Math 8590 – Homework 5 (Due Friday Nov 16)

Please hand in your solution to 1 problem from those below.

1. Prove the $O(\sqrt{h})$ convergence rate for monotone finite difference scheme using inf- and sup-convolutions, instead of the doubling variables argument. Your proof should be similar to the alternative proof of the $O(\sqrt{\varepsilon})$ rate in vanishing viscosity (from Section 8 in class notes).

2. Write code in your favorite programming language to solve the shape from shading problem in $n = 1$ dimension. You can use fast marching or fast sweeping to solve the eikonal equation, and decide on an appropriate boundary condition.

3. Write code in your favorite programming language to simulate an $n = 1$ dimensional homogenization problem of the form

$$u_\varepsilon + |u'_\varepsilon(x)| = f\left(\frac{x}{\varepsilon}\right),$$

on the domain $(0, 1)$ with homogeneous Dirichlet conditions, where f is 1-periodic. Solve the rapidly oscillating equation for u_ε. Try to find nonconstant f for which you can solve the cell problems and compute explicitly $\overline{H}(p)$. Then solve the effective equation for u, comparing results for small ε.