NEARBY CYCLES OF AUTOMORPHIC ÉTALE SHEAVES, II — ERRATA

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(1) In Sec. 3, our definition $G^c$ following [Mil90, Ch. III] is incorrect in general, when $Z(G)^o$ is not necessarily split over a CM field as in [Mil90 (II.2.1.4)]. The correct definition should be that $G^c$ is the quotient of $G_Q$ by the minimal subtorus $Z_s(G)$ of the center $Z(G)$ such that the torus $Z(G)^o/Z_s(G)$ has the same split ranks over $Q$ and $R$.

(2) In Sec. 3, the reference [Mil90, Ch. III, Sec. 6, Rem. 6.1], which claims that the Galois finite étale cover $X_{H(\ell^r)} \otimes_Z Q \to X_{H} \otimes_Z Q$ in (3.2) has Galois group exactly $H_{\ell}^c/U_{\ell}(\ell^r)^c$, is incorrect in general. (We thank Yihang Zhu for asking us about this reference and discussing with us about its validity.)

To see this, let $T := \ker(G \to G^c)$, which is the torus $Z_s()$ as above, by the definition of $G^c$. (But we will not need to know the precise definition of $T$.) The claim in [Mil90, Ch. III, Sec. 6, Rem. 6.1] would be valid only if, for nontrivial $T$, the cardinalities of $T(Q)\setminus T(\mathbb{A}^\infty)/H_T$ (which is a finite set by [Bor63, Thm. 5.1]) remain unchanged for all sufficiently small open compact subgroups $H_T$ of $T(\mathbb{A}^\infty)$. This implies that the closure $T(Q)$ of $T(Q)$ has finite index in $T(\mathbb{A}^\infty)$, but contradicts the fact that $T(Q)$ has infinite index in $T(\mathbb{A}^\infty)$ for every nontrivial torus $T$ over $Q$. (See [PR94, Prop. 7.13(2)], which explains that the same failure occurs, more generally, for algebraic groups over number fields that are connected but not simply-connected.)

This does not affect the construction of automorphic étale sheaves for representations of $G^c$, since all we need is that the Galois group is a quotient of $H_{\ell}/U_{\ell}(\ell^r(m))$ by construction, and admits $H_{\ell}^c/U_{\ell}(\ell^r(m))^c$ as a quotient. In particular, in (3.3), the contraction product can be formed using the action of $H_{\ell}/U_{\ell}(\ell^r(m))$ instead, whose pullback to $X_{H_{\ell}(\ell^r(m))} \otimes_Z Q$ is still isomorphic to $V_{0,\ell^m}$ by construction. The remainder of Sec. 3 is unaffected.

REFERENCES


