Adaptive Redirected Walking in a Virtual World

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ABSTRACT

Redirected walking enables the physical exploration of large virtual environments within the confines of more limited physical spaces. In this paper, we present a novel method for adaptively determining an appropriate mapping from a linear path in a larger virtual environment into a curved path in a smaller tracked space that simultaneously integrates the use of both rotational and translational offsets. We evaluate the method using a software simulator that redirects a user walking on random paths between 16 randomly selected target locations in a virtual environment that is twice as wide as the available tracked space. We find that our controller has a 77% success rate using an average rotational offset of 0.5 degrees per step and an average translational gain of 1.5.

Index Terms: I.3.7 [Computer Graphics]: Three-Dimensional Graphics—Virtual Reality; H.5.2 [Information Interfaces and Presentation]: User Interfaces—Input Devices and Strategies;

1 INTRODUCTION

In the real world, when we want to move from one place to another within a moderately-sized area, we typically get there by walking. In virtual environments, it has been shown that people subconsciously feel a greater sense of presence in a virtual world [16], and that they are generally better able to maintain their sense of orientation within the virtual space with respect to portions of the environment that are out of view [11, 15, 7, 10], when they are enabled to move through the virtual environment using direct, physical walking than when they are required to use alternative locomotion methods that control the change in viewpoint through non-natural, indirect or non-active means.

Redirected walking, originally proposed in 2001 by Razzaque et al. [9], offers the potential to enable people to use direct real walking to actively explore a virtual space that is larger than the physically available tracked area. Redirection is typically accomplished by dynamically injecting small amounts of scene rotation as a person walks, in addition to larger amounts of rotational gain when he actively turns his head, in order to cause the virtual environment to rotate, relative to the physical environment, in such a way as to lead the user to walk in a direction in the physical space that follows a circular path around the center of the tracked area and avoids approaching its boundaries too closely. Scaled translational gain [17, 4] can also be used to enable people to traverse larger virtual distances within the confines of smaller physical area, and it has been suggested that these two techniques could be used in common to more robustly enable the illusion of a natural and unrestricted free exploration capability [13].

Studies by Steinicke et al. [12] have determined conservative thresholds for the amount of rotational and translational gain that can be safely applied to a person's motion in a virtual environment without their noticing, using an experimental protocol in which subjects were actively attending to the scene manipulation while moving. In further investigations, Neth et al. [5] studied the dynamic nature of some of these sensitivities and found that people were less likely to notice the injection of subtle amounts of scene rotation when they were walking more slowly.

While it is most desirable to implement redirection in such a way as to avoid making people overtly aware of unnatural irregularities in the mapping between their actions in the real world and their perceptions in the virtual world, in practice it is often impossible to satisfy all of the necessary constraints to enable this functionality. Nevertheless, Peck et al. [8] have found that navigation and wayfinding are better supported by a locomotion method that allows redirected free walking through an immersive virtual environment combined with the use of distractors when an overt reorienting intervention is needed than when travel through the environment is accomplished by walking-in-place and turning or via the use of a joystick. In addition, Hodgson et al. [3] have found, through studies that test peoples' memory for the spatial locations of prominent landmarks, not only that redirection does not significantly interfere with people's ability to maintain their sense of orientation in the virtual world, but also that, even when people are overtly aware that the virtual environment is not behaving naturally, they still fail to recognize that they are being turned around in the real world while exploring the virtual environment.

A variety of different approaches have been used to define an appropriate amount and direction of rotational and/or translational offset to apply, at any particular moment in time, to the movement of the virtual world with respect to the users movement in the real world. The proposed methods roughly fall into two distinct categories: a) methods that consider the users future movement through the virtual environment as a discrete set of paths, and either assume knowledge of or attempt to predict their end goal or intended direction of travel in order to derive an appropriate alternative path through the real world [6, 14, 2]; and b) methods that require only the information that is instantaneously available at a single moment in time.

Among the methods in the second category, it has been proposed: to inject a small constant amount of rotation of the virtual world with respect to the real world over time, such as 1-3° per second, independent of the movement of the user, as well as to rotate the users heading by a statically or dynamically determined amount as he walks, and to apply large rotational gains at times when the user is purposefully turning their body and/or head, in order to reorient them in a direction that leads away from the boundaries of the tracked space and towards a path that circulates around its center [9, 1, 5]. It has also been suggested to apply a moderate amount of constant translational gain while the user is walking [13, 5] in order to allow him to traverse greater distances in the virtual world with respect to the real world, independently of the other manipulations. Our approach falls into the first category of methods, but, unlike its predecessors, seeks a solution that uses a combination of subtle rotational and translational offsets to accomplish the redirection objectives. Our primary objective in the development of this redirection controller is to enable people to freely move about in an immersive virtual environment that is several times larger than our available tracked area while minimizing, to the greatest extent possible, the chance that they will enter a *failure state* where they cannot proceed as they intended.

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2 THEORETICAL SETUP

Without undue loss of generality, our model represents the physical environment as the largest circular area that fits completely within our tracked space, and it represents the virtual environment as the smallest circular area that completely contains the full extent of the accessible area within the virtual world. Figure 1 shows a real world (blue circle) over which a virtual world (green circle) has been superimposed.

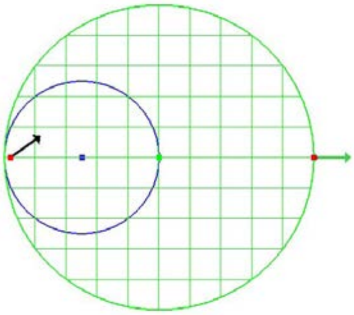


Figure 1: The superimposed bounding circles for the real world (blue) and virtual world (green).

We observe that when a user is close to the boundary of the physical space, it is advantageous for him to also be close to the boundary of the accessible area of the virtual environment, in order to avoid the possibility that he might choose to walk in a direction in the virtual world that would take him outside of the bounds in the real world. One possible method of ensuring such a relationship would be to constrain the virtual world to rotate around the real world as the user moves about, for instance by initializing the placement of the virtual world, relative to the real world, in an optimal configuration given the user's starting position in the room and then, as the user begins to walk around, keeping the virtual world 'pinned' to the real world at the point along the edge of the real world that is intersected by the radius that passes through the user's current location. Such a solution will actually work quite well in the case that the user decides to walk around the edges of the virtual space, but it degenerates as he walks towards the center of the real or virtual room. Fortunately, the importance of the attachment constraint also declines as the user walks away from the edges of the tracked area; in fact, when the user is located at the center of the physical space, the ideal configuration between real and virtual environments would have him centered in the virtual space as well. Our approach to the design of our redirected walking controller was inspired by the idea that it might be promising to consider the problem of redirection from an exocentric perspective as a question of beginning with the virtual and real environments relatively positioned in a 'good' state and then seeking to maintain the goodness of the configuration at subsequent points in time by a method that balances these two disjoint constraints.

3 OUR CONTROLLER

We now explain, through a simple example, how our controller works. The goal is to try to walk along the straight line indicated by the left arrow in the virtual world while simultaneously translating and rotating this world so that you arrive at the perimeter of both worlds at the same time. Many solutions to this can be found in general by choosing different rotation and translation speeds. Figure 2 show several frames of a solution for a specific choice of these speeds and indicates that in general you will walk along a curved path in the real world.

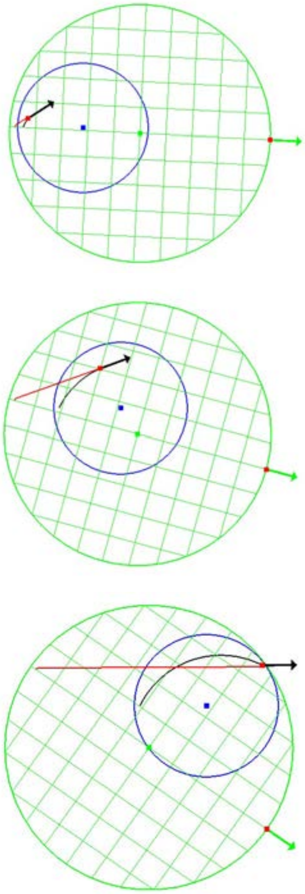


Figure 2: Three stages of walking. Top: the user sets out in the direction indicated by the black arrow. Middle: the red line shows the user's straight line path in the virtual world; the black line shows his actual path in the real world. Bottom: the user's arrival at the target endpoint in the virtual environment

First we consider the theoretical equations and constraints that define this problem. It is not possible in general to solve these equations with constraints exactly, so in the next section we define an algorithm for approximating a solution. Let R_r and R_v denote the real and virtual world radii, respectively, and let $\bar{x}(t)$ denote the straight line path in virtual world coordinates in the direction of the unit vector $\bar{v}$. Then

$$\bar{x}(t) = V(t)\bar{v} + \bar{x}(0) \tag{1}$$

where $V(t)$ is the distance traveled along this line, $V(0) = 0$ and for some t_p, $V(t_p) = R_v$. Also we want $V'(t) > 0$ for all t so that the motion is always in the direction of $\bar{v}$.

Now suppose we are rotating the virtual world so that at time t the angle rotated is $\theta(t)$. The direction we wish to rotate should be the same as the sense in which $\bar{v}$ starting at $\bar{x}(0)$ is rotating (see Figure 3). If we let $\bar{v} = \langle v_1, v_2 \rangle$ and $\bar{x}(0) = \langle x_1, x_2 \rangle$ we define

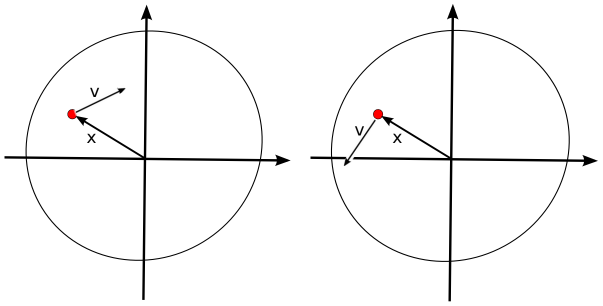
$$\text{Sense}(\bar{x}(0), \bar{v}) = \text{sgn}(x_1 v_2 - x_2 v_1) \tag{2}$$

and require that for all t,

$$\text{sgn}(\theta'(t)) = \text{Sense}(\bar{x}(0), \bar{v}) \tag{3}$$

Let

$$M(t) = \begin{pmatrix} \cos(\theta(t)) & -\sin(\theta(t)) \\ \sin(\theta(t)) & \cos(\theta(t)) \end{pmatrix} \tag{4}$$



(a) negative (clockwise) rotation (b) positive (counterclockwise) rotation

Figure 3: An illustration of the Rotation Sense.

be the associated rotation matrix.

We also want the center of the virtual world to be moving opposite to the direction of walking motion. We can achieve this by requiring that the center of the virtual world $\bar{c}(t)$ satisfies

$$\bar{c}'(t) = -T(t)M(t)\bar{v} \tag{5}$$

with $T'(t) > 0$. Then the real world path can be expressed as ;

$$\bar{u}(t) = M(t)\bar{x}(t) + \bar{c}(t) \tag{6}$$

Finally, there are several constraints we need to maintain:

$$|\bar{u}'(t)| = r \tag{7}$$

$$|\bar{u}(t)| \leq R_r \tag{8}$$

$$|\bar{x}(t_p)| = R_v \text{ implies } |\bar{u}(t_p)| = R_r \tag{9}$$

The first constraint moves $\bar{u}(t)$ at constant speed r, the second constraint keeps $\bar{u}(t)$ inside the real world circle and the third constraint forces the virtual and real paths to arrive at their respective perimeters at the same time.

In general, there is no tractable method for finding the relationship among $V(t)$, $\theta(t)$ and $T(t)$ satisfying equations (1) - (9), or even determining if a solution exists. The situation is simplified if we assume that $\theta(t)$ and $T(t)$ are linear, that is, for constants θ_s and T_s

$$\theta(t) = \theta_s t + \theta_0 \tag{10}$$

$$T(t) = T_s t \tag{11}$$

Even in this case, it may not be possible to find $V(t)$ exactly. However, under this linearity condition, in most cases, we can find a reasonable approximation to $V(t)$.

4 APPROXIMATING $V(t)$

Suppose in the current state the real position is $\bar{u}_0$, the virtual position is $\bar{x}_0$, and the virtual world has already been rotated by θ_0 and translated to $\bar{c}_0$ and we assume the linearity conditions in equations (10) and (11) in the previous section.

We start by taking a virtual step of unknown size V over the time interval Δt. Then approximately:

$$\bar{x}(\Delta t) \approx V\bar{v} + \bar{x}_0$$

The new center is approximately:

$$\bar{c}(\Delta t) \approx \bar{c}_0 - T_s \Delta t M(\Delta t)\bar{v}$$

and hence our new real world position is approximately:

$$\bar{u}(\Delta t) \approx M(\Delta t)(V\bar{v} + \bar{x}_0) + \bar{c}_0 - T_s \Delta t M(\Delta t)\bar{v}$$

In order to enforce the first constraint from the previous section (eq.(7)), we require that

$$|\bar{u}(\Delta t) - \bar{u}_0| = r\Delta t.$$

i.e. we are moving at a constant speed r in the real world.

This is equivalent to

$$(\bar{u}(\Delta t) - \bar{u}_0) \circ (\bar{u}(\Delta t) - \bar{u}_0) = (r\Delta t)^2 \tag{12}$$

where $\circ$ denotes the dot product.

If we let

$$\bar{y}_1 = M(\Delta t)\bar{v}$$

$$\bar{y}_2 = M(\Delta t)(\bar{x}_0 - T_s \Delta t \bar{v}) + \bar{c}_0 - \bar{u}_0$$

we can write equation (12) as

$$(y_1 \circ y_1)V^2 + 2(y_1 \circ y_2)V + y_2 \circ y_2 - (r\Delta t)^2 = 0 \tag{13}$$

Further, since $\bar{v}$ is a unit vector and M is a rotation matrix, $y_1 \circ y_1 = 1$ this reduces further to

$$V^2 + 2(y_1 \circ y_2)V + y_2 \circ y_2 - (r\Delta t)^2 = 0 \tag{14}$$

This is a quadratic equation in V and may not have a real positive solution. If there are two positive solutions, we choose the larger one.

Whether or not we can find a solution for V depends on the rotation and translations speeds (θ_s and T_s in eq.(10) and eq.(11)). Our algorithm for finding these speeds is summarized as follows:

1. For each new direction vector $\bar{v}$, iterate through a sequences of guesses for θ_s and T_s searching for a solution for V. When a solution is found we

2. Follow the path in the real-world created by taking steps of size V in the $\bar{v}$ direction in the virtual world while at the same time rotating and translating the virtual world by θ_s and T_s to see if we arrive at the real world and virtual world perimeters at the same time (Constraint in eq.(9)). If this occurs within some tolerance we

3. Choose the solution that minimizes the ratio of the virtual and real distances walked. The idea is to make the virtual speed as close as possible to the real speed.

4. Failure occurs when there is no solution to eq.(14)) somewhere along the way. Note that although this failure situation may occur mathematically, in actual practice a walker will in all likelihood change direction before reaching the perimeter, so the real failure rate may be much lower.

5 SIMULATING ADAPTIVE WALKING

We have developed a software simulator that can be used to test how well our method works. This simulator has the capability to include other methods of redirected walking so it can be used as a method comparison tool, although we have not yet done so. The simulator creates a real-world defined as a circle of a given diameter and a virtual world circle of twice the real-world diameter. Sixteen point locations are randomly placed in the virtual circle (Figure 4) and a random walk is computed that typically visits points many times before all have been visited at least once (Figure 5) .

The algorithm described in the previous section is run and we track whether or not the walk can be completed before *Failure* occurs. The frequency with which this happens depends on how big we allow the rotation amount per step to be. We ran our algorithm in the simulator for 100 different random locations of the 16 points and 100 different random walks to visit these points for a total of 10,000 test cases. The results are summarized in Figure 5. In particular, note that although the virtual world is twice the size of the real one, the walker's virtual speed is only 50% larger than the real speed.

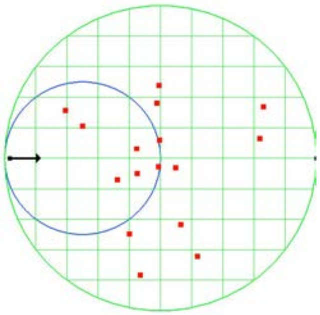
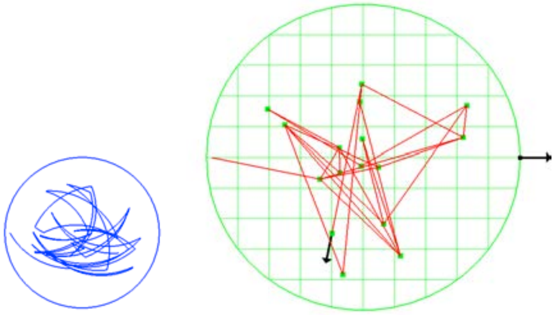


Figure 4: The simulator setup, showing superposition of both worlds.



(a) Real Path (b) Virtual Path

Figure 5: The paths taken in the real and virtual worlds during the random walk.

Success Rate	77%
Average Rotation Angle Per Step	0.5 degrees
Average Virtual/Real Step Ratio	1.5
Largest Virtual/Real Step Ratio	2.5

Figure 6: 10,000 Random Walks Results

6 FUTURE WORK

We are currently working on integrating this control algorithm into our virtual environments software so that we can test it under more realistic conditions of practical use. Our present algorithm does not take advantage of the possibility to apply large rotational gains at points where the user is rapidly turning his head, and our next goal in the development process is to extend the controller to incorporate this capability. We also plan to assess the performance of our controller in situations where the virtual environment is quite a bit larger than just twice the size of the available tracked area. Once our controller is functioning robustly, we will be able to conduct controlled experiments with human subjects to investigate the relative effects of using increasing amounts of redirection on peoples' ability to successfully perform automatic spatial updating and to maintain an accurate sense of where they are in the virtual space, in relationship to portions of the environment that are out of view. In those studies, we plan to additionally compare the impact of redirection applied both when people are walking and when they are physically driving around using a motorized wheelchair.

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