TOROIDAL COMPACTIFICATIONS OF PEL-TYPE KUGA FAMILIES — ERRATA

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(1) In Def. 1.1, in the definition of $\mathrm{G}(R)$, the condition should be "$\forall x, y \in L \underset{\mathbb{Z}}{\otimes} R$", and the parenthetical remark "(If $L \neq \{0\}$ ...)" should be "(If $L \neq \{0\}$ and R is flat over $\mathbb{Z}$, then the value of r is uniquely determined by g. Hence there is little that we lose when suppressing r from the notation. However, this suppression is indeed an abuse of notation in general. For example, when $L = \{0\}$, we have $\mathrm{G} = \mathbf{G}_{\mathrm{m}}$.)"

(2) In paragraph 2 after Def. 1.15, "$\Phi_{\mathcal{H}} = (X, Y, \phi, \varphi_{-2,n}, \varphi_{0,n})$" should be "$\Phi_{\mathcal{H}} = (X, Y, \phi, \varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$".

(3) In the second paragraph following Def. 1.15, and later, "$\mathcal{H}_n$" etc should be denoted "H_n" etc.

(4) In Def. 1.17, should first define $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ to be the quotient of $\coprod \mathsf{M}_n^{\mathsf{Z}_n}$ by H_n, where the disjoint union is over representatives $(\mathsf{Z}_n, \Phi_n, \delta_n)$ (with the same (X, Y, ϕ)) in $(\mathsf{Z}_{\mathcal{H}}, \Phi_{\mathcal{H}}, \delta_{\mathcal{H}})$, and then define $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ to be the (finite étale) quotient of $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ by the subgroup of Γ_ϕ stabilizing $\Phi_{\mathcal{H}}$ (which is called $\Gamma_{\Phi_{\mathcal{H}}}$ later in Def. 1.23). In the remainder of the article, the morphism $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}} \to \mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ should be described instead as an abelian scheme torsor over the finite étale cover $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ of $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$, and similar for objects such as $\widetilde{\mathsf{M}}_{\widetilde{\mathcal{H}}}^{\widetilde{\Phi}_{\widetilde{\mathcal{H}}}}$. (See the errata to [3], a published revision of [2], on the author's website.)

(5) In (2) of Def. 1.24, should require moreover that each σ_k appearing in the closure of σ_j is a face of σ_j.

(6) In (4) of Def. 1.27, "$x, y \in \mathbf{S}_{\Phi_{\mathcal{H}}}$" should be "$x, y \in \mathbf{P}_{\Phi_{\mathcal{H}}}$".

(7) In Cond. 1.29, "γ acts as the identity" should be "a power of γ acts as the identity", and "containing σ_j" should be "containing $\gamma\overline{\sigma}_j \cap \overline{\sigma}_j$".

(8) In paragraph 3 of (2) of Thm. 1.41, $\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma}$ is incorrectly described. It should be "where the formal algebraic stack $\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma}$ (before quotient by $\Gamma_{\Phi_{\mathcal{H}},\sigma}$, the subgroup of $\Gamma_{\Phi_{\mathcal{H}}}$ formed by elements mapping σ to itself) admits a canonical structure as the completion of an affine toroidal embedding $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}(\sigma)$ (along its σ-stratum $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\sigma}$) of a torus torsor $\Xi_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ over an abelian scheme torsor $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ over a finite étale cover $\mathsf{M}_{\mathcal{H}}^{\Phi_{\mathcal{H}}}$ of the smooth algebraic stack $\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$".

(9) In paragraph 2 of (5) of Thm. 1.41, "formally étale" should be "étale" (i.e., formally étale and of finite type).

(10) In the displayed equations in Cor. 2.13, "G" and "$G^\vee$" should be "$G_{\mathsf{M}_{\mathcal{H}}}$" and "$G^\vee_{\mathsf{M}_{\mathcal{H}}}$", respectively.

(11) In Thm. 2.15, the assertion that there exists a directed partially ordered set $\mathbf{K}_{Q,\mathcal{H},\Sigma}$ parameterizing the toroidal compactifications is correct, but

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the definition of $\mathbf{K}_{Q,\mathcal{H},\Sigma}$ (and the binary relation on it) is flawed and does not make $\mathbf{K}_{Q,\mathcal{H},\Sigma}$ a directed partially ordered set as desired. There are two ways to fix this. An easier one is to weaken the assertion and only claim that there is a set with a reflexive and transitive binary relation. (If we consider the equivalence relation defined by asserting $(\widetilde{\mathcal{H}},\widetilde{\Sigma},\widetilde{\sigma}) \sim (\widetilde{\mathcal{H}}',\widetilde{\Sigma}',\widetilde{\sigma}')$ when $\widetilde{\mathcal{H}} = \widetilde{\mathcal{H}}'$, $\widetilde{\Sigma} = \widetilde{\Sigma}'$ and $[(\widetilde{\Phi}_{\widetilde{\mathcal{H}}},\widetilde{\delta}_{\widetilde{\mathcal{H}}},\widetilde{\sigma})] = [(\widetilde{\Phi}'_{\widetilde{\mathcal{H}}'},\widetilde{\delta}'_{\widetilde{\mathcal{H}}'},\widetilde{\sigma}')]$, then we may still talk about an induced partial order, which is however not directed in general.) A more elaborate one is to introduce an equivalence relation so that the equivalence classes do carry a natural directed partial order. This is carried out in the forthcoming article [4]. (This does not affect most of the assertions in the remainder of this article, but does affect some applications requiring a precise description of $\mathbf{K}_{Q,\mathcal{H},\Sigma}$ and $\succ$.)

(12) In (3e) of Thm. 2.15, "Griffith transversality" should be "Griffiths transversality".

(13) In Section 3A, $\widetilde{\mathcal{H}}$ should satisfy the following conditions:
- $\mathrm{Gr}^{\widetilde{\mathrm{Z}}}_{-1}(\widetilde{\mathcal{H}}\cap\widetilde{\mathrm{P}}'_{\widetilde{\mathrm{Z}}}(\hat{\mathbb{Z}}^\square)) = \mathrm{Gr}^{\widetilde{\mathrm{Z}}}_{-1}(\widetilde{\mathcal{H}}\cap\widetilde{\mathrm{P}}_{\widetilde{\mathrm{Z}}}(\hat{\mathbb{Z}}^\square)) = \mathcal{H}$, where $\widetilde{\mathrm{P}}'_{\widetilde{\mathrm{Z}}}(\hat{\mathbb{Z}}^\square)$ is the subgroup of $\widetilde{\mathrm{P}}_{\widetilde{\mathrm{Z}}}(\hat{\mathbb{Z}}^\square)$ consisting of elements inducing trivial actions on $\mathrm{Gr}^{\widetilde{\mathrm{Z}}}_0$. (Both equalities are conditions. Then $\mathcal{H}$ is a direct factor of $\mathrm{Gr}^{\widetilde{\mathrm{Z}}}(\widetilde{\mathcal{H}}\cap\widetilde{\mathrm{P}}_{\widetilde{\mathrm{Z}}}(\hat{\mathbb{Z}}^\square))$.)
- The splitting $\widetilde{\delta}$ defines a (group-theoretic) splitting of the surjection $\widetilde{\mathcal{H}}\cap\widetilde{\mathrm{P}}'_{\widetilde{\mathrm{Z}}}(\hat{\mathbb{Z}}^\square) \twoheadrightarrow \mathcal{H}$ induced by $\mathrm{Gr}^{\widetilde{\mathrm{Z}}}_{-1}$.

(14) In paragraph 2 of Section 3B, the "faces of $[(\widetilde{\Phi}_{\widetilde{\mathcal{H}}},\widetilde{\delta}_{\widetilde{\mathcal{H}}},\widetilde{\sigma})]$" should be "equivalence classes $[(\breve{\Phi}_{\widetilde{\mathcal{H}}},\breve{\delta}_{\widetilde{\mathcal{H}}},\breve{\tau})]$ having $[(\widetilde{\Phi}_{\widetilde{\mathcal{H}}},\widetilde{\delta}_{\widetilde{\mathcal{H}}},\widetilde{\sigma})]$ as a face".

(15) In (2) after (3.7), "$\mathsf{M}_{\mathcal{H}}$" should be "$\mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$". Here $\widetilde{\mathsf{M}}^{\breve{\Phi}_{\widetilde{\mathcal{H}}}}_{\widetilde{\mathcal{H}}} \cong \mathsf{M}^{\Phi_{\mathcal{H}}}_{\mathcal{H}}$ as finite étale covers of $\widetilde{\mathsf{M}}^{\breve{\mathsf{Z}}_{\widetilde{\mathcal{H}}}}_{\widetilde{\mathcal{H}}} \cong \mathsf{M}^{\mathsf{Z}_{\mathcal{H}}}_{\mathcal{H}}$ by the above two conditions satisfied by $\widetilde{\mathcal{H}}$.

(16) In Section 3B and later, for consistency, "$\mathfrak{X}_{\breve{\Phi}_{\widetilde{\mathcal{H}}},\breve{\delta}_{\widetilde{\mathcal{H}}},\breve{\sigma},\breve{\tau}}$" should be denoted "$\widetilde{\mathfrak{X}}_{\breve{\Phi}_{\widetilde{\mathcal{H}}},\breve{\delta}_{\widetilde{\mathcal{H}}},\breve{\sigma},\breve{\tau}}$", and "$\mathfrak{X}_{\breve{\Phi}_{\widetilde{\mathcal{H}}},\breve{\delta}_{\widetilde{\mathcal{H}}},\breve{\tau}}$" should be denoted "$\widetilde{\mathfrak{X}}_{\breve{\Phi}_{\widetilde{\mathcal{H}}},\breve{\delta}_{\widetilde{\mathcal{H}}},\breve{\tau}}$".

(17) In the definition of $\underline{\mathrm{KS}}_{\widetilde{T}_{\mathsf{S}_0}/\mathsf{S}_0}$ after (3.13), "λ" should be "$\lambda_{\widetilde{T}_{\mathsf{S}_0}}$".

(18) In Section 4A, the definition of the group $\Gamma_{\widetilde{\Phi}_{\widetilde{\mathcal{H}}},\Phi_{\mathcal{H}}}$ is incorrectly stated. It should be defined as the subgroup of $\Gamma_{\Phi_{\mathcal{H}},\tau}$ consisting of elements acting trivially on $\widetilde{X}$ and $\widetilde{Y}$, which can be identified as a subgroup of $\mathrm{Hom}_{\mathcal{O}}(\widetilde{X},X)$ of finite index (prime to $\square$).

(19) In paragraph 2 of Section 4A, after the item (3), the "cusp labels" should be "equivalence classes". In the last sentence of the same paragraph, for consistency, "translates" should be "translations".

(20) In paragraph 5 of Section 4A and later (except in Section 6C), there are instances of both Ψ and $\widetilde{\Psi}$ with similar subscripts. The Ψ's without $\widetilde{\ }$ are typographical errors.

(21) In the definition of h before Lem. 4.6, "$C_{\Phi_{\mathcal{H}},\mathsf{Z}_{\mathcal{H}}}$" should be "$C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$".

(22) In (1) and (4) of Lem. 4.6 (and at several places below), the h should be h_τ; and the morphisms should be over $\mathfrak{X}_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}},\tau}$ instead of $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$.

(23) The proof of Lem. 4.9 should be modified. It suffices to reduce to the case of principal levels, in which case the abelian scheme torsors $\widetilde{C}_{\breve{\Phi}_{\widetilde{\mathcal{H}}},\breve{\delta}_{\widetilde{\mathcal{H}}}}$ and $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ are indeed abelian schmes.

(24) In the second paragraph of the proof of Lem. 4.9, the second morphism $\to \mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$ is redundant.

(25) In the paragraph after the proof of Lem. 4.9, the nerves of the open coverings of the formal schemes (such as the open covering $\{\mathfrak{U}_{\check{\tau}}\}_{\check{\tau}\in\Sigma_{\check{\Phi}_{\widetilde{\mathcal{H}}},\check{\sigma},\tau}}$ of $\widetilde{\mathfrak{X}}_{\check{\Phi}_{\widetilde{\mathcal{H}}},\check{\delta}_{\widetilde{\mathcal{H}}},\check{\sigma},\tau}$, which was inconsistently denoted as $\mathfrak{X}_{\check{\Phi}_{\widetilde{\mathcal{H}}},\check{\delta}_{\widetilde{\mathcal{H}}},\check{\sigma},\tau}$) are naturally identified with those of the closed coverings defined by closures of cones in the unions of cones (such as the closed covering $\{\check{\tau}^{\mathrm{cl}}\}_{\check{\tau}\in\Sigma_{\check{\Phi}_{\widetilde{\mathcal{H}}},\check{\sigma},\tau}}$ of $\widetilde{\mathfrak{N}}_{\check{\sigma},\tau}$); it was incorrect to directly identify the former with unions of cones like $\widetilde{\mathfrak{N}}_{\check{\sigma},\tau}$. For our purpose it is easier to work with the latter. (Thus we turned the cohomology of the structural sheaves to the cohomology of spaces which are unions of cones.)

(26) In the paragraph preceding (4.10), it is not appropriate to call $\mathscr{H}^d(\mathscr{M})$ a "local system" as it is not locally constant. It is a coefficient system on the nerve, which can be viewed as a constructible sheaf on the union of cones. The remainder of the argument applies nevertheless.

(27) In the proof of Lem. 4.16, it should be $\underset{\check{\ell}\not\in\check{\tau}^\vee}{\cup}\check{\tau}^{\mathrm{cl}} = \underset{\check{\ell}\not\in\check{\tau}^\vee}{\cup}\check{\tau}$, the complement of $\underset{\check{\ell}\in\check{\tau}^\vee}{\cup}\check{\tau}^{\mathrm{cl}} = \underset{\check{\ell}\in\check{\tau}^\vee}{\cup}\check{\tau}$, that is a *contractible or empty* subset of $\widetilde{\mathfrak{N}}_{\check{\sigma},\tau}$ for any given $\check{\ell}\in\check{\sigma}^\perp$ (being *convex* up to homotopy equivalence; the argument is standard but the convexity is, however, not literally true).

(28) In the proof of Lem. 4.16, since the nerves involve infinitely many open subsets, we should explain why we can work weight-by-weight as in [1, Ch. I, §3]. This is because, up to replacing the cone decompositions with locally finite refinements not necessarily carrying $\Gamma_{\check{\Phi}_{\widetilde{\mathcal{H}}}}$-actions (which is harmless for proving this lemma), we can compute the cohomology as a limit using unions of finite cone decompositions on expanding convex polyhedral subcones (by proving inductively that the cohomology of one degree lower has the desired properties, using [5, Thm. 3.5.8]); then we can consider the associated graded pieces defined by the completions, and work weight-by-weight, because taking cohomology commutes with taking infinite direct sums for Čech complexes defined by finite coverings, as desired.

(29) In the sentence after (4.34), the N' should be N.

(30) In the last paragraph preceding Rem. 4.42, "Griffith transversality" should be "Griffiths transversality".

(31) In Section 4D, the tensor products are not denoted over the correct bases. The schemes N and $\mathsf{M}_{\mathcal{H}}$ should be replaced with their compactifications $\mathsf{N}^{\mathrm{tor}}$ and $\mathsf{M}_{\mathcal{H}}^{\mathrm{tor}}$, respectively.

(32) In Section 5B, first sentence, Diff^{-1} should be $\mathrm{Diff}^{-1}_{\mathcal{O}'/\mathbb{Z}}$.

(33) In the paragraph preceding Lem. 5.16, (both instances of) "formally étale" should be "étale" (i.e., formally étale and of finite type).

(34) The proof of Lem. 5.18 should be modified (as in the proof of Lem. 4.9). It suffices to reduce to the case of principal levels, in which case the abelian scheme torsors $\widetilde{C}_{\check{\Phi}_{\widetilde{\mathcal{H}}},\check{\delta}_{\widetilde{\mathcal{H}}}}$ and $C_{\Phi_{\mathcal{H}},\delta_{\mathcal{H}}}$ are indeed abelian schmes, so that the kernel C is defined.

(35) In Def. 6.2, the pairings should have the subscript "can."

(36) In the beginning of Section 6C, "$(\mathsf{Z}_n, \Phi_n, \delta_n)$" should be "$(\mathsf{Z}_{\mathcal{H}}, \Phi_{\mathcal{H}}, \delta_{\mathcal{H}})$", and "$[(\mathsf{Z}_n, \Phi_n, \delta_n)]$" should be "$[(\mathsf{Z}_{\mathcal{H}}, \Phi_{\mathcal{H}}, \delta_{\mathcal{H}})]$".
(37) In Rem. 6.18, "$\mathsf{M}_n^{\mathsf{Z}_n}$" should be "$\mathsf{M}_{\mathcal{H}}^{\mathsf{Z}_{\mathcal{H}}}$".

References

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