CS 2011: Practice Quiz 2 SOLUTION

Summer 2018

University of Minnesota

Quiz period: 15 minutes

Points available: 20

Problem 1 (10 pts): Show that for real numbers x, y $x \leq \frac{x+y}{2}$ OR $y \leq \frac{x+y}{2}$. For full credit, include one of the following in your proof.

- Clearly defined cases which are proved independently
- OR Proper use of the notion of "Without loss of generality".

SOLUTION: Without loss of generality, assume that $x \leq y$. This will lead to the first inequality, $x \leq \frac{x+y}{2}$ holding while the reasoning would be identical with the opposite assumption.
Re-arranging the original inequality gives

$$x \leq \frac{x+y}{2} \tag{1}$$

$$2x \leq x+y \tag{2}$$

$$x \leq y \tag{3}$$

This last inequality is given as a fact in the assumption which we know to be true proving the case. Alternatively, one might transform the fact $x < y$ into the given equality.

Problem 2 (10 pts): Show that following equivalence involving sets A, B holds.

$$A - B = A \cap \overline{B}$$

You may use any method to do show the equivalence so long as you reasoning is explained.

SOLUTION: Below is a series of transformations in set builder notation the equivalence.

$$\begin{aligned}
A - B &= \{x | x \in A \land x \notin B\} && \text{Def. of set difference} && (4)\\
&= \{x | x \in A \land x \in \overline{B}\} && \text{Def. of set complement} && (5)\\
&= \{x | x \in (A \cap \overline{B})\} && \text{Def. of set intersection} && (6)\\
&= A \cap \overline{B} && \text{Simplification} && (7)
\end{aligned}$$