NEARBY CYCLES OF AUTOMORPHIC ÉTALE SHEAVES, II — ERRATA

KAI-WEN LAN AND BENOÎT STROH

(1) In Sec. 3, our definition G^c following [Mil90, Ch. III] is incorrect in general, when $Z(\mathrm{G})^\circ$ is not necessarily split over a CM field as in [Mil90, (II.2.1.4)]. The correct definition should be that G^c is the quotient of $\mathrm{G}_{\mathbb{Q}}$ by the minimal subtorus $Z_s(\mathrm{G})$ of the center $Z(\mathrm{G})$ such that the torus $Z(\mathrm{G})^\circ/Z_s(\mathrm{G})$ has the same split ranks over $\mathbb{Q}$ and $\mathbb{R}$.

(2) In Sec. 3, the reference [Mil90, Ch. III, Sec. 6, Rem. 6.1], which claims that the Galois finite étale cover $\mathsf{X}_{\mathcal{H}(\ell^r)} \underset{\mathbb{Z}}{\otimes} \mathbb{Q} \to \mathsf{X}_{\mathcal{H}} \underset{\mathbb{Z}}{\otimes} \mathbb{Q}$ in (3.2) has Galois group exactly $\mathcal{H}_\ell^c/\mathcal{U}_\ell(\ell^r)^c$, is incorrect in general. (We thank Yihang Zhu for asking us about this reference and discussing with us about its validity.)

To see this, let $\mathrm{T} := \ker(\mathrm{G} \to \mathrm{G}^c)$, which is the torus $Z_s()$ as above, by the definition of G^c. (But we will not need to know the precise definition of T.) The claim in [Mil90, Ch. III, Sec. 6, Rem. 6.1] would be valid only if, for nontrivial T, the cardinalities of $\mathrm{T}(\mathbb{Q})\backslash\mathrm{T}(\mathbb{A}^\infty)/\mathcal{H}_{\mathrm{T}}$ (which is a finite set by [Bor63, Thm. 5.1]) remain unchanged for all sufficiently small open compact subgroups $\mathcal{H}_{\mathrm{T}}$ of $\mathrm{T}(\mathbb{A}^\infty)$. This implies that the closure $\overline{\mathrm{T}(\mathbb{Q})}$ of $\mathrm{T}(\mathbb{Q})$ has finite index in $\mathrm{T}(\mathbb{A}^\infty)$, but contradicts the fact that $\overline{\mathrm{T}(\mathbb{Q})}$ has infinite index in $\mathrm{T}(\mathbb{A}^\infty)$ for *every* nontrivial torus T over $\mathbb{Q}$. (See [PR94, Prop. 7.13(2)], which explains that the same failure occurs, more generally, for algebraic groups over number fields that are connected but not simply-connected.)

This does not affect the construction of automorphic étale sheaves for representations of G^c, since all we need is that the Galois group is a quotient of $\mathcal{H}_\ell/\mathcal{U}_\ell(\ell^{r(m)})$ by construction, and admits $\mathcal{H}_\ell^c/\mathcal{U}_\ell(\ell^{r(m)})^c$ as a quotient. In particular, in (3.3), the contraction product can be formed using the action of $\mathcal{H}_\ell/\mathcal{U}_\ell(\ell^{r(m)})$ instead, whose pullback to $\mathsf{X}_{\mathcal{H}(\ell^{r(m)})} \underset{\mathbb{Z}}{\otimes} \mathbb{Q}$ is still isomorphic to $\underline{V}_{0,\ell^m}$ by construction. The remainder of Sec. 3 is unaffected.

References

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University of Minnesota, Minneapolis, MN 55455, USA
Email address: kwlan@math.umn.edu

C.N.R.S. and Institut de Mathématiques de Jussieu–Paris Rive Gauche, 75252 Paris Cedex 05, France
Email address: benoit.stroh@imj-prg.fr