VANISHING THEOREMS FOR TORSION AUTOMORPHIC SHEAVES ON COMPACT PEL-TYPE SHIMURA VARIETIES — ERRATA

KAI-WEN LAN AND JUNECUE SUH

(1) In Def. 1.1, the parenthetical remark "(If $L \neq \{0\}$, then the value of r is uniquely determined by g.)" is incorrect and should be "(This is an abuse of notation, because r is not always determined by g.)"
(2) In Lem. 1.20, "an object in $W \in \mathrm{Rep}_R(\mathrm{M}_1)$" should be "an object $W \in \mathrm{Rep}_R(\mathrm{M}_1)$".
(3) In the second paragraph of Section 2.1, "simple" should be "indecomposable", and "every projective $\mathcal{O}_1$-module" should be more precisely "every finitely generated projective $\mathcal{O}_1$-module".
(4) In the third paragraph in Section 2.4, "reductive group scheme G over $\mathrm{Spec}(R_1)$" should be "reductive group scheme G_1 over $\mathrm{Spec}(R_1)$".
(5) In the paragraphs preceding Def. 2.26 and 2.27, the action of the distribution algebras are redundant.
(6) In the two paragraphes after Def. 2.29, we should only define the split objects over $\mathbb{Z}_{(p)}$, but not $\mathbb{Z}$, to avoid saying that split (even) orthogonal groups are reductive at 2. And, again, the action of distribution algebras are redundant. Moreover, "minimal among admissible lattices" should be more precisely only "minimal among the admissible lattices containing the same highest weight vector".
(7) In Section 4.1, when defining and studying the Hodge filtration on $\underline{H}^i_{\mathrm{dR}}(A^n/\mathsf{M}_{\mathcal{H},1})$, all instances of "$\Omega^\bullet_{A^n/\mathsf{S}_1}$" should be "$\Omega^\bullet_{A^n/\mathsf{M}_{\mathcal{H},1}}$".
(8) The assertion in Prop. 7.10 that "$\underline{W}_\nu$ has trivial tensor square as a line bundle over $\mathsf{M}_{\mathcal{H},1}$ if its coefficients $(k_\tau)_{\tau\in\Upsilon}$ of ν satisfy $k_\tau + k_{\tau\circ c} = 0$" is too strong. The correct assertion is that "$\underline{W}_\nu$ defines a torsion element in the Picard group of $\mathsf{M}_{\mathcal{H},1}$ if its coefficients $(k_\tau)_{\tau\in\Upsilon}$ of ν satisfy the condition that $k_\tau + k_{\tau\circ c} = 0$". The simplest proof is to use the complex fiber of $\mathsf{M}_{\mathcal{H}}$, which we spell out as follows, for the convenience of the reader:

Proof. Suppose that the condition in the proposition holds. Then the representation W_ν is trivial after pullback to the complexification of the maximal compact subgroup of $\mathrm{G}(\mathbb{R})$, and hence the pullback $\underline{W}_{\nu,\mathbb{C}}$ of $\underline{W}_\nu$ under any ring homomorphism $R_1 \to \mathbb{C}$ is trivial, by the comparison in [1, §5.2]. Suppose R is any discrete valuation ring finite flat over R_1 such that $K := \mathrm{Frac}(R)$ is Galois over $K_1 = \mathrm{Frac}(R_1)$, and such that the connected components of $\mathsf{M}_{\mathcal{H},K} = \mathsf{M}_{\mathcal{H},1} \underset{R_1}{\otimes} K$ are geometrically connected. Let k and ϖ denote the residue field and uniformizer of R, respectively. Let M to be any connected component of $\mathsf{M}_{\mathcal{H},1} \underset{R_1}{\otimes} R$, and let $\underline{W}$ denote

Published in Duke Math. J. 161 (2012), no. 6, pp. 1113–1170, doi:10.1215/00127094-1548452.

the pullback of $\underline{W}_\nu$ to M. By taking norms with respect to the action of $\mathrm{Gal}(K/K_1)$, it suffices to show that $\underline{W}$ is trivial. Since the structural morphism $\mathsf{M}_{\mathcal{H}} \to \mathsf{S}_0 = \mathrm{Spec}(\mathcal{O}_{F_0,(p)})$ is proper and smooth, all fibers of $\mathsf{M} \to \mathrm{Spec}(R)$ are geometrically integral, so that $H^0(\mathsf{M}, \mathscr{O}_{\mathsf{M}}) \cong R$. Since $\underline{W}_{\nu,\mathbb{C}}$ is trivial, both $H^0(\mathsf{M}, \underline{W})$ and $H^0(\mathsf{M}, \underline{W}^\vee)$ are nonzero. Suppose s and t are nonzero elements of these two groups, respectively, whose product st defines an element of $H^0(\mathsf{M}, \mathscr{O}_{\mathsf{M}}) \cong R$. Let $V(s)$ (resp. $V(t)$) denote the closed subsets of M where the morphism $\mathscr{O}_{\mathsf{M}} \to \underline{W}$ (resp. $\underline{W} \to \mathscr{O}_{\mathsf{M}}$) defined by s (resp. t) fails to be an isomorphism. Suppose $st = \varpi r$ for some $r \in R$, so that $\mathsf{M} \underset{R}{\otimes} k \subset V(s) \cup V(t)$. Since $\mathsf{M} \underset{R}{\otimes} k$ is integral, either $\mathsf{M} \underset{R}{\otimes} k \subset V(s)$ and $s = \varpi s'$ for some $s' \in H^0(\mathsf{M}, \underline{W})$, or $\mathsf{M} \underset{R}{\otimes} k \subset V(t)$ and $t = \varpi t'$ for some $t' \in H^0(\mathsf{M}, \underline{W}^\vee)$. Up to replacing s with s' or t with t', and by repeating this process, we may assume that $st \in R^\times$, in which case $V(s) = \emptyset = V(t)$, and so $\underline{W}$ is trivial, as desired. $\square$

(9) In Thm. 8.7(1), both instances of "$\mathrm{X}_{\mathrm{G}_1}^{+,<_{\mathrm{w}}p}$" should be "$\mathrm{X}_{\mathrm{G}_1}^{+,<_{\mathrm{re}}p}$" (which are the ones used in Cor. 7.4, on which this statement is based). Also, "$A^i_\nu(\mathcal{H}; R) = 0$ for every $i > d - l(w(\nu))$ (resp. $i < d - l(w(\nu))$)" should be "$A^i_\nu(\mathcal{H}; R) = 0$ for every $i > d - l(w(\nu - \nu_+))$ (resp. $i < d - l(w(\nu + \nu_-))$)".

(10) In Thm. 8.20, "$A^i_\nu(\mathcal{H}; \mathbb{C}) = 0$ for every $i > d - l(w(\nu))$ (resp. $i < d - l(w(\nu))$)" should be "$A^i_\nu(\mathcal{H}; \mathbb{C}) = 0$ for every $i > d - l(w(\nu - \nu_+))$ (resp. $i < d - l(w(\nu + \nu_-))$)".

(11) In Rem. 8.21, "$\mu(\nu)$ is regular" should be "ν is cohomological and $\mu(\nu)$ is regular".

References

[1] K.-W. Lan, *Comparison between analytic and algebraic constructions of toroidal compactifications of PEL-type Shimura varieties*, J. Reine Angew. Math. **664** (2012) 163–228.

Current address: University of Minnesota, Minneapolis, MN 55455, USA
Email address: kwlan@math.umn.edu
Current address: Harvard University, Cambridge, MA 02138, USA
Email address: jsuh@math.harvard.edu