Math 5588 – Homework 10 (Due Thursday April 13)

1. Compute the six elemental stiffness values $k_{ij}^s = k_{ji}^s$ for an equilateral triangle.

2. Suppose your domain is triangulated by equilateral triangles. Calculate the nonzero values of $k_{ij} = \sum_{s=1}^{M} k_{ij}^s$ in the case that vertex i is an interior vertex (that is, not on the boundary).

3. Solve Burger's equation $u_t + uu_x = 0$ with initial condition $u(x,0) = x$.

4. Solve $u_t + u^2 u_x = 0$ with $u(x,0) = 2 + x$. Sketch the characteristics.

5. Solve Burger's equation $u_t + uu_x = 0$ with initial condition $u(x,0) = f(x)$ where

$$f(x) = \begin{cases} 2, & \text{if } x < -1 \\ 1, & \text{if } -1 < x < 0 \\ 0, & \text{if } x > 0. \end{cases}$$

[Hint: Proceed as we did in class by sketching the characteristics and then using the Rankine-Hugoniot Condition to determine the speed of the shock curves.]