Math 1272: Calculus II
Final exam review

Instructor: Jeff Calder
Office: 538 Vincent
Email: jcalder@umn.edu

http://www-users.math.umn.edu/~jwcalder/1272S19

Final Exam Information

- May 9, 12pm–3pm
- 18 questions
  - 8 multiple choice (a)–(e) Bubble sheets
  - 10 written questions
- Exam covers all sections listed on the schedule

  http://www-users.math.umn.edu/~jwcalder/1272S19/schedule.html

  except for **8.3, 9.4, 9.6, 10.5, orthogonal trajectories**.
- Midterm exams with solutions are available on the course website (linked above). This can be helpful for review.

Chapter 7

Topics: Integration (by parts, substitution, partial fractions, trig substitution, trig integrals, approximate integration)

Exampe: Determine the value of

$$\int_0^1 x^2 \cos x \, dx.$$

IBP

$u = x^2$, $dv = \cos x \, dx$ | $u = 2x$, $dv = \sin x \, dx$

$du = 2x$, $v = \sin x$ | $du = 2\,dx$, $v = -\cos x$

$$\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx$$

$$= x^2 \sin x - \left(-2x\cos x - \int -\cos x \, 2\,dx \right)$$

$$= x^2 \sin x + 2x\cos x - 2\int \cos x \, dx$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + C$$

$$\int_0^1 x^2 \cos x \, dx = x^2 \sin x + 2x\cos x - 2\sin x \Big]_0^1$$

$$= \sin(1) + 2\cos(1) - 2\sin(1)$$

$$= 2\cos(1) - \sin(1).$$

Chapter 8

Topics: Arclength, surface of revolution,

Exampe: Find the area of the surface obtained by rotating the curve

$$y = x^3, \quad 0 \le x \le 2$$

about the x-axis.

$$S = 2\pi \int_0^2 y \sqrt{1 + \left[\frac{dy}{dx}\right]^2} \, dx \quad \text{(Formula sheet)}.$$

$$= 2\pi \int_0^2 x^3 \sqrt{1 + (3x^2)^2} \, dx$$

$$= 2\pi \int_0^2 x^3 \sqrt{1 + 9x^4} \, dx$$

$u = 1 + 9x^4$, $\quad du = 36x^3\,dx$

$x=0,\ u=1$ $\quad dx = \dfrac{du}{36x^3}$

$x=2,\ u = 1 + 9 \cdot 16$
$= 145$

$$= 2\pi \int_1^{145} x^3 \sqrt{u}\, \frac{du}{36x^3}$$

$\begin{array}{r} 16 \\ 9 \\ \hline 144 \end{array}$

$$= \frac{2\pi}{36} \int_1^{145} \sqrt{u}\, du = \frac{\pi}{18} \frac{2}{3} u^{3/2} \Big]_1^{145}$$

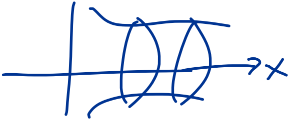
$$= \frac{\pi}{27} \left((145)^{3/2} - 1 \right)$$

$$S = 2\pi \int_a^b y \sqrt{1 + \left[\frac{dy}{dx}\right]^2}\, dx$$

radius

$y = f(x)$

rotate around x-axis

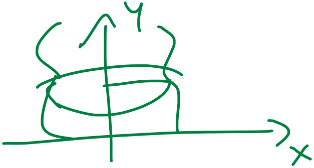


$$S = 2\pi \int_a^b x \underbrace{\sqrt{1 + \left[\frac{dx}{dy}\right]^2}\, dy}_{ds}$$

$$ds = \sqrt{dx^2 + dy^2}$$

$x = g(y)$

rotate around y-axis



Rotate $y = f(x)$ about y-axis

$$S = 2\pi \int_a^b x \underbrace{\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy}$$

$$ds = \sqrt{dx^2 + dy^2}$$

$$= \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = 2\pi \int_a^b x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Chapter 9

Topics: Differential equations (Euler's method, separable and linear equations)

Example: Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{x \sin x}{y}, \quad = f(x)g(y)$$

satisfying $y(0) = -1$.

Separate variables $\int y\,dy = \int x \sin x \, dx$

IBP

$$\frac{1}{2}y^2 = \left[\begin{array}{l} u = x \\ dv = \sin x\, dx \\ v = -\cos x \\ du = dx \end{array}\right] = -x\cos x - \int -\cos x \, dx$$

$$\frac{1}{2}y^2 = -x\cos x + \sin x + C$$

$$\frac{1}{2} = \frac{1}{2}(y(0))^2 = -0\cdot\cos(0) + \sin(0) + C$$

$$\longrightarrow \quad C = \frac{1}{2}$$

$$\frac{1}{2}y^2 = -x\cos x + \sin x + \frac{1}{2}$$

$$y^2 = -2x\cos x + 2\sin x + 1$$

$$\boxed{y = -\sqrt{1 - 2x\cos x + 2\sin x}}$$

$$y(0) = -\sqrt{1 - 0 + 0} = -\sqrt{1}$$

$$= -1$$

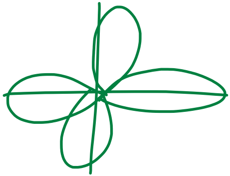
Chapter 10

Topics: Parametric equations, polar coordinates, area and arclength in polar coordinates.

Example: Set up (but do not evaluate) the integral computing the arclength of one loop of the four-leaved rose

$$r = \cos(2\theta).$$

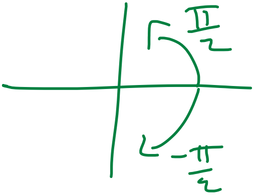
$$\frac{dr}{d\theta} = -2\sin(2\theta)$$



$\theta : \cos(2\theta) = 0$

$2\theta = \frac{\pi}{2} \longrightarrow \theta = \frac{\pi}{4}$

$2\theta = -\frac{\pi}{2} \longrightarrow \theta = -\frac{\pi}{4}$



$$L = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{(\cos(2\theta))^2 + (-2\sin(2\theta))^2}\, d\theta$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{\cos^2(2\theta) + 4\sin^2(2\theta)}\, d\theta.$$

Chapter 11

Topics: Sequences and series, tests for convergence, power series, Taylor and Maclaurin series

Example: Determine whether the series

$$n=1 \quad 2-\frac{2}{\sqrt{2}}+\frac{2}{\sqrt{3}}-\frac{2}{\sqrt{4}}+\cdots$$

converges absolutely, converges conditionally, or ~~diverges~~.

General form $\displaystyle\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}(-1)^{n+1}$

$$= -2\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

Alt. Series Test

$b_n = \frac{1}{\sqrt{n}}$

1. positive $b_n > 0$ ✓
2. Decreasing $b_{n+1} < b_n$ ✓
3. $\lim_{n\to\infty} b_n = 0$ ✓

$\longrightarrow$ Converges by Alt series test

Absolute convergence

$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}$, $a_n = \frac{2(-1)^{n+1}}{\sqrt{n}}$

Diverges by p-test since $p = \frac{1}{2} < 1$

$\longrightarrow$ Series converges conditionally

Chapter 12

Topics: Dot and cross products, lines and planes.

Example: Find a vector orthogonal (perpendicular) to both $\mathbf{a} = \langle 1, 2, 3 \rangle$ and $\mathbf{b} = \langle 1, 0, 1 \rangle$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= \hat{i}\,(2-0) - (1-3)\,\hat{j} + (0-2)\,\hat{k}$$

$$= 2\hat{i} + 2\hat{j} - 2\hat{k} = \langle 2, 2, -2 \rangle$$

Example: Find an equation for the plane containing the lines

$$x = 1 + t,\ \ y = 1 - t,\ \ z = 1$$

and

$$x = 1 + 2t,\ \ y = 1 + 3t,\ \ z = 1 + t.$$