Math 126 – Homework 1 (Due Friday Sept 4)

1. Course logistics.
   (a) Read the *entire* course syllabus: https://math.berkeley.edu/~jcalder/126F15 When are the midterm and final exams? What is the homework policy? How will your final grade be computed.
   (b) Sign up for Piazza to get course announcements and participate in classroom discussions with other students: https://piazza.com/berkeley/spring2015/math126
   (c) Fill out background courses poll on Piazza.

2. For each of the following equations, state the order and whether it is nonlinear, linear inhomogeneous, or linear homogeneous. Give a brief justification.
   (a) $u_t - u_{xx} + x = 0$.
   (b) $(x+y)u_{xy} + 2xu_y = x^2$.
   (c) $u_{xx} = e^u$.
   (d) $u_{txy} - u_{xx}u_{yy} + u_x = x^3$.
   (e) $u_t + u_{xxxxxx} - \sqrt{1+u^2} = 0$.
   (f) $u_t + u_x + u_y + u/xy = 0$.

3. Show that the difference between two solutions of an inhomogeneous linear equation $\mathcal{L}u = g$ with the same g is a solution of the homogeneous equation $\mathcal{L}u = 0$.

4. (a) Verify that $u(x,y) = \rho x + y/\rho$ is a solution of $u_x u_y = 1$.
   (b) By trial and error, find two different solutions of $u_x u_y = u$ in the domain $x \geq 0$ and $y \geq 0$ that satisfy $u(x,0) = u(0,y) = 0$ for all $x \geq 0$ and $y \geq 0$.

5. Find the general solution of the equation $u_x + u_y = 1$.

6. (a) Show that any continuously differentiable function u satisfying $xu_x + yu_y = 0$ on the entire plane $\mathbb{R}^2$ must be constant.
   (b) Find a solution of $xu_x + yu_y = 0$ on the punctured plane $\mathbb{R}^2 \setminus (0,0)$ that is not constant.

7. Derive the equation of a one-dimensional diffusion in a medium that is moving along the x-axis to the right at constant speed $V > 0$.

8. Recall Schrödinger's equation

$$-ihu_t = \frac{h^2}{2m}\Delta u + \frac{e^2}{r}u.$$

   Show that if $\iiint |u|^2 dxdydz = 1$ at $t = 0$, then the same is true at later times. [Hint: Differentiate the integral with resepct to t. You may assume that u and $\nabla u \to 0$ fast enough as $(x,y,z) \to \infty$. Recall that u is complex-valued.]

9. Consider the Neumann problem

$$\left.\begin{aligned} -\Delta u &= f(x,y,z) \quad && \text{in } D \\ \frac{\partial u}{\partial n} &= 0 && \text{on } \partial D. \end{aligned}\right\}$$

(a) If a solution exists, is it unique?

(b) Use the divergence theorem and the PDE to show that

$$\iiint_D f(x,y,z)\, dxdydz = 0$$

is a necessary condition for the Neumann problem to have a solution.

(c) Give a physical interpretation of (a) and (b) for either heat flow or diffusion.