Introduction to the Mathematics of Image and Data Analysis

Math 5467, Spring 2015

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Time: Tue & Thu, 1:25 – 2:40PM

Office Hours: Tue 2:40-3:40PM & Thu 11:00AM – 12:00PM

Location: Amundson Hall, Room 116

Course description and objectives

The course will introduce students to mathematical techniques that have been very successful in analyzing signals and images. In terms of techniques we will mainly focus on Fourier and Wavelet analysis and some practical numerical algorithms, whereas in terms of data we will concentrate on images. We will try to understand related mathematical theory and modeling, some relevant topics in numerical analysis, problems in image analysis and gain some practical experience implementing the methods to real data.

Prerequisites

Linear algebra, Calculus (Multivariable calculus is recommended), Basic background of probability and analysis (recommended).

Course structure, workload and grades

Grade will be based on the following components:
10% Homework
10% Project
10% Class Participation
20% Exam 1
20% Exam 2
30% Final Exam

Homework will be assigned weekly and needs to be submitted on Thursdays.
A project will be assigned individually and can include either a report on a relevant research paper or write up of software for a relevant problem.
Class participation includes attendance and contribution to class discussion (either by active participation or by preparing or sharing relevant slides or applets).
Exam1 will tentatively take place during the class of Tue Mar. 3rd.
Exam2 will tentatively take place during the class of Tue Apr. 14th.
Final exam will take place on Wed, May 13th, 1:30-3:30pm (tentatively in classroom).

Preliminary Syllabus

Below is a list of topics. I plan to cover most of them (but not all of them) depending on class background and interests and the need to supplement further mathematical background.

- Introduction: Quick introduction to data and image analysis (the emphasis on this class will be on the latter).
- Singular value decomposition and applications: SVD, pseudoinverses, matrix l_2 and F_r norms, relation to least squares and orthogonal least squares, linear regression and the bias-variance trade off, compression of images.
- Hilbert spaces and normed spaces: Properties, orthogonal bases for L_2, in particular Fourier series, other norms and their effect on analysis of signals and images.
- Basic Fourier analysis and Image Analysis in the frequency domain: Fourier transform and its properties (1d and 2d), high and low pass frequency domain filters, the fast Fourier transform.
- Convolution and low/high pass spatial filters: Properties, approximate identities, smoothing and sharpening images in the spatial domain, use of differential operators (e.g. Laplacian) in the spatial and frequency domain.
- Image restoration (application): model for image restoration, noise reduction by spatial and frequency filtering, inverse filtering and Wiener filtering.
- Mathematical perspective and background on wavelets: Orthonormal bases generated by a single function (Gabor, Haar, Shannon), Balian-Low Theorem, Local sine and cosine bases, Lemarie-Meyer wavelets, MRA, constructions of wavelets from MRA
- More on wavelets and their application to image analysis: Subband coding, the continuous and discrete wavelet transforms (including the 2-dimensional case), fast wavelet transform, wavelet packets, wavelets via lifting, application to images.

- Image compression: image compression models, basics of information theory, error-free compression, lossy compression (including wavelet compression), JPEG.
- Sparse approximation and compressed sensing

References

Unfortunately, there is no single textbook that covers the course material with similar emphases. I will try to provide additional notes.
Relevant textbooks that you may find helpful:

1. Digital Image Processing by R. C. Gonzalez and R. E. Woods, Third Edition. It covers some of the practical aspects of imaging and I use some of its exercises. Though my mathematical explanations will differ from this book.
2. Digital Image Processing Using Matlab by R. C. Gonzalez, R. E. Woods and S. L. Eddins. This book discusses Matlab functions and their use for implementing some of the problems discussed in class.
3. Applied Numerical Linear Algebra by James W. Demmel. I will use the relevant chapter of this book for teaching SVD.
4. Fourier Analysis: An Introduction by E. M. Stein and R. Shakarchi. This book treats Fourier analysis formally assuming only beginning knowledge of mathematical analysis.
5. A First Course on Wavelets by E. Hernandez and G. Weiss.I will use it in order to supplement the theory of wavelets.
6. Ripples in Mathematics: The Discrete Wavelet Transform by A. Jensen and A. Cour-Harbo. Another perspective on wavelets (more applied)

Additional references and research papers will be provided in class