Math 1272: Calculus II
12.1 Three dimensional coordinate systems

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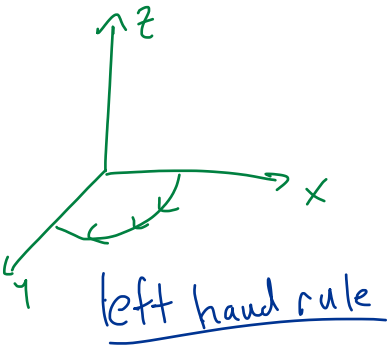
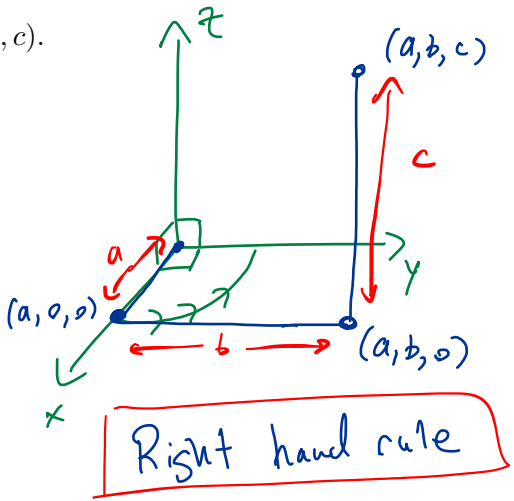
3D space

Every point P in 3D space is represented in rectangular (cartesian) coordinates by three numbers

$$P = (a, b, c).$$

- $a = x$-coordinate
- $b = y$-coordinate
- $z = z$-coordinate



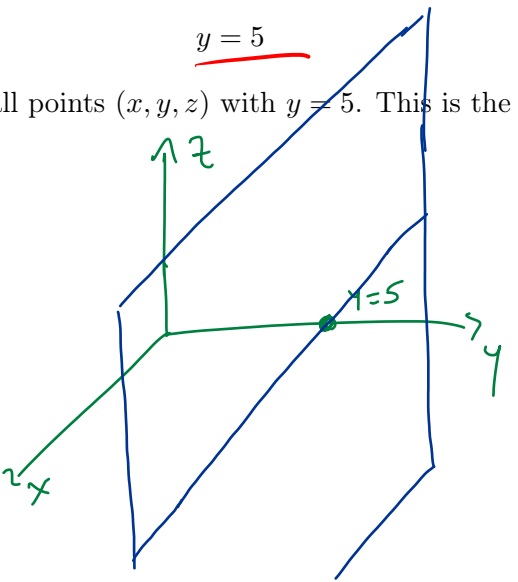


Surfaces

Surfaces in $\mathbb{R}^3$ are defined by equations involving x, y, z. For example

$$y = 5$$

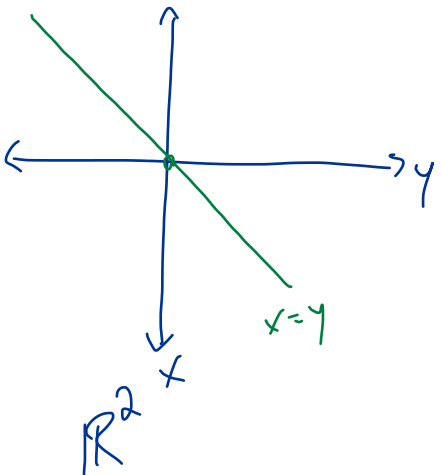
refers to the set of all points (x, y, z) with $y = 5$. This is the plane.

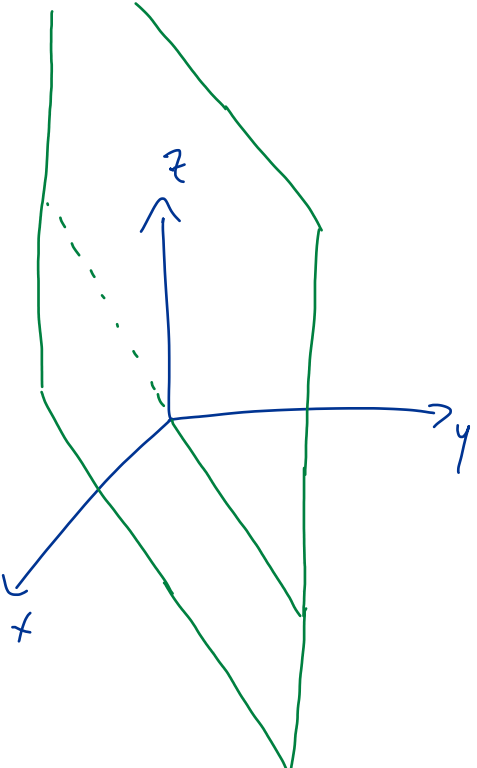


Example: Sketch the curve/surface

$$x = y$$

in $\mathbb{R}^2$ and $\mathbb{R}^3$.

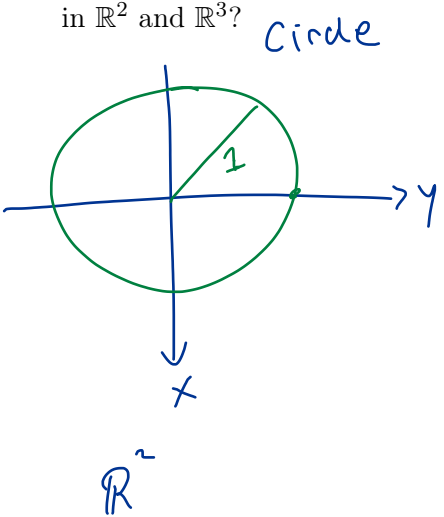


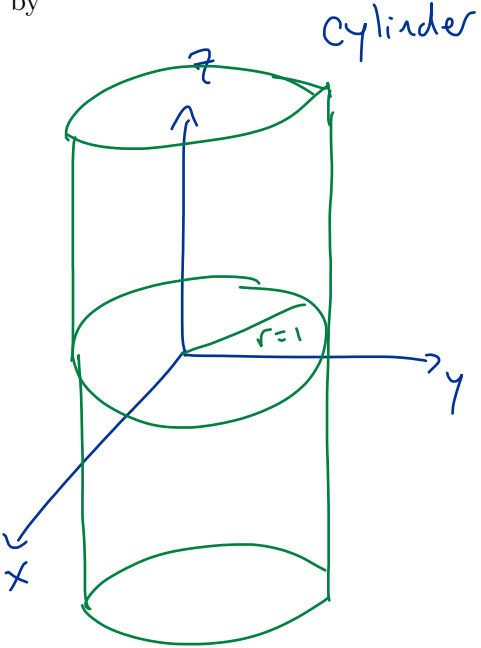


Example: What curve/surface is described by

$$x^2 + y^2 = 1$$

in $\mathbb{R}^2$ and $\mathbb{R}^3$?



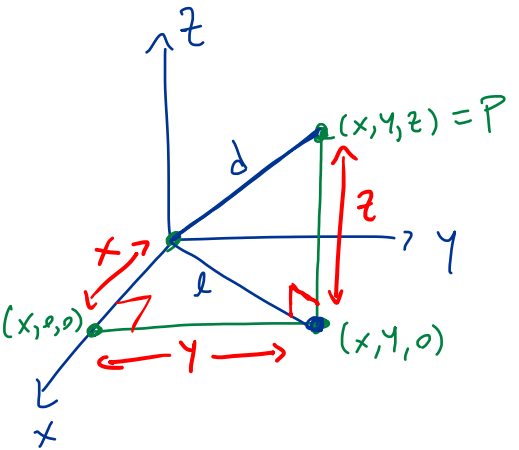


Example: What curve/surface is described by

$$x^2 + y^2 = 1$$

in $\mathbb{R}^2$ and $\mathbb{R}^3$?

Question: What is the distance between a point $P = (x, y, z)$ and the origin $(0, 0, 0)$ in $\mathbb{R}^3$?



$$\ell^2 = x^2 + y^2$$

$$d^2 = \ell^2 + z^2$$

$$d^2 = x^2 + y^2 + z^2$$

$$d = \sqrt{x^2 + y^2 + z^2}$$

Distance formula

The distance between the points $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

x^2 y^2 z^2

Example: Find the distance between $(2, -1, 7)$ and $(1, -3, 5)$.

$$d = \sqrt{(2-1)^2 + (-1-(-3))^2 + (7-5)^2}$$

$$= \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$$

Exercise: What is the equation for a sphere with radius r centered at (h, k, l)?

Sphere is all (x, y, z) such that

$$\sqrt{(x-h)^2 + (y-k)^2 + (z-l)^2} = r$$

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

$$x^2 - 2xh + h^2 + y^2 - 2ky + k^2 + z^2 - 2lz + l^2 = r^2$$

Exercise: Show that

$$x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$$

is the equation for a sphere, and find the center and radius.

Complete the square:

$$x^2 + 4x = (x+2)^2 - 4$$

$$y^2 - 6y = (y-3)^2 - 9$$

$$z^2 + 2z = (z+1)^2 - 1$$

$$(x+2)^2 + (y-3)^2 + (z+1)^2 - 14 + 6 = 0$$

$$(x+2)^2 + (y-3)^2 + (z+1)^2 = 8 = r^2$$

Sphere of radius $r = \sqrt{8}$ centered at $(-2, 3, -1)$.