Robust Locally Linear Analysis with Applications

to Image Denoising and Blind Inpainting

Yi (Grace) Wang

School of Mathematics
University of Minnesota

May 21, 2012

Joint work with Arthur Szlam and Gilad Lerman

Data analysis: massive and high-dimensional data sets

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  - Web data;
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- Can work with low-dimensional and sparse structures. e.g.,
  - Low rank;
  - Sparsity.

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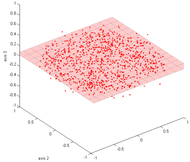

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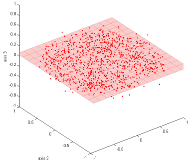

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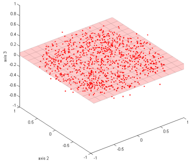

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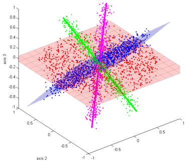

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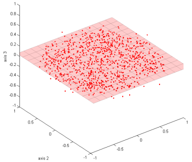

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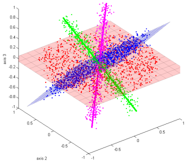

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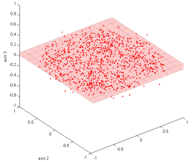

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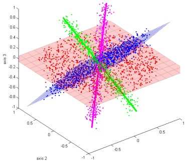

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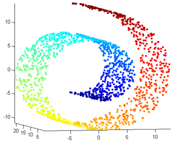

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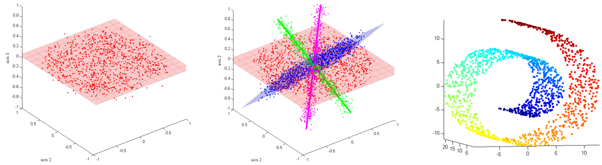

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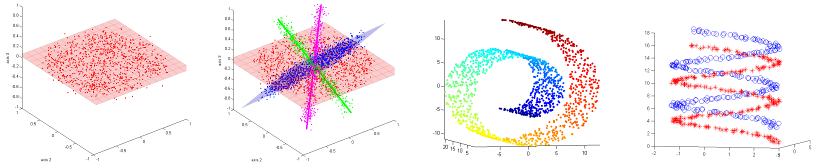

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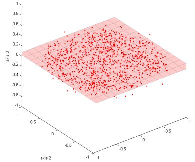

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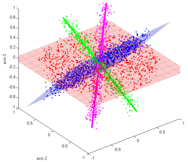

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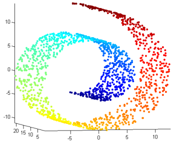

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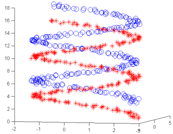

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Natural image represented by multiple subspaces [Yu, Sapiro and Mallat 2010]

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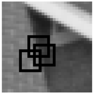
Data matrix formed by stacking overlapping patches into columns:



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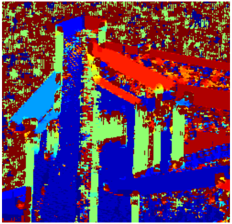

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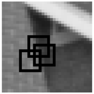

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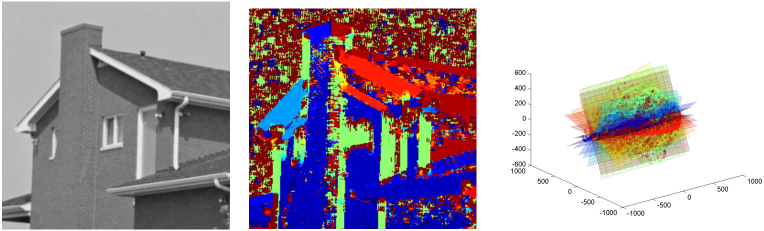

(a) *House* (b) Clustered patches (c) Projection on 3D

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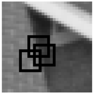

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(c) Sctraches + Impls noise

Outline

Recover a single subspace from data with corruptions

Definition

Given data sampled from a low-dimensional subspace, possibly corrupted with Gaussian noise and impulsive noise, the goal is to recover the underlying low-dimensional subspace.

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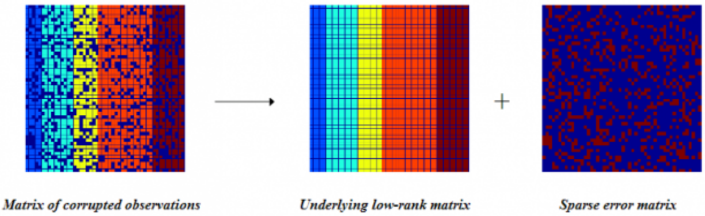

from Nuit Blanche

Existing methods

- Principle component pursuit(PCP) [Candès, Li, Ma and Wright. 2009]
  - $\mathbf{X} = \mathbf{L} + \mathbf{S}$:

$$\min_{\mathbf{L},\mathbf{S}} \|\mathbf{L}\|_* + \lambda \|\mathbf{S}\|_1 \tag{1}$$

    - ★ $\|\cdot\|_*$: nuclear norm, i.e. sum of singular values.
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- A variant of PCP: Low Rank Matrix Fitting(LMaFit) (Wen, Yin and Zhang. 2010)
  - $\mathbf{X} \approx \mathbf{B}_{m\times d}\mathbf{C}_{d\times n} + \mathbf{S}$:
$$\min_{\mathbf{B},\mathbf{C},\mathbf{S}} \|\mathbf{S} + \mathbf{BC} - \mathbf{X}\|_F^2 + \lambda\|\mathbf{S}\|_1. \tag{3}$$

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  - ALS iterates between solving for $\mathbf{B}$,$\mathbf{C}$ and $\mathcal{I}$.

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- K-ALS algorithm **converges**.

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Main Theorem

For simplicity, we denote $(\{\mathbf{B}_k\}_{k=1}^K, \{\mathbf{C}_k\}_{k=1}^K, \{\mathcal{I}_k\}_{k=1}^K, [\mathbf{X}_1, \cdots, \mathbf{X}_K])$ by Ω and $J(\Omega^t)$ is the objective function for K-ALS at step t.

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4. *either Ω^t converges or the accumulation points form a continuum.*

Sketch of the proof

Key steps:

- The algorithm is **strictly monotonic** w.r.t. the energy function.
  - Suffices to show the algorithm is *monotone* and *single-valued*.
  - $J(\Omega^{t+1}) \leq J(\Omega^t)$.
  - Regularized by: $\min_{\mathbf{c}} \|\tilde{\mathbf{B}}\mathbf{c} - \tilde{\mathbf{x}}\|_2^2 + \lambda\|\mathbf{c}\|_2^2$. ($\tilde{\mathbf{x}}$: uncorrupted elements in $\mathbf{x}$; $\tilde{\mathbf{B}}$: the corresponding rows of $\mathbf{B}$.)

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- The iterates produced by the algorithm lie in a **compact** set.
  - $\|\mathbf{B}\|_F^2$ and $\|\mathbf{C}\|_F^2$ are bounded by $J(\Omega^0)$.

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  - Regularized by: $\min_{\mathbf{c}} \|\tilde{\mathbf{B}}\mathbf{c} - \tilde{\mathbf{x}}\|_2^2 + \lambda\|\mathbf{c}\|_2^2$. ($\tilde{\mathbf{x}}$: uncorrupted elements in $\mathbf{x}$; $\tilde{\mathbf{B}}$: the corresponding rows of $\mathbf{B}$.)
- The iterates produced by the algorithm lie in a **compact** set.
  - $\|\mathbf{B}\|_F^2$ and $\|\mathbf{C}\|_F^2$ are bounded by $J(\Omega^0)$.
- The algorithm is **closed**.
  - By continuity of J w.r.t. $\mathbf{B}$ and $\mathbf{C}$.

Outline

1. Introduction
   - Data analysis: massive data sets and high dimensions
   - Image denoising and blind inpainting
2. Algorithms
3. Mathematical Analysis
4. Applications

Removing impulsive noise and blind inpainting

Recover images corrupted with

1. i.i.d. additive Gaussian noise with standard deviation σ,
2. a percentage of p_0 random corruptions at random pixels (**impulsive noise**).
3. and for **blind inpainting** (inpaint without info of locations), further degraded by scratches.

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Process images by:

1. Forming the actual data matrix $\mathbf{X}$ by stacking the vectors representing overlapping 8×8 patches as columns.
2. Transforming estimated $\tilde{\mathbf{X}}$ back to the image (after enhancing it) by averaging values of all coordinates representing the same pixel.

Examples revisit

(a) Impulsive noise denoising (b) Blind inpainting

Methods to compare with:

- K-PCP(capped): learning K-subspaces by PCP(capped).

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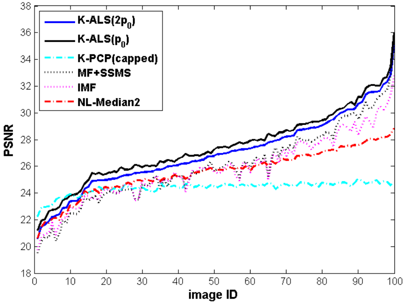
- NL-Median: a variant of non-local means ([Buades, Coll and Morel 2005]).

Results of removing impulsive noise, PSNR on 100 images



(a) $p_0 = 5\%, \sigma = 20$

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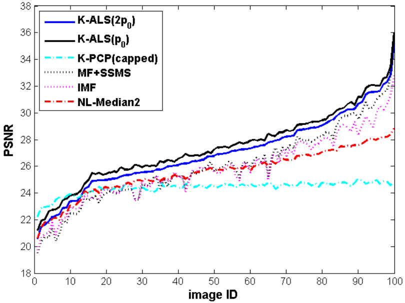

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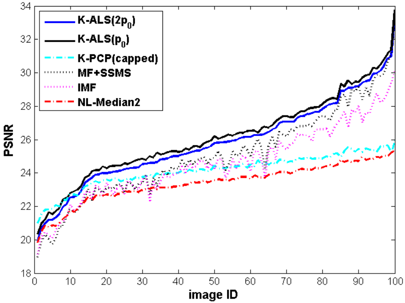
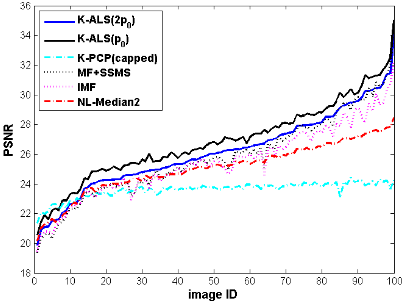


(b) $p_0 = 5\%, \sigma = 30$

Results of removing impulsive noise, PSNR on 100 images



(a) $p_0 = 10\%, \sigma = 20$

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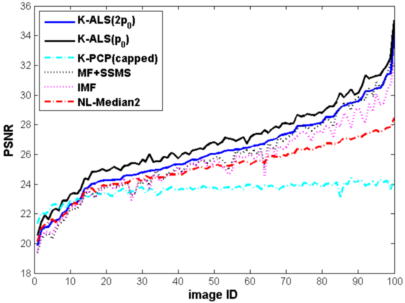

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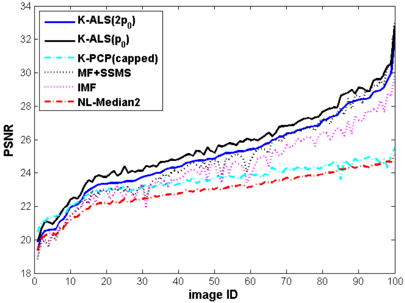


(b) $p_0 = 10\%, \sigma = 30$

Visualization for removing impulsive noise

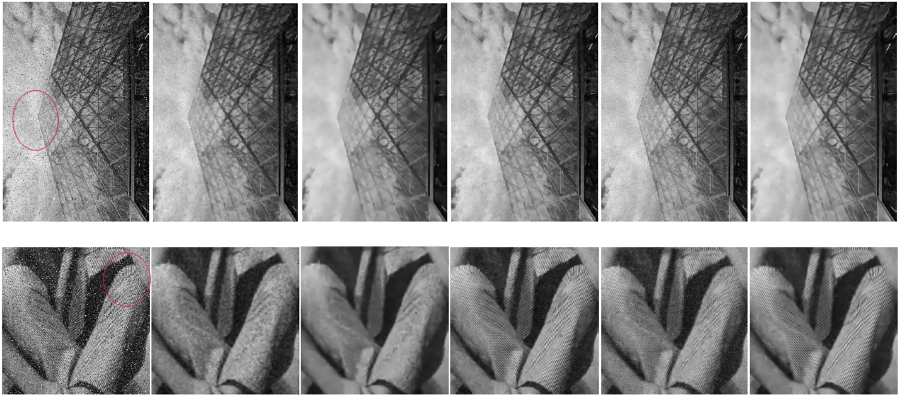
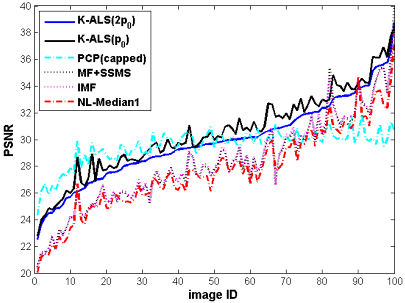


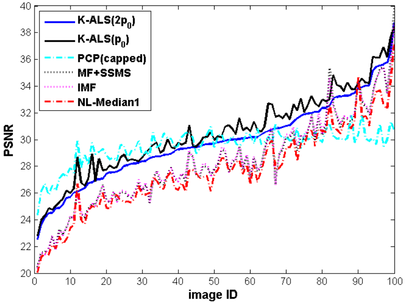
Figure: From left to right: noisy images, IMF, MF+SSMS, NL-Median1, K-PCP(capped) and K-ALS($2p_0$).

Results of blind inpainting, PSNR on 100 images



(a) $p_0 = 0\%, \sigma = 5$

Results of blind inpainting, PSNR on 100 images



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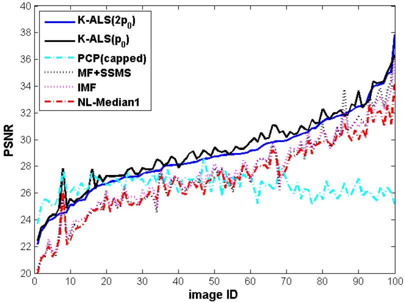
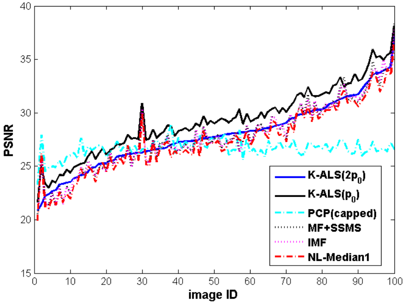


(b) $p_0 = 0\%, \sigma = 10$

Results of blind inpainting, PSNR on 100 images



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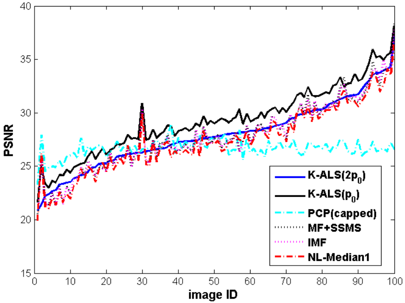
Results of blind inpainting, PSNR on 100 images



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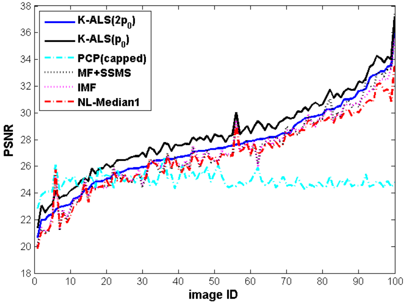


(b) $p_0 = 5\%, \sigma = 10$

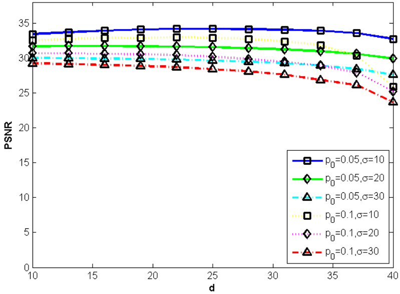
Visualization for blind inpainting

Figure: From left to right, from top to bottom: noisy images, IMF, MF+SSMS, NL-Median2, PCP(capped) and K-ALS($2p_0$).

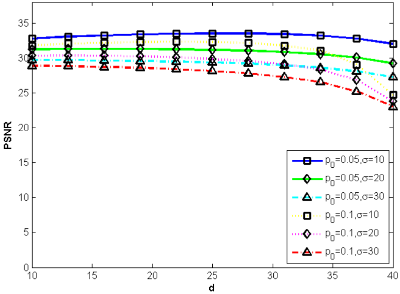
Thank You!

- **Contact: wangx857@umn.edu**
- **Preprint and code are available on**
  `http://www.math.umn.edu/~wangx857/`

Effect of d of K-ALS algorithm on denoising.

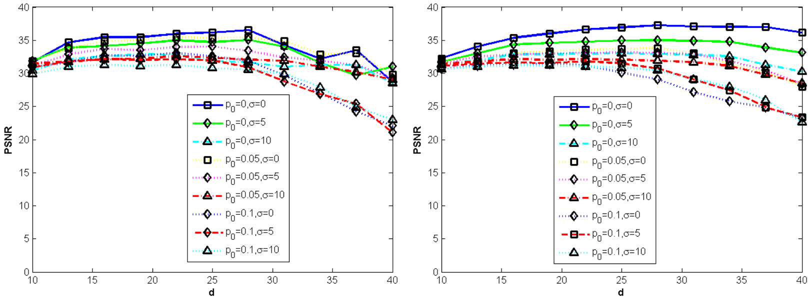


(a) *House*



(b) *Lena*

Effect of *d* of *K*-ALS algorithm on blind inpainting.

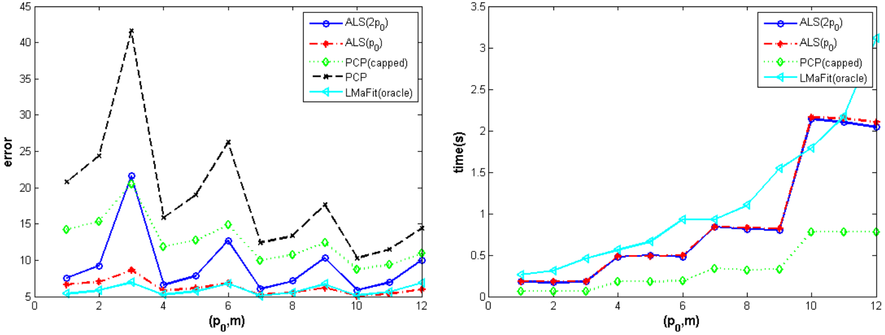


(a) *House* (b) *Lena*

Initialization

(a) *At direction* 30°

Numerical simulations with a single subspace



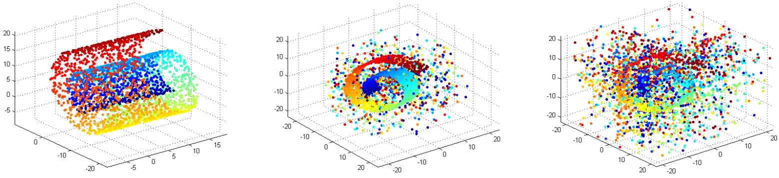
(a) Relative fitting error (b) Computing time

Figure: The computing time of PCP is about 200 times that of ALS.

PCP(capped): PCP capped at d of rank for computational efficiency.

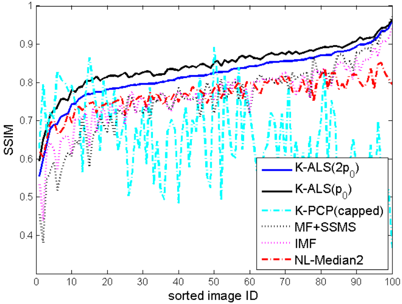
Challenges in very noisy cases

It is difficult when data is largely corrupted by Gaussian noise, outliers, **impulsive noise** **(corrupted at coordinates)**, and etc.



(a) Clean (b) 40% outliers (c) 40% impulsive noise

Results of removing impulsive noise, SSIM on 100 images



(a) $p_0 = 5\%, \sigma = 20$

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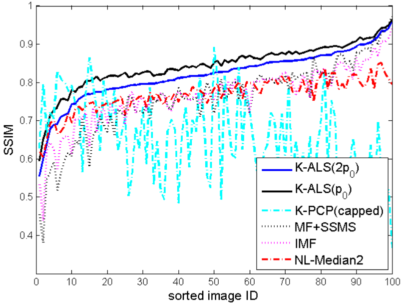

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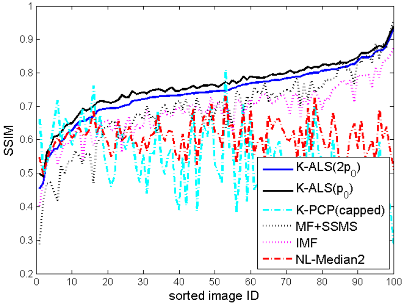
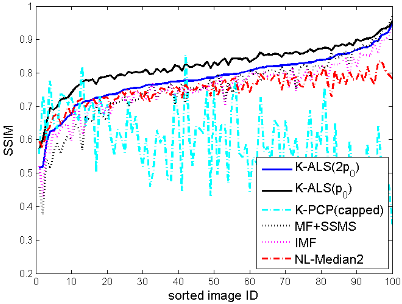


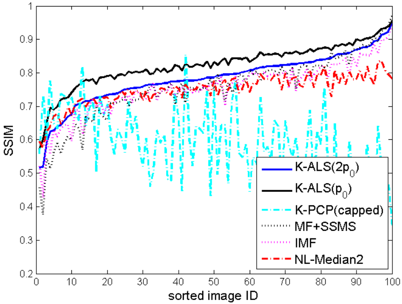
(b) $p_0 = 5\%, \sigma = 30$

Results of removing impulsive noise, SSIM on 100 images



(a) $p_0 = 10\%, \sigma = 20$

Results of removing impulsive noise, SSIM on 100 images



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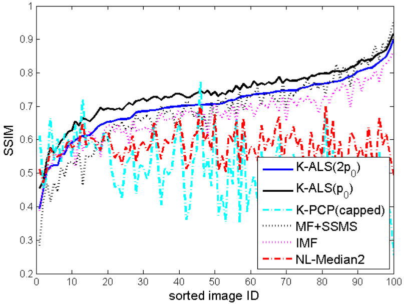
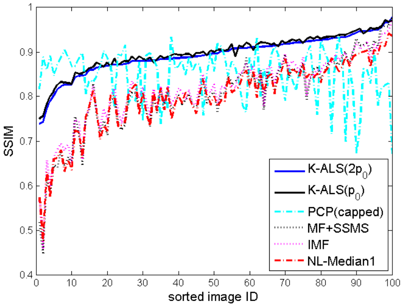


(b) $p_0 = 10\%, \sigma = 30$

Results of blind inpainting, SSIM on 100 images



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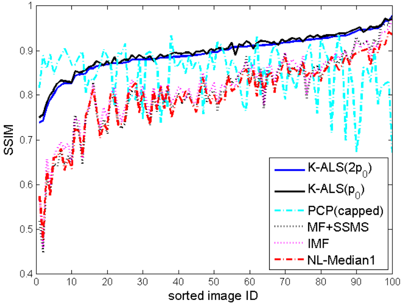

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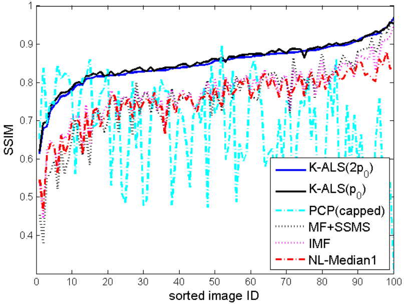

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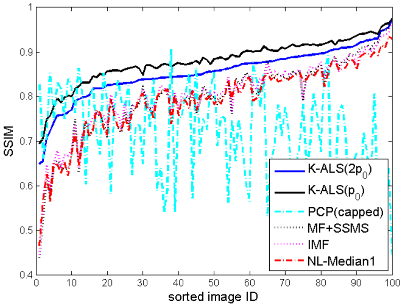

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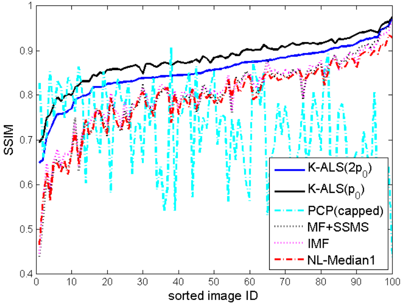

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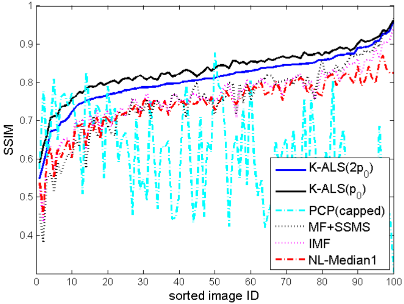

(b) $p_0 = 5\%, \sigma = 10$

Complexity

$O(m^2 n) + O(Kdmn) + O(d^4 mn)$: where m is the number of pixels in each patch, n is number of patches, K is the number of subspaces, and d is the intrinsic dimension of the subspace.