

CS 2011: Practice Quiz 5 SOLUTION

Summer 2018

University of Minnesota

Quiz period: 15 minutes

Points available: 20

Problem 1 (10 pts): The letters **ABCDEF** are randomly arranged without replacement. Determine what the **probability** that the fixed string **BAD** appears in the random arrangement. You may give your answer in a symbolic form involving permutations / combinations and compute a decimal value for the probability.

SOLUTION: The total number of random orderings w/o replacement is permutations of the 6 characters so $P(6,6) = 6!$.

*To compute the number of possible strings containing **BAD** in that order, treat it as a single character α giving 4 total characters to arrange: **CEFA**. The number of ways to arrange this are $P(4,4) = 4!$.*

This gives a probability of $\frac{4!}{6!} = \frac{1}{5 \cdot 6} = \frac{1}{30} = 0.0333\bar{3}$

*NOTE that the above solution technique only works when all 6 characters are arranged. If 5 of the 6 characters were being arranged, it is possible that not all letters in **BAD** would be selected which would require a different solution.*

Problem 2 (10 pts): The operation counts for an algorithm are described by the following recurrence relation.

$$f(n) = 3f(n/4) + 6$$

Give a big-O estimate for the overall runtime of the algorithm. Show your reasoning and any mathematical tools you employ.

SOLUTION: The Master Theorem applies to this recurrence relation associated constants

$$a = 3, b = 4, d = 1$$

. This means Case 3 of the Master theorem applies meaning f is $O(n^{\log_b a}) = O(n^{\log_4 3})$.