ARITHMETIC COMPACTIFICATIONS OF PEL-TYPE SHIMURA VARIETIES — ERRATA

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(1) In Def. 1.2.1.6, in the definition of $\mathrm{G}(R)$, the condition should be "$\forall x, y \in L \underset{\mathbb{Z}}{\otimes} R$", and the parenthetical remark "(If $L \neq \{0\}$ ...)" should be "(If $L \neq \{0\}$ and R is flat over $\mathbb{Z}$, then the value of r is uniquely determined by g. Hence there is little that we lose when suppressing r from the notation. However, this suppression is indeed an abuse of notation in general. For example, when $L = \{0\}$, we have $\mathrm{G} = \mathbf{G}_{\mathrm{m}}$.)"
(2) In (1.2.1.10) and the previous displayed equation: "$[\tau] : F \hookrightarrow \mathbb{Q}_{[\tau]}$" should be "$[\tau] : F \to \mathbb{Q}_{[\tau]}$".
(3) In Def. 1.2.1.21, "integrable $\mathcal{O}$-lattice" should be simply "$\mathcal{O}$-lattice".
(4) In paragraph 1 of the proof of Prop. 1.2.2.3, "$\mathbf{Sym}_{\varrho}(L_1, L_2) \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_p$" should be "$\mathbf{Sym}^{\epsilon}_{\varrho}(L_1, L_2) \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_p$".
(5) In the proof of Cor. 1.3.1.6, "$h(m_A(x_1, x_2)) = m_G(h(x_1), f(x_2))$" should be "$h(m_A(x_1, x_2)) = m_G(h(x_1), h(x_2))$".
(6) In Def. 1.3.2.1, "for some $\mathcal{M}$ over S" should be "for some $\mathcal{M}$ over T".
(7) In Def. 1.3.2.19, N should be a section of $(\mathbb{Z}_{>0})_S$ (rather than a global constant).
(8) In 2. of Def. 1.4.1.2, "$\mathbb{Z}^{\times}_{(\Box)}$-polarization" should be "prime-to-$\Box$ polarization".
(9) In 2. of Def. 1.4.1.4, "$\mathbb{Z}^{\times}_{(\Box)}$-polarization" should be "prime-to-$\Box$ polarization".
(10) In 5. of Def. 1.4.2.1, "rational principle level-$\mathcal{H}$ structure" should be "rational level-$\mathcal{H}$ structure".
(11) In the proof of Lem. 2.1.1.1, "$u := u \underset{\tilde{R}}{\otimes} R$" should be only "$u \underset{\tilde{R}}{\otimes} R$".
(12) In Def. 2.1.2.1, "the isomorphism classes" should be "the set of isomorphism classes".
(13) In paragraph 3 of the proof of Prop. 2.1.2.2, towards the end, "$\mathrm{Aut}_{\tilde{S}}(\tilde{U}_\alpha|_{U_{\alpha\beta},S})$" should be "$\mathrm{Aut}_{\tilde{S}}(\tilde{U}_\alpha|_{U_{\alpha\beta}}, S)$".
(14) In 3. of Prop. 2.1.3.2, "$\mathsf{o}(f; \mathsf{m}_{\tilde{X}} + \tilde{X}, \mathsf{m}_{\tilde{Y}} + \tilde{Y}, S \hookrightarrow \tilde{S}) = \mathsf{o}(f; \tilde{X}, \tilde{Y}, S \hookrightarrow \tilde{S}) - df(\mathsf{m}_{\tilde{X}}) + f^*(\mathsf{m}_{\tilde{Y}})$" should be "$\mathsf{o}(f; \mathsf{m}_{\tilde{X}} + \tilde{X}, \mathsf{m}_{\tilde{Y}} + \tilde{Y}, S \hookrightarrow \tilde{S}) = \mathsf{o}(f; \tilde{X}, \tilde{Y}, S \hookrightarrow \tilde{S}) + df(\mathsf{m}_{\tilde{X}}) - f^*(\mathsf{m}_{\tilde{Y}})$".

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(15) In the proof of Prop. 2.1.3.2:
   (a) In paragraph 1, "By smoothness of f" should be "By smoothness of $\tilde{Y}$".
   (b) In paragraph 4, "$c'_{\alpha\beta} = c_{\alpha\beta} - df(m_{\tilde{X},\alpha\beta}) + f^*(m_{\tilde{Y},\alpha\beta})$" should be "$c'_{\alpha\beta} = c_{\alpha\beta} + df(m_{\tilde{X},\alpha\beta}) - f^*(m_{\tilde{Y},\alpha\beta})$".

(16) In the second displayed equation of Cor. 2.1.4.4, the left-hand side should be "$H^1(X, f^*\underline{\mathrm{Der}}_{X/T} \underset{\mathscr{O}_T}{\otimes} \mathscr{I})$".

(17) In Def. 2.1.5.2, "invertible sheaves $\tilde{\mathcal{L}}$ over $\tilde{X}$ such that $\tilde{\mathcal{L}} \underset{\mathscr{O}_{\tilde{S}}}{\otimes} \mathscr{O}_S \cong \mathcal{L}$ over X" should be "pairs $(\tilde{\mathcal{L}}, \psi)$ such that $\tilde{\mathcal{L}}$ is an invertible sheaf over $\tilde{X}$ and such that $\psi : \tilde{\mathcal{L}} \underset{\mathscr{O}_{\tilde{S}}}{\otimes} \mathscr{O}_S \to \mathcal{L}$ is an isomorphism over X".

(18) In the last paragraph of the proof of Prop. 2.1.5.3, "$\xi^*_{\alpha\beta}(\tilde{l}_{\alpha\beta})$ and $(\xi'_{\alpha\beta})^*(\tilde{l}_{\alpha\beta})$ become the same $l_{\alpha\beta}$ modulo $\mathscr{I}$" should be "$\xi^*_{\alpha\beta}(\tilde{l}_{\beta\gamma})$ and $(\xi'_{\alpha\beta})^*(\tilde{l}_{\beta\gamma})$ become the same $l_{\beta\gamma}$ modulo $\mathscr{I}$".

(19) In Cor. 2.1.5.15, the "$\underline{\mathrm{Lie}}_{X^\vee/S} \underset{\mathscr{O}_S}{\otimes} \underline{\mathrm{Lie}}_{X/S}$" in the commutative diagram should be "$\underline{\mathrm{Lie}}_{X^\vee/S} \underset{\mathscr{O}_S}{\otimes} \underline{\mathrm{Lie}}_{X^\vee/S}$".

(20) At the end of paragraph 2 of Sec. 2.1.6, and in the last paragraph preceding Prop. 2.1.6.1, "$R^i\pi_*\Omega^\bullet_{U_\alpha/S}$ is trivial for all $i > 0$" should be "$R^i\pi_*\Omega^q_{U_\alpha/S}$ is trivial for all $i > 0$ and all q".

(21) In paragraph 3 of Sec. 2.1.6, in the first two displayed equations, the indices should start with "α_0" instead of "α_1".

(22) In paragraph 3 of the proof of Prop. 2.1.6.4, "$x^{(1,0)}_{\alpha\beta} := \tilde{f}^*_\alpha(y^{(1,0)}_{\alpha\beta}) + T_{\alpha\beta}(y^{(1,0)}_\beta)$" should be "$x^{(1,0)}_{\alpha\beta} := \tilde{f}^*_\alpha(y^{(1,0)}_{\alpha\beta}) + T_{\alpha\beta}(y^{(0,1)}_\beta)$".

(23) In the proof of Prop. 2.2.2.5, paragraph 2, in the last sentence, "$\tilde{g} \circ j_1 = \mathrm{Id}_{\tilde{A}}$ and $\tilde{g} \circ j_2 = \tilde{e} \circ \tilde{\pi}$ do lift the morphisms $g \circ j_1$ and $g \circ j_2$" should be "$\mathrm{Id}_{\tilde{A}}$ and $\tilde{e} \circ \tilde{\pi}$ do lift the morphisms $g \circ j_1 = \mathrm{Id}_A$ and $g \circ j_2 = e \circ \pi$".

(24) In the proof of Prop. 2.2.2.5, paragraph 3, in the displayed equation, "$\mathrm{pr}_2^* \underline{H}^1(A_0, \mathscr{O}_{A_0})$" should be "$\mathrm{pr}_2^* H^1(A_0, \mathscr{O}_{A_0})$"; after the displayed equation, "the pullback from one of the two factors $\mathrm{pr}_i^* H^1(A_0 \underset{S_0}{\times} A_0, g_0^*(\underline{\mathrm{Der}}_{A_0/S_0})) \underset{k}{\otimes} I$" should be "a sum of elements of the two factors $[\mathrm{pr}_i^* H^1(A_0, \mathscr{O}_{A_0})] \underset{k}{\otimes} \mathrm{Lie}_{A_0/S_0} \underset{k}{\otimes} I$".

(25) In the paragraph following (2.2.3.6), "$[(A_{\tilde{R}}, f_{0,\tilde{R}})] \in \mathsf{Def}_{A_0}(q)^{-1}([(A_R, f_{0,R})])$" should be "$[(A_{\tilde{R}}, f_{0,\tilde{R}})] \in \mathsf{Def}_{A_0}(r)^{-1}([(A_R, f_{0,R})])$". At the end of the paragraph, "$\mathsf{Def}_{A_0}(r)^{-1}([(A_{\tilde{R}}, f_{0,\tilde{R}})])$" should be "$\mathsf{Def}_{A_0}(r)^{-1}([(A_R, f_{0,R})])$".

(26) In the first displayed equation after (2.2.3.8), "$\mathsf{Def}_{A_0}(p)([(A_{\tilde{R}}, \lambda_{\tilde{R}}, f_{0,\tilde{R}})])$" should be "$\mathsf{Def}_{A_0}(r)([(A_{\tilde{R}}, \lambda_{\tilde{R}}, f_{0,\tilde{R}})])$".

(27) In paragraph 2 of the proof of Prop. 2.2.4.1, "$j_2(x) = (x, x)$" should be "$j_2(x) = (e, x)$".

(28) In the proof of Thm. 3.3.2.4, the reference "[61, X]" should be "[61, IX]".
(29) In paragraph 2 of Section 4.3.3, "adic injections" should be "continuous injections".
(30) In paragraph 1 after Rem. 4.5.5.4, "adic injections" should be "continuous injections".
(31) In part 2 of the description of the data on the tuple $(A, \lambda_A, \underline{X}, \underline{Y}, \phi, c, c^\vee, \tau)$, all instances of "$X$" and "$Y$" should be "$\underline{X}$" and "$\underline{Y}$", respectively, and "ring morphism" should be "ring homomorphism".
(32) In Lem. 5.1.1.1, the terminology of the "underlying groups X and Y of the étale sheaves $\underline{X}$ and $\underline{Y}$" might be confusing, and should better be replaced with the respective values of $\underline{X}$ and $\underline{Y}$ over a finite étale covering of S trivializing them.
(33) In part 2 of Def. 5.1.1.3, should replace the first sentence with the following: "The étale sheaves $\underline{X}$ and $\underline{Y}$ are equipped with ring homomorphisms $i_{\underline{X}} : \mathcal{O} \to \mathrm{End}_S(\underline{X})$ and $i_{\underline{Y}} : \mathcal{O} \to \mathrm{End}_S(\underline{Y})$, respectively, making them étale sheaves of $\mathcal{O}$-lattices of the same $\mathcal{O}$-multirank (see Definition 1.2.1.11)."
(34) In Prop. 5.1.2.2 and 5.1.2.4, and in Def. 5.1.2.6, "X is the underlying $\mathcal{O}$-lattice of $\underline{X}$" should better be "X is the $\mathcal{O}$-lattice given by the value of $\underline{X}$ over some geometric point over η".
(35) In paragraph 4 of the proof of Prop. 5.1.2.4, the parenthetical remark "which is the restriction of the complex conjugation under any homomorphism $\mathcal{O}_F \hookrightarrow F \overset{\tau}{\hookrightarrow} \mathbb{C}$" should be "which is compatible with the complex conjugation under any homomorphism $\mathcal{O}_F \hookrightarrow F \overset{\tau}{\to} \mathbb{C}$".
(36) In paragraphs 5 and 7 of the proof of Prop. 5.1.2.4, should work with the constant values X and Y of $\underline{X}$ and $\underline{Y}$, respectively, over the geometric point $\bar{\eta} = \mathrm{Spec}(K^{\mathrm{sep}})$ over $\eta = \mathrm{Spec}(K)$.
(37) In Prop. 5.2.3.3, should assume that $\underline{X}$ and $\underline{Y}$ are constant with values X and Y, respectively.
(38) In Def. 5.2.3.6, should remove "with underlying $\mathcal{O}$-module N a finitely generated $\mathcal{O}$-module" because it is confusing and never used.
(39) In paragraph 4 of the proof of Prop. 5.2.3.9, in the second last line, "$\underline{\mathrm{Hom}}(N, Z)$" should be "$\underline{\mathrm{Hom}}_{\mathcal{O}}(N, Z)$".
(40) In the paragraph preceding Cor. 5.2.3.11, "$\underline{\mathrm{Hom}}_{\mathcal{O}}(\underline{Y}, A)$" should be "$\underline{\mathrm{Hom}}_{\mathcal{O}}(Y, A)$".
(41) In paragraph 1 of Sec. 5.2.7, should replace the sentence "For simplicity, let us continue to assume that $\underline{X}$ and $\underline{Y}$ are constant with values X and Y, respectively" with "For simplicity, let us continue to assume that $\underline{X}$ and $\underline{Y}$ are constant with values X and Y, respectively".
(42) In Def. 5.4.2.6, should first define $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ to be the quotient of $\coprod \mathsf{M}_n^{\mathsf{Z}_n}$ by H_n, where the disjoint union is over representatives $(\mathsf{Z}_n, \Phi_n, \delta_n)$ (with the same (X, Y, ϕ)) in $(\mathsf{Z}_{\mathcal{H}}, \Phi_{\mathcal{H}}, \delta_{\mathcal{H}})$, and then define $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ to be the (finite étale) quotient of $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ by the subgroup of Γ_ϕ stabilizing $\Phi_{\mathcal{H}}$ (which is called $\Gamma_{\Phi_{\mathcal{H}}}$ later in Def. 6.2.4.1). (See below for the precise places for

$\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ to be used. Also, the previously definition of $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ as a moduli only for the abelian parts was not useful and should be abandoned.)

(43) In Def. 5.4.2.8, should replace the rather discrete object $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in $\alpha_{\mathcal{H}}^{\natural} = (\mathsf{Z}_{\mathcal{H}}, \varphi_{-2,\mathcal{H}}, \varphi_{-1,\mathcal{H}}, \varphi_{0,\mathcal{H}}, \delta_{\mathcal{H}}, c_{\mathcal{H}}, c_{\mathcal{H}}^{\vee}, \tau_{\mathcal{H}})$ with a subscheme $(\varphi_{-2,\mathcal{H}}^{\sim}, \varphi_{0,\mathcal{H}}^{\sim})$ of $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}}) \underset{\mathsf{Z}_{\mathcal{H}}}{\times} \varphi_{-1,\mathcal{H}}$, where $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ is (indeed a discrete object) as in Def. 5.4.2.1 above, and where $(\varphi_{-2,\mathcal{H}}^{\sim}, \varphi_{0,\mathcal{H}}^{\sim})$ is an étale-locally-defined H_n-orbits which surjects under the two projections to the orbits defining $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ and $\varphi_{-1,\mathcal{H}}$. In this case we say that $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ is induced by $(\varphi_{-2,\mathcal{H}}^{\sim}, \varphi_{0,\mathcal{H}}^{\sim})$. (Then, by the universal property of $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ because of its very construction, the torus part $(\mathsf{Z}_{\mathcal{H}}, \Phi_{\mathcal{H}}^{\sim} = (X, Y, \phi, \varphi_{-2,\mathcal{H}}^{\sim}, \varphi_{0,\mathcal{H}}^{\sim}), \delta_{\mathcal{H}})$ and abelian part $(A, \lambda_A, i_A, \varphi_{-1,\mathcal{H}})$ of $(A, \lambda_A, i_A, \underline{X}, \underline{Y}, \phi, c, c^{\vee}, \tau, [\alpha_{\mathcal{H}}^{\natural}])$ canonically define a morphism $S = \operatorname{Spec}(R) \to \mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$.)

(44) In Lem. 5.4.2.10, the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in the second displayed object should be denoted $(\varphi_{-2,\mathcal{H}}^{\sim}, \varphi_{0,\mathcal{H}}^{\sim})$, and it should be added in the sentence that $(\varphi_{-2,\mathcal{H}}^{\sim}, \varphi_{0,\mathcal{H}}^{\sim})$ induces the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in the given $\Phi_{\mathcal{H}}$. (See (43) above.) It should be clarified that the assertion of uniqueness up to isomorphism allows isomorphisms inducing automorphisms of $(X, Y, \phi, \varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$.

(45) In Prop. 5.4.3.8 and Def. 5.4.3.9, "$\mathcal{H}' \subset \mathcal{H}$" should be "$\mathcal{H}$ and $\mathcal{H}'$".

(46) In 2. of Def. 6.1.1.10, should require moreover that each σ_k appearing in the closure of σ_j in C is a face of σ_j.

(47) In the second last paragraph of Section 6.2.1, "formally étale" should be "étale" (i.e., formally étale and of finite type).

(48) In the paragraph preceding Def. 6.2.5.23, "formally étale" should be "étale" (i.e., formally étale and of finite type).

(49) In the second paragraph of Section 6.2.4, the wording should be changed to reflect the changes made in Def. 5.4.2.8.

(50) In Section 6.2.4, and the construction for general levels is not correctly deduced from the construction for principle levels.

In the displayed equation preceding (6.2.4.3), the definition $H_{n,\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}} \ltimes \mathrm{U}_{\mathsf{Z}_n}^{\mathrm{ess}}} := H_{n,\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}}} \ltimes H_{n,\mathrm{U}_{\mathsf{Z}_n}^{\mathrm{ess}}}$ is wrong. It should be following Def. 5.3.1.11 faithfully by viewing the the semidirect product $\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}} \ltimes \mathrm{U}_{\mathsf{Z}_n}^{\mathrm{ess}}$ as a subgroup of $\mathrm{G}^{\mathrm{ess}}(\mathbb{Z}/n\mathbb{Z})$. (And later $\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}} \ltimes \mathrm{U}_{1,\mathsf{Z}_n}^{\mathrm{ess}} = (\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}} \ltimes \mathrm{U}_{\mathsf{Z}_n}^{\mathrm{ess}})/\mathrm{U}_{2,\mathsf{Z}_n}^{\mathrm{ess}}$ should be viewed as a subquotient.) In Lem. 6.2.4.6, should consider $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ and $H'_{n,\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}}}$ instead of $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ and $H_{n,\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}}}$, respectively. In the paragraph following Lem. 6.2.4.6, the $\mathrm{Z}_{\mathsf{Z}_n}^{\mathrm{ess}}$ and $\mathrm{Z}_{\mathsf{Z}_n}^{\mathrm{ess}}/\mathrm{U}_{2,\mathsf{Z}_n}^{\mathrm{ess}}$ should be $\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}} \ltimes \mathrm{U}_{\mathsf{Z}_n}^{\mathrm{ess}}$ and $\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}} \ltimes \mathrm{U}_{1,\mathsf{Z}_n}^{\mathrm{ess}}$, respectively. As a result, the image $H'_{n,\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}}}$ of $H_{n,\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}} \ltimes \mathrm{U}_{\mathsf{Z}_n}^{\mathrm{ess}}}$ in $\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}}$ might be smaller than $H_{n,\mathrm{G}_{h,\mathsf{Z}_n}^{\mathrm{ess}}}$ in general. Hence, in the bottom-right vertical arrow in (6.2.4.3), the quotient $\mathsf{M}_n^{\mathsf{Z}_n}/H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{\mathsf{Z}_n}}$ should be replaced with $\mathsf{M}_n^{\mathsf{Z}_n}/H'_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}$.

In Prop. 6.2.4.7 and the remainder of Ch. 6, the morphism $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}\to\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ should be replaced with $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}\to\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$. (See (42) above.) The latter is an abelian scheme torsor, not exactly an abelian scheme. We should define $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}\to C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}\to\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ as the equivariant quotient of $\coprod\Xi_{\Phi_n,\delta_n}\to\coprod C_{\Phi_n,\delta_n}\to\coprod\mathsf{M}_n^{\Phi_n}$ by H_n, where the disjoint unions are over representatives $(\mathsf{Z}_n,\Phi_n,\delta_n)$ (with the same (X,Y,ϕ)) in $(\mathsf{Z}_{\mathcal{H}},\Phi_{\mathcal{H}},\delta_{\mathcal{H}})$, which carries compatible actions of $\Gamma_{\Phi_{\mathcal{H}}}$. (By construction, $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}=\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ when, for some (and hence every) choice of a representative $(\mathsf{Z}_n,\Phi_n,\delta_n)$ in $(\mathsf{Z}_{\mathcal{H}},\Phi_{\mathcal{H}},\delta_{\mathcal{H}})$, the image of $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{\mathsf{Z}_n}}$ in $\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}$ is $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}$; i.e., when the image of $H_{n,\mathrm{P}^{\mathrm{ess}}_{\mathsf{Z}_n}}$ in $\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\times\mathrm{G}^{\mathrm{ess}}_{l,\mathsf{Z}_n}$ is the direct product $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}\times H_{n,\mathrm{G}^{\mathrm{ess}}_{l,\mathsf{Z}_n}}$; the abelian scheme torsor $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}\to\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ is an abelian scheme when, for some (and hence every) choice of a representative $(\mathsf{Z}_n,\Phi_n,\delta_n)$ in $(\mathsf{Z}_{\mathcal{H}},\Phi_{\mathcal{H}},\delta_{\mathcal{H}})$, the splitting of the canonical homomorphism $\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{1,\mathsf{Z}_n}\twoheadrightarrow\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}$ defined by δ_n induces a splitting of the canonical homomorphism $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{1,\mathsf{Z}_n}}\twoheadrightarrow H'_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}$, and hence an isomorphism $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{1,\mathsf{Z}_n}}\cong H'_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}\ltimes H_{n,\mathrm{U}^{\mathrm{ess}}_{1,\mathsf{Z}_n}}$.) It should be noted that, by definition, $\Gamma_{\Phi_{\mathcal{H}}}$ acts compatibly on $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ and $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$, but trivially on $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$; and the canonical morphism $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}\to\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ induces a canonical isomorphism $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}/\Gamma_{\Phi_{\mathcal{H}}}\xrightarrow{\sim}\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$. In (6.2.4.8), "$\underline{\mathrm{Pic}}_e(C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}})$" should be "$\underline{\mathrm{Pic}}(C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})$". In the proof of Prop. 6.2.5.18, when computing the sheaves of differentials by applying Prop. 2.3.5.2, it is harmless to replace $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ with $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ because $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ is finite étale over $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$.

(51) In the displayed object after (6.2.4.2), the $(\varphi_{-2,\mathcal{H}},\varphi_{0,\mathcal{H}})$ should be denoted $(\varphi^{\sim}_{-2,\mathcal{H}},\varphi^{\sim}_{0,\mathcal{H}})$, and it should be added in the sentence that $(\varphi^{\sim}_{-2,\mathcal{H}},\varphi^{\sim}_{0,\mathcal{H}})$ induces the $(\varphi_{-2,\mathcal{H}},\varphi_{0,\mathcal{H}})$ in $\Phi_{\mathcal{H}}$. (See (43) above.)

(52) In part (b) of 3. of Prop. 6.2.5.8, "$y\in Y_\sigma$" should be "$0\neq y\in Y_\sigma$".

(53) In the definition of (6.2.5.9), the invertible sheaf $\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell)$ does not have to be rigidified. (Hence it is harmless to replace "$\underline{\mathrm{Pic}}_e(C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}})$" with "$\underline{\mathrm{Pic}}(C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})$" in the correction of (6.2.4.8) above.)

(54) In the proof of Prop. 6.2.5.11, "for every discrete valuation υ of K" should be more precisely "for every discrete valuation υ of K such that $\upsilon(R)\geq 0$".

(55) In the proof of Prop. 6.2.5.18, all instances of "$\Omega^1_{\overline{\Xi}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/\mathsf{S}_0}$" should be "$\Omega^1_{\overline{\Xi}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/\mathsf{S}_0}[d\log\infty]$", and all instances of "$\Omega^1_{\overline{\Xi}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}}$" should be "$\Omega^1_{\overline{\Xi}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}}[d\log\infty]$".

(56) In (6.2.5.22), the $(\varphi_{-2,\mathcal{H}},\varphi_{0,\mathcal{H}})$ should be denoted $(\varphi^{\sim}_{-2,\mathcal{H}},\varphi^{\sim}_{0,\mathcal{H}})$, and it should be remarked that $(\varphi^{\sim}_{-2,\mathcal{H}},\varphi^{\sim}_{0,\mathcal{H}})$ induces the $(\varphi_{-2,\mathcal{H}},\varphi_{0,\mathcal{H}})$ in $\Phi_{\mathcal{H}}$. (See (43) above.)

(57) In Cond. 6.2.5.25, "γ acts as the identity" should be "a power of γ acts as the identity", and "containing σ_j" should be "containing $\gamma\overline{\sigma}_j \cap \overline{\sigma}_j$".
(58) In Rem. 6.2.5.26, given the above correction of Cond. 6.2.5.25, "γ acts as the identity" should be "some power of γ acts as the identity".
(59) In Prop. 6.2.6.7, line -2, "over $\mathsf{M}'_{\mathcal{H}}$" should be dropped.
(60) In Step 1 of Construction 6.3.1.1, the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in the second displayed object should be denoted $(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$, and it should be added in the sentence that $(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$ induces the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in the given $\Phi_{\mathcal{H}}$. (See (43) above.) Similarly, the $(\varphi^{\dagger}_{-2,\mathcal{H}}, \varphi^{\dagger}_{0,\mathcal{H}})$ in the second last displayed object should be denoted $(\varphi^{\dagger,\sim}_{-2,\mathcal{H}}, \varphi^{\dagger,\sim}_{0,\mathcal{H}})$, and it should be added in the sentence that $(\varphi^{\dagger,\sim}_{-2,\mathcal{H}}, \varphi^{\dagger,\sim}_{0,\mathcal{H}})$ induces the $(\varphi^{\dagger}_{-2,\mathcal{H}}, \varphi^{\dagger}_{0,\mathcal{H}})$ in $\Phi^{\dagger}_{\mathcal{H}}$. The uniqueness of the objects in $\mathrm{DD}^{\text{fil.-spl.}}_{\mathrm{PEL},\mathsf{M}_{\mathcal{H}}}(R)$ or $\mathrm{DD}^{\text{fil.-spl.}}_{\mathrm{PEL},\mathsf{M}_{\mathcal{H}}}(R^{\dagger})$ are only up to isomorphism inducing automorphisms on $\Phi_{\mathcal{H}}$ or $\Phi^{\dagger}_{\mathcal{H}}$.
(61) In Prop. 6.3.1.6, "formally étale" should be "étale" (i.e., formally étale and of finite type).
(62) In 6. of Prop. 6.3.1.6, in the second paragraph, the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in $\alpha^{\natural}_{\mathcal{H}} = (\mathsf{Z}_{\mathcal{H}}, \varphi_{-2,\mathcal{H}}, \varphi_{-1,\mathcal{H}}, \varphi_{0,\mathcal{H}}, \delta_{\mathcal{H}}, c_{\mathcal{H}}, c^{\vee}_{\mathcal{H}}, \tau_{\mathcal{H}})$ should be denoted $(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$, and it should be added in the sentence that $(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$ induces the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in $\Phi_{\mathcal{H}}$. (See (43) above.) Also, "Let $(\mathsf{Z}_{\mathcal{H}}, \Phi_{\mathcal{H}}, \delta_{\mathcal{H}})$ be a representative of this cusp label" should be "Suppose $(\mathsf{Z}_{\mathcal{H}}, \Phi_{\mathcal{H}}, \delta_{\mathcal{H}})$ is a representative of this cusp label".
(63) In Cor. 6.3.1.8, should assume that S_{for} is noetherian.
(64) In Cor. 6.3.1.14, should assume that k is of finite type over $\tilde{k}$.
(65) In Cor. 6.3.1.18, should assume that f induces an isomorphism between separable closures of residue fields.
(66) In the proof of Prop. 6.3.2.1, should denote the $\Phi_{\mathcal{H}}$ in the two displayed degeneration data by two different notations (other than the prescribed $\Phi_{\mathcal{H}}$), and remark that they can be approximated because they are discrete in nature. (See (43) above.)
(67) In Rem. 6.3.2.8, "$({}^{\diamond}G, {}^{\diamond}\lambda, {}^{\diamond}i, {}^{\diamond}\alpha_{\mathcal{H}})$" should be "$(G, \lambda, i, \alpha_{\mathcal{H}})$".
(68) In the proof of Prop. 6.3.3.11, should use both $(A^{\ddagger}, \lambda_{A^{\ddagger}}, i_{A^{\ddagger}}, \varphi^{\ddagger}_{-1,\mathcal{H}})$ and the $\Gamma_{\Phi^{\ddagger}_{\mathcal{H}}}$-orbit of $\Phi^{\sim,\ddagger}_{\mathcal{H}} = (X^{\ddagger}, Y^{\ddagger}, \phi^{\ddagger}, \varphi^{\sim,\ddagger}_{-2,\mathcal{H}}, \varphi^{\sim,\ddagger}_{0,\mathcal{H}})$ to determine a morphism $\mathrm{Spec}(R) \to \mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$.
(69) In step 2 of the proof of Prop. 6.3.3.13, for $i = 1, 2$, the $(\varphi_{-2,\mathcal{H},i}, \varphi_{0,\mathcal{H},i})$ in $\alpha^{\natural}_{\mathcal{H},i}$ should be denoted $(\varphi^{\sim}_{-2,\mathcal{H},i}, \varphi^{\sim}_{0,\mathcal{H},i})$, and it should be added in the sentence that $(\varphi^{\sim}_{-2,\mathcal{H},i}, \varphi^{\sim}_{0,\mathcal{H},i})$ induces the $(\varphi_{-2,\mathcal{H},i}, \varphi_{0,\mathcal{H},i})$ in $\Phi_{\mathcal{H},i}$. (See (43) above.)
(70) In Rem. 6.3.3.16, "descends" should be "descend".
(71) In 2. of Thm. 6.4.1.1, $\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma}$ is incorrectly described. It should be "$\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma}$ (before quotient by $\Gamma_{\Phi_{\mathcal{H}},\sigma}$) admits a canonical structure as the completion of an affine toroidal embedding $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\sigma)$ (along its

σ-stratum $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma}$) of a torus torsor $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ over an abelian scheme torsor $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ over a finite étale cover $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ of the algebraic stack $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$".

(72) In 5. of Thm. 6.4.1.1, "formally étale" should be "étale" (i.e., formally étale and of finite type). In the corresponding paragraph of the proof, the first instance of "formally étale" (in the parenthetical remark) should be "étale", while the second instance can be removed harmlessly.

(73) In 6. of Thm. 6.4.1.1, the $\lambda_A^\ddagger$ and $i_A^\ddagger$ should be denoted $\lambda_{A^\ddagger}$ and $i_{A^\ddagger}$; just to clarify, the condition for $\overline{\sigma}^\ddagger$ to contain all $\upsilon \circ B^\ddagger$ means for all υ centered at the same given geometric point $\bar{s}$.

(74) In paragraph 4 of the proof of Thm. 6.4.1.1: "$(\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma)$-stratum" should be "$[(\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma)]$-stratum".

(75) In Section 7.1.2, paragraph 1, it is a mistake to call $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ a "moduli scheme" because it is not necessarily a scheme.

(76) In the proof of Lem. 7.1.2.1, "formally étale" should be "étale" (i.e., formally étale and of finite type).

(77) In Prop. 7.1.2.13 and its proof, should remark that it is constant along the fibers because it is also invariant under $\Gamma_{\Phi_{\mathcal{H}}}$, and we know $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}/\Gamma_{\Phi_{\mathcal{H}}} \cong \mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$.

(78) In the proof of Cor. 7.2.3.11, "$\mathsf{M}_{\mathcal{H}}^{\mathrm{tor}}|_{\mathsf{Z}_{[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)]}} : \mathsf{Z}_{[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)]} \to \mathsf{Z}_{[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}})]}$" should be "$\oint_{\mathcal{H}}|_{\mathsf{Z}_{[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)]}} : \mathsf{Z}_{[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)]} \to \mathsf{Z}_{[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}})]}$".

(79) In Cor. 7.2.3.14, the proof can be slightly weakened to allow $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}} \to \mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ to be an abelian scheme torsor.

(80) At the end of the proof of Prop. 7.2.3.16, should replace "$(\underline{\mathrm{FJ}}^{(0)}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^{\mathrm{Aut}(\bar{x})}$" with "$((\underline{\mathrm{FJ}}^{(0)}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^{\wedge}_{\bar{x}})^{\mathrm{Aut}(\bar{x})\times\Gamma_{\Phi_{\mathcal{H}}}}$".

(81) In 4. of Thm. 7.2.4.1, should simply say that $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ is as in Definition 5.4.2.6, without saying that it represents a moduli problem.

(82) In 5. of Thm. 7.2.4.1, should say instead that $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ is an abelian scheme torsor over the finite étale cover $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ over the algebraic stack $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ over the coarse moduli space $[\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}]$ (which is a scheme). (See above.)

(83) In the proof of 3. of Prop. 7.2.4.3, the assertion that "the action of $\Gamma_{\Phi_{\mathcal{H}}}$ is just a permutation of Fourier–Jacobi coefficients" is not literally true. What is true is a more elaborate assertion: By the construction of $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ (see (42) above), there exists a finite index normal subgroup $\Gamma'_{\Phi_{\mathcal{H}}}$ of $\Gamma_{\Phi_{\mathcal{H}}}$ such that $\Gamma'_{\Phi_{\mathcal{H}}}$ acts trivially on $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$, and such that the induced action of $\Gamma_{\Phi_{\mathcal{H}}}/\Gamma'_{\Phi_{\mathcal{H}}}$ on $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ makes $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}} \to \mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ an étale $(\Gamma_{\Phi_{\mathcal{H}}}/\Gamma'_{\Phi_{\mathcal{H}}})$-torsor. For each ℓ_0 in $\mathbf{P}^\vee_{\Phi_{\mathcal{H}}}$, consider its stabilizer $\Gamma'_{\Phi_{\mathcal{H}},\ell_0}$ in $\Gamma'_{\Phi_{\mathcal{H}}}$. Then the correct statement (which suffices for the argument of the proof, because $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}} \to \mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ is an étale $(\Gamma_{\Phi_{\mathcal{H}}}/\Gamma'_{\Phi_{\mathcal{H}}})$-torsor) is that the formation of $\Gamma'_{\Phi_{\mathcal{H}},\ell_0}$-invariants in $(\underline{\mathrm{FJ}}^{(\ell_0)}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^{\wedge}_{\bar{x}}$ commutes with the base change from S to $\bar{s}$ under the assumption that the condition (7.2.4.6) is satisfied. (When $\mathcal{H}$ is neat, it can be shown that $\Gamma'_{\Phi_{\mathcal{H}},\ell_0}$ acts trivially on $(\underline{\mathrm{FJ}}^{(\ell_0)}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^{\wedge}_{\bar{x}}$.)

(84) In the proof of 4. of Prop. 7.2.4.3, for the assertions involving only S, should first reduce to the case where S is local, and define S_1 to be the localization of S_0 at the image under $S \to \mathsf{S}_0$ of the closed point of S.

(85) In 4. of Def. 7.3.1.1, "$x, y \in \mathbf{S}_{\Phi_\mathcal{H}}$" should be "$x, y \in \mathbf{P}_{\Phi_\mathcal{H}}$".

(86) In Lem. 7.3.1.7, "$K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$" should be "$K^\vee_{\mathsf{pol}_{\Phi_{\mathcal{H}'}}}$".

(87) The literal statements of Lem. 7.3.1.9, which we cited almost verbatim from Faltings–Chai (Ch. V, Lem. 5.5), are unfortunately incorrect. For example, if $\mathbf{P}^+_{\Phi_\mathcal{H}} = \mathbb{R}_{>0} = \sigma$, then there are no other top-dimensional cones at all, and hence the lemma asserts that $\sigma^\vee = \{0\}$—but $\sigma^\vee$ is certainly nonzero. This error was inherited from a similar error in Ash–Mumford–Rapoport–Tai (Ch. IV, Sec. 2, p. 330). To fix this:

(a) Rewrite the statements of Lem. 7.3.1.9 as follows: "Suppose $\sigma \in \Sigma_{\Phi_\mathcal{H}}$, and suppose $\sigma_1, \ldots, \sigma_r$ are the one-dimensional faces of σ. For each $1 \leq j \leq r$, consider the unique $y_j \in \sigma_j$ such that $\mathbf{S}^\vee_{\Phi_\mathcal{H}} \cap \sigma_j = \mathbb{Z}_{\geq 1} \cdot y_j$, so that $K_{\mathsf{pol}_{\Phi_\mathcal{H}}} \cap \sigma_j = \mathbb{R}_{\geq 1} \cdot (\mathsf{pol}_{\Phi_\mathcal{H}}(y_j)^{-1} y_j)$, and let $L_j := \{x \in \mathbf{S}_{\Phi_\mathcal{H}} \underset{\mathbb{Z}}{\otimes} \mathbb{R} : \langle x, y_j \rangle = \mathsf{pol}_{\Phi_\mathcal{H}}(y_j)\}$. Then each $L_j \cap K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$ is a top-dimensional face of $K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$, whose vertices are in $\mathbf{S}_{\Phi_\mathcal{H}} \cap K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$ because $y_j \in \mathbf{S}^\vee_{\Phi_\mathcal{H}}$ and $\mathsf{pol}_{\Phi_\mathcal{H}}$ takes integral values on $\mathbf{S}^\vee_{\Phi_\mathcal{H}}$, and the intersection $\underset{1 \leq j \leq r}{\cap} (L_j \cap K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}})$ defines a face of $K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$ (which we consider dual to σ). Suppose $d \geq 1$ is any integer, and suppose $\ell_0 \in \mathbf{S}_{\Phi_\mathcal{H}} \cap d \cdot \big(\underset{1 \leq j \leq r}{\cap} (L_j \cap K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}})\big)$ does not lie on any proper face of $d \cdot \big(\underset{1 \leq j \leq r}{\cap} (L_j \cap K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}})\big)$. Then there exist $\ell_1, \ldots, \ell_n \in \mathbf{S}_{\Phi_\mathcal{H}} \cap K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$ (which are not necessarily vertices of $K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$) such that $\mathbb{R}_{\geq 0} \cdot \sigma^\vee = \sum\limits_{\ell \in \mathbf{S}_{\Phi_\mathcal{H}} \cap (d \cdot K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}})} \mathbb{R}_{\geq 0} \cdot (\ell - \ell_0) = \sum\limits_{1 \leq i \leq n} \mathbb{R}_{\geq 0} \cdot (d \cdot \ell_i - \ell_0)$."

(b) Modify the proof of 2 of Thm. 7.3.3.4 as follows:

(i) In paragraph 6, replace the first two sentences with just one sentence: "Suppose $\ell_{\text{gen}} \in K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$." And replace "for every vertex $d_0 \cdot \ell_{\text{gen}}$ of $d_0 \cdot K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$" with "for every $\ell_{\text{gen}} \in K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$".

(ii) In paragraph 7, replace "vertices $d_0 \cdot \ell_i$ of $d_0 \cdot K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$" with "elements ℓ_i of $K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$".

(iii) Remove paragraphs 8 and 9 completely.

(iv) Replace the first three sentences of paragraph 10 with the following: "Suppose ℓ_0 lies on the face of $d_0 \cdot K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$ dual to some $\tau \in \Sigma_{\Phi_\mathcal{H}}$ as in Lemma 7.3.1.9, so that there exist $\ell_1, \ldots, \ell_n \in \mathbf{S}_{\Phi_\mathcal{H}} \cap K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$ (which are not necessarily vertices of $K^\vee_{\mathsf{pol}_{\Phi_\mathcal{H}}}$) such that $\mathbb{R}_{\geq 0} \cdot \tau^\vee = \sum\limits_{1 \leq i \leq n} \mathbb{R}_{\geq 0} \cdot (d_0 \cdot \ell_i - \ell_0)$. By (7.3.3.6) (with $\ell_{\text{gen}} = \ell_i$ there, for each i), we see that $\upsilon(\Psi_{\Phi_\mathcal{H}, \delta_\mathcal{H}}(\ell)) \geq 0$ for all $\ell \in \tau^\vee$."

(88) In Prop. 7.3.2.3, "$\imath' \to (f')^{-1}\mathcal{I}' \cdot \mathscr{O}_{\tilde{W}'}$" should be "$(f')^{-1}\mathcal{I}' \cdot \mathscr{O}_{\tilde{W}'} \to \imath'$", and "$\imath^{\otimes d_0} \to f^{-1}\mathcal{I}^{(d_0)} \cdot \mathscr{O}_{\tilde{W}}$" should be "$f^{-1}\mathcal{I}^{(d_0)} \cdot \mathscr{O}_{\tilde{W}} \to \imath^{\otimes d_0}$".

(89) In Def. 7.3.3.1, "$\mathbb{Z}_{\geq 0}$-generator" should be "$\mathbb{Z}_{>0}$-generator".

(90) In Thm. 7.3.3.4, 1., and in its proof, all instances of "$\jmath^{\otimes d_0}_{\mathcal{H},\mathsf{pol}} \to \oint_{\mathcal{H}}^{-1} \mathcal{J}^{(d_0)}_{\mathcal{H},\mathsf{pol}} \cdot \mathscr{O}_{\mathsf{M}^{\mathrm{tor}}_{\mathcal{H}}}$" should be "$\oint_{\mathcal{H}}^{-1} \mathcal{J}^{(d_0)}_{\mathcal{H},\mathsf{pol}} \cdot \mathscr{O}_{\mathsf{M}^{\mathrm{tor}}_{\mathcal{H}}} \to \jmath^{\otimes d_0}_{\mathcal{H},\mathsf{pol}}$".

(91) In the proof of 1 of Thm. 7.3.3.4:
   (a) In paragraph 2, "$\mathbb{Z}_{\geq 0}$-generator" should be "$\mathbb{Z}_{>0}$-generator".
   (b) In paragraph 4, "ample line bundle $\underset{1\leq i\leq r}{\otimes} (\mathrm{pr}_i^*(\mathrm{Id}_A, \lambda_A)^*\mathcal{P}_A)^{\otimes e_i}$" should be just "line bundle $\underset{1\leq i\leq r}{\otimes} (\mathrm{pr}_i^*(\mathrm{Id}_A, \lambda_A)^*\mathcal{P}_A)^{\otimes e_i}$", the various instances of "Hom" should be "$\underline{\mathrm{Hom}}$", and "$\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell_0)$ is ample over $\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$" should be "$\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell_0)$ is relatively ample over $\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$".
   (c) In paragraph 5, "$(d \cdot \ell_0 + \tau^\vee)/\Gamma_{\Phi_{\mathcal{H}}}$" should be "$(\Gamma_{\Phi_{\mathcal{H}}} \cdot (d \cdot \ell_0 + \tau^\vee))/\Gamma_{\Phi_{\mathcal{H}}}$".
   (d) In paragraph 6, "structural sheaf of $\underset{\ell\in\tau^\vee}{\hat{\oplus}} (\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell))^\wedge_{\bar{x}}$" should be "$\mathscr{O}_{\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\tau}} \cong \underset{\ell\in\tau^\vee}{\hat{\oplus}} (\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell))^\wedge_{\bar{x}}$"

(92) In the proof of 2 of Thm. 7.3.3.4:
   (a) In paragraph 2, the morphism $y : \mathrm{Spf}(V) \to \mathsf{M}^{\mathrm{tor}}_{\mathcal{H}}$ should be required to induce morphisms $\mathrm{Spec}(V) \to \Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\sigma)$ and $\mathrm{Spec}(V) \to \mathsf{M}^{\mathrm{tor}}_{\mathcal{H}}$ mapping the generic point of $\mathrm{Spec}(V)$ to $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ and $\mathsf{M}_{\mathcal{H}}$, respectively. Also, "$\bar{y}$ is uniquely determined by $\bar{z}$" should be "there are only finitely many $\bar{y}$ inducing the same $\bar{z}$".
   (b) In (7.3.3.5), "$\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma}$" should be "$(\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma})^\wedge_{\bar{x}}$".
   (c) In paragraph 5, all instances of "$d_0 \cdot K^\vee_{\mathsf{pol}_{\Phi_{\mathcal{H}}}}$" should be "$\mathbf{S}_{\Phi_{\mathcal{H}}} \cap (d_0 \cdot K^\vee_{\mathsf{pol}_{\Phi_{\mathcal{H}}}})$".
   (d) In the second last paragraph, it is literally incorrect to consider the pullbacks to $(\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma})^\wedge_{\bar{x}}$ of sections of $\mathscr{O}_{\mathfrak{U}_f}$ and of the (coherent ideal) pullback of $(\mathcal{J}^{(d_0)}_{\mathcal{H},\mathsf{pol}})^\wedge_{\bar{x}}$ to $\mathfrak{U}_f$. To fix this:
      (i) In paragraph 5 (of the proof), add the following sentences: "Without loss of generality, we may and we shall assume that $f^{(\ell)} \neq 0$ exactly when $\ell \in \Gamma_{\Phi_{\mathcal{H}}} \cdot \ell_0$. Let $\mathfrak{V}_{f^{(\ell_0)}}$ denote the maximal open formal subscheme of $(C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^\wedge_{\bar{x}}$ over which $f^{(\ell_0)}$ is a generator of the pullback of $\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell_0)$, and let $\mathfrak{W}_{f^{(\ell_0)}}$ denote the preimage of $\mathfrak{V}_{f^{(\ell_0)}}$ under the canonical morphism $(\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma})^\wedge_{\bar{x}} \to (C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^\wedge_{\bar{x}}$. Then the proof of 1 of Theorem 7.3.3.4 shows that $\mathfrak{W}_{f^{(\ell_0)}}$ is the preimage of $\mathfrak{U}_f$ under the canonical morphism $(\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma})^\wedge_{\bar{x}} \to \mathrm{Bl}_{(\mathcal{J}^{(d_0)}_{\mathcal{H},\mathsf{pol}})^\wedge_{\bar{x}}}((\mathsf{M}^{\min}_{\mathcal{H}})^\wedge_{\bar{x}})$."
      (ii) In the second last paragraph, "pullback of sections of $\mathscr{O}_{\mathfrak{U}_f}$ to $(\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma})^\wedge_{\bar{x}}$" should be "pullback of sections of $\mathscr{O}_{\mathfrak{U}_f}$ to the open formal subscheme $\mathfrak{W}_{f^{(\ell_0)}}$ of $(\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma})^\wedge_{\bar{x}}$"; "pullback

of sections of $(\mathcal{J}^{(d_0)}_{\mathcal{H},\mathsf{pol}})^{\wedge}_{\bar{x}}$ to $(\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma})^{\wedge}_{\bar{x}}$" should be "pullback to $\mathfrak{W}_{f(\ell_0)}$ of sections of the (coherent ideal) pullback of $(\mathcal{J}^{(d_0)}_{\mathcal{H},\mathsf{pol}})^{\wedge}_{\bar{x}}$ to $\mathfrak{U}_f$"; "pullback of $(\mathcal{J}^{(d_0)}_{\mathcal{H},\mathsf{pol}})^{\wedge}_{\bar{x}}$ to $(\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma})^{\wedge}_{\bar{x}}$" should be "pullback of $(\mathcal{J}^{(d_0)}_{\mathcal{H},\mathsf{pol}})^{\wedge}_{\bar{x}}$ to $\mathfrak{W}_{f(\ell_0)}$"; and all instances of "sections in $\hat{\bigoplus}_{\ell\in\sigma^\vee}(\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell))^{\wedge}_{\bar{x}}$" should be "sections of $\hat{\bigoplus}_{\ell\in\sigma^\vee}(\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell))^{\wedge}_{\bar{x}}$ over $\mathfrak{V}_{f(\ell_0)}$".

(iii) In the last paragraph, both instances of "$(\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma})^{\wedge}_{\bar{x}}$" should be "$\mathfrak{V}_{f(\ell_0)}$".

(e) In the last paragraph of the proof, both instances of "$(\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^{\wedge}_{\bar{x}}$" should be "$(\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell))^{\wedge}_{\bar{x}}$", and should only assert that $\bar{z}$ determines a compatible collection of morphisms $\{\Gamma(\mathfrak{V}_{f(\ell_0)},(\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell))^{\wedge}_{\bar{x}})\to k\}$ for ℓ's in a finite index subgroup of $\sigma^\perp$.

(93) In Def. A.1.2.1, 2., (ii): "for each three objects $X, Y, Z \in \operatorname{Ob}\mathsf{C}$" should be "for each two objects $X, Y \in \operatorname{Ob}\mathsf{C}$".

(94) In the second paragraph of Def. A.7.2.8, "$U \to \mathsf{X}$" should be "$U \to \mathsf{Y}$".

(95) In 5. of Thm. B.3.7, "of finite type S" should be "of finite type over S".

(96) In the third paragraph of the proof of Thm. B.3.11, "show that ξ is formally étale" should be "show that $\widetilde{\xi}$ is formally étale", and "Krull dimension of U and X" should be "Krull dimensions of X and X'".

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