Math 1272: Calculus II
9.2 Direction Fields and Euler's Method

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Direction Fields

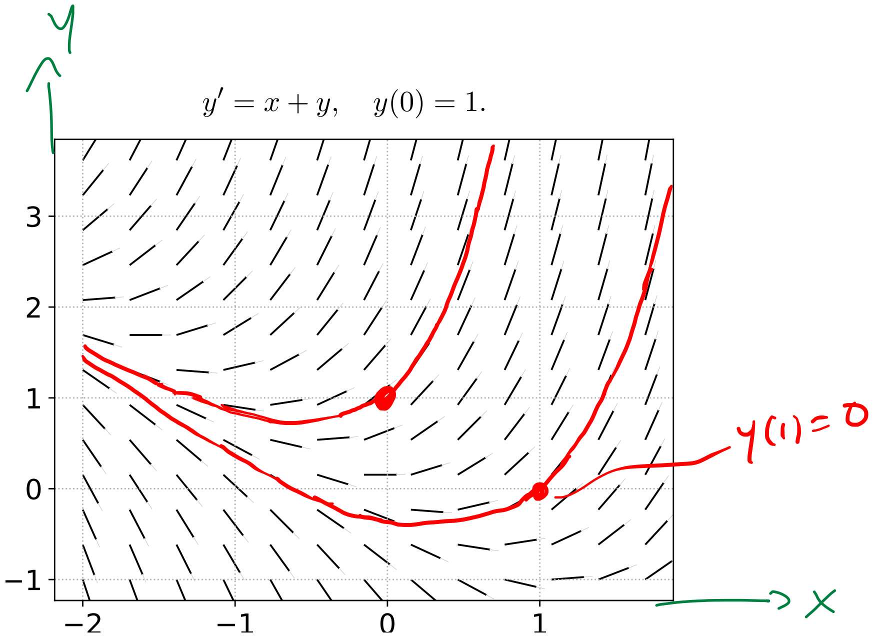
Consider the differential equation

$$y' = x + y, \quad y(0) = 1.$$

The equation says the *slope* of y is $x + y$.

$$\frac{dy}{dx} = x + y$$

$$y' = x + y, \quad y(0) = 1.$$



Direction Fields

For a differential equation of the form

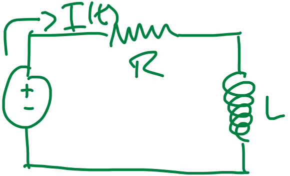
$$y' = F(x, y)$$

$F(x, y)$ is the slope. A **Direction Field** is a sketch consisting of short line segments of slope $F(x, y)$ at points (x, y) in the plane.

The solution can be sketched by tracing the direction field.

Sketch the direction field for $y' = x^2 + y^2 - 1$ and sketch a solution.

Equillibrium Solution $I(t) = 5$

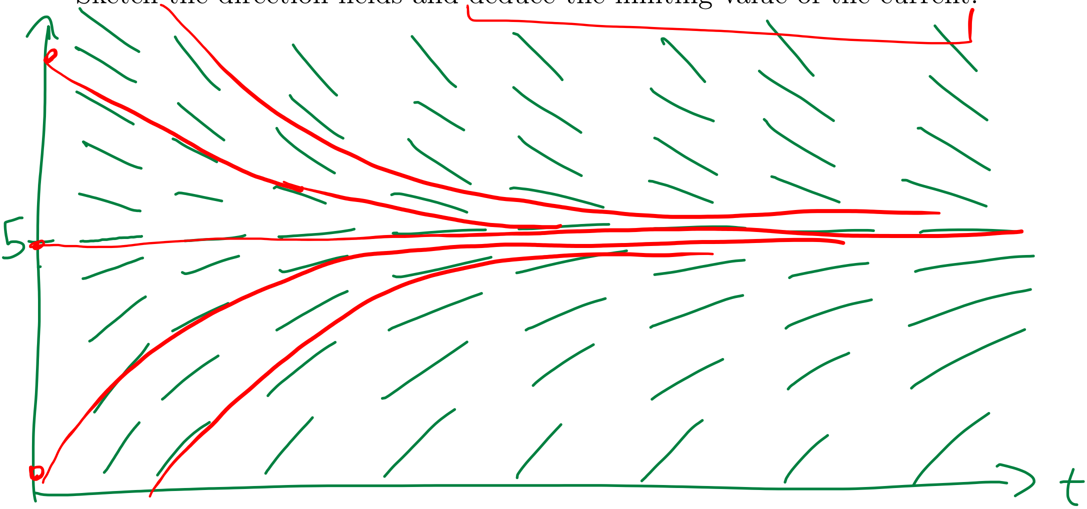


Equations for electric current in a circuit

$$\frac{dI}{dt} = 15 - 3I.$$

I

Sketch the direction fields and deduce the limiting value of the current.



Equations of the form

$$y' = F(y)$$

are called **autonomous**. The slope depends only on y.

Euler's method

Consider a differential equation

$$y' = F(x, y) \quad \text{with } y(0) = a.$$

$$y(x+h) \simeq y(x) + h \frac{dy}{dx}(x)$$

$$= y(x) + h\, y'(x)$$

$$= y(x) + h\, F(x, y(x))$$

Euler's method

Approximate values for the solution of the inital value problem

$$y' = F(x, y) \quad y(x_0) = y_0$$

with step size h at $x_n = x_{n-1} + h$ are

$$y_n = y_{n-1} + hF(x_{n-1}, y_{n-1}).$$

Use Euler's method with step size 0.1 to construct a table of approximate values for the solution of the initial-value problem

$$y' = 2x + 3y \quad y(0) = 1.$$

n	x	y	y'
0	0	1	3
1	0.1	1.3	4.1
2	0.2	1.71	5.53
3	0.3	2.263	
4			
5			

$y_1 = y_0 + h\underbrace{F(x_0, y_0)}_{y_0'}$

$= 1 + 0.1 \cdot 3 = 1.3$

$y_1' = 2(0.1) + 3(1.3) = 4.1$

$y_2 = y_1 + 0.1 \cdot (4.1)$

$= 1.3 + 0.41 = 1.71$

$y_2' = 2(0.2) + 3(1.71)$

$= 5.53$

$$y_3 = y_2 + 0.1(5.53)$$

$$= 1.71 + 0.553$$

$$= 2.263$$

Use Euler's method with step size 0.05 to construct a table of approximate values for the solution of the initial-value problem

$$y' = \sout{2x + 3y} \quad y(0) = 1.$$

$x + y$

$h = 0.05$

n	x	y	y' = x + y
0	0.00	1	1
1	0.05	1.05	1.10
2	0.10	1.105	1.205
3	0.15		
4	0.20		
5	0.25		

$y_1 = y_0 + h(x_0 + y_0)$

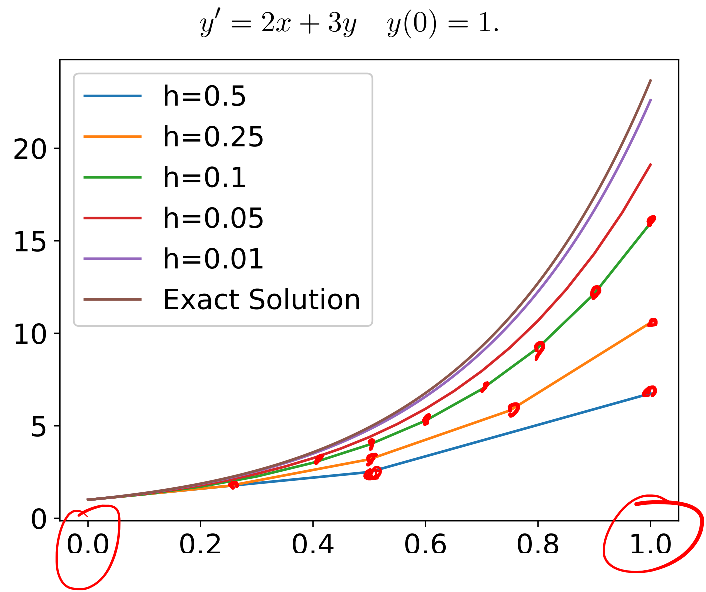
$= 1 + 0.05(0 + 1)$

$= 1.05$

$y_2 = y_1 + h(x_1 + y_1)$

$= 1.05 + 0.05(1.10)$

$= 1.05 + .055 = 1.105$


$y' = 2x + 3y \quad y(0) = 1.$
h=0.5
h=0.25
h=0.1
h=0.05
h=0.01
Exact Solution
0
5
10
15
20
0.0
0.2
0.4
0.6
0.8
1.0