Arithmetic compactifications of PEL-type Shimura varieties — Errata

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Unless for the purpose of citations, the original version should not be read. Please read the latest revision.

Even for the purpose of citations, the reader is encouraged to consult the revision whenever possible, to make sure she or he is not citing an incorrectly stated result, definition, or remark. There are a lot of folklore statements in this subject which are not entirely correct. Some of them might even have good reasons not to be correct. So it is especially important to make sure the desired statements are indeed proved.

The numbering of results and pages below refer to the double-spaced version of the thesis submitted to the university. (We shall avoid using page numbers whenever possible.) We have also recorded some improvements which are not necessarily corrections.

- Under the protest of more than one prominent readers, the convention that schemes are *separated* are replaced with the convention that schemes are *quasi-separated*. This incurs no substantial change in the arguments. (Presumably one can drop the quasi-separateness assumption as well, if one also drops the quasi-separateness assumptions in the definitions of algebraic spaces and algebraic stacks. We did not do this due to practical concerns about lack of references.)
- Notations and Conventions:

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- The correct definition is that $\mathbb{Z}_{(\Box)}$ is the unique localization of $\mathbb{Z}$ (at the multiplicative subset of $\mathbb{Z}$ generated by nonzero integers prime-to-$\Box$) having $\Box$ as its set of height one primes.
- The geometric points should be spectra of algebraically closed fields, because some of the statements involving them do not hold for merely separably closed ones.

- Prop. 1.1.1.9, 1.: It is necessary to add the assumption that $\hat{R}$ is a noetherian domain. Moreover, reference to Thm. 11.5 in [112] should be given. (This part of the section is modified in a revision.)
- Proof of Prop. 1.1.1.12, last displayed equation: "$x_k y_j$" should be "$y_k x_j$".
- Proof of Prop. 1.1.1.20, paragraph -2, line 2: "$\mathcal{O} \underset{R}{\otimes} R_{\mathfrak{p}}$" should be "$\mathcal{O} \underset{R}{\otimes} \hat{R}_{\mathfrak{p}}$".
- Def. 1.1.1.19 is a misprint. It should be removed.
- Lem. 1.1.2.1: the parameter "$\tau : E \to K_{[\tau]}$" in the displayed equation should be "$[\tau] : E \to K_{[\tau]}$".
- Cor. 1.1.2.5, 1.: "unique $C \underset{k}{\otimes} K$-module $W_{[\tau]}$" should be "unique simple $C \underset{k}{\otimes} K$-module $W_{[\tau]}$".
- (1.1.2.6): The modules $W_{\tau'}$ may appear with a common multiplicity $s_{[\tau]}$ greater than 1 in the decomposition. The multiplicity $s_{[\tau]}$ is 1 if $C \underset{k}{\otimes} K$ is a product of matrix algebras containing $E \underset{k}{\otimes} K$ as its center. We clarified this mistake (and made corrections accordingly in what follows). (See in particular the correction for Lem. 1.2.5.10 below.)
- Paragraph 2 following Cor. 1.1.2.5: "unique $C \underset{k}{\otimes} K^{\mathrm{sep}}$-module" should be "unique simple $C \underset{k}{\otimes} K^{\mathrm{sep}}$-module".
- Def. 1.1.2.13:
  - The locally noetherian hypothesis on S is unnecessary.
  - The extra comma in the definition of $\mathscr{O}_S[\mathscr{L}^\vee]$ should be removed.
- Def. 1.1.2.17:

- Line 1: The locally noetherian hypothesis on S is unnecessary.
- Line 2: "on which $\mathcal{O}$ has an action" should be more precisely "on which $\mathcal{O}$ acts by maps of $\mathscr{O}_S$-modules".

- Sec. 1.1.3, paragraph 2 following Rem. 1.1.3.2, the discriminant Disc should better be $\mathrm{Disc}_{\mathcal{O}/R_0}$, not $\mathrm{Disc}_{\mathcal{O}/\mathbb{Z}}$. The condition $p \nmid \mathrm{Disc}$ is too strong. (We have made necessary modifications in a revision to make sure that the redefinitions are performed consistently.) Accordingly, Λ should be the complete noetherian local R_0-algebra with residue filed k, such that a complete noetherian local algebra with residue field k is an R_0-algebra if and only if it is a Λ-algebra. When $R_0 = \mathbb{Z}$ and $\mathrm{char}(k) = p > 0$, this is exactly $W(k)$.
- Sec. 1.1.3, paragraph 3 following Rem. 1.1.3.2, the decompositions for $\mathcal{O}_{F,\tau}$ and $\mathcal{O}_\tau$ should use "$\prod$", not "$\coprod$".
- Line -5 from Convention 1.1.3.3: Prop. 1.1.1.17 implies that $\mathcal{O}_\tau$ is a matrix algebra only when $p > 0$, or when $p = 0$ but k is already algebraically closed.
- Lem. 1.1.4.1 is redundant because it follows trivially from perfectness of the trace pairing $\mathrm{Tr}_{\mathcal{O}/R_0}$. (The proof is misleading too: the bases $\{e_i\}_{1\leq i\leq t}$ and $\{f_i\}_{1\leq i\leq t}$ in the proof are not directly related to the bases in the statement of the lemma, unless a change of coordinate is performed.) This is dropped in a revision.
- Sec. 1.1.5, paragraph 2, lines 4–5: The description in terms of matrix algebras should be dropped (unless one assumes $\mathcal{O}_k$ is a product of matrix algebras when $p = 0$).
- Proof of Lem. 1.1.5.2, paragraph 3, line -3: "$\mathrm{End}_\Lambda(M)$" should be "$\mathrm{End}_R(M)$".
- Lem. 1.1.5.5: The codomain of $\langle\,\cdot\,,\,\cdot\,\rangle$ should be R instead of $\mathcal{O}_R$. The condition that $r = r^\star$ is redundant.
- Sec. 1.1.5, starting with Lem. 1.1.5.7, all statements in the case that $\star$ is nontrivial on F are made under the assumption that F^+ is simple (for simplicity). It is a mistake not to have stated this assumption.
- Proof of Lem. 1.1.5.9, paragraph 5, line 2: The sentence "If $i = 0, \ldots$" should be removed.

- Proof of Cor. 1.1.5.10: "$\langle x, \delta' y\rangle = -\langle \delta' y, \gamma x\rangle = -\langle y, (\delta')^\star \gamma x\rangle = -\langle y, \delta' x\rangle$" should be "$\langle x, \delta' y\rangle = \langle \delta' y, \gamma x\rangle = \langle y, (\delta')^\star \gamma x\rangle = -\langle y, \delta' x\rangle$".
- Cor. 1.1.5.14, line 5: "$\langle \cdot , \cdot \rangle_2 := \langle \cdot , \cdot \rangle_1 \circ a$" should be "$\langle \cdot , \cdot \rangle_2 := \langle \cdot , \cdot \rangle_1 \circ (a \times \mathrm{Id})$".
- Proof of Cor. 1.1.5.14, lines 2–3: "$\langle x, y\rangle_3 = \langle x, a(y)\rangle_1$" should be "$\langle x, y\rangle_3 := \langle a(x), y\rangle_1$".
- Lem. 1.1.5.16: "$M_{\tau,R}^{\oplus m_\tau}$" should be "$\oplus\, M_{\tau,R}^{\oplus m_\tau}$".
- Proof of Lem. 1.1.5.16: Explicit use of matrices is removed in the revision (otherwise we need to assume that $\mathcal{O}_k$ is a matrix algebra when $p = 0$). The proof is simplified in the revision.
- Sec. 1.2.1, paragraph 2, line 1: "invariant under $\star$" should be clarified as "mapped to itself under $\star$".
- Def. 1.2.1.5:
  - Displayed equation: The condition should be "$\forall x, y \in L \underset{\mathbb{Z}}{\otimes} R$".
  - Line -7: "second fact" should be "second factor".
  - Lines -4– -1: The parenthetical remark "(If $L \neq \{0\} \ldots$)" should be "(If $L \neq \{0\}$ and R is flat over $\mathbb{Z}$, then the value of r is uniquely determined by g. Hence there is little that we lose when suppressing r from the notation. However, this suppression is indeed an abuse of notation in general. For example, when $L = \{0\}$, we have $\mathrm{G} = \mathbf{G}_{\mathrm{m}}$.)"
- Rem. 1.2.1.9: The reference to Prop. 1.2.3.11 is imprecise.
- Displayed equation -1 before (1.2.1.10): "$[\tau] : F \hookrightarrow \mathbb{Q}_{[\tau]}$" should be "$[\tau] : F \to \mathbb{Q}_{[\tau]}$".
- Line -2 before (1.2.1.10): "embeddings $F \hookrightarrow \mathbb{Q}^{\mathrm{sep}}$" should be "homomorphisms $F \to \mathbb{Q}^{\mathrm{sep}}$".
- (1.2.1.10): "$[\tau] : F \hookrightarrow \mathbb{Q}_{[\tau]}$" should be "$[\tau] : F \to \mathbb{Q}_{[\tau]}$".
- Line 1 after (1.2.1.10): "consisting of elements in B commuting with F_τ" should be "of B containing F_τ as its center".
- Prop. 1.2.1.14: The notation k used in $k \in B \underset{\mathbb{Q}}{\otimes} \mathbb{R}$ is already used in the expression $B \cong \mathrm{M}_k(D)$ etc.. We changed the former case of k to b.

- Def. 1.2.1.15, 3., line 3: "$B \underset{F,\tau}{\otimes} \mathbb{R} \cong \mathrm{M}_k(\mathbb{C})$" should be "$B \underset{F,\tau}{\otimes} \mathbb{C} \cong \mathrm{M}_k(\mathbb{C})$".
- Rem. 1.2.1.16 is imprecise (and wrong when certain simple factors of B act trivially on $(L, \langle \cdot, \cdot \rangle, h)$). This is corrected in a revision.
- Def. 1.2.1.19: The noetherian hypothesis on the ring "R" should be dropped.
- Def. 1.2.1.20: The definition of multi-ranks for integrable $\mathcal{O} \underset{\mathbb{Z}}{\otimes} R$-modules does not work for general R (and the statement involving $M \underset{\mathbb{Z}}{\otimes} k$ certainly does not make sense). We modified Def. 1.2.1.20 (and split it into several definitions) in the revision so that we only consider cases where R is either flat over $\mathbb{Z}$, or a complete noetherian local ring with good residue characteristic.
- Lem. 1.2.1.23: The definition of "$\mathcal{O}_{[\tau],R} := \mathrm{End}_{\mathcal{O}_{F,R}}(M_{[\tau],R})$" is wrong when $p = 0$ but $\mathcal{O}_k$ is not a matrix algebra. One can define alternatively that $\mathcal{O}_{[\tau],R}$ is the image of $\mathcal{O}_R$ in $\mathrm{End}_{\mathcal{O}_{F,R}}(M_{[\tau],R})$.
- Lem. 1.2.1.25: We need to assume that m_τ is a multiple of s_τ when $p = 0$ but $\mathcal{O}_k$ is not a matrix algebra.
- Proof of Prop. 1.2.2.1:
  - Paragraph -4 from the end, line 3: "bc^{-1}" should be "ac^{-1}".
  - Paragraph -3 from the end: In line 3, "$\mathrm{M}_k(\mathbb{R})$" should be "$\mathrm{M}_{2k}(\mathbb{R})$". The introduction of a field "such as $\mathbb{C}$" and the statements following it are confusing and now modified in a revision.
- Proof of Prop. 1.2.2.4:
  - Paragraph 1, line 3: "$\mathbf{Sym}_\varrho(L_1, L_2) \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_p$" should be "$\mathbf{Sym}^\epsilon_\varrho(L_1, L_2) \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_p$".
  - Paragraph 3, line 2 after displayed equation: "$\mathbf{Sym}(L_p)$" should be "$\mathbf{Sym}(L_\Lambda)$".
- Sec. 1.2.3: The running assumption that $p \nmid \mathrm{Disc}$ is not enough to ensure that $\mathcal{O}_\Lambda$ is a matrix algebra when $p = 0$. Thus we need to add the assumption that $\mathcal{O} \underset{\mathbb{Z}}{\otimes} k$ is a product of matrix algebras when $p = 0$.

- Sec. 1.2.3, paragraph 2:
  - Line 5: "$N \cong \mathcal{O}_{F,R}^{\oplus n_\tau}$" should be "$N \cong \bigoplus_\tau \mathcal{O}_{F_\tau,R}^{\oplus n_\tau}$".
  - Line 6: "$M_0 \underset{\mathcal{O}_{F,R}}{\otimes} N \cong M_\tau^{\oplus n_\tau}$" should be "$M_0 \underset{\mathcal{O}_{F,R}}{\otimes} N \cong \bigoplus_\tau M_{\tau,R}^{\oplus n_\tau}$".
- Lem. 1.2.3.1, line 3: "$N = \mathcal{O}_{F_\tau,R}^{\oplus m_\tau}$" should be "$N = \bigoplus_\tau \mathcal{O}_{F_\tau,R}^{\oplus m_\tau}$".
- Proof of Lem. 1.2.3.2:
  - Paragraph 1, line 3: "$x \mapsto x^\star = c\,{}^t x c$" should be "$x \mapsto x^\star = c\,{}^t x c^{-1}$".
  - Paragraph 2, line 7: "${}^t c^\diamond = -c$" should be "${}^t c^\diamond = c$". Since $x^\diamond$ is the conjugate of ${}^t x$ by $\left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$ for any $x \in \mathrm{M}_2(\mathcal{O}_{F,R})$, the proof can be fixed by replacing the statement "given by $(x, y) \mapsto \mathrm{Tr}_{\mathcal{O}_{F,R}/R}({}^t x^\diamond c^{-1} y)$ for $x, y \in \mathcal{O}_{F,R}^{\oplus 2k}$" with "given by $(x, y) \mapsto \mathrm{Tr}_{\mathcal{O}_{F,R}/R}({}^t x d^{-1} y)$ for $x, y \in \mathcal{O}_{F,R}^{\oplus 2k}$", for some d such that ${}^t d = -d$.
- The modules in Def. 1.2.3.3 and 1.2.3.4, and in the paragraph preceding it, "rank one" should be "multi-rank (1)". The generator $x = (x_{[\tau]})$ has entries $x_{[\tau]}$ noncanonically given by Lemma 1.2.1.23, and that choice of x is fixed and maintained in Def. 1.2.3.3 and 1.2.3.4 (for simplicity).
- Proof of Lem. 1.2.3.8, line 1: "$\phi(x) = x$" should be "$\phi(rx) = rx$ for any $r \in \mathcal{O}_{\tau,R}$".
- Proof of Lem. 1.2.3.8, line -1: "$\phi(x) = x$ and "$\phi(y) = y$" should be "$\phi(ax) = ax$ and $\phi(by) = by$ for any $a \in \mathcal{O}_{\tau\circ c,R}$ and $b \in \mathcal{O}_{\tau,R}$".
- Proof of Prop. 1.2.3.7, paragraph 11: β should be taken to be in $\mathcal{O}_F$, Tr_{F/F^+} should be used instead of $\mathrm{Tr}_{\mathcal{O}_{F,\Lambda}/\mathcal{O}_{F^+,\Lambda}}$, and $\mathrm{Diff}^{-1}_{\mathcal{O}_{F,\Lambda}/\mathcal{O}_{F^+,\Lambda}}$ should be written $(\mathrm{Diff}^{-1}_{\mathcal{O}_F/\mathcal{O}_{F^+}})_\Lambda$. "$\alpha := \beta - \beta^\star$ is a unit" should be "$\alpha := \beta - \beta^\star$ is a unit in $\mathcal{O}_{F,\Lambda}$".
- Lem. 1.2.3.8, line 5: "to $\mathcal{O}_\tau$" should be "to $\mathcal{O}_{\tau,R}$".
- Lem. 1.2.3.9, line 5: "to $\mathcal{O}_\tau$" should be "to $\mathcal{O}_{\tau,R}$".
- Cor. 1.2.3.10, paragraph 2, line -2: "Helsel's" should be "Hensel's".
- Prop. 1.2.3.11, line -1: "simple factor $\mathrm{H}^{\mathrm{ad}}(k^{\mathrm{sep}})$" should be "simple factor of $\mathrm{H}^{\mathrm{ad}}(k^{\mathrm{sep}})$".

- Proof of Prop. 1.2.3.11, paragraph -2, line -2: "$\mathrm{G}_{[\tau]}(\tilde{R})$" should be "$\mathrm{H}_{[\tau]}(\tilde{R})$".
- Lem. 1.2.4.3, line 4: "$R' \hookrightarrow R$" should be "$R' \to R$".
- Prop. 1.2.4.6, line 1: "Assumptions on k and Λ as in Lemma 1.2.4.5" should be "Assumptions on k and Λ as above".
- Lem. 1.2.5.3: "$m_{[\tau]}$ is even" should be "$m_\tau = s_{[\tau]} m_{[\tau]}$ is even", with $s_{[\tau]}$ defined in the revision for the reason stated above for (1.1.2.6).
- Cor. 1.2.5.6: Both "$\mathrm{Tr}_{\mathbb{Q}}$" should be "$\mathrm{Tr}_{\mathbb{C}}$".
- Lem. 1.2.5.10 and the paragraph preceding it are incorrect and incur unnecessary complications. The correct definition of L_0 is that there exists a finite extension F_0' of F_0 and an $\mathcal{O} \underset{\mathbb{Z}}{\otimes} \mathcal{O}_{F_0'}$-submodule of V_0, such that $L_0 \underset{\mathcal{O}_{F_0'}}{\otimes} \mathbb{C} \cong V_0$. We may assume that F_0' is unramified at any prescribed finite set of primes unramified in $\mathcal{O}$. Other claims we have made, including especially the existence of the map (1.2.5.11), are incorrect and unnecessary for later arguments. We have removed all later references to the incorrect statements in a revision.
- Cor. 1.2.5.12: We should use both Lem. 1.2.5.10 and the definition of reflex fields. (So $\mathrm{Det}_{\mathcal{O}|V_0}$ is an element in $\mathcal{O}_{F_0}[\mathcal{O}^\vee]$ because it is in both $\mathcal{O}_{F_0'}[\mathcal{O}^\vee]$ and $F_0[\mathcal{O}^\vee]$.)
- (1.2.5.15) used the incorrect (1.2.5.11) and hence removed. The argument is modified to allow a finite étale extension $\Lambda \hookrightarrow \Lambda'$. We have replaced most occurrences of Λ in this section with Λ', which is harmless for the application to the proof of Prop. 2.2.4.11 (concerning the formal smoothness of our moduli problems), the only place the results in this section will be needed.
- Lem. 1.2.5.19: All instances of "$\mathcal{O} \underset{\mathbb{Z}}{\otimes} \mathbb{R}$-module" should be "$\mathcal{O} \underset{\mathbb{Z}}{\otimes} R$-module".
- Proof of Prop. 1.2.5.20, line -1: "$\mathcal{O} \underset{\mathbb{Z}}{\otimes} \mathbb{R}$-module" should be "$\mathcal{O} \underset{\mathbb{Z}}{\otimes} R$-module".
- Proof of Prop. 1.2.5.22:
  - Paragraph -4, line -3: "β" should be "$\alpha^{-1}\beta$".

- Paragraph -2, line -1: "$\tilde{g} := \begin{pmatrix} \tilde{\alpha} & 0 \\ & (\tilde{A}^\vee)^{-1} \end{pmatrix}$" should be "$\tilde{g} := \left(\begin{smallmatrix} \tilde{\alpha} & 0 \\ & (\tilde{\alpha}^\vee)^{-1} \end{smallmatrix}\right)$".
- Paragraph -1, line -1: Both instances of "B" should be "β".

- All noetherian hypotheses in Sec. 1.2.6 should be removed (at the expense of assuming finiteness of the number of nonzero filtered pieces).
- All occurrences of $\mathtt{Z}$ in Sec. 1.2.6 should be $\mathtt{F}$.
- Sec. 1.2.6, paragraph 3, line 2: "$\mathtt{F} = \mathtt{F}_{-i}$" should be "$\mathtt{F} = \{\mathtt{F}_{-i}\}$".
- Cor. 1.2.6.5, line 3: "automatically admissible" should be "automatically split".
- Paragraph 1 after Def. 1.2.6.7, "$\langle \cdot, \cdot \rangle$" should be "$\langle \cdot, \cdot \rangle_M$".
- "Def. 1.2.6.9" should be "Rem. 1.2.6.9".
- Thm. 1.3.1.3:
  - Line 3: "partially ordered" should be "directed partially ordered".
  - 2., line 3: "J" should be "I".
  - 3., displayed equation: both instances of "$\underline{\mathrm{Hom}}$" should be Hom.
- The separateness assumption is only made for q in Prop. 1.3.1.4.
- Proof of Prop. 1.3.1.4:
  - Lines 2 and 5: "$f \circ p \circ e$" should be "$f \circ e \circ p$"
  - Line -8 from the end: "complementing $p^{-1}(T)$" should be "complementing $p^{-1}(s)$". (A better explanation of how the claimed closed subset can be chosen is included in a revision of the document.)
  - Line -4 from the end: The statement "p is an open map as it is flat" is not correct in general. We should assume that "p is open" in the statement of Prop. 1.3.1.4.
- Def. 1.3.1.15, line 4: "prime-to-$\Box$ isogeny" should be "prime-to-$\Box$ quasi-isogeny".
- Rem. 1.3.1.19, 1.3.1.20, and 1.3.1.21 are not correct when the base scheme has infinitely many connected components.
- Def. 1.3.2.1: "for some $\mathcal{M}$ on S" should be "for some $\mathcal{M}$ on T".

- Rem. 1.3.2.5, line 1: "over ." should be "over S".
- Paragraph preceding Prop. 1.3.2.18 is an oversimplification. We added reference to EGA that ampleness is an open condition over locally noetherian bases.
- Def. 1.3.2.23: N should be a section of $(\mathbb{Z}_{>0})_S$ (rather than a global constant).
- The title of Sec. 1.3.3 should be "Endomorphism Structures".
- Def. 1.3.3.1:
  - To be precise, $\mathbb{Q}$ or R should be $\mathbb{Q}_S$ or R_S, the group of locally constant functions valued in $\mathbb{Q}$ or R, respectively.
  - The λ-Rosati involutions should be properly introduced.
  - line 3 after (1.3.3.2): "it has to fix $\mathcal{O}$" should be "it has to fix the image of $\mathcal{O}\underset{\mathbb{Z}}{\otimes} R$".
- Rem. 1.3.3.4, line 3: "$r \in \mathbb{Q}^\times_{>0}$" should be "$r \in \mathbb{Z}^\times_{(\Box),>0}$".
- Rem. 1.3.4.1: It is incorrect to claim that h is determined by $(L\underset{\mathbb{Z}}{\otimes}\mathbb{R}, \langle\,\cdot\,,\,\cdot\,\rangle)$ up to conjugacy. What is correct is that only the $\mathrm{G}(\mathbb{R})$-conjugacy class of h is needed in order to define the conditions in Definitions 1.2.5.4 and 1.3.4.2. We have decided to make the choice of h more explicit throughout the book to avoid possible confusions incurred.
- Def. 1.3.5.1, 3.: "$\{l^{\mathbb{Z}_{\geq 0}}\}$" should be "$l^{\mathbb{Z}_{\geq 0}}$".
- Paragraph preceding Lem. 1.3.5.2, line 7: "define $\mathrm{V}^\Box(f)$" should be "define $\mathrm{V}^\Box(N^{-1}f)$".
- Lem. 1.3.5.2, line 1 after displayed equation: "$\mathrm{V}(f)^{-1}(A')$" should be "$\mathrm{V}(f)^{-1}(\mathrm{T}^\Box A')$".
- References are supplied for Prop. 1.3.5.3.
- Cor. 1.3.5.4, line 1 after displayed equation: "$\mathrm{V}(f_{\bar{s}})^{-1}(A'_{\bar{s}})$" should be "$\mathrm{V}(f_{\bar{s}})^{-1}(\mathrm{T}^\Box A'_{\bar{s}})$".
- Rem. 1.3.5.5, line 3: "principal polarization" should be "prime-to-$\Box$ polarization".
- The second appearance of the diagram in Def. 1.3.6.1 is redundant.

- Rem. 1.3.6.2: "$(L^{\#}\underset{\mathbb{Z}}{\otimes}\hat{\mathbb{Z}}^{\square})/(L\underset{\mathbb{Z}}{\otimes}\hat{\mathbb{Z}}^{\square})$" should be "$((L^{\#}\underset{\mathbb{Z}}{\otimes}\hat{\mathbb{Z}}^{\square})/(L\underset{\mathbb{Z}}{\otimes}\hat{\mathbb{Z}}^{\square}))_S$".
- Lem. 1.3.7.3: It is necessary to assume that the base scheme is connected. (This does not effect subsequent exposition because this lemma is never used. It is included only to supply a motivation for Def. 1.3.7.2.)
- Def. 1.3.7.4, line -2: "$\nu(H)$-orbit" should be "$\nu(H_n)$-orbit".
- Construction 1.3.7.10, line -7: "definition" should be "by definition".
- Def. 1.4.1.2, 2.: "$\mathbb{Z}^{\times}_{(\square)}$-polarization" should be "prime-to-$\square$ polarization".
- Def. 1.4.1.4, 2.: "$\mathbb{Z}^{\times}_{(\square)}$-polarization" should be "prime-to-$\square$ polarization".
- Def. 1.4.1.8, line 2: "$\mathrm{Z}(\hat{\mathbb{Z}}^{\square})$" should be "$\mathrm{G}(\hat{\mathbb{Z}}^{\square})$".
- Lem. 1.4.1.10 and Cor. 1.4.1.11 should be moved to Ch. 2 because they used Lem. 2.2.2.1, a consequence of the rigidity of abelian schemes. (This is better for the exposition.)
- Proof of Cor. 1.4.1.11, line -3 from the end: "preserves any" should be "preserves the $\mathcal{H}$-orbit of any".
- Thm. 1.4.1.12 and Cor. 1.4.1.13: The phrase "representable by an algebraic stack" is replaced with "is an algebraic stack" because it is less misleading (and more consistent with our convention in the appendices). Similar changes have been made globally.
- Sec. 1.4.2, line 2: "$(L\underset{\mathbb{Z}}{\otimes}\hat{\mathbb{Z}}^{\square}, \langle\cdot,\cdot\rangle)$" should be "$(L\underset{\mathbb{Z}}{\otimes}\mathbb{A}^{\infty,\square}, \langle\cdot,\cdot\rangle)$".
- Def. 1.4.2.1 and Def. 1.4.2.4:
  - "groupoid $\mathsf{M}_{\mathcal{H}}(S)$" should be "groupoid $\mathsf{M}^{\mathrm{rat}}_{\mathcal{H}}(S)$".
  - The description of i should be: "$i : \mathcal{O}\underset{\mathbb{Z}}{\otimes}\mathbb{Z}_{(\square)} \to \mathrm{End}_S(A)\underset{\mathbb{Z}}{\otimes}\mathbb{Z}_{(\square)}$ defines an $\mathcal{O}\underset{\mathbb{Z}}{\otimes}\mathbb{Z}_{(\square)}$-structure of (A,λ)."
  - "rational principle level-$\mathcal{H}$ structure" should be "rational level-$\mathcal{H}$ structure".
  - "$\sim_{\mathbb{Z}_{(\square)}\text{-isog.}}$" should be "$\sim_{\mathbb{Z}^{\times}_{(\square)}\text{-isog.}}$".

- “$f \circ i(b) = i'(b) \circ f$ for all $b \in \mathcal{O} \underset{\mathbb{Z}}{\otimes} \hat{\mathbb{Z}}^{\Box}$” should be “$f \circ i(b) = i'(b) \circ f$ for all $b \in \mathcal{O} \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_{(\Box)}$”.

- Rem. 1.4.2.7 is clarified in a revision.
- In Construction 1.4.3.1 and Proof of 1.4.3.4, “representing a class” is unnecessary because we are working with category fibred in groupoids, not functors of sets of isomorphism classes.
- Rem. 1.4.3.8: We meant to enlarge L' but to retain $\mathcal{O}$. (This is clarified in the revision.) As for the tacitly implied need of $\mathcal{O}'$ to be invariant under *, there is nowhere we really need this, so we reformulated the remark in the revision to make this clear.
- Rem. 1.4.3.11:
  - Lines 11–13: “Note that the characteristic zero fiber ...” should be “When $\Box$ has only finitely many elements, the characteristic zero fiber ...”.
  - Last 5 lines: The statements on Kottwitz's result is affirmative only when B does not involve simple factors of type D.
- Rem. 1.4.3.13, line -6 from the end: “PEL$\mathcal{O}$-lattice” should be “$\mathcal{O}$-lattice”.
- Proof of Prop. 1.4.4.1:
  - Line 6: “lifting conditions” should be ”symplectic-liftability conditions”.
  - Line -2: “liftability condition” should be “symplectic-liftability condition”.
  - Line -2: “$\Box_1|1$” should be “$\Box_1|p$”.
- We now removed the vague sentence in Rem. 1.4.4.3 that the isomorphism might not necessarily be true. Certainly, it is true for those simple Shimura varieties defined by Kottwitz. We decided to remove the vague sentence because we do not have any concrete example in mind.
- In Sec. 2.1, the notation $\underline{H}^i$ is used by mistake in the theory of obstructions — from the context (and proofs given) it is clear that we meant global sections, not higher direct images.

- Proof of Lem. 2.1.1.1: "$u := u \underset{\tilde{R}}{\otimes} R$" should be only "$u \underset{\tilde{R}}{\otimes} R$".
- Lem. 2.1.1.2: We should assume that Y is smooth over S. (The condition we need is that $\Omega^1_{Y/S}$ is locally free.)
- Cor. 2.1.1.3: We should assume that $Z \to S$ is smooth, instead of only being flat.
- Cor. 2.1.1.4: Once we assume that Y is smooth over S in Lem. 2.1.1.2, we no longer need to assume moreover that Y is flat over S.
- Def. 2.1.2.1: "the isomorphism classes" should be "the set of isomorphism classes".
- Prop. 2.1.2.2, statement 1: "$\mathsf{o}(X; S \hookrightarrow \tilde{S}) \neq 0$" should be "$\mathsf{o}(X; S \hookrightarrow \tilde{S}) = 0$".
- Proof of Prop. 2.1.2.2:
  - Paragraph 2, line 1 after displayed equation 2: "$\xi^{-1}_{\alpha\gamma} = \xi_{\alpha\gamma}$" should be "$\xi^{-1}_{\alpha\gamma} = \xi_{\gamma\alpha}$", and "$c_{\alpha\beta\gamma} \underset{S}{\times} S$" should be "$c_{\alpha\beta\gamma} \underset{\tilde{S}}{\times} S$".
  - Paragraph 3, line 1 after displayed equation -1: "$\mathrm{Aut}_{\tilde{S}}(\tilde{U}_\alpha|_{U_{\alpha\beta},S})$" should be "$\mathrm{Aut}_{\tilde{S}}(\tilde{U}_\alpha|_{U_{\alpha\beta}}, S)$"
  - Paragraph -3, line -2: "1-cochain" should be "1-coboundary".
- Prop. 2.1.3.2, statement 1: "$\mathsf{o}(f; \tilde{X}, \tilde{Y}, S \hookrightarrow \tilde{S}) \neq 0$" should be "$\mathsf{o}(f; \tilde{X}, \tilde{Y}, S \hookrightarrow \tilde{S}) = 0$".
- Prop. 2.1.3.2, statement 3: "$\mathsf{o}(f; \mathsf{m}_{\tilde{X}} + \tilde{X}, \mathsf{m}_{\tilde{Y}} + \tilde{Y}, S \hookrightarrow \tilde{S}) = \mathsf{o}(f; \tilde{X}, \tilde{Y}, S \hookrightarrow \tilde{S}) - df(\mathsf{m}_{\tilde{X}}) + f^*(\mathsf{m}_{\tilde{Y}})$" should be "$\mathsf{o}(f; \mathsf{m}_{\tilde{X}} + \tilde{X}, \mathsf{m}_{\tilde{Y}} + \tilde{Y}, S \hookrightarrow \tilde{S}) = \mathsf{o}(f; \tilde{X}, \tilde{Y}, S \hookrightarrow \tilde{S}) + df(\mathsf{m}_{\tilde{X}}) - f^*(\mathsf{m}_{\tilde{Y}})$".
- Proof of Prop. 2.1.3.2:
  - Paragraph 1, line 4: "By smoothness of f" should be "By smoothness of $\tilde{Y}$".
  - Paragraph 4, displayed equation 2: "$c'_{\alpha\beta} = c_{\alpha\beta} - df(m_{\tilde{X},\alpha\beta}) + f^*(m_{\tilde{Y},\alpha\beta})$" should be "$c'_{\alpha\beta} = c_{\alpha\beta} + df(m_{\tilde{X},\alpha\beta}) - f^*(m_{\tilde{Y},\alpha\beta})$".
- Cor. 2.1.4.4, displayed equation -1: the left-hand side should be "$H^1(X, f^* \underline{\mathrm{Der}}_{X/T} \underset{\mathscr{O}_T}{\otimes} \mathscr{I})$".

- Def. 2.1.5.3: "invertible sheaves $\tilde{\mathcal{L}}$ over $\tilde{X}$ such that $\tilde{\mathcal{L}} \underset{\mathscr{O}_{\tilde{S}}}{\otimes} \mathscr{O}_S \cong \mathcal{L}$ over X" should be "pairs $(\tilde{\mathcal{L}}, \psi)$ such that $\tilde{\mathcal{L}}$ is an invertible sheaf over $\tilde{X}$ and such that $\psi : \tilde{\mathcal{L}} \underset{\mathscr{O}_{\tilde{S}}}{\otimes} \mathscr{O}_S \to \mathcal{L}$ is an isomorphism over X".
- Prop. 2.1.5.4, statement 1: "$\mathsf{o}(\mathcal{L}; \tilde{X}, S \hookrightarrow \tilde{S}) \neq 0$" should be "$\mathsf{o}(\mathcal{L}; \tilde{X}, S \hookrightarrow \tilde{S}) = 0$".
- Proof of Prop. 2.1.5.3, paragraph -1, lines 5–6 after displayed equation 5: "$\xi_{\alpha\beta}^*(\tilde{l}_{\alpha\beta})$ and $(\xi'_{\alpha\beta})^*(\tilde{l}_{\alpha\beta})$ become the same $l_{\alpha\beta}$ modulo $\mathscr{I}$" should be "$\xi_{\alpha\beta}^*(\tilde{l}_{\beta\gamma})$ and $(\xi'_{\alpha\beta})^*(\tilde{l}_{\beta\gamma})$ become the same $l_{\beta\gamma}$ modulo $\mathscr{I}$".
- In Prop. 2.1.5.4 and its proof, "$\underline{\mathrm{Pic}}(X/S) \cong \underline{H}^1(X, \mathscr{O}_X^\times)$" should be "$\mathrm{Pic}(X) \cong H^1(X, \mathscr{O}_X^\times)$".
- Proof of Prop. 2.1.5.4, displayed equation 5: "$(1 + h_{\alpha\beta\gamma})$" should be "$(1 - h_{\alpha\beta\gamma})$".
- Line 1 after (2.1.5.7): "over $U_{\alpha\beta}$" should be "over $U_{\alpha\beta\gamma}$".
- Cor. 2.1.5.10:
  - "Suppose moreover that X is a flat group scheme over S" should be "Suppose X is a smooth group scheme over S".
  - The definition "$\underline{\mathrm{Lie}}_{\underline{\mathrm{Pic}}(X/S)/S} := \underline{\mathrm{Pic}}(X/S)(\mathscr{O}_S[\varepsilon]/(\varepsilon^2))$" in the displayed equation is wrong. The correct definition is in the proof of Cor. 2.1.5.10.
- Cor. 2.1.5.17: "$\underline{\mathrm{Lie}}_{X^\vee/S} \underset{\mathscr{O}_S}{\otimes} \underline{\mathrm{Lie}}_{X/S}$" in the commutative diagram should be "$\underline{\mathrm{Lie}}_{X^\vee/S} \underset{\mathscr{O}_S}{\otimes} \underline{\mathrm{Lie}}_{X^\vee/S}$".
- Sec. 2.1.6, paragraph 2, line 5: "$R^i \pi_* \Omega^\bullet_{U_\alpha/S}$ is trivial for all $i > 0$" should be "$R^i \pi_* \Omega^q_{U_\alpha/S}$ is trivial for all $i > 0$ and all q".
- Sec. 2.1.6, paragraph 3, the indices should start with "α_0" instead of "α_0".
- Proof of Prop. 2.1.6.4, paragraph 2, displayed equation 2: "$x_{\alpha\beta}^{(1,0)} := \tilde{f}_\alpha^*(y_{\alpha\beta}^{(1,0)}) + T_{\alpha\beta}(y_\beta^{(1,0)})$" should be "$x_{\alpha\beta}^{(1,0)} := \tilde{f}_\alpha^*(y_{\alpha\beta}^{(1,0)}) + T_{\alpha\beta}(y_\beta^{(0,1)})$".
- Sec. 2.1.7, paragraph 1, line 8: "defined by $\mathscr{I}^2 = 0$" should be "defined by $\mathscr{I}^2$".

- Paragraph after (2.1.7.2), line 3: "$\underline{\mathrm{Ext}}^1_{\mathscr{O}_S}(\Omega^1_{X/S}, f^*(\Omega^1_{S/\mathsf{U}}))$" should be "$\mathrm{Ext}^1_{\mathscr{O}_X}(\Omega^1_{X/S}, f^*(\Omega^1_{S/\mathsf{U}}))$".
- Sec. 2.2.1, paragraph 2, line 4: "p is ramified in F" should be "p is unramified in F".
- Rem. 2.2.1.5: The last sentence on $\underline{H}$ should be removed.
- Prop. 2.2.2.5: "e" should be "$e_{\tilde{A}}$" or $\tilde{e}$, and $A := \tilde{A} \underset{\tilde{S}}{\times} S$ and e_A should be explicitly defined.
- Proof of Prop. 2.2.2.5,
  - Line 1 after displayed equation 4: "$\tilde{g} \circ j_1 = \mathrm{Id}_{\tilde{A}}$ and $\tilde{g} \circ j_2 = \tilde{e} \circ \tilde{\pi}$ do lift the maps $g \circ j_1$ and $g \circ j_2$" should be "$\mathrm{Id}_{\tilde{A}}$ and $\tilde{e} \circ \tilde{\pi}$ do lift the maps $g \circ j_1 = \mathrm{Id}_A$ and $g \circ j_2 = e \circ \pi$".
  - Line 3 after displayed equation 5: "a (relative) constant sheaf..." should be "canonically isomorphic to the pullback of $\underline{\mathrm{Lie}}_{A_0/S_0} \ldots$". (Several instances of "$\underline{\mathrm{Lie}}_{A_0/S_0}$" in this proof should be "Lie_{A_0/S_0}".)
  - Line 3 of displayed equation 6: "$\mathrm{pr}_2^* \underline{H}^1(A_0, \mathscr{O}_{A_0})$" should be "$\mathrm{pr}_2^* H^1(A_0, \mathscr{O}_{A_0})$".
  - Line 2 after displayed equation 6: "the pullback from one of the two factors $\mathrm{pr}_i^* H^1(A_0 \underset{S_0}{\times} A_0, g_0^*(\underline{\mathrm{Der}}_{A_0/S_0})) \underset{k}{\otimes} I$" should be "a sum of elements of the two factors $[\mathrm{pr}_i^* H^1(A_0, \mathscr{O}_{A_0})] \underset{k}{\otimes} \mathrm{Lie}_{A_0/S_0} \underset{k}{\otimes} I$"
- Proof of Prop. 2.2.3.4, line -3 preceding displayed equation -2: "is a constant sheaf and" should be removed, and the remainder of the paragraph should be modified accordingly.
- Line -3 preceding (2.2.3.6): "by some $[(A_R, f_{0,R})]$" should be "by some $(A_R, f_{0,R})$".
- Displayed equation 2 after (2.2.3.6): "$[(A_{\tilde{R}}, f_{0,\tilde{R}})] \in \mathsf{Def}_{A_0}(q)^{-1}([(A_R, f_{0,R})])$" should be "$[(A_{\tilde{R}}, f_{0,\tilde{R}})] \in \mathsf{Def}_{A_0}(r)^{-1}([(A_R, f_{0,R})])$". At the end of the same paragraph, "$\mathsf{Def}_{A_0}(r)^{-1}([(A_{\tilde{R}}, f_{0,\tilde{R}})])$" should be "$\mathsf{Def}_{A_0}(r)^{-1}([(A_R, f_{0,R})])$".
- Proof of Prop. 2.2.3.7, line 1 after displayed equation 2: "by some $[(A_R, \lambda_R, f_{0,R})]$" should be "by some $(A_R, \lambda_R, f_{0,R})$".
- Displayed equation 1 after (2.2.3.8): "$\mathsf{Def}_{A_0}(p)([(A_{\tilde{R}}, \lambda_{\tilde{R}}, f_{0,\tilde{R}})])$" should be "$\mathsf{Def}_{A_0}(r)([(A_{\tilde{R}}, \lambda_{\tilde{R}}, f_{0,\tilde{R}})])$".

- Proof of Prop. 2.2.3.9, line 1: "$\mathsf{Def}_{(A_0,\lambda_0)}$ is a subfunctor of $\mathsf{Def}_{(A_0,\lambda_0,i_0)}$" should be "$\mathsf{Def}_{(A_0,\lambda_0,i_0)}$ is a subfunctor of $\mathsf{Def}_{(A_0,\lambda_0)}$".
- Proof of Prop. 2.2.4.1:
  - Paragraph 1, line -2: "$c_2 := \{c_{\alpha\beta\gamma} \underset{S}{\times} c_{\alpha'\beta'\gamma'}\}$" should be "$c_2 := \{\mathrm{pr}_1^*(c_{\alpha\beta\gamma}) + \mathrm{pr}_2^*(c_{\alpha'\beta'\gamma'})\}_{\alpha\alpha'\beta\beta'\gamma\gamma'}$".
  - Paragraph 2, line 2: "$j_2(x) = (x,x)$" should be "$j_2(x) = (e,x)$".
  - Displayed equation 4: the second and third instances of "$\underline{\mathrm{Der}}_{A_0 \underset{S_0}{\times} A_0/S_0}$" should be "$\mathrm{Lie}_{A_0 \underset{S_0}{\times} A_0/S_0}$".
  - Displayed equation 5: Some of the parentheses are misplaced or missing.
- Cor. 2.2.4.3, line 2: "smooth algebra" should be "formally smooth algebra".
- Cor. 2.2.4.7, line 2: "smooth algebra" should be "formally smooth algebra".
- Paragraph 1 after Cor. 2.2.4.7, lines 4–5: "$\mathcal{P}_A\ A \underset{S}{\times} A^\vee$" should be "$\mathcal{P}_A$ over $A \underset{S}{\times} A^\vee$".
- (2.2.4.9) is imprecise (might not be true if we also consider the actions of $\mathcal{O}$) and not really needed later. It is removed in the revision.
- Proof of Prop. 2.2.4.11, paragraph 3: "$\mathcal{O}_{F_0,(\square)} \to k$ of finite type" should be "$\mathcal{O}_{F_0,(\square)} \to \Lambda$ whose composition with $\Lambda \to k$ is of finite type". Moreover, we have modified the statements according to the corrections we have made on Lem. 1.2.5.10 and (1.2.5.15) in a revision.
- Rem. 2.2.4.12, line 8: "the normalizer of the maximal isotropic subspace V_0 in $L \underset{\mathbb{Z}}{\otimes} \mathbb{C}$" should be "the normalizer of some maximal isotropic subspace in $L \underset{\mathbb{Z}}{\otimes} \mathbb{C}$ isomorphic to $V_0^\vee$".
- Cor. 2.2.4.15, line 2: "smooth algebra" should be "formally smooth algebra".
- Thm. 2.2.4.16:
  - Line 2: "smooth algebra" should be "formally smooth algebra".
  - Line 3: Reference to Cor. 2.2.4.13 should be to Cor. 2.2.4.15.

- Paragraph preceding Thm. 2.3.1.5, line -3 from the end: "For such a scheme $\mathfrak{X}$" should be "For any such formal scheme $\mathfrak{X}$".
- Section 2.3.2, paragraph 2, line 3: "$R_{A_0}/\mathfrak{m}_R^{i+1}$" should be "$R/\mathfrak{m}_R^{i+1}$".
- Proof of Prop. 2.3.2.1:
  - Line 2: "prorepresented over" should be "prorepresented by".
  - Line 6: Reference to Thm. 2.3.1.2 should be to Thm. 2.3.1.5.
- In order to apply Thm. B.3.12 in Sec. 2.3.3, we need S_0 to have either only one point or infinitely many points. In other words, we need the set $\Box$ to be either empty or infinite. When $\Box$ is nonempty but finite, one way to fix this is to use the fact that there is a representable open immersion from the moduli problem to the pullback of another moduli problem defined using some infinite $\Box'$. This is fixed in a revision.
- Rem. 2.3.3.1: This empty remark should be removed. (All subsequent numberings in the same sections are then adjusted in a revision.)
- Proof of Prop. 2.3.4.2, paragraph 2, line 1: "restricting the map to local rings of points of finite type" should be "replacing S with completions of any étale presentation of $\mathsf{M}_{\mathcal{H}}$ at points of finite type". To be precise, after replacing S we also need to replace $\Omega^1_{S/\mathsf{S}_0}$ with its completion.
- Def. 3.1.1.5, lines 6–7: The statement "(which is defined locally ...)" should be removed.
- Thm. 3.1.2.5, line 1 after the displayed equation: "all four" should be "all three". Similarly, other numbers in the theorem should be reduced by one.
- In the paragraph preceding Example 3.1.3.6, the statement that $\mathcal{M}$ is an open subscheme of $\overline{\mathcal{M}}$ require more conditions. To avoid introducing the theory of toroidal embeddings here, we have removed this statement in a revision.
- Paragraph 1 after Assumption 3.1.2.7, line 3: "(necessarily unique)" should be removed.
- Paragraph 1 after Convention 3.1.2.9, line 1 after displayed equation 1: "H is commutative" should be "H is of multiplicative type".
- Displayed equation 2 after Convention 3.1.2.9: "$\chi \in \underline{X}(H) \cong \mathbb{Z}$" should be "$\chi \in \underline{X}(H)$".

- Proof of 3.1.4.1: All instances of "$\mathscr{O}_G$" should be "$\mathscr{O}_{\mathcal{M}}$".
- Line 1 after Cor. 3.1.4.4: "unique" should be removed.
- Lem. 3.2.2.12: We need the assumption that Z and W are geometrically reduced. (It is mentioned in [26] but not explicitly in [59, VII, 4.1], where the result is stated without proof.)
- Prop. 3.2.3.1, line -1: "identity section of A" should be "identity section of G".
- Prop. 3.2.4.2 and the paragraph preceding it: All instances of "ξ" should be "$\tilde{a}$", and "$x \in K(\mathcal{L})$" should be "$a \in K(\mathcal{L})$".
- Line -3 preceding Prop. 3.2.5.3: "*connect fibers*" should be "*connected geometric fibers*".
- Prop. 3.2.5.3: "all torus" should be "each torus".
- Rem. 3.3.1.5: "$T[n]_s$", "$G[n]_s$", and "$A[n]_s$" should be "$T[n]_s$", "$G[n]_s$", and "$A[n]_s$", respectively.
- Rem. 3.3.1.6: The explanation for the extendability of $G[n]_s$ to a subscheme of $G[n]_U$ is misleading. It is corrected in a revision.
- Thm. 3.3.2.4, paragraph 2, line 9: "$\mathcal{D}(\mathcal{L})$" should be "$\mathcal{D}_2(\mathcal{L})$".
- Proof of Thm. 3.3.2.4: the reference "[59, X]" should be "[59, IX]".
- Sec. 3.3.3, line 5: "projective system" should be "inductive system".
- Cor. 3.3.3.4, 3., line 1: "all torus" should be "all tori".
- In Prop. 3.3.3.11:
  - (3.3.3.12): "$\mathrm{CUB}_{S_{\mathrm{for}}}(G^\natural, \mathbf{G}_{\mathrm{m},S_{\mathrm{for}}})$" should be "$\mathrm{CUB}_S(G^\natural, \mathbf{G}_{\mathrm{m},S})$".
  - Line 3 after (3.3.3.12): "all torus" should be "all tori".
- Paragraph after Prop. 3.3.3.11, line -1: "descent to A_{for}" should be more precisely "descent to A_{for} over some finite étale extension of S". Since this is still not the precise argument (and might possibly suggest a wrong argument), we removed this sentence in the revision.
- Paragraph 2 after Lem. 3.4.1.8:
  - Line 2: "$H^{\mu}_{U/S}$" should be "H^{μ}_{U}".
  - Line 1 after the commutative diagram: "locally Henselian" should be "Henselian local".

- Thm. 3.4.2.6, end of 4.: "$\mathrm{e}_S^{\mathcal{L}_\eta}$ induces a perfect duality between K_S^μ and K_S/K_S^{f}" should be simply "K_S^μ is trivial".
- Rem. 3.4.4.3 should be stated over the generic point η of S.
- Def. 4.2.1.1: The invertible sheaf $\mathcal{L}$ should be ample, not just over η (as in Faltings-Chai).
- Paragraph 2 after Def. 4.2.1.1, line -2: "$\underline{\mathrm{Pic}}_e(A/S)$" should be "$\underline{\mathrm{Pic}}_e^0(A/S)$".
- Paragraph 3 after Def. 4.2.1.1, line -2: "of $G^\natural$ defined by the dual $G^\vee$" should be "of the dual $G^\vee$".
- Lem. 4.2.1.3: It is more precise to denote "$\tau \circ (\mathrm{Id}_{\underline{Y}} \times \phi)$" by "$(\mathrm{Id}_{\underline{Y}} \times \phi)^*\tau$". (Similar instances should be corrected accordingly.)
- Def. 4.2.1.4, line 5: "$(c^\vee(y) \times c\phi(y))^*\mathcal{P}_A^{\otimes -1} \to \mathscr{O}_S$" should be "$(c^\vee(y) \times c\phi(y))^*\mathcal{P}_A \to \mathscr{O}_S$".
- Def. 4.2.1.7:
  - It should be explained that τ^{-1} is the tensor inverse of τ. This is clarified in a revision.
  - In 6., "$(\underline{X} \underset{S}{\times} \underline{Y})_\eta$" should be "$(\underline{Y} \underset{S}{\times} \underline{X})_\eta$".
- Thm. 4.2.1.8: The claim that $\mathrm{F}_{\mathrm{ample}}$ detects isomorphisms should be postponed until Thm. 4.5.4.12 is proved. (This is an issue inherited from Faltings-Chai.) See the corrections to Rem. 4.2.12 and Lem. 4.3.1.15 below.
- Rem. 4.2.1.10 and 4.2.1.11: The independence of ι and τ on the choice of polarizations is now supplied in the revision, in the new Section 4.5.5.
- Rem. 4.2.1.12 should be removed (or postponed). See the correction for Lem. 4.3.1.15 below.
- Sec. 4.2.2, displayed equation 7: "$\mathscr{O}_X$" should be "$\mathscr{O}_\chi$".
- Sec. 4.2.2, paragraph -1, line -2: "$\tau : \mathbf{1}_{\underline{Y} \underset{S}{\times} \underline{X},\eta} \xrightarrow{\sim} (c^\vee \times c)^*\mathcal{P}_{A,\eta}$" should be "$\tau : \mathbf{1}_{\underline{Y} \underset{S}{\times} \underline{X},\eta} \xrightarrow{\sim} (c^\vee \times c)^*\mathcal{P}_{A,\eta}^{\otimes -1}$".
- Paragraph -3 preceding Lem. 4.2.3.1, line -1: "$\mathcal{D}_3(\mathrm{Id}_\chi)$" should be "$\mathcal{D}_3(\mathrm{Id}_{\chi_0})$".
- Paragraph 1 after Lem. 4.2.4.3:

- Lines 1–2: "such that $I_v := I \underset{R}{\otimes} R_v \subsetneq R_v$" should be "such that $R \subset R_v$ and $I_v := I \underset{R}{\otimes} R_v \subsetneq R_v$".
- Line 3: "that has center on" should be "that are nonnegative on R and has center on".

- Sec. 4.3.1: all instances of "$\varinjlim_i$" should be "$\varprojlim_i$".
- Sec. 4.3.1, line 1 after displayed equation 2: "$\underline{Y} = \underline{\mathbf{X}}(T)$" should be "$\underline{Y} = \underline{\mathbf{X}}(T^\vee)$".
- Paragraph -2 preceding Rem. 4.3.1.1, ampleness of $\mathcal{L}_\eta$ and $\mathcal{M}_{\chi,\eta}$ are irrelevant. The relations $\Gamma(G_\eta, \mathcal{L}_\eta) \cong \Gamma(G, \mathcal{L}) \underset{R}{\otimes} K$ and $\Gamma(A_\eta, \mathcal{M}_{\chi,\eta}) \cong \Gamma(A, \mathcal{M}_\chi) \underset{R}{\otimes} K$ are true because $R \hookrightarrow K$ is flat.
- Rem. 4.3.1.7, line 4: "$g : S \to G^\natural$" should be "$g : S \to G$".
- Proof of Prop. 4.3.1.8, displayed equation 3 should be "$\psi(y_1 + y_2) = \psi(y_1)\psi(y_2)\tau(y_1, \phi(y_2))$".
- Lem. 4.3.1.15 is placed at the wrong place. The proof of Lem. 4.3.1.15 uses Cor. 4.3.4.2 and the proof of Thm. 4.5.4.12. One can check that there is no circulation of logic after all, but the wrong location and lack of proper reference are mistakes. This is relocated to Sec. 4.5.4 in a revision.
- Sec. 4.3.2, paragraph 1, the following simplifying assumption should be added (in order to quote Thm. 3.4.2.6): "Since (4.3.1.4) is about equalities, we may localize and make the convenient assumption that R is complete local."
- Paragraph 2 after the paragraph containing (4.3.2.4), lines 2–3: "Since $K(\mathcal{L})^\mu$ is commutative" should be "Since $K(\mathcal{L})^\mu$ is of multiplicative type".
- The comma after the diagram preceding Lem. 4.3.2.6 should be a period.
- Proof of Lem. 4.3.2.7: The commutative diagram should be numbered, and the reference "(4.3.2)" in the last line should be pointed to this diagram instead.
- Proof of Lem. 4.3.2.10 used the fact that $\Gamma(G_\eta, \mathcal{L}_\eta)_{\bar{\chi}} \cong \Gamma((\lambda_A^* G^{\vee,\natural})_\eta, (\mathcal{L}_{\bar{\chi}}^\natural)_\eta) \neq 0$, but was not stated explicitly.

- Sec. 4.3.3, paragraph 1: Last sentence should be omitted because the proof of Thm. 4.2.1.8 will be finished in Sec. 4.3.4, not Sec. 4.3.3.
- Sec. 4.3.3, paragraph 2:
  - Line 1: "the points in" is redundant.
  - Line 3: "adic injection" should be "continuous injection".
  - Line 5: "See also ..." should be removed. See the corrections for Proof of Prop. 4.5.2.18 and Rem. 4.5.2.19 below.
- Lem. 4.3.3.2:
  - Paragraph 1, line 4: "of H" should be "of H (resp. H')".
  - Diagram, second row: "π" should be placed between $H^\natural$ and A.
  - Paragraph 2, line 2: "which a cubical" should be "which determines a cubical".
- Lem. 4.3.3.6: In the proof it is the point $\mathcal{L}_\eta \underset{\mathscr{O}_{G_\eta}}{\otimes} [-1]^*\mathcal{L}_\eta^{\otimes -1}$ of $G_\eta^\vee$ that should be divided by 2. For the paragraph that follows, the lemma is not precise enough: By translation by torsion points, we can and we should assert that $(\mathcal{L}')^\natural$ descends to some symmetric $\mathcal{M}'$.
- Paragraph 1 after proof of Lem. 4.3.3.6, line 3: "$\mathcal{L} \cong \pi^*\mathcal{M}$" should be "$\mathcal{L}^\natural \cong \pi^*\mathcal{M}$".
- Proof of (4.3.1.11): The notations "$-$" representing empty slots should be "$\cdot$". (In fact, they are unnecessary.)
- In Lem. 4.3.4.1, it is more appropriate to refer to Cor. 4.3.1.13, because the proof of Thm. 4.2.1.8 is not finished yet. Moreover, the proof is incomplete for the cases $\tau = \tau_1$ or $\tau = \tau_2$. We removed these cases because we will only need the case $\tau_1 = \tau_2$.
- Proof of Lem. 4.3.4.1: First paragraph after the displayed equation, line 5: The notations "$-$" representing empty slots should be "$\cdot$".
- Proof of Lem. 4.3.4.3:
  - Line 5: "$f_{Y^\vee}$" should be "$f_{T^\vee}$".
  - Equation 3: "$\psi_2(y)$" should be "$\psi_2(f_Y(y))$".
  - Equation 4: "$\sigma^{\mathcal{M}_1}_{f_X(\chi)+\phi_2(y)}$" should be "$\sigma^{\mathcal{M}_1}_{f_X(\chi)+\phi_1(y)}$".

- Paragraph 1 after Def. 4.4.1, line 1: “with $\mathcal{L}_\eta$ ample over G_η” is incorrect. The correct statement is that $\mathcal{L}$ is ample over the whole G. Also, it should be emphasized that we have used not just the normality of S, but also its noetherian property, when citing Raynaud's result.
- Rem. 4.4.7 is better clarified in a revision.
- Rem. 4.4.8 is false: To allow the polarization to be doubled, the datum ϕ has to be doubled as well. This is corrected in a revision. (This is also incorrectly stated in Faltings-Chai.)
- Proof of Lem. 4.4.16, paragraph 2, line 3: “$\mathcal{F}^\natural \cong \mathcal{L}_1^\natural \underset{\mathscr{O}_A}{\otimes} \mathcal{L}_2^\natural$” should be “$\mathcal{F}^\natural \cong \mathcal{L}_1^\natural \underset{\mathscr{O}_A}{\otimes} (\mathcal{L}_2^\natural)^{\otimes -1}$”.
- Def. 4.5.1.2
  - (4), line 4: “to sections of $P^\natural$” should be “a section of $P^\natural$”.
  - (iii), line 4–5: “S-valued point $S \to A$” should be “S_v-valued point $S_v \to A$”, where $S_v = \mathrm{Spec}(R_v)$ and where R_v is the valuating ring of v. To avoid confusion, we should denote this S_v-valued point as $\tilde{x}_v$, not just x_v.
- Rem. 4.5.1.4: The reference to “[47, Prop. 3.3]” should be “[47, Ch. III, Prop. 3.3]”. Also, “connected components” should be ”irreducible components”.
- Construction 4.5.1.5: The “$\sum\limits_{n\geq 0}$” in the definition of $\mathcal{S}_1$ should be “$\bigoplus\limits_{n\geq 0}$”. The action $\tilde{S}_y$ of Y on $\mathcal{S}_2$ should be on $\mathcal{S}_{2,\eta}$.
- Proof of Lem. 4.5.1.7, displayed equation -1: “z” should be “α”.
- Proof of Lem. 4.5.1.9:
  - Paragraph 3, line 3 after displayed equation 1: “$y \in Y \underset{\mathbb{Z}}{\otimes} \mathbb{R}$” should be “$y \in Y \underset{\mathbb{Z}}{\otimes} \mathbb{Q}$”.
  - Paragraph -1, line -1: “n_j” should be “n_{y_j}”.
- Prop. 4.5.1.11: Remove the word “affine”.
- Proof of Prop. 4.5.1.14:

  - Paragraph 2: The old completeness condition in Mumford and Faltings-Chai is stated accidentally. The second sentence of this paragraph should be replaced by the new condition in the revision.
  - Paragraph 3, displayed equation: "R" should be "R_v". (See Def. 4.5.1.2 above.)

- Rem. 4.5.1.15: All instances of "transforms" should be "translations".
- Proof of Lem. 4.5.2.5, line 5: "dominating" should be "dominant".
- Proof of Prop. 4.5.2.6, line -4: "> 0" should be "≥ 0".
- Proof of Prop. 4.5.2.8, line 3: "$S_y \circ T_t = T_t \circ S$" should be "$S_y \circ T_t = T_t \circ S_y$".
- In proof of Prop. 4.5.2.10, the notations of R_v and $\tilde{R}_v$ are confusing and changed to $R_{\bar{v}}$ and R_v, respectively. Other notations are changed accordingly. The claim that both x and $G_0^\natural$ lie in the image of $(P_{\bar{v}}^\natural)'_0 \to P_0^\natural$ is imprecise (and inherited from Mumford's original article). The proof have been revised.
- Lem. 4.5.2.11, "$-nv'(\varpi) \geq v'(x_{v'}^*(\mathscr{O}_\chi)) \geq nv'(\varpi)$" should be "$-nv'(\varpi) \leq v'(x_{v'}^*(\mathscr{O}_\chi)) \leq nv'(\varpi)$".
- Proof of Lem. 4.5.2.11, paragraph 1, line -2: "$P_{v'}^\natural$" should be "$P_v^\natural$".
- Rem. 4.5.2.12 is misleading and should be removed.
- Proof of Lem. 4.5.2.13, paragraph -2, line -5: "Zariski's connected theorem" should be "Zariski's connectedness theorem".
- Construction 4.5.2.15:
  - Reference to Mumford's paper should be to p. 253.
  - Step 1, line 3: "n" should be "i".
  - Step 3, line 2: "of C" should be "of P".
  - Step 3: The last sentence should be "the formal completion G_{for} of G is canonically isomorphic to $G_{\mathrm{for}}^\natural$".

- Prop. 4.5.2.16, condition 2 (and later in the proof): "subschemes $C_1 \subset p^{-1}(C_2)$" should be "formal subschemes $C_{1,\mathrm{for}} \subset p^{-1}(C_{2,\mathrm{for}})$".
- Proof of Prop. 4.5.2.16: To be clear, "$\mathscr{P}^i$" should be "$\mathrm{Gr}^i\,\mathscr{P}$". In statement 1 in the proof, "$\mathscr{O}_S[1/g]$" should be "$\mathscr{O}_{P_1,x}[1/h]$".

- Proof of Prop. 4.5.2.18, line 2: "that S is Nagata" should be more precisely "that S is excellent, and in particular Nagata". We replaced this with the simpler phrase "that S is excellent" in a revision.
- Rem. 4.5.2.19, line 3: The sentence "Moreover, ..." should be removed.
- Lem. 4.5.3.3, line 3: "integral" should be "integrable".
- Paragraph 2 after Lem. 4.5.3.3, step 1, line 2: "as $G_1^\natural$ is" should be "as $G_{1,\eta}^\natural$ is".
- Prop. 4.5.3.4: "$(W_1^\natural \underset{S}{\times} S_0) \cap S_y(U_0) \neq \emptyset$" should be "$(W_1^\natural \underset{S}{\times} S_0) \cap S_y(U_0) = \emptyset$".
- Thm. 4.5.3.5 and its proof: "$\iota_2 \circ h = f \circ \iota_1$" should be "$\iota_2 \circ h = f^\natural \circ \iota_1$".
- Proof of Cor. 4.5.3.6, line -2: "$(G_1, \mathcal{L}_1) \xrightarrow{\sim} (G_2, \mathcal{L}_2)$" should be "$G_1 \xrightarrow{\sim} G_2$".
- Proof of Thm. 4.5.3.9, paragraph -1, line -4: "the part of the torus $G_0^\natural$" should be "the semi-abelian scheme $G_0^\natural$".
- Sec. 4.5.4, paragraph 1:
  - Line 2: "$\mathrm{DD}_{\mathrm{ample}}^{\mathrm{split},*}$" should be "$\mathrm{DD}_{\mathrm{ample}}^{\mathrm{split}}$".
  - The claim that "$\mathrm{F}_{\mathrm{ample}}$ sends $(G, \mathcal{L})$ to ..." should be removed.
  - Line 3 after displayed equation 1: "$\mathrm{M}_{\mathrm{ample}}^{*}$" should be "$\mathrm{M}_{\mathrm{ample}}^{\mathrm{split},*}$".
- Def. 4.5.4.1, line 1: "subcategory" should be "full subcategory".
- Prop. 4.5.4.2: The claim that "$\mathrm{F}_{\mathrm{ample}}$ sends $(G, \mathcal{L})$ to ..." should be removed.
- Construction 4.5.4.11: The verification of the formula for $\mathcal{D}_2(\mathcal{P}_A)$ has some obvious typos.
- Proof of Thm. 4.5.4.12:
  - All instances of "$y^\vee$" should be "$c^\vee(y)$".
  - Paragraph 1, line 1: "$\chi(\mathcal{N}_\eta) = \deg(f_A)$" should be "$\chi(\mathcal{N}_\eta)^2 = \deg(f_A)$".
  - Paragraph 2:
    * Line 4: "$\Gamma(G_{\mathrm{for}}, \mathscr{O}_{G,\mathrm{for}}) = \hat{\bigoplus}_{\chi \in X} \Gamma(A, \mathscr{O}_\chi)$" should be "$\Gamma(G_{\mathrm{for}}, \mathcal{L}_{\mathrm{for}}^\natural) = \hat{\bigoplus}_{\chi \in X} \Gamma(A, \mathcal{M} \underset{\mathscr{O}_A}{\otimes} \mathscr{O}_\chi)$".

    * Line 5: "$\sigma_\chi(s) \in \Gamma(A, \mathscr{O}_\chi)$" should be "$\sigma_\chi(s) \in \Gamma(A, \mathcal{M} \underset{\mathscr{O}_A}{\otimes} \mathscr{O}_\chi)$".
    * Near the end, "we may identify $\Gamma(G_\eta, \mathcal{L}_\eta)$ with the K-subspace V of" should be "we may identify $\Gamma(G_\eta, \mathcal{L}_\eta)$ as a K-subspace of the K-subspace V of". We have rewritten this sentence in a revision to make it easier to understand.
  - Paragraph 3, line 5: "G-invariant" should be "$G^\natural$-invariant".
  - Paragraph -1, line -1: "ϕ" should be "f_Y", and "$\chi(\mathcal{L}_\eta^\natural)$" should be "$\chi(\mathcal{L}_\eta)$".

- Proof of Cor. 4.5.4.15: One should reduce to the case for "$\mathrm{DD}^{\mathrm{split},*}_{\mathrm{ample}}$", not "$\mathrm{DD}^{*}_{\mathrm{ample}}$", and one should refer to the proof of Thm. 4.5.4.12, not to Prop. 4.5.4.2.
- Lem. 4.5.4.17: We mean, λ_η is a polarization.
- A new section Sec. 4.5.5 on independence of ι and τ on the choice of polarizations is added. (This is needed to fix Sec. 5.1.1.)
- A new section Sec. 4.5.6 on two-step constructions is added for better explanation of a some later step.
- Most of Sec. 4.6 was written with the implicit assumption that S is affine, which unfortunately was not stated. This does not affect the conclusion at the end because the local results globalizes. The reader should refer to the revision for an improved exposition.
- Proof of Prop. 4.6.1.1, line -1: "Proposition 2.1.3.2" should be "Proposition 2.1.2.2".
- Def. 4.6.2.1, line 5: "on Z" should be "on H".
- Paragraph 2 after Def. 4.6.2.7, line 2: "$(S_1 \hookrightarrow S)^*\mathscr{O}_{S_1}$" should be "$(S_1 \hookrightarrow S)_*\mathscr{O}_{S_1}$".
- Def. 4.6.2.8 and the preceding paragraphs: "$\Omega^1_{S/\mathsf{U}}$" should be replaced with the image of the canonical morphism "$\Omega^1_{S/\mathsf{U}} \to (S_1 \hookrightarrow S)_*\Omega^1_{S_1/\mathsf{U}}$".
- Line -4 preceding Prop. 4.6.2.9: "$(\Omega^1_{S/\mathsf{U}}[d\log\infty])//\Omega^1_{S/\mathsf{U}}$" should be "$(\Omega^1_{S/\mathsf{U}}[d\log\infty])/\Omega^1_{S/\mathsf{U}}$".
- In Sec. 4.6.3, all instances of $\Omega^1_{S/\mathsf{S}_0}$ over $S = \mathrm{Spec}(R)$ should be replaced with $\widehat{\Omega}^1_{S/\mathsf{S}_0}$, the completion of $\Omega^1_{S/\mathsf{S}_0}$ with respect to the ideal of

definition of R. (This is a convention we inherited from Faltings-Chai, but we decided that it is more clear to spell out the difference.)

- Line 2 after (4.6.3.18): The T-action does not commute with Y-action, and we do not need this. We removed this in the revision.
- Line 1 after Lem. 5.1.1.1: "an $\mathbb{Z}$-algebra" should be "a $\mathbb{Z}$-algebra".
- Description of the data on the tuple $(A, \lambda_A, \underline{X}, \underline{Y}, \phi, c, c^\vee, \tau)$ following Lem. 5.1.1.1:
  - Part 2, paragraph 2, the compatibility "${}^t i_{\underline{X}}^{\mathrm{op}}(b) = i_T(b^\star)$ (resp. ${}^t i_{\underline{Y}}(b) = i_{T^\vee}^{\mathrm{op}}(b^\star)$)" should be "${}^t i_{\underline{X}}^{\mathrm{op}}(b) = i_T(b)$ (resp. ${}^t i_{\underline{Y}}(b) = i_{T^\vee}^{\mathrm{op}}(b)$)". The clause "and the natural anti-isomorphism $\mathcal{O} \to \mathcal{O}^{\mathrm{op}} : b \mapsto b^\star$." should be removed from the sentence. (The paragraph should also be combined with the next paragraph.)
  - Part 2, paragraphs 2 and 3: All instances of "X" and "Y" should be "$\underline{X}$" and "$\underline{Y}$", respectively.
  - Part 2, paragraph 3: "ring morphism" should be "ring homomorphism".
  - Part 4 uses implicitly Lem. 4.3.4.3 and the fact that τ does not depend on the choice of polarization (mentioned in Rem. 4.2.1.11). (Then Lem. 5.1.1.1 can be moved to a later section, because $\mathrm{M} : \mathrm{DD} \to \mathrm{DEG}$ is now an equivalence of categories.)
- Lem. 5.1.1.3: The terminology of the "underlying groups X and Y of the étale sheaves $\underline{X}$ and $\underline{Y}$" might be confusing, and should better be replaced with the respective values of $\underline{X}$ and $\underline{Y}$ over a finite étale covering of S trivializing them.
- Def. 5.1.1.5, part 2: Should replace the first sentence with the following: "The étale sheaves $\underline{X}$ and $\underline{Y}$ are equipped with ring homomorphisms $i_{\underline{X}} : \mathcal{O} \to \mathrm{End}_S(\underline{X})$ and $i_{\underline{Y}} : \mathcal{O} \to \mathrm{End}_S(\underline{Y})$, respectively, making them étale sheaves of $\mathcal{O}$-lattices of the same $\mathcal{O}$-multirank (see Definition 1.2.1.11)."
- Sec. 5.1.2, paragraph 1, line 1: "Section 1.2.1" should be "Section 1.2.5".
- Prop. 5.1.2.1 and 5.1.2.2 and their proofs have many inaccuracies and have to be corrected due to the mistake mentioned in (1.1.2.6) above.

They have been rewritten in the revision. The original statements can be corrected if we make the following changes (and correct many other typos):

- Replace the $\mathcal{O}_\mathbb{R}$-multi-rank $(r_{[\tau]})$ of W by the $\mathcal{O}_\mathbb{C}$-multi-rank (r_τ) of $W \underset{\mathbb{R}}{\otimes} \mathbb{C}$, and replace all instances of $r_{[\tau]}$ (resp. modules over $\mathcal{O} \underset{\mathbb{Z}}{\otimes} \mathbb{R}_{[\tau]}$) by r_τ (resp. modules over $\mathcal{O} \underset{\mathbb{Z}}{\otimes} \mathbb{C}$).
- Use modules $W_{[\tau]_c} := W_\tau$ if τ is real, and $W_{[\tau]_c} := W_\tau \oplus W_{\tau \circ c}$ if τ is complex, to replace $W_{[\tau]}$.
- In Prop. 5.1.2.2, paragraph 1, line 5, and similar occurrences in the remaining exposition, it seems better to use "$\mathrm{Hom}_\mathbb{R}(X \underset{\mathbb{Z}}{\otimes} \mathbb{R}, \mathbb{R}(1)) \hookrightarrow L \underset{\mathbb{Z}}{\otimes} \mathbb{R}$" instead of "$\mathrm{Hom}_\mathbb{R}(X \underset{\mathbb{Z}}{\otimes} \mathbb{R}, \mathbb{R}) \hookrightarrow L \underset{\mathbb{Z}}{\otimes} \mathbb{R}$". (This change of notations is incorporated in a revision.)
- In Prop. 5.1.2.2, paragraph 1, lines 5–6, and similar occurrences in the remaining exposition, "X is the underlying $\mathcal{O}$-lattice of $\underline{X}$" should better be "X is the $\mathcal{O}$-lattice given by the value of $\underline{X}$ over some geometric point over $\underline{X}$".
- Emphasize that $(\mathrm{Gr}^{\mathrm{Z}}_{-1,\mathbb{R}}, \langle \cdot, \cdot \rangle_{11,\mathbb{R}}, h_{-1})$ has the same reflex field F_0 as $(L \underset{\mathbb{Z}}{\otimes} \mathbb{R}, \langle \cdot, \cdot \rangle, h)$ does (which is implicit in the proof).
- In proof of Prop. 5.1.2.2:
    * Paragraph 3, line 1: "$\mathcal{O}$-anti-linear" should be "$\mathcal{O}$-linear".
    * Paragraph 3, lines 2–5: It is incorrect (and unnecessary) to mention $F = F^+$ or not, and to mention the embedding $\tau : F \hookrightarrow \mathbb{C}$, because F is not assumed to be a field.
    * Paragraph 3, line 3: "on $\mathrm{Lie}_{G^\vee_\eta/\eta}$" should be "on $\mathrm{Lie}^\vee_{G^\vee_\eta/\eta}$".
    * Paragraph 4: Should work with the constant values X and Y of $\underline{X}$ and $\underline{Y}$, respectively, over the geometric point $\bar{\eta} := \mathrm{Spec}(K^{\mathrm{sep}})$ over $\eta = \mathrm{Spec}(K)$.
    * Paragraph 4, line -4 from the end: "$\mathcal{O} \underset{\mathbb{Z}}{\otimes} k(\eta)$-submodules" should be more precisely "$\mathcal{O} \underset{\mathbb{Z}}{\otimes} k(\eta)$-subquotients".
    * Modules over $\mathcal{O} \underset{\mathbb{Z}}{\otimes} k(\eta)$ should be replaced by modules over $\mathcal{O} \underset{\mathbb{Z}}{\otimes} k(\eta)^{\mathrm{sep}}$ so that signatures are indeed defined.

* $*$ Definition of h_{-1} on $(\mathrm{Gr}^{\mathtt{Z}}_{-1,\mathbb{R}}, \langle\,\cdot\,,\,\cdot\,\rangle_{11,\mathbb{R}})$ should be supplied.

- Sec. 5.2.1, paragraph 2, line 6: "could working with" should be "could work with".
- Proof of Lem. 5.2.2.3:
  - Line 5: "$\mathtt{Z}_{-2}$ and $\mathtt{Z}_0$" should be "$\mathrm{Gr}^{\mathtt{Z}}_{-2}$ and $\mathrm{Gr}^{\mathtt{Z}}_0$".
  - Lines 6 and -2: "$\mathtt{Z}_{-1}$ should be "$\mathrm{Gr}^{\mathtt{Z}}_{-1}$.
- One important byproduct of the proof of Lem. 5.2.2.4 should be pointed out in the exposition: Under the technical assumption that the PEL-type $\mathcal{O}$-lattice $(L, \langle\,\cdot\,,\,\cdot\,\rangle)$ satisfies Condition 1.4.3.9, the $\mathcal{O}$-actions on T, A, and Y all extend to some maximal order $\mathcal{O}'$ in B containing $\mathcal{O}$, compatible with $c : X \to A^\vee$ and $c^\vee : Y \to A$. We included this as a corollary to Lem. 5.2.2.4 in a revision. As a consequence, the possible lattices X and Y we can consider in Def. 5.2.7.9 and all dependent definitions later must allow the extendability of actions of $\mathcal{O}$ to some maximal order. This is automatically implied by Condition 1.4.3.9 and the existence of δ, but it is far from obvious for the readers to notice it. We have added remarks in later sections.
- Proof of Lem. 5.2.2.4:
  - Paragraph 2, line 4, the notation $\sqrt{I' \cdot R'}$ should be replaced with $\mathrm{rad}(I' \cdot R')$, for consistency of notation.
  - Paragraph 2, lines 5–7: The construction of $G^\vee$ using Lem. 3.4.3.3 is incorrect, and in fact unnecessary. Since the action of $\mathcal{O}$ on L extends to an action of $\mathcal{O}'$ on L, for some $\eta' = \mathrm{Spec}(K')$ as in the beginning of this paragraph, $\mathcal{O}'$ induces endomorphisms of $G_{\eta'}$. Hence it suffices to quote Prop. 3.3.1.5 to show that $\mathcal{O}'$ also define endomorphisms on $(G')^\natural := G^\natural \underset{S}{\times} S'$ and $T' := T \underset{S}{\times} S'$. (The details of these are supplied in a revision.)
- Paragraph -1 before Rem. 5.2.2.7, line -2 before expression for δ_n, "$i \leq 0$" should be "$-i \leq 0$".
- Def. 5.2.2.8, paragraph 2, line 3: "$\mathtt{Z}$ and $\mathtt{Z}$" should be "$\mathtt{Z}$ and $\mathtt{Z}'$".
- In the proof of Lem. 5.2.3.2, we asserted that the schematic closure $K_{\tilde{S}}$ of $K_{\tilde{\eta}}$ in $G_{\tilde{S}}$, which require some further justification. For simplicity,

we have rearranged the logical order in the argument, and introduced a simplifying assumption on $\tilde{S}$, so that the proof no longer relies on the flatness assertion.

- Def. 5.2.3.4: "étale locally over $\tilde{\eta}$" should be "over $\bar{\eta}$". Rem. 5.2.3.5 is unnecessary and should be removed.
- Def. 5.2.3.6: Should remove "with underlying $\mathcal{O}$-module N a finitely generated $\mathcal{O}$-module" because it is confusing and never used.
- Prop. 5.2.3.8, 4., paragraph 2, line 1: The terminology "(fiber-wise) identity component" (and the references given there) is not false, but could be misleading. What seems more clear is the "fiber-wise geometric identity component". More precisely, the proper smooth group scheme $\underline{\mathrm{Hom}}_{\mathcal{O}}(N, Z)$ is the extension of a finite étale group scheme E by an abelian scheme. We call this abelian scheme the "fiber-wise geometric identity component" for obvious reasons. We have made necessary corrections and modifications for this in a revision. We also introduce the notion of "group schemes of fiber-wise geometric components" and identify E as such a group scheme.
- Proof of Prop. 5.2.3.8:
  - Paragraph 3: Also, the argument counting the ranks (not dimensions) of the fibers of $\underline{\mathrm{Hom}}_{\mathcal{O}}(N, Z)$ is insufficient and misleading because the base might not be reduced. It suffices to argue that over an étale extension that $\underline{\mathrm{Hom}}(N, Z[m])$ becomes a constant group scheme, the compatibility with $\mathcal{O}$ is an open and closed condition.
  - Paragraph 4, lines -3 and -2: All instances of "$\underline{\mathrm{Hom}}(N, Z)$" should be "$\underline{\mathrm{Hom}}_{\mathcal{O}}(N, Z)$".
  - Paragraph 6: It is necessary to pass to an isogeny $Z \to Z'$. This makes the diagram (and related arguments) obsolete. The details have been supplied in a revision.
  - Paragraph -1, line -2: "$\underline{\mathrm{Hom}}_{\mathcal{O}}(L/N, Z)$" should be "$\underline{\mathrm{Hom}}_{\mathcal{O}}(N/L', Z)$".
- Line -1 before (5.2.3.9): The notation "$-$" representing empty slot should be "$\cdot$".

- Cor. 5.2.3.12, line 2: "$(\hat{c}, \hat{c}^\vee, \hat{\tau}) = \{(c_n, c_n^\vee, \tau_n)\}_{n|m, \square\nmid m}$ of $(c, c^\vee, \tau)$" should be "$(\hat{c}, \hat{c}^\vee, \hat{\tau}) = \{(c_m, c_m^\vee, \tau_m)\}_{n|m, \square\nmid m}$ of $(c, c^\vee, \tau)$".
- Prop. 5.2.4.8, line 2: "$a, a' \in K$" should be "$a \in A[n]$ and $a' \in \times K(\mathcal{M}^{\otimes n})$".
- Sec. 5.2.5, paragraph 3: The simplifying assumption that c_n extends to S can be confusing. It is removed in a revision.
- Sec. 5.2.5, displayed equation 3: "$[n]_A^*(\underset{\chi\in X}{\oplus} \mathscr{O}_\chi|_{A[n]}) \hookrightarrow \underset{\chi\in X}{\oplus} \mathscr{O}_\chi|_{A[n]}$" should be "$[n]_A^*(\underset{\chi\in X}{\oplus} \mathscr{O}_\chi)|_{A[n]} \hookrightarrow \underset{\chi\in X}{\oplus} \mathscr{O}_\chi|_{A[n]}$".
- Sec. 5.2.5, displayed equation 4: "$\underset{\chi\in X}{\oplus} [n]_A^* \mathscr{O}_\chi|_{A[n]}$" should be "$\underset{\chi\in X}{\oplus} ([n]_A^* \mathscr{O}_\chi)|_{A[n]}$".
- Sec. 5.2.5, after the third diagram, line 1: "image of $\underset{\chi\in X}{\oplus} \mathscr{O}_\chi|_{A[n]}$ lies in $\mathscr{O}_{\frac{1}{n}\bar{0}}|_{A[n]}$" should be "image of $\underset{\chi\in X}{\oplus} \mathscr{O}_\chi|_{A[n]}$ lies in $\mathscr{O}_{\frac{1}{n}\bar{0}}$".
- Sec. 5.2.5, diagram -1: All "$\mathscr{O}_S$" should be "$\mathscr{O}_{S,\eta}$".
- Sec. 5.2.6: Definitions and facts involving c_n (and other objects defined over η) should be stated only over η, and "$\mathcal{D}_2(\mathcal{M})_A$" should be "$\mathcal{D}_2(\mathcal{M}_\eta)$" (when stated over η).
- Sec. 5.2.6, end of paragraph 3: It is unnecessary (and invalid) to refer to the proof of Thm. 4.5.4.12 (which requires the existence of a relatively complete model).
- Sec. 5.2.6, statements for $\mathrm{e}^{\lambda_\eta}(t, t')$ and $\mathrm{e}^{\lambda_\eta}(t, a)$: It is less confusing to replace the values "$= 0$" (i.e., in additive terms) by "$= 1$" (i.e., in multiplicative terms).
- Sec. 5.2.6, calculation of $\mathrm{e}^{\lambda_\eta}(t, y) = (\phi(y))(t)$, last sentence: "on $\mathcal{L}_\eta^\natural$" should be "on $(\mathcal{L}_\eta^\natural)^{\otimes n}$".
- Sec. 5.2.7, paragraph 1: Should replace the sentence "For simplicity, let us continue to assume that $\underline{X}$ and $\underline{Y}$ are constant with values X and Y, respectively" with "For simplicity, let us continue to assume that $\underline{X}$ and $\underline{Y}$ are constant with values X and Y, respectively".
- Lem. 5.2.7.5:

- Line 2: "Let $(\mathrm{Gr}^{\mathtt{Z}}_{-1}, \langle\,\cdot\,,\,\cdot\,\rangle_{11})$ be some lifting of $(\mathrm{Gr}^{\mathtt{Z}}_{-1,n}, \langle\,\cdot\,,\,\cdot\,\rangle_{11,n})$" should be "Let $(\mathrm{Gr}^{\mathtt{Z}}_{-1}, \langle\,\cdot\,,\,\cdot\,\rangle_{11})$ be induced by some fully symplectic lifting $\mathtt{Z}$ of $\mathtt{Z}_n$"
- Line 4: "a PEL-type $\mathcal{O}$-lattice $(L^{\mathtt{Z}_n}, \langle\,\cdot\,,\,\cdot\,\rangle^{\mathtt{Z}_n})$ such that" should be "a PEL-type $\mathcal{O}$-lattice $(L^{\mathtt{Z}_n}, \langle\,\cdot\,,\,\cdot\,\rangle^{\mathtt{Z}_n})$ satisfying Cond. 1.4.3.9 such that".

- Displayed equation 2 after (5.2.7.7): "$OPTPT[n]_{\bar{\eta}}$" should be "$\mathrm{T}^{\square}\, T_{\bar{\eta}}$".
- Paragraph 2 after Def. 5.2.7.11, line 2: "$\mathtt{w}_n : \mathrm{Gr}^{\mathtt{W}}_n \xrightarrow{\sim} \mathrm{Gr}^{\mathtt{W}}_n$" should be "$\mathtt{z}_n : \mathrm{Gr}^{\mathtt{Z}}_n \xrightarrow{\sim} \mathrm{Gr}^{\mathtt{Z}}_n$".
- Statement 4 before Prop. 5.3.1.1, last sentence: " $\nu(\hat{(}f))$ replaced with $\nu(\hat{f}') \circ \nu(g)$" should be "$\nu(\hat{f}') = \nu(\hat{f}) \circ \nu(g)$".
- Def. 5.3.1.3, definition of $\mathrm{U}^{\mathrm{ess}}_{1,\mathtt{Z}_n}$, the first line should be "$(g_{21,n}, g_{10,n}) \in \mathrm{Hom}_{\mathcal{O}}(\mathrm{Gr}^{\mathtt{Z}}_{-1,n}, \mathrm{Gr}^{\mathtt{Z}}_{-2,n}) \times \mathrm{Hom}_{\mathcal{O}}(\mathrm{Gr}^{\mathtt{Z}}_{0,n}, \mathrm{Gr}^{\mathtt{Z}}_{-1,n})$".
- Paragraph 2 after Def. 5.3.1.9: The definition of $\mathtt{Z}_{H_n}$ is incorrect. We need to allow it to be a possibly non-constant étale scheme over η.
- Paragraph 3 after Def. 5.3.1.9: The definition of $\varphi_{-1,n}$ contains many typos. It should be simply an étale-locally-defined $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathtt{Z}_n}}$-orbit of principal level-n structures of $(A_\eta, \lambda_{A,\eta}, i_{A,\eta})$ of type $(\mathrm{Gr}^{\mathtt{Z}}_{-1}, \langle\,\cdot\,,\,\cdot\,\rangle_{11})$.
- Paragraph 6 after Def. 5.3.1.9, line -4: The correct sentence should be: "Then we know that d_n, $d_n^\vee$, and e_n are determined respectively by $g_{21,n}$, $g_{10,n}$, and $g_{20,n}$."
- Def. 5.3.1.12: "level-$\mathcal{U}$" should be "level-$\mathcal{H}$".
- Def. 5.3.1.14: "level-$\mathcal{U}$" should be "level-$\mathcal{H}$".
- Def. 5.3.1.16: "$\mathrm{DEG}_{\mathrm{PEL},\mathsf{M}_{\mathcal{H}}}$" should be "$\mathrm{DD}_{\mathrm{PEL},\mathsf{M}_{\mathcal{H}}}$".
- Def. 5.3.2.1, 5.: "level-n" should be "level-$\mathcal{H}$".
- Def. 5.4.1.1: The extensibility of $c_n|_X$ and $c_n^\vee|_Y$ to c and $c^\vee$ over S is a condition. This is clarified in a revision.
- Rem. 5.4.1.2: All instances of "DEG" should be "DD".
- Lem. 5.4.1.12, line 5: "in $\mathrm{DD}_{\mathrm{PEL},\mathsf{M}_n}$" should be more precisely "in $\mathrm{DD}_{\mathrm{PEL},\mathsf{M}_n}$ (with $\underline{X}$ and $\underline{Y}$ constant with values respectively X and Y)".

- Lem. 5.4.1.14, 4., paragraph 2, line 1: "$\mathsf{Z}_{0,n} \twoheadrightarrow \mathsf{Z}_{0,n}/\mathsf{Z}_{-1,0} = \mathrm{Gr}^{\mathsf{Z}}_{0,n}$" should be "$\mathsf{Z}_{0,n} \twoheadrightarrow \mathsf{Z}_{0,n}/\mathsf{Z}_{-1,n} = \mathrm{Gr}^{\mathsf{Z}}_{0,n}$".
- Def. 5.4.1.15: The reference to Lem. 5.4.2.11 should be to Lem. 5.4.1.14.
- Def. 5.3.2.1, 2.: "$\lambda : G \xrightarrow{\sim} G^\vee$" should be "$\lambda : G \to G^\vee$". It is stated as a homomorphism but typed as an isomorphism by mistake. (Unfortunately, this mistake has been inherited by copy-and-paste in some other articles.)
- Def. 5.4.2.6:
  - Line 2: "PEL-type $\mathcal{O}$-lattices determined by" should be "PEL-type $\mathcal{O}$-lattices $(L^{\mathsf{Z}_n}, \langle\,\cdot\,,\,\cdot\,\rangle^{\mathsf{Z}_n})$ determined by".
  - Lines 4–5: "$\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n} \cong \mathrm{G}^{\mathrm{ess}}_{(L^{\Phi_n},\langle\,\cdot\,,\,\cdot\,\rangle^{\Phi_n},h^{\mathsf{Z}_n})}(\mathbb{Z}/n\mathbb{Z})$" should be "$\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n} \cong \mathrm{G}^{\mathrm{ess}}_{(L^{\mathsf{Z}_n},\langle\,\cdot\,,\,\cdot\,\rangle^{\mathsf{Z}_n},h^{\mathsf{Z}_n})}(\mathbb{Z}/n\mathbb{Z})$".
  - Line 6: "surjection $\mathrm{G}_{(L^{\mathsf{Z}_n},\langle\,\cdot\,,\,\cdot\,\rangle^{\mathsf{Z}_n})} \twoheadrightarrow \mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}$" should be "surjection $\mathrm{G}_{(L^{\mathsf{Z}_n},\langle\,\cdot\,,\,\cdot\,\rangle^{\mathsf{Z}_n})}(\hat{\mathbb{Z}}^\Box) \twoheadrightarrow \mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}$".
  - Should first define $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ to be the quotient of $\coprod \mathsf{M}^{\mathsf{Z}_n}_n$ by H_n, where the disjoint union is over representatives $(\mathsf{Z}_n, \Phi_n, \delta_n)$ (with the same (X, Y, ϕ)) in $(\mathsf{Z}_{\mathcal{H}}, \Phi_{\mathcal{H}}, \delta_{\mathcal{H}})$, and then define $\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$ to be the (finite étale) quotient of $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ by the subgroup of Γ_ϕ stabilizing $\Phi_{\mathcal{H}}$ (which is called $\Gamma_{\Phi_{\mathcal{H}}}$ later in Def. 6.2.4.1). (See below for the precise places for $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ to be used. Also, the previously definition of $\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$ as a moduli only for the abelian parts was not useful and should be abandoned.)
- Def. 5.4.2.8:
  - 3., line 3: "and descent" should be "and perform descent".
  - The extensibility of $c_{\mathcal{H}}|_X$ and $c^\vee_{\mathcal{H}}|_Y$ to c and $c^\vee$ over S is a condition. This is clarified in a revision.
  - "$\tau := \tau_n|_{\mathbf{1}_{\underline{Y} \underset{S}{\times} \underline{X},\eta}}$" should be "$\tau := \tau_{\mathcal{H}}|_{\mathbf{1}_{Y\times X,\eta}}$".
  - Should replace the rather discrete object $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in $\alpha^\natural_{\mathcal{H}} = (\mathsf{Z}_{\mathcal{H}}, \varphi_{-2,\mathcal{H}}, \varphi_{-1,\mathcal{H}}, \varphi_{0,\mathcal{H}}, \delta_{\mathcal{H}}, c_{\mathcal{H}}, c^\vee_{\mathcal{H}}, \tau_{\mathcal{H}})$ with a subscheme $(\varphi^\sim_{-2,\mathcal{H}}, \varphi^\sim_{0,\mathcal{H}})$ of $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}}) \underset{\mathsf{Z}_{\mathcal{H}}}{\times} \varphi_{-1,\mathcal{H}}$, where $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ is (indeed a discrete object) as in Def. 5.4.2.1 above, and

where $(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$ is an étale-locally-defined H_n-orbit which surjects under the two projections to the orbits defining $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ and $\varphi_{-1,\mathcal{H}}$. In this case we say that $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ is induced by $(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$. (Then, by the universal property of $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ because of its very construction, the torus part $(\mathsf{Z}_{\mathcal{H}}, \Phi^{\sim}_{\mathcal{H}} = (X, Y, \phi, \varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}}), \delta_{\mathcal{H}})$ and abelian part $(A, \lambda_A, i_A, \varphi_{-1,\mathcal{H}})$ of $(A, \lambda_A, i_A, \underline{X}, \underline{Y}, \phi, c, c^{\vee}, \tau, [\alpha^{\natural}_{\mathcal{H}}])$ canonically define a morphism $S = \operatorname{Spec}(R) \to \mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$.)

- Rem. 5.4.2.9: "$\mathrm{DEG}^{\text{fil.-spl.}}_{\mathrm{PEL},\mathsf{M}_{\mathcal{H}}} \to \mathrm{DEG}_{\mathrm{PEL},\mathsf{M}_{\mathcal{H}}}$" should be "$\mathrm{DD}^{\text{fil.-spl.}}_{\mathrm{PEL},\mathsf{M}_{\mathcal{H}}} \to \mathrm{DD}_{\mathrm{PEL},\mathsf{M}_{\mathcal{H}}}$".
- Lem. 5.4.2.10: The object is only unique up to isomorphisms allowing automorphisms of $(X, Y, \lambda, \varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$.
- Section 5.4.3, paragraph 4, line 6: The flatness of K_S requires some explanation. (This incurs some reorganization of the argument in the revision.)
- Section 5.4.3, paragraph 5, line -2: "$(\mathsf{Z}_{\mathcal{H}'}, \Phi_{\mathcal{H}'}, \varphi_{0,\mathcal{H}'}), \delta_{\mathcal{H}'})$" should be "$(\mathsf{Z}_{\mathcal{H}'}, \Phi_{\mathcal{H}'}, \delta_{\mathcal{H}'})$".
- Section 5.4.3, paragraph 6, lines 1–2: We do not need the condition $n|m$, but we need to assume that $g^{-1}\mathcal{U}^{\Box}(m)g \subset \mathcal{U}^{\Box}(n)$.
- Section 5.4.3, paragraph 9, lines -4 and -3: "$K_{\bar{\eta}} = \hat{\alpha}(g(L \underset{\mathbb{Z}}{\otimes} \hat{\mathbb{Z}}^{\Box})/(L \underset{\mathbb{Z}}{\otimes} \hat{\mathbb{Z}}^{\Box}))$" should be "$K_{\bar{\eta}} = \hat{\alpha}((N^{-1}g(L \underset{\mathbb{Z}}{\otimes} \hat{\mathbb{Z}}^{\Box}))/(L \underset{\mathbb{Z}}{\otimes} \hat{\mathbb{Z}}^{\Box}))$", "$K^{\mathrm{f}}_{\bar{\eta}} = \hat{\alpha}(g(\mathsf{Z}'_{-1})/\mathsf{Z}_{-1})$" should be "$K^{\mathrm{f}}_{\bar{\eta}} = \hat{\alpha}((N^{-1}g(\mathsf{Z}'_{-1}))/\mathsf{Z}_{-1})$", and "$K^{\mu}_{\bar{\eta}} = \hat{\alpha}(g(\mathsf{Z}'_{-2})/\mathsf{Z}_{-2})$" should be "$K^{\mu}_{\bar{\eta}} = \hat{\alpha}((N^{-1}g(\mathsf{Z}'_{-2}))/\mathsf{Z}_{-2})$".
- In (5.4.3,2), "$\mathrm{Gr}^{\mathsf{Z}'_{\mathbb{A}^{\infty,\Box}}}_{-2}$" should be "$\mathrm{Gr}^{\mathsf{Z}'_{\mathbb{A}^{\infty,\Box}}}_{0}$", and "$\mathrm{Gr}_{-2}(g)$" should be "$\mathrm{Gr}_0(g)$".
- Paragraph between Lem. 5.4.3.4 and Prop. 5.4.3.5, lines 15–16: "in $Y \underset{\mathbb{Z}}{\otimes} \hat{\mathbb{Z}}^{\Box}$" should be "in $Y \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_{(\Box)}$".
- Prop. 5.4.3.5:
  - 2., line 1: "the filtration" should be "any filtration".
  - 4., line 3: "in $Y \underset{\mathbb{Z}}{\otimes} \hat{\mathbb{Z}}^{\Box}$" should be "in $Y \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_{(\Box)}$".

  - Line -2: “$\delta_{\mathcal{H}}$” should be “$\delta'_{\mathcal{H}}$”, and choice of $\delta'_{\mathcal{H}}$ should have been mentioned.

- Def. 6.1.1.4: “in $\underline{X}(H)^{\vee}_{\mathbb{R}}$” should be omitted, and “$\underline{X}(H)^{\vee}_{\mathbb{R}}$” should be attached to the end of the sentence.
- Def. 6.1.1.7 and 6.1.1.8: The notation H for the supporting hyperplane should be modified.
- Def. 6.1.1.12, 2.: Should require moreover that each σ_k appearing in the closure of σ_j in C is a face of σ_j.
- Def. 6.1.2.3: “any rational polyhedral cone” should be “any nondegenerate rational polyhedral cone”.
- Thm. 6.1.2.8:

  - 2., line 3: “over S” should be “over Z”, and “view $\mathcal{M}$ as” should be “view $\overline{\mathcal{M}}_{\Sigma}$ as”.
  - 4.: “By subdividing . . . ” is incorrect. The correct statement we need requires the notion of boundary components, and is made more precise in the revision of Cond. 6.2.5.18 below.

- Paragraph 2 in Sec. 6.2.1, line 1: “simple algebra” should be ”semisimple algebra”.
- Paragraph -1 preceding Def. 6.2.1.1, line 2: “leaves invariant” should be “leave invariant”.
- Paragraph -2 in Sec. 6.2.1, line -8: “formally étale” should be “étale” (i.e., formally étale and of finite type).
- Prop. 6.2.2.1: The locally noetherian assumption is unnecessary and might cause (minor) compatibility concerns. It is removed in a revision.
- In Prop. 6.2.2.5, the notions of “(fiber-wise) identity components” and “(fiber-wise) component groups” are wrong notions and are replaced with the notions of “fiber-wise geometric identity components” and “group scheme of fiber-wise geometric connected components” in a revision. (See the correction for Prop. 5.2.3.8 above.) The notation, the proof, and the consequent results are modified accordingly.
- Cor. 6.2.2.6 is abandoned because the language is problematic (see Prop. 6.2.2.5 above), and because it is not needed.

- Paragraph -1 preceding Rem. 6.2.2.9, line 3: "$S' \to \dddot{C}_{\Phi_1}$" is étale but might not be finite.
- Paragraph 1 after Rem. 6.2.2.9, line 2: "$\iota : \underline{Y} \to \dddot{G}$" should be "$\iota : \underline{Y} \to G^{\natural}$".
- Paragraph -2 in Sec. 6.2.2, line -2: "$S \to \dddot{\Xi}_{\Phi_1}/\Gamma_{\Phi_1}$" should be "$S \to \dddot{\Xi}_{\Phi_1}$".
- Paragraph after displayed equation 1 in Sec. 6.2.3, "such that a suitable formal complete ..." should be removed.
- Paragraph after Lem. 6.2.3.1, displayed equation 1: "$\tau_n(\frac{1}{n}y, \phi(y'))\tau_n(\frac{1}{n}y, \phi(y'))^{-1} = a_{\Phi_n,\delta_n}(\frac{1}{n}y, \frac{1}{n}y')$" should be "$\tau_n(\frac{1}{n}y, \phi(y'))\tau_n(\frac{1}{n}y', \phi(y))^{-1} = a_{\Phi_n,\delta_n}(\frac{1}{n}y, \frac{1}{n}y')$".
- In Prop. 6.2.3.2, the notions of "(fiber-wise) identity components" and "(fiber-wise) component groups" are wrong notions and are replaced with the notions of "fiber-wise geometric identity components" and "group scheme of fiber-wise geometric connected components" in a revision. (See the corrections to Prop. 5.2.3.8 and 6.2.2.5 above.) The notation, the proof, and the consequent results are modified accordingly.
- In the statements and proof of Prop. 6.2.3.2, those "$\dfrac{1}{n}$" should certainly be "$\frac{1}{n}$".
- Proof of Prop. 6.2.3.2:
  - Paragraph 1, displayed equation 1 should be from "$\underline{\mathrm{Hom}}_{\mathcal{O}}(\frac{1}{n}X, A)$", not "$\underline{\mathrm{Hom}}_{\mathcal{O}}(\frac{1}{n}X, A)^{\circ}$".
  - Paragraph 3, displayed equation 1 should be "$\dddot{C}^{\circ\circ\circ}_{\Phi_n} \to \underline{\mathrm{Hom}}_{\mathcal{O}}(Y, A^{\vee})^{\circ}$" instead of the written one.
- Cor. 6.2.3.3 is abandoned because the language is problematic (see Prop. 6.2.3.2 above), and because it is not needed.
- Lem. 6.2.3.4: We now say "the fibers $\dddot{C}_{\Phi_n,b_n}$ of $\partial_n^{(1)}$ are (possibly empty) proper smooth subschemes of $\dddot{C}_{\Phi_n}$ over $\mathsf{M}_n^{\mathsf{Z}_n}$" instead of "the fibers $\dddot{C}_{\Phi_n,b_n}$ of $\partial_n^{(1)}$ are (possibly empty) unions of fiber-wise connected components of $\dddot{C}_{\Phi_n}$ over $\mathsf{M}_n^{\mathsf{Z}_n}$". (The statement is still correct but we want to remove references to connected components. See the corrections to Prop. 5.2.3.8, 6.2.2.5, and 6.2.3.2 above.)

- Paragraph 1 after Lem. 6.2.3.4, line -1: “nonempty” means ”has a section”.
- Lem. 6.2.3.7 is incorrect (and not needed). We have removed Lem. 6.2.3.7, Cor. 6.2.3.8, and Def. 6.2.3.9 in a revision. Nevertheless, the proof of Lem. 6.2.3.7 still works for Cor. 6.2.3.11. This corollary is all we need and have been turned into a lemma by itself.
- The expressions following the definition of $\dddot{\mathbf{S}}^{(n)}_{\Phi_n}$ are incorrect (and not needed).
- Lem. 6.2.3.10 is incorrect (and not needed).
- In Cor. 6.2.3.11, the functor “Spec” should be “$\underline{\mathrm{Spec}}$”. (See also the comments above for Lem. 6.2.3.7.)
- Lem. 6.2.3.14, line 4: “$\tilde{b}_m \in \underline{\mathrm{Hom}}_{\mathcal{O}}(\frac{1}{n}X/\phi(Y), A^\vee) \underset{\underline{\mathrm{Hom}}_{\mathcal{O}}(Y,A^\vee)}{\times} \{e\}$” should be “$\tilde{b}_m \in \underline{\mathrm{Hom}}_{\mathcal{O}}(\frac{1}{m}X/\phi(Y), A^\vee) \underset{\underline{\mathrm{Hom}}_{\mathcal{O}}(Y,A^\vee)}{\times} \{e\}$”.
- Proof of Lem. 6.2.3.14, line 2: “$n|m$, $\Box \nmid m$” should be “$n|l$, $\Box \nmid l$”.
- Rem. 6.2.3.15, line 4: “in $\mathrm{DD}^{\text{fil.-spl.}}_{\mathrm{PEL},\mathsf{M}_n}$” should be removed, as one of them is not.
- Proof of Lem. 6.2.3.16, line -2: “abelian schemes” should be “abelian scheme”.
- Cor. 6.2.3.18: “$\dddot{C}_{\Phi_n,0}$” should be “$\dddot{C}^{\circ}_{\Phi_n}$”.
- Paragraph after Cor. 6.2.3.18: “$|_{\dddot{C}_{\Phi_m,b_m}}$” should be “$|_{\dddot{C}^{\mathrm{com.}}_{\Phi_m,b_m}}$”.
- Cor. 6.2.3.20, line -1: “$\dddot{C}_{\Phi_n,b_n}$” should be “$\dddot{C}^{\mathrm{com.}}_{\Phi_n,b_n}$”.
- The statements of Lem. 6.2.3.21 and Cor. 6.2.3.22 have been suitably modified to reflect the changes above (such as the removal of Lem. 6.2.3.7, Cor. 6.2.3.8, and Def. 6.2.3.9). The proof of Lem. 6.2.3.21 has been modified accordingly.
- Proof of Lem. 6.2.3.21, after definition of $\dddot{\Xi}_{\Phi_m,0}$, “the fiber $(\partial^{(1)}_m)^{-1}(0)$” should be “the fiber $(\partial^{(0)}_m)^{-1}(0)$”.
- Prop. 6.2.3.25:
  - The locally noetherian assumption is unnecessary and might cause (minor) compatibility concerns. It is removed in a revision.
  - 4., displayed equation: “$\varphi_{-1,n}$” should be “$\varphi_{-1,m}$”.

- Sec. 6.2.4: The construction for general levels is not correctly deduced from the construction for principle levels.

  It is a mistake to consider the action of $H_{n,\mathrm{Z}^{\mathrm{ess}}_{\mathsf{Z}_n}}$. We should consider the action of $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{\mathsf{Z}_n}}$, which is defined as in Def. 5.3.1.9 by viewing the the semidirect product $\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{\mathsf{Z}_n}$ as a subgroup of $\mathrm{G}^{\mathrm{ess}}(\mathbb{Z}/n\mathbb{Z})$. (And later $\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{1,\mathsf{Z}_n}=(\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{\mathsf{Z}_n})/\mathrm{U}^{\mathrm{ess}}_{2,\mathsf{Z}_n}$ should be viewed as a subquotient.) In Lem. 6.2.4.6, should consider $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ and $H'_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}$ instead of $\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$ and $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}$, respectively. In the paragraph following Lem. 6.2.4.6, the $\mathrm{Z}^{\mathrm{ess}}_{\mathsf{Z}_n}$ and $\mathrm{Z}^{\mathrm{ess}}_{\mathsf{Z}_n}/\mathrm{U}^{\mathrm{ess}}_{2,\mathsf{Z}_n}$ should be $\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{\mathsf{Z}_n}$ and $\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{1,\mathsf{Z}_n}$, respectively. As a result, the image $H'_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}$ of $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{\mathsf{Z}_n}}$ in $\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}$ might be smaller than $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}$ in general. Hence, the top-right vertical arrow in (6.2.4.3) should be replaced with $\Xi_{\Phi_n,\delta_n}/H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{\mathsf{Z}_n}}\to C_{\Phi_n,\delta_n}/H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{1,\mathsf{Z}_n}}$, and the bottom-right arrow there should be replaced with $\mathsf{M}^{\mathsf{Z}_n}_n/H'_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}\to\mathsf{S}_0$.

  In Prop. 6.2.4.7 and the remainder of Ch. 6, the morphism $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}\to\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$ should be replaced with $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}\to\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$. (See Def. 5.4.2.6 above.) The latter is an abelian scheme torsor, not exactly an abelian scheme. We should define $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}\to C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}\to\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ as the equivariant quotient of $\coprod\Xi_{\Phi_n,\delta_n}\to\coprod C_{\Phi_n,\delta_n}\to\coprod\mathsf{M}^{\Phi_n}_n$ by H_n, where the disjoint unions are over representatives $(\mathsf{Z}_n,\Phi_n,\delta_n)$ (with the same (X,Y,ϕ)) in $(\mathsf{Z}_{\mathcal{H}},\Phi_{\mathcal{H}},\delta_{\mathcal{H}})$, which carries compatible actions of $\Gamma_{\Phi_{\mathcal{H}}}$. (By construction, $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}=\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$ when the image of $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{\mathsf{Z}_n}}$ in $\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}$ is $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}$; i.e., when, for some (and hence every) choice of a representative $(\mathsf{Z}_n,\Phi_n,\delta_n)$ in $(\mathsf{Z}_{\mathcal{H}},\Phi_{\mathcal{H}},\delta_{\mathcal{H}})$, the image of $H_{n,\mathrm{P}^{\mathrm{ess}}_{\mathsf{Z}_n}}$ in $\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\times\mathrm{G}^{\mathrm{ess}}_{l,\mathsf{Z}_n}$ is the direct product $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}\times H_{n,\mathrm{G}^{\mathrm{ess}}_{l,\mathsf{Z}_n}}$; the abelian scheme torsor $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}\to\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ is an abelian scheme when, for some (and hence every) choice of a representative $(\mathsf{Z}_n,\Phi_n,\delta_n)$ in $(\mathsf{Z}_{\mathcal{H}},\Phi_{\mathcal{H}},\delta_{\mathcal{H}})$, the splitting of the canonical homomorphism $\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{1,\mathsf{Z}_n}\twoheadrightarrow\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}$ defined by δ_n induces a splitting of the canonical homomorphism $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{1,\mathsf{Z}_n}}\twoheadrightarrow H'_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}$, and hence an isomorphism $H_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}\ltimes\mathrm{U}^{\mathrm{ess}}_{1,\mathsf{Z}_n}}\cong H'_{n,\mathrm{G}^{\mathrm{ess}}_{h,\mathsf{Z}_n}}\ltimes H_{n,\mathrm{U}^{\mathrm{ess}}_{1,\mathsf{Z}_n}}$.) It should be noted that, by definition, $\Gamma_{\Phi_{\mathcal{H}}}$ acts compatibly on $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ and $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$, but trivially on $\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$; and the canonical morphism $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}\to\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$ induces a canonical isomorphism $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}/\Gamma_{\Phi_{\mathcal{H}}}\xrightarrow{\sim}\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$. In the proof of Prop. 6.2.5.14, when computing the sheaves of differentials by applying Prop. 2.3.4.2, it is harmless to replace $\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$ with $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ because

$\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ is finite étale over $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$.

- Sec. 6.2.4, paragraph 2, line 2: "(principal)" is redundant (and makes no sense).
- Sec. 6.2.4, paragraph 2: The wording should be changed to reflect the changes made in Def. 5.4.2.8.
- Paragraph 2 after Def. 6.2.4.1, part 5: "level-$\mathcal{H}$" should be "level-$\mathcal{H}_h$", where $\mathcal{H}_h$ is as in Def. 5.4.2.6.
- Displayed equation 1 after (6.2.4.2): The "$(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$" should be denoted "$(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$", and it should be added in the sentence that $(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$ induces the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in $\Phi_{\mathcal{H}}$.
- (6.2.4.3) has to be revised because of the mistake of using $H_{n,\mathsf{Z}^{\mathrm{ess}}_{\mathsf{Z}_n}}$.
- Prop. 6.2.4.7: The locally noetherian assumption is unnecessary and might cause (minor) compatibility concerns. It is removed in a revision.
- Def. 6.2.5.3, line -1: "$\mathbb{Z}$-bilinear" should be "$\mathbb{R}$-bilinear".
- Prop. 6.2.5.7, 3., (b), line -1: "$y \in Y_\sigma$" should be "$0 \neq y \in Y_\sigma$".
- Definition of I_ℓ before Prop. 6.2.5.8 is not precise. It is better (and easier) to define it using pullbacks of the rigidified invertible sheaf $\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell)$ over $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$, defined by the $E_{\Phi_{\mathcal{H}}}$-torsor structure of $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$.
- Prop. 6.2.5.8:
  - Paragraph 1, line 1: R does not have to be local.
  - Paragraph 2, line 3: υ has to satisfy $\upsilon(R) \geq 0$.
- In proof of Prop. 6.2.5.14: The terminology of filtered pieces and graded pieces are sometimes mixed up. All instances of "$\Omega^1_{\overline{\Xi}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/\mathsf{S}_0}$" should be "$\Omega^1_{\overline{\Xi}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/\mathsf{S}_0}[d\log\infty]$", and all instances of "$\Omega^1_{\overline{\Xi}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}}$" should be "$\Omega^1_{\overline{\Xi}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}}[d\log\infty]$". In paragraph -4, the morphism (6.2.5.15) is not supposed to be an isomorphism. It is meant to induce an isomorphism between the top graded pieces.
- (6.2.5.16): the source "$\underline{\mathrm{Lie}}^\vee_{T/C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}} \underset{\mathscr{O}_{C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}}}{\otimes} \underline{\mathrm{Lie}}^\vee_{T^\vee/C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}}$" should be "$\underline{\mathrm{Lie}}^\vee_{T/\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}} \underset{\mathscr{O}_{\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}}}{\otimes} \underline{\mathrm{Lie}}^\vee_{T^\vee/\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}}$".

- Displayed equations -2 and -1 preceding Def. 6.2.5.17: The $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ should be denoted $(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$, and it should be remarked that $(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$ induces the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in $\Phi_{\mathcal{H}}$. (See above.)
- Paragraph -1 preceding Def. 6.2.5.17: Some of the R_1's and R_2's are incorrectly specified. Moreover, "formally étale" should be "étale" (i.e., formally étale and of finite type).
- Cond. 6.2.5.18 is incorrectly stated: We only need a power of γ to act as the identity on the smallest *admissible boundary component* of $\mathbf{P}_{\Phi_{\mathcal{H}}}$ containing $\gamma\bar{\sigma}_j \cap \bar{\sigma}_j$, where an admissible boundary component of $\mathbf{P}_{\Phi_{\mathcal{H}}}$ is defined to be the image of $\mathbf{P}_{\Phi'_{\mathcal{H}}}$ under the embedding $(\mathbf{S}_{\Phi'_{\mathcal{H}}})^{\vee}_{\mathbb{R}} \hookrightarrow (\mathbf{S}_{\Phi_{\mathcal{H}}})^{\vee}_{\mathbb{R}}$ defined by some surjection $(\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}) \twoheadrightarrow (\Phi'_{\mathcal{H}}, \delta'_{\mathcal{H}})$. Linear subspace of $(\mathbf{S}_{\Phi_{\mathcal{H}}})^{\vee}_{\mathbb{R}}$ is not the correct terminology, and "containing σ_j" is too much.
- Rem. 6.2.5.19: Given the above correction of Cond. 6.2.5.18, "γ acts as the identity" should be "a power of γ acts as the identity".
- Lem. 6.2.5.20:
  - Lines 3–4: "an algebraic space and $\mathcal{H}$ is neat" should be "a formal algebraic space when $\mathcal{H}$ is neat".
  - The modifier "formal" is missing in front of algebraic spaces and algebraic stacks, not only in this lemma (and its proof) but also in the paragraph following it.
- Prop. 6.2.6.5, line 4: "(uniquely)" should be dropped.
- Prop. 6.2.6.7, line -2: "over $\mathsf{M}'_{\mathcal{H}}$" should be dropped.
- Construction 6.3.1.1 is flawed:
  - X_s and Y_s should be respectively $X(s) = X/X_s$ and $Y(s) = Y/Y_s$ to avoid contradicting usages.
  - Completeness of the base schemes is necessary for defining objects in $\mathrm{DEG}_{\mathrm{PEL},\mathsf{M}_{\mathcal{H}}}$, yet it is important to globalize the result over general excellent normal algebraic stacks.
  - The claim on the definition of $\underline{B}(G)$ towards the end is misleading.

  The construction is corrected in a revision, referring to a new Section 4.5.5 added on two-step constructions.

- Prop. 6.3.1.2:
  - Line 2: "formally étale" should be "étale" (i.e., formally étale and of finite type).
  - 3.: "$\underline{\mathbf{S}}(\underline{\Phi}_{\mathcal{H}})$" should be "$\underline{\mathbf{S}}_{\underline{\Phi}_{\mathcal{H}}(\,^{\diamond}G)}$".
  - 4.: "$\underline{B} : \underline{\mathbf{S}}(\underline{\Phi}_{\mathcal{H}})(\,^{\diamond}G) \to \underline{\mathrm{Inv}}(S)$" should be "$f^*(\underline{B}) : \underline{\mathbf{S}}_{\underline{\Phi}_{\mathcal{H}}(\,^{\diamond}G)} \to \underline{\mathrm{Inv}}(S)$".
  - 5.: "$\Omega^1_{\mathrm{Spec}(R)/\mathsf{S}_0}[d\log\infty]$" should be "$\widehat{\Omega}^1_{\mathrm{Spec}(R)/\mathsf{S}_0}[d\log\infty]$", the completion of $\Omega^1_{\mathrm{Spec}(R)/\mathsf{S}_0}[d\log\infty]$ with respect to the topology of R defined by I. The normal crossing divisor $\mathrm{Spec}(R/I)$ should be the normal crossing divisor on $\mathrm{Spec}(R)$ induced by that of $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\sigma) - \Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$.
  - 6.:
    * Paragraph 2, line 1: "base scheme R" should be "base ring R".
    * Paragraph 2, line 2: "Section 6.2.1" should be "Section 5.2.1".
    * Paragraph 2, displayed equation 2: The $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in $\alpha^{\natural}_{\mathcal{H}} = (\mathsf{Z}_{\mathcal{H}}, \varphi_{-2,\mathcal{H}}, \varphi_{-1,\mathcal{H}}, \varphi_{0,\mathcal{H}}, \delta_{\mathcal{H}}, c_{\mathcal{H}}, c^{\vee}_{\mathcal{H}}, \tau_{\mathcal{H}})$ should be denoted $(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$, and it should be added in the sentence that $(\varphi^{\sim}_{-2,\mathcal{H}}, \varphi^{\sim}_{0,\mathcal{H}})$ induces the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in $\Phi_{\mathcal{H}}$. (See above.)
    * Paragraph 3, line 3: "$\upsilon : (\mathrm{Frac}(\mathrm{Spec}(R)))^{\times} \to \mathbb{Z}$" should be "$\upsilon : \underline{\mathrm{Inv}}(\mathrm{Spec}(R)) \to \mathbb{Z}$".
    * Paragraph 3, line 8: "$\mathrm{Spec}(K) \to \Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/\Gamma_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$" should be "$\mathrm{Spec}(K) \to \Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}/\Gamma_{\Phi_{\mathcal{H}},\sigma}$".
    * Paragraph 3, lines 11–12: "formal schemes" should be "formal algebraic stacks".
- Cor. 6.3.1.4 and its proof: "$\Omega^1_{\mathrm{Spec}(R)/\mathsf{S}_0}[d\log\infty]$" should be "$\widehat{\Omega}^1_{\mathrm{Spec}(R)/\mathsf{S}_0}[d\log\infty]$" as in Prop. 6.3.1.2 above.
- The statement of Cor. 6.3.1.4 is imprecise — we need to clarify what strata-preserving means, and we also need more assumptions on R. We have corrected Cor. 6.3.1.4, and revised Cor. 6.3.1.10, Def. 6.3.1.11, Cor. 6.3.1.14 accordingly.
- Rem. 6.3.1.13, line 5: "$\underline{\mathbf{S}}(\underline{\Phi}_{\mathcal{H}})$" should be "$\underline{\mathbf{S}}_{\underline{\Phi}_{\mathcal{H}}(\,^{\diamond}G)}$".

- Prop. 6.3.2.1:
  - 3., line 7: "formal good" should be "good formal".
  - 4.: "$\Omega^1_{S/\mathsf{S}_0}[d\log\infty]$" should be "$\widetilde{\Omega}^1_{S/\mathsf{S}_0}[d\log\infty]$", the sheaf of universal finite module differentials, and the remainder of the statement should be corrected accordingly.
- Proof of Prop. 6.3.2.1: Should denote the $\Phi_{\mathcal{H}}$ in the two displayed degeneration data by two different notations (other than the prescribed $\Phi_{\mathcal{H}}$), and remark that they can be approximated because they are discrete in nature. (See above.)
- (6.3.2.3): "${}^\diamondsuit\sigma_{\chi+\phi(y)}({}^\diamondsuit s)$" should be "${}^\diamondsuit\sigma_{\chi+2\phi(y)}({}^\diamondsuit s)$".
- Proof of Lem. 6.3.2.4, line 4: "$x_i \to \infty$" should be "$x_i \to 0$".
- Prop. 6.3.2.5, 3., (a), line 2: "formal good" should be "good formal".
- In Rem. 6.3.2.11, "$({}^\diamondsuit G, {}^\diamondsuit\lambda, {}^\diamondsuit i, {}^\diamondsuit\alpha_{\mathcal{H}})$" should be "$(G,\lambda,i,\alpha_{\mathcal{H}})$".
- Prop. 6.3.2.13: The notation "R" should be replaced with "R_{alg}". At the end of 1., "$\Phi_{\mathcal{H}}$" should be "$\underline{\Phi}_{\mathcal{H}}$".
- In proof of Prop. 6.3.2.13, the role played by Construction 6.3.1.1 and the help of the new Sec. 4.5.5 are mentioned.
- Def. 6.3.2.15, line 2: "equivalence class $[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)]$ of $[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)]$" should be "equivalence class $[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)]$ of $(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)$".
- Def. 6.3.3.6, line 4: "if $r > r'$" should be "if $r' > r$".
- Prop. 6.3.3.7:
  - R should be assumed to be noetherian.
  - 3., paragraph 2, line 1: "$\underline{\mathbf{S}}_{\underline{\Phi}_{\mathcal{H}}}(G^\dagger)$" should be "$\underline{\mathbf{S}}_{\underline{\Phi}_{\mathcal{H}}(G^\dagger)}$".
- Proof of Prop. 6.3.3.7:
  - In paragraph 4, up to line 3: It should be emphasized that $(G^\dagger,\lambda^\dagger,i^\dagger,\alpha^\dagger_{\mathcal{H}}) \to \mathrm{Spec}(R)$ determines (by the universal properties of $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ and $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$) not only a morphism $\mathrm{Spec}(K) \to \Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$, but also a morphism $\mathrm{Spec}(R) \to C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ compatible with the (relatively affine) structural morphism $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}} \to C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$.

- Paragraph 1 after Rem. 6.3.3.10, "$\mathsf{R}^{[0]}_{\mathcal{H}} \to \mathsf{U}^{[0]}_{\mathcal{H}} \underset{\mathsf{S}_0}{\times} \mathsf{U}^{[0]}_{\mathcal{H}}$" should be "$\mathsf{R}^{[0]}_{\mathcal{H}} \to \mathsf{U}_{\mathcal{H}} \underset{\mathsf{S}_0}{\times} \mathsf{U}_{\mathcal{H}}$".
- Proof of Prop. 6.3.3.11, step 2: For $i = 1, 2$, the $(\varphi_{-2,\mathcal{H},i}, \varphi_{0,\mathcal{H},i})$ in $\alpha^{\natural}_{\mathcal{H},i}$ should be denoted $(\varphi^{\sim}_{-2,\mathcal{H},i}, \varphi^{\sim}_{0,\mathcal{H},i})$, and it should be added in the sentence that $(\varphi^{\sim}_{-2,\mathcal{H},i}, \varphi^{\sim}_{0,\mathcal{H},i})$ induces the $(\varphi_{-2,\mathcal{H},i}, \varphi_{0,\mathcal{H},i})$ in $\Phi_{\mathcal{H},i}$. (See above.)
- Proof of Prop. 6.3.3.11, step 4, paragraph 2, line 2: "$\mathsf{U}^{[0]}_{\mathcal{H}} \underset{\mathsf{S}_0}{\times} \mathsf{U}^{[0]}_{\mathcal{H}}$" should be "$\mathsf{U}_{\mathcal{H}} \underset{\mathsf{S}_0}{\times} \mathsf{U}_{\mathcal{H}}$".
- Cor. 6.3.3.17, line 2: "descends" should be "descend", and "degenerating family $\mathsf{M}^{\mathrm{tor}}_{\mathcal{H}}$" should be "$\mathsf{M}^{\mathrm{tor}}_{\mathcal{H}}$".
- Proof of Prop. 6.3.3.18, paragraph 2:
  - Line 8: "principal polarization" should be "polarization".
  - Line 10: "an isomorphism $\lambda_V : G_V \xrightarrow{\sim} G_V^{\vee}$" should be "a homomorphism $\lambda_V : G_V \to G_V^{\vee}$".
- Thm. 6.4.1.1:
  - 2., paragraph 2: the description of a torus-torsor etc should be for $\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma}$, not for $\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma}/\Gamma_{\Phi_{\mathcal{H}},\sigma}$, and should be "$\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma}$ (before quotient by $\Gamma_{\Phi_{\mathcal{H}},\sigma}$) admits a canonical structure as the completion of an affine toroidal embedding $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\sigma)$ (along its σ-stratum $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma}$) of a torus torsor $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ over an abelian scheme torsor $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ over a finite étale cover $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ of the algebraic stack $\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$".
  - 3., line 2: "each stratum of $\mathsf{D}_{\infty,\mathcal{H}}$ is open dense in an intersection of components of $\mathsf{D}_{\infty,\mathcal{H}}$" should be "each connected component of a stratum of $\mathsf{M}^{\mathrm{tor}}_{\mathcal{H}} - \mathsf{M}_{\mathcal{H}}$ is open dense in an intersection of irreducible components of $\mathsf{D}_{\infty,\mathcal{H}}$".
  - 4., line -2 from the end: "the Cartier divisor" should be "the relative Cartier divisor".
  - 5.: The strata-preserving property can be formulated using Prop. 6.3.1.6, without introducing the notion of good formal models.
  - 6.:
    - Paragraph 2:

      - Line 1: "any morphism $\mathrm{Spec}(V) \to S$" should be "any dominant morphism $\mathrm{Spec}(V) \to S$".
      - Displayed equation: "$\lambda_A^\ddagger$" and "$i_A^\ddagger$" should be denoted "$\lambda_{A^\ddagger}$" and "$i_{A^\ddagger}$"
    * Paragraph 3:
      - Line 1: "$\underline{\mathbf{S}}_{\underline{\Phi}_{\mathcal{H}}}(G^\ddagger)$" should be "$\underline{\mathbf{S}}_{\underline{\Phi}_{\mathcal{H}}(G^\ddagger)}$".
      - Just to clarify, the condition for $\overline{\sigma}^\ddagger$ to contain all $\upsilon \circ B^\ddagger$ means for all υ centered at the same given geometric point $\bar{s}$.

- Proof of Thm. 6.4.1.1:
  - Paragraph 1, line 1: "$/\Gamma_{\Phi_{\mathcal{H}},\sigma}$" should be "$\Gamma_{\Phi_{\mathcal{H}},\sigma}$".
  - Paragraph 2, line 3: "finite étale" is incorrect and unnecessary — properties of objects over the algebraic stack $\mathsf{M}_{\mathcal{H}}^{\mathrm{tor}}$ are by definition given by the corresponding properties or objects of the étale presentation $\mathsf{U}_{\mathcal{H}}$ with descent datum over $\mathsf{R}_{\mathcal{H}}$.
  - Paragraph 4, line 8: "$(\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma)$-stratum" should be "$[(\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma)]$-stratum".
- Def. 6.4.2.1, line 8: "(f_X, f_Y)" should be "(γ_X, γ_Y)". (There are other similar typos in this section.)
- Def. 6.4.2.6, line 5: "in $\mathbf{P}_{\Phi_{\mathcal{H}}}$" should be "in $\mathbf{P}_{\Phi_{\mathcal{H}'}}$".
- In Sec. 6.4.3 and in later definitions of Hecke actions, we have never really used the assumption that $\mathcal{H}' \subset \mathcal{H}$. Hence we should assume that $\mathcal{H}' \subset g\mathcal{H}g^{-1}$ instead of $\mathcal{H}' \subset \mathcal{H} \cap g\mathcal{H}g^{-1}$. (This is not a mistake, but a redundant assumption.) We have removed this redundancy in the revision.
- Sec. 6.4.3, paragraph 2: More precisely, $G'_{\mathsf{M}_{\mathcal{H}'}}$ can be realized as a quotient of $G_{\mathsf{M}_{\mathcal{H}'}}$ by a finite *étale* group scheme $K_{\mathsf{M}_{\mathcal{H}'}}$ over $\mathsf{M}_{\mathcal{H}'}$ of rank prime-to-$\Box$, and this allows us to realize G' as a quotient of G by the quasi-finite étale closure K of $K_{\mathsf{M}_{\mathcal{H}'}}$ in G.
- Def. 6.4.3.1, line -4: "$\Phi'_{\mathcal{H}}$-translate" should be "$\Gamma_{\Phi'_{\mathcal{H}}}$-translate".
- Rem. 6.4.3.4:
  - Line 3: "of Let" should be "of".

- Line 8: "Σ' to Σ" should be "Σ to Σ'".
- Line -2: "$(\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma')$" should be "$(\Phi'_{\mathcal{H}}, \delta'_{\mathcal{H}}, \sigma')$".

- Rem. 6.4.3.6, line 6: "both $\mathbb{R}$-rank" should be "$\mathbb{R}$-rank".
- In all of Ch. 7, $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ should be replaced with $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$. (See above.)
- Lem. 7.1.1.4: We need to assume that $\mathcal{E}$ is of the form $\mathcal{E} = \mathcal{E}_0 \underset{\mathcal{O}_{F_0,(\square)}}{\otimes} M$, where $\mathcal{E}_0$ is a locally free sheaf over $\mathsf{M}_{\mathcal{H},\Sigma}^{\mathrm{tor}}$, and where M is a module over $\mathcal{O}_{F_0,(\square)}$. Then in the proof, we need to adopt the reduction step in the proof of Prop. 7.1.2.15 (from general M to the case $M = \mathcal{O}_{F_0,(\square)}/\mathfrak{n}$ for some ideal $\mathfrak{n}$ of $\mathcal{O}_{F_0,(\square)}$).
- Sec. 7.1.2, paragraph 1:
  - Line 8: "support" should be removed.
  - Line 10: "by some $(L^{\mathsf{Z}_{\mathcal{H}}}, \langle \cdot\,, \cdot \rangle^{\mathsf{Z}_{\mathcal{H}}})$" should be "by some $(L^{\mathsf{Z}_{\mathcal{H}}}, \langle \cdot\,, \cdot \rangle^{\mathsf{Z}_{\mathcal{H}}})$ (as in Lemma 5.2.7.5)".

- Proof of Lem. 7.1.2.1, line 2: "formally étale" should be "étale" (i.e., formally étale and of finite type).
- Paragraph 1 after Rem. 7.1.2.6:
  - Line 3: "$\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\sigma_2) \hookrightarrow \Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\sigma_1)$" should be "$\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\sigma_1) \hookrightarrow \Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\sigma_2)$". Similar changes should be made in what follows.
  - Line 5: "canonical morphism $\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma_2}/\Gamma_{\Phi_{\mathcal{H}},\sigma_2} \hookrightarrow \mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma_1}/\Gamma_{\Phi_{\mathcal{H}},\sigma_1}$" should be "canonical morphism $\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma_1}/\Gamma_{\Phi_{\mathcal{H}},\sigma_1} \to \mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma_2}/\Gamma_{\Phi_{\mathcal{H}},\sigma_2}$".

- Cor. 7.1.2.13:
  - Line 2: "the semi-subgroup of elements in $\mathbf{S}_{\Phi_{\mathcal{H}}}$ that pairs positively with some element in $\mathbf{P}_{\Phi_{\mathcal{H}}}$, or equivalently" is incorrect and should be removed.
  - In line 3 and in the proof, should remark that it is constant along the fibers because it is also invariant under $\Gamma_{\Phi_{\mathcal{H}}}$, and we know $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}/\Gamma_{\Phi_{\mathcal{H}}} \cong \mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$.

- Prop. 7.2.1.1, line 7: "relative" should be removed.

- Proof of Prop. 7.2.1.2
  - Line 2: "C' over C" should be "C' finite over C".
  - Line 4: "$A \underset{C}{\times} \tilde{C} \underset{C}{\times} C' \to S \underset{C}{\times} C'$" should be "$A \underset{C}{\times} \tilde{C} \underset{C}{\times} C' \to \tilde{C} \underset{C}{\times} C'$".
- Sec. 7.2.2, paragraph 2, after displayed equation 1, line 1: "$r_0 \geq 1$" should be "$r_0 \geq 0$". Similar changes should be made in Sec. 7.2.3.
- Paragraph -1 preceding Lem. 7.2.2.1, line -3: "f^{st}" should be "Y^{st}". (In the revision the notation Y is replaced with W, to avoid conflict with the notation for character groups.)
- Lem. 7.2.2.1, line 1: "f^{st}" should be "Y^{st}".
- Paragraph 1 after proof of Lem. 7.2.2.1, displayed equation 1: "$\mathcal{O}(1)$" should be "$\mathcal{O}(1)^{\otimes k}$".
- Prop. 7.2.2.3: To apply the Künneth formula, we need to assume that Z and at least one of $\mathcal{E}$ and $\mathcal{F}$ are all flat over S.
- Proof of Prop. 7.2.2.3: Resolution for $\mathscr{I} \otimes \mathrm{pr}_1^*(\mathcal{M}^{\otimes a}) \otimes \mathrm{pr}_2^*(\mathcal{M}^{\otimes b})$ should be for $\mathscr{I}$. In the last line, "M_0" should be "k_0".
- Cor. 7.2.2.4 and 7.2.2.5: We need Z to be flat over S.
- Cor. 7.2.2.6: We need Y^{st} to be flat over S, and "Z" should be "Y^{st}".
- Proof of Cor. 7.2.2.6, paragraph 2
  - Line 1: "Z^{st}" should be "Y^{st}".
  - Displayed equation 1: "$\mathcal{L}^{\otimes k}$" should be "$\mathcal{L}^{\otimes N_1 k}$".
- We should assume that Z_1 and Z_2 are both locally noetherian. To define $\tilde{f}$ using the universal property of the normalization, we need to f to induce dominant morphisms from irreducible components of Z_1 to irreducible components of Z_2.
- Sec. 7.2.3, paragraph 2, line -1: In order to apply Corollary 7.2.2.6, we need to explain that $\mathsf{M}_{\mathcal{H}}^{\min}$ is flat over $\mathsf{S}_0 = \mathrm{Spec}(\mathcal{O}_{F_0,(\Box)})$, which can be justified after Proposition 7.2.3.2. We have added this in the revision.
- Paragraph -1 preceding Lem. 7.2.3.4: The restriction $\oint_{\mathcal{H}} |_{\mathsf{Z}_{[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)]}} : \mathsf{Z}_{[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)]} \to \mathsf{M}_{\mathcal{H}}^{\min}$ is proper only when σ is top-dimensional in $\mathbf{P}_{\Phi_{\mathcal{H}}}^{+} \subset$

$(\mathbf{S}_{\Phi_{\mathcal{H}}})^\vee_{\mathbb{R}}$, and to explain this assertion we need Proposition 7.2.3.7 and Corollary 7.2.3.8. Hence we should relocate Lem. 7.2.3.4 and state it after Corollary 7.2.3.8 as another corollary.

- Prop. 7.2.3.11 is incorrect when the PEL datum we started with is not $\mathbb{Q}$-simple, and its proof contains some typos and incorrect claims. The correct statement is that, if we take $\mathsf{M}^1_{\mathcal{H}}$ to be open subscheme of $\mathsf{M}^{\min}_{\mathcal{H}}$ formed by the union of the strata of $\mathsf{M}^{\min}_{\mathcal{H}}$ at most one, then the pullback to $\mathsf{M}^1_{\mathcal{H}}$ of the canonical surjection $[\oint_{\mathcal{H}}] : [\mathsf{M}^{\mathrm{tor}}_{\mathcal{H}}] \twoheadrightarrow \mathsf{M}^{\min}_{\mathcal{H}}$ induced by $\oint_{\mathcal{H}}$ is an isomorphism. The proof can be slightly weakened to allow $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}} \to \mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ to be an abelian scheme torsor.
- Prop. 7.2.3.14: The definition of $\mathrm{Aut}(\bar{x})$ uses the identification of $\bar{x}$ as a geometric point of $\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$. This is clarified in a revision.
- Proof of Prop. 7.2.3.14: At the end, should replace "$(\underline{\mathrm{FJ}}^{(0)}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^{\mathrm{Aut}(\bar{x})}$" with "$((\underline{\mathrm{FJ}}^{(0)}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^{\wedge}_{\bar{x}})^{\mathrm{Aut}(\bar{x})\times\Gamma_{\Phi_{\mathcal{H}}}}$".
- Thm. 7.2.4.1:
  - Statement 2: The definition of $\omega^{\min}$ is ambiguous. Since we have $N_0 = 1$ with a unique choice of $\mathcal{O}(1)$ when $\mathcal{H}$ is neat, we shall define $\omega^{\min}$ to be this unique $\mathcal{O}(1)$.
  - Statement 3, paragraph 2, line 3: "determined by $\omega^{\min}$" should be "determined by ω^{tor}".
  - Statement 4, paragraph 2: Should simply say that $\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$ is as in Def. 5.4.2.6, without saying that it represents a moduli problem.
  - Statement 5, paragraph 1: The restriction is only a surjection. This surjection is smooth when $\mathcal{H}$ is neat, and is proper if σ is top-dimensional in $\mathbf{P}^+_{\Phi_{\mathcal{H}}} \subset (\mathbf{S}_{\Phi_{\mathcal{H}}})^\vee_{\mathbb{R}}$. Should say instead that $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ is an abelian scheme torsor over the finite étale cover $\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ over the algebraic stack $\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$ over the coarse moduli space $[\mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}]$ (which is a scheme). (See above.)
- The proof of Thm. 7.2.4.1 has to be modified according to the correction of Prop. 7.2.3.11 above, using the newly introduced subscheme $\mathsf{M}^1_{\mathcal{H}}$ to be open subscheme of $\mathsf{M}^{\min}_{\mathcal{H}}$. Also, the completions along $\bar{x}$ and $\tilde{x}_\sigma$ do not make sense and have to be replaced with pullbacks to completions of strict local rings.

- Cor. 7.2.4.3, Def. 7.2.4.4, and Cor. 7.2.4.9 are flawed for general M. (we need assumptions as in Prop. 7.2.4.5.) They are corrected in the revision.
- Rem. 7.2.4.8: The claim of liftability at the end is, unfortunately, not justified. It is a mistake and should be removed.
- Cor. 7.2.4.9: The flatness assumption of M (over some M_0) is not necessary.
- Prop. 7.2.5.1:
  - Paragraph 2, line 2: "$\mathsf{M}^{\min}_{\mathcal{H},\Sigma'}$" should be "$\mathsf{M}^{\min}_{\mathcal{H}}$".
  - Paragraph 2, line 4: "$(\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma')$" should be "$(\Phi_{\mathcal{H}}, \delta_{\mathcal{H}})$".
- Proof of Prop. 7.2.5.1, paragraph 1, line 8: "$\mathsf{M}^{\mathrm{tor}}_{\mathcal{H}}$" should be "$\mathsf{M}^{\mathrm{tor}}_{\mathcal{H}',\Sigma}$".
- Cor. 7.2.5.2: Need the normality of $\mathcal{H}'$ as a subgroup of $\mathcal{H}$.
- Def. 7.3.1.1, 2.: The condition should be "$\mathsf{pol}_{\Phi_{\mathcal{H}}}$ is linear (in the above sense) on a rational polyhedral cone σ in $\mathbf{P}_{\Phi_{\mathcal{H}}}$ if and only if σ is contained in some cone σ_j in $\Sigma_{\Phi_{\mathcal{H}}}$".
- Def. 7.3.1.1, 4.: "$x, y \in \mathbf{S}_{\Phi_{\mathcal{H}}}$" should be "$x, y \in \mathbf{P}_{\Phi_{\mathcal{H}}}$".
- Prop. 7.3.1.2
  - 2., line 2 after displayed equation: "$\mathbb{R}_{\geq 0} \cdot K_{\mathsf{pol}_{\Phi_{\mathcal{H}}}} \supset \overline{\mathbf{P}}_{\Phi_{\mathcal{H}}}$" should be "$\mathbb{R}_{\geq 0} \cdot K_{\mathsf{pol}_{\Phi_{\mathcal{H}}}} \supset \mathbf{P}_{\Phi_{\mathcal{H}}}$".
  - 3.: All instances of "$(\mathbf{P}^{\vee}_{\Phi_{\mathcal{H}}}) \underset{\mathbb{Z}}{\otimes} \mathbb{R}$" should be "$\mathbb{R}_{\geq 0} \cdot \mathbf{P}_{\Phi_{\mathcal{H}}}$".
  - 5., line -2: "(s_X, S_Y)" should be "(s_X, s_Y)".
- For Lem. 7.3.1.6 to be useful, we need to assert explicitly that the cone decomposition remains smooth for the new integral structure defined by the level $\mathcal{H}'$.
- Lem. 7.3.1.7: "$K^{\vee}_{\mathsf{pol}_{\Phi_{\mathcal{H}}}}$" should be "$K^{\vee}_{\mathsf{pol}_{\Phi_{\mathcal{H}'}}}$", and γ should satisfy $\gamma \neq 1$.
- Lem. 7.3.1.8: The literal statements of this lemma, which we cited almost verbatim from Faltings–Chai (Ch. V, Lem. 5.5), are unfortunately flawed. For example, if $\mathbf{P}^{+}_{\Phi_{\mathcal{H}}} = \mathbb{R}_{>0} = \sigma$, then there are no other top-dimensional cones at all, and hence the lemma asserts that $\sigma^{\vee} = \{0\}$—but $\sigma^{\vee}$ is certainly nonzero. This error was inherited from a

similar error in Ash–Mumford–Rapoport–Tai (Ch. IV, Sec. 2, p. 330). We have corrected this lemma (and accordingly modified the later arguments using it) in a revision.

- Def. 7.3.2.2: In the beginning, we need the image of the canonical morphism $f^*\mathcal{I} \to \mathscr{O}_{\tilde{W}}$, rather than $f^*\mathcal{I}$ itself, to be an invertible $\mathscr{O}_{\tilde{W}}$-ideal. (This was once adopted as some abuse of language, but we have later decided to use the precise notation.) Later, to define $\mathrm{NBl}_{\mathcal{I}}(f)$ using the universal property of the normalization, we need to f to induce dominant morphisms from irreducible components of $\tilde{W}$ to irreducible components of W.
- Prop. 7.3.2.3:
  - Line 2 after the diagram: "$W_1 \to f_{1,*}\tilde{W}_1$ and $W_2 \to f_{2,*}\tilde{W}_2$" should be "$\mathscr{O}_{W_1} \to f_{1,*}\mathscr{O}_{\tilde{W}_1}$ and $\mathscr{O}_{W_2} \to f_{2,*}\mathscr{O}_{\tilde{W}_2}$".
  - Line 2 of paragraph -1: "$\imath_1 \xrightarrow{\sim} f_1^*\mathcal{I}_1$" should be "$f_1^*\mathcal{I}_1 \xrightarrow{\sim} \imath_1$".
  - Line -3 of paragraph -1: "$\imath_2^{\otimes k} \xrightarrow{\sim} f_2^*\mathcal{I}_2^{(d_0)}$" should be "$f_2^*\mathcal{I}_2^{(d_0)} \xrightarrow{\sim} \imath_2^{\otimes d_0}$".
- Def. 7.3.3.1: It is imprecise to call identify irreducible components of $\mathsf{D}_{\infty,\mathcal{H}}$ with schemes of the form $\overline{\mathsf{Z}}_{[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)]}$, because the latter are not irreducible in general. We should consider the irreducible components of $\overline{\mathsf{Z}}_{[(\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma)]}$ instead. Also, "$\mathbb{Z}_{\geq 0}$-generator" should be "$\mathbb{Z}_{>0}$-generator".
- Cond. 7.3.3.3: γ should satisfy $\gamma \neq 1$.
- Paragraph -1 before Thm. 7.3.3.4: "$\mathsf{M}_{\mathcal{H}}$" should be "$\mathsf{M}_{\mathcal{H}}^{\mathrm{tor}}$", and this last sentence should be placed as the second last sentence in Thm. 7.3.3.4 (before Condition 7.3.3.3 is mentioned).
- Thm. 7.3.3.4, 1.: "$\jmath_{\mathcal{H},\mathsf{pol}}^{\otimes d_0} \to \oint_{\mathcal{H}}^{-1} \mathcal{J}_{\mathcal{H},\mathsf{pol}}^{(d_0)} \cdot \mathscr{O}_{\mathsf{M}_{\mathcal{H}}^{\mathrm{tor}}}$" should be "$\oint_{\mathcal{H}}^{-1} \mathcal{J}_{\mathcal{H},\mathsf{pol}}^{(d_0)} \cdot \mathscr{O}_{\mathsf{M}_{\mathcal{H}}^{\mathrm{tor}}} \to \jmath_{\mathcal{H},\mathsf{pol}}^{\otimes d_0}$".
- Proof of 1. of Thm. 7.3.3.4:
  - All instances of "$\jmath_{\mathcal{H},\mathsf{pol}}^{\otimes d_0} \to \oint_{\mathcal{H}}^{-1} \mathcal{J}_{\mathcal{H},\mathsf{pol}}^{(d_0)} \cdot \mathscr{O}_{\mathsf{M}_{\mathcal{H}}^{\mathrm{tor}}}$" should be "$\oint_{\mathcal{H}}^{-1} \mathcal{J}_{\mathcal{H},\mathsf{pol}}^{(d_0)} \cdot \mathscr{O}_{\mathsf{M}_{\mathcal{H}}^{\mathrm{tor}}} \to \jmath_{\mathcal{H},\mathsf{pol}}^{\otimes d_0}$".
  - Paragraph 2, line 3: "$\mathbb{Z}_{\geq 0}$-generator" should be "$\mathbb{Z}_{>0}$-generator".
  - Paragraph 2, displayed equation 1: "$\sigma_0^\vee \cap \mathbf{S}_{\Phi_{\mathcal{H}}}$" should be simply "$\sigma_0^\vee$".

- Paragraph 4, lines 6–7: The sentence "In particular, the canonical morphism $(\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(d\cdot\ell_0))^\wedge_{\bar{x}} \to (\mathrm{p}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^*(\underline{\mathrm{FJ}}^{(d\cdot\ell_0)}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^\wedge_{\bar{x}}$ is an isomorphism." is incorrect (and not needed).
- Paragraph 5, displayed equation 2: "$f = \sum_{[\ell]\in(\ell_0+(\sigma_0)^\vee)/\Gamma_{\Phi_{\mathcal{H}}}} f^{[\ell]}$" should be "$f = \sum_{[\ell]\in(\Gamma_{\Phi_{\mathcal{H}}}\cdot(d\cdot\ell_0+(\sigma_0)^\vee))/\Gamma_{\Phi_{\mathcal{H}}}} f^{[\ell]}$".
- Paragraph 6: In the first sentence, "structural sheaf of $\hat{\bigoplus}_{\ell\in\tau^\vee}(\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell))^\wedge_{\bar{x}}$" should be "$\mathscr{O}_{\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\tau}} \cong \hat{\bigoplus}_{\ell\in\tau^\vee}(\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell))^\wedge_{\bar{x}}$". The correct definition for $f^{[d\cdot\ell_0]}$ to be a *leading subseries* of f is that $f - f^{[d\cdot\ell_0]}$ has a higher degree than $f^{[d\cdot\ell_0]}$ in the natural grading defined by the ideal of definition of $\hat{\bigoplus}_{\ell\in(\sigma_0)^\vee}(\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell))^\wedge_{\bar{x}}$. Similarly, the correct definition for $f^{(d\cdot\ell_0)}$ to be the *leading term* of $f^{[d\cdot\ell_0]}$ is that $f^{[d\cdot\ell_0]} - f^{(d\cdot\ell_0)}$ (or equivalently $f - f^{(d\cdot\ell_0)}$) has a higher degree than $f^{(d\cdot\ell_0)}$ in the natural grading defined by the ideal of definition of $\hat{\bigoplus}_{\ell\in(\sigma_0)^\vee}(\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\ell))^\wedge_{\bar{x}}$. (These are abused terminologies because the leading subseries or terms might be zero.) The formal scheme $(\mathfrak{X}_{\Phi_{\mathcal{H}},\mathcal{H}}(\sigma_0))^\wedge_{\bar{x}}$ should be $(\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma_0})^\wedge_{\bar{x}}$. At the end of the paragraph, "$(\mathrm{p}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^*(\underline{\mathrm{FJ}}^{(d\cdot\ell_0)}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}})^\wedge_{\bar{x}} \cong (\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(d\cdot\ell_0))^\wedge_{\bar{x}}$" should be simply "$(\Psi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(d\cdot\ell_0))^\wedge_{\bar{x}}$". All instances of "$(\mathscr{J}_{\mathcal{H},\mathsf{pol}})^\wedge_{\bar{x}}$" should be "$(\mathscr{J}^{(d)}_{\mathcal{H},\mathsf{pol}})^\wedge_{\bar{x}}$".

- Proof of 2. of Thm. 7.3.3.4:
  - Overall, the proof by writing down a degeneration data is flawed and unnecessary. (It is flawed because we cannot determine a global choice for data coming from the abelian part. This can be fixed by working only locally, but with little modification it is possible to avoid this argument as a whole.) We have revised the proof by directly showing that the morphism is quasi-finite.
  - Paragraph 4, line 5: "$\mathrm{Bl}_{(\mathscr{J}^{(d_0)}_{\mathcal{H},\mathsf{pol}})^\wedge_{\bar{x}}}((\mathsf{M}^{\min}_{\mathcal{H}})^\wedge_{\bar{x}})$" should be "$\mathrm{NBl}_{(\mathscr{J}^{(d_0)}_{\mathcal{H},\mathsf{pol}})^\wedge_{\bar{x}}}((\mathsf{M}^{\min}_{\mathcal{H}})^\wedge_{\bar{x}})$".
  - Paragraph 6: all instances of "$d_0\cdot K^\vee_{\mathsf{pol}_{\Phi_{\mathcal{H}}}}$" should be "$\mathbf{S}_{\Phi_{\mathcal{H}}}\cap(d_0\cdot K^\vee_{\mathsf{pol}_{\Phi_{\mathcal{H}}}})$".

  - Paragraph -3, line 2: "ℓ_{gen} is a vertex ℓ_{gen}" should be "ℓ_{gen} is a vertex".

- A substantial part of the two appendices have to be rewritten. (Please consult the revision for corrections and improvements.)
- Throughout Appendix A, the notation "$-$" representing empty slots should be "$\cdot$".
- A.1.1, paragraph 1, line 2: "Zermelo-Frankel" should be "Zermelo-Fraenkel".
- Def. A.1.2.1, 2., (b): "for any three objects $X, Y, Z \in \operatorname{Ob} \mathsf{C}$" should be "for any two objects $X, Y \in \operatorname{Ob} \mathsf{C}$".
- Paragraph 1 after Def. A.2.4 should be removed because it is irrelevant.
- Rem. A.2.5: The convention that schemes are "*separated preschemes*" are replaced with schemes are "*quasi-separated preschemes*".
- Paragraph following Rem. A.2.5: All instances of "fidélement" should be "fidèlement".
- Sec. A.2, paragraph -2, line 2: "finitely many" should be removed.
- In Example A.4.5: The notations X' and X'' etc should be replaced with simply X and X' etc. (They were inconsistent.)
- In Lem. A.6.1.4: The assumption of smoothness on f is redundant (and not used in the proof).
- In Def. A.6.1.6, paragraph 2, line 2: "$U \to \mathsf{X}$" should be "$U \to \mathsf{Y}$".
- In Def. A.6.1.14: "any two open" should be "any two nonempty open".
- Def. A.6.4.1, 2., line 3: "S-morphisms from s to X" should be "isomorphism classes of S-morphisms from $\bar{s}$ to X".
- Paragraph 1 after Rem. A.6.4.2, 1., paragraph 2: $H \subset \operatorname{Aut}(x)$ should be the subgroup of automorphisms of the object x in X over k that extends to automorphisms of the object $\operatorname{Spec}(\mathscr{O}^h_{\mathsf{X},x}) \to \mathsf{X}$ of X.
- The paragraph after Notation B.1.1 contains too many unfortunate typos. We removed the whole paragraph in a revision because it is unimportant after all.
- Cor. B.1.1.11: "$p - ring$" should be "p-ring".

- Lem. B.1.1.16: The scheme S need to be excellent. If the characteristic of k (namely of $k(s)$) is zero, then Λ is the completion of the ring $\hat{\mathscr{O}}_{S,s} \underset{k(s)}{\otimes} k$ at the maximal ideal determined by $\mathrm{Spec}(k) \to S$. If the characteristic k is a prime number p, then Λ is the completion of the ring $\hat{\mathscr{O}}_{S,s} \underset{A_k(s)}{\otimes} A_k$ at the maximal ideal determined by $\mathrm{Spec}(k) \to S$.
- Proof of Prop. B.3.5, line 1: "of the lemma" should be "of the proposition".
- Thm. B.3.10, condition 4, line 4: "any ξ" should be "a ξ".
- Proof of Thm. B.3.12:
  - Paragraph 2:
    - Line 1: Should clarify that ξ and U comes from some algebraization in the proof of Theorem B.8.
    - Line 4: The notion of "entire" and its definition in the parenthesis (from line 4 to line 7, with references to EGA) are not appropriate. We now simply use "normal and integral". All subsequent instances of "entire" are replaced.
    - Line -5: "Theorem 4" should be "Theorem B.8".
- The missing/problematic index entries related to Det are fixed.
- The problematic (and indeed redundant) index entry for $\mathrm{KS}_{(G^\natural,\iota)/S/\mathsf{U}}|_{S_1}$ has been removed.

There are some other typos we have corrected but not recorded in this document. Please contact us whenever there are any suspicious instances.