Math 5587 Midterm II

Prof. Jeff Calder
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Name:_____

Instructions:

1. I recommend looking over the problems first and starting with those you feel most comfortable with.

2. Unless otherwise noted, be sure to include explanations to justify each step in your arguments and computations. For example, make sure to check that the hypotheses of any theorems you use are satisfied.

3. All work should be done in the space provided in this exam booklet. Cross out any work you do not wish to be considered. Additional white paper is available if needed.

4. Books, notes, calculators, cell phones, pagers, or other similar devices are not allowed during the exam. Please turn off cell phones for the duration of the exam. You may use the formula sheet attached to this exam.

5. If you complete the exam within the last 15 minutes, please remain in your seat until the examination period is over.

6. In the event that it is necessary to leave the room during the exam (e.g., fire alarm), this exam and all your work must remain in the room, face down on your desk.

Problem	Score
1	/10
2	/10
3	/10
4	/10
5	/10
Total:	/50

1. Answer each question below. No justification is required for (b) and (c).

[10 points]

(a) [4 points] Explain briefly why separation of variables and Fourier series techniques are unlikely to provide a solution for

$$u_t - ku_{xx} + u^2 = 0.$$

(b) [3 points] Give an example of a function $f(x)$ whose Fourier series does not converge uniformly.

(c) [3 points] Give an example of a sequence of functions f_n that converges pointwise to some function f, but does not converge uniformly. Make sure to identify the pointwise limit f of your sequence, and the domain of your functions.

2. [10 points] Solve the heat equation $u_t - u_{xx} = 0$ on the rectangle $0 < x < \pi$ and $t > 0$ with initial condition $u(x, 0) = 1 - x$ and Dirichlet boundary conditions $u(0, t) = u(\pi, t) = 0$ for $t > 0$.

3. [10 points] Solve the wave equation $u_{tt} = u_{xx}$ on the rectangle $-\pi < x < \pi$, $t > 0$ with periodic boundary conditions $u(-\pi, t) = u(\pi, t)$ and $u_x(-\pi, t) = u_x(\pi, t)$, initial position $u(x, 0) = 1$ and initial velocity $u_t(x, 0) = 1$.

4. [10 points] Use separation of variables to find a family of solutions of $u_{tx} - u_{xx} = 0$ on the entire plane $\mathbb{R}^2$ that satisfy the boundary condition $u(0, t) = 0$ for all t.

5. [10 points] Let f be a 2π-periodic infinitely differentiable function satisfying

$$f(x) + f'(x) + f''(x) + \cdots + f^{(m)}(x) = 0$$

for all $x \in \mathbb{R}$, where m is a positive integer and $f^{(m)}$ denotes the mth derivative of f. Show that

$$f(x) = \begin{cases} A\cos(x) + B\sin(x), & \text{if } m \in \{3, 7, 11, 15, 19, \dots\}, \\ 0, & \text{otherwise.} \end{cases}$$

where A and B are arbitrary constants. The condition on m can also be stated as $m = 4k - 1$ for a positive integer k. [Hint: Equate the Fourier series coefficients on both sides.]

Scratch paper

Formula Sheet

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(nx) + B_n \sin(nx) = \sum_{n=-\infty}^{\infty} c_n e^{inx},$$

$$\int_{-\pi}^{\pi} \cos(nx) \cos(mx)\, dx = \int_{-\pi}^{\pi} \sin(nx) \sin(mx)\, dx = \begin{cases} \pi, & \text{if } n = m \\ 0, & \text{otherwise.} \end{cases}$$

$$\int_{-\pi}^{\pi} \cos(nx) \sin(mx)\, dx = 0 \quad \text{and} \quad \int_{-\pi}^{\pi} e^{inx} e^{-imx}\, dx = \begin{cases} 2\pi, & \text{if } n = m \\ 0, & \text{otherwise.} \end{cases}$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} f(x)^2\, dx = \sum_{n=-\infty}^{\infty} |c_n|^2.$$

$$\frac{1}{\pi} \int_{-\pi}^{\pi} f(x)^2\, dx = \frac{1}{2} A_0^2 + \sum_{n=1}^{\infty} A_n^2 + B_n^2.$$