CS 310: Recursion and Tree Traversals

Chris Kauffman

Week 10-1

Announcement

Mason Women in Computer Science: A Networking Event

- The event will take place in Sub 1 – 3B on Monday, November 14th 10am-12pm.
- We are organizing an event to encourage networking and community involvement among women undergraduate and graduate students and faculty.
- The event is indeed open to all students and faculty, though we do want to encourage underrepresented students in CS foremost.

– Amarda Shehu & Foteini Baldimtsi

Logistics

HW 2 Due Last Night

Any last words?

HW 3

- Likely up tonight/tomorrow
- Basic Spreadsheet
- Involves walking a binary tree of expressions
- Later, implement a *Directed Acyclic Graph*, ensure it has no cycles
- Maps and Sets show up *a lot*

Reading

- Weiss Ch. 7 Recursion
- Weiss Ch 18 General Trees
- Weiss Ch 19 BSTs

Today

- Finish Maps/Sets
- Tree Traversals
- Recursive traversals
- Recursion practice for tree properties

Ordering

List property

There is a well defined ordering of first, next, last objects in the data structure,

- Wide ranging uses
- Supported in List data structure (`LinkedList`, `ArrayList`)
- Supported structurally in Lists
- *A property of the Data Structure*

Sorting property

There is a well defined ordering relation over all possible data of a type

- "bigger than" "less than" "equal to" are well defined
- A property of the *Data*
- A data structure can try to mirror the data ordering structurally
- Useful for searching, walking through stored data in order

Sorted Lists

Definition is straight-forward

- "Smallest" things are structurally "first", "Biggest" last
- Ordering on elements (`Comparable`/`Comparator`)
- `add`/`insert` put elements in proper place

Question: For a sorted `List L`, what is the complexity of `L.insert(x)` which preserves sorting?

`L` is an `ArrayList`

How long to

- find insertion location?
- complete insertion?
- traverse elements in order (e.g. for printing)?

`L` is a `LinkedList`

How long to

- find insertion location?
- complete insertion?
- traverse elements in order (e.g. for printing)?

Alternatives to the Linear Data Structures

Hash Tables

- Abandon list property
- Abandon sorting property
- $O(1)$ insertion/retrieval
- $O(N)$ traversal, not ordered

Trees

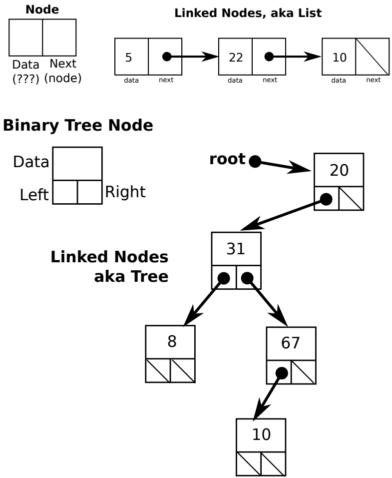
- Abandon list property
- Preserve sorting property
- $O(\log N)$ insertion/retrieval
- $O(N)$ traversal, ordered
- Commonly Binary Trees
- Other variants

Roots

Source

- Next few sessions we'll talk about roots
- For simplicity, we'll call them trees

Mutated Nodes



Node structures should be familiar for linked lists

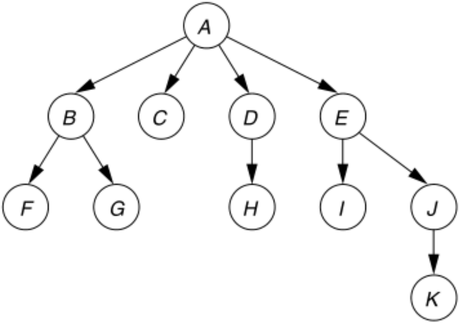
- Singly linked: `next/data`
- Doubly linked: `next/previous/data`

Trees use `Nodes` as well

- `children`, `data`, possibly `parent`
- Arbitrary Trees: `List<Node>` of `children`
- Binary Trees: `left` and `right` children

Tree Properties of Interest

- Root of tree
- Leaves
- Data at nodes
- Size (number of nodes)
- Height of tree
- Depth of a node



Node	Height	Depth
A	3	0
B	1	1
C	0	1
D	1	1
E	2	1
F	0	2
G	0	2
H	0	2
I	0	2
J	1	2
K	0	3

An Apropos Quote

You spend years writing code without recursion and then one day you have to write functions that operate on trees and realize recursion is amazing.
–Kevin DeRonne

Recursion Warm-Up

Write two versions of Singly Linked List `length()` function.

`int length(Node n)`

1. Iterative
2. Recursive

Compare and contrast runtime and memory complexity

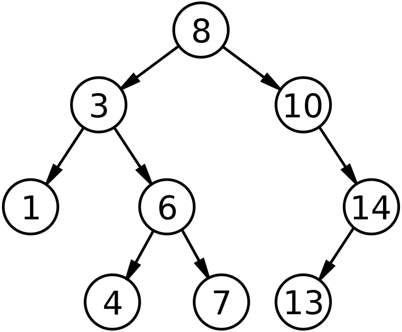
```
class Node<T> {
  T data; Node<T> next;
  public Node(T d, Node<T> n){
    this.data=d; this.next=n;
  }
}

// Singly linked
// No header/auxiliary/dummy nodes
class SimpleList<T>{
  Node<T> head;  // null When empty
  public int length(){
    return length(this.head);
  }
  public static <T>
  int length(Node<T> n){
    // Iterative version?
    // Recursive version?
  }
}
```

Binary Tree

Binary Tree Nodes

```
class Node<T>{
  T data;
  Node<T> left, right;
}
void main(){
  Node root = new Node();
  root.data = 8;
  root.left = new Node();
  root.right= new Node();
  root.left.data = 3;
  root.right.data= 10;
  root.left.left = new Node();
  ...
```

Structure



Recursive Example: Binary Tree Size Method

Tree Nodes

```
class Node<T>{
  T data;
  Node<T> left, right;
}
```

Usage

```
Tree<Integer> myTree = new Tree();
// add some stuff to myTree
int s = myTree.size();
```

Exercise

- Define a recursive `t.height()`
- `t.height()` is the longest path from root to leaf
- Empty tree has height=0

`int size(Node<T> t)`

Number of nodes in tree t

```
public Tree<T>{
  Node<T> root;

  // Entry point
  public int size(){
    return size(this.root);
  }
  // Recursive helper
  public static <T>
    int size( Node<T> t ){
    if(t == null){
      return 0;
    }
    int sL = size(t.left);
    int sR = size(t.right);
    return 1 + sL + sR;
  }
}
```

Recursive Implementation of height()

Slight difference of definitions from textbook

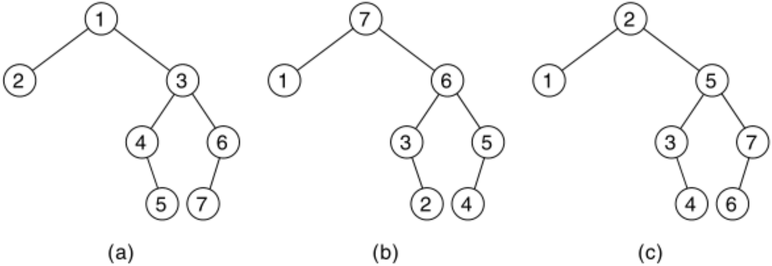
- Empty tree has size=0 and height=0
- 1-node tree has size=1 and height=1

```
// Depth of deepest node
public Tree<T>{
  Node<T> root;
  public int height(){
    return height(this.root);
  }

  public static <T>
    int height( Node<T> t ){
    if(t == null){
      return 0;
    }
    int hL = height(t.left);
    int hR = height(t.right);
    int bigger = Math.max(hL,hR);
    return 1+bigger;
  }
}
```

The Many Ways to Walk

No list property: several orders to traverse tree

- (a) Pre-order traversal (parent, left, right)
- (b) Post-order traversal (left, right, parent)
- (c) In-order traversal (left, parent, right)



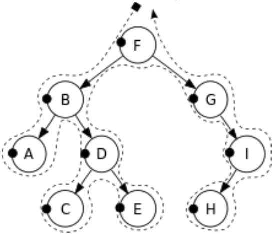
Picture shows the order nodes will be visited in each type of traversal

The Many Ways to Walk

No list property: several orders to traverse tree

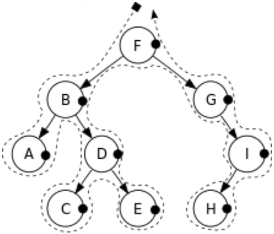
Pre-order traversal

parent, left, right



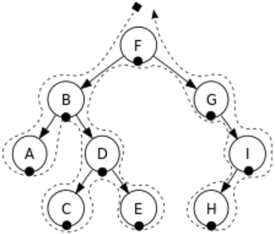
Post-order traversal

left, right, parent

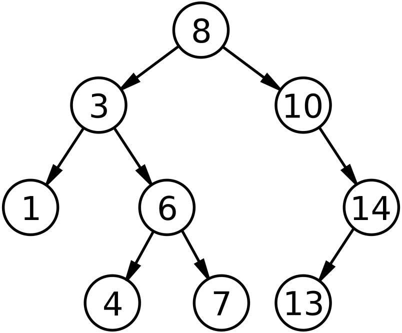


In-order traversal

left, parent, right



Walk This Tree



Show

- (a) Pre-order traversal (parent, left, right)
- (b) Post-order traversal (left, right, parent)
- (c) In-order traversal (left, parent, right)

Which one "sorts" the numbers?

Implementing Traversals for Binary Trees

```
class Tree<T>{
  private Node<T> root;

  public void printPreOrder(){
    preOrder(this.root);
  }
  private static void
  preOrder(Node<T> t){
    ... print(t.data) ...
  }

  public void printInOrder(){ }
  private static void
  inOrder(Node<T> t){ }

  public void printPostOrder(){ }
  private static void
  postOrder(Node<T> t){ }
}
```

```
class Node<T> {
  T data;
  Node<T> left, right;
}
```

Implement Print Traversals

- `preOrder(this.root)`
- `postOrder(this.root)`
- `inOrder(this.root)`

2 Ways

- Recursively (first)
- Iteratively (good luck...)

Recursive Implementation of Traversals

```
inOrder(Node t){
  if(t != null){
    inOrder(t.left);
    print(t.data);
    inOrder(t.right);
  }
}
```

```
preOrder(Node t){
  if(t != null){
    print(t.data);
    preOrder(t.left);
    preOrder(t.right);
  }
}
```

```
postOrder(Node t){
  if(t != null){
    postOrder(t.left);
    postOrder(t.right);
    print(t.data);
  }
}
```

Evaluate

- Correct?
- Time complexity?
- Space complexity?
- What makes this so easy?

Distribution Code

Today's code distribution contains demos of recursive methods

`SimpleList.java`

Demos recursive version of list length

`Tree.java`

Contains a very simple tree example that demos

- `size()`
- `height()`
- Traversals: Pre-order, In-order, Post-order

JGrasp helpful

- Visualize list/tree
- Step through recursive methods
- Use debugger to watch call stack and position in tree

Iterative Implementation?

Compare to Iterative Implementation of Traversals

```
// Pseudo-code for post order print
void postOrder(root){
  Stack s = new Stack();
  s.push( {root, DOLEFT });
  while(!s.empty()){
    {tree, action} = s.popTop();
    if(tree == null){
      // do nothing;
    }
    else if(action == DOLEFT){
      s.push({tree, DORIGHT});
      s.push({tree.left, DOLEFT});
    }
    else if(action == DORIGHT){
      s.push({tree, DOTHIS});
      s.push({tree.right, DOLEFT});
    }
    else if(action == DOTHIS){
      print(tree.data);
    }
    else{
      throw new YouScrewedUpException();
    }
  }
}
```

- No call stack
- Use an explicit stack
- Auxilliary data action

DOLEFT	work on left subtree
DORIGHT	work on right subtree
DOTHIS	process data for current

Evaluate

- Correct?
- Time complexity?
- Space complexity?

Weiss's Traversals

Implemented as iterators

- See TestTreeIterators.java
- Uses BinaryTree.java and BinaryNode.java
- Must preserve state accross advance() calls

```
BinaryTree<Integer> t = new BinaryTree<Integer>( );
... // fill tree

TreeIterator<AnyType> itr = new PreOrder<Integer>( t );
for( itr.first( ); itr.isValid( ); itr.advance( ) ){
  System.out.print( " " + itr.retrieve( ) );
}
```

- Much more complex to understand but good for you
- Play with some of these in a debugger if you want more practice

General Notes

Iterative Traversal Implementation Notes

- Can augment tree nodes to have a `parent` pointer

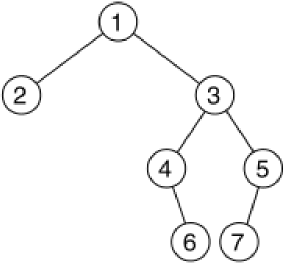
```
class Node<T>{
  T data; Node left, right, parent;
}
```

- Enables stackless, iterative traversals with great cleverness

Iterative vs Recursive Tree Methods

- Multiple types of traversals of `T`
- Other Tree methods: `T.find(x)`, `T.add(x)`, `T.remove(x)`
- Recursive implementations are simpler to code but will cost more memory
- Iterative methods are possible and save memory at the expense of tricky code

Level-order Traversal



Level Order Traversal: 1 2 3 4 5 6 7

- Top level first (depth 1: 1)
- Then next level (depth 2: 2 3)
- etc.

This is a bit trickier

- Need an auxilliary data structure: Queue
- Does recursion help?