Math 1901
Freshman Seminar

Mathematical Climate Models

Fall 2024
1:00 - 2:15 Mondays and Wednesdays
Vincent Hall 213

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course website
https://www-users.cse.umn.edu/~mcgehee/Course/Math1901/

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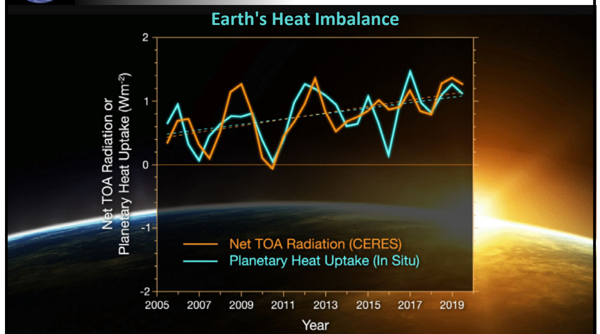

https://www.nasa.gov/centers-and-facilities/langley/joint-nasa-noaa-study-finds-earths-energy-imbalance-has-doubled/

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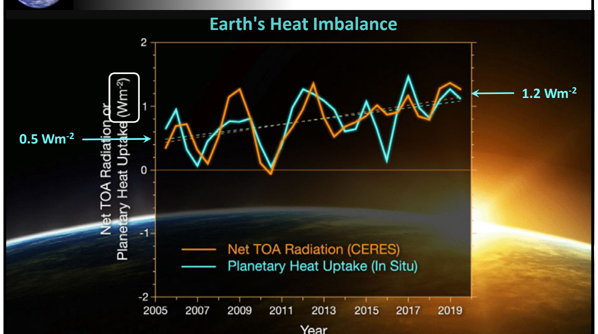

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Math 1901
Earth's Heat Imbalance

James Hansen

James Hansen arrested at a demonstration outside the White House, August 29, 2011

Hansen giving testimony before the United States Congress in 1988.

https://en.wikipedia.org/wiki/James_Hansen

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www.sciencemag.org SCIENCE VOL 308 3 JUNE 2005

RESEARCH ARTICLES

Earth's Energy Imbalance: Confirmation and Implications

James Hansen,[1,2*] Larissa Nazarenko,[1,2] Reto Ruedy,[3] Makiko Sato,[1,2] Josh Willis,[4] Anthony Del Genio,[1,5] Dorothy Koch,[1,2] Andrew Lacis,[1,5] Ken Lo,[3] Surabi Menon,[6] Tica Novakov,[6] Judith Perlwitz,[1,2] Gary Russell,[1] Gavin A. Schmidt,[1,2] Nicholas Tausnev[3]

Our climate model, driven mainly by increasing human-made greenhouse gases and aerosols, among other forcings, calculates that Earth is now absorbing 0.85 ± 0.15 watts per square meter more energy from the Sun than it is emitting to space. This imbalance is confirmed by precise measurements of increasing ocean heat content over the past 10 years. Implications include (i) the expectation of additional global warming of about 0.6°C without further change of atmospheric composition; (ii) the confirmation of the climate system's lag in responding to forcings, implying the need for anticipatory actions to avoid any specified level of climate change; and (iii) the likelihood of acceleration of ice sheet disintegration and sea level rise.

equal forcing by CO_2 (9). F_e is an energy flux change arising in response to an imposed forcing agent. It is constant throughout the atmosphere, because it is evaluated after atmospheric temperature has been allowed to adjust to the presence of the forcing agent.

The largest forcing is due to well-mixed greenhouse gases (GHGs)—CO_2, CH_4, N_2O, CFCs (chlorofluorocarbons)—and other trace gases, totaling 2.75 W/m² in 2003 relative to the 1880 value (Table 1). Ozone (O_3) and stratospheric H_2O from oxidation of increasing CH_4 bring the total GHG forcing to 3.05 W/m² (9). Estimated uncertainty in the total GHG forcing is ~15% (*11*, *12*).

Atmospheric aerosols cause climate forcings by reflecting and absorbing radiation, as well as through indirect effects on cloud cover and cloud albedo (*11*). The aerosol scenario in our model uses estimated anthropogenic emissions from fuel use statistics and includes temporal changes in fossil-fuel use technologies (*13*). Our parameterization of aerosol

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Ice sheet melting to raise sea level 1 meter. Assume ice starts at –10°C and ends at mean ocean surface temperature (+15°C). Energy required is 100 cal/g (80 cal/g for melting). Energy for 1 meter of sea level: 100g/cm² × 100cal/g × 4.19 joules/cal × area Earth × 0.7 ~1.5 × 10^{23} joules ~ **9.3 W yr/m²**.

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Land surface warming by 1°C: The depth of penetration of a thermal wave into the Earth's crust in 10 years, weighted by ΔT, is ~10 m. With density ~ 3 g/cm³, heat capacity ~ 0.2 cal/g/°C, and 0.29 fractional land coverage, land heat storage is 10^3 cm ×3 g/cm³ × 0.2 cal/g/°C × 1°C ×4.19 joules/cal × area Earth × 0.29 ~ 0.37 × 10^{22} joules ~ **0.23 W yr**. [In a century the depth of penetration is ~3 times more than in a decade, so heat storage in a century due to 1°C warming is ~ **0.7 W yr/m²**.]

[1]Note that 1 W sec = 1 joule, # sec/year ~ π×10^7, area Earth ~ 5.1×10^{18} cm², 1 W yr over full Earth ~ 1.61×10^{22} joules, ocean fraction of Earth ~ 0.7, 1 calorie ~ 4.19 joules.

James Hansen, et al, *Earth's Energy Imbalance: Confirmation and Implications*, SCIENCE 308 (2005), p. 1431

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What is a watt-year?

A ***watt*** is a unit of power, or energy per unit time, i.e., ***one joule per second***.

A year is about 3.14×10^7 seconds, so a ***watt-year*** is about **3.14×10^7 *joules*.**

What is a watt-year per square meter?

About **3.14×10^7 *joules per square meter*.**

If the heat imbalance at the Earth's surface is one watt per square meter, then the energy imbalance over the course of a year is about **3.14×10^7 *joules per square meter*.**

Earth's surface area: about ***5.1×10^{14} square meters***.

If the heat imbalance at the Earth's surface is one watt per square meter, then the energy absorbed over the whole Earth is about ***1.61×10^{22} joules = 16.1 zJ (zettajoules)***.

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How much heat does it take to melt enough of the ice sheets to raise the sea level by one meter?

Assumption: Ice starts at -10°C and ends up as ocean water at +15°C.
Hansen: About 1.5×10^{23} joules ≈ ***9.3 Wyr/m^2***

How long does it take to melt enough of the ice sheets to raise the sea level by one meter?

Assume a heat imbalance of 1 W/m2.

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How long does it take to melt enough of the ice sheets to raise the sea level by one meter?

If the heat imbalance at the Earth's surface is one watt per square meter, and if all the heat imbalance goes toward melting the ice sheets, then the time required to raise the sea level one meter is ***9.3 years***.

meters per year: 1/9.3 = ***0.108 m/yr***
= 1.08 m/decade = 10.8 m/century

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Suppose that all the heat imbalance goes to melting the ice sheets.

It takes 9.3 Wyr/m^2 to turn ice sheets into 1 meter of ocean. If the heat imbalance is 1 W/m^2, the sea level would rise at the rate of 1/9.3 meters per year, or about 0.108 meters per year, or 1.08 meters per decade, or 10.8 meters per century.

How long would it take to melt all the ice?

Assume that, if all the ice melted, it would raise the sea level by 70 meters.

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How long would it take to melt all the ice?

70 meters/0.1098 meters per year = **650 years**.

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Melting all the ice sheets would cause a sea level rise of about 70 meters and would take about 650 years at the current imbalance.

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Melting all the ice sheets would cause a sea level rise of about 70 meters and would take about 650 years at the current imbalance of 1 W/m^2.

What if the imbalance was 2 W/m^2?

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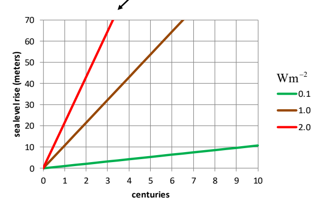
Melting all the ice sheets would cause a sea level rise of about 70 meters and would take about 650 years at the current imbalance of 1 W/m^2.

What if the imbalance was 2 W/m^2?
325 years



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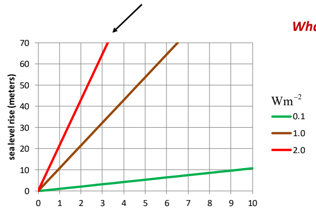
What if the imbalance was 2 W/m^2?
325 years

What if the imbalance was 0.1 W/m^2?



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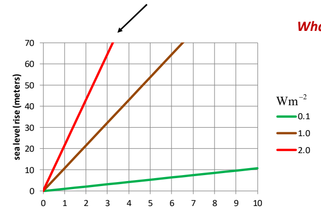
325 years

What if the imbalance was 0.1 W/m^2?
6500 years



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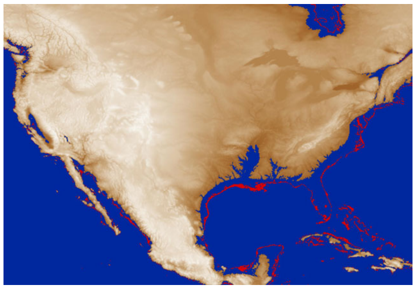
The USA in the Ice Free Earth
Computer Simulation, Clarence Lehman, Univ. Mn. 2006



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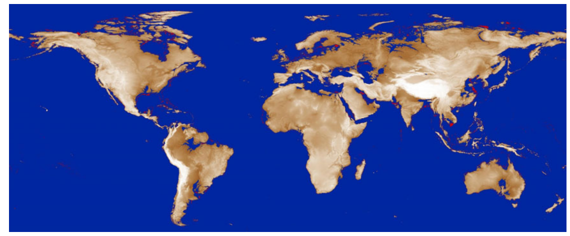
The Modern Ice Free Earth
Computer Simulation, Clarence Lehman, Univ. Mn. 2006



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The Good News

- Even if all the current heat imbalance goes into melting the ice sheets, it will take 650 years to melt all the ice.
- The ocean seems to be absorbing most of the heat imbalance, so we have some time before the ice sheets start melting seriously.

The Bad News

- 700 years is a long time, but that is actually a rate of 1 meter of sea level rise per decade. At that rate it may be impossible for coastal cities to adapt.
- As we have seen recently, ocean surface temperature rise is itself a serious problem.
- Once the sea level begins to rise, it could go at a rate of 1 meter per decade, so we should start preparing to adapt to that.

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warming the oceans

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How much heat does it take to raise the temperature of the top kilometer of the oceans by 1°C?

Assumption: Warm the top kilometer of the oceans by 1°C
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Now imagine a scenario where the temperature of the top kilometer of the ocean must increase by 0.5°C before any of the ice sheets can melt.

If the heat imbalance is 1 W/m², how long before the sea level starts to rise?

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It takes 93 years to raise the temperature by 1°C, so it would take 46.5 years to raise the temperature of a kilometer of ocean by 0.5°C.

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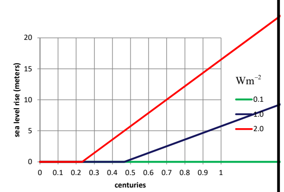
If the heat imbalance is 2 W/m², how long before the sea level starts to rise?

Half as long: 23 years.



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melting the sea ice

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With an imbalance of 1 W/m², how long would it take to melt all the sea ice?

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If the heat imbalance at the Earth's surface is one watt per square meter, and if all the heat imbalance goes toward melting the sea ice, then all the sea ice would be gone in ***1.3 years***.

Won't happen before 2025!

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Sea ice melting (all sea ice on planet). Assume ice starts at –10°C and ends at mean ocean surface temperature (+15°C), and that sea ice covers 4% of the planet with mean thickness 2.5 m. Energy required is 250 g cm² × 100 cal/g (80 cal/g for melting) × 4.19 joules/cal × 0.04 × area Earth ~ 2.14 × 10^{22} joules ~ **1.3 W yr/m²**.

Air warming by 1°C. The Earth's atmospheric mass is ~ 10 m of water. Heat capacity of air ~ 0.24 cal/g/°C. Energy to raise air temperature 1°C: 1°C × 1000 g/cm² × 0.24 cal/g/°C × 4.19 joules/cal × area Earth ~ 0.26 ×10^{22} joules ~ **0.32 W yr/m²**.

Land surface warming by 1°C: The depth of penetration of a thermal wave into the Earth's crust in 10 years, weighted by ΔT, is ~10 m. With density ~ 3 g/cm³, heat capacity ~ 0.2 cal/g/°C, and 0.29 fractional land coverage, land heat storage is 10^3 cm ×3 g/cm³ × 0.2 cal/g/°C × 1°C ×4.19 joules/cal × area Earth × 0.29 ~ 0.37 × 10^{22} joules ~ **0.23 W yr**. [In a century the depth of penetration is ~3 times more than in a decade, so heat storage in a century due to 1°C warming is ~ **0.7 W yr/m²**.]

[1]Note that 1 W sec = 1 joule, # sec/year ~ $\pi \times 10^7$, area Earth ~ 5.1×10^{18} cm², 1 W yr over full Earth ~ 1.61×10^{22} joules, ocean fraction of Earth ~ 0.7, 1 calorie ~ 4.19 joules.

James Hansen, et al, *Earth's Energy Imbalance: Confirmation and Implications*, SCIENCE 308 (2005), p. 1431

Math 1901 10/2/2024

Math 1901
Earth's Heat Imbalance

Suppose the ocean stopped absorbing 1 W/m² and instead that heat went into the atmosphere.
How long would it take to increase the air temperature by 1°C?

Math 1901 10/2/2024

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So it would take **0.32 years**, or about **117 days**, or **January 26, 2025**.

The air temperature would go up 3°C every year.

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3.2 years, or December 2027.

The ocean is still absorbing most of the imbalance, so we are good.
Will it continue?

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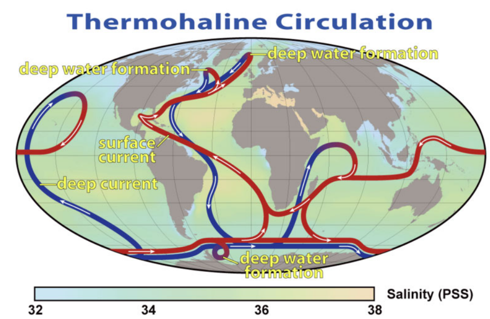
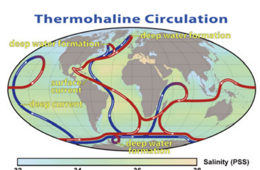
Earth's Heat Imbalance



Math 1901
Earth's Heat Imbalance

The ocean transports energy around the planet, mostly moving heat from the equatorial regions to the polar regions.

It also mixes the shallow ocean with the deep ocean, maintaining some kind of balance across the planet.



Currently, the ocean is slowly moving heat from the surface to the deep ocean.

Will it continue?

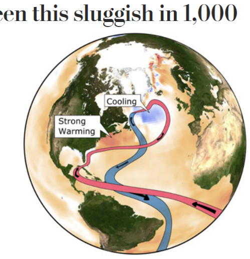
Math 1901
Earth's Heat Imbalance

The Washington Post

April 11, 2018

The oceans' circulation hasn't been this sluggish in 1,000 years. That's bad news.

The Atlantic Ocean circulation that carries warmth into the Northern Hemisphere's high latitudes is slowing down because of climate change, a team of scientists asserted Wednesday, suggesting one of the most feared consequences is already coming to pass.



Nature volume 556, pages191–196 (2018)

Math 1901
Earth's Heat Imbalance

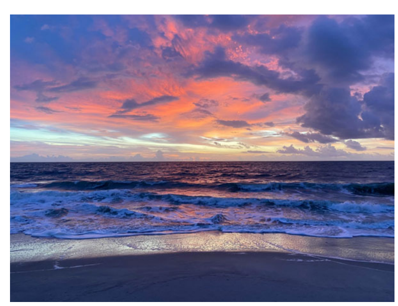
Current Atlantic Meridional Overturning Circulation weakest in last millennium

The Atlantic Meridional Overturning Circulation (AMOC)—one of Earth's major ocean circulation systems—redistributes heat on our planet and has a major impact on climate. Here, we compare a variety of published proxy records to reconstruct the evolution of the AMOC since about AD 400. A fairly consistent picture of the AMOC emerges: **after a long and relatively stable period, there was an initial weakening starting in the nineteenth century, followed by a second, more rapid, decline in the mid-twentieth century, leading to the weakest state of the AMOC occurring in recent decades.**

NATURE GEOSCIENCE | VOL 14 | MARCH 2021 | 118–120 | www.nature.com/naturegeoscience118

Math 1901
Earth's Heat Imbalance

The oceans cover 70% of Earth's surface. What they do affects the whole planet.



https://oceanconservancy.org/