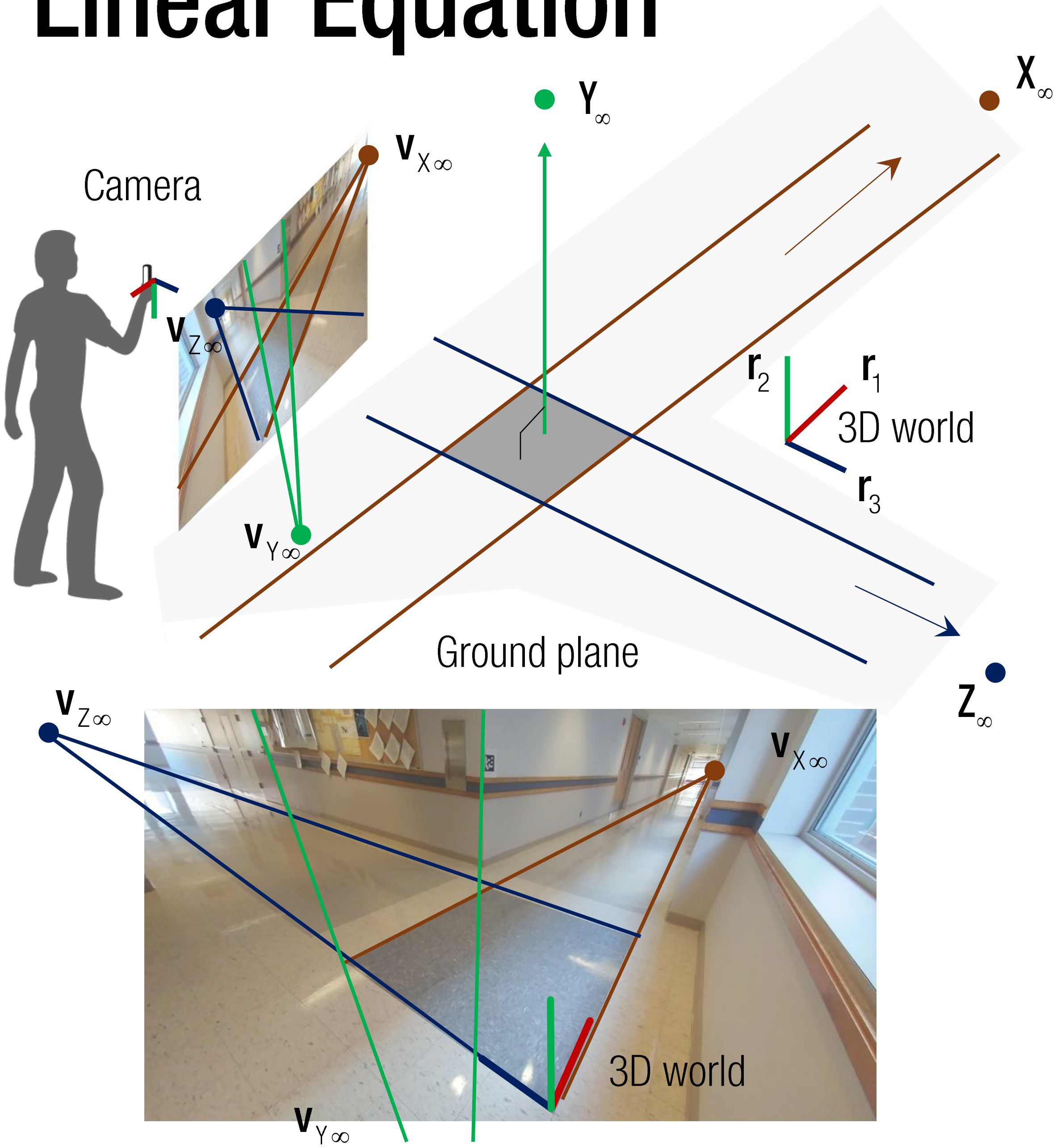
: 3 unknowns and 3 equations

$$\rightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_i\right)^T\left(\mathbf{K}^{-1}\mathbf{v}_j\right) = \mathbf{v}_i^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_j \qquad i,j = X,Y,Z, \text{ and } i \neq j$$

$$\rightarrow \quad \mathbf{v}_i^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \mathbf{v}_j :$$

Linear in b

Linear Equation



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

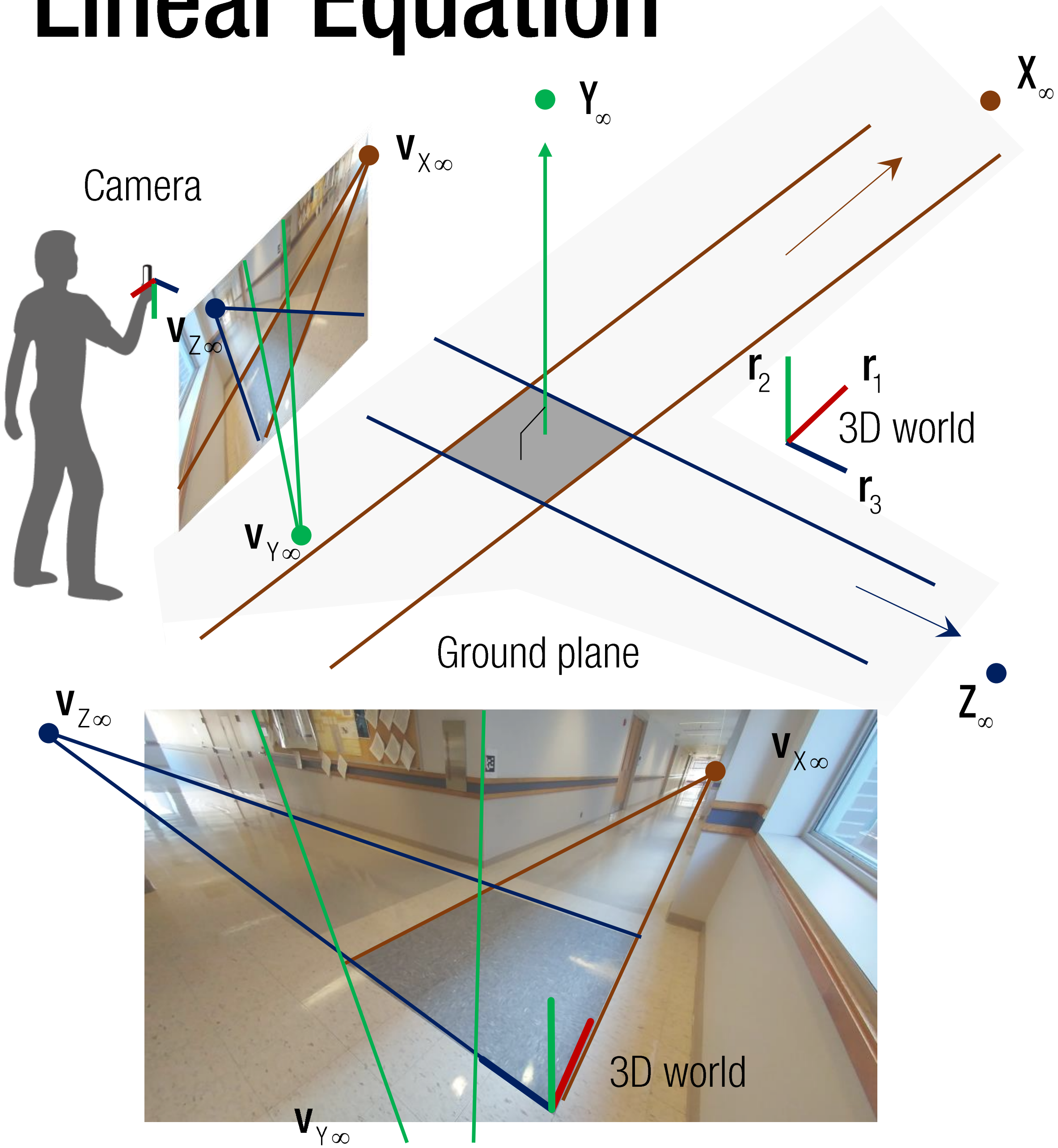
: 3 unknowns and 3 equations

$$\rightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_{i}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{j}\right)=\mathbf{v}_{i}^{T}\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_{j} \qquad i,j=X,Y,Z, \text{ and } i\neq j$$

$$\rightarrow \quad \mathbf{v}_{i}^{T}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\mathbf{v}_{j}=\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^{T}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix}=0$$

Linear in b

Linear Equation



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

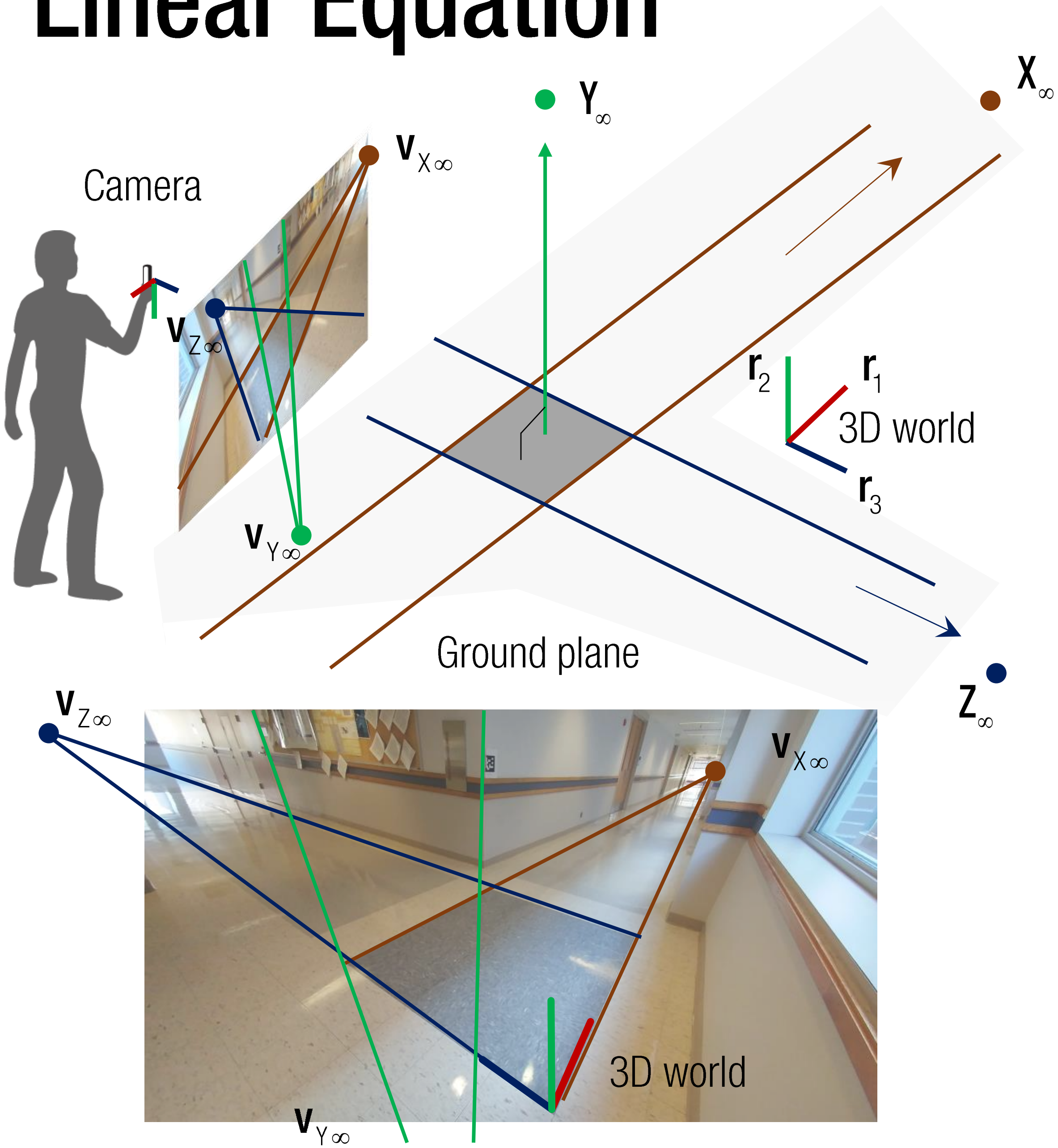
$$\rightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_{i}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{j}\right)=\mathbf{v}_{i}^{T}\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_{j} \qquad i,j=X,Y,Z, \text{ and } i\neq j$$

$$\rightarrow \quad \mathbf{v}_{i}^{T}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\mathbf{v}_{j}=\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^{T}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix}=0$$

Linear in b

$$\rightarrow \quad \begin{bmatrix} u_i u_j + v_i v_j & u_i + u_j & v_i + v_j & 1 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}=0$$

Linear Equation



$$(\mathbf{K}^{-1}\mathbf{v}_{X\infty})^T(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Y\infty})^T(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Z\infty})^T(\mathbf{K}^{-1}\mathbf{v}_{X\infty}) = 0$$

: 3 unknowns and 3 equations

$$\rightarrow \quad (\mathbf{K}^{-1}\mathbf{v}_i)^T(\mathbf{K}^{-1}\mathbf{v}_j) = \mathbf{v}_i^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_j \qquad i,j = X,Y,Z, \text{ and } i \neq j$$

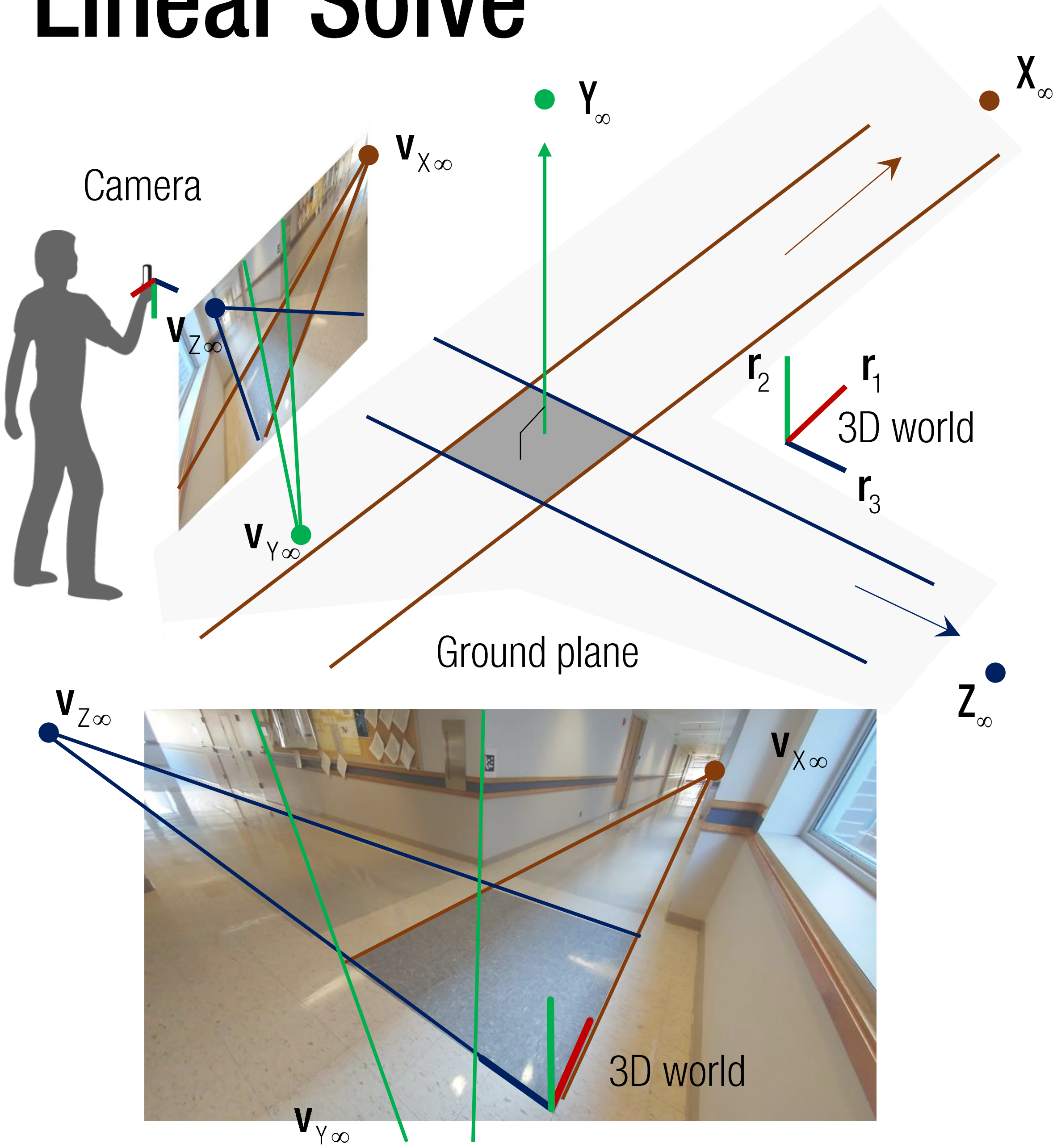
$$\rightarrow \quad \mathbf{v}_i^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \mathbf{v}_j = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix} = 0$$

Linear in b

$$\rightarrow \quad \begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = 0$$

3x4

Linear Solve



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

→ $\left(\mathbf{K}^{-1}\mathbf{v}_i\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_j\right)=\mathbf{v}_i^{T}\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_j$ $\quad i,j=X,Y,Z,$ and $i\neq j$

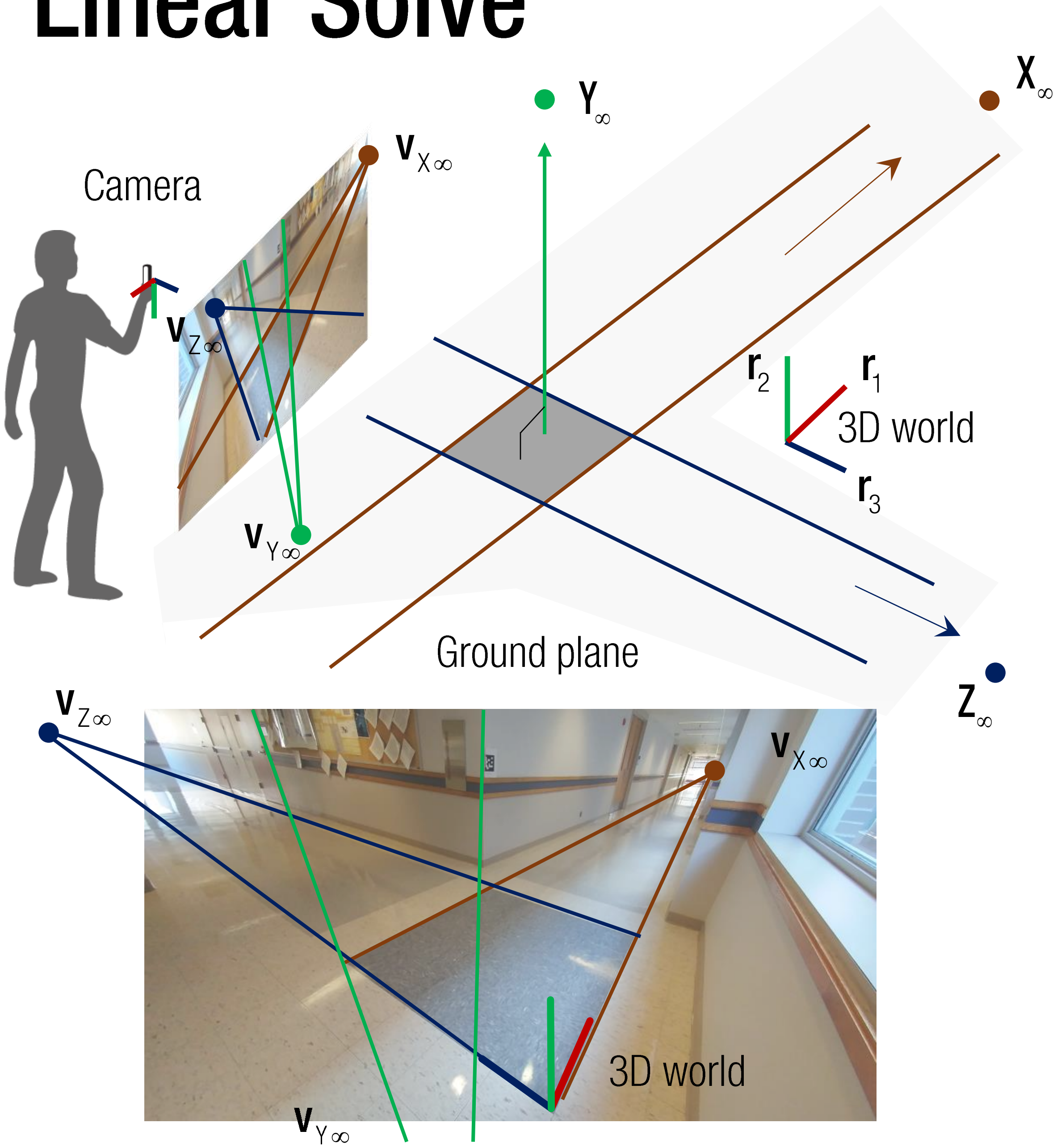
→ $$\mathbf{v}_i^{T}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\mathbf{v}_j=\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^{T}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix}=0$$

Linear in b

→ $$\underbrace{\begin{bmatrix} u_1u_2+v_1v_2 & u_1+u_2 & v_1+v_2 & 1 \\ u_3u_2+v_3v_2 & u_3+u_2 & v_3+v_2 & 1 \\ u_1u_3+v_1v_3 & u_1+u_3 & v_1+v_3 & 1 \end{bmatrix}}_{\mathbf{A}}\underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{\mathbf{x}}=\mathbf{0}$$

3x4

Linear Solve



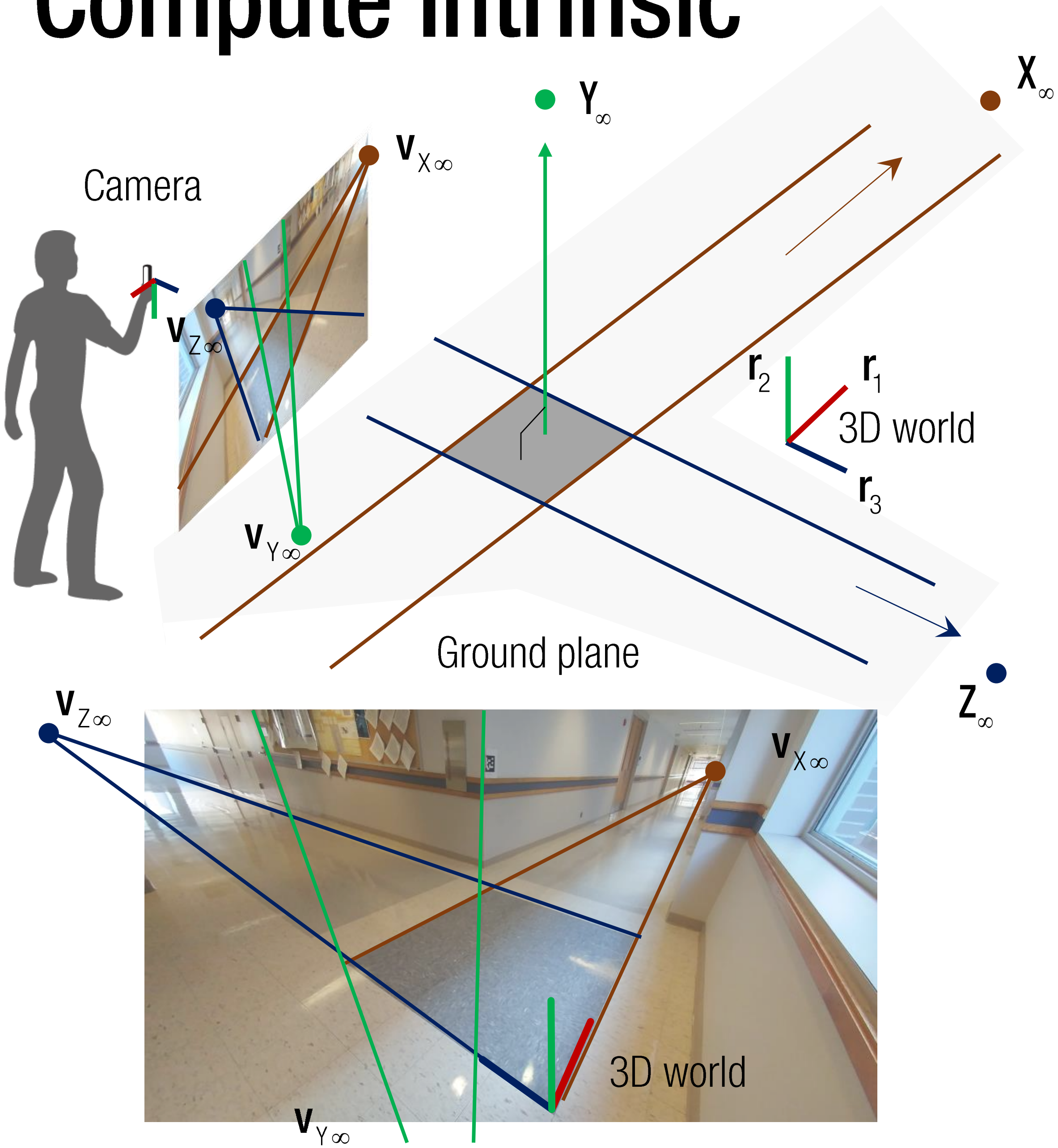
$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

$$\rightarrow \quad \mathbf{x}=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

where $b_1=\frac{1}{f^2},\quad b_2=-\frac{p_x}{f^2},\quad b_3=-\frac{p_y}{f^2},\quad b_4=\frac{p_x^{\,2}}{f^2}+\frac{p_y^2}{f^2}+1$

Compute Intrinsic



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

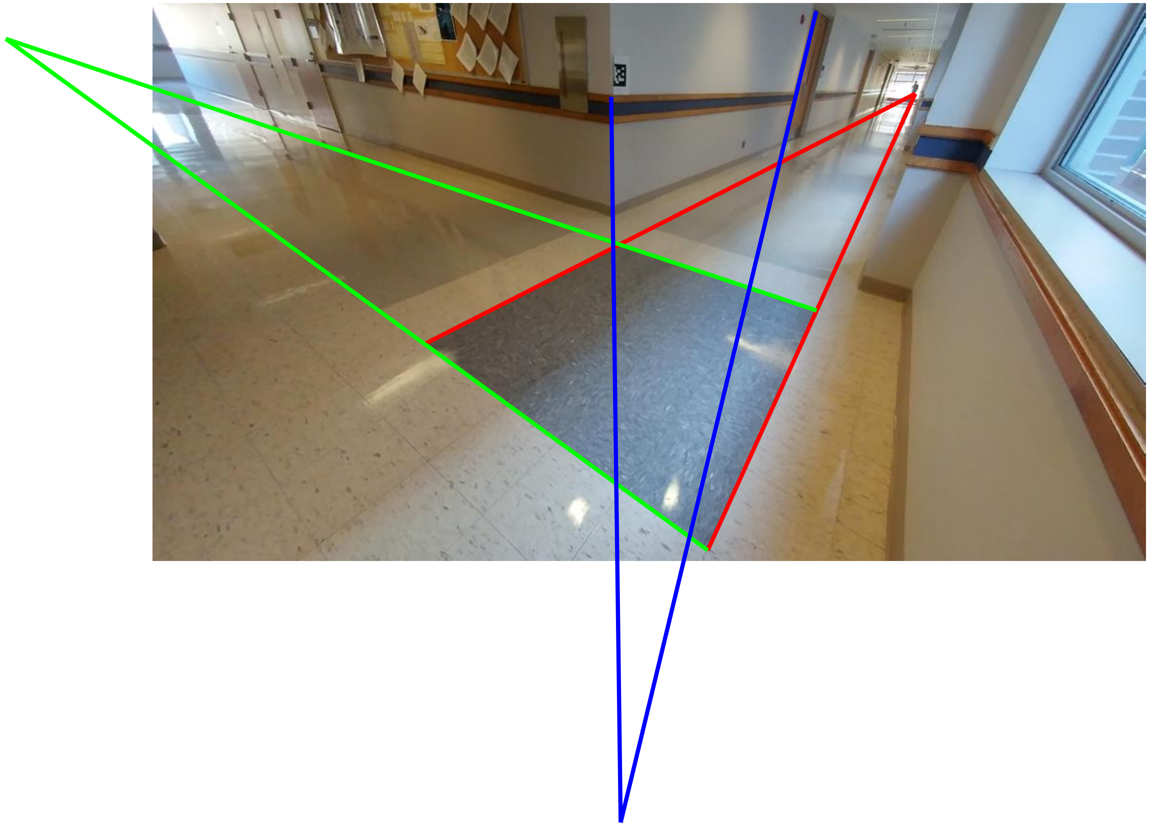
: 3 unknowns and 3 equations

$$\rightarrow \quad \mathbf{x}=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

where $b_1=\frac{1}{f^2},\quad b_2=-\frac{p_x}{f^2},\quad b_3=-\frac{p_y}{f^2},\quad b_4=\frac{p_x^2}{f^2}+\frac{p_y^2}{f^2}+1$

$$\rightarrow \quad p_x=-\frac{b_2}{b_1},\quad p_y=-\frac{b_3}{b_1},\quad f=\sqrt{\frac{b_4}{b_1}-(p_x^2+p_y^2)}$$

Camera Calibration



K =

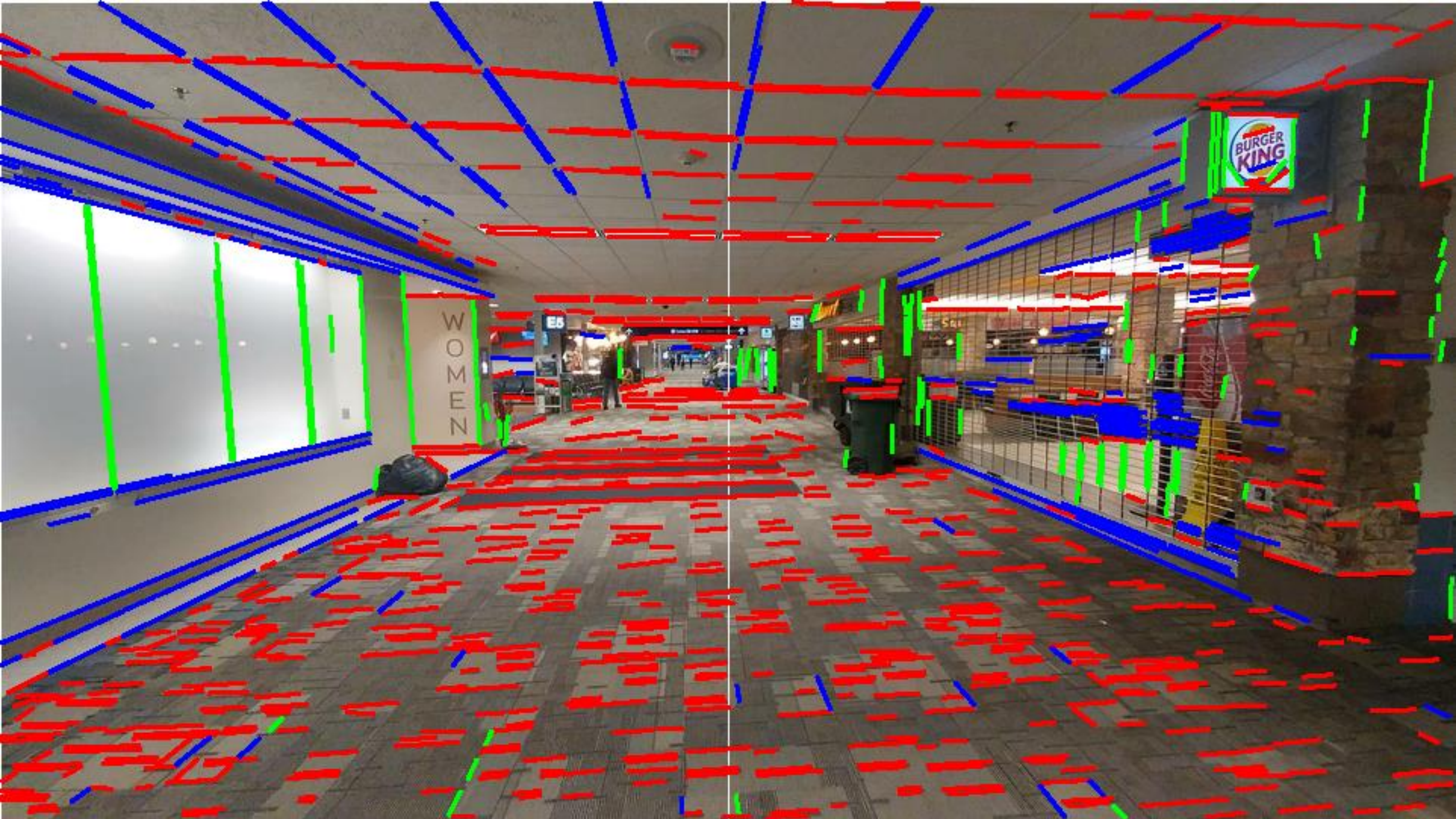
1317.2	0	1931.8
0	1317.2	1146.1
0	0	1

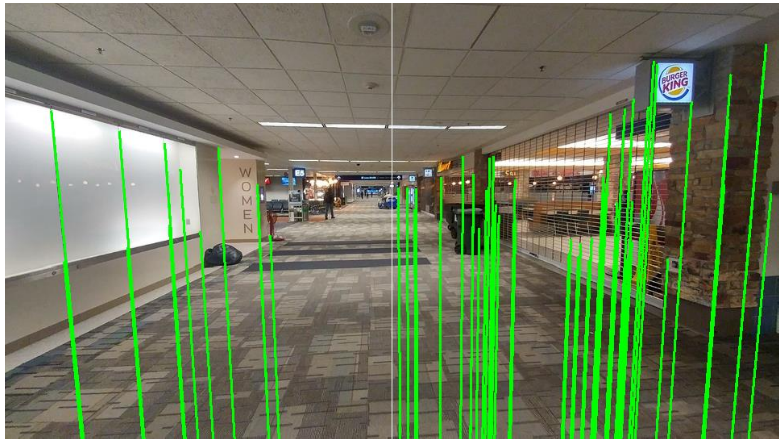
$f = 1224$

$p_x = \text{size(im,2)}/2 = 1920$

$p_y = \text{size(im,1)}/2 = 1080$

Previous manual estimate


WOMEN
BURGER KING
E6



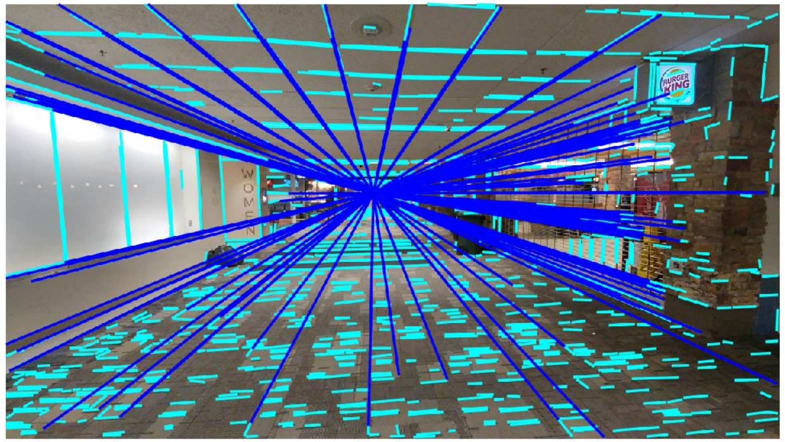
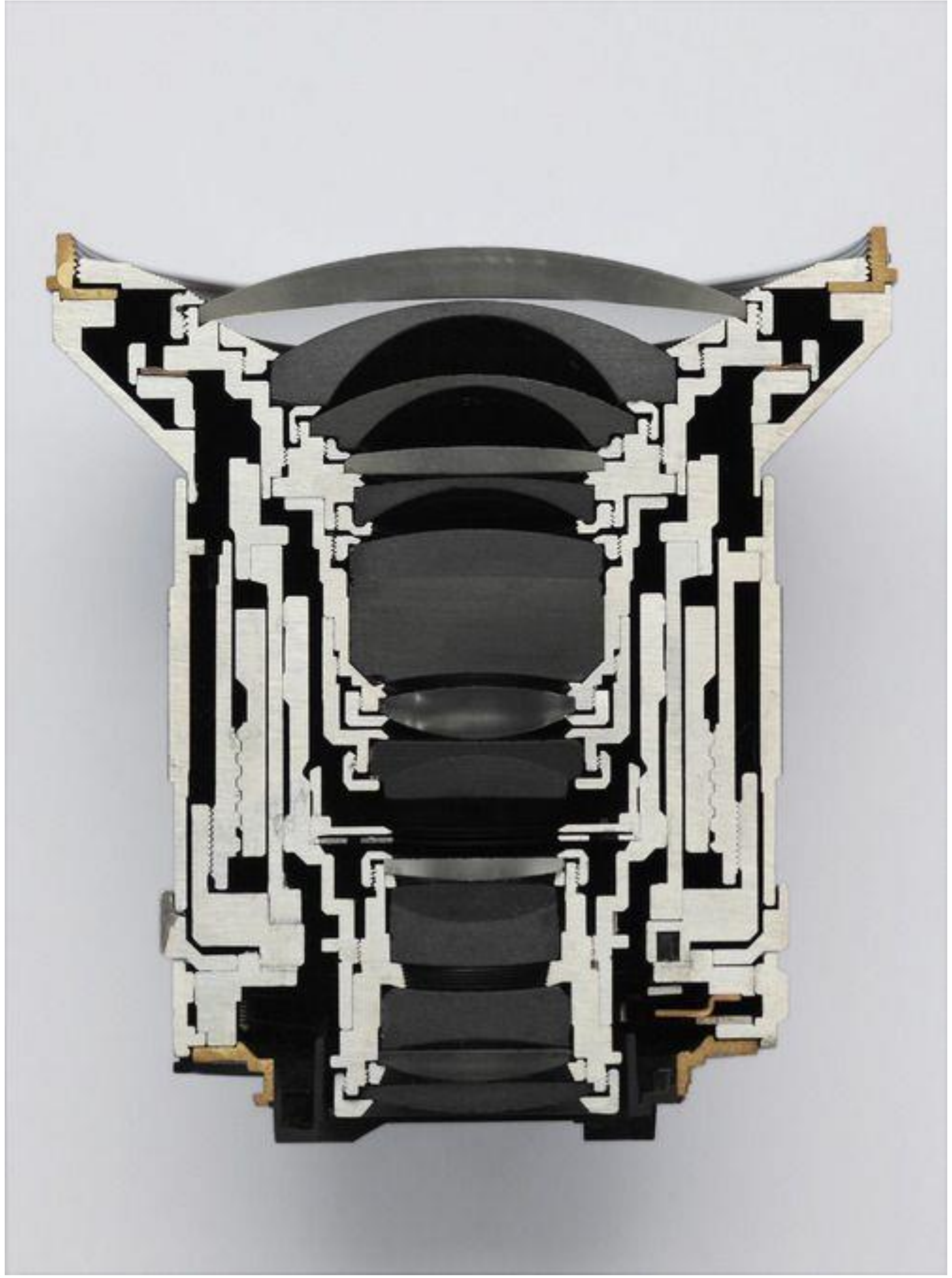
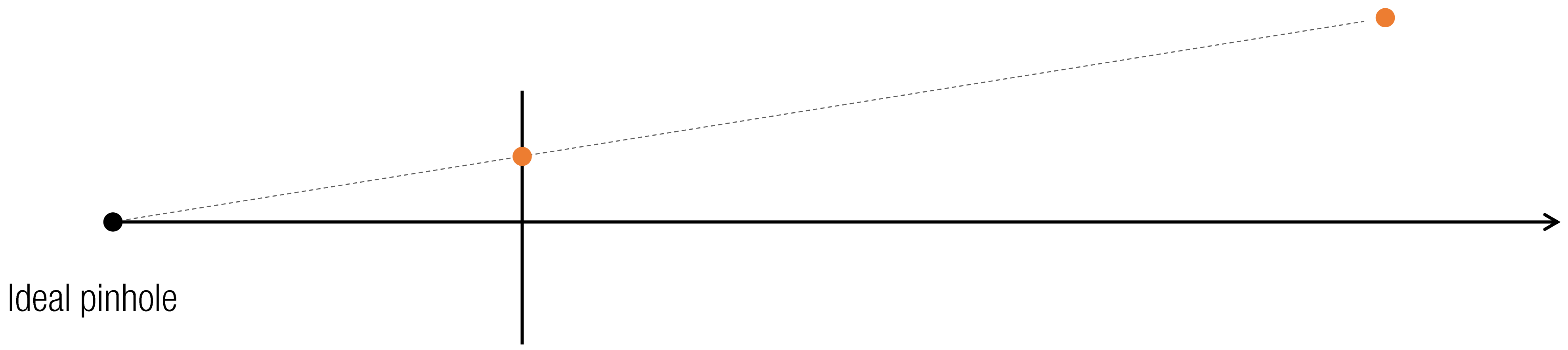


Image size: 960x540

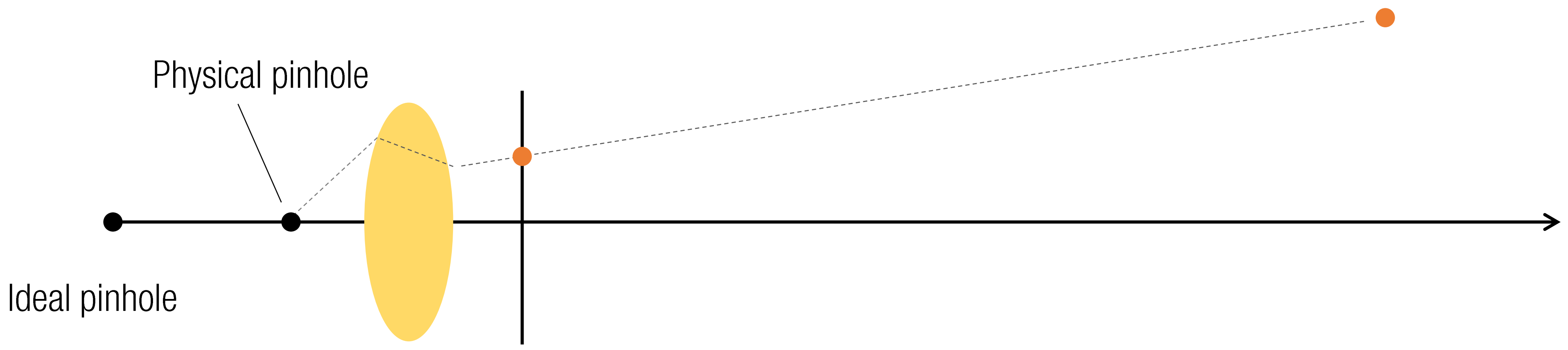
$$K = \begin{bmatrix} 296.0220 & 0 & 477.8139 \\ 0 & 296.0220 & 255.3901 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

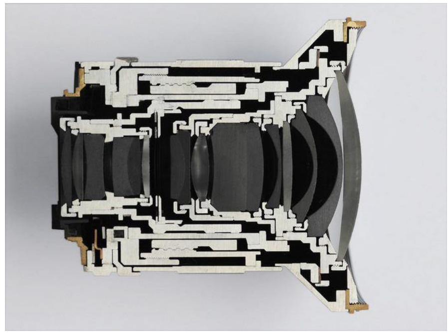


Focal Point

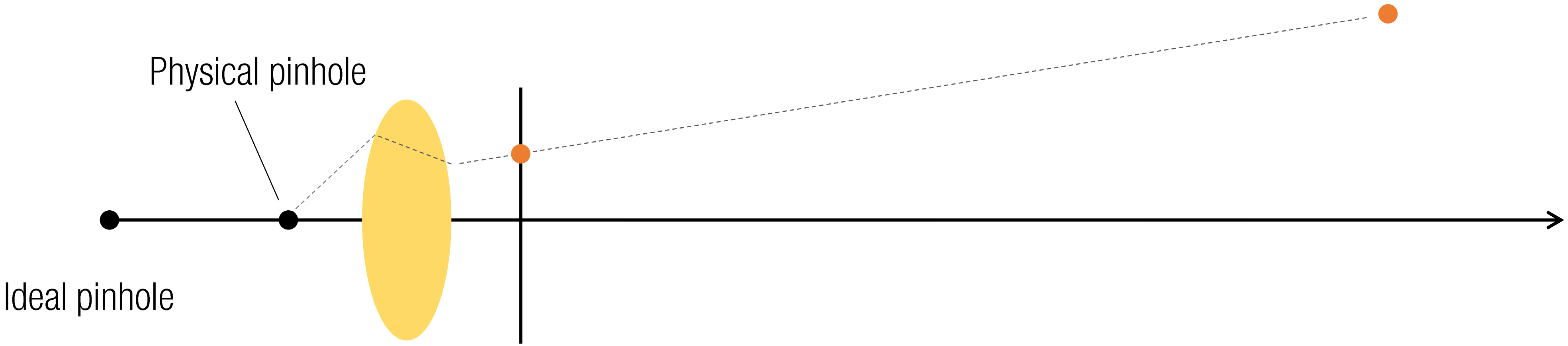


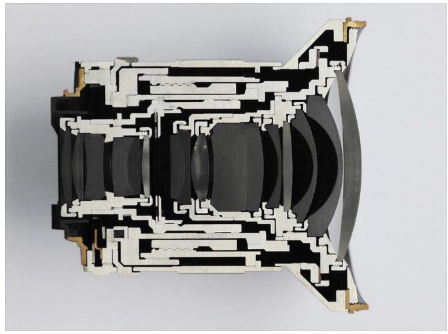
Focal Point





Focal Point



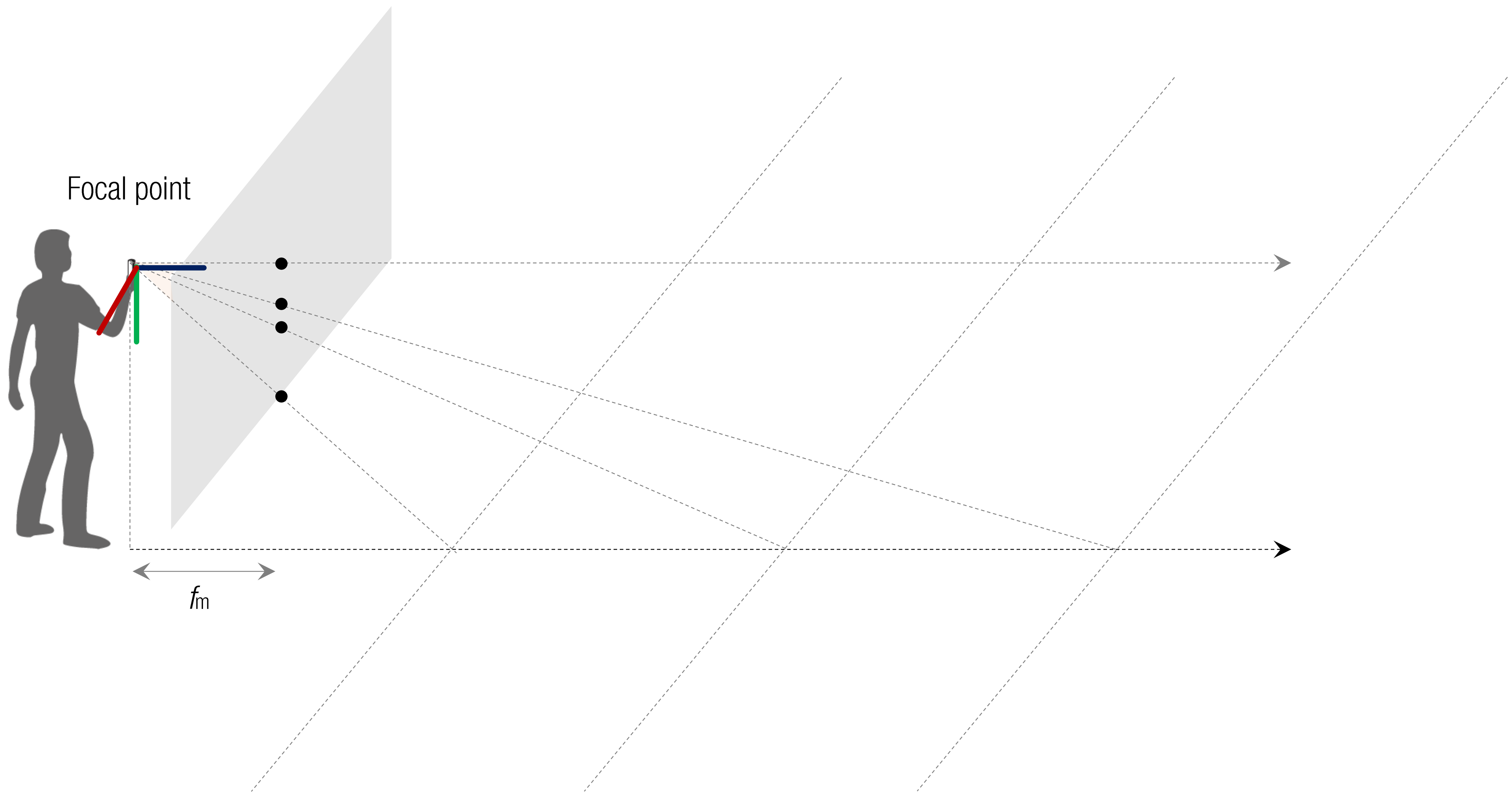


Where is the ideal pinhole?

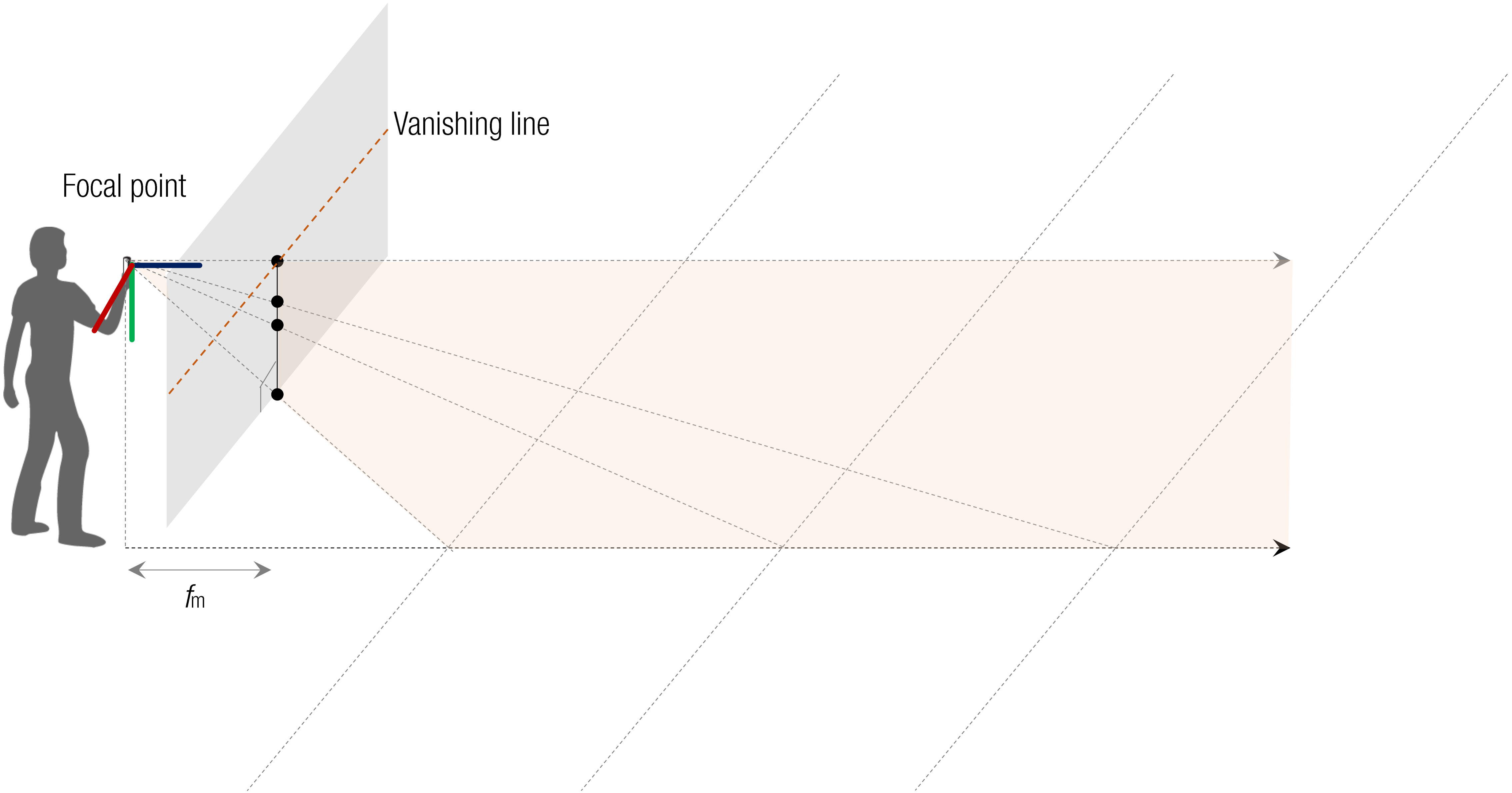
Focal point
f_m

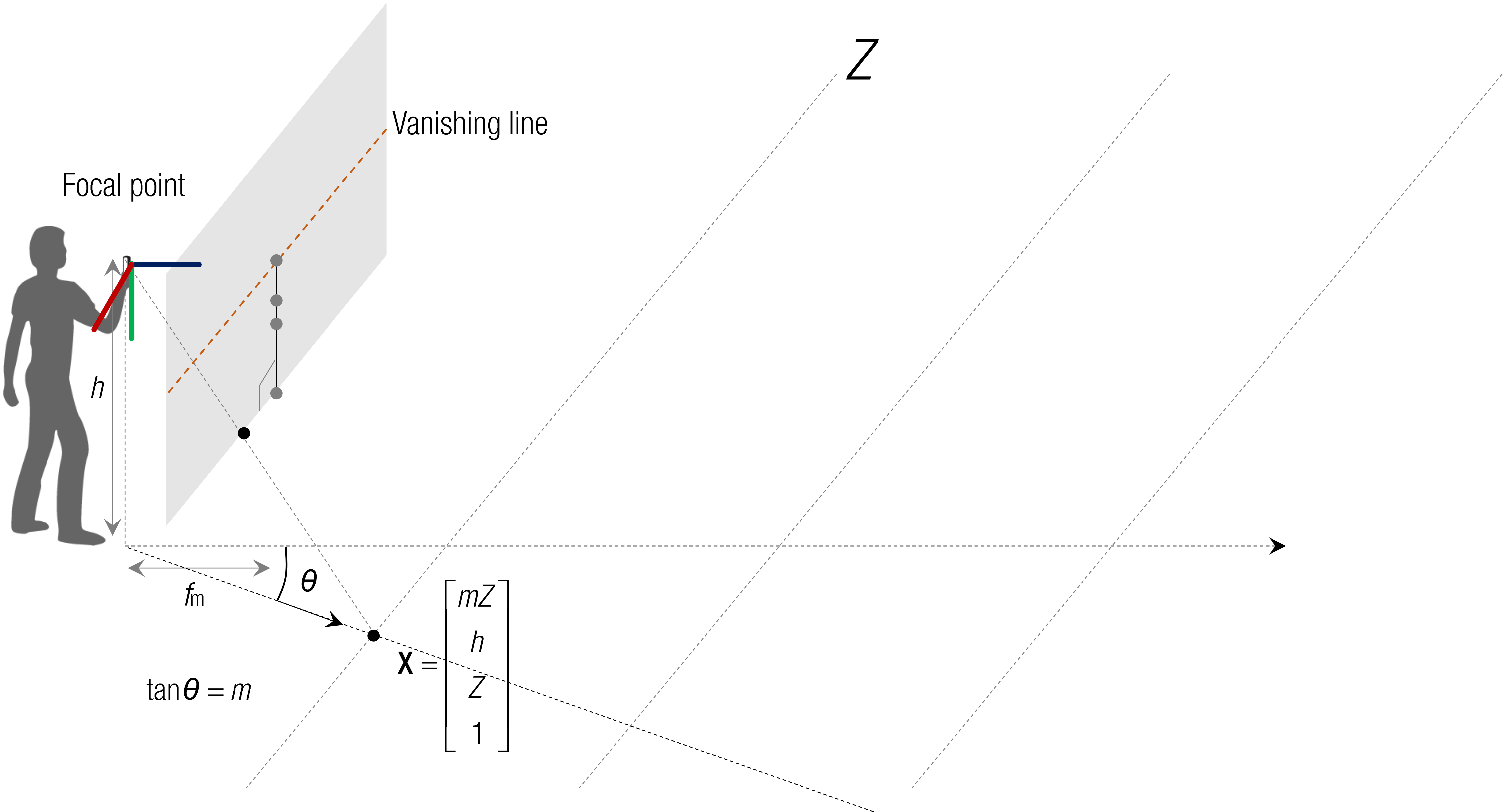

Focal point
f_m


Focal point
f_m

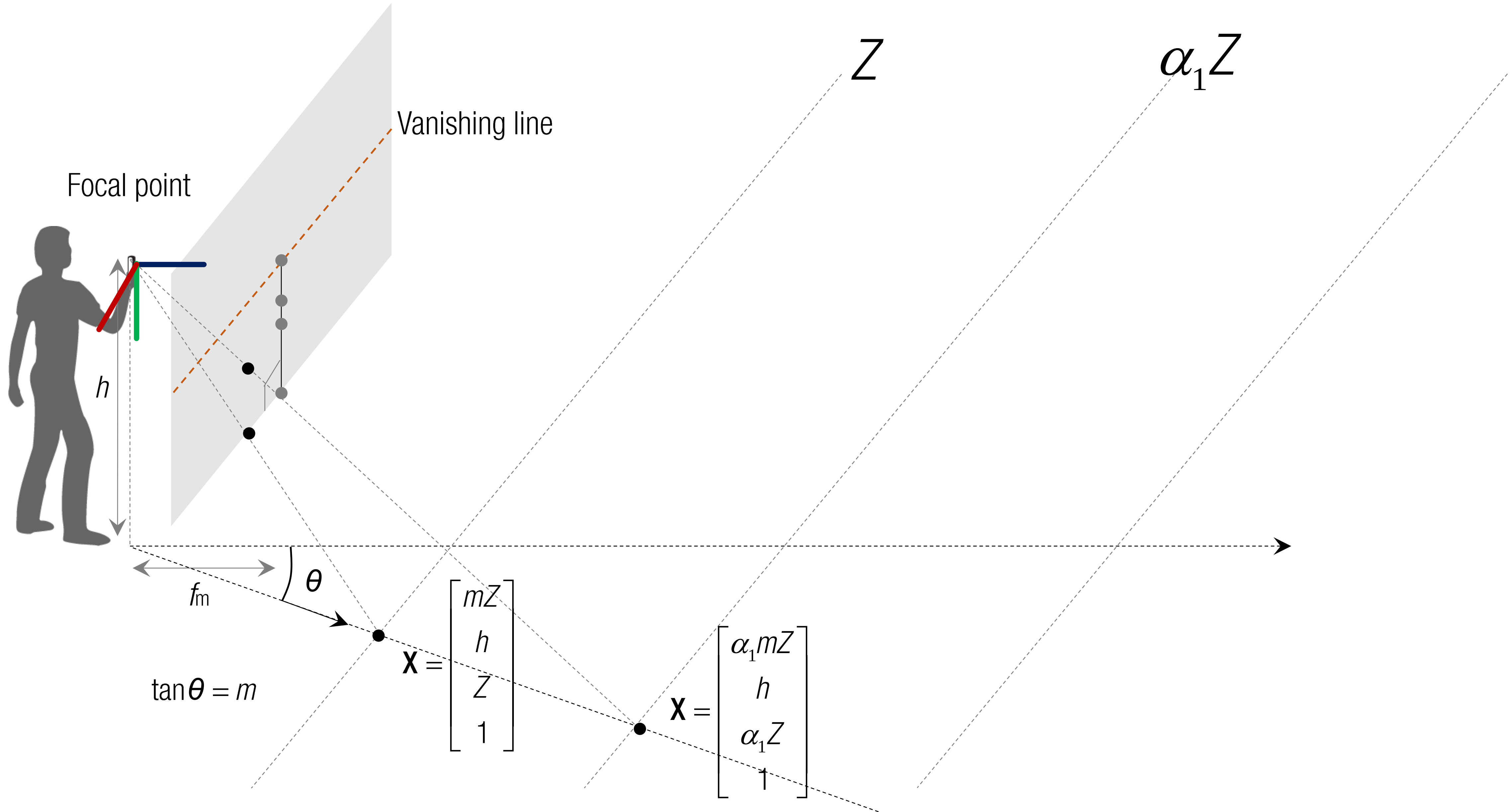


Focal point
f_m

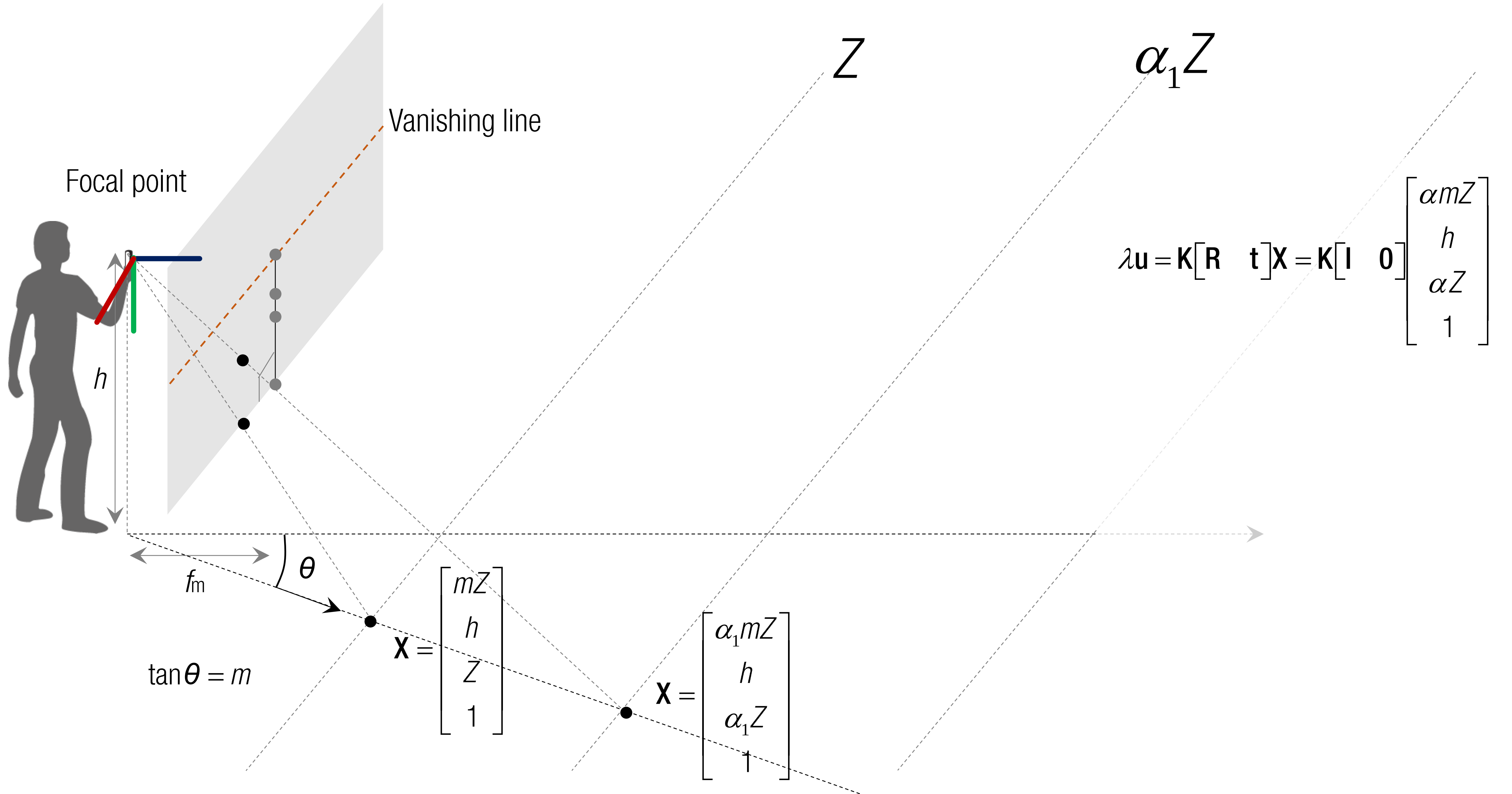

Vanishing line
Focal point
f_m



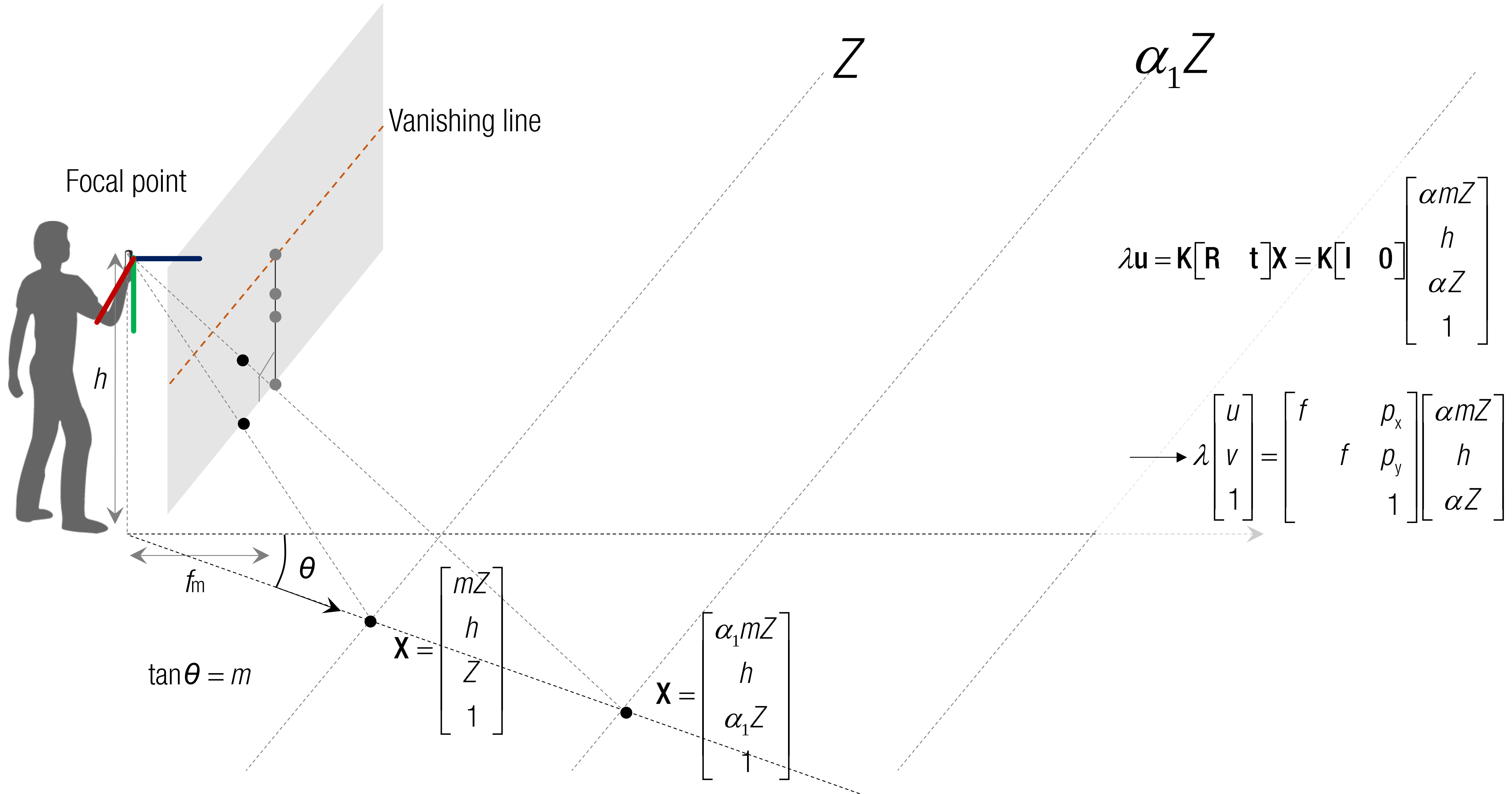
Z
Vanishing line
Focal point
h
f_m
θ
$\mathbf{X} = \begin{bmatrix} mZ \\ h \\ Z \\ 1 \end{bmatrix}$
$\tan\theta = m$



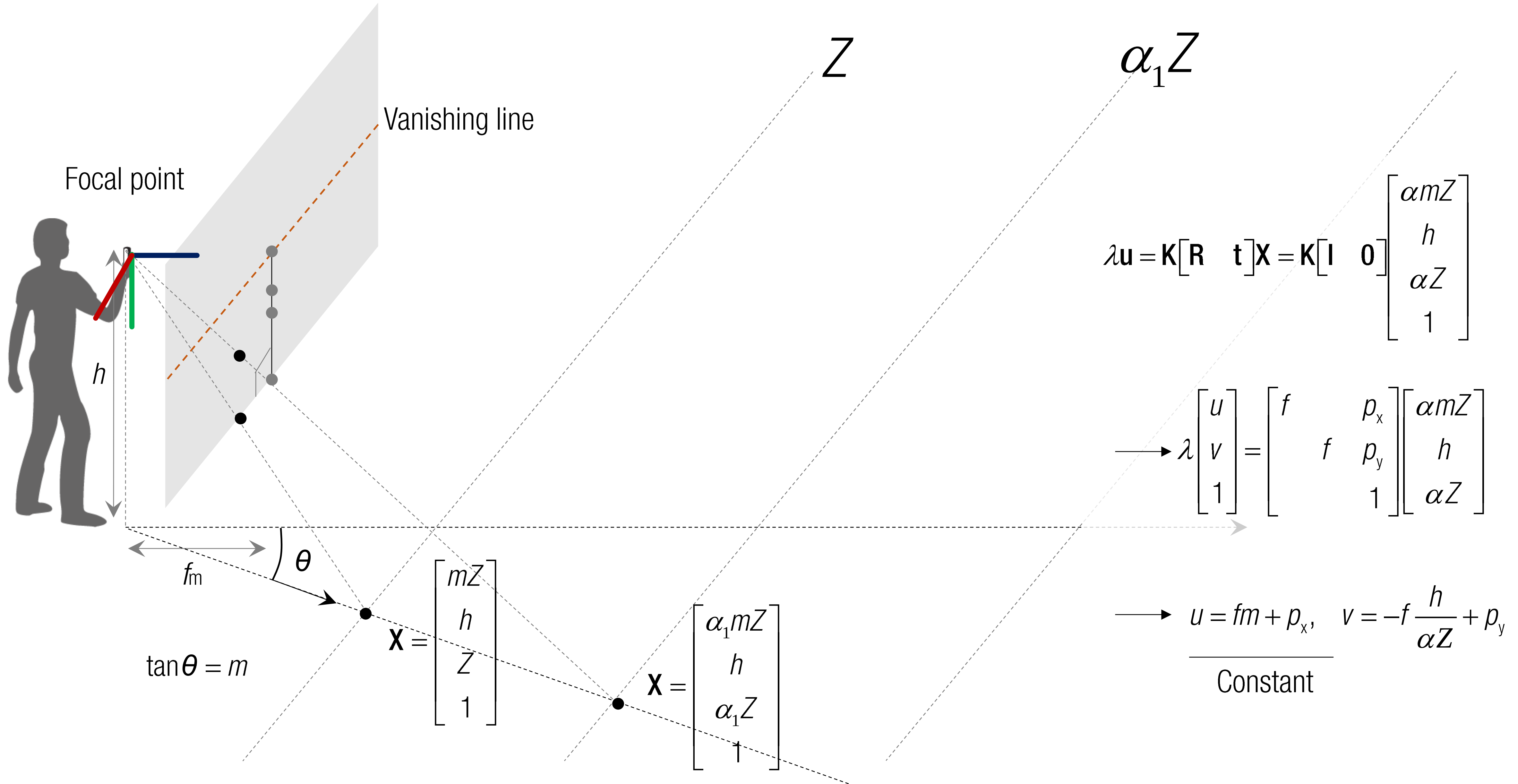
Focal point
Vanishing line
Z
$\alpha_1 Z$
h
f_m
θ
$\tan\theta = m$
$\mathbf{X} = \begin{bmatrix} mZ \\ h \\ Z \\ 1 \end{bmatrix}$
$\mathbf{X} = \begin{bmatrix} \alpha_1 mZ \\ h \\ \alpha_1 Z \\ 1 \end{bmatrix}$

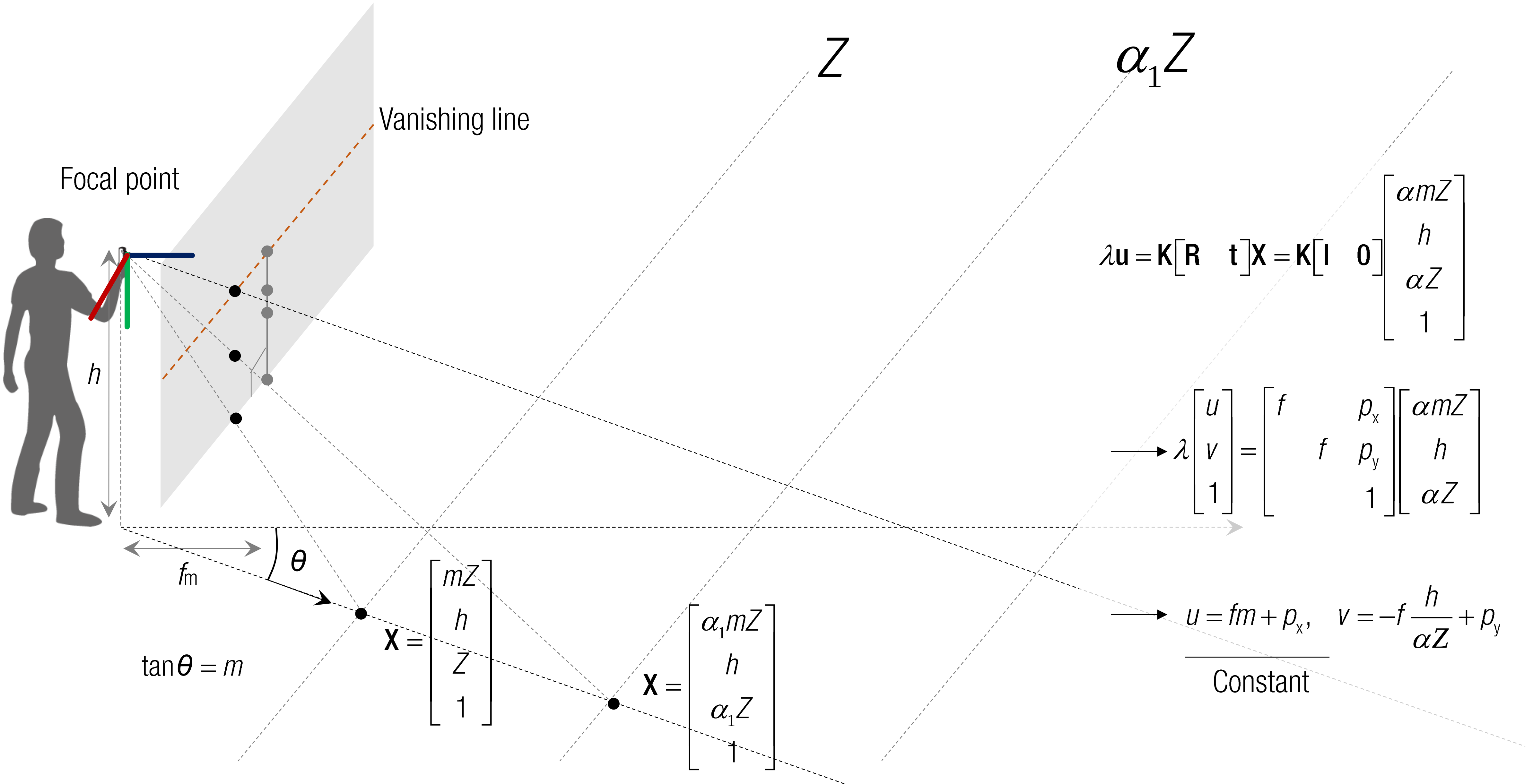


Z
α1Z
Vanishing line
Focal point
λu = K[R t]X = K[I 0][αmZ h αZ 1]
h
θ
fm
X = [mZ h Z 1]
X = [α1mZ h α1Z 1]
tanθ = m

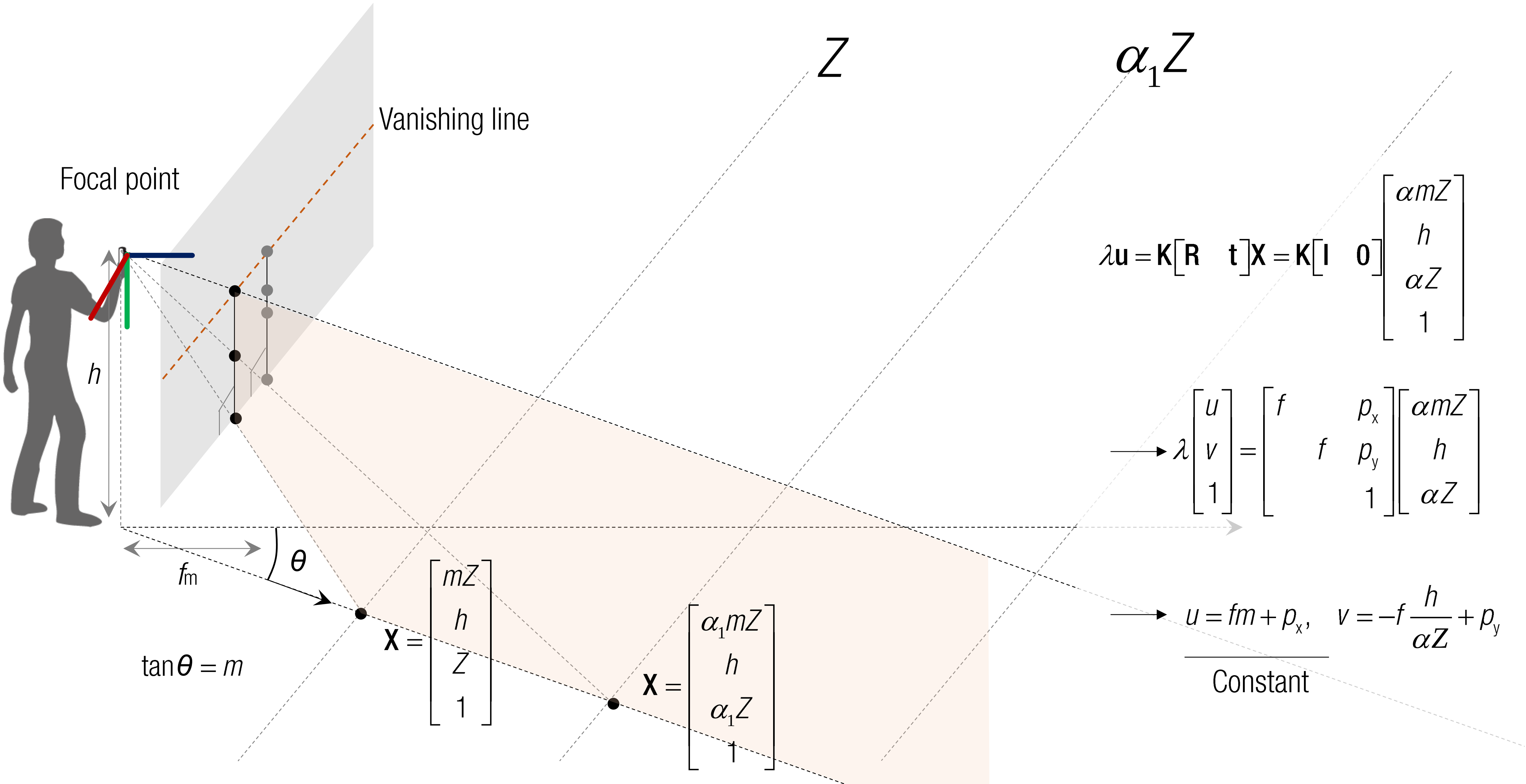


Focal point
Vanishing line
Z
$\alpha_1 Z$
h
f_m
θ
$\tan\theta = m$
$\mathbf{X} = \begin{bmatrix} mZ \\ h \\ Z \\ 1 \end{bmatrix}$
$\mathbf{X} = \begin{bmatrix} \alpha_1 mZ \\ h \\ \alpha_1 Z \\ 1 \end{bmatrix}$
$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\mathbf{X} = \mathbf{K}\begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \alpha mZ \\ h \\ \alpha Z \\ 1 \end{bmatrix}$
$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\begin{bmatrix} \alpha mZ \\ h \\ \alpha Z \end{bmatrix}$

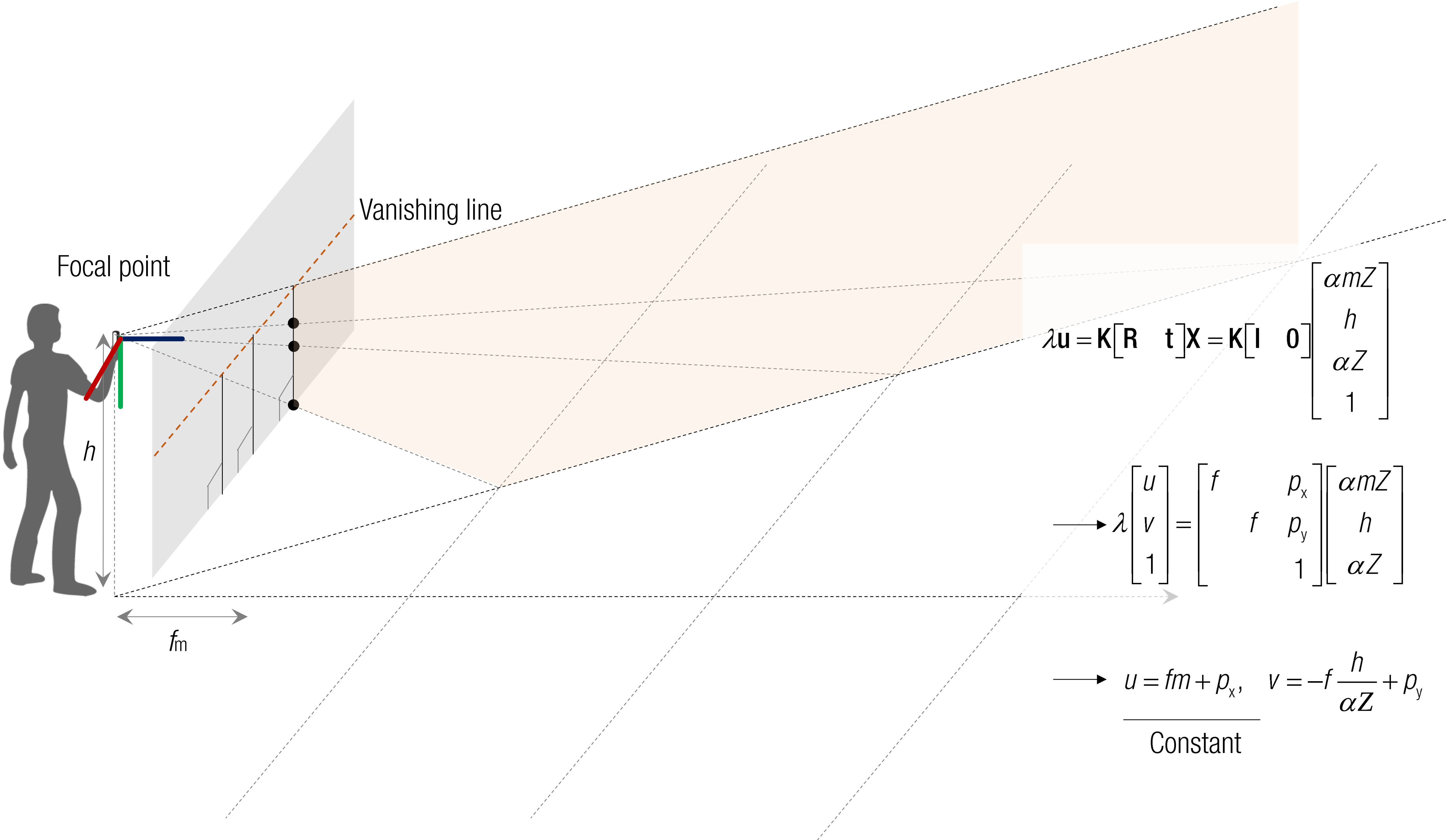

Focal point
Vanishing line
Z
α1Z
h
fm
θ
tanθ = m
X = [mZ h Z 1]
X = [α1mZ h α1Z 1]
λu = K[R t]X = K[I 0][αmZ h αZ 1]
λ[u v 1] = [f px; f py; 1][αmZ h αZ]
u = fm + px, v = −f h/(αZ) + py
Constant



Z
$\alpha_1 Z$
Vanishing line
Focal point
h
fm
θ
$\tan\theta = m$
$\mathbf{X} = \begin{bmatrix} mZ \\ h \\ Z \\ 1 \end{bmatrix}$
$\mathbf{X} = \begin{bmatrix} \alpha_1 mZ \\ h \\ \alpha_1 Z \\ 1 \end{bmatrix}$
$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\mathbf{X} = \mathbf{K}\begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \alpha mZ \\ h \\ \alpha Z \\ 1 \end{bmatrix}$
$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\begin{bmatrix} \alpha mZ \\ h \\ \alpha Z \end{bmatrix}$
$u = fm + p_x, \quad v = -f\dfrac{h}{\alpha Z} + p_y$
Constant



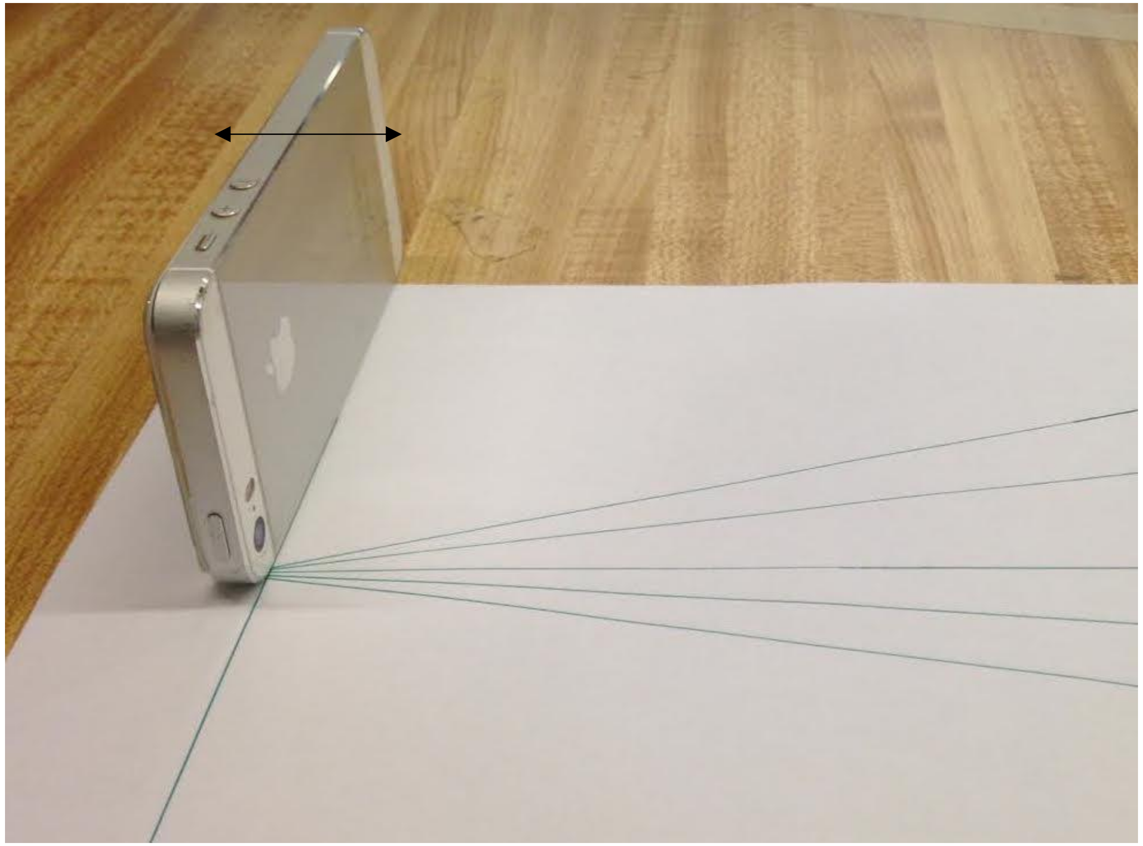
Focal point
Vanishing line
Z
$\alpha_1 Z$
h
fm
θ
$\tan\theta = m$
$\mathbf{X} = \begin{bmatrix} mZ \\ h \\ Z \\ 1 \end{bmatrix}$
$\mathbf{X} = \begin{bmatrix} \alpha_1 mZ \\ h \\ \alpha_1 Z \\ 1 \end{bmatrix}$
$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\mathbf{X} = \mathbf{K}\begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \alpha mZ \\ h \\ \alpha Z \\ 1 \end{bmatrix}$
$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\begin{bmatrix} \alpha mZ \\ h \\ \alpha Z \end{bmatrix}$
$u = fm + p_x, \quad v = -f\dfrac{h}{\alpha Z} + p_y$
Constant

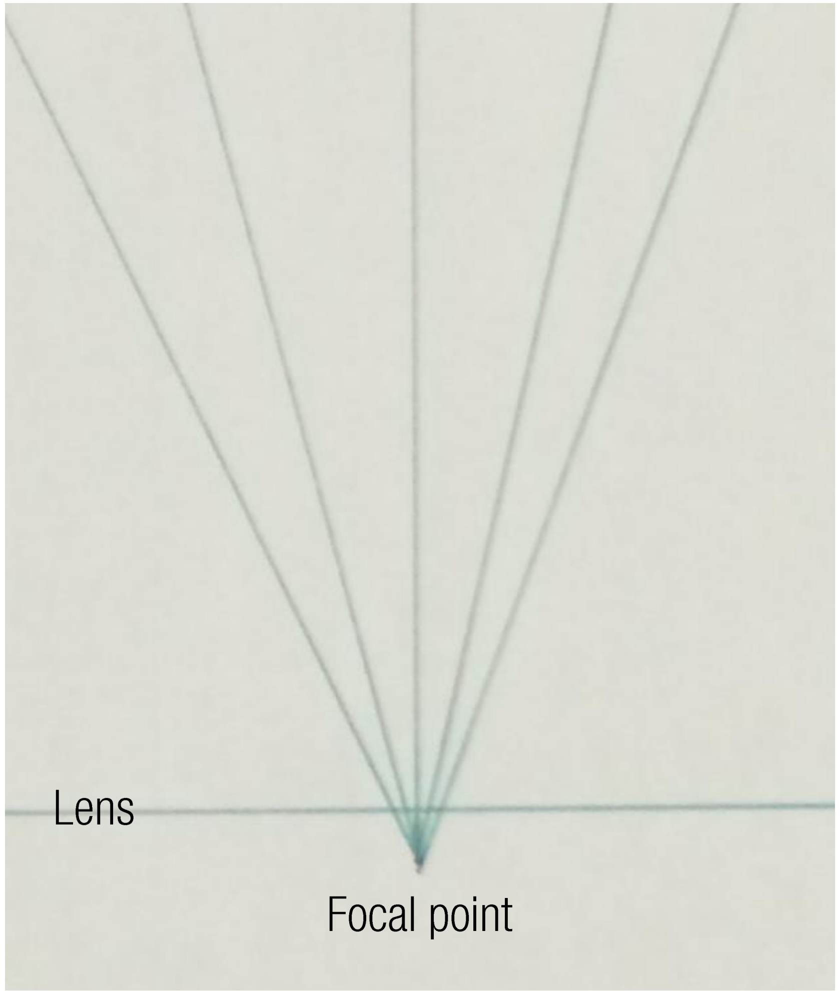


$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\alpha m Z \\ h \\ \alpha Z \\ 1\end{bmatrix}$$

$$\rightarrow \lambda\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix}f & & p_x \\ & f & p_y \\ & & 1\end{bmatrix}\begin{bmatrix}\alpha m Z \\ h \\ \alpha Z\end{bmatrix}$$

$$\rightarrow \underbrace{u = fm + p_x,}_{\text{Constant}} \quad v = -f\frac{h}{\alpha Z} + p_y$$




Lens
Focal point