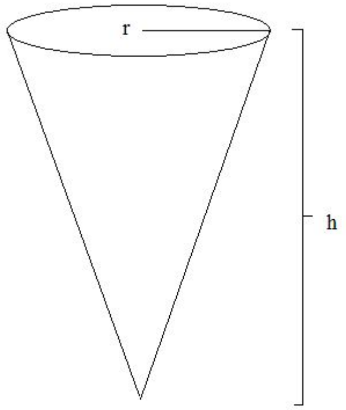
MATH 1271

Tuesday 3 March 2015

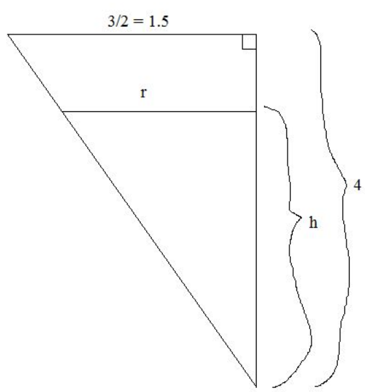
Groupwork

1. A conical paper cup 3 inches across the top and 4 inches deep is full of water. The cup springs a lead at the bottom and loses water at the rate of 2 cubic inches per minute. How fast is the water level dropping at the instant when the water is exactly 3 inches deep? Express the answer in inches per minute. [Hint: The formula for volume of a cone of base radius r and height h is $\frac{\pi}{3}r^2h$.]

   We start by drawing a picture:

   We have the formula $V = \frac{\pi}{3}r^2h$ but if we were to differentiate this with respect to time, we will pick up a $\frac{dr}{dt}$ term and we do not know how the radius is changing with respect to time. So instead we will write r in terms of h. We will do this by similar triangles below:

   Then we have that $\frac{r}{h} = \frac{1.5}{4} = \frac{3}{8}$ so $r = \frac{3h}{8}$. Substituting this into the equation for volume yields

$$
\begin{aligned}
V &= \frac{\pi}{3}(\frac{3h}{8})^2h \\
&= \frac{\pi}{3}(\frac{9}{64})h^3 \\
\frac{dV}{dt} &= \pi(\frac{9}{64})h^2\frac{dh}{dt}
\end{aligned}
$$

   But we know that $\frac{dV}{dt} = -2$ (because we are losing water) and $h = 3$ so plugging in these values and rearranging the above equation leads to our solution

$$
\frac{dh}{dt} = \boxed{\frac{-128}{81\pi}\text{in/min}}
$$

2. Determine the approximate linearization for $f(x) = \sqrt[3]{x}$ at $x = 8$. Use linearization to approximate the value of $\sqrt[3]{8.05}$ and $\sqrt[3]{25}$. Which do you think is closer to the true value?

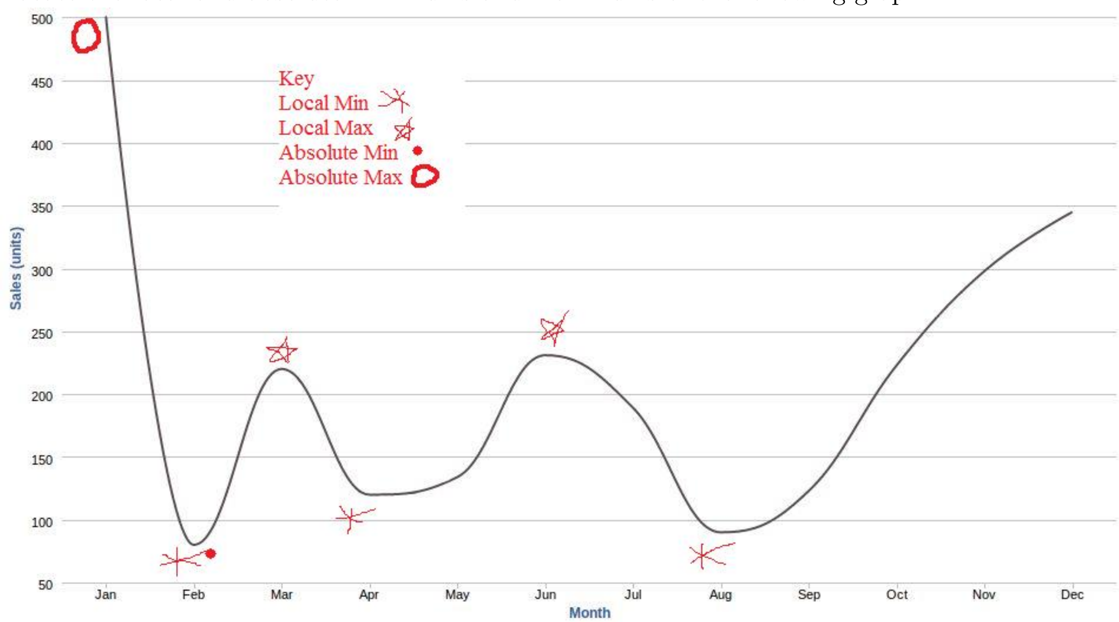
$$L(x) = 2 + \frac{1}{12}(x - 8)$$
$$L(8.05) = 2.0041\bar{6}$$
$$L(25) = 3.41\bar{6}$$

Of course the linearization of $\sqrt[3]{8.05}$ at $x = 8$ will be closer than the linearization of $\sqrt[3]{8.05}$ at $x = 8$ since 8.05 is closer to 8 than 25 is to 8.

3. Compute dy and Δy if $y = \cos(x^2 + 1) - x$ as x changes from $x = 2$ to $x = 2.03$.

$$dy = 0.08507$$
$$\Delta y = 0.08358$$

4. Locate the local and absolute minimums and maximums of the following graph:



5. Determine the absolute extrema for the function $f(x) = 2x^3 + 3x^2 - 12x + 4$ on $[-4, 2]$.

Absolute Minimum is at -4 and is -28

Absolute Maximum is at -2 and is 24

6. A ladder 13 feet long is leaning against the side of a building. If the foot of the ladder is pulled away from the building at a constant rate of 8 inches per second, how fast is the area of the triangle formed by the ladder, the building, and the ground changing (in feet squared per second) at the instant when the top of the ladder is 12 feet above the ground?

Since there is an example that is very similar to this one in the textbook and in the lecture notes, I will just give you the answer and ask you to fill in the details yourself.

$$\boxed{\frac{119}{36}\text{ft}^2/\text{sec}}$$

7. Use a linear approximation (or differentials to estimate the number $\ln(e + 0.01)$.

$$\ln(e + 0.01) \approx 1.01$$

8. Find the critical numbers of $\dfrac{x-4}{3x^2 - 5x + 2}$.

The critical numbers are $\{-2, \frac{1}{3}, 4 + \sqrt{10}, 4 - \sqrt{10}\}$