CS 2011: Practice Quiz 1 SOLUTION
Summer 2018
University of Minnesota

Quiz period: 15 minutes
Points available: 40

Show that the following two statements are logically equivalent.

- $(p \to q) \lor (p \to r)$
- $p \to (q \lor r)$

Problem 1 (10 pts): Show the equivalence using a Truth Table.
SOLUTION:

p	q	r	$(p \to q) \lor (p \to r)$	$p \to (q \lor r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

Problem 2 (10 pts): Show the equivalence using a derivation. For full credit mention the laws/rules taken at each step to reach the next.

SOLUTION:

$$\begin{aligned}
(p \to q) \lor (p \to r) &\equiv (\neg p \lor q) \lor (\neg p \lor r) && \textit{Equivalence of } A \to B \textit{ and } \neg A \lor B \\
&\equiv (\neg p \lor \neg p) \lor (q \lor r) && \textit{Associative and Commutative Laws} \\
&\equiv \neg p \lor (q \lor r) && \textit{Idempotent Law} \\
&\equiv p \to (q \lor r) && \textit{Equivalence of } A \to B \textit{ and } \neg A \lor B
\end{aligned}$$

Problem 3 (10 pts): *Determine whether the following two statements are logically equivalent OR provide a counterexample demonstrating that they are not logically equivalent.*

- $\forall x (P(x) \to Q(x))$
- $(\forall x P(x)) \to (\forall x Q(x))$

SOLUTION: NOT EQUIVALENT. Here is one counter example but there are many others.
Define: $P(x) : x$ is Even; $Q(x) : x$ is Odd and quantify over the integers.
The first statement, $\forall x (P(x) \to Q(x))$, is false as Eveness never implies Oddness for integers.
The second statement, $(\forall x P(x)) \to (\forall x Q(x))$, is true. To see this, note that the hypothesis is false: NOT all integers are Even. This makes the entire statement true as the truth table for implication indicate that it is true when the hypothesis is false.
Since there are definitions for $P(x), Q(x)$ and a domain of quantification for which the two statements have different truth values, they are NOT EQUIVALENT.

Problem 4 (10 pts): *Determine if the following reasoning is correct or identify a flaw in it.*

- *Fact: Students in 2011 earn bonus credit if they participate in class.*
- *Fact: Amy has some bonus credit in 2011.*
- *Conclusion: Amy participated in class.*

Hint: reviewing the syllabus may give some insight on this one.

SOLUTION: This is not valid reasoning and is an example of the fallacy of "Affirming the Conclusion". Creating a symbolic version makes this obvious:

- p *: Amy participates in class*
- q *: Amy gets bonus credit*

Then the reasoning is

- *Fact:* $p \to q$
- *Fact:* q
- *Conclusion:* p

There is no valid rule of inference which allows this jump making the reasoning faulty.