Math 8385: Calculus of Variations and Minimal Surfaces (Fall 2019)

Instructor: Jeff Calder
MWF: 9:05am–9:55am, Akerman Hall 317

The calculus of variations is a field of mathematics concerned with minimizing (or maximizing) functionals (that is, real-valued functions whose inputs are functions). The calculus of variations has a wide range of applications in physics, engineering, applied and pure mathematics, and is intimately connected to partial differential equations (PDEs).

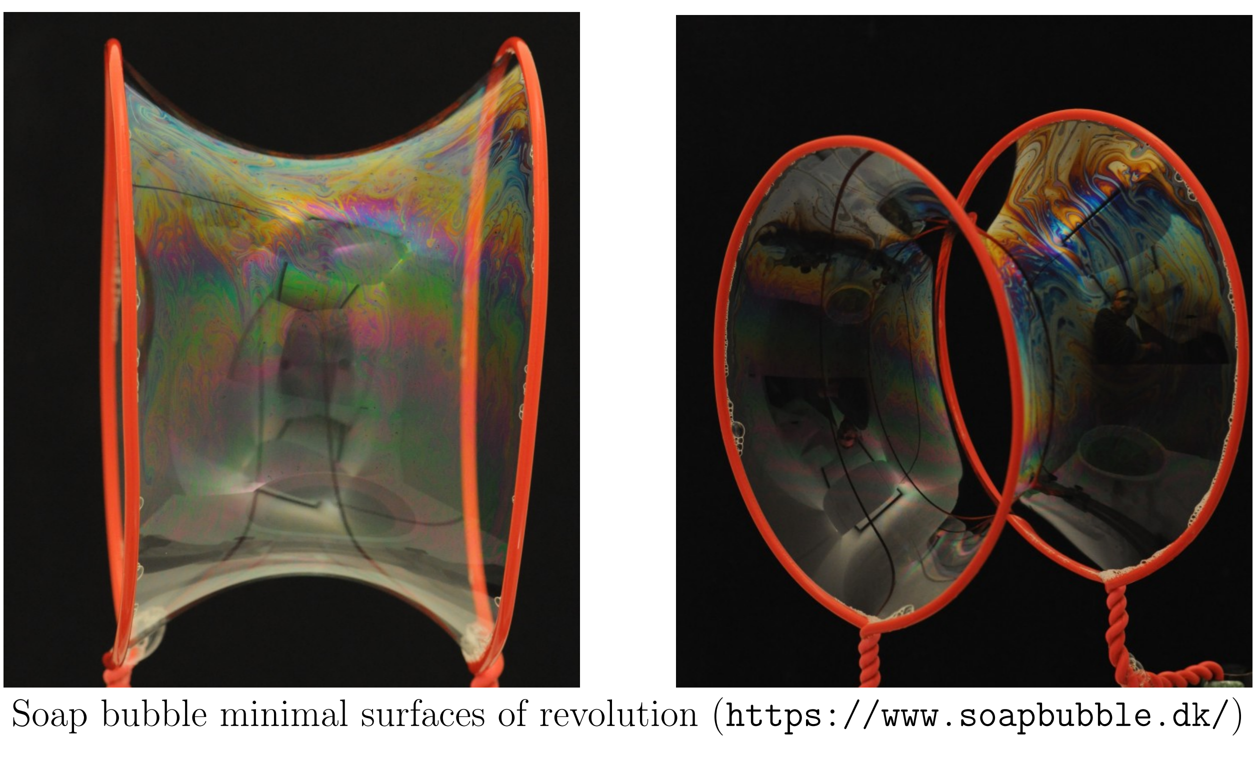
For example, a classical problem in the calculus of variations is finding the shortest path between two points. The notion of length need not be Euclidean, or the path may be constrained to lie on a surface, in which case the shortest path is called a geodesic. In physics, Hamilton's principle states that trajectories of a physical system are critical points of the action functional. Critical points may be minimums, maximums, or saddle points of the action functional. In computer vision and image processing, problems of image segmentation and denoising have been very successfully formulated as problems in the calculus of variations, and many algorithms in machine learning can be viewed as discrete calculus of variations problems on graphs.

This course will cover basic theory and applications of calculus of variations. We work from the lecture notes at the link below, which will be updated throughout the course.

`http://www-users.math.umn.edu/~jwcalder/CalculusOfVariations.pdf`

The first part of the course will cover classical one dimensional calculus of variations problems, including minimal surfaces of revolution, the isoperimetric inequality, and the brachistochrone problem, which were some of the early motivating problems in the field. We will then study multivariable problems, and rigorously study the direct method in the calculus of variations for proving existence of minimizers. Along the way we will discuss applications to image processing, computer vision, and machine learning. At the end of the semester, we'll cover some recent works on using Γ-convergence to prove discrete to continuum convergence results in graph-based machine learning.

Grades will be determined based on three homework assignments. The prerequisites are a graduate course in real analysis and at least one semester of a graduate course in PDEs. Please contact the instructor (`jwcalder@umn.edu`) if you are interested in the course and do not have the prerequisites.



Soap bubble minimal surfaces of revolution (`https://www.soapbubble.dk/`)