



# *Multiview Camera Calibration*

Hyun Soo Park

# Camera Intrinsic Parameter

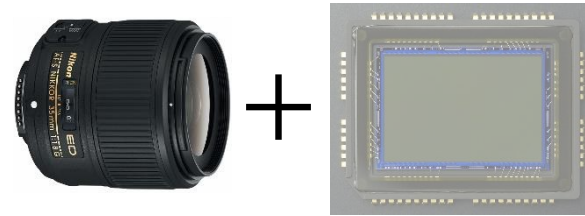


Pixel space

Metric space

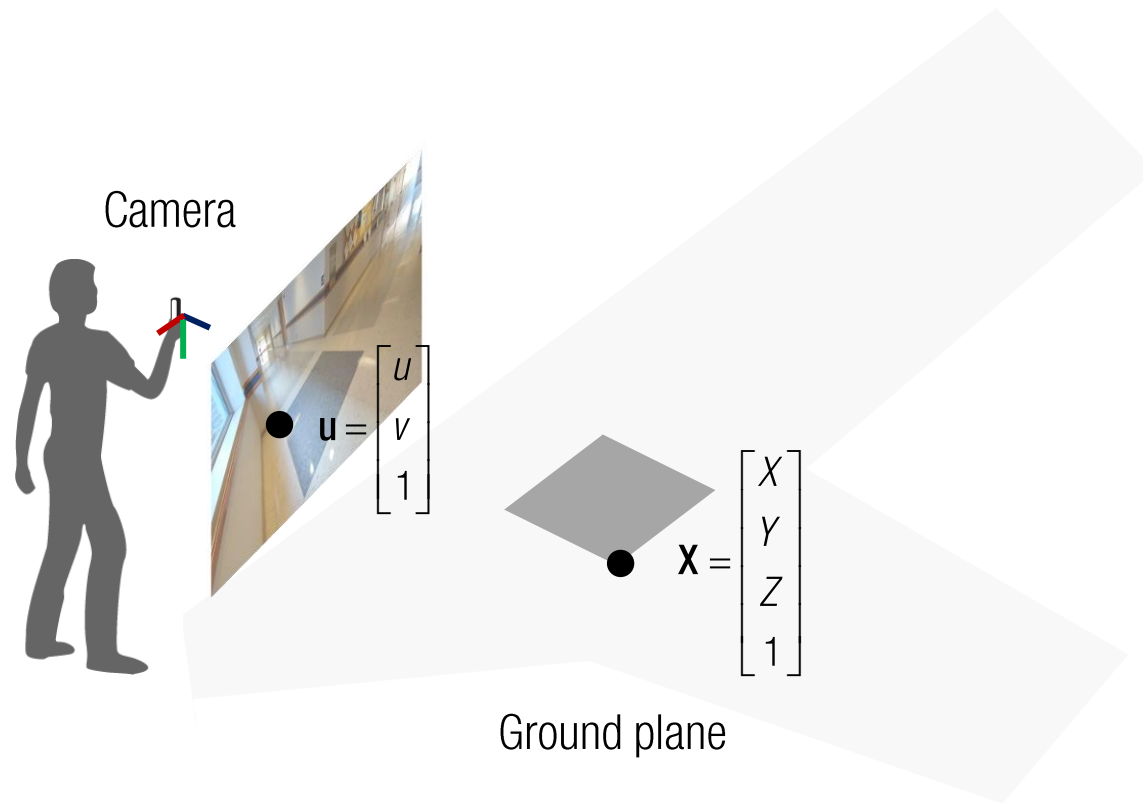
$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & t_y & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

**K**



Camera intrinsic parameter  
: metric space to pixel space

# Camera Calibration in Pixel Space



$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} [\mathbf{R} \quad \mathbf{t}] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 ( $\mathbf{K}$ ) + 6F ( $\mathbf{R}$  and  $\mathbf{t}$ ) + 3P ( $\mathbf{X}$ )

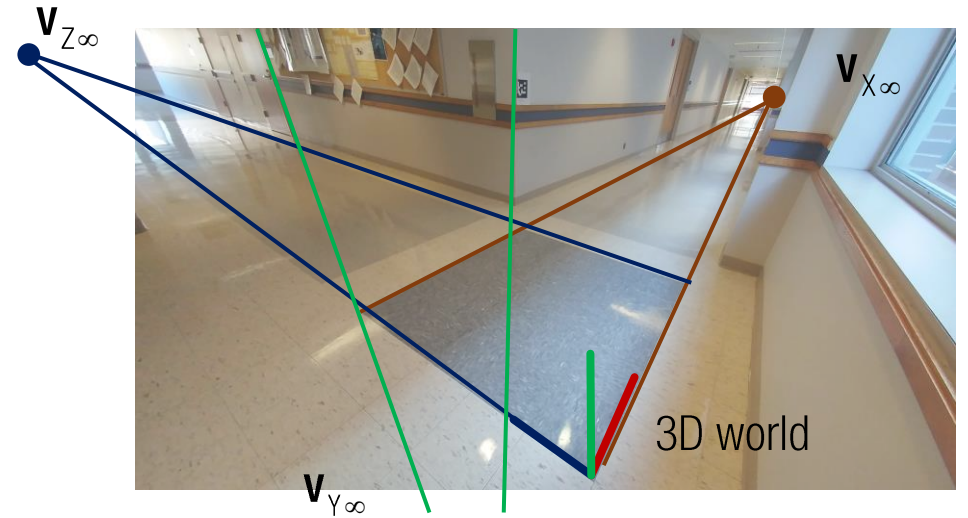
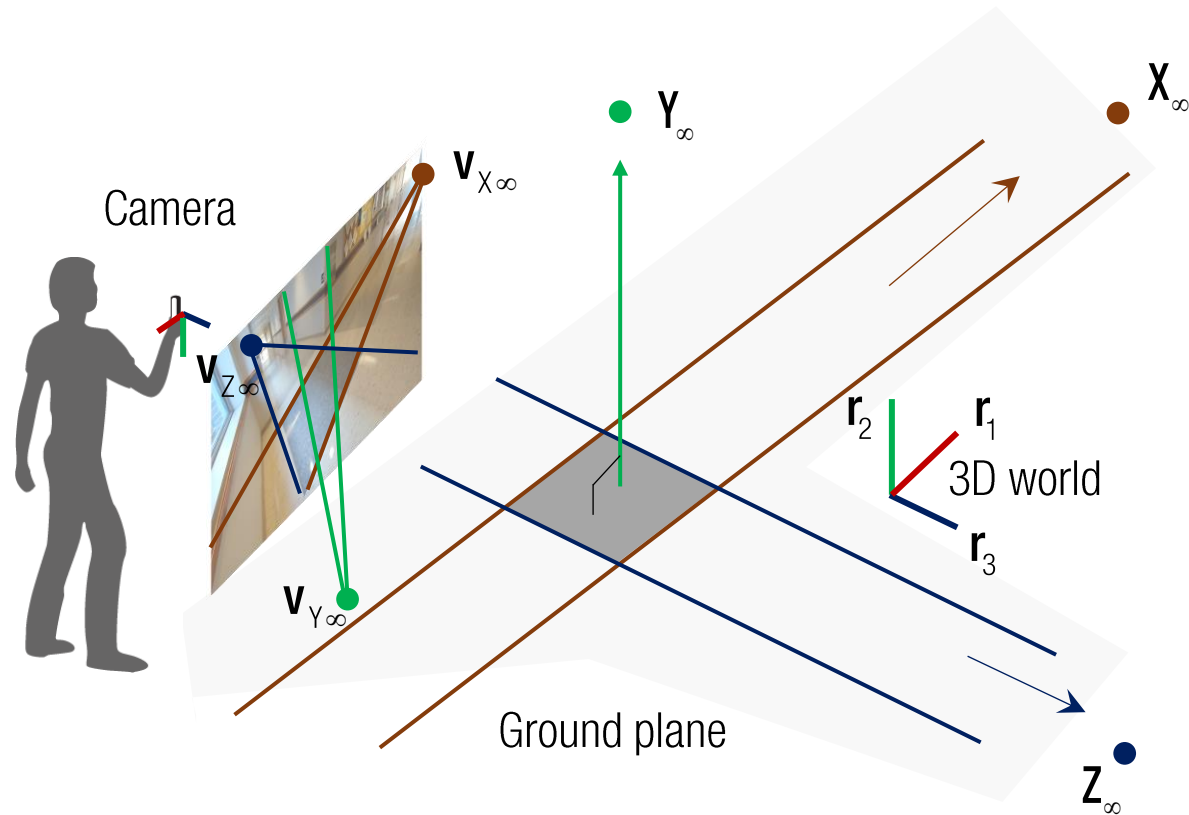
# of equations: 2P

where  $F$  is # of images and  $P$  is # of points.

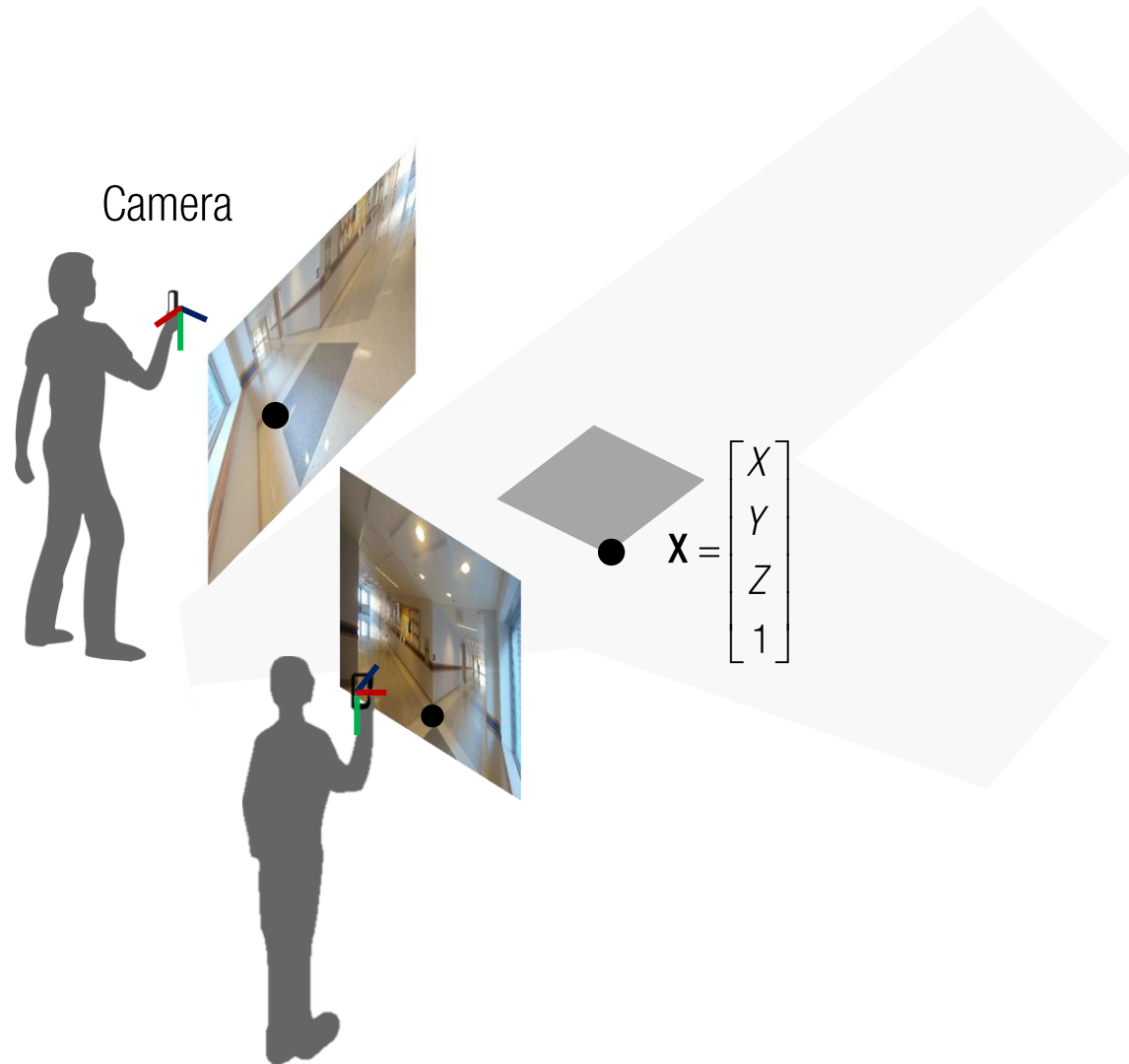
# of unknowns > # of equations

What do we know about the scene?

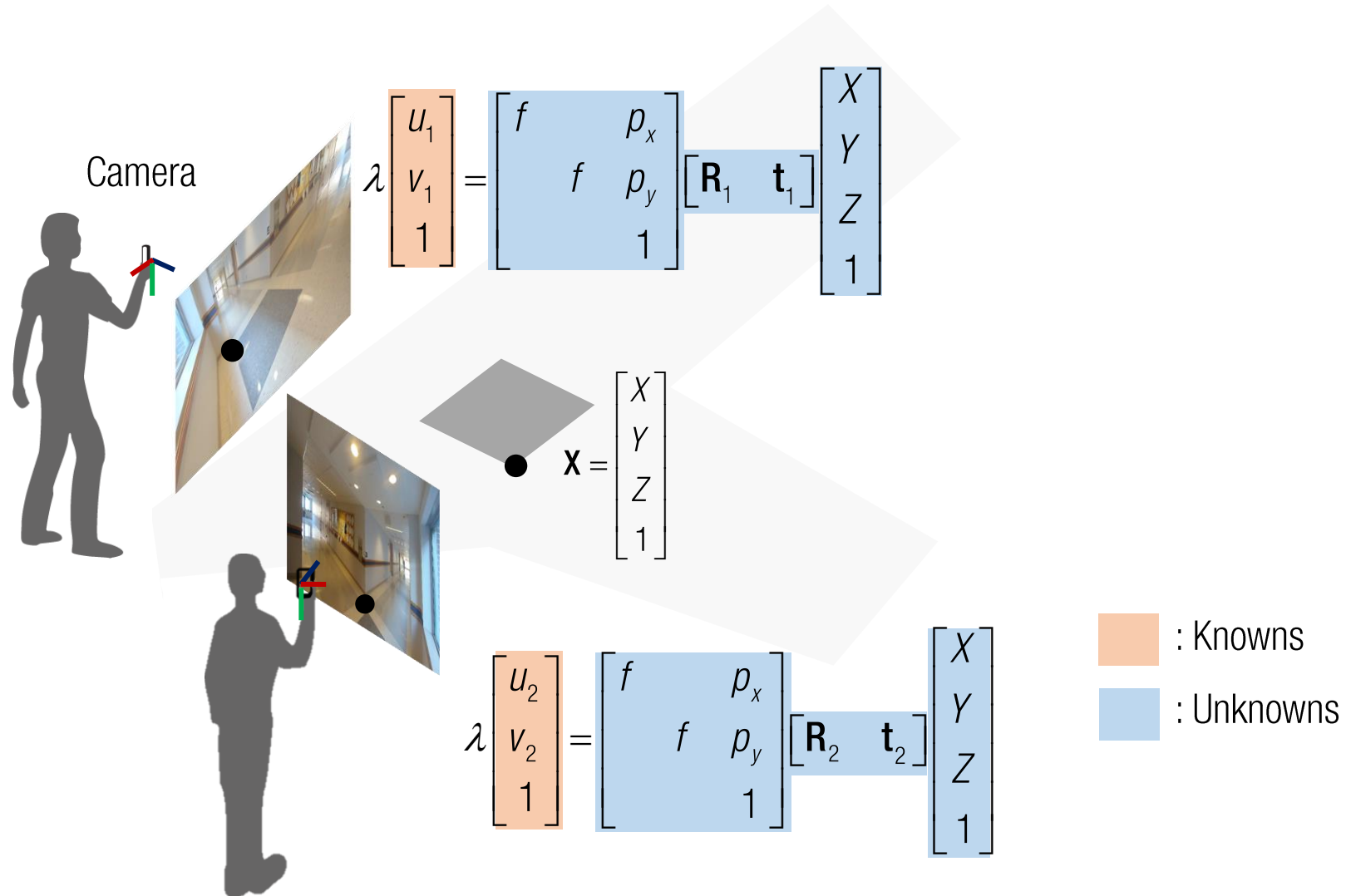
# Camera Calibration via Vanishing Points



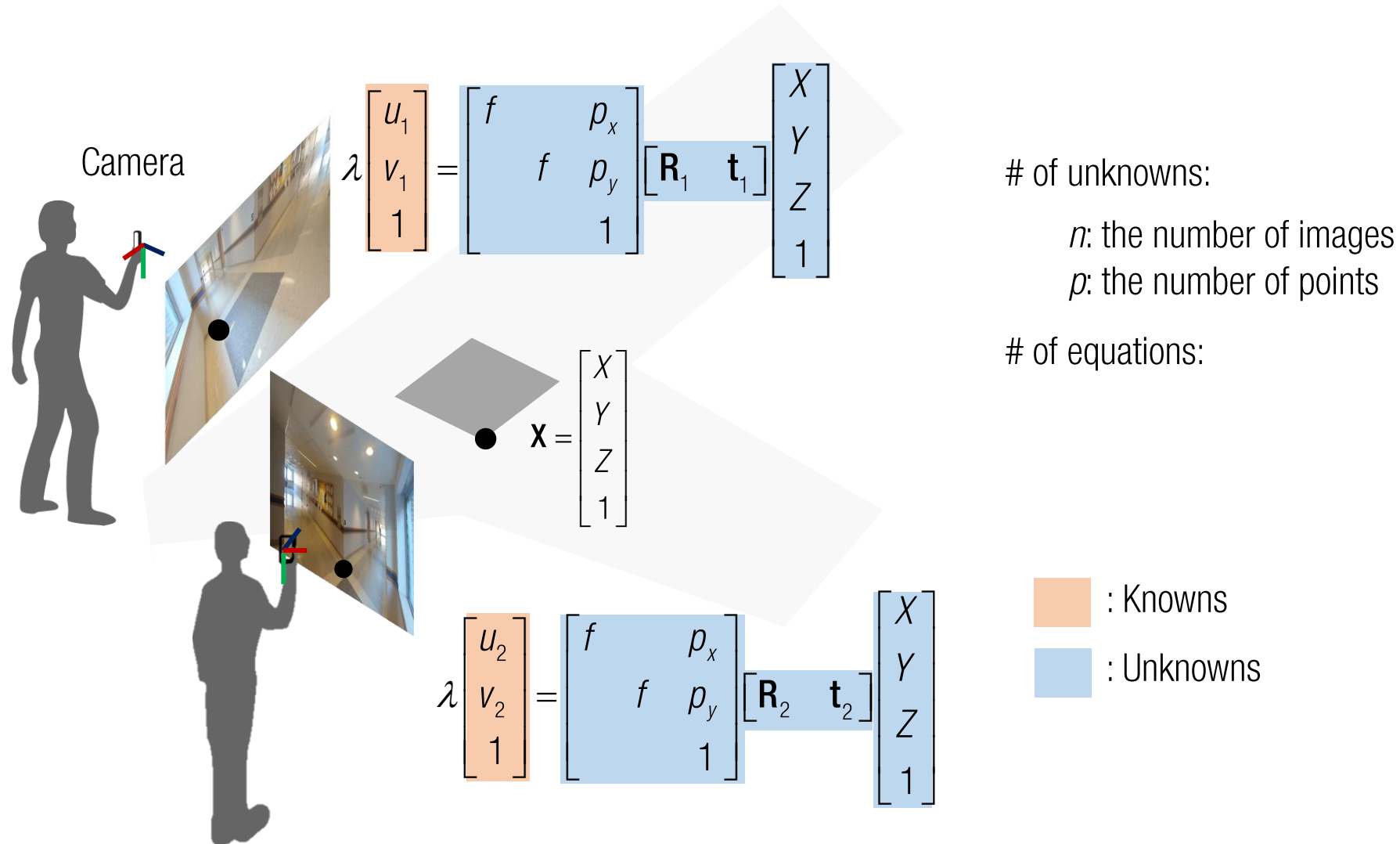
# Multiview Camera Calibration



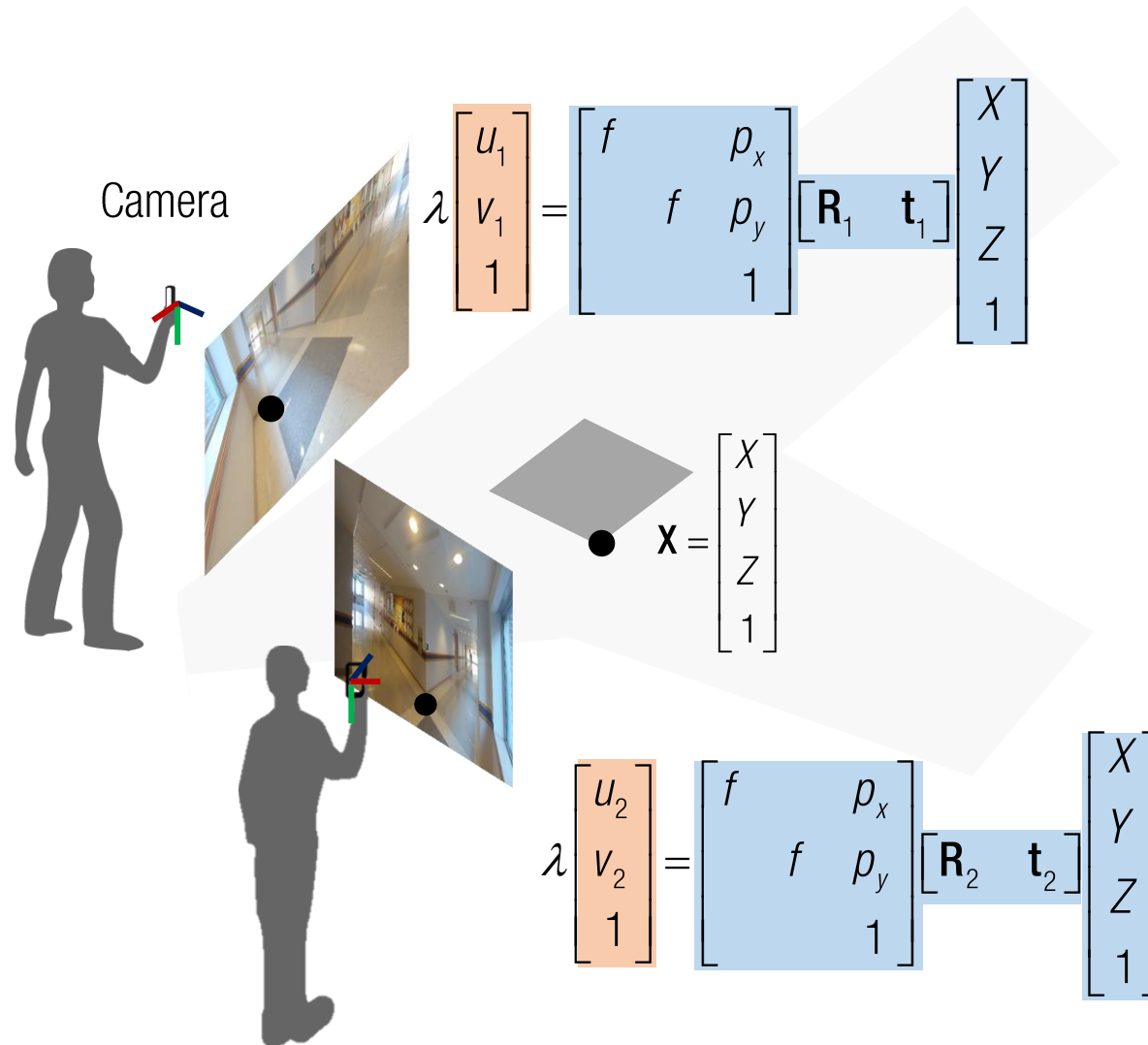
# Multiview Camera Calibration



# Multiview Camera Calibration



# Multiview Camera Calibration



# of unknowns:  $3 (\mathbf{K}) + 6n (\mathbf{R} \text{ and } \mathbf{t}) + 3p (\mathbf{X})$

$n$ : the number of images

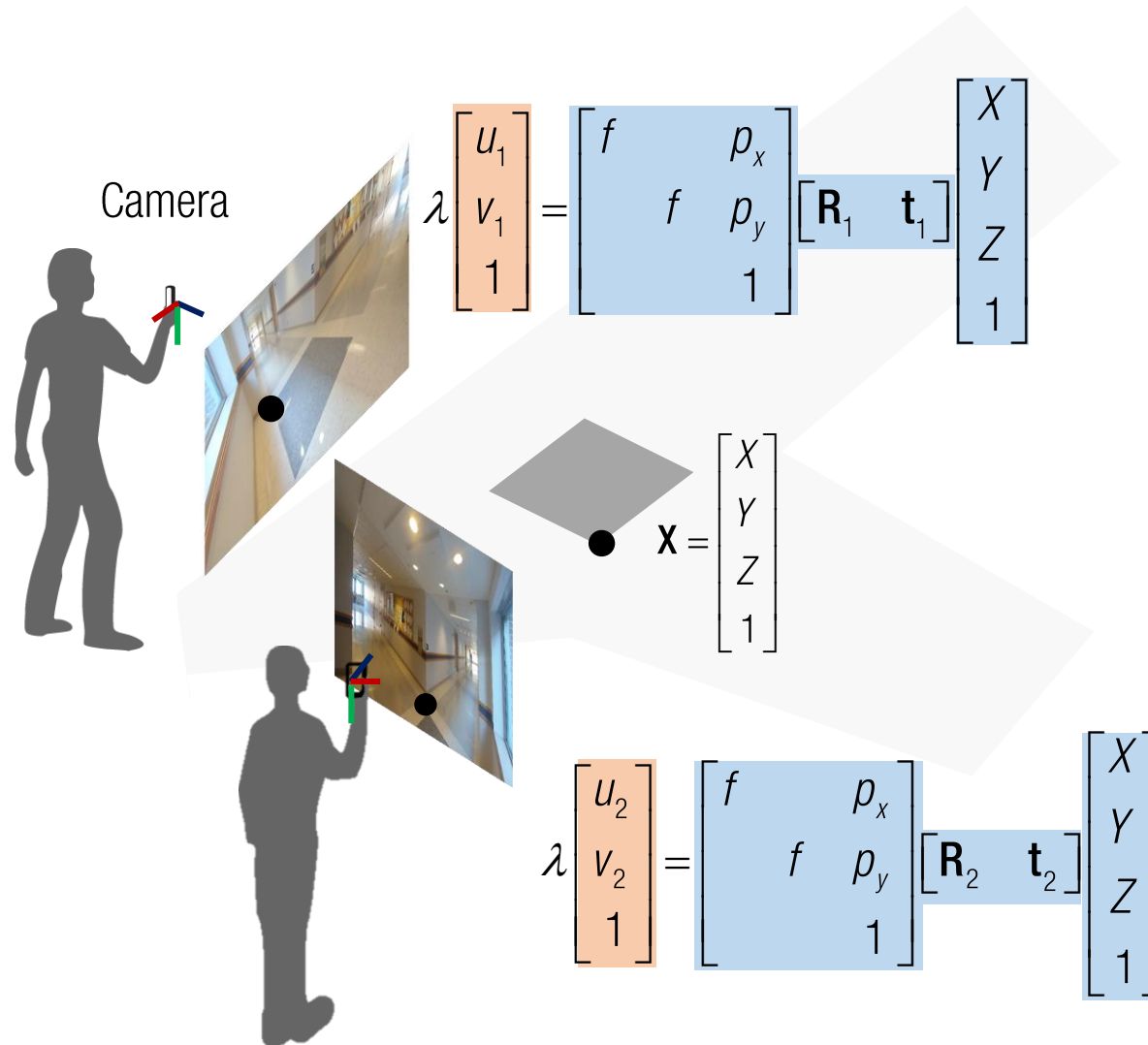
$p$ : the number of points

# of equations:

: Knowns

: Unknowns

# Multiview Camera Calibration



# of unknowns:  $3 (\mathbf{K}) + 6n (\mathbf{R} \text{ and } \mathbf{t}) + 3p (\mathbf{X})$

$n$ : the number of images

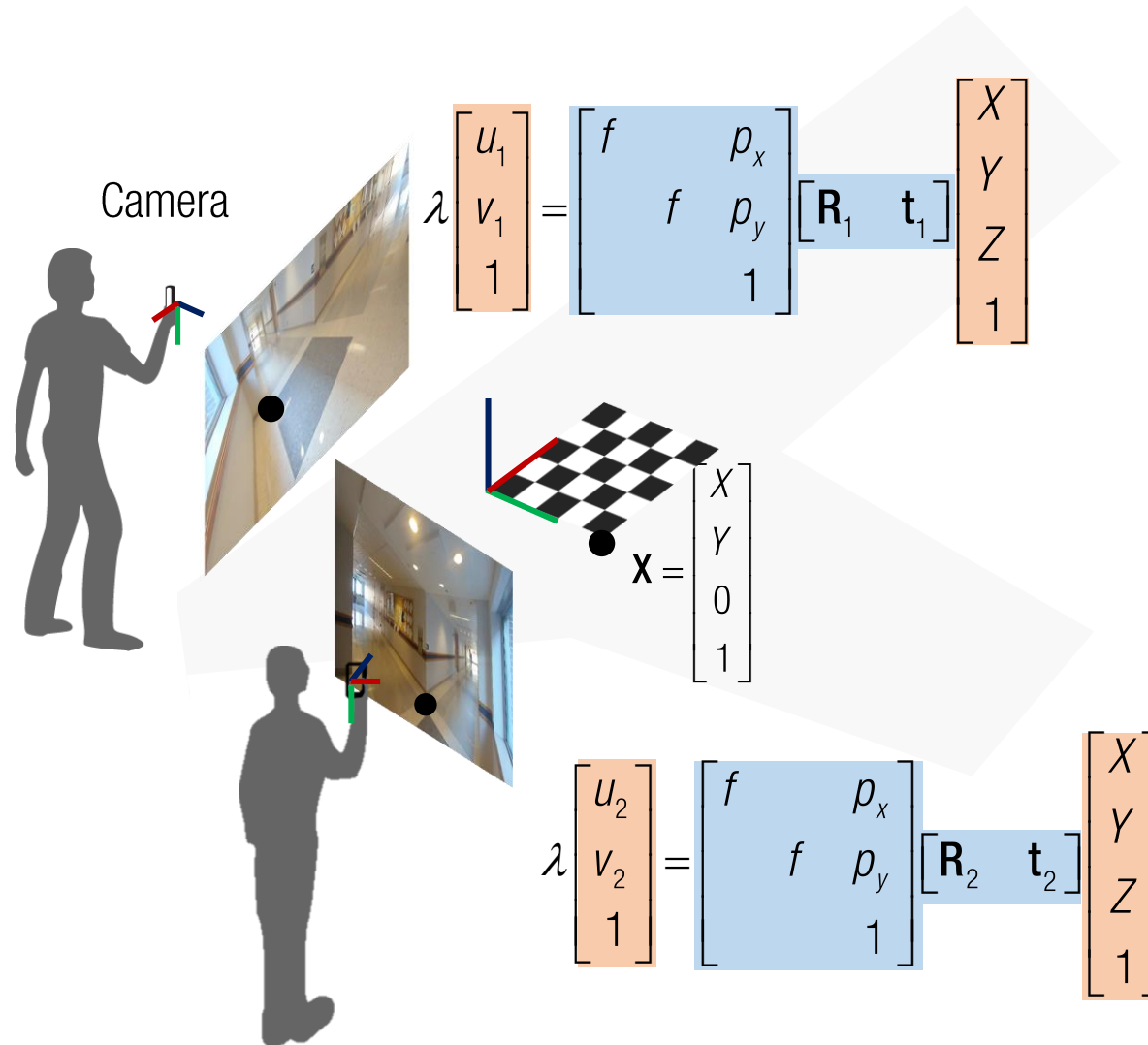
$p$ : the number of points

# of equations:  $2np$

: Knowns

: Unknowns

# Insight: Known Common 3D Points



# of unknowns:  $3 (\mathbf{K}) + 6n (\mathbf{R}$  and  $\mathbf{t})$

$n$ : the number of images

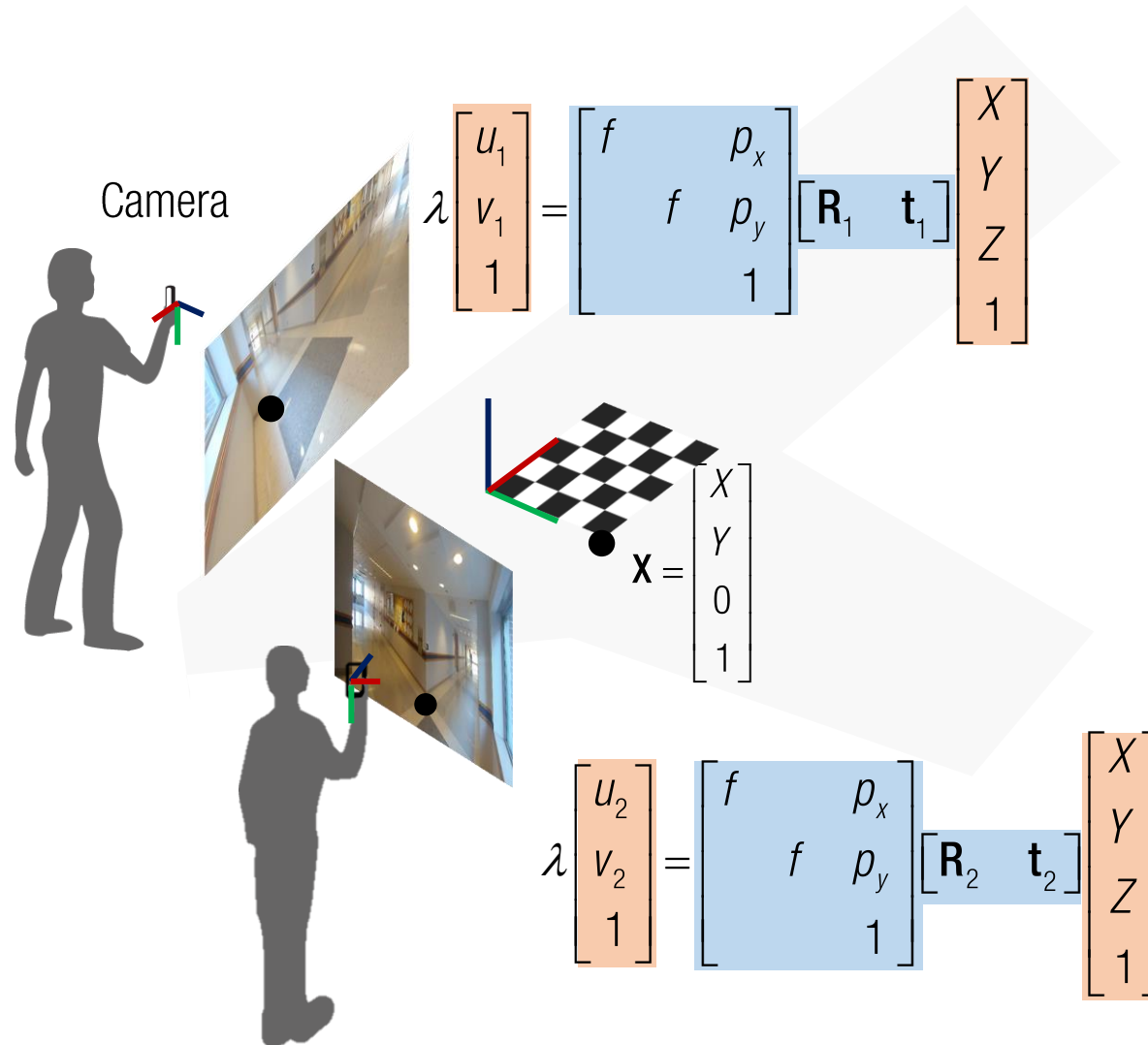
$p$ : the number of points

# of equations:  $2np$

: Knowns

: Unknowns

# Insight: Known Common 3D Points



# of unknowns: 3 (**K**) + 6*n* (**R** and **t**)

*n*: the number of images

*p*: the number of points

# of equations: 2*np*

We can solve for **K**, **R**, **t** if  $3 + 6n < 2nm$

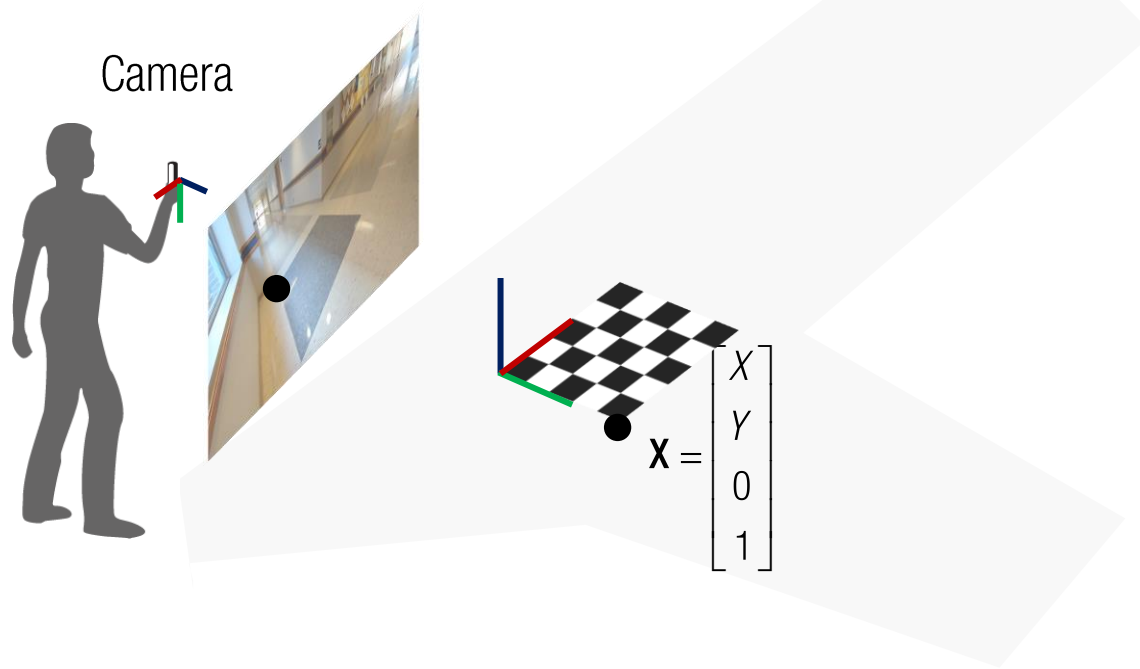
: Knowns

: Unknowns

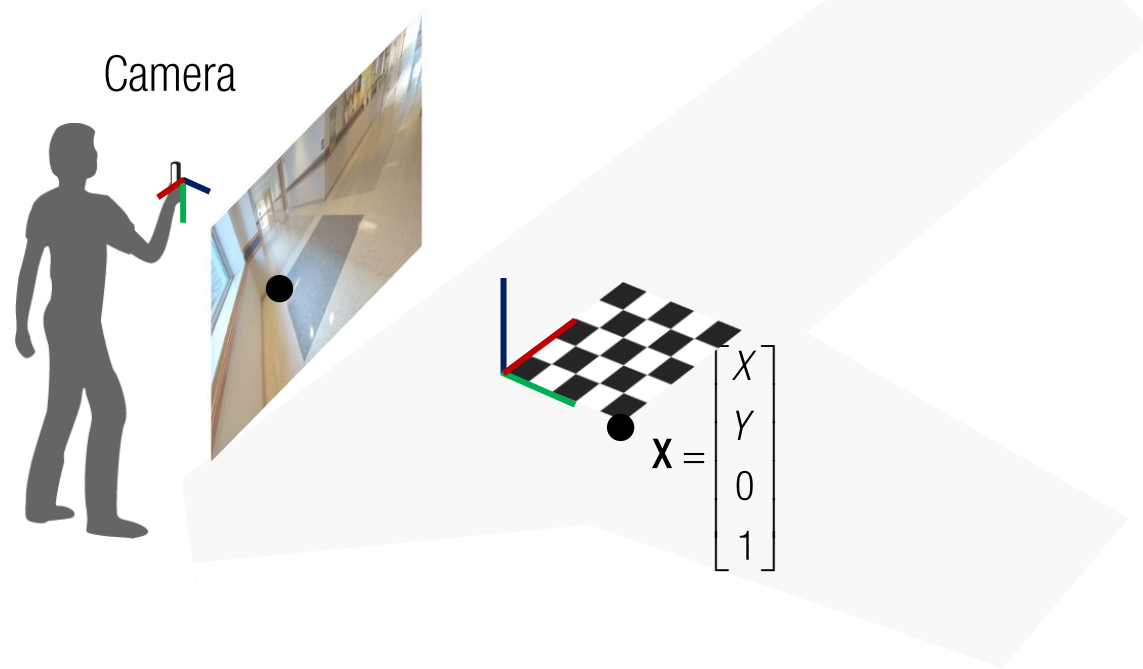
# Homography

Points in 2D plane are mapped to an image with homography:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$



# Homography

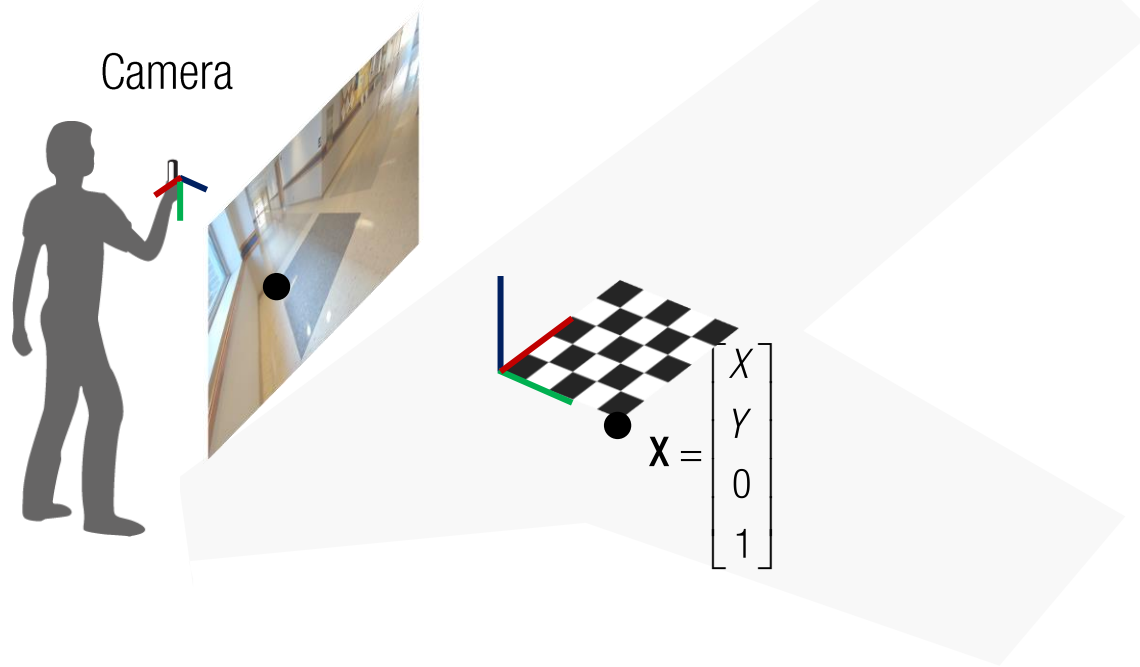


Points in 2D plane are mapped to an image with homography:

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$$= \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

# Homography

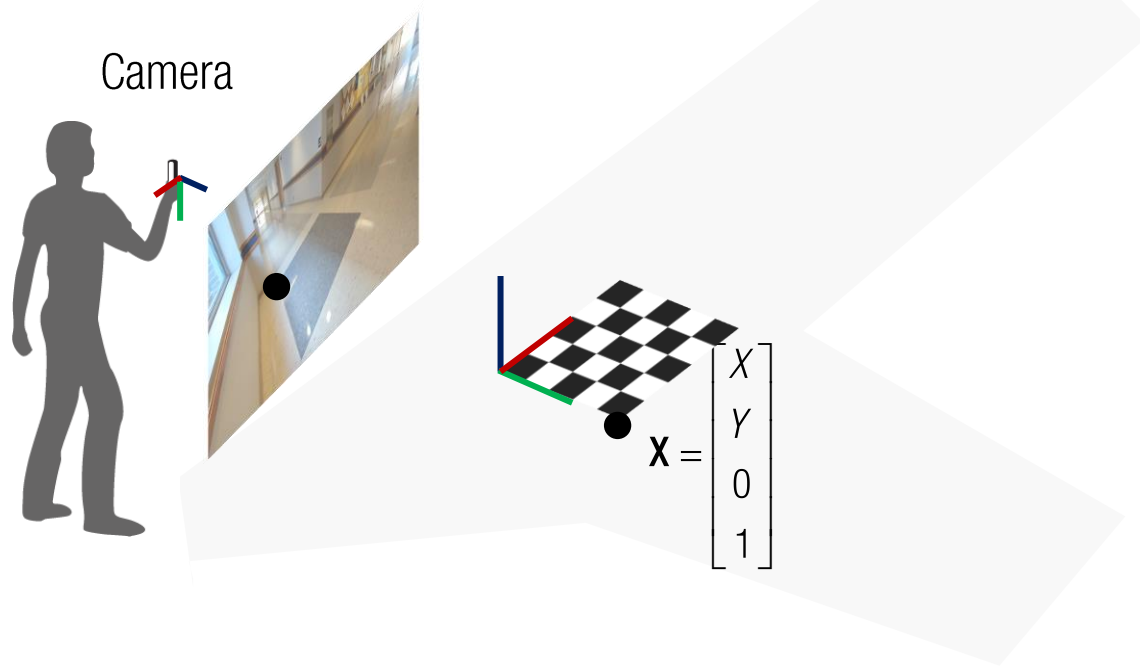


Points in 2D plane are mapped to an image with homography:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

# Homography



Points in 2D plane are mapped to an image with homography:

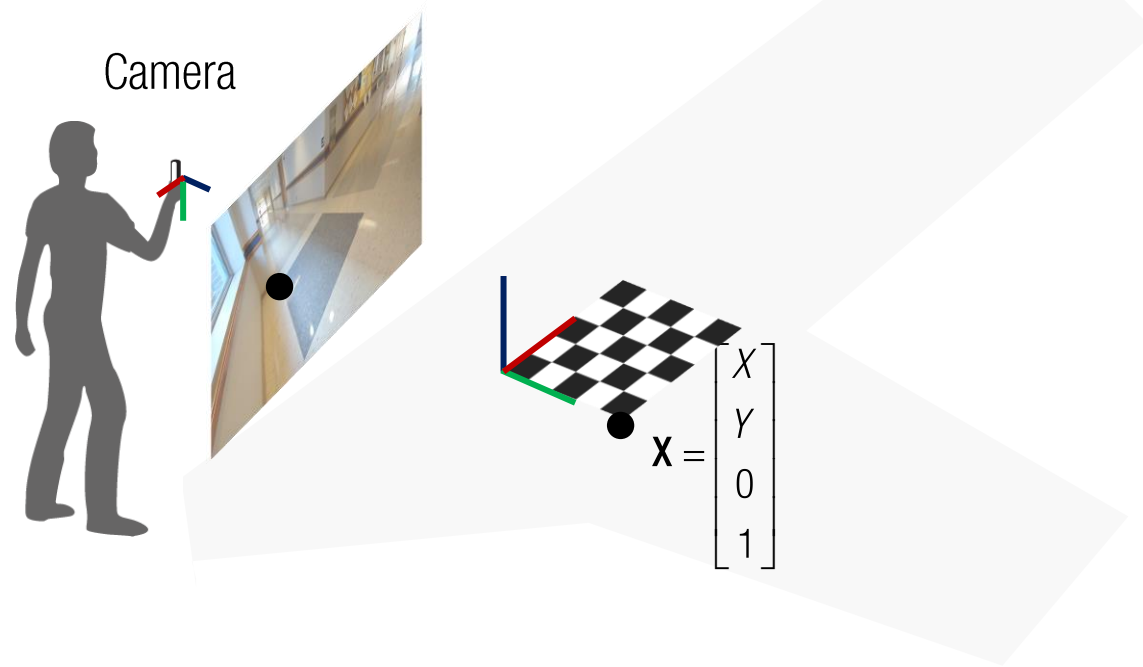
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

1. Compute homography

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

# Homography



Points in 2D plane are mapped to an image with homography:

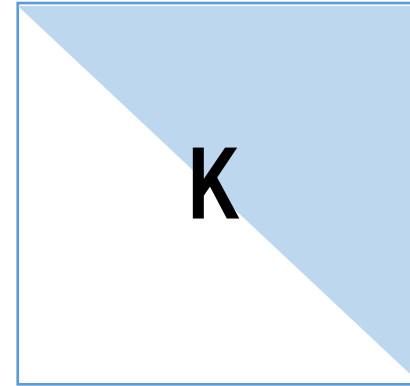
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = H \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

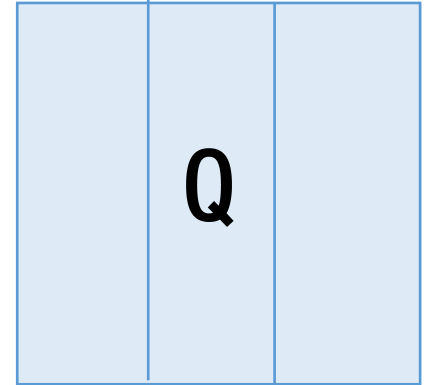
$$H = \begin{bmatrix} \text{K} & \text{Q} \end{bmatrix}$$

# Method1: RQ Decomposition

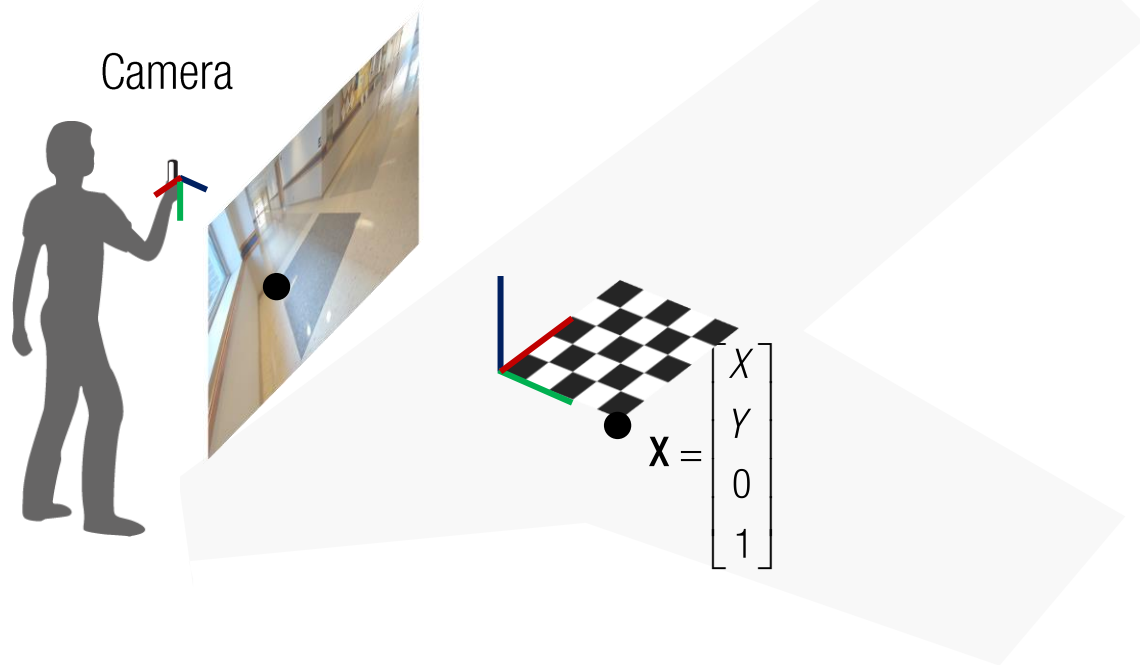
$$H =$$



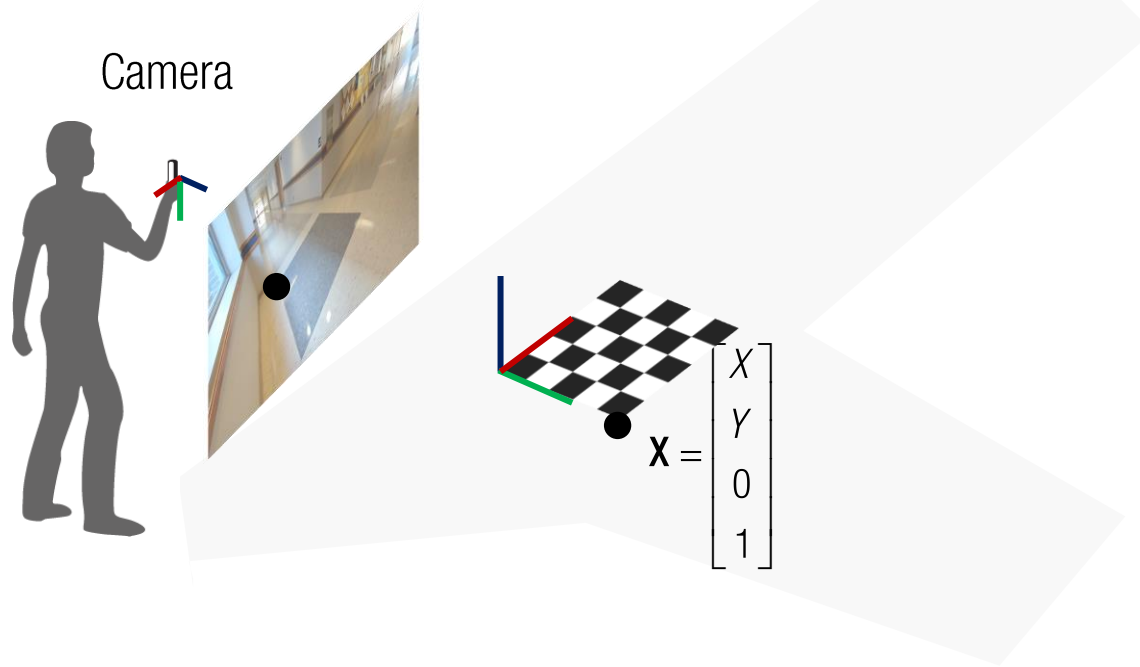
Upper triangle matrix



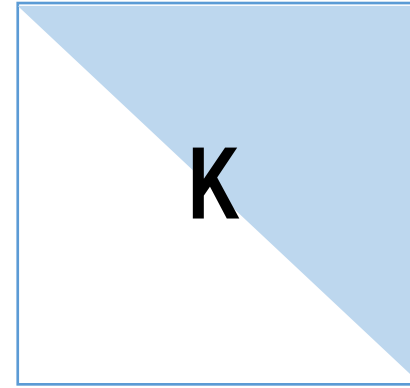
Othogonal matrix



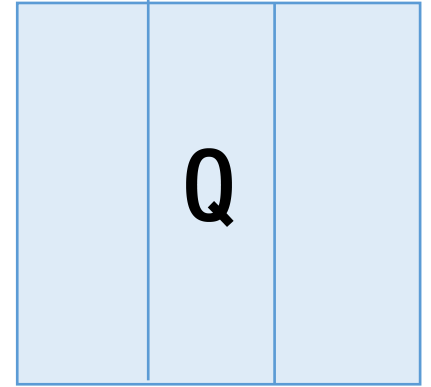
# Method1: RQ Decomposition



$$H =$$

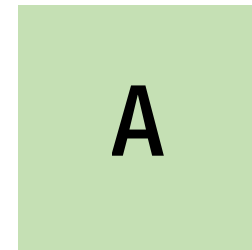


Upper triangle matrix

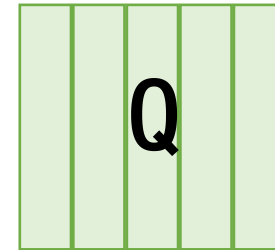


Othogonal matrix

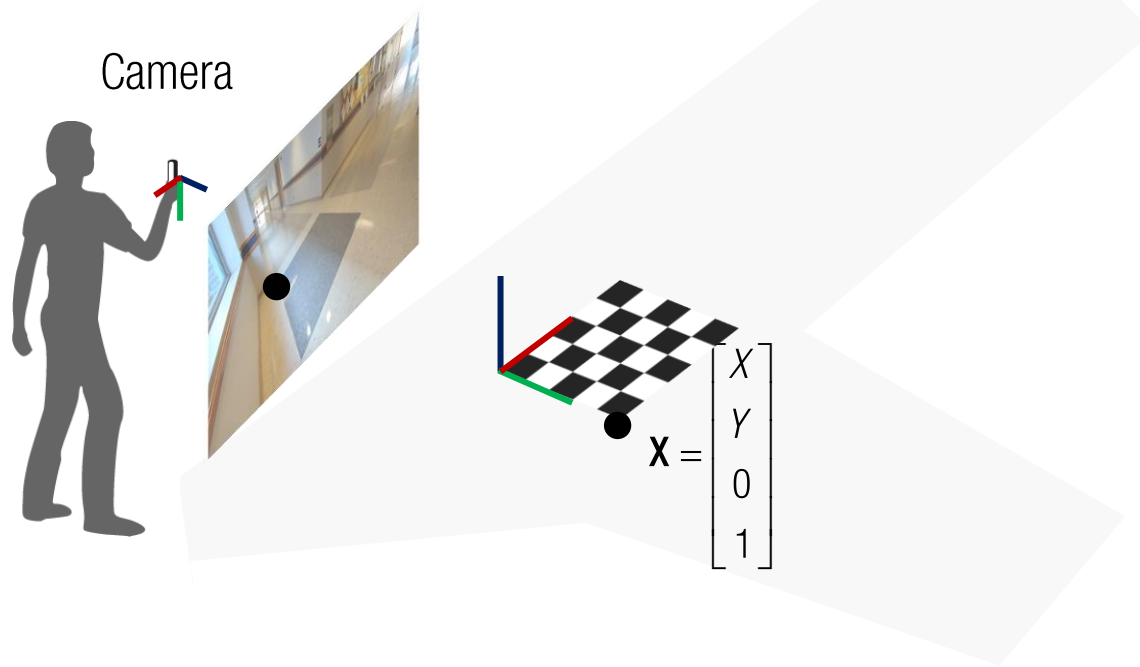
QR decomposition:



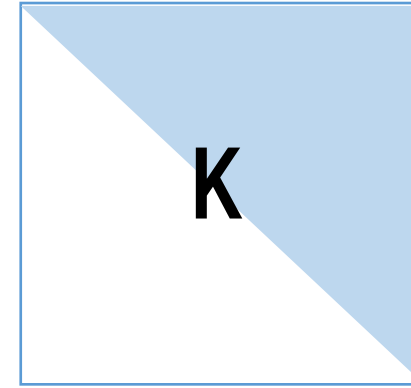
=



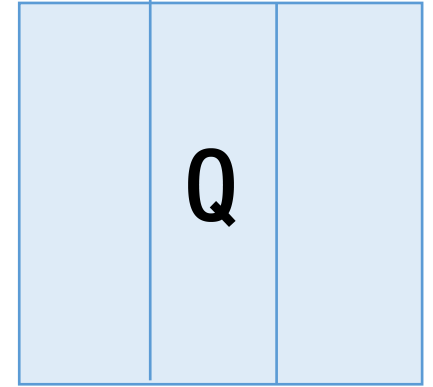
# Method1: RQ Decomposition



$$H =$$

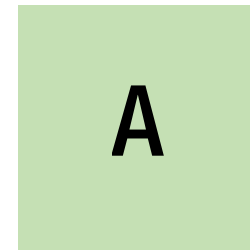


Upper triangle matrix

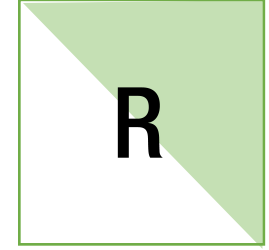
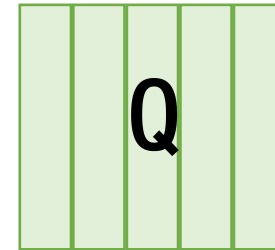


Othogonal matrix

QR decomposition:



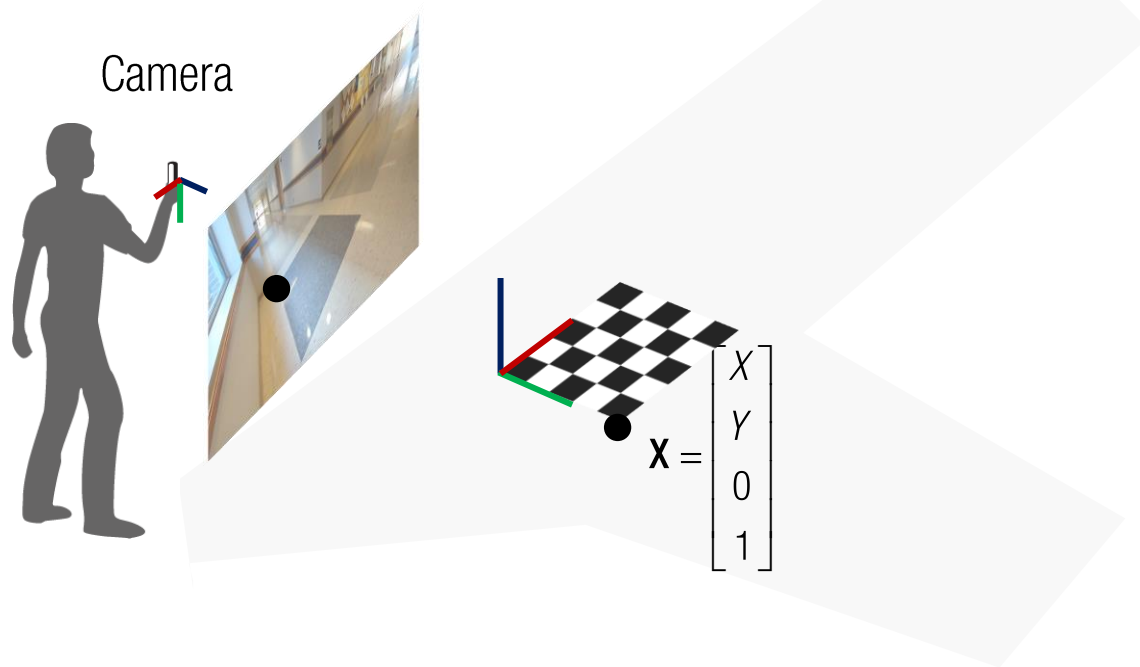
=



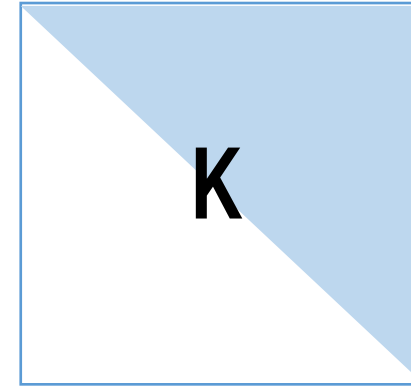
MATLAB

`[Q R] = qr(A)`

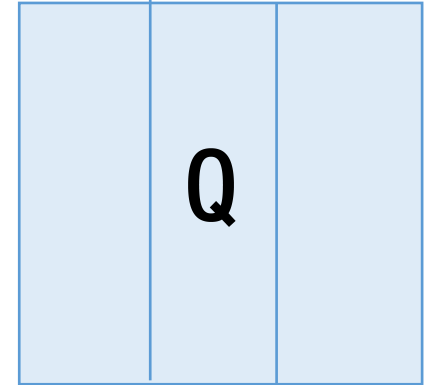
# Method1: RQ Decomposition



$$H =$$



Upper triangle matrix



Othogonal matrix

QR decomposition:

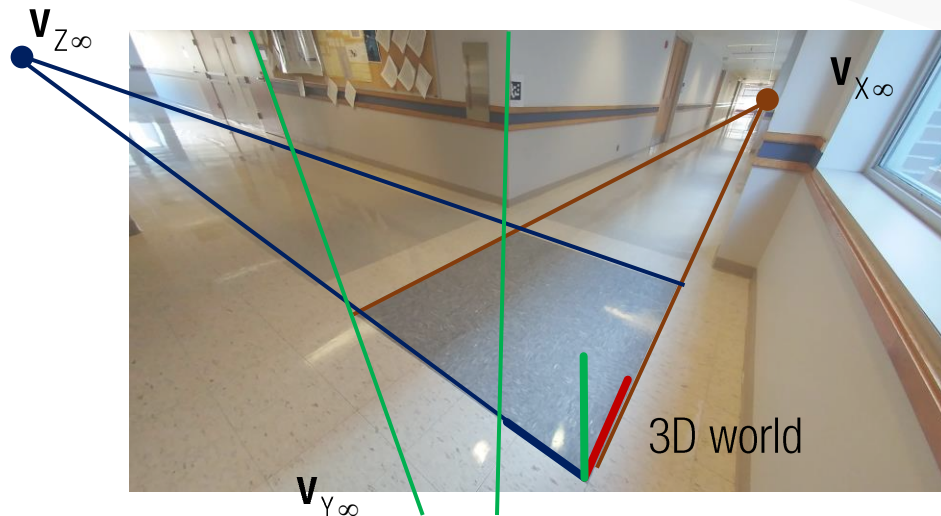
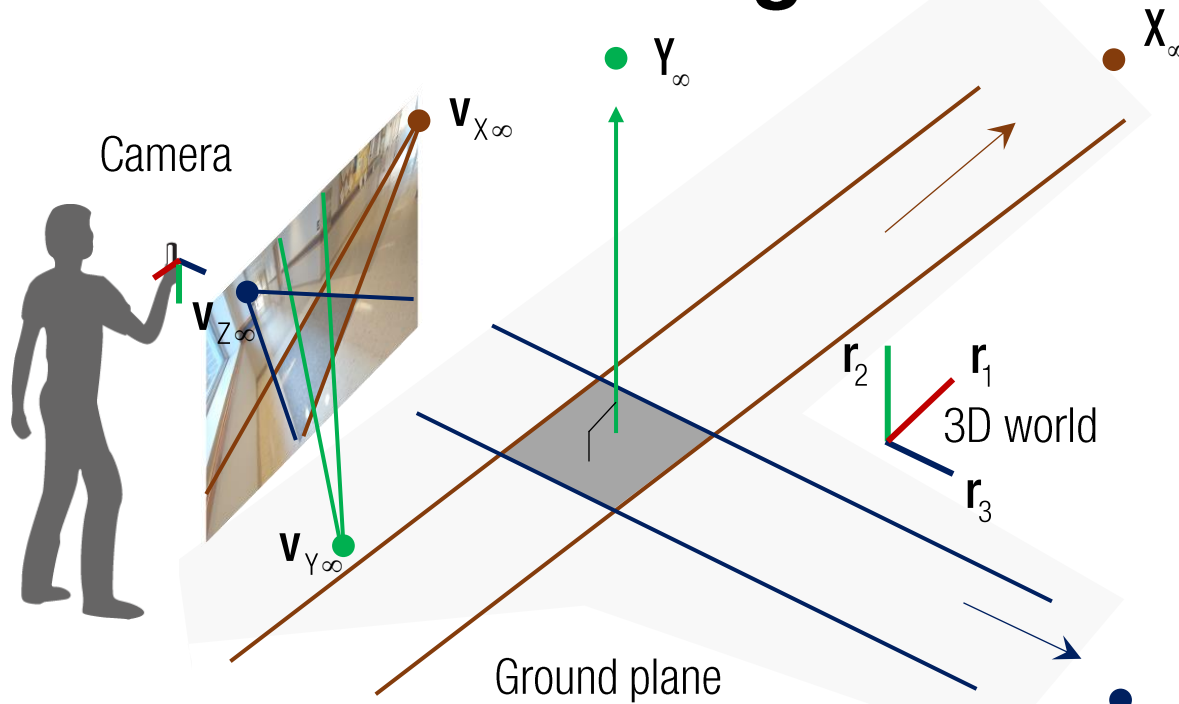
$$A = Q R$$

MATLAB

`[Q R] = qr(A)`

How to convert **QR** to **RQ**?

# Recall: Vanishing Points



$$\lambda \mathbf{v}_{X_\infty} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} \mathbf{R} \mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z_\infty} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} \mathbf{R} \mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y_\infty} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} \mathbf{R} \mathbf{Y}_\infty$$

$$\lambda \mathbf{K}^{-1} \mathbf{v}_{X_\infty} = \mathbf{R} \mathbf{X}_\infty \quad \lambda \mathbf{K}^{-1} \mathbf{v}_{Y_\infty} = \mathbf{R} \mathbf{Y}_\infty \quad \lambda \mathbf{K}^{-1} \mathbf{v}_{Z_\infty} = \mathbf{R} \mathbf{Z}_\infty$$

Note that the camera extrinsic is still unknown ( $\mathbf{R}$  and  $\mathbf{t}$ ).

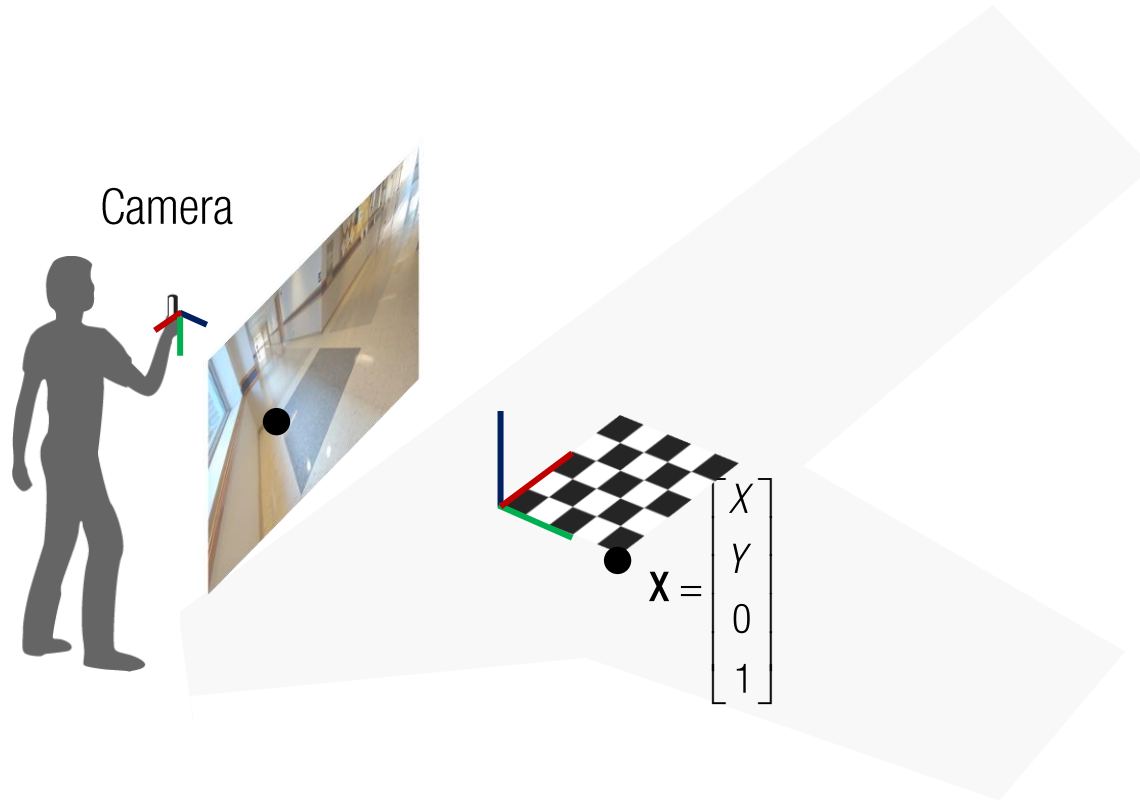
Known property of points at infinity:

$$\begin{aligned} (\mathbf{X}_\infty)^\top (\mathbf{Y}_\infty) &= 0 & (\mathbf{R} \mathbf{X}_\infty)^\top (\mathbf{R} \mathbf{Y}_\infty) &= 0 \\ (\mathbf{Y}_\infty)^\top (\mathbf{Z}_\infty) &= 0 & \longleftrightarrow & (\mathbf{R} \mathbf{Y}_\infty)^\top (\mathbf{R} \mathbf{Z}_\infty) = 0 \\ (\mathbf{Z}_\infty)^\top (\mathbf{X}_\infty) &= 0 & & (\mathbf{R} \mathbf{Z}_\infty)^\top (\mathbf{R} \mathbf{X}_\infty) = 0 \end{aligned}$$

$$(\mathbf{K}^{-1} \mathbf{v}_{X_\infty})^\top (\mathbf{K}^{-1} \mathbf{v}_{Y_\infty}) = (\mathbf{K}^{-1} \mathbf{v}_{Y_\infty})^\top (\mathbf{K}^{-1} \mathbf{v}_{Z_\infty}) = (\mathbf{K}^{-1} \mathbf{v}_{Z_\infty})^\top (\mathbf{K}^{-1} \mathbf{v}_{X_\infty}) = 0$$

: 3 unknowns and 3 equations

# Method2: Rotation



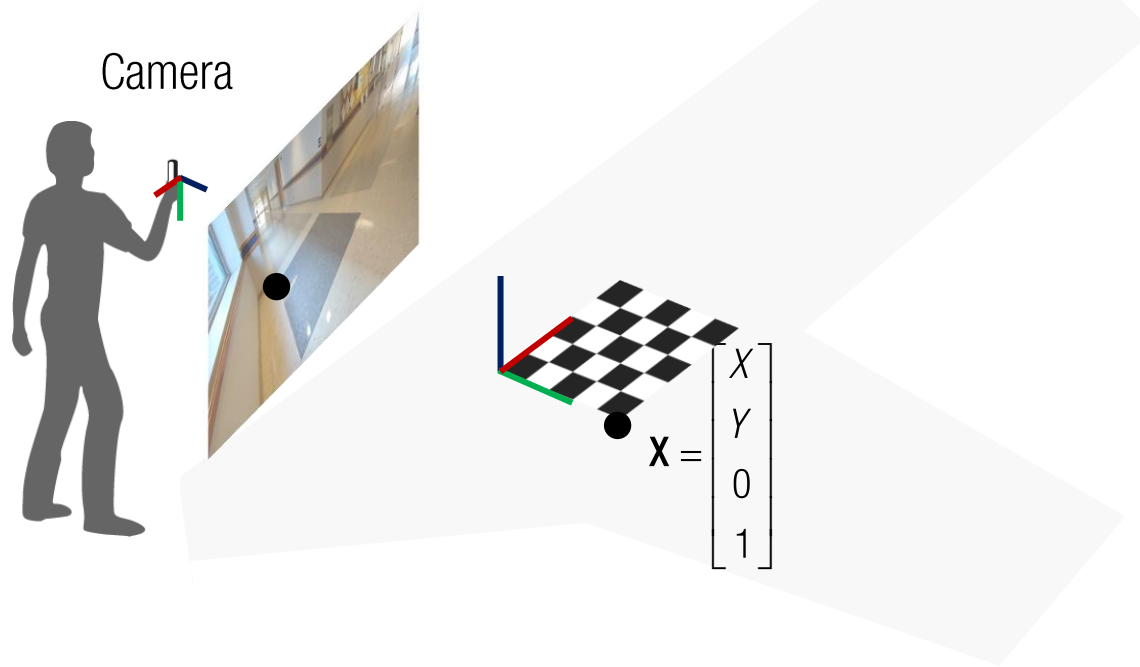
Homography factorization:

: Knowns

: Unknowns

$$\begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} f & p_x & \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

# Method2: Rotation



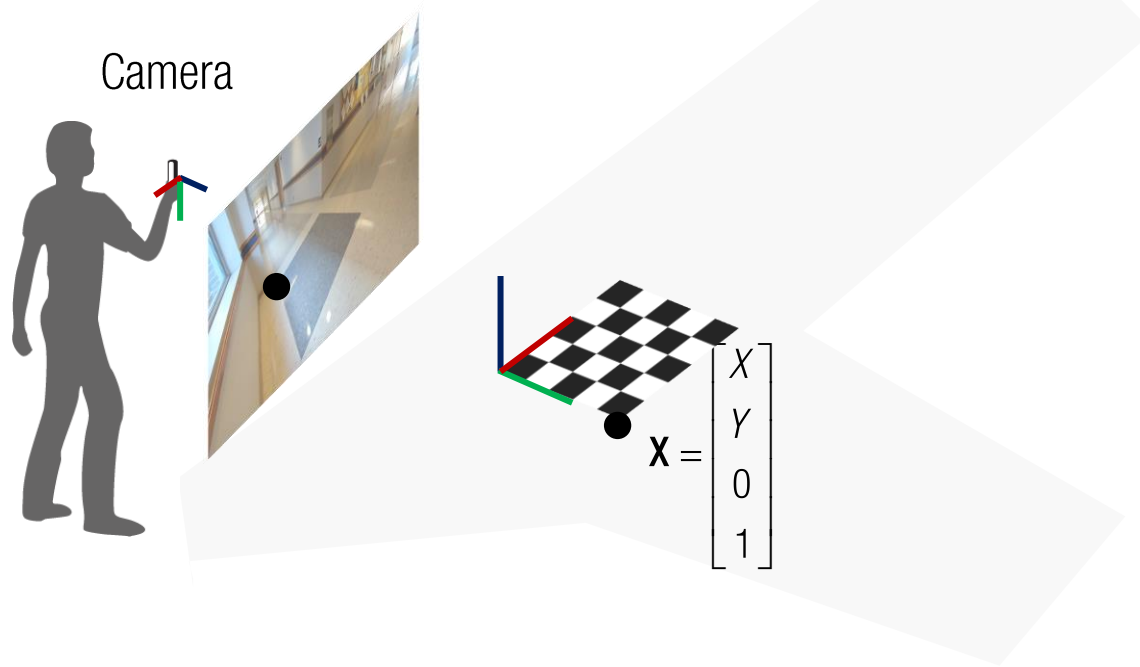
Homography factorization:

: Knowns

: Unknowns

$$\begin{pmatrix} \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}^{-1} \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} \end{pmatrix}$$

# Method2: Rotation



Homography factorization:

: Knowns

: Unknowns

$$\begin{pmatrix} \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}^{-1} \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

$$r_1 = K^{-1} h_1$$

$$r_2 = K^{-1} h_2$$

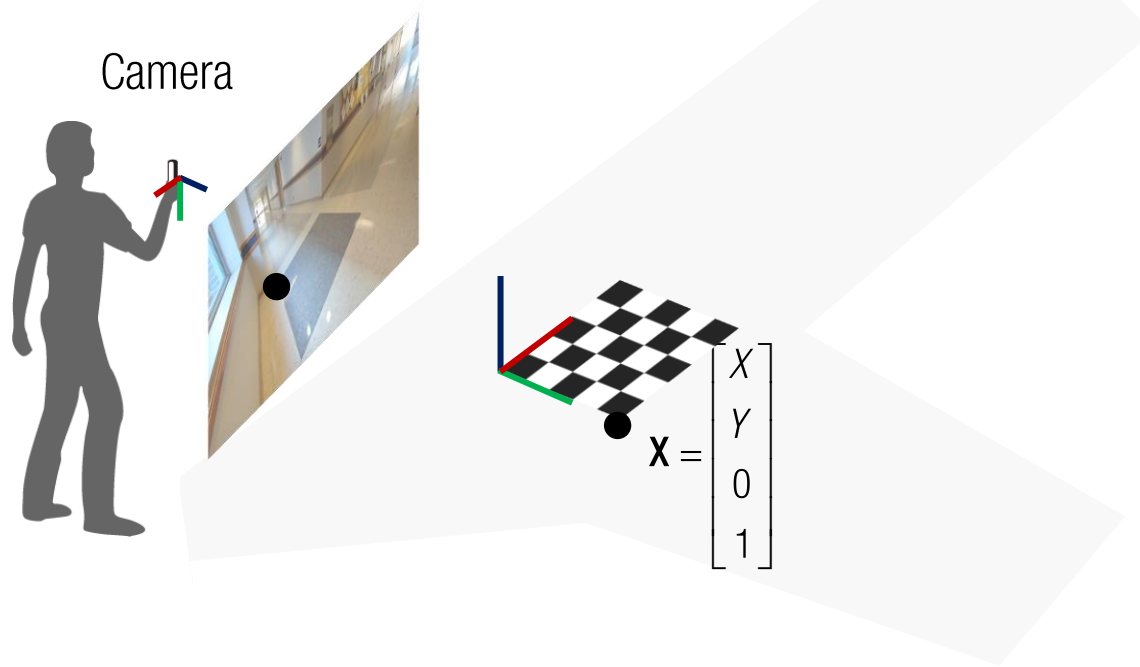
$$t = K^{-1} h_3$$

# Method2: Rotation

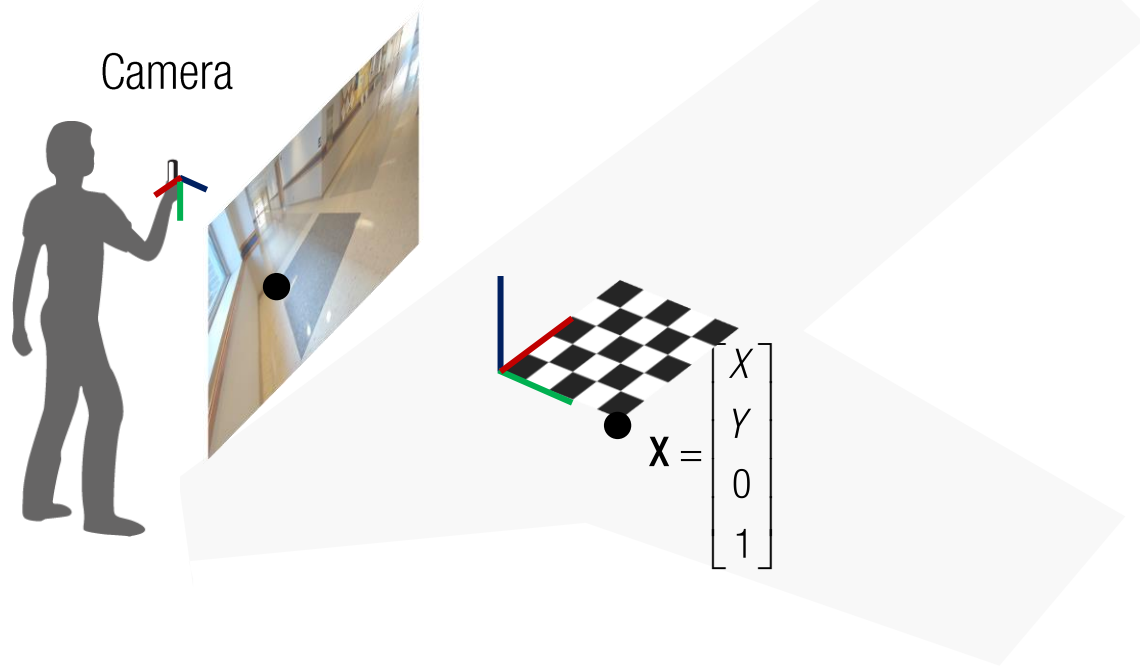
$$\mathbf{r}_1 = \mathbf{K}^{-1} \mathbf{h}_1 \quad \mathbf{r}_2 = \mathbf{K}^{-1} \mathbf{h}_2 \quad \mathbf{t} = \mathbf{K}^{-1} \mathbf{h}_3$$

Orthogonality of rotation matrix property:

$$\mathbf{r}_1^T \mathbf{r}_2 = 0 \quad \|\mathbf{r}_1\| = 1 \quad \|\mathbf{r}_2\| = 1$$



# Method2: Rotation



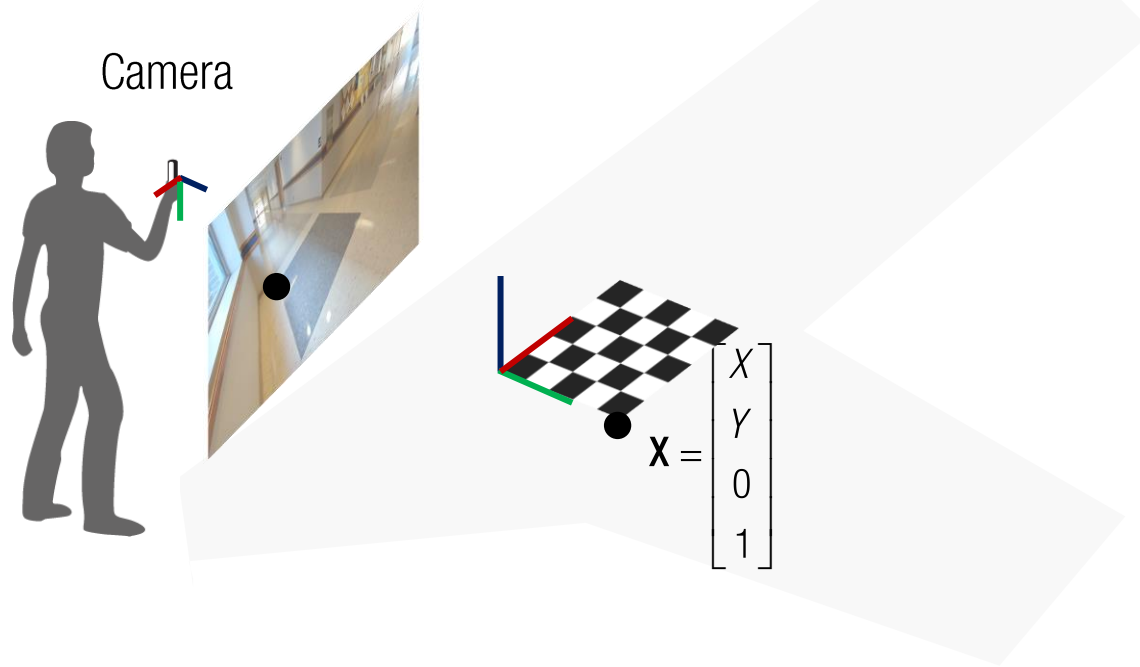
$$\mathbf{r}_1 = \mathbf{K}^{-1} \mathbf{h}_1 \quad \mathbf{r}_2 = \mathbf{K}^{-1} \mathbf{h}_2 \quad \mathbf{t} = \mathbf{K}^{-1} \mathbf{h}_3$$

Orthogonality of rotation matrix property:

$$\mathbf{r}_1^T \mathbf{r}_2 = 0 \quad \|\mathbf{r}_1\| = 1 \quad \|\mathbf{r}_2\| = 1$$

$$\longrightarrow (\mathbf{K}^{-1} \mathbf{h}_1)^T (\mathbf{K}^{-1} \mathbf{h}_2) = \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

# Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1} \mathbf{h}_1 \quad \mathbf{r}_2 = \mathbf{K}^{-1} \mathbf{h}_2 \quad \mathbf{t} = \mathbf{K}^{-1} \mathbf{h}_3$$

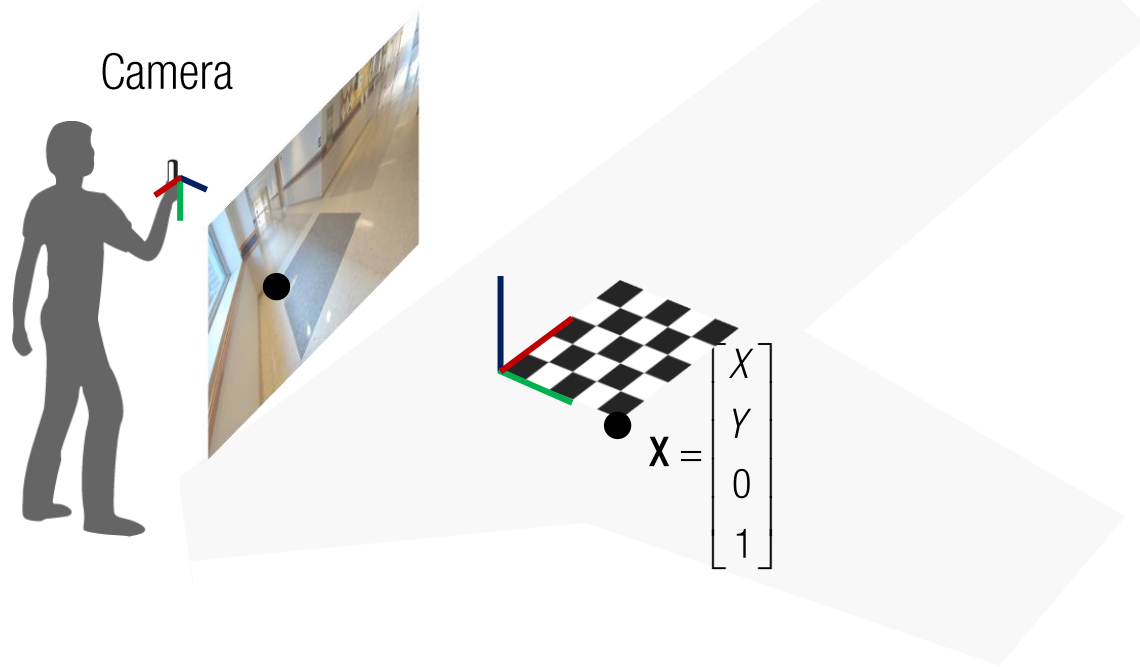
Orthogonality of rotation matrix property:

$$\mathbf{r}_1^T \mathbf{r}_2 = 0 \quad \|\mathbf{r}_1\| = 1 \quad \|\mathbf{r}_2\| = 1$$

$$\longrightarrow (\mathbf{K}^{-1} \mathbf{h}_1)^T (\mathbf{K}^{-1} \mathbf{h}_2) = \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

$$\|\mathbf{K}^{-1} \mathbf{h}_1\| = \|\mathbf{K}^{-1} \mathbf{h}_2\| \quad \text{or,} \quad \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2$$

# Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1} \mathbf{h}_1 \quad \mathbf{r}_2 = \mathbf{K}^{-1} \mathbf{h}_2 \quad \mathbf{t} = \mathbf{K}^{-1} \mathbf{h}_3$$

Orthogonality of rotation matrix property:

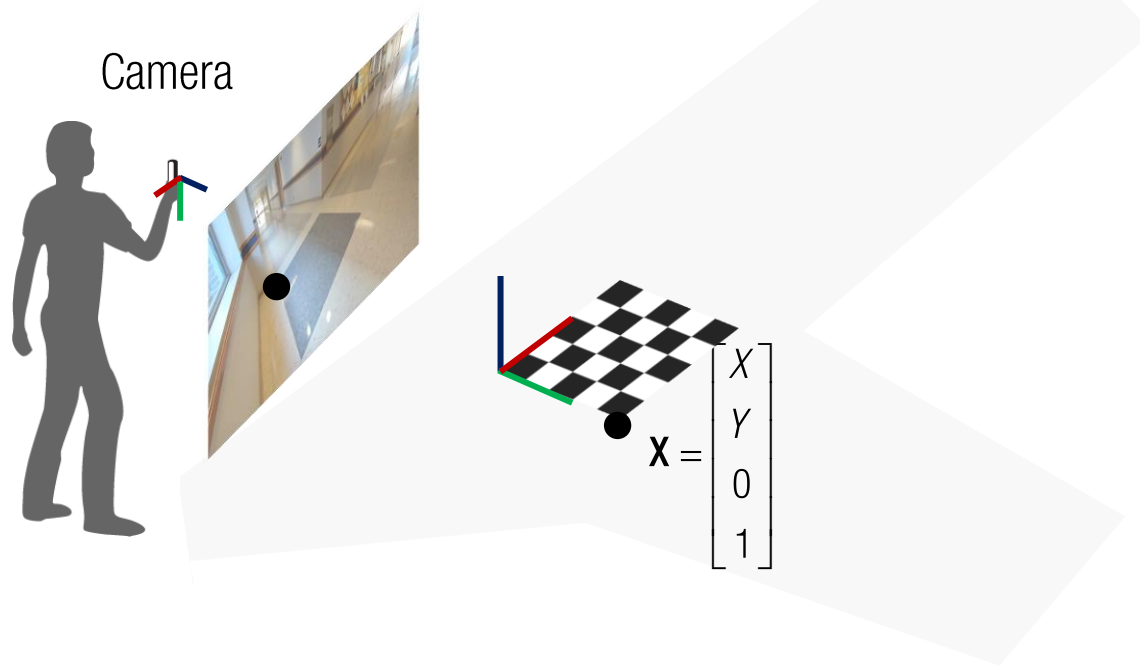
$$\mathbf{r}_1^T \mathbf{r}_2 = 0 \quad \|\mathbf{r}_1\| = 1 \quad \|\mathbf{r}_2\| = 1$$

$$\longrightarrow (\mathbf{K}^{-1} \mathbf{h}_1)^T (\mathbf{K}^{-1} \mathbf{h}_2) = \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

$$\|\mathbf{K}^{-1} \mathbf{h}_1\| = \|\mathbf{K}^{-1} \mathbf{h}_2\| \quad \text{or,} \quad \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2$$

$$\mathbf{K}^{-T} \mathbf{K}^{-1} =$$

# Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1} \mathbf{h}_1 \quad \mathbf{r}_2 = \mathbf{K}^{-1} \mathbf{h}_2 \quad \mathbf{t} = \mathbf{K}^{-1} \mathbf{h}_3$$

Orthogonality of rotation matrix property:

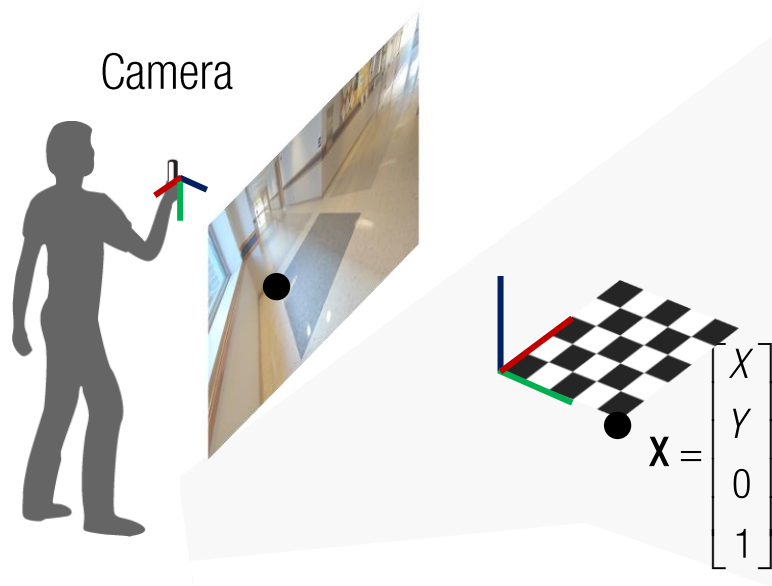
$$\mathbf{r}_1^T \mathbf{r}_2 = 0 \quad \|\mathbf{r}_1\| = 1 \quad \|\mathbf{r}_2\| = 1$$

$$\longrightarrow (\mathbf{K}^{-1} \mathbf{h}_1)^T (\mathbf{K}^{-1} \mathbf{h}_2) = \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

$$\|\mathbf{K}^{-1} \mathbf{h}_1\| = \|\mathbf{K}^{-1} \mathbf{h}_2\| \quad \text{or,} \quad \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2$$

$$\mathbf{K}^{-T} \mathbf{K}^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix} \begin{bmatrix} 1/f & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix}$$

# Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1} \mathbf{h}_1 \quad \mathbf{r}_2 = \mathbf{K}^{-1} \mathbf{h}_2 \quad \mathbf{t} = \mathbf{K}^{-1} \mathbf{h}_3$$

Orthogonality of rotation matrix property:

$$\mathbf{r}_1^T \mathbf{r}_2 = 0 \quad \|\mathbf{r}_1\| = 1 \quad \|\mathbf{r}_2\| = 1$$

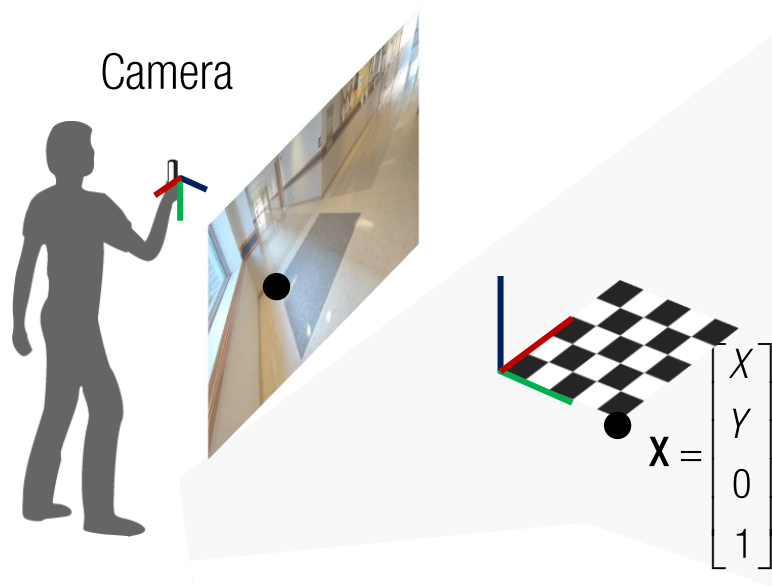
$$\longrightarrow (\mathbf{K}^{-1} \mathbf{h}_1)^T (\mathbf{K}^{-1} \mathbf{h}_2) = \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

$$\|\mathbf{K}^{-1} \mathbf{h}_1\| = \|\mathbf{K}^{-1} \mathbf{h}_2\| \quad \text{or,} \quad \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2$$

$$\mathbf{K}^{-T} \mathbf{K}^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix} \begin{bmatrix} 1/f & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix} = \begin{bmatrix} b_1 & b_2 & \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}$$

$$\text{where } b_1 = \frac{1}{f^2}, \quad b_2 = -\frac{p_x}{f^2}, \quad b_3 = -\frac{p_y}{f^2}, \quad b_4 = \frac{p_x^2}{f^2} + \frac{p_y^2}{f^2} + 1$$

# Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1} \mathbf{h}_1 \quad \mathbf{r}_2 = \mathbf{K}^{-1} \mathbf{h}_2 \quad \mathbf{t} = \mathbf{K}^{-1} \mathbf{h}_3$$

Orthogonality of rotation matrix property:

$$\mathbf{r}_1^T \mathbf{r}_2 = 0 \quad \|\mathbf{r}_1\| = 1 \quad \|\mathbf{r}_2\| = 1$$

$$\longrightarrow (\mathbf{K}^{-1} \mathbf{h}_1)^T (\mathbf{K}^{-1} \mathbf{h}_2) = \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

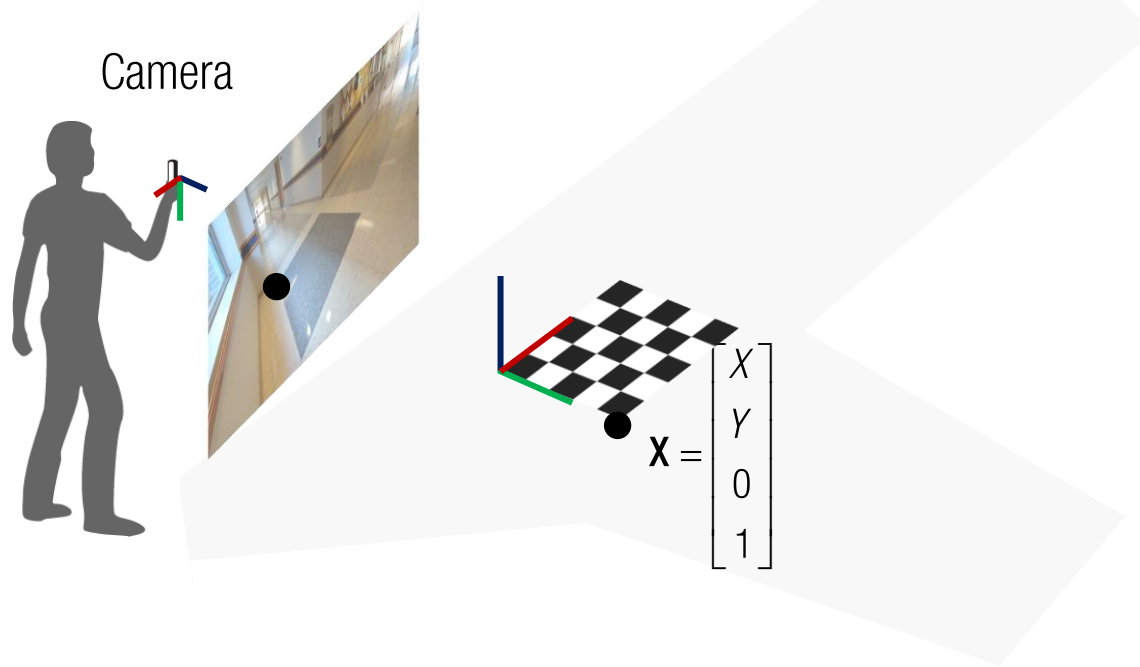
$$\|\mathbf{K}^{-1} \mathbf{h}_1\| = \|\mathbf{K}^{-1} \mathbf{h}_2\| \quad \text{or,} \quad \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2$$

$$\mathbf{K}^{-T} \mathbf{K}^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix} \begin{bmatrix} 1/f & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}}_{\mathbf{B}}$$

$$\text{where } b_1 = \frac{1}{f^2}, \quad b_2 = -\frac{p_x}{f^2}, \quad b_3 = -\frac{p_y}{f^2}, \quad b_4 = \frac{p_x^2}{f^2} + \frac{p_y^2}{f^2} + 1$$

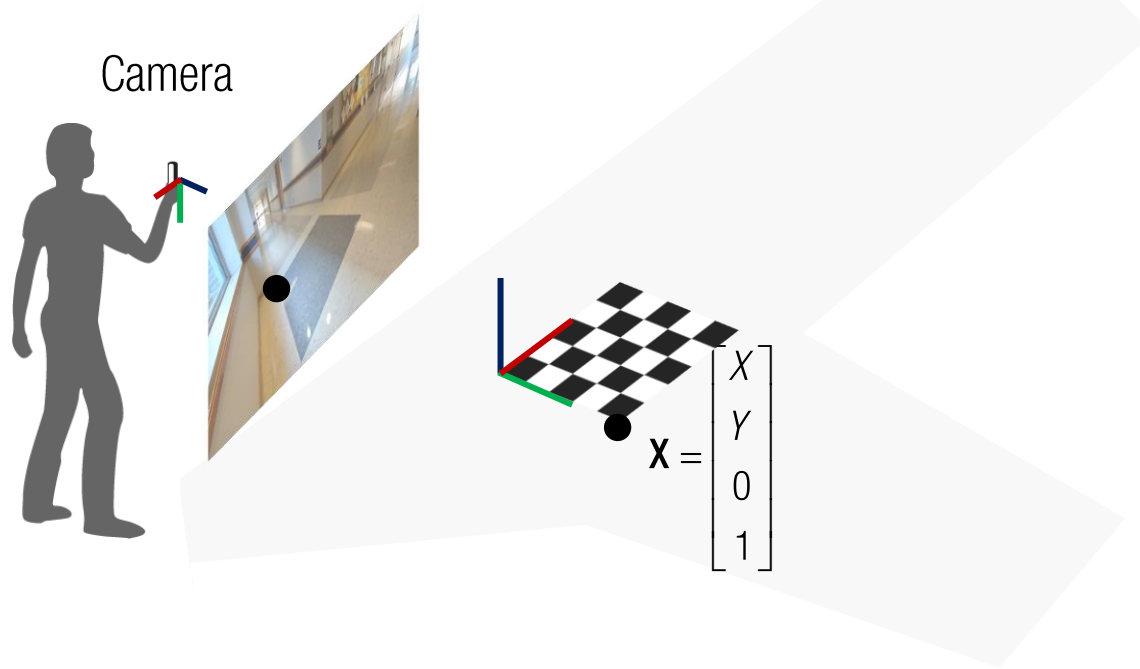
$$\text{Linear in } \mathbf{B}: \quad \mathbf{h}_1^T \mathbf{B} \mathbf{h}_2 = 0 \quad \mathbf{h}_1^T \mathbf{B} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{B} \mathbf{h}_2$$

# Method2: Rotation



$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

# Method2: Rotation

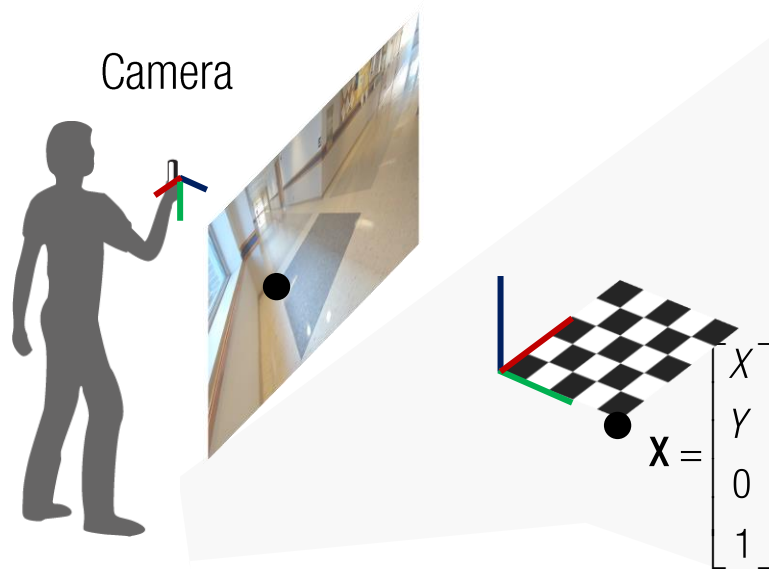


$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{B} \mathbf{h}_2$$

$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

# Method2: Rotation



$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

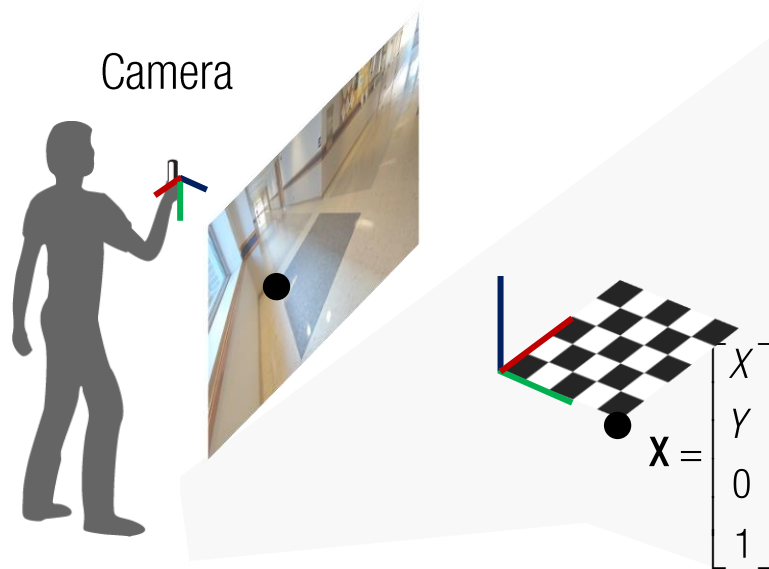
$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{B} \mathbf{h}_2$$

$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11}h_{32} + h_{31}h_{12} & h_{21}h_{32} + h_{31}h_{22} & h_{32}h_{31} \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11}h_{31} - h_{12}h_{32}) & 2(h_{21}h_{31} - h_{22}h_{32}) & h_{31}^2 - h_{32}^2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \mathbf{0}$$

2x4

# Method2: Rotation



$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{B} \mathbf{h}_2$$

$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11}h_{32} + h_{31}h_{12} & h_{21}h_{32} + h_{31}h_{22} & h_{32}h_{31} \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11}h_{31} - h_{12}h_{32}) & 2(h_{21}h_{31} - h_{22}h_{32}) & h_{31}^2 - h_{32}^2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \mathbf{0}$$

2x4

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

# Method2: Rotation

$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

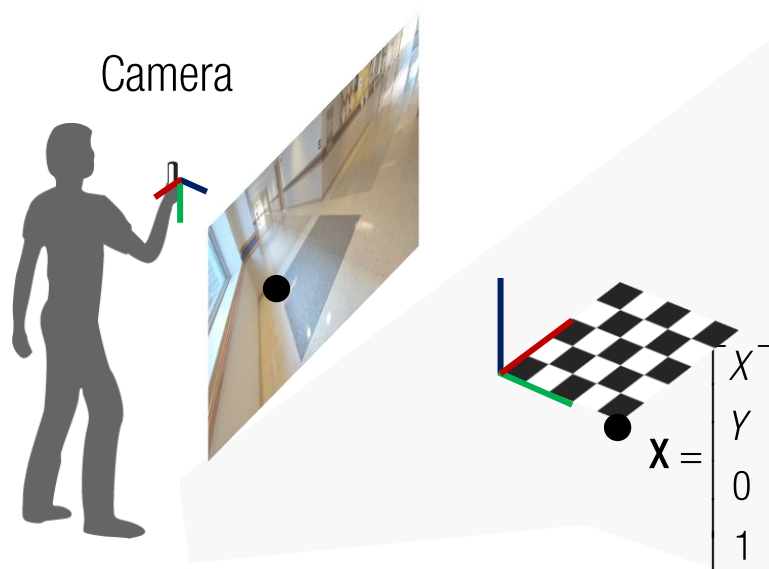
$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{B} \mathbf{h}_2$$

$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

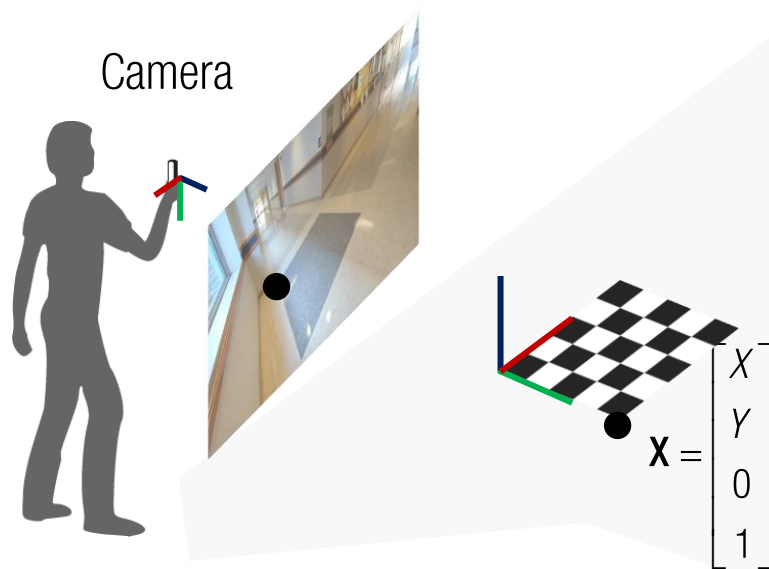
$$\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11}h_{32} + h_{31}h_{12} & h_{11}h_{32} + h_{31}h_{22} & h_{32}h_{31} \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11}h_{31} - h_{12}h_{32}) & 2(h_{21}h_{31} - h_{22}h_{32}) & h_{31}^2 - h_{32}^2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

2x4

Each image produces 2 equations and therefore,  $\mathbf{x}$  can be computed with minimum 2 images.



# Method2: Rotation



$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{B} \mathbf{h}_2$$

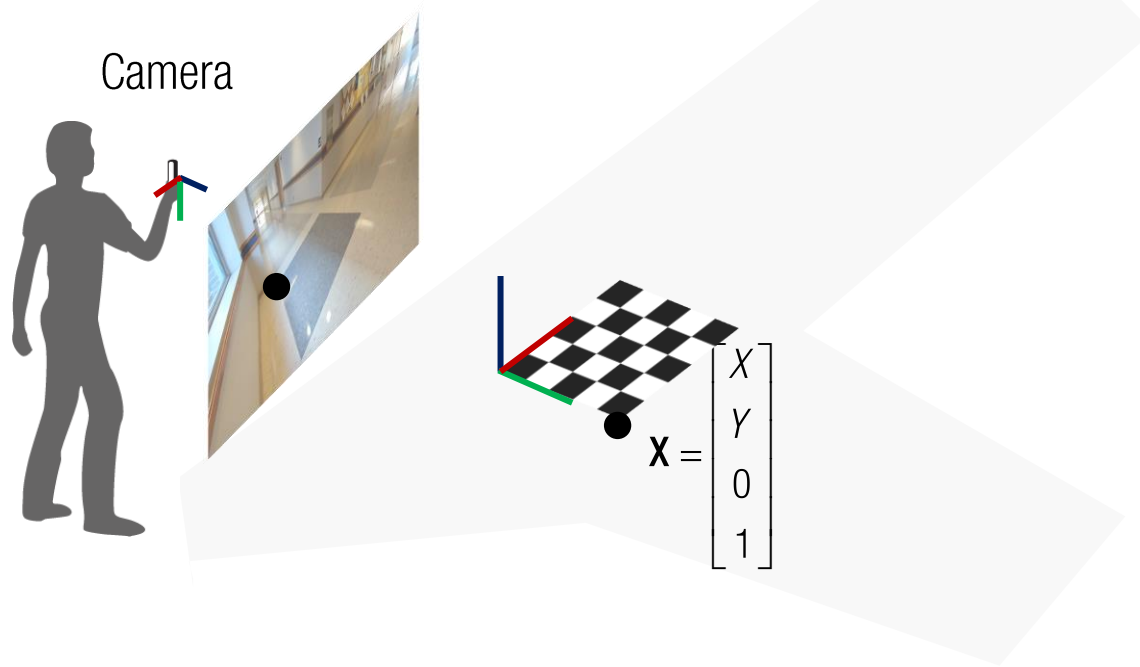
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^T \begin{bmatrix} b_1 & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11}h_{32} + h_{31}h_{12} & h_{11}h_{32} + h_{31}h_{22} & h_{32}h_{31} \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11}h_{31} - h_{12}h_{32}) & 2(h_{21}h_{31} - h_{22}h_{32}) & h_{31}^2 - h_{32}^2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

2x4

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

# Method2: Rotation



Homography factorization:

: Knowns

: Unknowns

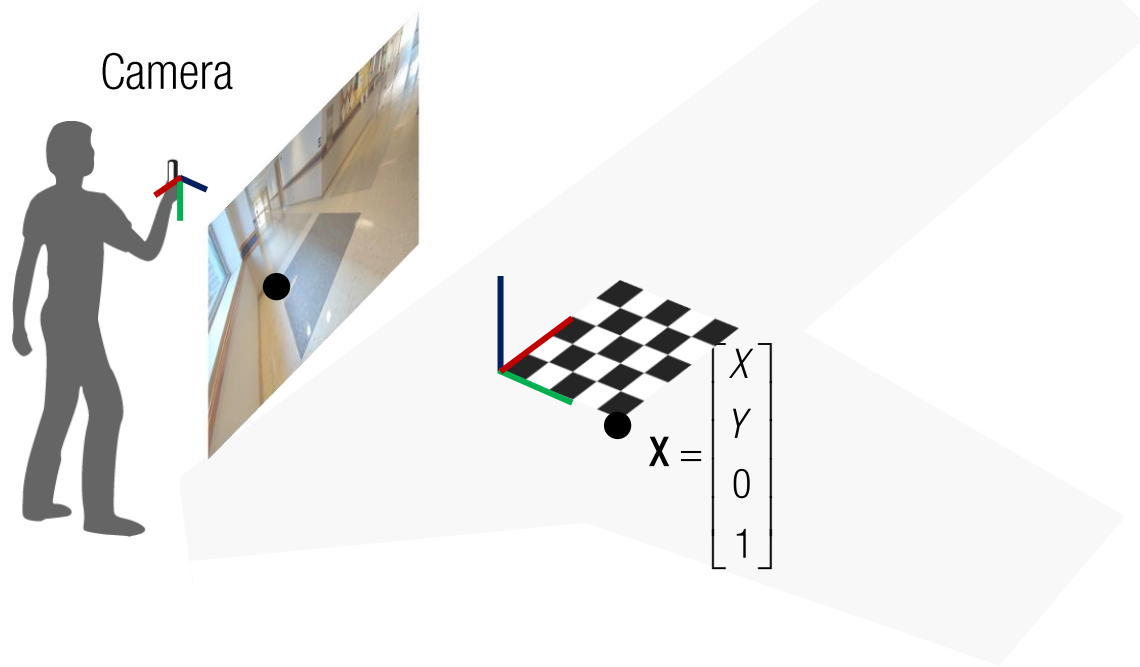
$$\left( \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

$$r_1 = K^{-1} h_1$$

$$r_2 = K^{-1} h_2$$

$$r_3 = K^{-1} h_3$$

# Method2: Rotation



Homography factorization:

: Knowns

: Unknowns

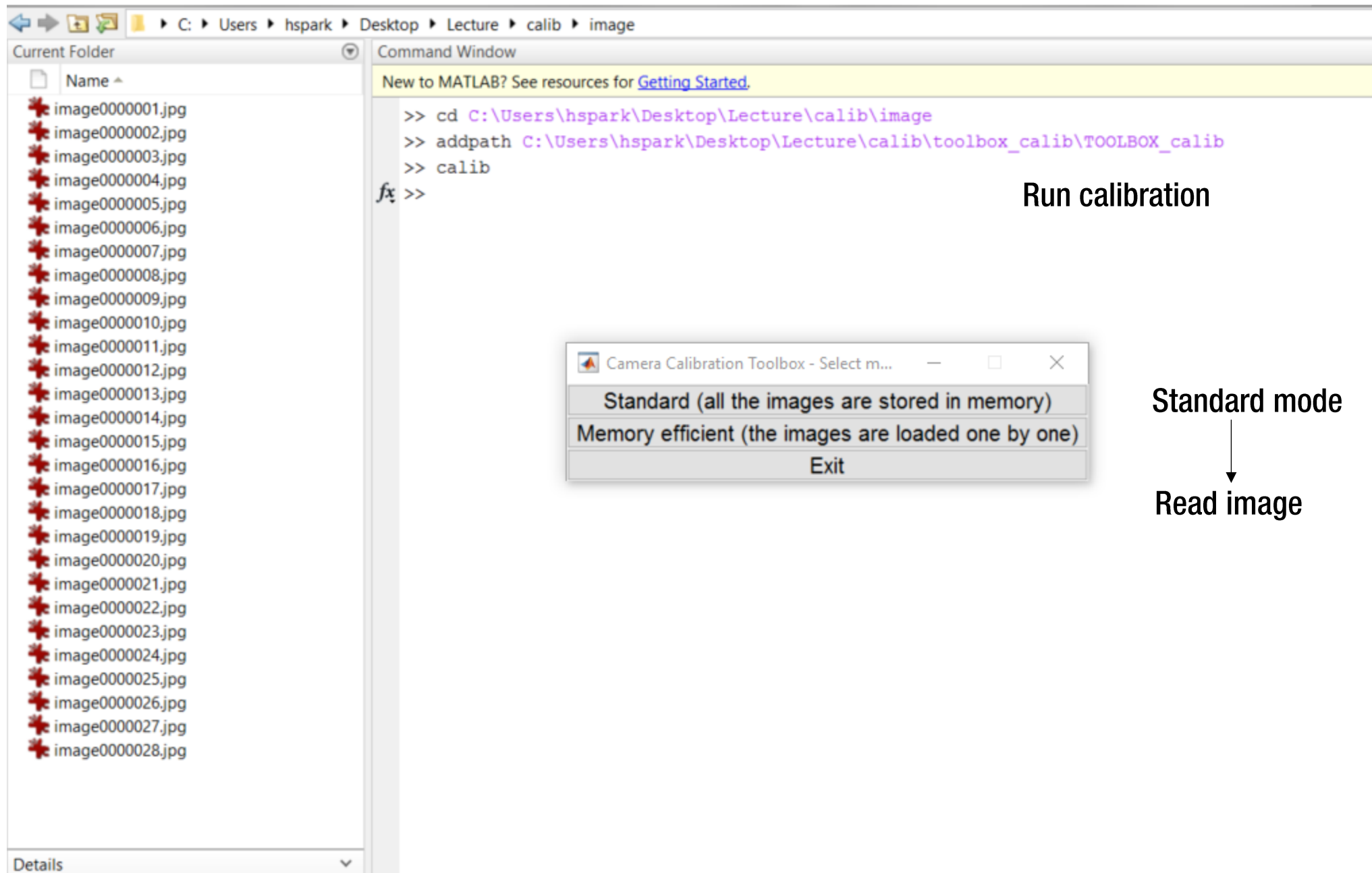
$$\begin{pmatrix} \begin{bmatrix} f & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \end{pmatrix}^{-1} \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

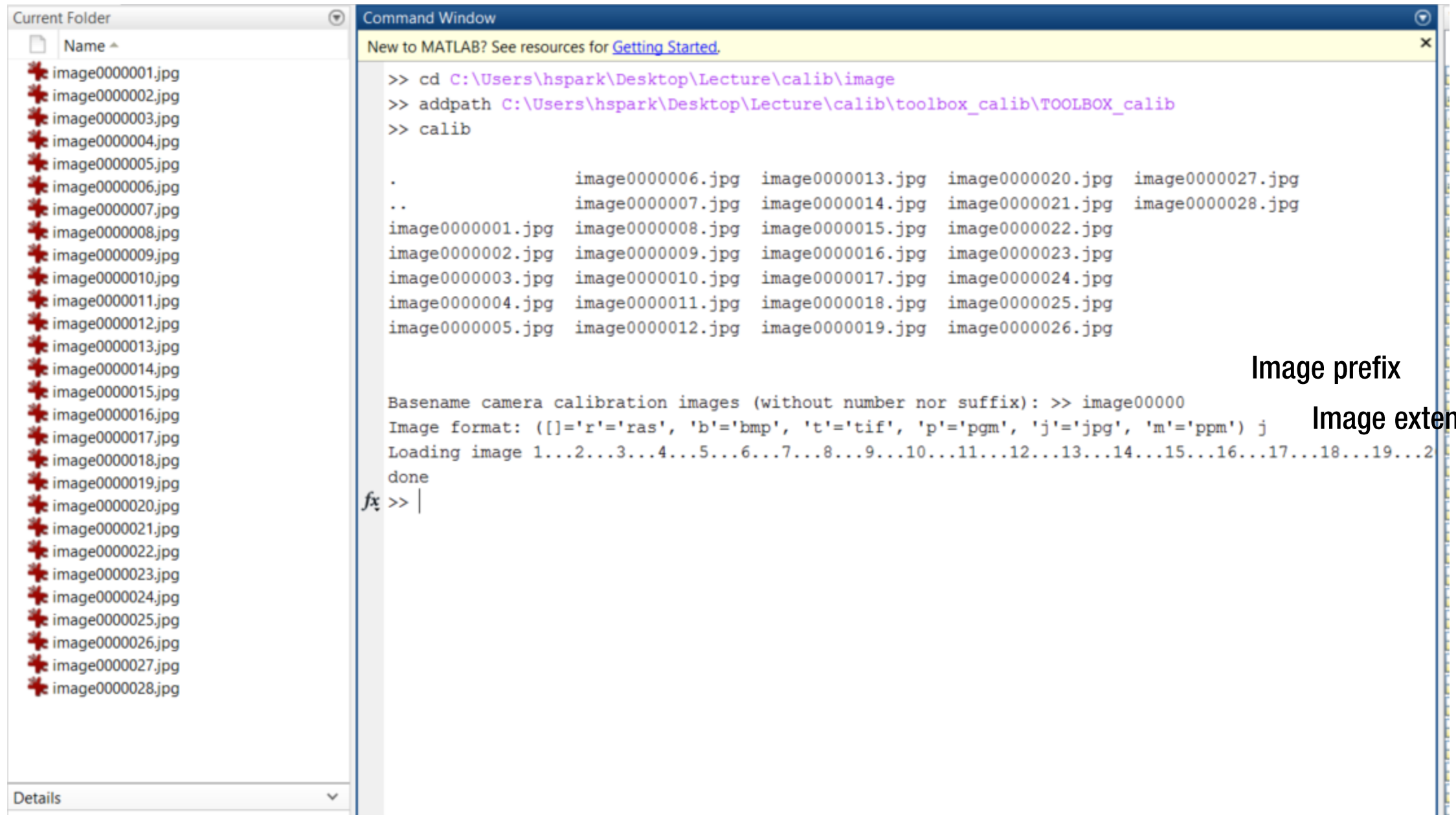
$$\mathbf{r}_1 = \mathbf{K}^{-1} \mathbf{h}_1 \quad \mathbf{r}_2 = \mathbf{K}^{-1} \mathbf{h}_2 \quad \mathbf{r}_3 = \mathbf{K}^{-1} \mathbf{h}_3$$

$$\mathbf{r}_1 = \frac{\mathbf{K}^{-1} \mathbf{h}_1}{\|\mathbf{K}^{-1} \mathbf{h}_1\|}, \quad \mathbf{r}_2 = \frac{\mathbf{K}^{-1} \mathbf{h}_2}{\|\mathbf{K}^{-1} \mathbf{h}_2\|}, \quad \mathbf{t} = \frac{\mathbf{K}^{-1} \mathbf{h}_3}{\|\mathbf{K}^{-1} \mathbf{h}_1\|}, \quad \mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

Divided by constant factor







Current Folder

Name ^

image0000001.jpg  
image0000002.jpg  
image0000003.jpg  
image0000004.jpg  
image0000005.jpg  
image0000006.jpg  
image0000007.jpg  
image0000008.jpg  
image0000009.jpg  
image0000010.jpg  
image0000011.jpg  
image0000012.jpg  
image0000013.jpg  
image0000014.jpg  
image0000015.jpg  
image0000016.jpg  
image0000017.jpg  
image0000018.jpg  
image0000019.jpg  
image0000020.jpg  
image0000021.jpg  
image0000022.jpg  
image0000023.jpg  
image0000024.jpg  
image0000025.jpg  
image0000026.jpg  
image0000027.jpg  
image0000028.jpg

Details

Command Window

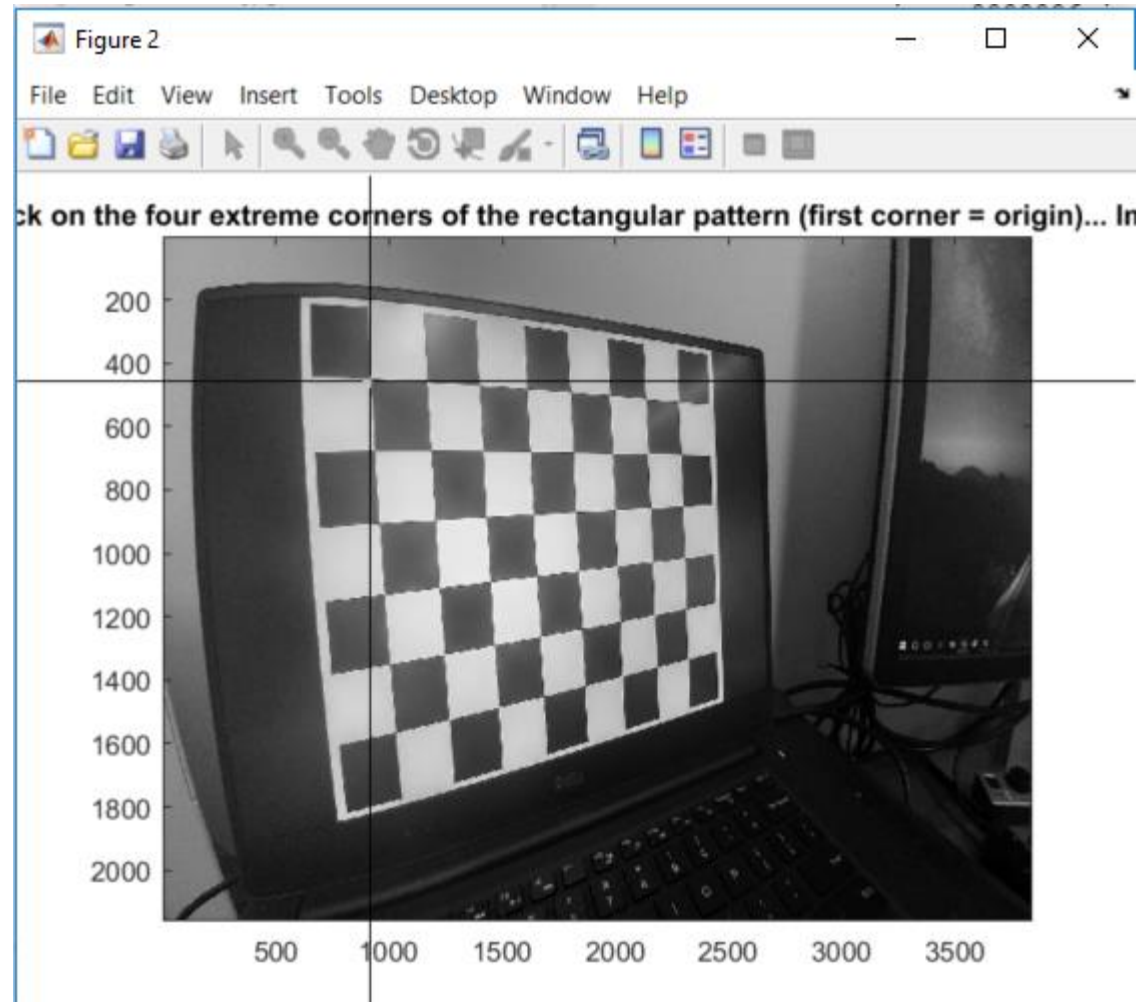
New to MATLAB? See resources for [Getting Started.](#)

>> cd C:\Users\hspark\Desktop\Lecture\calib\image  
>> addpath C:\Users\hspark\Desktop\Lecture\calib\toolbox\_calib\TOOLBOX\_calib  
>> calib

Camera Calibration Toolbox - Standard Version

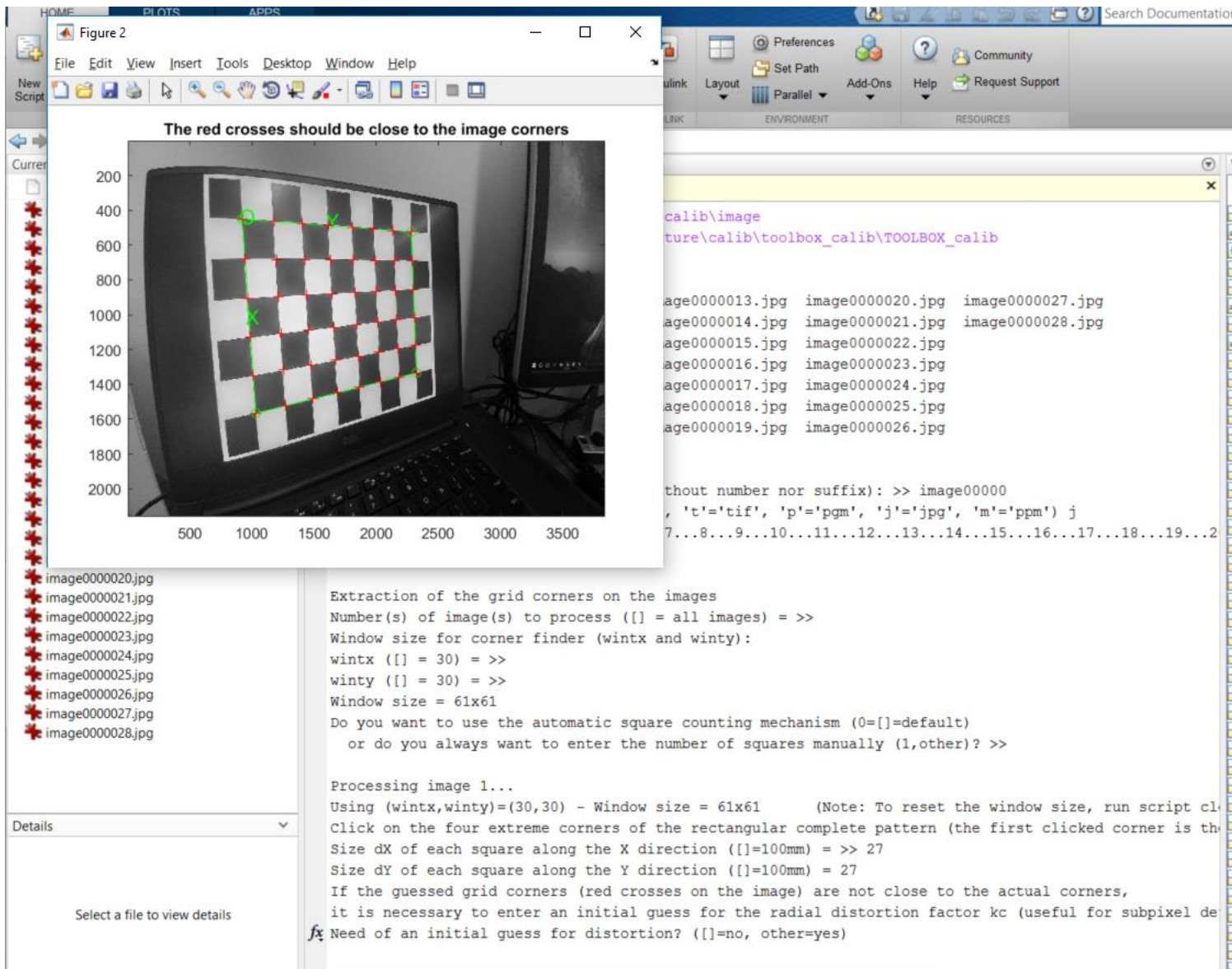
Image names	Read images	Extract grid corners	Calibration
Show Extrinsic	Reproject on images	Analyse error	Recomp. corners
Add/Suppress images	Save	Load	Exit
Comp. Extrinsic	Undistort image	Export calib data	Show calib results

image0000005.jpg image0000012.jpg image0000019.jpg image0000026.jpg  
  
Basename camera calibration images (without number nor suffix): >> image00000  
Image format: ([]='r'='ras', 'b'='bmp', 't'='tif', 'p'='pgm', 'j'='jpg', 'm'='ppm') j  
Loading image 1...2...3...4...5...6...7...8...9...10...11...12...13...14...15...16...17...18...19...20  
done  
fx >>



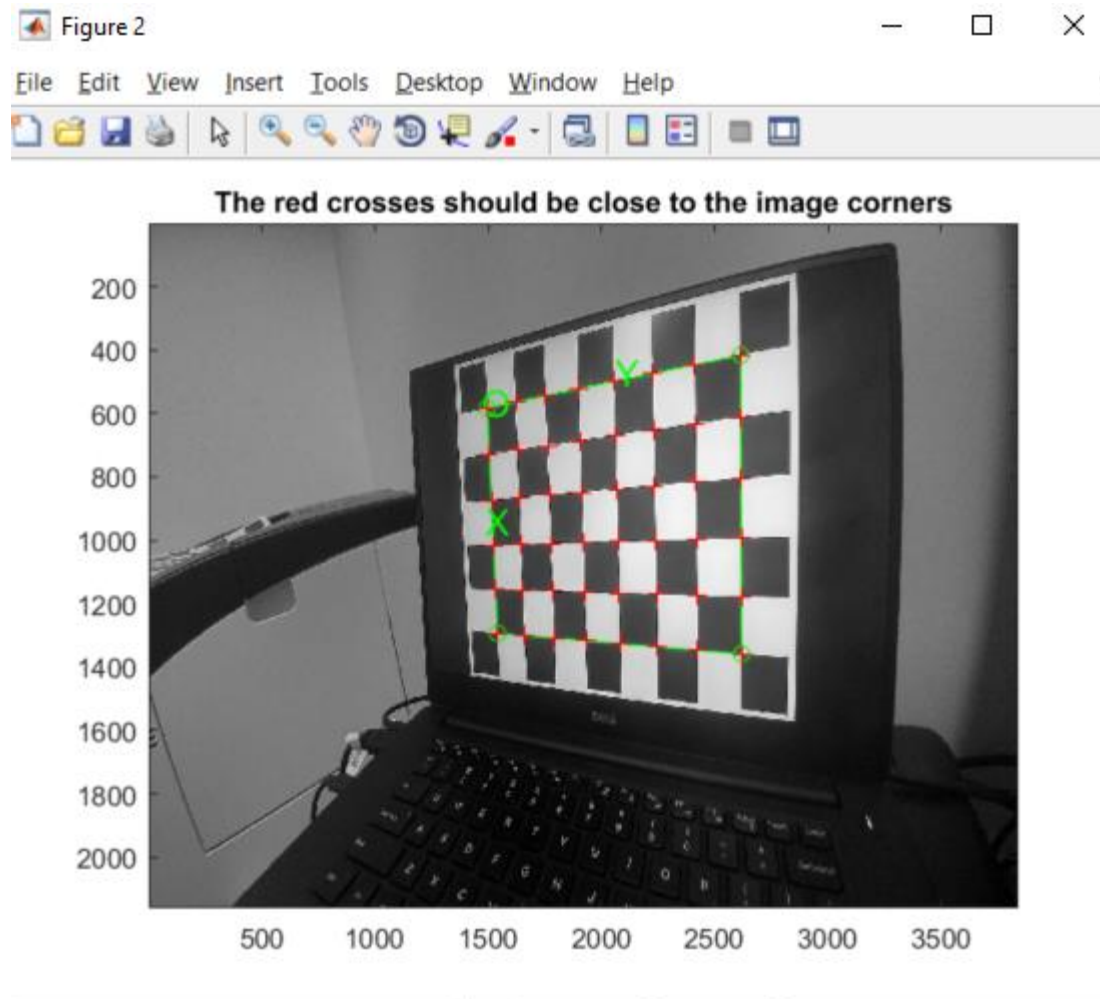
Click four corner in the following order:

1. Top left
2. Top right
3. Bottom right
4. Bottom left



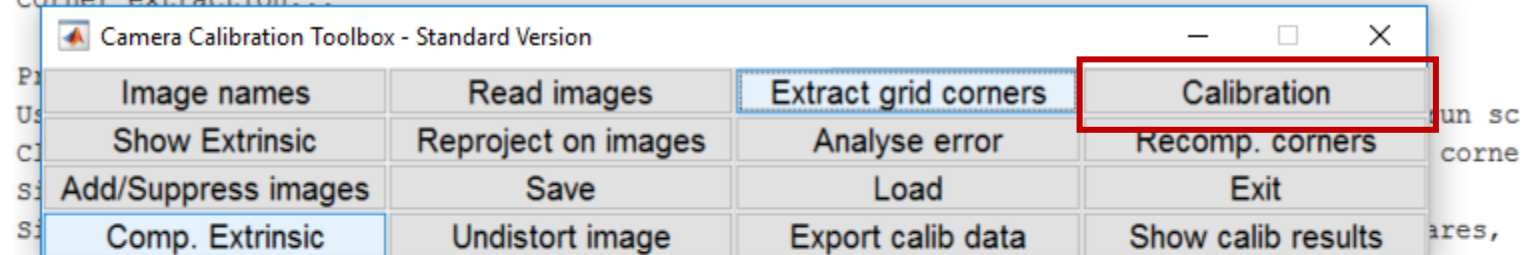
Default mode (press Enter)

Set grid size (27mm)



Need of an initial guess for distortion? ([])=no, other=yes)

Corner extraction...



If the guessed grid corners (red crosses on the image) are not close to the actual corners, it is necessary to enter an initial guess for the radial distortion factor  $k_c$  (useful for subp  
Need of an initial guess for distortion? ([])=no, other=yes)  
Corner extraction...

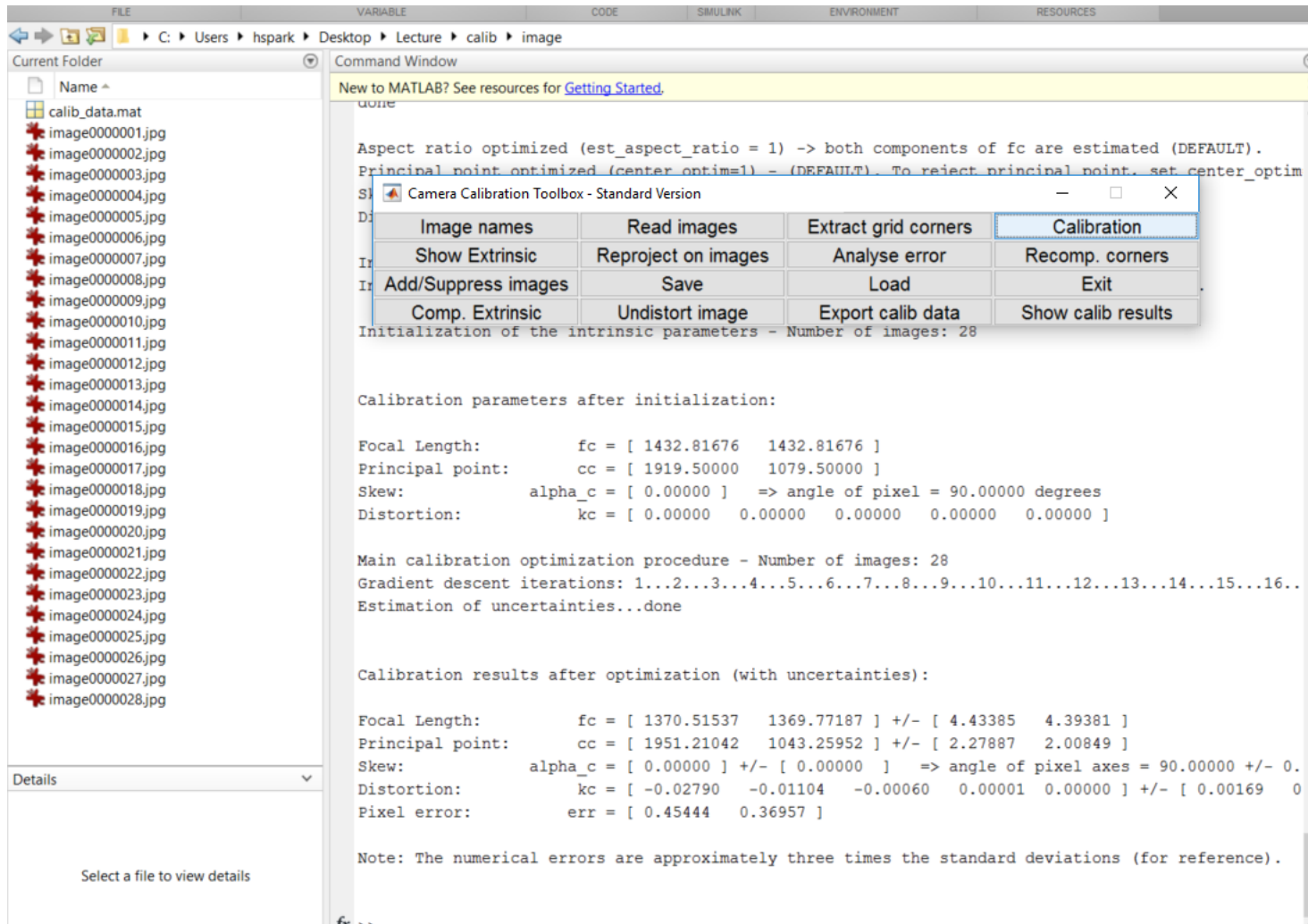
Processing image 27...

Using (wintx,winty)=(30,30) - Window size = 61x61 (Note: To reset the window size, run sc

Click on the four extreme corners of the rectangular complete pattern (the first clicked corne

Size of each square along the X direction: dX=27mm

Size of each square along the Y direction: dY=27mm (Note: To reset the size of the squares,

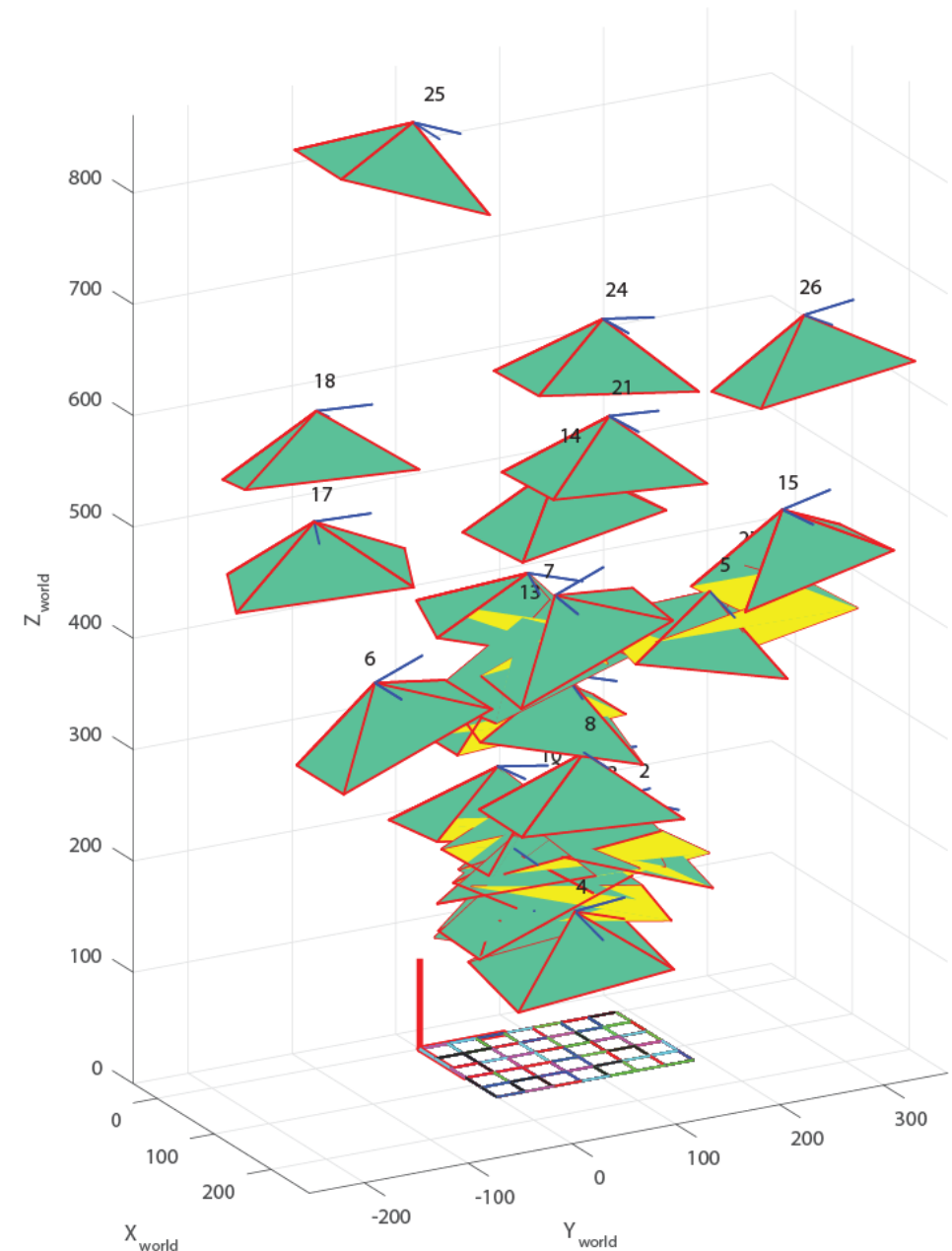


Cf) calibration with vanishing points

K =

$$\begin{bmatrix} 1317.2 & 0 & 1931.8 \\ 0 & 1317.2 & 1146.1 \\ 0 & 0 & 1 \end{bmatrix}$$

Extrinsic parameters (world-centered)



FILE VARIABLE CODE SIMULINK ENVIRONMENT RESOURCES

C:\Users\hspark\Desktop\Lecture\calib\image

Current Folder

calib\_data.mat

image0000001.jpg

image0000002.jpg

image0000003.jpg

image0000004.jpg

image0000005.jpg

image0000006.jpg

image0000007.jpg

image0000008.jpg

image0000009.jpg

image0000010.jpg

image0000011.jpg

image0000012.jpg

image0000013.jpg

image0000014.jpg

image0000015.jpg

image0000016.jpg

image0000017.jpg

image0000018.jpg

image0000019.jpg

image0000020.jpg

image0000021.jpg

image0000022.jpg

image0000023.jpg

image0000024.jpg

image0000025.jpg

image0000026.jpg

image0000027.jpg

image0000028.jpg

Command Window

New to MATLAB? See resources for [Getting Started](#).

done

Aspect ratio optimized (est\_aspect\_ratio = 1) -> both components of fc are estimated (DEFAULT).  
Principal point optimized (center\_optim=1) - (DEFAULT). To reject principal point, set center\_optim

Camera Calibration Toolbox - Standard Version

Image names	Read images	Extract grid corners	Calibration
Show Extrinsic	Reproject on images	Analyse error	Recomp. corners
Add/Suppress images	Save	Load	Exit
Comp. Extrinsic	Undistort image	Export calib data	Show calib results

Initialization of the intrinsic parameters - Number of images: 28

Calibration parameters after initialization:

Focal Length:  $fc = [ 1432.81676 \quad 1432.81676 ]$

Principal point:  $cc = [ 1919.50000 \quad 1079.50000 ]$

Skew:  $\alpha_c = [ 0.00000 ] \Rightarrow \text{angle of pixel} = 90.00000 \text{ degrees}$

Distortion:  $kc = [ 0.00000 \quad 0.00000 \quad 0.00000 \quad 0.00000 \quad 0.00000 ]$

Main calibration optimization procedure - Number of images: 28

Gradient descent iterations: 1...2...3...4...5...6...7...8...9...10...11...12...13...14...15...16...

Estimation of uncertainties...done

Calibration results after optimization (with uncertainties):

Focal Length:  $fc = [ 1370.51537 \quad 1369.77187 ] \pm [ 4.43385 \quad 4.39381 ]$

Principal point:  $cc = [ 1951.21042 \quad 1043.25952 ] \pm [ 2.27887 \quad 2.00849 ]$

Skew:  $\alpha_c = [ 0.00000 ] \pm [ 0.00000 ] \Rightarrow \text{angle of pixel axes} = 90.00000 \pm 0.$

Distortion:  $kc = [ -0.02790 \quad -0.01104 \quad -0.00060 \quad 0.00001 \quad 0.00000 ] \pm [ 0.00169 \quad 0.00169 \quad 0.00169 \quad 0.00169 \quad 0.00169 ]$

Pixel error:  $err = [ 0.45444 \quad 0.36957 ]$

Note: The numerical errors are approximately three times the standard deviations (for reference).

Select a file to view details

