Math 222A – Homework 7

1. Evans: Section 5.10, Problem 1
2. Evans: Section 5.10, Problem 3 (Problem 7 in 1st edition)
3. Evans: Section 5.10, Problem 4 (Problems 5 and 6 in 1st edition) [Hint: Set $v(x) = \int_0^x u'(t)\,dt$, where u' is the *weak* derivative of u. You may use the fact from analysis that v is absolutely continuous, hence differentiable almost everywhere (this follows from the integrability of u'). Show that $v = u$ almost everywhere.]
4. Evans: Section 5.10, Problem 8 (Problem 14 in 1st edition)
5. Evans: Section 5.10, Problem 9 (Problem 8 in 1st edition)
6. Evans: Section 5.10, Problem 10 (a) (Problem 9 in 1st edition)
7. Evans: Section 5.10, Problem 11 (Problem 10 in 1st edition)
8. Verify that the extension operator is a bounded linear operator $E : W^{2,p}(U) \to W^{2,p}(\mathbb{R}^n)$ whenever ∂U is C^2. In other words, show that the "higher-order reflection" used in the proof gives a function $\overline{u} \in W^{2,p}(B)$ and there exists a constant C such that

$$\|\overline{u}\|_{W^{2,p}(B)} \leq C\|u\|_{W^{2,p}(B^+)},$$

where C does not depend on u. [You don't need to verify that the rest of the proof works beyond this.]