1. Consider Laplace’s equation with potential function $c$:

$$-\Delta u + cu = 0,$$

and the divergence structure equation:

$$-\text{div}(aDv) = 0,$$

where the function $a$ is positive.

(a) Show that if $u$ solves (1) and $w > 0$ also solves (1), then $v := u/w$ solves (2) for $a := w^2$.

(b) Conversely, show that if $v$ solves (2), then $u := va^{1/2}$ solves (1) for some potential $c$.

2. Evans: Section 6.6, Problem 2 (Problem 1 in 1st edition)

3. Evans: Section 6.6, Problem 3 (Problem 2 in 1st edition)

4. Evans: Section 6.6, Problem 4 (Problem 3 in 1st edition)

5. Explain how to define $u \in H^1(U)$ to be a weak solution of Poisson’s equation with *Robin boundary conditions*:

$$\begin{cases}
-\Delta u = f & \text{in } U \\
u + \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial U.
\end{cases}$$

Discuss the existence and uniqueness of a weak solution for a given $f \in L^2(U)$.

6. Suppose $U$ is connected and $\partial U$ consists of two disjoint, closed sets $\Gamma_1$ and $\Gamma_2$. Define what it means for $u$ to be a weak solution of Poisson’s equation with *mixed Dirichlet-Neumann boundary conditions*:

$$\begin{cases}
-\Delta u = f & \text{in } U \\
u = 0 & \text{on } \Gamma_1 \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \Gamma_2
\end{cases}$$

Discuss the existence and uniqueness of weak solutions.